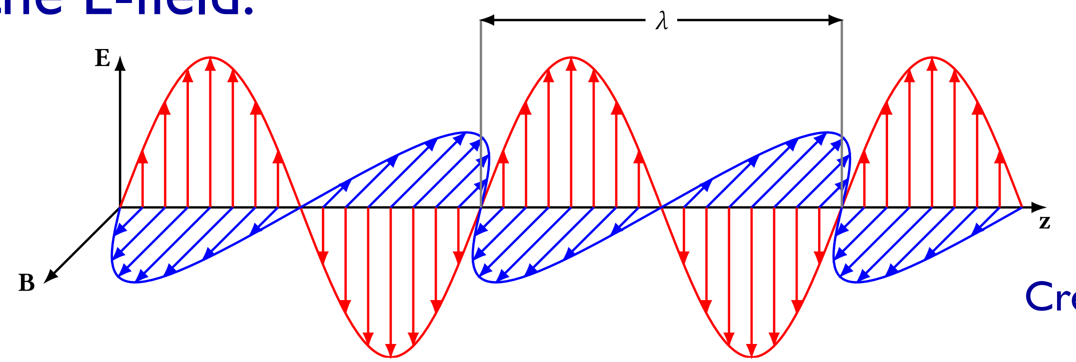
Interferometric Polarimetry – an Introduction

Rick Perley
National Radio Astronomy Observatory
Socorro, NM

Polarization – What is it?

- EM radiation is a transverse wave (in the far field), comprising propagating electric and magnetic fields.
- The E and B fields are orthogonal, and directly connected, so we normally think of the E-field.

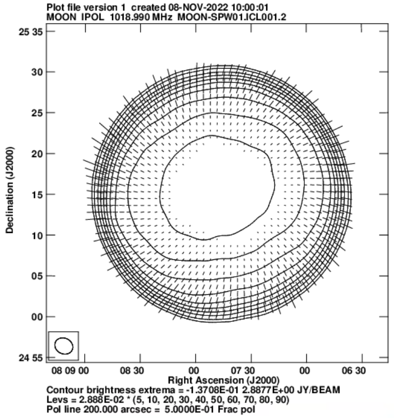


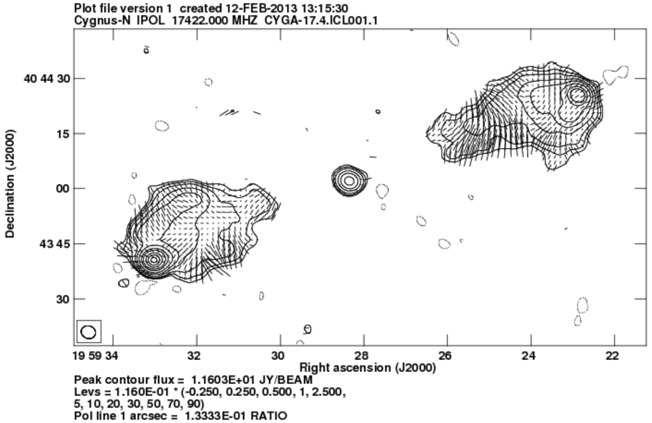
Credit: Wikipedia

- Being a transverse wave, the E-field comprises two orthogonal components
- These components propagate independently.
- Polarimetry refers to the characteristics of these two components.
  - Their amplitudes, and the phase relation between them.

Polarimetry – Why Do It?

- Measuring the polarization gives us additional information into the physical processes at play.
- Examples:
  - Synchrotron radiation – orientation and strength of magnetic fields.
  - Zeeman splitting – strength of fields.
  - Electron scattering
  - Faraday rotation (of linear polarization due to magnetic fields)
  - Polarization of radiation from thermal bodies – measures the material refractive index.





Visualizing Polarization -- My 'magic screen'

- It helps to be able to visualize the incoming electric/magnetic fields.
  - Note: For the detectors we are considering, electron motion is low, so the B-field component of the EM wave is unimportant.
  - We thus considering only the E-field component of the EM wave.
- Imagine a 'magic screen', which you hold up to intercept incoming radiation.
- The magic screen has 'visible electrons', which are reacting to the electric fields passing through.
- What will you see?
- For wideband data – it's a mess, with random motions .
  - But if you watch closely, you may note that the motions are not completely random, but may prefer certain position angles.
- For mathematical analysis, it is again useful to consider a minutely narrow bandwidth, for which the magic electron motion becomes quite simple.

The General Case -- Elliptical

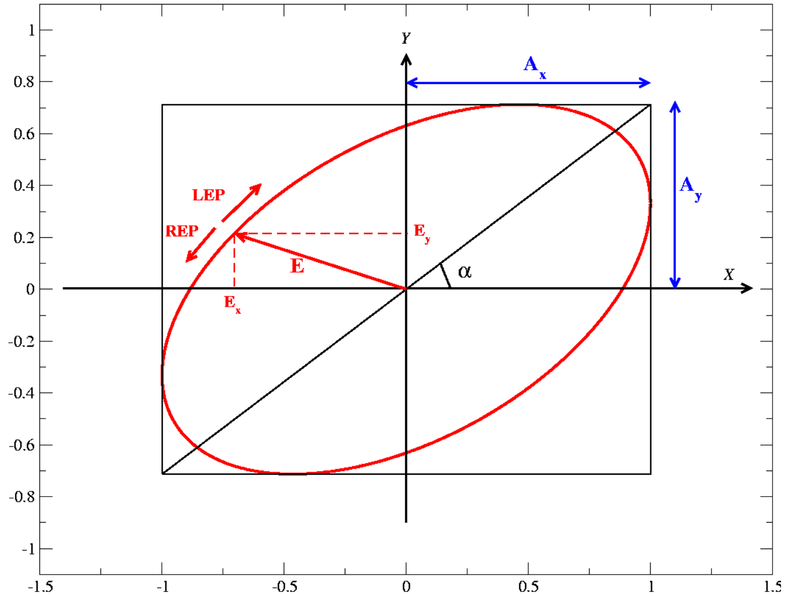
- The description of polarization usually begins with utilizing the 'quasi-monochromatic approximation'.
- Imagine analysis of radiation passed through a very narrow filter – say, 1 Hz
- The characteristics of the field are then quasi-stable for ~1 second.
- Maxwell's equations then tell us the electric field describes an ellipse.

In general, three parameters are needed to describe the ellipse.

- A_x – X-axis amplitude max
- A_y – Y-axis amplitude max
- α = atan(A_y/A_x) – an angle describing the orientation

If the E vector is rotating (as seen by the observer):

- Clockwise, the wave is Left Elliptically Polarized:
- Anti-clockwise, the wave is Right Elliptically Polarized.

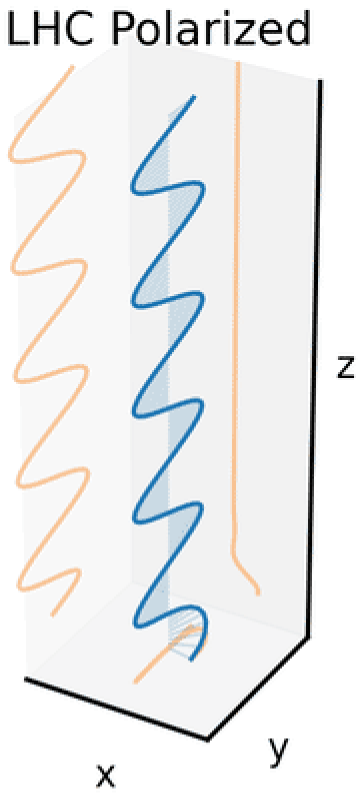


Linear and Circular Bases

- It is easy to conceptualize an elliptically polarized propagating wave as the sum of two orthogonal linear components: E_x and E_y.
  - There are three factors: the two amplitudes, and the phase difference ϕ_{xy} between them.
  - This phase difference describes how far 'behind' the 'Y' component sinusoid is behind the 'X' component.
- But we can also describe the elliptical wave in terms of two oppositely rotating circular components.
- Again – three factors: E_r , E_l , and the phase ϕ_{rl} between them.
- This is sufficient for the monochromatic case, but in general, radiation is broad-band, originating from an uncountably large number of electrons.
- This results in partial polarization, for which we need a fourth parameter.

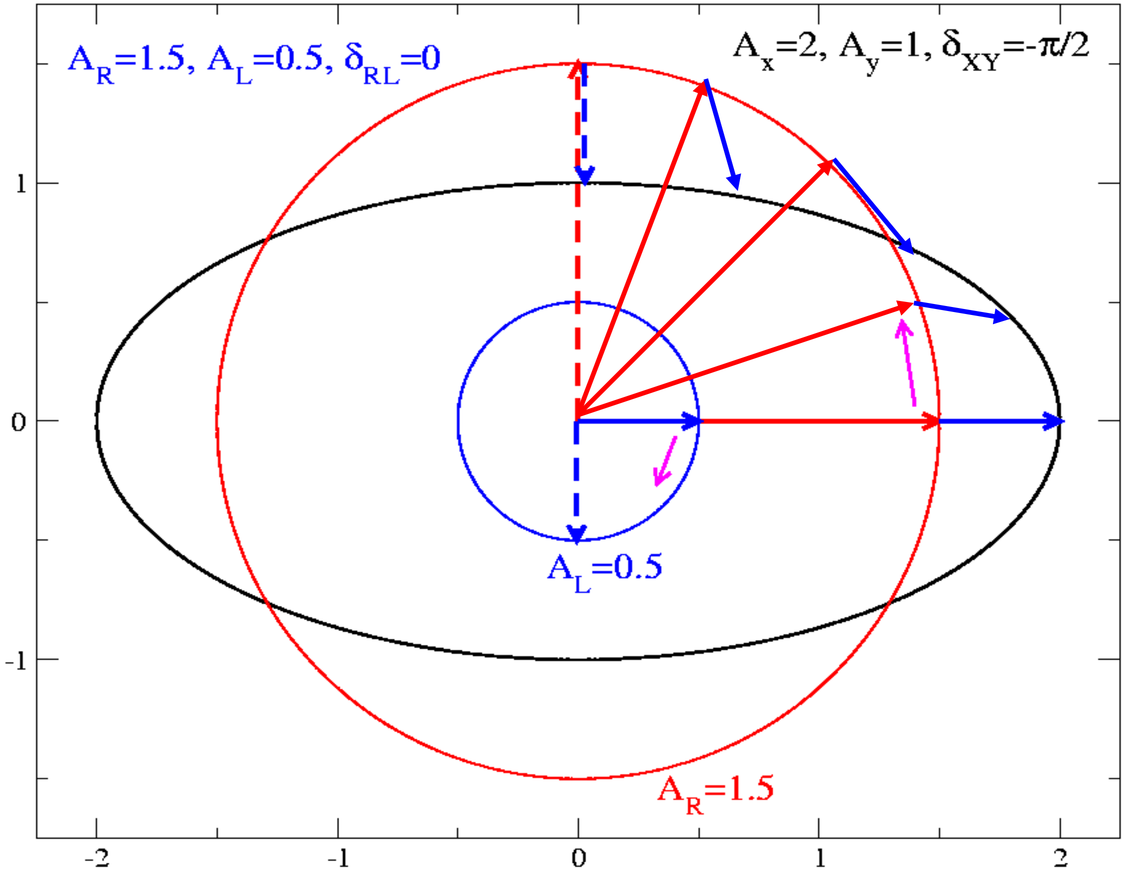
Linear Basis – Various polarization states.

