Fundamental Radio Interferometry
II. Real Interferometers

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Topics

- This lecture extends the basic theory to include various practical details.
- We will cover the effects of:
  - Real Sensors (aka ‘antennas’)
  - Finite bandwidths
  - Rotating reference frames (source motion)
  - Finite time averaging
  - Local Oscillators and Frequency Downconversion
- But I won’t discuss polarization, sensitivity or calibration. These are topics for following lectures.

Review

- In the previous lecture, I set down the principles of Fourier synthesis imaging.
- I showed:

$$V_\upsilon(\mathbf{b}) = R_C - iR_S = \iint I_\nu(\mathbf{s}) e^{-2\pi i\nu \mathbf{b}\cdot\mathbf{s}/c} d\Omega$$

  where the intensity $I_\nu(\mathbf{s})$ is a real function, and the visibility V(**b**) is complex and Hermitian (V(**b**) = V*(-**b**))
- The model used for the derivation was idealistic – not met in practice. We presumed:
  - Isotropic Sensors (no dependence on direction)
  - Monochromatic radiation
  - Stationary reference frame (no motion)
  - No frequency conversions
- I now relax, in turn, these restrictions.

Real Sensors

- The idealized isotropic sensors assumed earlier don't exist.
- Real sensors (antennas) have a directional voltage gain pattern $A(\mathbf{s}, \mathbf{s}_0)$, where **s** is a general direction, and $\mathbf{s}_0$ is the antenna pointing direction.
- The gain pattern is easily incorporated into the formalism, once we realize that it attenuates the actual sky brightness. We can write:

$$V_\upsilon(\mathbf{s}_0, \mathbf{b}) = \iint G_1(\mathbf{s}, \mathbf{s}_0) G_2^*(\mathbf{s}, \mathbf{s}_0) I_\nu(s) e^{-2\pi i \nu\, \mathbf{b}\cdot\mathbf{s}/c} d\Omega$$

- Here, G_1 and G_2 are the complex voltage gain functions for the two antennas. (Complex means it contains both amplitude and phase).
- Note that if the antenna patterns are identical, and pointing in the same direction, then G_{1G2}^{*} = G^2, the power gain response.
- The effect on the output image is an attenuated (modified) representation of the true sky:

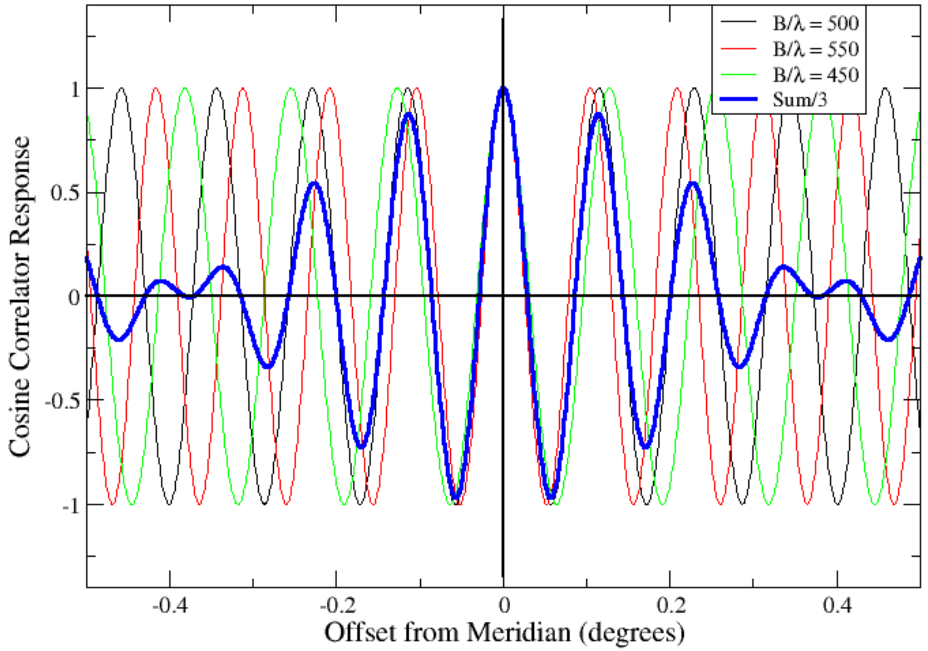
$$I_{mod} = A^2 I$$

- We can correct for this in the image by dividing by the beam inverse.
  - This is in general not the best solution, for a number of reasons.

Effect of Finite Bandwidth

- Real interferometers must have a finite, non-zero, bandwidth.
- Each frequency component generates a fringe pattern with angular separation of λ/B . (B = the baseline length – a fixed number).
- All fringe patterns have a maximum at the n=0 fringe (meridional plane).
- The patterns get increasingly out of step as n gets larger.

  - A simple illustration – three wavelength components from the same physical baseline.
  - Baselines of u=450, 500, and 550 wavelengths.
  - The net result (thick blue line) is the (normalized) sum over all components.

