PARTICLE ACCELERATION AT RELATIVISTIC SHOCKS AND CURRENT SHEETS

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Abstract

Simple kinematic theories of particle acceleration at relativistic shocks lead to the prediction of a high-energy spectral index of -1.1 for the energy flux of synchrotron photons. However, several effects can change this picture. In this paper I discuss this theory briefly and mention recent developments concerning shock formation and magnetic field generation. An additional mechanism must operate at lower energy and current sheets present a possible alternative acceleration site; I summarize some recent work on the relativistic version of these objects.

1 Introduction

Many particle acceleration processes have been studied in connection with the non-thermal emission observed in astrophysical objects. An aspect shared by most of them is that the energy released at spatially localized sites of entropy generation, such as shock fronts, boundary layers and current sheets is used directly or indirectly to feed the energetic particle population. In this paper, I briefly describe two such scenarios — acceleration at relativistic shocks, and acceleration at relativistic current sheets — and elaborate on some of the recent work performed on them. This selection is guided by the theme of this conference: the close connection between X-ray and radio emission, at least in the case of extra-galactic sources, has led to the conclusion that we are dealing with intrinsically *relativistic* systems, involving bulk motion with large Lorentz factor even on spatial scales of kiloparsecs. Relativistic shocks are, therefore, obvious candidate acceleration sites. Relativistic current sheets are less well-known, and do not, as yet, have a generally accepted definition. However, as I will show below, they have much in common with relativistic shocks and should also be considered as promising candidate sites.

2 Relativistic shocks

The kinematic problem of particle acceleration at a relativistic shock, i.e., that of finding the distribution of a collection of test particles undergoing small-angle, random, elastic (in the plasma frame) deflections in the vicinity of a discontinuity in the (relativistic) plasma velocity is well-understood. An analytic method based on an eigenvalue decomposition is available which gives the spectrum and angular dependence of the distribution function at energies well above those of injection for arbitrary shock speeds (Kirk, et al., 2000). In addition, Monte-Carlo simulations have been performed, finding results which are in good agreement with the analytic approach (Bednarz & Ostrowski, 1998; Achterberg, et al., 2001).

These results are illustrated in Fig. 1 and 2. Well above the injection energy the phase-space density f is a power-law in momentum: $f \propto p^{-s}$ and at the shock front the angular dependence is well-approximated by the simple expression

$$f \quad \propto \quad (1-\mu_{\rm s}u)^{-s}\exp\left(-\frac{1+\mu_{\rm s}}{1-\mu_{\rm s}u}\right) \qquad (1)$$

where $\mu_{\rm s}$ is the cosine of the angle between the shock normal and the particle velocity, measured in the frame in which the shock is at rest and the upstream plasma flows along the shock normal at speed $c\vec{u}$. Figure 1 shows the compression ratio and the high-energy power-law index s as a function of the spatial component of the 4-speed Γu of the upstream plasma, where $\Gamma = (1-u^2)^{-1/2}$.

An interesting aspect of these results is that the power-law index tends asymptotically to the value $s \approx 4.23$

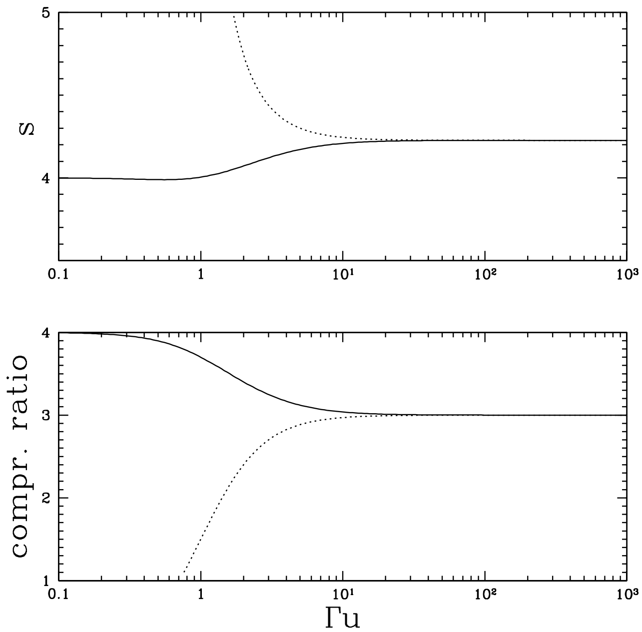

Figure 1: The high-energy power-law index s (upper panel) and compression ratio (lower panel) as a function of the spatial component of the upstream four speed Γu. The dotted line refers to a shock in a gas with negligible rest-mass and the solid line to a strong shock (i.e., cold upstream medium) in an ideal gas with adiabatic index 5/3.