- This nice animation (from Wikipedia) shows how linear and circularly polarized waves can be decomposed into orthogonal linear components.

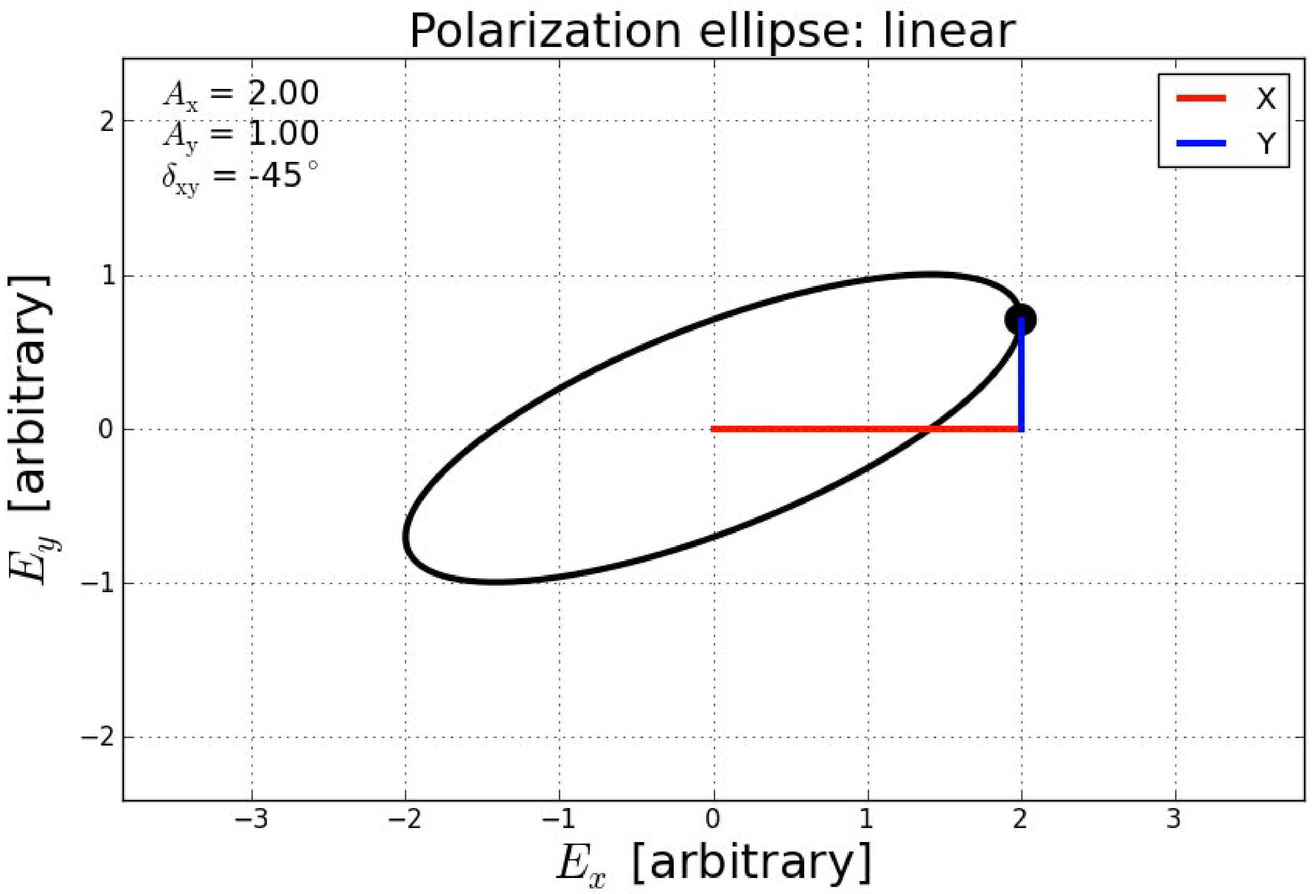


Circular Basis Example

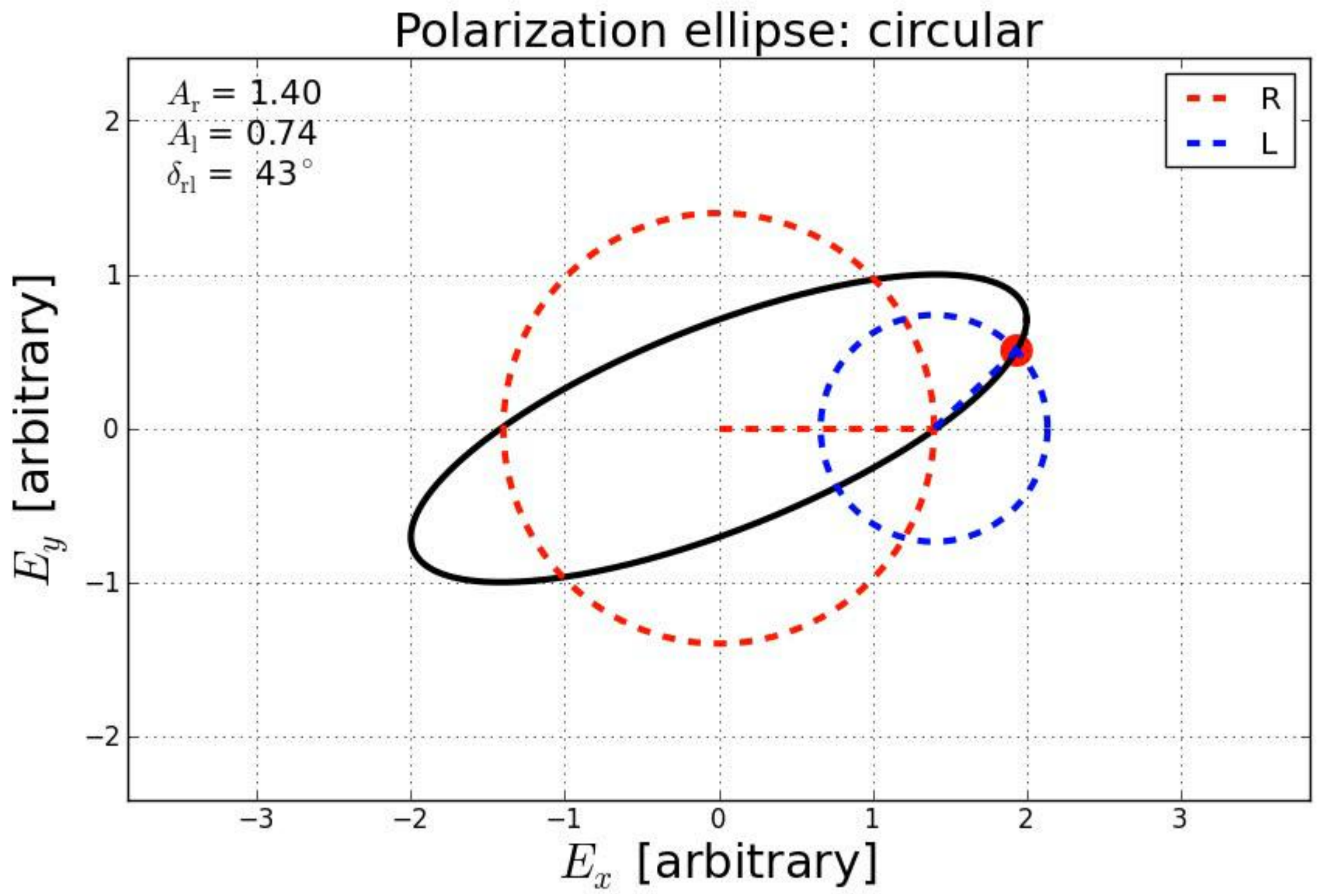
- The polarization ellipse (black) can be decomposed into an X-component of amplitude 2, and a Y-component of amplitude 1 which lags by ¼ turn.
- It can alternatively be decomposed into a counterclockwise (RCP) rotating vector of length 1.5 (red), and a clockwise rotating (LCP) vector of length 0.5 (blue).



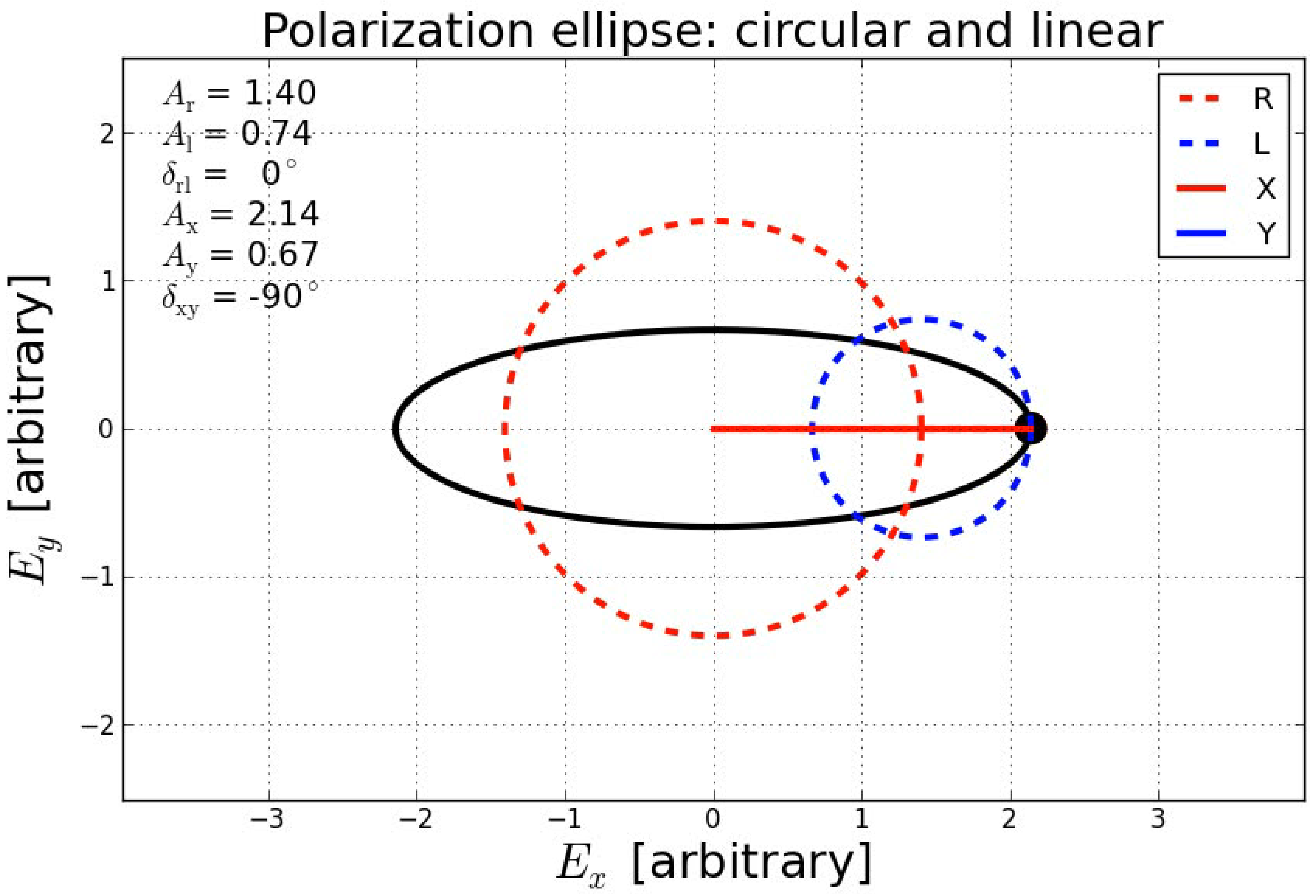
- Elliptical Wave, decomposed into orthogonal linear components.