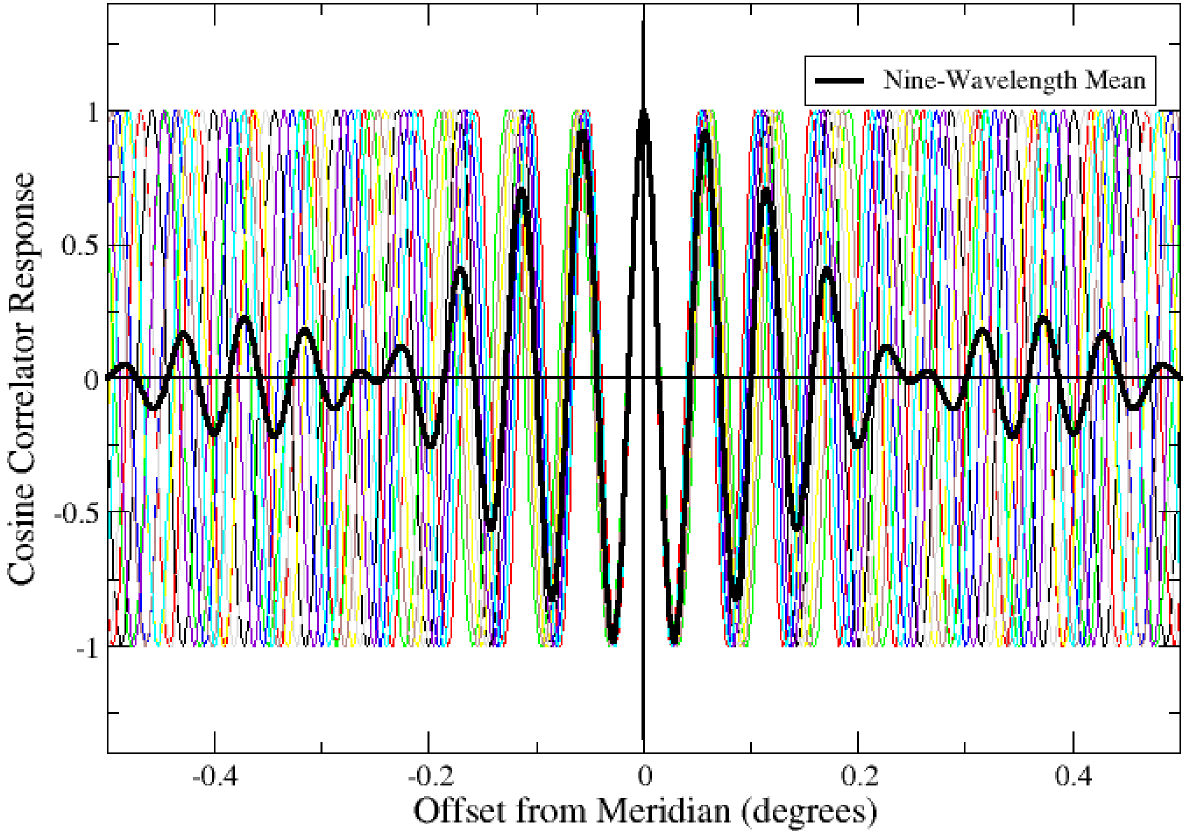


Bandwidth Effect (cont.)

- With more components (nine equal ones in this case), the summed response (thick black lilne) begins to look like a 'wave packet'.
- This shows the 'COS' component – the SIN component is shifted so the null is at the origin.

The black trace is what we actually would observe.

Note that I added the nine equal components to make this response. In fact, the bandpass is not flat, and has phase and amplitude variations – the actual sum is complex.



The Effect of Bandwidth -- Analysis.

- To find the finite-bandwidth response, we integrate our fundamental response over a frequency response G(ν), of width $\Delta\nu$, centered at ν_0

$$V = \iint \left(\frac{1}{\Delta \nu} \int_{\upsilon_0 - \Delta\upsilon/2}^{\upsilon_0 + \Delta\upsilon/2} I(\mathbf{s}, \upsilon) G_1(\upsilon) G_2^*(\upsilon) e^{-i2\pi\upsilon\tau_g} d\upsilon \right) d\Omega$$

- If the source intensity does not vary over the bandwidth, and the gain parameters G_1 and G_2 are square (= 1.0) and real, then

$$V = \iint I_\upsilon(\mathbf{s}) \frac{\sin(\pi\tau_g\Delta\upsilon)}{\pi\tau_g\Delta\upsilon} e^{-2i\pi\nu_0\tau_g} d\Omega = \iint I_\upsilon(\mathbf{s}) \mathrm{sinc}(\tau_g\Delta\upsilon) e^{-2i\pi\nu_0\tau_g} d\Omega$$

where the **fringe attenuation function**, sinc(x), is defined as:

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

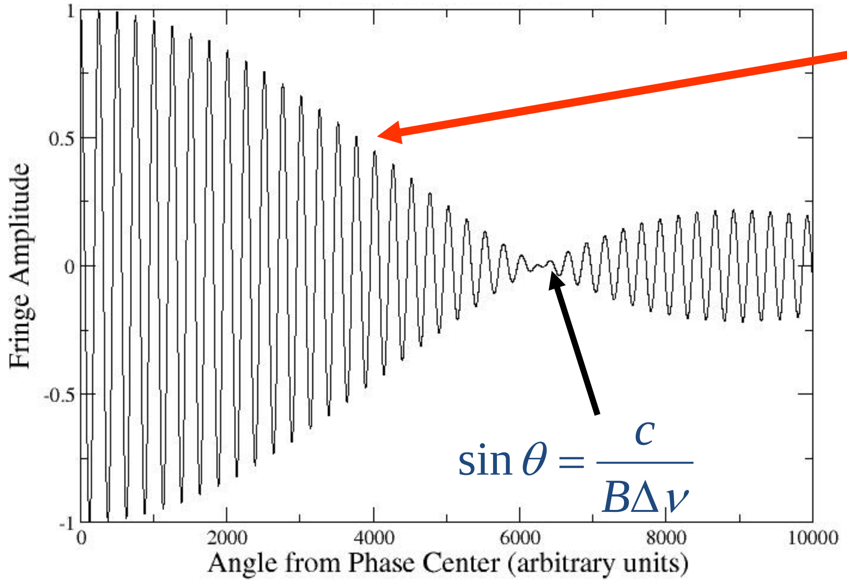
The attenuation function has nulls at x = +/- n. Or, when τ_g = +/- (n/$\Delta\nu$)