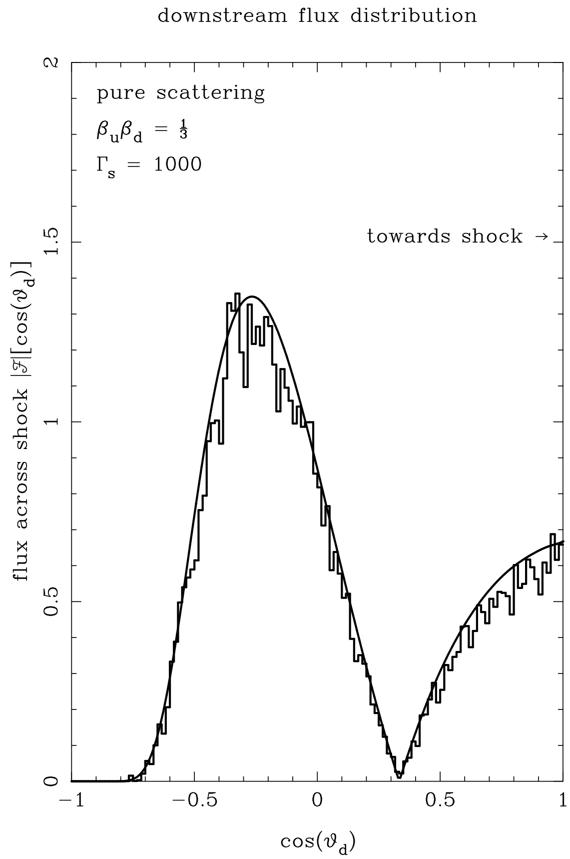

Figure 2: A comparison (from Achterberg, et al. (2001)) between a Monte-Carlo simulation and the analytic result for the particle flux across a shock front as a function of the cosine of the angle θ_d between the particle velocity and the shock normal measured in the frame in which the downstream plasma is at rest. $\theta_d = 0$ corresponds to motion along the normal from downstream to upstream. Jump conditions for a relativistic gas are used and the upstream plasma has a Lorentz factor $\Gamma = 1000$.

for large shock Lorentz factors (or, equivalently, upstream Lorentz factors), independent of the equation of state of the plasma. This asymptotic value is essentially fixed by the compression ratio of the shock and depends only weakly on the form of the scattering operator used to describe the small-angle deflections (Kirk, et al., 2000).

In Monte-Carlo treatments, it is possible to investigate more general forms of the scattering operator, whilst retaining the effect of a non-vanishing average magnetic field (Ostrowski, 1993; Achterberg, et al., 2001). Provided the turbulence remains strong, little difference is found. However, as expected, the acceleration mechanism becomes less effective as the turbulence diminishes (Ostrowski & Bednarz, 2002). Explicit calculations of particle motion in a completely random magnetic field (with vanishing average component) have been performed by Ballard & Heavens (1992) and Casse, et al. (2002). They have been used to compute the acceleration around a relativistic shock for Lorentz factors $\Gamma \leq 5$ (Ballard & Heavens, 1992) and, more recently, for $\Gamma \leq 100$ (Lemoine & Pelletier, 2003). The latter find good agreement with the analytic result on the asymptotic power-law index.

In contrast with the situation in non-relativistic shocks (Malkov & Drury, 2001), the nonlinear modification of relativistic shock does not affect the *asymptotic* power-law index. There are two reasons for this: firstly, isotropized, accelerated particles behave like a relativistic gas with adiabatic index 4/3, so that the overall compression ratio of an ultra-relativistic shock front remains 3, even when a significant part of the overall energy and momentum flux is carried by these particles. Secondly, the asymptotic power-law index in the test-particle picture is *soft* (i.e., $s > 4$). This means that it is possible to consider a Lorentz factor above which the test-particle approximation is valid, because the energy density in the remaining accelerated particles is indeed small. Nevertheless, a strong nonlinear effect can be exerted by particles of lower energies, whose mean free path to scattering is comparable to the size of internal structures in the shock transition (Ellison & Double, 2002).