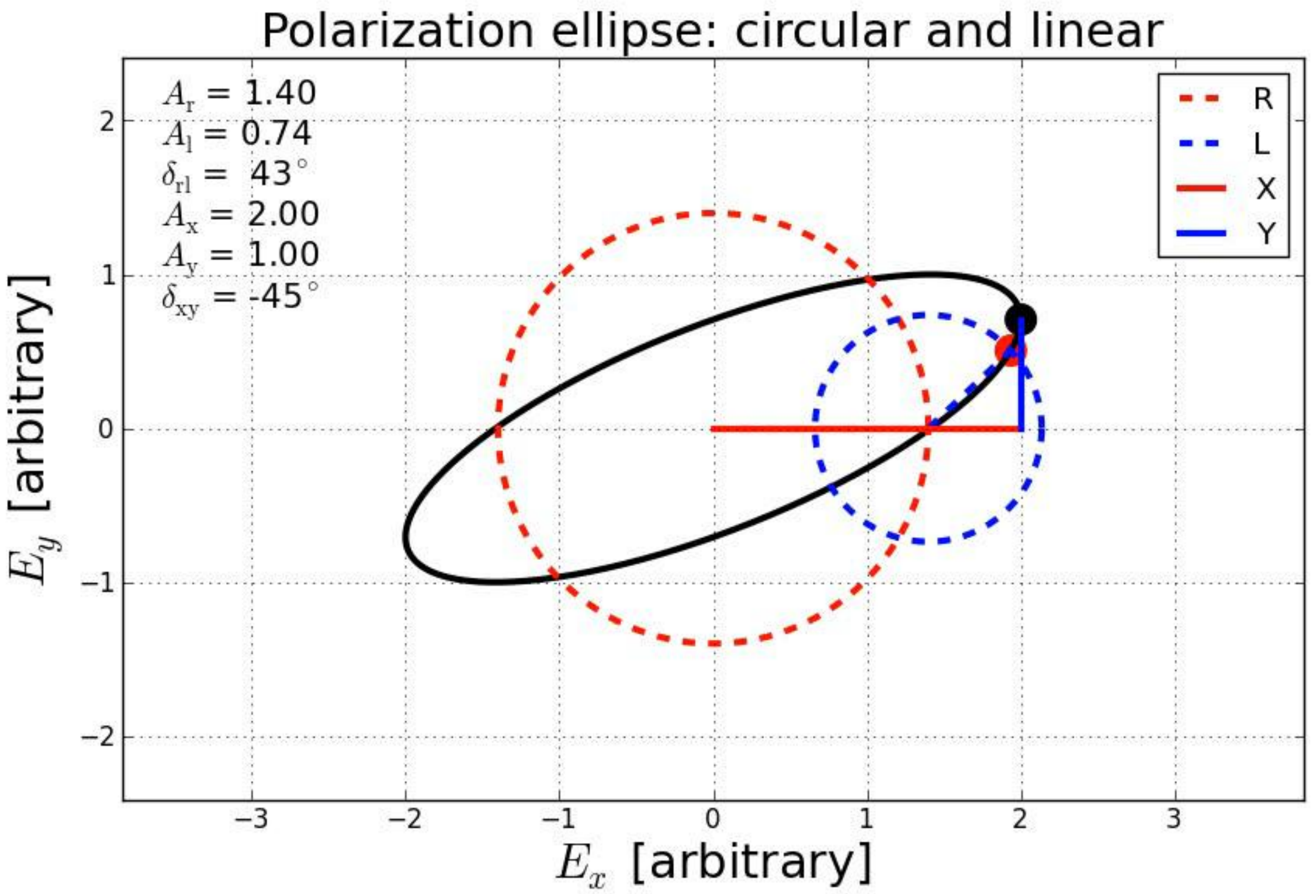
- The same wave, decomposed into orthogonal circular components



- Both decompositions, for a horizontal ellipse



- Both decompositions, for a tilted ellipse.



Stokes Parameters – Definition

- Perfectly monochromatic EM waves have an E-vector which traces a perfect ellipse in a fixed plane.
- We utilize in radio astronomy the parameters defined by George Stokes (1852), and introduced to astronomy by Chandrasekhar (1946):

$$
\begin{aligned}
I &= A_X^2 + A_Y^2 &&= A_R^2 + A_L^2 \\
Q &= A_X^2 - A_Y^2 &&= 2A_R A_L \cos\delta_{RL} \\
U &= 2A_X A_Y \cos\delta_{XY} &&= -2A_R A_L \sin\delta_{RL} \\
V &= -2A_X A_Y \sin\delta_{XY} &&= A_R^2 - A_L^2
\end{aligned}
$$

Units of power:
Jy, or Jy/beam

where A_X and A_Y are the cartesian amplitude components of the E-field, and δ_{XY} is the phase lag between them, and

A_R and A_L are the opposite circular amplitude components of the E-field, and δ_{RL} the phase lag between them.

- By (IAU) convention, the 'X' axis points to the NCP, the 'Y' axis to the east.
- Also by IAU convention, LCP has the E-vector rotating clockwise for approaching radiation.
- Monochromatic radiation is 100% polarized: $I^2 = Q^2 + U^2 + V^2$

Stokes Parameters -- Definition

- The Stokes Parameters (named for George Stokes, 1842) are now commonly used to describe astronomical signal polarization.
- They have units of spectral power, or brightness.
- I describes the brightness (‘total power’).
- Q and U describe the linear polarization:

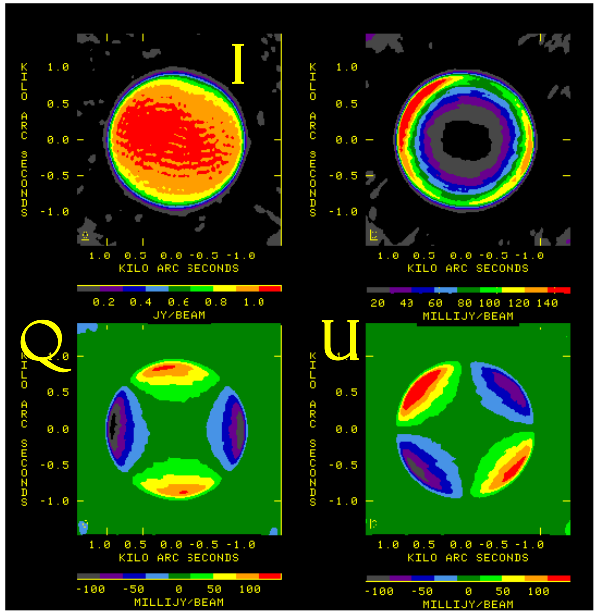
  +Q => vertical EVPA, -Q => horizontal EVPA

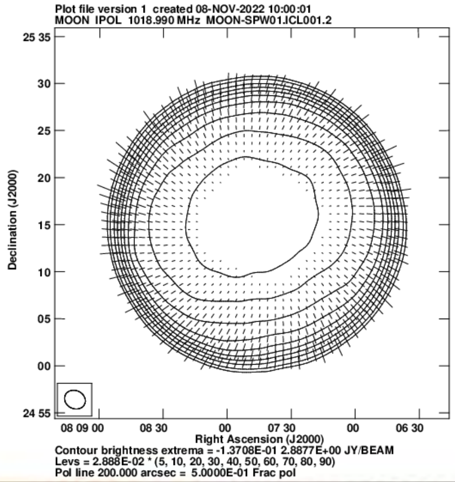
  +U => EVPA at 45deg, -U => EVPA at -45 deg

  EVPA: $\chi = 0.5 \arctan(U/Q)$
- V describes circular polarization:
  - +V => Right CP, -V => Left CP
- In general, the signal is a mixture of Q, U, and V.

$$I^2 > Q^2 + U^2 + V^2 \qquad P = \sqrt{Q^2 + U^2}$$

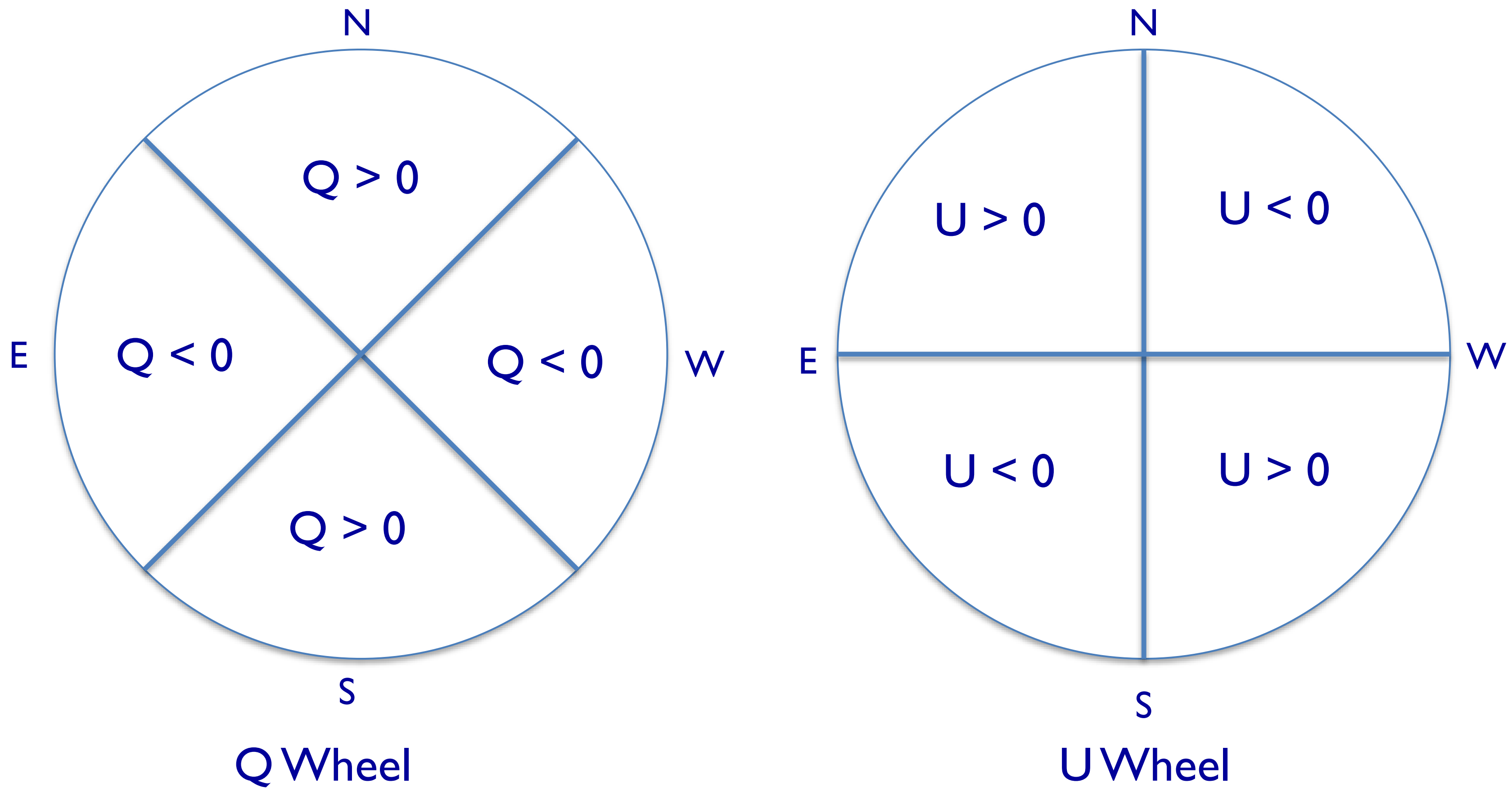
- Note: Q, U, and V can be negative!