Bandwidth Effect Example

- For a square bandpass, the bandwidth attenuation reaches a null when $\tau_g \Delta\nu = 1$, or

$$\sin\theta = \frac{\lambda}{B}\frac{\upsilon_0}{\Delta\upsilon} = \frac{c}{B\Delta\nu}$$

- For the Jansky VLA, $\Delta\nu$ = 1 MHz, and B = 35 km, (A configuration) then the null occurs at about 30 arcminutes off the meridian.
- A 10% loss in fringe amplitude occurs at an offset of ~7.5 arcminutes.
- This is a real loss of visibility amplitude – because the effect depends on offset angle and baseline length, it effectively limits the resolution.



Fringe Attenuation function:

$$\mathrm{sinc}(\tau_g \Delta\upsilon) = \mathrm{sinc}\left(\frac{B\Delta\upsilon}{c}\sin\theta\right)$$

Number of fringes between peak and null:

$$N \sim \frac{c}{B\Delta\upsilon}\frac{B}{\lambda} \sim \frac{\upsilon}{\Delta\upsilon}$$

Observations off the Meridian

- In our basic scenario -- stationary source, stationary interferometer -- the effect of finite bandwidth will strongly attenuate the fringe amplitudes from sources not on the meridional plane.
- Since each baseline has its own meridional plane, the only point on the sky free of attenuation for all baselines is a small angle around the zenith.
- Hence, for our model (fixed) interferometer, we can only observe objects within a few arcminutes of the central fringe.
- Suppose we wish to observe an object far from the meridional plane? What to do?
  - Reducing the bandwidth is not the answer! (1 kHz needed …)
- Best way is to shift the entire 'fringe pattern' to the position of interest by adding time delay to the signal coming from the antenna closer to the source.

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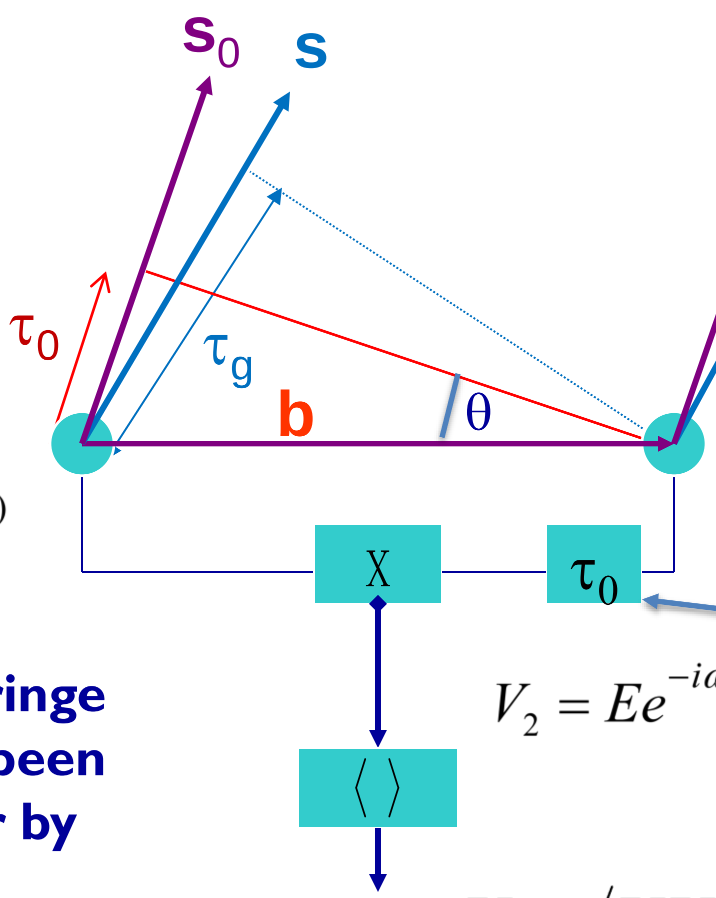
Adding Time Delay

Two delays:

