# ANALYTICAL MODELING OF INFANT RADIO SOURCES

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# Abstract

We extend previous analytical models of FRII-type radio source growth to model very young sources. The new model describes the source evolution in the regime before self-similarity and when ballistic growth of the jet must be considered. Dynamical and spectral results are given showing the evolution from a highly overdense jet which punches its way forward through the surrounding medium to the stage where its progress is slowed by the surroundings and the cocoon begins to inflate. We follow the source evolution by calculating the synchrotron emissivity and the frequency at which the spectrum peaks at various stages of growth.

# 1 Introduction

Radio sources provide an important heating mechanism for the interstellar medium and intracluster medium as well as a potential probe of their environment and triggering mechanism.

Whilst the dynamics of mature, highly luminous classical double radio sources are well understood, less has been done to model the dynamics of their smaller, fainter counterparts. Observationally these smaller sources present themselves as compact steep spectrum (CSS) and gigahertz peaked spectrum (GPS) sources. In one scenario these sources are very young and the GPS sources owe their convex spectral shapes to the self absorption of their synchrotron emission when they are so dense as to be optically thick. See O'Dea (1998) for a review of GPS and CSS properties. Here we use analytical models of very young radio sources to predict the evolution of observable quantities.

Previous work has seen the development of a model for the large-scale evolution of radio sources in which the sources grow self-similarly (Falle, 1991; Kaiser & Alexander, 1997). Alexander (2000) extended this model to account for a King-type density profile and thereby modeled the source evolution on scales down to a few kpc. Here we present a model for the earliest stages of source evolution where the jet is uncollimated. Initially the jet pushes its way ballistically into the surrounding medium. As the jet length approaches the characteristic length scale  $L_2$  its forward expansion slows and the source begins to inflate a more substantial cocoon - this phase resembles that first discussed by Scheuer (1974) in his Model A. Once the jet length reaches a few times  $L_2$  the pressure in the cocoon becomes sufficient to collimate the jet, Scheuer's model becomes invalid and the evolution changes to the selfsimilar phase. Here we present an approximate solution describing the radio source which is valid from the beginning of its life through to the stage where Scheuer's model breaks down (at radio sources of sizes of a kpc or so).

We assume that the radio source has an axisymmetric jet with a constant opening angle so that it forms a cone (see Fig. 1). Throughout this work we assume that the sources are expanding into a medium of constant density,  $\rho_0$ . We also assume that the velocity of the particles in the jet,  $v_j$ , the jet power, Q and the jet opening angle  $\Omega$  are constant in time as the source grows. We further assume that the sound speed in the cocoon is so large that the pressure is constant across the cocoon.

#### **2** Our approximate solution

Details of the model and solution will appear elsewhere. We find, for the source size (or jet length):

$$D/L_2 = 1/K_3(\sqrt{(2K_3v_jt/L_2) + 1} - 1),$$
 (1)

where D is the source length; t is the age of the source;  $L_2$  is the characteristic scale length given by:

$$L_2 = \sqrt{8Q/\rho_0 v_j^3},$$
 (2)



Figure 1: The main components of the model we present here.

and  $K_3$  is a constant proportional to the square-root of the jet opening solid-angle:  $K_3 \propto \Omega^{1/2}$ .

The source expansion rate is given by

$$\dot{D} = v_j (1 + K_3 D / L_2)^{-1}.$$
 (3)

For small values of D the expansion rate is equal to the jet velocity and the growth is ballistic.

Writing the dimensionless source size,  $x \equiv D/L_2$  the volume of the cocoon is given by:

$$V_c = a_1 \Omega^{3/4} L_2^3 x^2 [(a_3/a_1)^{2/3} \Omega^{-1/2} + x]^{3/2}, \quad (4)$$

and the pressure in the cocoon is

$$P_{c} = a_{2} \Omega^{-1/4} (Q/v_{j}L_{2}^{2}) x^{-1} [(a_{2}/a_{4})^{2} \Omega^{-1/2} + x]^{-1/2}.$$
(5)

The volume of the hotspot is given by

$$V_h = C_h \Omega^{3/2} L_2^3 x^3, (6)$$

and the pressure in the hotspot is

$$P_h = a_h \Omega^{-1} (Q/v_j L_2^2) [1/K_3 + x]^{-2}.$$
 (7)

The constants  $a_1$  to  $a_4$  and  $a_h$  are dependent on the ratios of specific heats for the plasma, the jet and the external medium and also on  $\Omega$ . With  $\Omega$  set to 0.1 steradians they have values within an order of magnitude of unity. These equations represent ballistic growth for small sizes or times ( $x \ll 1$  or  $t \ll L_2/K_3 v_j$ ) and tend to the Scheuer model at large sizes and long times. The equation for synchrotron emission does not directly depend upon the pressure, but if we assume that the energy density in the particles,  $u_p$  is proportional to the magnetic energy density,  $u_B$  we get the magnetic field in terms of the pressure and find:

$$j(\nu) \propto P_i^{(p+1)/4} \delta^{-(p+1)/2} \nu^{(1-p)/2}.$$
 (8)

Where  $j(\nu)$  is the synchrotron emissivity at a given frequency,  $\nu$ ,  $P_i$  is the pressure in the relevant source component (the cocoon or the hotspot) and  $\delta$  is the factor by which the magnetic field is below its equipartition value so that

$$u_B = u_p / \delta^2, \tag{9}$$

We can then, using the above equations, find the emissivity in terms of source size for the two main components of the source: the cocoon and the hotspot.