Stokes Parameters, cont.

- To help visualize the meaning of the Stokes parameters, it's useful to use a 'Stokes wheel'.

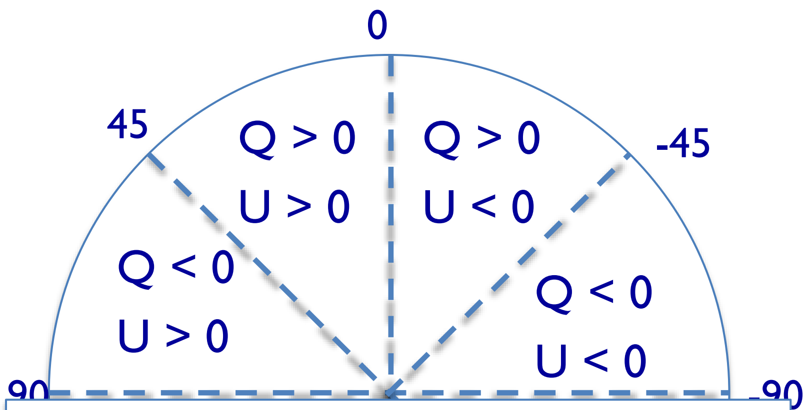


Linear Polarization Position Angle

- The Stokes parameters are real numbers, with units of Jy, of Jy/beam.
- If both Q and U are positive, then we know the EVPA (electric vector position angle) is between 0 and 45 degrees.
- The formal definition is:

$$\chi = 0.5 \arctan(U / Q)$$

- Note that the 0.5 factor arises because the EVPA is not a vector – it is an orientation.
- Rotation by 180 degrees results in the same orientation.



Stokes Visibilities for Interferometry

- You will all know that the Visibility Function, V(u,v), is related to the sky brightness by Fourier Transform:

  $$V\,(u,v) \Longleftrightarrow I\,(l,m)$$ (a Fourier Transform Pair)

- We now generalize this, and consider the Stokes brightness distributions of I, Q, U, and V.
- Define the **Stokes Visibilities** $\mathcal{I}$, $\mathcal{Q}$, $\mathcal{U}$, and $\mathcal{V}$, to be the Fourier Transforms of these brightness distributions.
- Then, the relations between these are:
- $\mathcal{I} \Longleftrightarrow I$, $\mathcal{Q} \Longleftrightarrow Q$, $\mathcal{U} \Longleftrightarrow U$, $\mathcal{V} \Longleftrightarrow V$
- **Stokes Visibilities** are complex functions of (u,v), while the **Stokes Images** are real functions of (l,m). All visibilities are Hermitian.
- Our task is now to measure these Stokes visibilities.

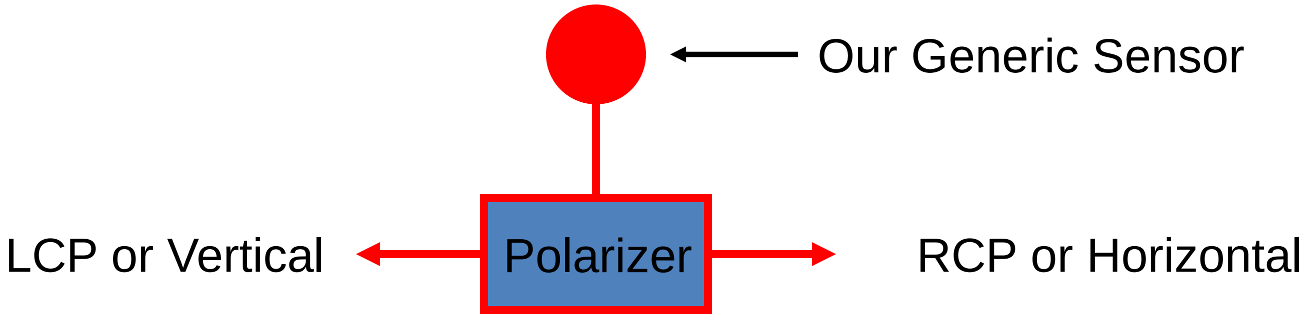
NRAO

Stokes Parameters and Stokes Visibilities

- So (you might say to yourself), that's all very nice, but how do we actually measure these Stokes Visibilities?
- Interesting fact (probably a theorem, but I don't actually know if there is one):
  - You cannot build an interferometer which **directly** produces Stokes visibilities.
- So we need a little more background.
- Although it may seem an oxymoron, in order to measure source polarization, we need to have polarized antennas.

Antennas are Polarized!

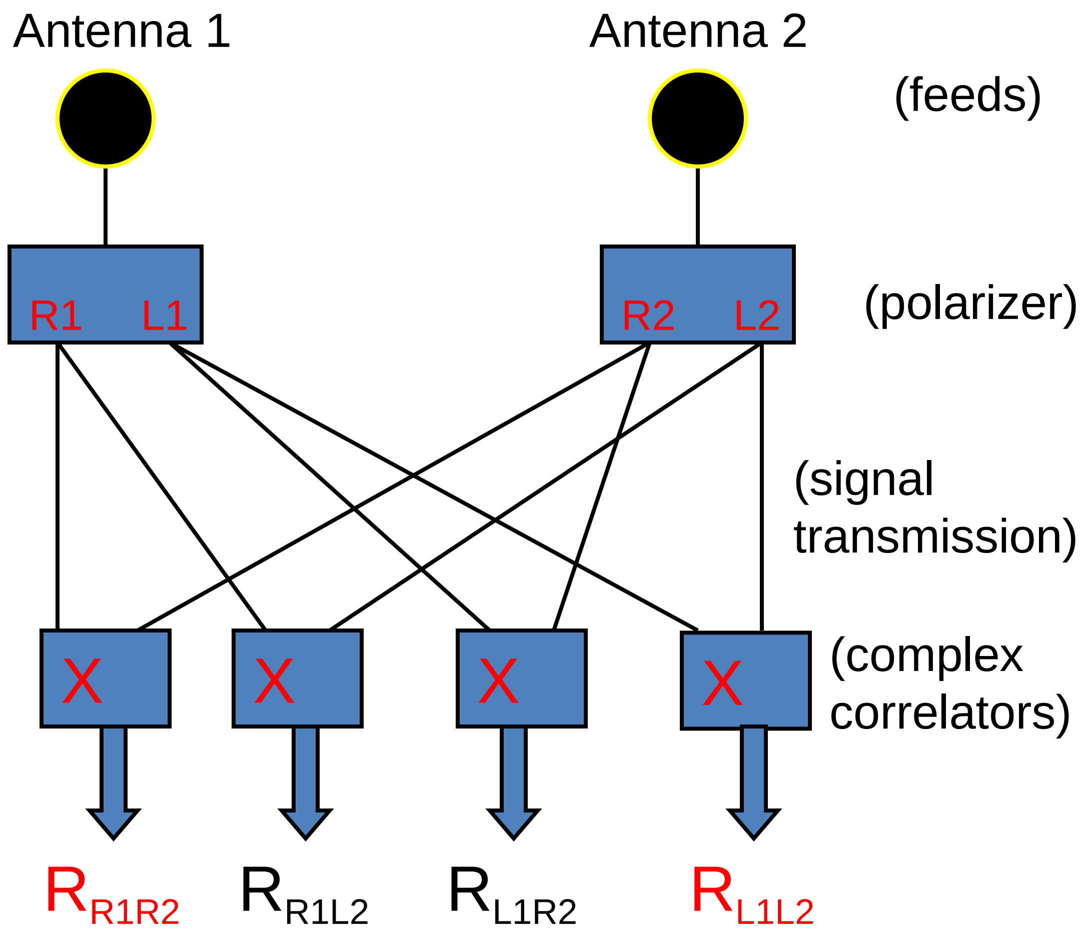
- The choice of basis for the description is important, since antenna/receiver systems are themselves naturally polarized.
- They are designed to output signals (voltages) proportional to the amplitude and phase of either the linear, or circular, components.
- They provide two simultaneous voltage signals whose values are (ideally) representations of the electric field components – either in a circular or linear basis.



- We have two antennas, each with two polarized outputs.
- We can then form four complex correlations.

Four Complex Correlations per Pair of Antennas

- Two antennas, each with two differently polarized outputs, produce four complex correlations.
- The 'RR' and 'LL' (or VV and HH) correlations are called the 'parallel hands'.
- The 'RL' and 'LR'(or VH and HV) correlations are called 'cross-hands'.

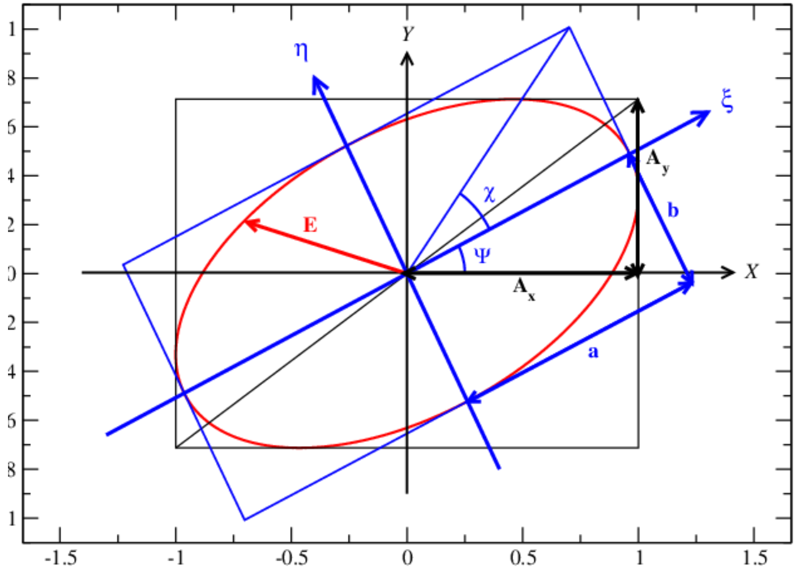


What is the relation between these correlations and Stokes parameters?