$$\tau_g = \mathbf{b}\cdot\mathbf{s}/c$$

$$\tau_0 = \mathbf{b}\cdot\mathbf{s}_0/c$$



$$V_1 = Ee^{-i\omega(t-\tau_g)}$$

$$V_2 = Ee^{-i\omega t}$$

$$V_2 = Ee^{-i\omega(t-\tau_0)}$$

$\mathbf{s}_0$ = reference (delay) direction

$\mathbf{s}$ = general direction vector

New box: Time delay

The entire fringe pattern has been shifted over by angle

$\sin\theta = c\tau_0/b$

$$V = \langle V_1 V_2^* \rangle = E^2 e^{-i[\omega(\tau_0-\tau_g)]}$$

$$= E^2 e^{i2\pi[\upsilon\mathbf{b}\cdot(\mathbf{s}-\mathbf{s}_0)/c]}$$

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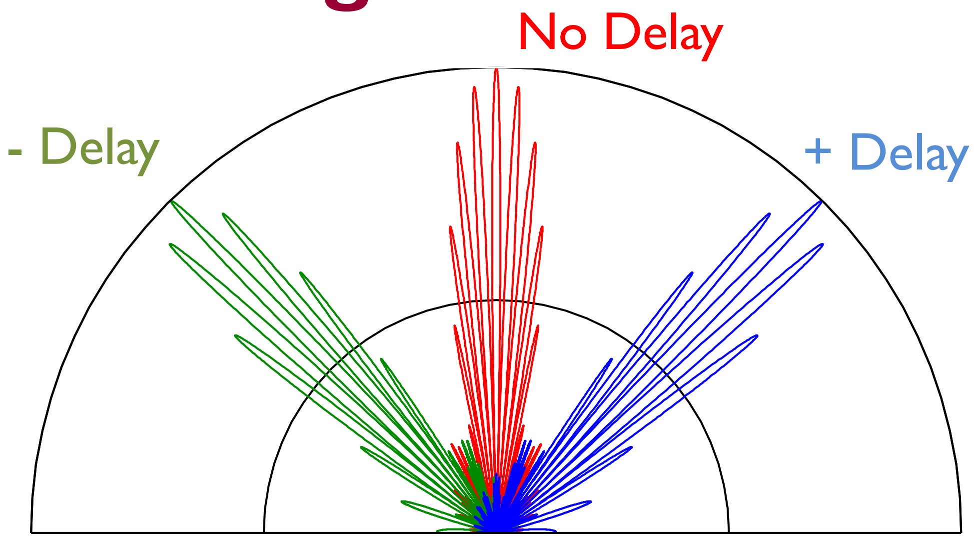
Illustrating Delay Tracking

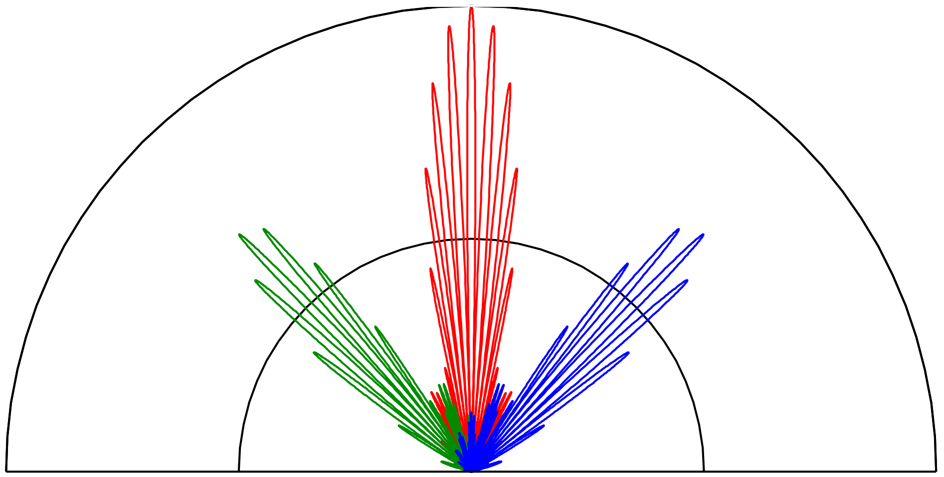
- Top Panel:

  Delay has been added and subtracted to move the delay pattern maximum to the source location, presuming an isotropic sensor.



- Bottom Panel:

  A cosinusoidal sensor pattern is added, to illustrate losses from a fixed sensor.



Observations from a Rotating Platform

- Most interferometers are built on the surface of the earth – a rotating platform. From the observer's perspective, sources move across the sky.

- Since we know how to adjust the interferometer delay to move its coherence pattern to the direction of interest, it is a simple step to continuously move the pattern to follow a moving source.

- All that is necessary is to continuously add time delay, with an accuracy $\delta\tau \sim 1/(10\Delta\nu)$ to minimize bandwidth loss.

- But there's one more issue to keep in mind, which is that of phase. The source is moving through the fringe pattern, which can be quite rapid.

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Phase Tracking …

- Adding time delay will prevent bandwidth losses for observations off the baseline's meridian.
- Delay insertion is finite – not continuously variable.
- Between delay settings, the source is moving through the interferometer pattern – a rapidly changing phase.
- The 'natural fringe rate' – due to earth's rotation, is given by

$$\upsilon_f = u\omega_e \cos\delta \quad \text{Hz}$$

- Where u = B/λ, the (E-W) baseline in wavelengths, and $\omega_e = 7.3\times10^{-5}$ rad/s is the angular rotation rate of the earth.
- For a million-wavelength baseline, $\nu_f \sim 70$ Hz – that's fast.
- If we leave things this way, we have to sample the output at least twice this rate. A lot of data!

Following a Moving Object.