Particle transport in astrophysical plasmas is usually dominated by interaction with fluctuations in the electromagnetic field produced collectively by the background plasma. However, there are strong indications that two-body *collisional* processes (including those with the photon gas, e.g., photo-pion production and Compton scattering) may be important for the acceleration and/or the thermalization of energetic particles in the inner parts of a γ-ray burst (GRB) fireball $r < 10^{16}$ cm (Derishev, et al., 2003; Stern, 2003). In all other applications, the plasma responsible for non-thermal emission can be assumed to be collisionless, so that in order to understand how a shock can form, it is necessary to identify a suitable instability which can lead to dissipation in the nonlinear regime. The currently most promising approach to this problem for relativistic shocks considers the nonlinear development of the Weibel instability (Yang, et al., 1993, 1994; Medvedev & Loeb, 1999), which generates downstream magnetic field perpendicular to the streaming motion of the plasma i.e., in the plane of the incipient shock. A full simulation of this situation has not yet been performed, but recent three-dimensional-PIC simulations of colliding plasma shells (Sakai, et al., 2000; Fonseca, et al., 2003; Silva, et al., 2003) suggest that magnetic field can be generated with a strength up to $\sigma \approx 1\%$. (Here the magnetization parameter σ is defined as the ratio of the magnetic energy density to twice the total enthalpy density (including rest mass) as measured in the plasma rest frame). This is encouraging, since it is roughly the level implied by spectral modeling of GRB after-glows (Panaitescu & Kumar, 2002).

The manner in which magnetic field is generated at the shock is certainly has a strong influence on the spectrum of accelerated particles. However, if we are interested only in high energy particles of long mean free path, the complex aspects of the problem can be by-passed: the power-law index predicted by the first-order Fermi mechanism can be calculated simply by modifying the shock jump conditions to account for the generated field. To do this, we consider time-averaged conditions, so that linear functions of the oscillating electromagnetic field vanish. The stress-energy tensor in the plasma frame is

$$T^{\mu\nu} = \left(w + \frac{B^2}{4\pi}\right) u^\mu u^\nu + \left(p + \frac{B^2}{8\pi}\right) g^{\mu\nu} - \frac{B^\mu B^\nu}{4\pi} \qquad (2)$$

(for notation see Kirk & Duffy (1999)) and the last term on the right hand side does not contribute to the fluxes across the shock front if the magnetic field lies in the shock plane. As a result, the jump conditions are the same as those of an unmagnetized fluid, provided the magnetic enthalpy density $B^2/4\pi$ and pressure $B^2/8\pi$ are taken into account (Lyubarsky, 2003). For a relativistic gas, this gives an effective adiabatic index

$$\gamma_{\rm eff} = \frac{4(1+\sigma)}{(3+\sigma)} \qquad (3)$$

leading to an asymptotic compression ratio of $1/(\gamma_{\rm eff} - 1)$ and a relative speed of the upstream medium with respect to the downstream medium corresponding to the Lorentz factor $\Gamma_{\rm rel} = \Gamma\sqrt{(2-\gamma_{\rm eff})/\gamma_{\rm eff}}$ (where Γ is the Lorentz factor of the shock front seen in the upstream medium). As σ increases, the compression ratio of the shock decreases and the high-energy power-law softens, as shown in Fig. 3. If magnetic field amplification indeed saturates at $\sigma \sim 1\%$, the asymptotic spectral index still remains close to 4.2, a value which fits nicely with the X-ray synchrotron emission from the termination shock of the wind from the Crab pulsar (Willingale, et al., 2001; Gallant, et al., 2002) and also seems to work well in models of blazar emission (Krawczynski, et al., 2000, 2002). However, these objects share the property that the synchrotron spectrum is much harder at lower frequencies. Since this is not a natural prediction of shock acceleration, it is interesting to speculate that it reflects the pre-acceleration mechanism, which is in any case needed in order to inject particles into the shock acceleration mechanism.

3 Relativistic current sheets

Possible pre-acceleration processes include the maser mechanism of Hoshino, et al. (1992) and the *destruction* of magnetic flux in the shock front (Lyubarsky, 2003). Another possibility, closely connected with the latter, is acceleration at relativistic current sheets (Kirk, 2004).