Antenna Polarization Characteristics

- Consider an antenna with two ports labelled 'R' and 'L' (or 'V' and 'H').
- Connect a monochromatic oscillator to this port, and go 'far far away' to measure the polarization properties of the radiated field.
- You will find, in general, and elliptical response, characterized by three parameters: An amplitude A, **an ellipticity** χ, and a **position angle** Ψ of the major axis.

$$\tan \chi = b / a$$



  - From analytic geometry, with:

$$\tan \alpha = A_y / A_x$$

  - We find $\tan 2\Psi = \tan 2\alpha \cos \delta_y$
  - And $\sin 2\chi = -\sin 2\alpha \sin \delta_y$

The response of the four correlations:

$$\begin{aligned}
R_{pq} = G_{pq}\{&[\cos(\Psi_p - \Psi_q)\cos(\chi_p - \chi_q) + i\sin(\Psi_p - \Psi_q)\sin(\chi_p + \chi_q)]\mathcal{I}/2 \\
&+[\cos(\Psi_p + \Psi_q)\cos(\chi_p + \chi_q) + i\sin(\Psi_p + \Psi_q)\sin(\chi_p - \chi_q)]\mathcal{Q}/2 \\
&-i[\cos(\Psi_p + \Psi_q)\sin(\chi_p - \chi_q) + i\sin(\Psi_p + \Psi_q)\cos(\chi_p + \chi_q)]\mathcal{U}/2 \\
&-[\cos(\Psi_p - \Psi_q)\sin(\chi_p + \chi_q) + i\sin(\Psi_p - \Psi_q)\cos(\chi_p - \chi_q)]\mathcal{V}/2\}
\end{aligned}$$

This is the remarkable expression derived by Morris, Radhakrishnan and Seielstad (1964), relating the output of a single complex correlator to the complex Stokes visibilities, where the antenna effects are described in terms of the polarization ellipses of the two antennas.

R_{pq} is the complex output from the interferometer, for polarizations p and q from antennas 1 and 2, respectively.

Ψ and χ are the antenna polarization major axis and ellipticity for polarizations p and q.

$\mathcal{I}$,$\mathcal{Q}$, $\mathcal{U}$, and $\mathcal{V}$ are the Stokes Visibilities

G_{pq} is a complex gain, including the effects of transmission and electronics

For pure systems, it's easy!

- Before giving up in despair, note that this interesting expression becomes very simple for antennas which are perfectly polarized.
- For pure linearly polarized antennas, the ellipticity is zero: $\chi = 0$. Then, for the two linearly polarized channels,
  - $\Psi_v = 0, \Psi_h = \pi/2$. (presuming equatorial feeds).
- While for perfectly circularly polarized antennas, we have: $\chi_r = -\pi/4, \chi_l = \pi/4$. (For perfectly circular feeds, Ψ has no meaning).
- Then, (exercise for the student), that wondrous expression from Morris et al. provides remarkably simple results:

For Pure Linearly Polarized Antennas

- Here are the expressions, assuming equatorial mounts (zero parallactic angle):

$$R_{V1V2} = \langle V_{V1} V^*_{V2} \rangle = (\mathcal{I} + \mathcal{Q})/2$$
$$R_{H1H2} = \langle V_{H1} V^*_{H2} \rangle = (\mathcal{I} - \mathcal{Q})/2$$
$$R_{V1H2} = \langle V_{V1} V^*_{H2} \rangle = (\mathcal{U} + i\mathcal{V})/2$$
$$R_{H1V2} = \langle V_{H1} V^*_{V2} \rangle = (\mathcal{U} - i\mathcal{V})/2$$

- For which the solutions for the Stokes Visibilities are dead-easy:

$$\mathcal{I} = R_{V1V2} + R_{H1H2}$$
$$\mathcal{Q} = R_{V1V2} - R_{H1H2}$$
$$\mathcal{U} = R_{V1H2} + R_{H1V2}$$
$$\mathcal{V} = -i(R_{V1H2} - R_{H1V2})$$

While for Pure Circular …

- Again the reduction is simple (again assuming zero parallactic angle):

$$R_{R1R2} = \langle V_{R1} V_{R2}^{*} \rangle = (\mathcal{I} + \mathcal{V})/2$$

$$R_{L1L2} = \langle V_{L1} V_{L2}^{*} \rangle = (\mathcal{I} - \mathcal{V})/2$$

$$R_{R1L2} = \langle V_{L1} V_{R2}^{*} \rangle = (\mathcal{Q} + i\mathcal{U})/2$$

$$R_{L1R2} = \langle V_{R1} V_{L2}^{*} \rangle = (\mathcal{Q} - i\mathcal{U})/2$$

- Giving for the visibilities:

$$\mathcal{I} = R_{R1R2} + R_{L1L2}$$

$$\mathcal{V} = R_{R1R2} - R_{L1L2}$$

$$\mathcal{Q} = R_{R1L2} + R_{L1R2}$$

$$\mathcal{U} = -i(R_{R1L2} - R_{L1R2})$$

Stokes Visibilities – Comparing Bases

- For simplicity, I omit (for this slide) the orientation of the dipoles, and presume they are aligned with the (α,δ) sky coordinates.
- These apply to equatorial-mounted antennas.

Perfect Circular	Perfect Linear
$\mathcal{I} = R_{R1R2} + R_{L1L2}$	$\mathcal{I} = R_{V1V2} + R_{H1H2}$
$\mathcal{V} = R_{R1R2} - R_{L1L2}$	$\mathcal{V} = i\left(R_{H1V2} - R_{V1H2}\right)$
$\mathcal{Q} = R_{R1L2} + R_{L1R2}$	$\mathcal{Q} = \left(R_{V1V2} - R_{H1H2}\right)$
$\mathcal{U} = i\left(R_{L1R2} - R_{R1L2}\right)$	$\mathcal{U} = \left(R_{V1H2} + R_{H1V2}\right)$

- All quantities here are complex valued.
- For both systems, Stokes ‘I’ is the sum of the parallel-hands correlations.
- Stokes ‘V’ is the difference of the crossed hand responses for linear, (good) and is the difference of the parallel-hand responses for circular (bad).
- Stokes ‘Q’ and ‘U’ have more complicated relations to the observed correlations.

Stokes Visibilities – General Case

- The more general form, which includes the orientation of the antenna w.r.t. the celestial coordinate frame (described by the 'parallactic angle') looks like these: (easily derived from that same expression):

Circular	Linear
$\mathcal{I} = R_{R1R2} + R_{L1L2}$	$\mathcal{I} = R_{V1V2} + R_{H1H2}$
$\mathcal{V} = R_{R1R2} - R_{L1L2}$	$\mathcal{V} = i\left(R_{H1V2} - R_{V1H2}\right)$
$\mathcal{Q} = e^{i2\Psi_P} R_{R1L2} + e^{-i2\Psi_P} R_{L1R2}$	$\mathcal{Q} = \left(R_{V1V2} - R_{H1H2}\right)\cos 2\Psi_P - \left(R_{V1H2} + R_{H1V2}\right)\sin 2\Psi_P$
$\mathcal{U} = i\left(e^{-i2\Psi_P} R_{L1R2} - e^{i2\Psi_P} R_{R1L2}\right)$	$\mathcal{U} = \left(R_{V1V2} - R_{H1H2}\right)\sin 2\Psi_P + \left(R_{V1H2} + R_{H1V2}\right)\cos 2\Psi_P$

- Note that in the circular system, the linear components (Q and U) are uniquely found in the cross-hand components, while in the linear system, they require all four correlations.
- Note that nearly always, the parallel hand correlations have much higher amplitudes than cross-hand.

NRAO

Circular vs. Linear?

- Both systems provide straightforward derivation of the Stokes' visibilities from the four correlations.
- Deriving useable information from differences of large values requires both good stability and good calibration. Hence
  - To do good circular polarization using circular system, or good linear polarization with linear system, we need special care and special methods to ensure good calibration.
- There are practical reasons to use linear:
  - Antenna polarizers are natively linear – extra components are needed to produce circular. This hurts performance. Linear is simpler, more sensitive, and purer.
  - These extra components are also generally of narrower bandwidth – it's harder to build circular systems with really wide bandwidth.
- One important practical reason favoring circular:
  - Calibrator sources are often significantly linearly polarized, but have imperceptible circular polarization.
  - Gain calibration is much simpler with circular feeds.

Calibration Troubles …

- To understand this last point, note that for the linear system:

$$R_{V1V2} = G_{V1}G^*_{V2}(\mathcal{I} + \mathcal{Q}\cos 2\Psi_P + \mathcal{U}\sin 2\Psi_P)/2$$
$$R_{H1H2} = G_{H1}G^*_{H2}(\mathcal{I} - \mathcal{Q}\cos 2\Psi_P - \mathcal{U}\sin 2\Psi_P)/2$$

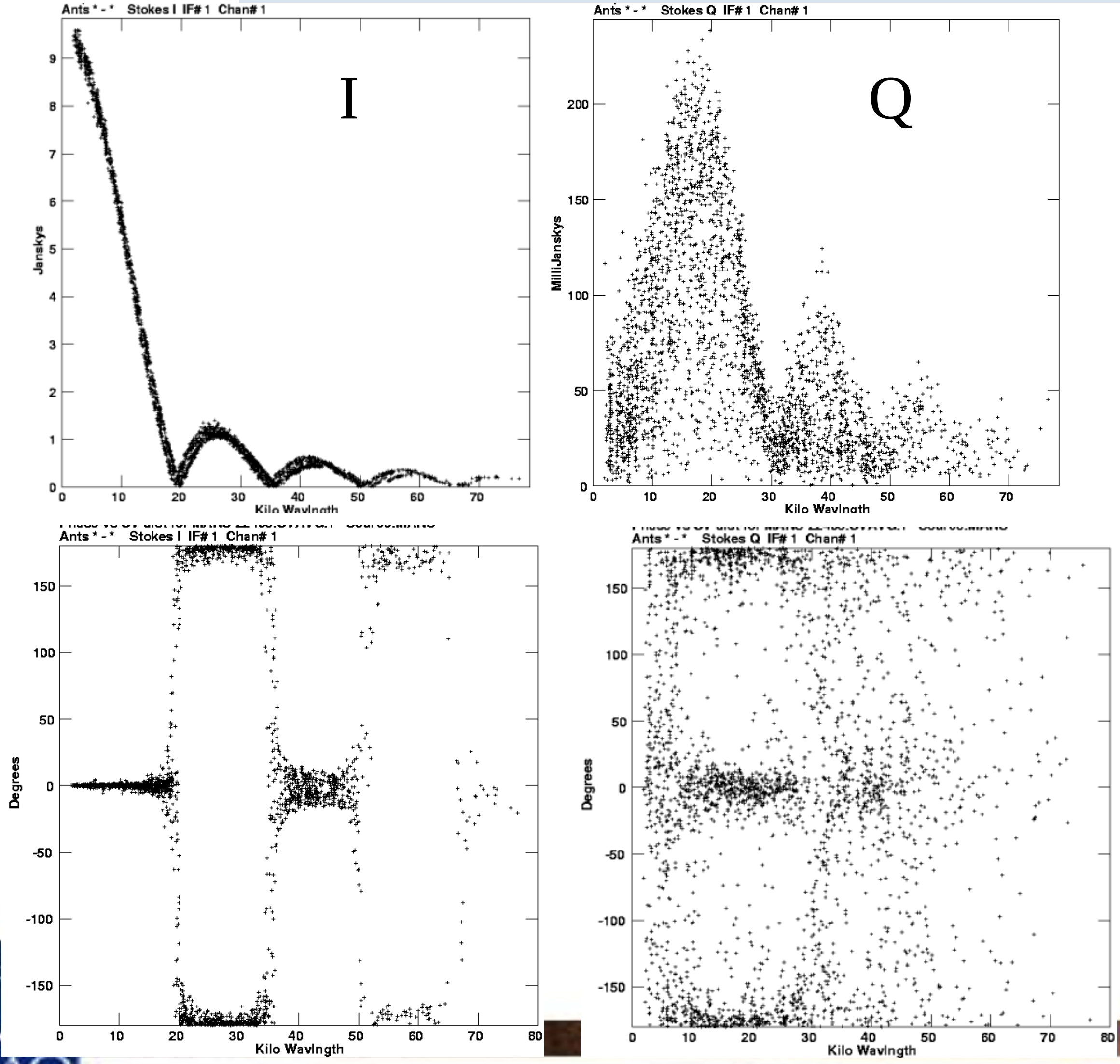
- To calibrate means to solve for the G_V and G_H terms.
- To do so requires knowledge of both Q and U.
- Virtually all calibrators have notable, and variable, linear pol.
- Meanwhile, for circular:

$$R_{R1R2} = G_{R1}G^*_{R2}(\mathcal{I} + \mathcal{V})/2$$
$$R_{L1L2} = G_{L1}G^*_{L2}(\mathcal{I} - \mathcal{V})/2$$

- Now we have *no* sensitivity to Q or U (good!). Instead, we have a sensitivity to V.
- But as it turns out – V is nearly always negligible for the 1000-odd sources that we use as standard calibrators.