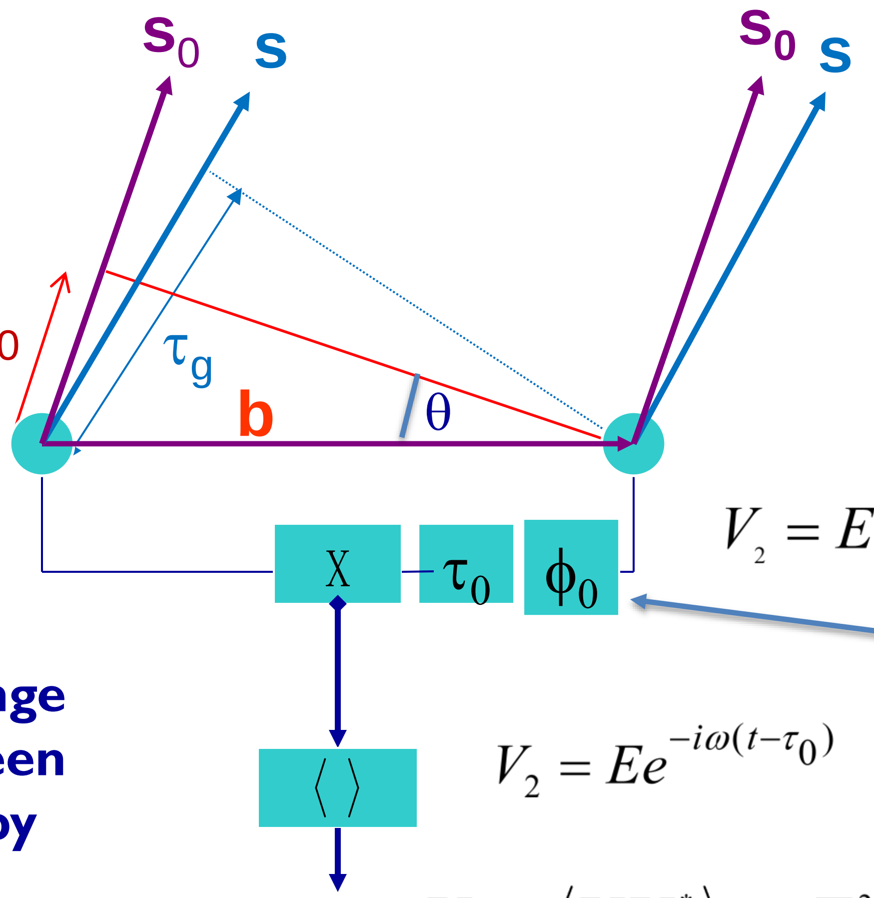
- There is **no useful information** in this fringe rate – it's simply a manifestation of the platform rotation.
- Tracking, or 'stopping' the fringes greatly slows down the post-correlation data processing/archiving needs.
- To 'stop' the fringes, we must shift the signal phases much quicker than the rate of delay adjustments.
- How fast? For a 1 million-wavelength baseline (VLA A-configuration, X-band, 2 MHz channelwidth)
  - Tracking delay: $$\upsilon_d >> \frac{\Delta\upsilon}{\upsilon}\frac{B}{\lambda}\omega_e \cos\delta \sim 0.02\ \mathrm{Hz}$$
  - Tracking fringe: $$\upsilon_f >> \frac{B}{\lambda}\omega_e \cos\delta \sim 70\ \mathrm{Hz}$$

Adding Phase Shift

Two times:

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$

$$\tau_0 = \mathbf{b} \cdot \mathbf{s}_0 / c$$



s_0 = reference (delay) direction

s = general direction vector

$$V_1 = E e^{-i\omega(t-\tau_g)}$$

$$V_2 = E e^{-i\omega t}$$

New box: Phase Shift

$$V_2 = E e^{-i\omega(t-\tau_0)}$$

The entire fringe pattern has been shifted over by angle

$\sin\theta = c\tau_0 / b$

$$V = \langle V_1 V_2^* \rangle = E^2 e^{-i[\omega(\tau_0 - \tau_g)]}$$

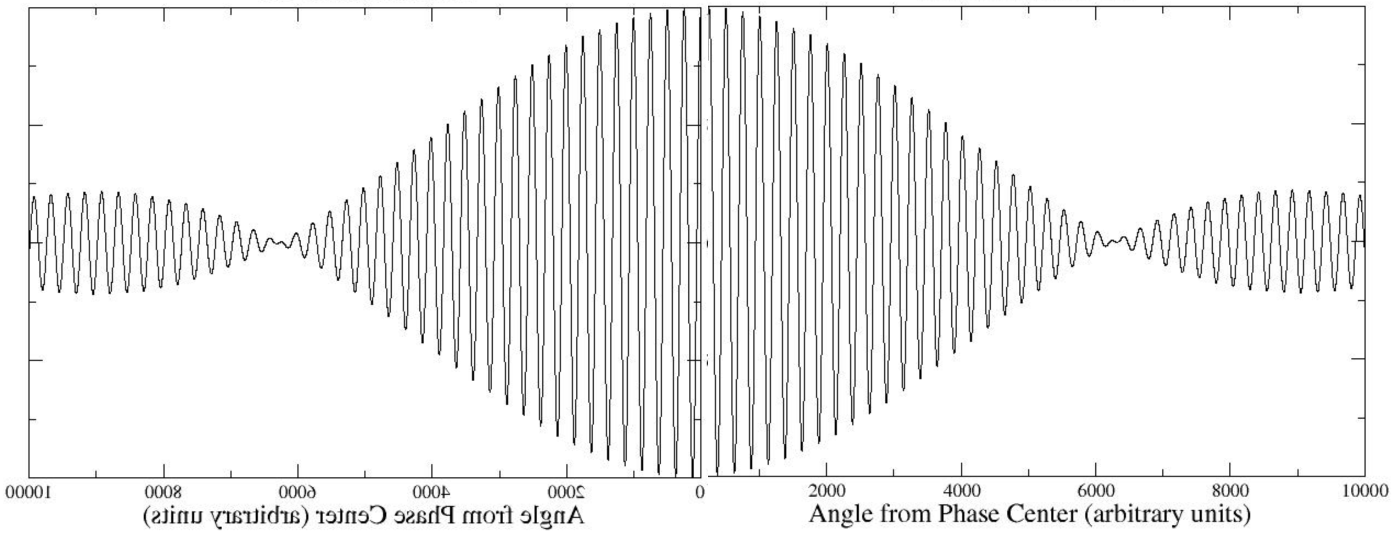
$$= E^2 e^{i2\pi[\upsilon \mathbf{b} \cdot (\mathbf{s} - \mathbf{s}_0)/c]}$$

Emphasis:

- Shown again is the fringe pattern of a real wide-band baseline.
- To avoid delay losses, we **must** re-set delays before the source moves too far down the delay pattern.
- To avoid ridiculous sample rates, we need to also shift the signal phase.
- These operations done in the correlator

Source moves this way

Tracking the phase requires much faster adjustments than maintaining amplitude. The process is much easier if the phase adjustment is done independently of delay (which it is).