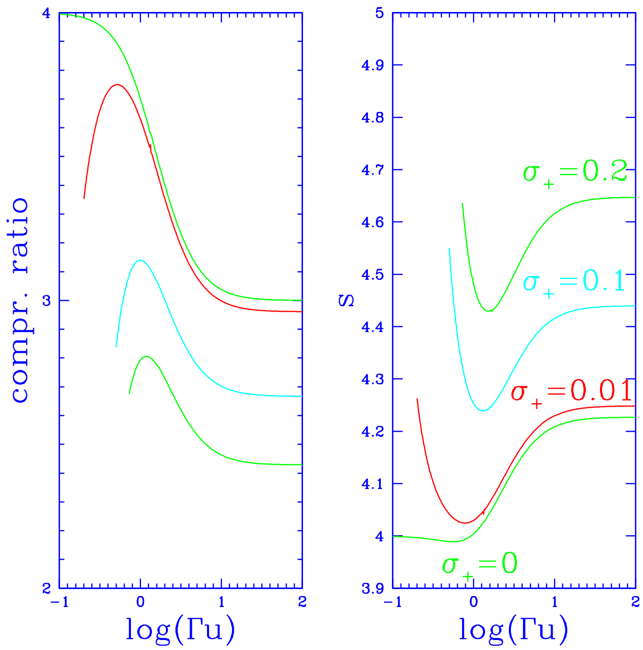

Figure 3: The compression ratio and power-law index s for a gas of adiabatic index 5/3 in which a magnetic field is generated downstream to the level $\sigma = \sigma_+$ (see text for notation).

The current sheets at which reconnection and particle acceleration takes place in astrophysics are relativistic in two senses: Firstly, the magnetization parameter, σ is large and the Alfvén speed $v_\mathrm{A} = c\sqrt{\sigma/(1+\sigma)}$ is close to c. Secondly, the geometry of the current sheet at which magnetic energy is dissipated and, hence, the field configuration, is dictated by a highly relativistic plasma flow. Particle acceleration depends crucially on both the magnetization parameter and the field configuration.

The relativistic effects associated with a large magnetization parameter are readily appreciated. On the other hand, the geometrical effects of a relativistic flow are more subtle. The situation is closely analogous to that of magneto-hydrodynamic shock fronts, which can be classified into "subluminal" and "superluminal" according to whether the speed of the intersection point of the magnetic field and the shock front is less or greater than c (Drury, 1983; Begelman & Kirk, 1990). In each case, a Lorentz transformation enables the shock to be viewed from a reference frame in which it has a particularly simple configuration: either a de Hofmann-Teller frame with zero electric field, or a frame in which the magnetic field is exactly perpendicular to the shock normal. In the case of a current sheet, the speed of the intersection point of the magnetic field lines and the sheet center-line is important. If it is subluminal, a transformation to a de Hofmann-Teller frame is possible, leading to the standard configuration for a non-relativistic sheet (Chen, 1992; Büchner & Zelenyi, 1989). Alternatively, for superluminal motion of the intersection point, which should be the rule for sheets in relativistic flows, a frame can be found in which the sheet is a true neutral sheet with no field lines linking through it. This is, in fact, the original configuration considered by Speiser (1965). For a relativistic sheet, however, it is the *generic* case, rather than a very special singular one.

Most discussions of reconnection treat a Sweet-Parker or Petschek configuration in which the length of the current sheet in the average field direction determines the dissipation rate. This is also true for recent analytic treatments that are relativistic in the sense that the effects of large σ are included (Lyutikov & Uzdensky, 2002; Lyutikov, 2003). But the vanishing of B_z in the generic relativistic case has important implications, since it is the linking field that can eject particles from the sheet, making it crucial for the determination of the spectrum of accelerated particles, and, especially, the maximum permitted energy (Litvinenko, 1999; Larrabee, et al., 2003). Relativistic current sheets, can extend over large distances along the field, depending on the nature of the boundary conditions. An example, drawn from the case of a striped pulsar wind (Coroniti, 1990; Lyubarsky & Kirk, 2001; Kirk & Skjæraasen, 2003), is shown in Fig. 4. If we assume that reconnection leads on average to a stationary field configuration, then as the spiral pattern moves outward, the linking field lines shown in the inset must move through the plasma at a speed sufficient to keep their average distance from the star constant. The striped spiral pattern depicted in the figure is expected to be established well outside the light cylinder, defined to be at radius $r = r_\mathrm{L}$, where the corotation speed reaches c. In this case, the magnetic chevrons, which must move a distance $2\pi r$ in each rotation of the spiral pattern, have a superluminal speed equal to cr/r_L. Transformation to the frame in which the sheet is a true neutral sheet involves a small boost in the x direction, and the resulting configuration has a typical dimension in the azimuthal direction of $\sim 2\pi r$.