$\mathcal{I}$ and $\mathcal{Q}$ Visibilities for Mars at 23 GHz

VLA, 23 GHz, 'D' Configuration, January 2006



Amplitude

- $|\mathcal{I}|$ is close to a J_0 Bessel function.
- Zero crossing at 20kλ tells us Mars' diameter ~ 10 arcsec.
- $|\mathcal{Q}|$ amplitude ~0 at zero baseline.
- $|\mathcal{Q}|$ zero at 30 kλ means polarization structures ~ 8 arcsec scale.

Phase

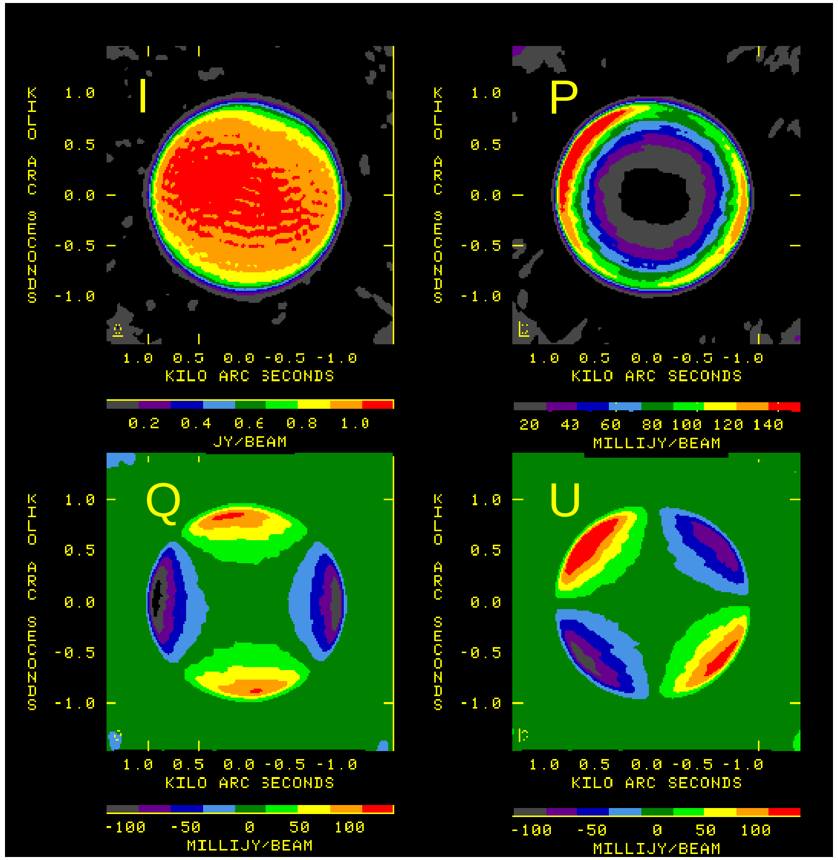
- $\mathcal{I}$ phase alternates between 0 and π.
- $\mathcal{Q}$ phase = both 0 and π in the 'main lobe' – this tells us there are both positive and negative structures, at different PA.

NRAO

Imaging – Polarization of the Moon

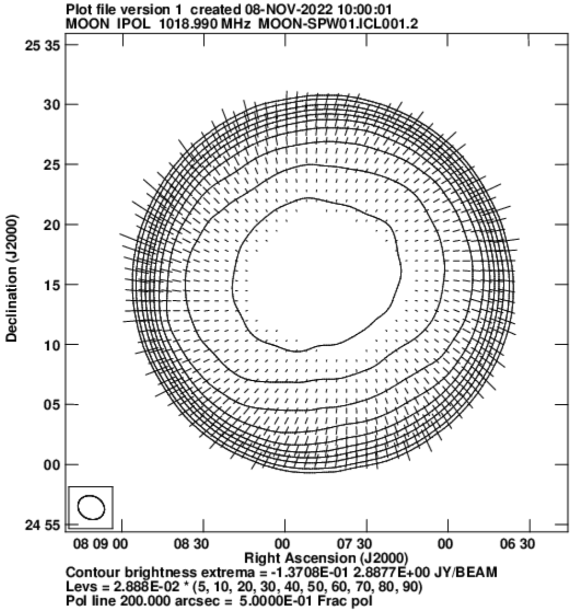
$$P = \sqrt{Q^2 + U^2}$$

- Shown here are the total intensity (I), polarized intensity (P), and Q and U images at 1040 MHz.
- The apparent elliptical brightness shape (in both I and P) is real – the moon is hotter around the equator – just like the earth.
- The edge-brightened polarization maximum is exactly as expected. See Perley & Butler, ApJSupl, **206**, 16 (2013) for details.
- The Q and U images tell us right away that the EVPAs are very nearly radial (as expected).



Example Images: Moon, Venus, Mars

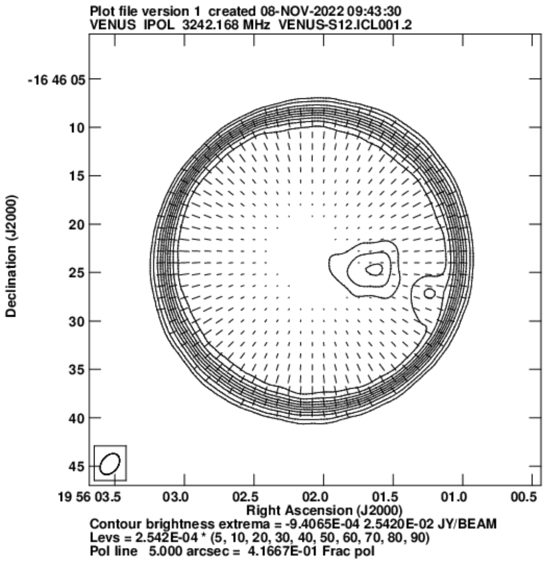
- Theory tells us that thermal radiation emitted from underneath the surface of a solid planet must be radially polarized, reaching about 30% near the limb.
- The maximum polarization depends on the dielectric constant of the material.
- We can use the observed position angle to calibrate our instruments.



Moon at 1.02 GHz

30 arcmin diameter

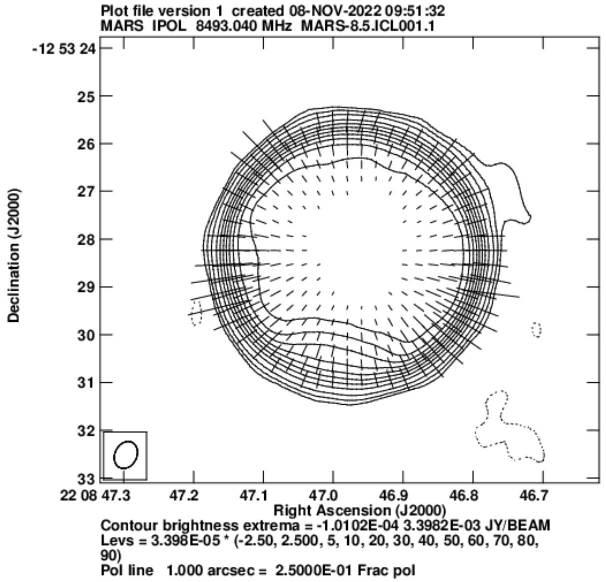
Limb darkening due to primary beam attenuation.



Venus at 3.24 GHz

30 arcsec

Cold regions are elevated terrain (Ovda and Thetis Regio)