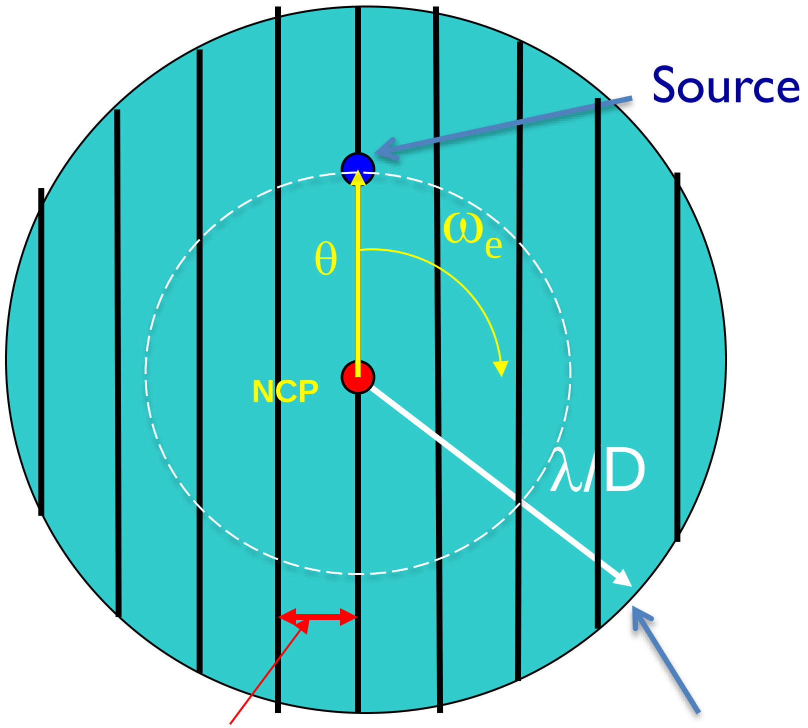


Differential Time Averaging Loss

- We can track a moving source, continuously adjusting the delay and phase to move the fringe pattern with the source.
- This does two good things:
  - Slows down the data recording needs
  - Prevents bandwidth delay losses.
- But, you cannot increase the time averaging indefinitely.
- The reason is that the fringe tracking mechanism is correct for only one point in the sky. All others have a different rate.
- So while you can reduce the observed fringe rate to zero for any given place, all other directions retain a differential rate.
- The limit to time averaging set by this differential.

Time-Averaging Loss Timescale

Simple derivation of fringe period, from observation at the NCP.



- Turquoise area is antenna primary beam on the sky – radius = λ/D
- Interferometer coherence pattern has spacing = λ/B
- Sources in sky rotate about NCP at angular rate:

  ω_e =7.3x10^{-5} rad/sec.

- Minimum time taken for a source at offset θ to move by λ/B is:

$$t = \frac{\lambda}{B}\frac{1}{\omega_E \theta}$$

- For a source at the primary beam, $\theta=\lambda/D$, so at that radius:

$$t = \frac{D}{B}\frac{1}{\omega_E}$$

- For the VLA in A configuration, t~10 seconds.

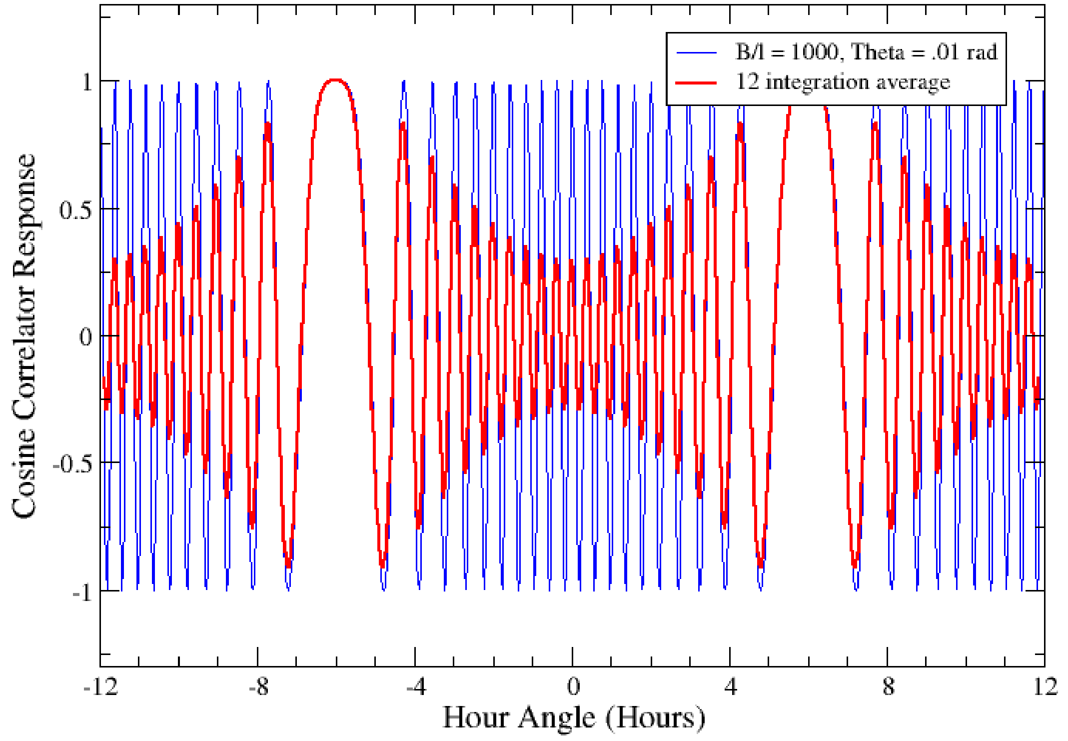
Illustrating Time Averaging Loss

- An object located away from the fringe tracking center moves through the fringe pattern as the earth rotates.
- It makes one cycle around in 24 hours.
- If we average the correlation products for too long a period, a loss in fringe amplitude will result.