Particle acceleration in current sheets with finite linking field (B_z) has been extensively investigated (Sy-

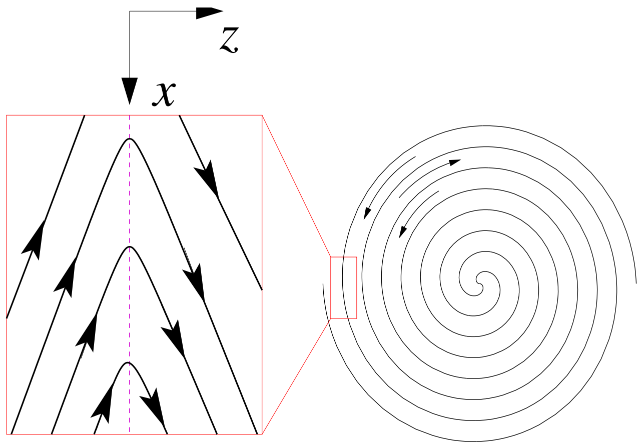

Figure 4: The striped pattern of a pulsar wind. A magnetic dipole embedded in the star at an oblique angle to the rotation axis introduces field lines of both polarities into the equatorial plane. The current sheet separating these regions is shown. In the inset, an almost planar portion of this sheet (dashed line) is shown, together with the magnetic field lines, assuming they undergo reconnection.

rovatskii, 1981). But in the generic, relativistic, configuration, the linking field can play no role in ejecting particles from the sheet. Instead, acceleration is controlled by the finite extent of the sheet in latitude, i.e., in the direction parallel to the electric field (E_y). This is limited not by the boundary conditions, but by local parameter values, as first described by Alfvén (1968). Assuming the plasma consists of cold electrons and positrons, and that $\sigma \gg 1$, the maximum Lorentz factor $\gamma_{\rm max}$ after acceleration is,

$$\gamma_{\rm max} = 2\sigma, \tag{4}$$

whereas a cold electron-proton plasma gives

$$\gamma_{\rm max} \approx \sigma \qquad \text{forprotons} \tag{5}$$

$$\gamma_{\rm max} \approx \sigma M/m \qquad \text{forelectrons,} \tag{6}$$

(Kirk, 2004) with M and m the proton and electron masses, respectively. It is interesting to note that in a plasma in which the magnetic field and particle *rest mass* are in rough equipartition ($\sigma \approx 1$), the upper limit given by Eq. (6) coincides with that quoted by Lesch & Birk (1997). However, this situation arises only in relativistic plasmas. In the interstellar medium, for example, $\sigma \approx 10^{-9}$ or smaller, in which case the upper limit on the energy gain reduces to $Mv_{\rm A}^2$. Standard estimates of the interstellar magnetic field and particle density (1 μG, 1 proton cm^{-3}) imply that electrons can be accelerated, at most, to only mildly relativistic energies. In this case, and in solar system applications, direct acceleration by the DC field may be masked by particle acceleration in the turbulence fed by reconnection or the associated shocks (Cargill, 2001).

The picture sketched above applies only to quasi-steady current sheets. However, Zelenyi & Krasnosel'skikh (1979) have shown that relativistic current sheets are unstable to the growth of the tearing mode and other instabilities are also likely to operate (see, for example, Daughton (1999)). On scale lengths comparable to the sheet thickness it is likely that an unsteady, oscillating component of B_z will be generated. Thus, locally, the non-relativistic picture may be relevant to the micro-structure of the sheet, although not in its standard 2-dimensional stationary incarnations. Particle-in-cell simulations can provide valuable insight here, provided they account for relativistic particle motion. Such simulations been performed by Zenitani & Hoshino (2001) and by Jaroschek, et al. (2004). Encouragingly, in view of observations of the hard synchrotron spectrum of the Crab and other objects, a very hard spectrum of energetic particles is observed. This can be understood in the framework of a box-model by identifying an "acceleration zone" near the sheet center, in which the electric field exceeds the magnetic field. The escape rate from this zone is then approximated as the time taken by a particle to complete one quarter of a revolution around the linking component of the magnetic field (Zenitani & Hoshino, 2001). Although relatively large values of the initial magnetization parameter can be treated, these simulations still suffer from the drawback that no spatial structure is permitted in the direction of the electric field. They are therefore not yet able to give us a full picture of quasi-stationary dissipation within a relativistic current sheet.

4 Summary

Although the details of the plasma physics remain obscure, simple kinematic considerations suggest that acceleration at shocks imprints a characteristic power-law index on the particle spectrum. In the case of non-relativistic shocks, this is p^{-4} for the phase space density, but is expected to be strongly modified by nonlinear effects (see Ellison, this conference). In the case of relativistic shocks, it is $p^{-4.2}$, and seems to be much less sensitive to nonlinear effects and also the effects of magnetic field generation.

A spectrum consistent with this prediction has been identified in a few objects, but observations also show that acceleration into a much harder spectrum is needed at low energies. Current sheets are in principle capable of producing particles with such a spectrum, but our understanding of them is still at a rather basic level.

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