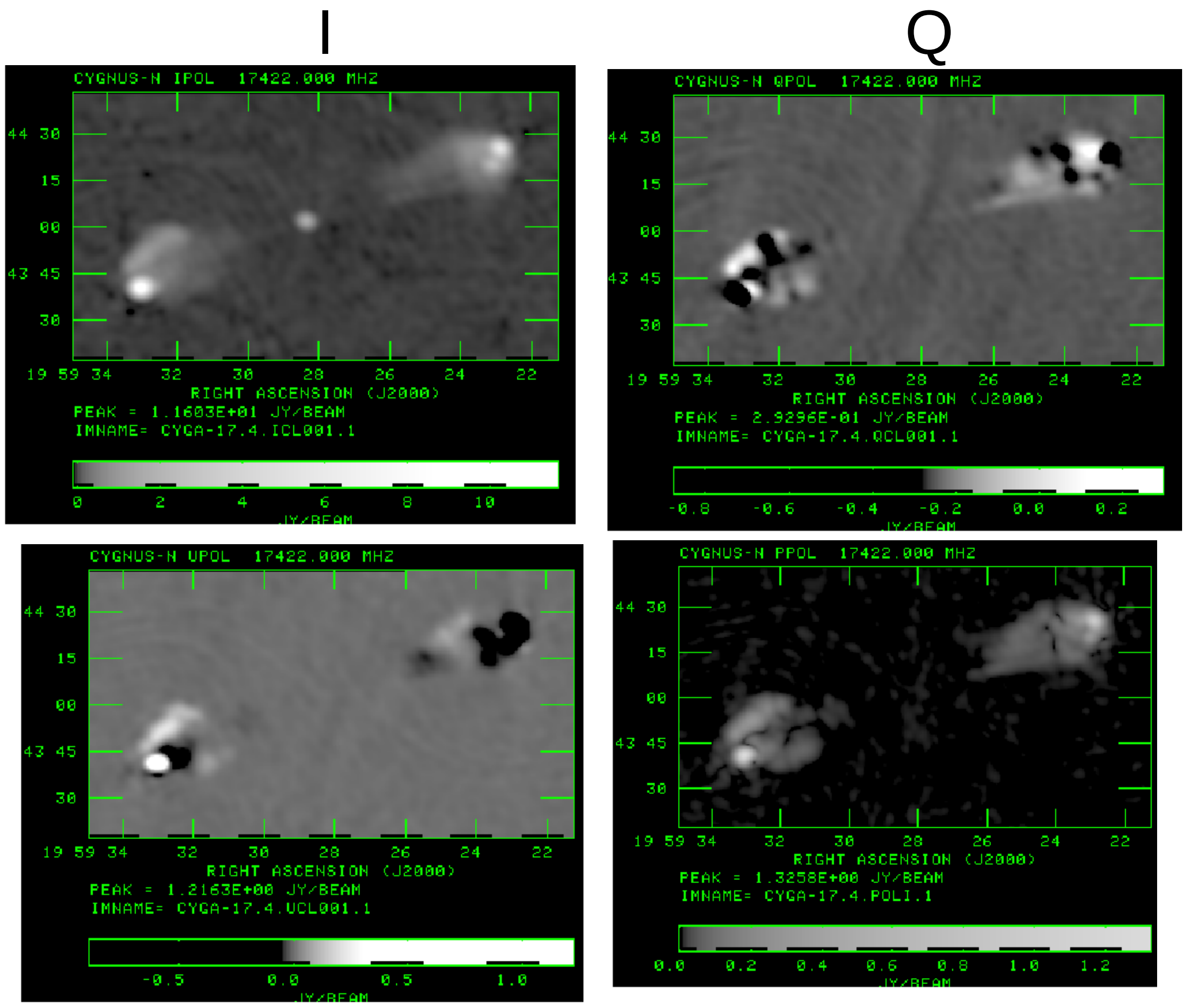


Mars at 8.49 GHz

5 arcsec

Cygnus A at 17.2 GHz

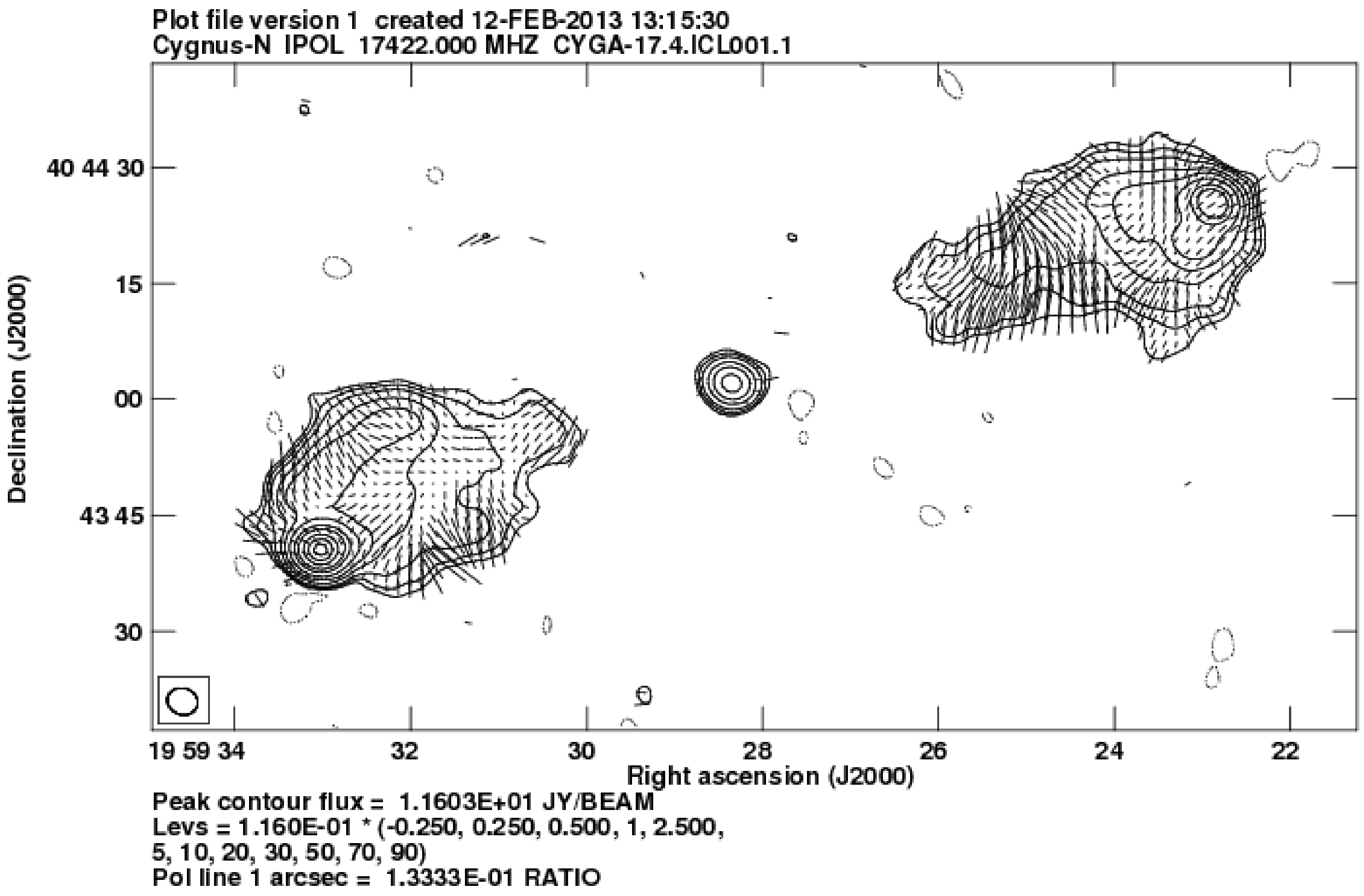
- Cygnus A is a luminous radio galaxy, one of the strongest sources in the sky.
- It is highly polarized at high (> 5 GHz) frequencies.
- Shown here are some D-configuration data, at 17.2 GHz.



U

$$P = \sqrt{Q^2 + U^2}$$

A more traditional representation.



Not as Simple as it Seems …

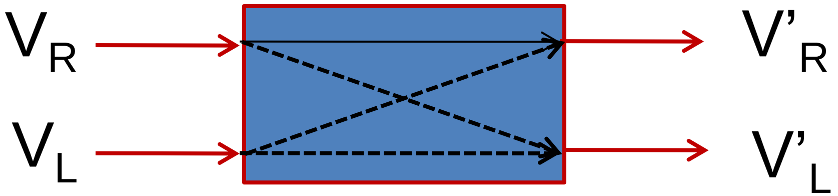
- From this, you may be led to think this is easy.
  - Add polarizers, cross-multiply, calibrate, image, and done!
- Sadly, the reality is a bit more complex.
  - The polarizers are not perfect.
  - Real electronics 'leak' signals from one polarization to the other.
- And – if this isn't enough …
  - Real antennas are differentially spatially polarized – their polarizations are a function of angle on the sky.
- Bottom line here is that the antenna output labelled (say) 'R' is not wholly 'R', but contains a little bit of 'L', and vice versa.
- This is an issue of design, and of the software needed to correct for the contamination.

Managing Impure Polarizers

- Sadly, despite the best efforts of our skilled engineers, antennas are not purely polarized.
- This means that the port labelled 'R', has a bit of 'L' in it, and vice versa.
- The analysis of such systems is commonly done via 'Jones Matrices'. The concept is simple
- Time to introduce Jones matrices…

Jones Matrix Algebra

- The analysis of how a real interferometer, comprising real antennas and real electronics, is greatly facilitated through use of Jones matrices.
- In this, we break up our general system into a series of 4-port components, each of which is presumed to be linear.
- Each component is represented like this:



- And write:

$$\begin{pmatrix} V'_{R'} \\ V'_{L} \end{pmatrix} = \begin{pmatrix} 1 & D_{LR} \\ D_{RL} & 1 \end{pmatrix} \begin{pmatrix} V_R \\ V_L \end{pmatrix}$$

- Or, in shorthand **V' = JV**
- The entire signal chain can be analyzed as a series of Jones matrices, one for each component, with a 'system' Jones matrix equal to the matrix product of each.

The Response Vectors

- If we define a column vector describing the four observed correlator outputs:

$$\mathbf{R}' = \begin{pmatrix} R'_{R1R2} \\ R'_{R1L2} \\ R'_{L1R2} \\ R'_{L1L2} \end{pmatrix}$$

- And we define another column vector descripting the four correct, true correlator outputs:

$$\mathbf{R} = \begin{pmatrix} R_{R1R2} \\ R_{R1L2} \\ R_{L1R2} \\ R_{L1L2} \end{pmatrix}$$

- These are the four quantities we need to form the Stokes visibilities.
- How are these vectors related?

The outer product (Kronecker product)

- These two column vectors are related by the outer product (Kronecker product) of the two Jones' matrices describing the antenna responses:

$$R'_{mn} = (J_m \otimes J_n^*) R_{mn}$$

- Which can be multiplied* out to show:

$$\begin{pmatrix} R'_{R1R2} \\ R'_{R1L2} \\ R'_{L1R2} \\ R'_{L1L2} \end{pmatrix} = \begin{pmatrix} 1 & D^*_{LR2} & D_{LR1} & D_{LR1}D^*_{LR2} \\ D^*_{RL2} & 1 & D_{LR1}D^*_{RL2} & D_{LR1} \\ D_{RL1} & D_{RL1}D^*_{LR2} & 1 & D^*_{LR2} \\ D_{RL1}D^*_{LR2} & D_{RL1} & D^*_{RL2} & 1 \end{pmatrix} \begin{pmatrix} R_{R1R2} \\ R_{R1L2} \\ R_{L1R2} \\ R_{L1L2} \end{pmatrix}$$

- The 4 x 4 matrix is often called the 'Mueller Matrix'. I prefer 'PMM' – Polarization Mixing Matrix.
- This expression can be inverted to recover the correct correlations, from which the Stokes visibilities are derived.

*See Thompson, Moran and Swenson, Chapter 4, for details

The Generalized Formulation (circular basis)

- But these ‘corrected’ (true) visibilities are related to the Stokes visibilities by the relations I have shown, so we can write:

$$\begin{pmatrix} R'_{R1R2} \\ R'_{R1L2} \\ R'_{L1R2} \\ R'_{L1L2} \end{pmatrix} = \begin{pmatrix} 1 & D^*_{LR2} & D_{LR1} & D_{LR1}D^*_{LR2} \\ D^*_{RL2} & 1 & D_{LR1}D^*_{RL2} & D_{LR1} \\ D_{RL1} & D_{RL1}D^*_{LR2} & 1 & D^*_{LR2} \\ D_{RL1}D^*_{LR2} & D_{RL1} & D^*_{RL2} & 1 \end{pmatrix} \begin{pmatrix} (\mathcal{I}+\mathcal{V})/2 \\ e^{-2i\Psi_P}(\mathcal{Q}+i\mathcal{U})/2 \\ e^{2i\Psi_P}(\mathcal{Q}-i\mathcal{U})/2 \\ (\mathcal{I}-\mathcal{V})/2 \end{pmatrix}$$

- The D’s are (unimaginatively) called the ‘D-terms’, and describe the amplitude and phase of the cross-over signals from R to L, and L to R.
- **Main Point:** The effect of an impure polarizer is to couple all four of the Stokes visibilities to all four cross-products.
- If the ‘D’ terms are known in advance, this matrix equation can be easily inverted, to solve for the Stokes visibilities in terms of the measured Rs, and the known Ds.