Illustrating time average loss.

Blue trace: the fringe amplitude with no averaging.

Red trace: Amplitude after averaging for 12 'samples'.



Time-Averaging Loss

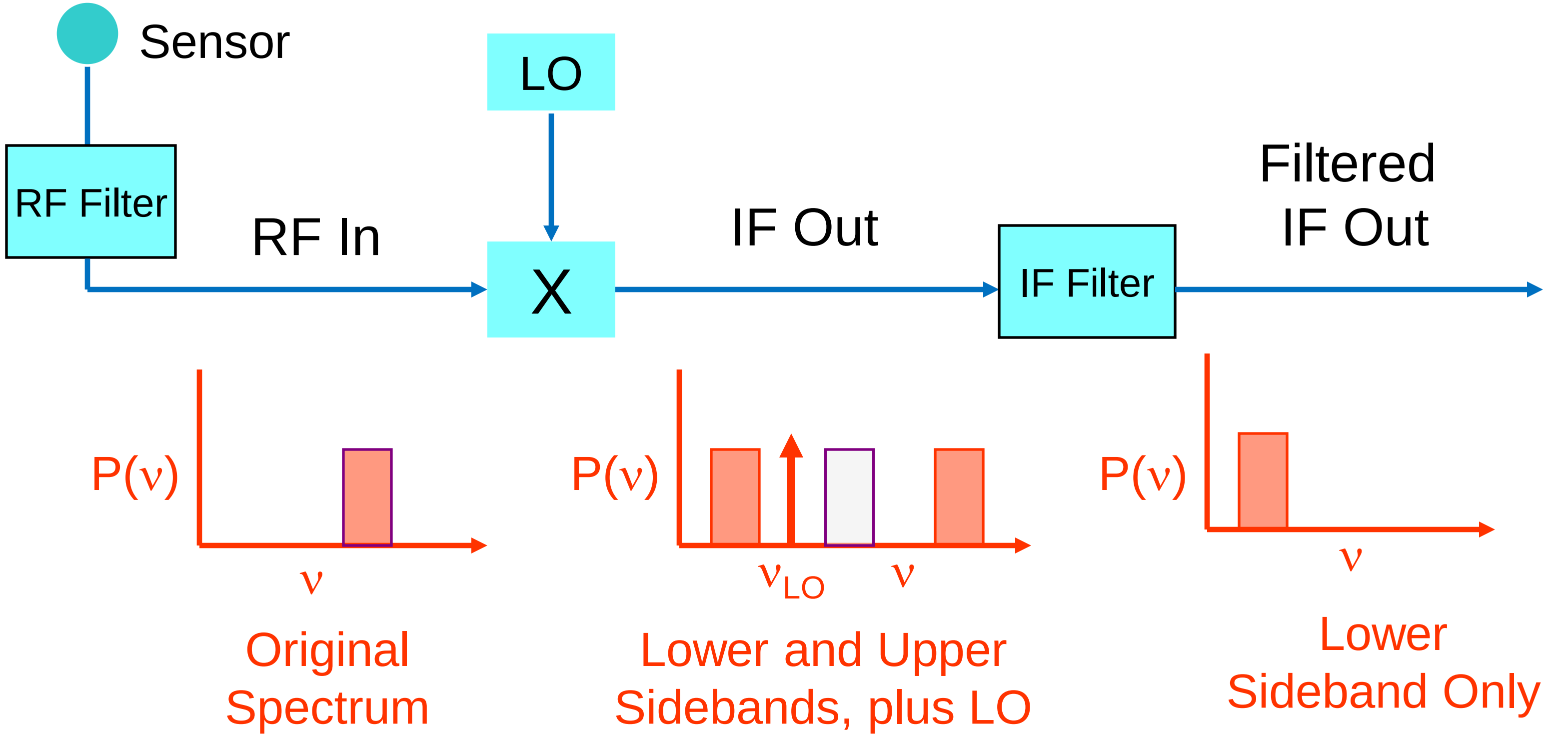
- So, what kind of time-scales are we talking about now?
- How long can you integrate before the differential motion destroys the fringe amplitude?
- **Case A:** VLA A-configuration: D = 25 m , and B = 35 km for source at edge of primary beam:
  - $t = D/(B\omega_e) = 10$ seconds. (independent of observing frequency).
- **Case B**: ngVLA: D = 18 m, B = 1000 km:
  - $t = D/(B\omega_e) = 250$ msec.
- Averaging for durations comparable to these will cause severe attenuation of the visibility amplitudes.
- To prevent 'delay losses', your averaging time must be much less than this.
  - Averaging time 1/10 of this value normally sufficient to prevent time loss.

The Heterodyne Interferometer: LOs, IFs, and Downcoversion

- This would be the end of the story (so far as the fundamentals are concerned) if all the internal electronics of an interferometer would work at the observing frequency (often called the 'radio frequency', or RF).
- Unfortunately, this cannot be done in general, as high frequency components are much more expensive, and generally perform more poorly than low frequency components.
- Thus, most radio interferometers use 'down-conversion' to translate the radio frequency information from the 'RF' to a lower frequency band, called the 'IF' in the jargon of our trade.
- For signals in the radio-frequency part of the spectrum, this can be done with almost no loss of information.
- But there is an important side-effect from this operation in interferometry which we now review.