The surface brightness and total flux at a given frequency can then be calculated (at frequencies for which the source is optically thin) by simply multiplying the emissivity by the appropriate line-of-sight distance or volume. The modeled source is axisymmetric and so we look at the surface brightness in two different orientations: side-on and end-on. In this formulation the hotspots have volume (Eq. 6)  $V_h \propto x^3$ , and since their cross-sectional area must relate to the area of the jet when it reaches the bow shock  $(A_j = \Omega x^2 L_2^2)$ the end-on line of sight length through the hotspot must be  $l_{\text{hotspot}(\text{end})} = C_h \Omega^{1/2} L_2 x$ , which is different from the side-on length by the factor  $C_h$  which is close to unity. We then model the hotspot as a cube, and the surface brightness is the same end-on or side-on.

We take the length of the cocoon to be proportional to the source size, which means that the thickness, or the side-on line of sight through the cocoon has length:  $l_{\rm coccon(side)} \propto x^{1/2}((a3/a1)^{2/3}\Omega^{-1/2} + x)^{3/4}$ . The value of the constant  $[(a3/a1)^{2/3}\Omega^{-1/2}]$  is about 2.75 so for  $x \ll 2.75$  the cocoon width grows only like  $x^{1/2}$ , but the length (as always) grows like x. For  $x \gg 2.75$  the cocoon width grows as  $x^{5/4}$ , so the cocoon's axial ratio falls.

We also calculate the absorption coefficient as a function of source size and frequency, and hence calculate the frequency at which a source of a given size becomes optically thick. We approximate the entire synchrotron spectrum by assuming a spectral index of  $\alpha$ = -5/2 in the optically thick region (where we have



Figure 2: The full synchrotron flux density for the whole source at various source sizes.

taken  $S \propto \nu^{-\alpha}$ ). By assuming that the exponent, p, in the electron energy distribution,  $n(E)dE \propto E^{-p}dE$ is equal to -2.4 we take the spectral index above the peak frequency to be  $\alpha = 0.7$ . We do not include any synchrotron losses. Figure 2 shows the model flux density, as a function of frequency, for sources at various sizes, where the cocoon magnetic energy density is one-tenth of its equipartition value and  $\Omega =$ 0.1. This figure shows the relative contributions to the flux density from the hotspot and the cocoon. Spectra are plotted for sources of size 0.01, 0.1, 1 and 10  $L_2$ . The hotspot does not contribute significantly for small sources (x < 1) but is the dominant emitting component at low frequencies for x = 1 and higher.

If the sources are resolved then the surface brightness density and not the total flux density is the observable quantity. Figure 3, 4 and 5 show the surface brightness spectra of the cocoon (seen end-on and side-on) and of the hotspot for values of x of 0.01, 0.1, 1 and 10.

It is interesting to note the importance that orientation has on the outcome for the cocoon — for small x, the surface brightness of the cocoon increases if viewed end-on, but falls if viewed side on. This is because initially the growth is ballistic, so the cocoon grows much faster along the direction of the jet than transverse to it, and the line of sight through the end of the cocoon therefore lengthens much faster than the line of sight through the side.

As the source expands ballistically, in its first stages, the cocoon becomes more and more prolate (the axial ratio increases as  $\sqrt{x}$ ) and the pressure falls inversely with the jet length. As a result, and as shown in Fig.



Figure 3: The synchrotron surface brightness spectrum of the cocoon, as seen end-on, at various stages in the source evolution, where the assumed magnetic field is  $1/\sqrt{10}$  of its equipartition value.



Figure 4: The synchrotron surface brightness spectrum of the cocoon, as seen from the side, for sources of different sizes where the assumed magnetic field is  $1/\sqrt{10}$  of its equipartition value.



Figure 5: The synchrotron surface brightness spectrum of the hotspot, for sources of different sizes.

3 and 4, viewed end-on the cocoon surface brightness is steady at high frequencies but viewed side-on the cocoon surface brightness decreases at high frequency. Figure 5 shows the evolving surface brightness spec-

tra for hotspot. The line of sight through the hotspot increases as x at all times, but initially the pressure in the hotspot is constant, so we see a rise in the surface brightness of the hotspot at all frequencies and as the optical depth rises the hotspot remains optically thick to higher frequencies. As the source leaves the purely ballistic regime the pressure in the hotspot begins to fall as  $x^{-2}$  (this change occurs at a jet length of around  $2L_2$  or so), so the optically thin surface brightness falls as  $x^{-1}$  and the peak frequency also falls.

# 3 Summary

In this work we have presented an approximate solution that describes the growth of radio sources from the beginning of their lives where the jet punches its way ballistically into the surrounding medium into a regime where the forward expansion of the bow shock slows and the cocoon is able to undergo significant lateral expansion. We follow the evolution of the synchrotron spectra and of the surface brightness of the main components. The development of a model that is able to extend to sources in the self-similar regime after jet collimation will be essential if we are to make predictions about the full source evolution and source counts at different frequencies.

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## References

Alexander, P. 2000, MNRAS, 319, 8 Falle, S. 1991, MNRAS, 250, 581 Kaiser, C., Alexander, P. 1997, MNRAS, 286, 215 O'Dea, C. P. 1998, PASP, 110, 493 Scheuer, P. 1974, MNRAS, 166, 513