Approximations for Good Polarizers

- Considerable simplification occurs if the polarizers are good.
- Typically, circular polarizers have |D| < 0.05.
- If |D|<<1, we can then ignore D*D products.
- Furthermore, if $|\mathcal{Q}|$ and $|\mathcal{U}|$ << $|\mathcal{I}|$, we can ignore products between them and the Ds. (OK for point sources --- not always ok for extended sources).
- And $\mathcal{V}$ can be safely assumed to be zero.
- These approximations then allow us write:

$$R_{R1R2} = \mathcal{I}/2$$

$$R_{L1L2} = \mathcal{I}/2$$

$$R_{R1L2} = \left[\left(D_{R1} + D^{*}_{L2}\right)\mathcal{I} + e^{-2i\Psi_P}\left(\mathcal{Q} + i\mathcal{U}\right)\right]/2$$

$$R_{L1R2} = \left[\left(D_{L1} + D^{*}_{R2}\right)\mathcal{I} + e^{2i\Psi_P}\left(\mathcal{Q} - i\mathcal{U}\right)\right]/2$$

‘Nearly’ Circular Feeds (small D approximation)

- We get:

$$R_{R1R2} = \mathcal{I}/2$$
$$R_{L1L2} = \mathcal{I}/2$$
$$R_{R1L2} = \left[\left(D_{R1} + D^*_{L2}\right)\mathcal{I} + e^{-2i\Psi_P}\left(\mathcal{Q} + i\mathcal{U}\right)\right]/2$$
$$R_{L1R2} = \left[\left(D_{L1} + D^*_{R2}\right)\mathcal{I} + e^{2i\Psi_P}\left(\mathcal{Q} - i\mathcal{U}\right)\right]/2$$

Contamination   Desired

- The cross-hand responses are contaminated by a term proportional to ‘$\mathcal{I}$’.
- |D| ~ 0.05 ~ $|\mathcal{Q}|/|\mathcal{I}|$ => the two terms are of comparable magnitude.
- To recover the linear polarization, we must determine these D-terms, and remove their contribution.

Nearly Perfectly Linear Feeds

- In this case, assume that the ellipticity is very small ($\chi \ll 1$), and that the two feeds ('dipoles') are nearly perfectly orthogonal.
- We then define a different set of D-terms:

$$D_V = \varphi_V - i\chi_V$$

$$D_H = -\varphi_H + i\chi_H$$

- The angles φ_V and φ_H are the angular offsets from the exact horizontal and vertical orientations, w.r.t. the antenna.

$$R_{V1V2} = (\mathcal{I} + \mathcal{Q}\cos 2\Psi_P + \mathcal{U}\sin 2\Psi_P)/2$$

$$R_{H1H2} = (\mathcal{I} - \mathcal{Q}\cos 2\Psi_P - \mathcal{U}\sin 2\Psi_P)/2$$

$$R_{V1H2} = [\mathcal{I}(D_{V1} + D^*_{H2}) - \mathcal{Q}\sin 2\Psi_P + \mathcal{U}\cos 2\Psi_P + i\mathcal{V}]/2$$

$$R_{H1V2} = [\mathcal{I}(D_{H1} + D^*_{V2}) - \mathcal{Q}\sin 2\Psi_P + \mathcal{U}\cos 2\Psi_P - i\mathcal{V}]/2$$

Always, the 'linear' D terms are << 'circular' D terms.

Measuring Cross-Polarization Terms

- Correction of the X-hand response for the 'leakage' is important, since the D-term amplitude is comparable to the source fractional polarization.
- There are two standard ways to proceed:
  1. Observe a calibrator source of known polarization (preferably zero!)
  2. Observe a calibrator of unknown polarization over an extended period with alt-az antennas.
- **Case 1: Calibrator source known to have zero polarization.**

$$R_{V1V2} = \mathcal{J}/2$$
$$R_{H1H2} = \mathcal{J}/2$$
$$R_{V1H2} = \mathcal{J}\left(D_{V1} + D^*_{H2}\right)/2$$
$$R_{H1V2} = \mathcal{J}\left(D_{H1} + D^*_{V2}\right)/2$$

- Then a single observation should suffice to measure the leakage terms.
- The procedure is the same for both linear and circular systems.

NRAO

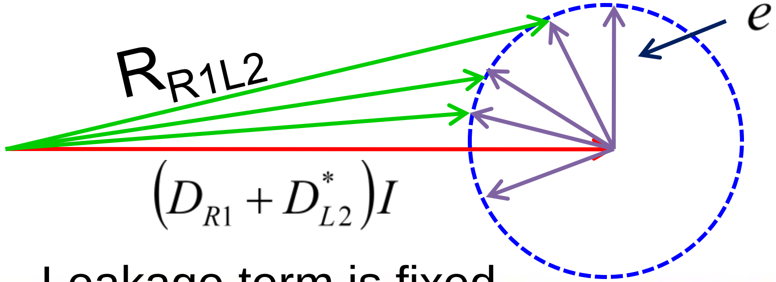
Determining Source and Antenna Polarizations

Case 2: Calibrator with significant (or unknown) polarization.

- You can determine both the (relative) D terms and the calibrator polarizations for an alt-az antenna by observing over a wide range of parallactic angle. (Conway and Kronberg first used this method.)

$$R_{L1R2} = \left[\left(D_{L1} + D^*_{R2}\right)\mathcal{I} + e^{2i\Psi_P}\left(\mathcal{Q} - i\mathcal{U}\right)\right]/2$$
$$R_{R1L2} = \left[\left(D_{R1} + D^*_{L2}\right)\mathcal{I} + e^{-2i\Psi_P}\left(\mathcal{Q} + i\mathcal{U}\right)\right]/2$$

- As time passes, Ψ_P changes in a known way.
- The source polarization term then rotates w.r.t. the antenna leakage term, allowing a separation.



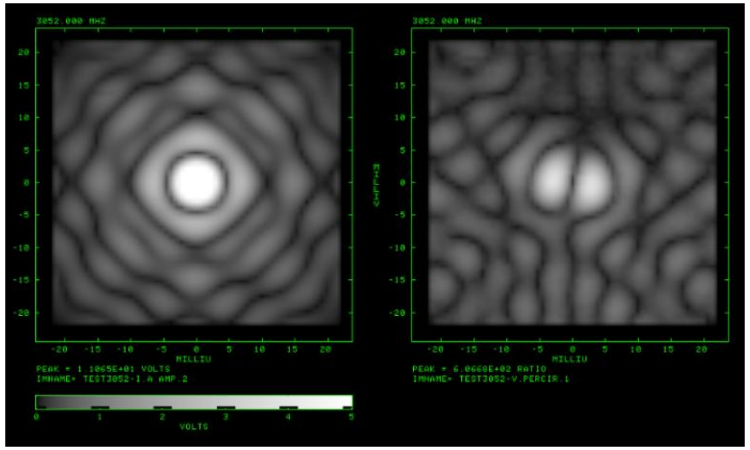
Source polarization rotates with parallactic angle

Leakage term is fixed

VLA's Polarized Beams at 3 GHz.

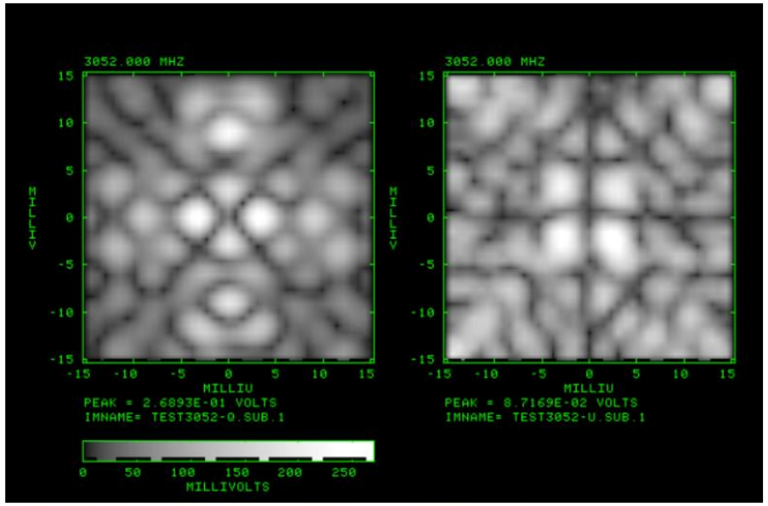
- The VLA's primary antenna response is significantly polarized.
- This is due primarily to asymmetries in the optical design.
- V polarization due to offset of the feed from axis of symmetry.
- Q, U polarizations due to parabolic reflector.
- These antenna-imposed signals must be removed from data to enable accurate wide-field astronomical polarimetry.

I



V

Q



U

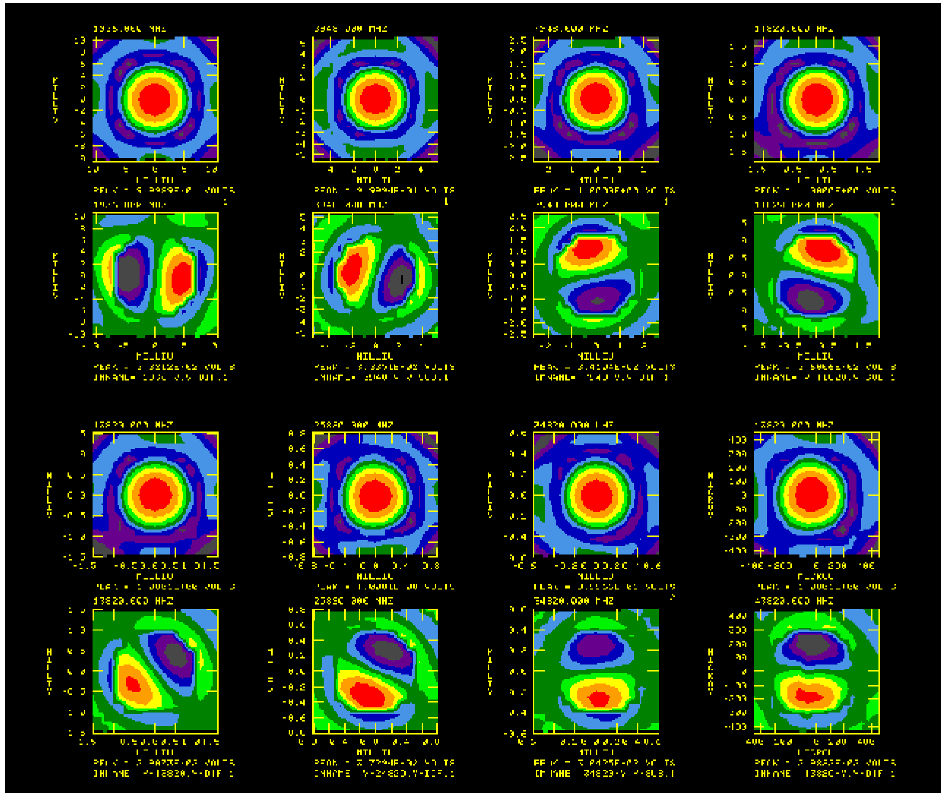
NRAO

VLA's I and V beams – all 8 bands

- I and V beam patterns for all eight JVLA bands.
- I beams (scaled) are all very similar.
- V beams rotate according to the position angle of the offset feed.
- V>0 => Red = RCP
- V< 0 => Purple = LCP
- Correction for beam polarization is difficult – done by 'A-Projection' in CASA.

L S C X

1.5 GHz 3.0 GHz 6 GHz 10 GHz



I

V

I

V

U K A Q

15 GHz 23 GHz 34 GHz 45 GHz

Summary Overview

- Interferometric polarimetry is based on the same principles as the single-polarization discussion held up until now.
- The principles are simple: The four combinations between the two polarized outputs of two antennas can be combined to produce four visibilities.
- These four visibilities, when transformed, give us the four Stokes images I, Q, U, V
- We utilize Stokes parameters because they all have the same units of brightness – easier to visualize to handle by the methods discussed at this summer school
- If the antennas produce pure orthogonal decompositions, then the combinations are simple and (even!) intuitive.
- Impure antennas do create complications in the analysis, but the approximate relations shown here are sufficient in nearly all cases.