Downconversion

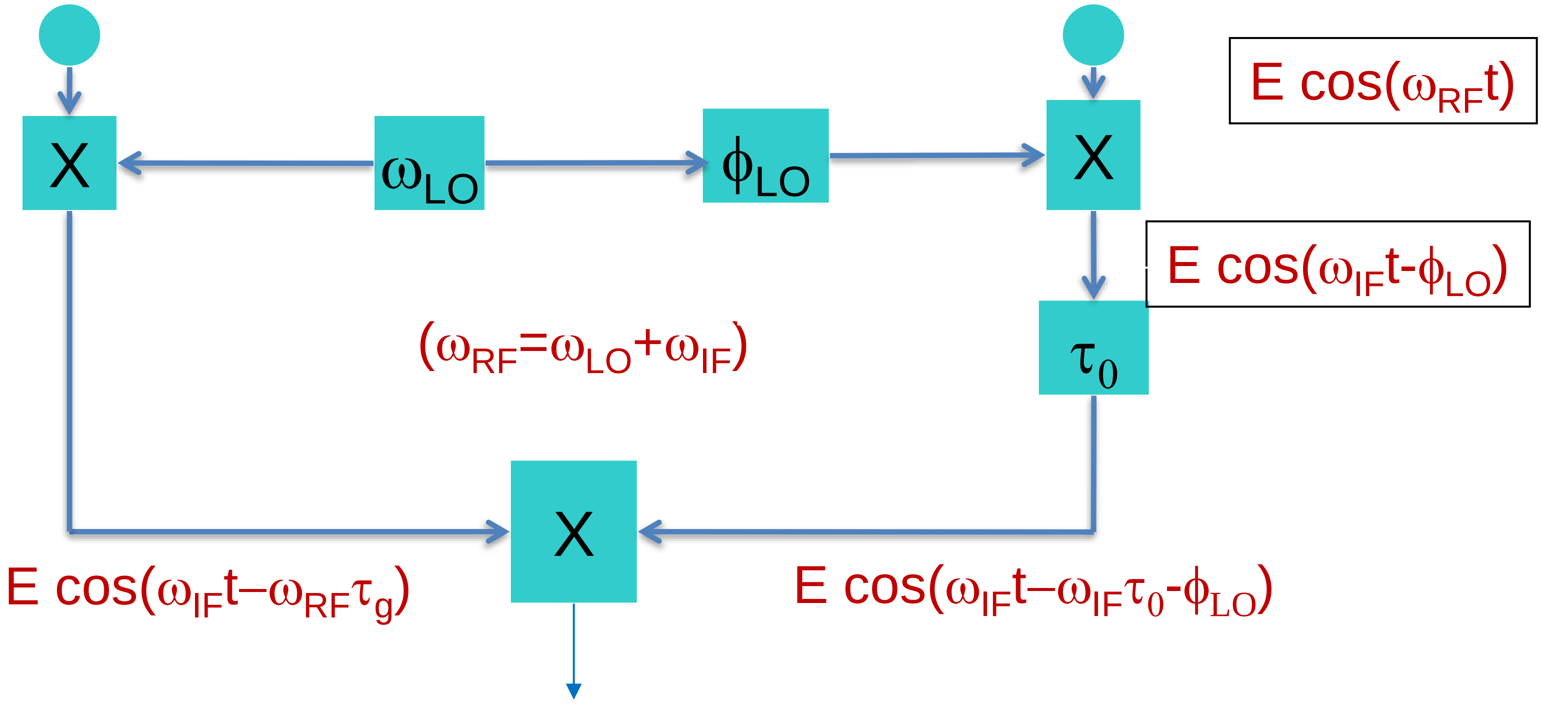
At radio frequencies, the spectral content within a passband can be shifted – with almost no loss in information, to a lower frequency through multiplication by a 'LO' signal – a pure sinusoid.



This operation preserves the amplitude and phase relations

Signal Relations, with LO Downconversion

- The RF signals are multiplied by a pure sinusoid, at frequency ν_{LO}
- We can add arbitrary phase ϕ_{LO} on one side.



$$V = E^2 e^{-i(\omega_{RF}\tau_g - \omega_{IF}\tau_0 - \phi_{LO})}$$

Recovering the Correct Visibility Phase

- Unfortunately, this downconversion process changes the phase of the correlated product. To see this, note:
- The correct phase (RF interferometer) is:

$$\omega_{RF}\left(\tau_g - \tau_0\right)$$

- The observed phase, with frequency downconversion is:

$$\omega_{RF}\tau_g - \omega_{IF}\tau_0 - \phi_{LO}$$

- These will be the same when the LO phase is set to:

$$\phi_{LO} = \omega_{LO}\tau_0$$

- Thus, to retrieve the correct phase, we must adjust the LO phase.
- This is necessary because the delay, τ_0, has been added in the IF portion of the signal path, rather than at the RF frequency at which the delay actually occurs.
- The phase adjustment of the LO compensates for the delay having been inserted at the IF , rather than at the RF.

The Three 'Centers' in Interferometry

- You are forgiven if you're confused by all these 'centers'.
- So let's review:

1. **Beam Tracking (Pointing) Center**: Where the antennas are pointing to. Normally follows the source over time. (But doesn't have to …)
2. **Delay Tracking Center:** The location for which the delays are being set for maximum wide-band coherence.
3. **Phase Tracking Center:** The location for which the system phase has been adjusted in order to track the coherence (fringe) pattern position.

- Note: Generally, we make all three the same – but in general, the phase and delay centers are separable.