Fundamentals of Radio Interferometry
1: The Basics

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Topics

- **Introduction:**
  - Motivation: Mapping the Sky
  - Why Interferometry?
- **The Basic Interferometer**
  - Simplifying Assumptions
  - 'Fringe' patterns
  - Response to a Point Source
    - Sine and Cosine Fringes
  - Response to Extended Emission
  - The Complex Correlator and Complex Visibilities
  - Illustrations to aid visualization
- **Important Note:**
  - The concepts I'll be speaking about are not difficult – but they are unfamiliar to beginners.

Our Goal: Mapping the Sky Brightness

- In astronomy, we wish to know the angular distribution of the emission (brightness, or specific intensity) of distant radiating sources.
  - This can be a function of frequency, polarization, and time.
- 'Angular Distribution' means we are interested in the **brightness distribution** of the emission.
- And this requires **resolving** the emission – utilizing an instrument of sufficient angular resolution.
- Because our targets are so far away, the emission is extremely weak, and of very small angular size.
  - Power from strongest radio source (Cygnus A) collected over the entire Earth in 1 GHz bandwidth ~1 watt!
  - 1 arcsecond = angular width of a US quarter at ~3 miles distance!
- Early (1950s) surveys of the radio sky employed single dishes.
- Nowadays, most (but not all!) observations are done with interferometers.

Why Interferometry?

- It's due to **Diffraction** – a consequence of the wave nature of light.
- Radio telescopes coherently sum electric fields over an aperture of size D. For this, diffraction theory applies –
- The angular resolution is: $\theta_{rad} \approx \lambda / D$

  Or, in practical units $\theta_{\rm arcsec} \approx 2\,\lambda_{\rm cm} / D_{\rm km}$
- To obtain 1 arcsecond resolution at a wavelength of 21 cm, we require an aperture of ~42 km – not feasible.
- The largest single, fully-steerable apertures are the 100-m antennas in Bonn, and Green Bank. Nowhere big enough.
- Can we synthesize a larger aperture with separated antennas?
- The technique of synthesizing a larger effective aperture through combinations of separated pairs of antennas is called 'aperture synthesis'.

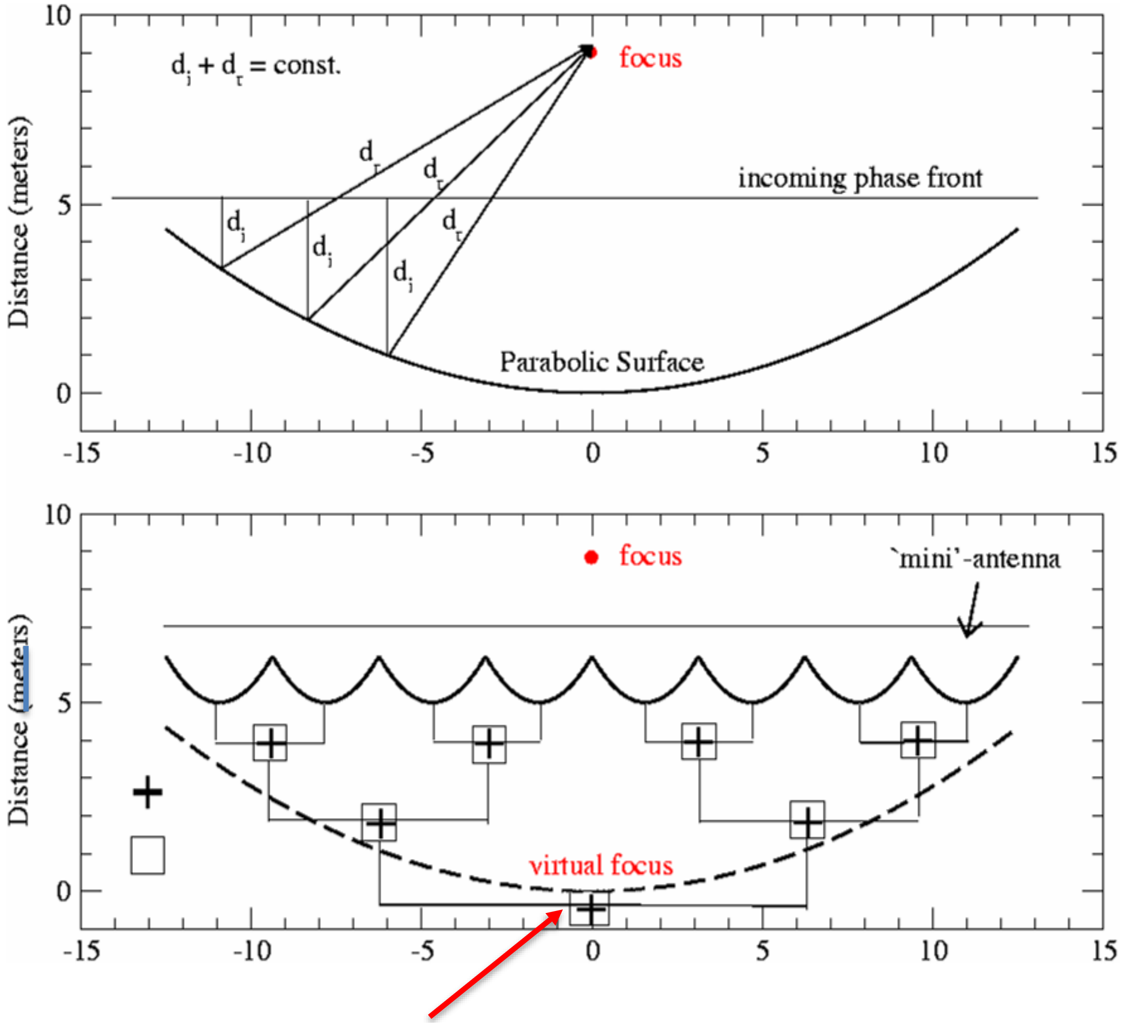
Interferometry – Basic Concepts

- A parabolic dish coherently sums EM fields at the focus.

- The same sum can be obtained by adding voltages from individual elements with a wired network.
    - Note – they need not be adjacent.

- This is the basic concept of interferometry – coherent summation of voltages from separated antennas.

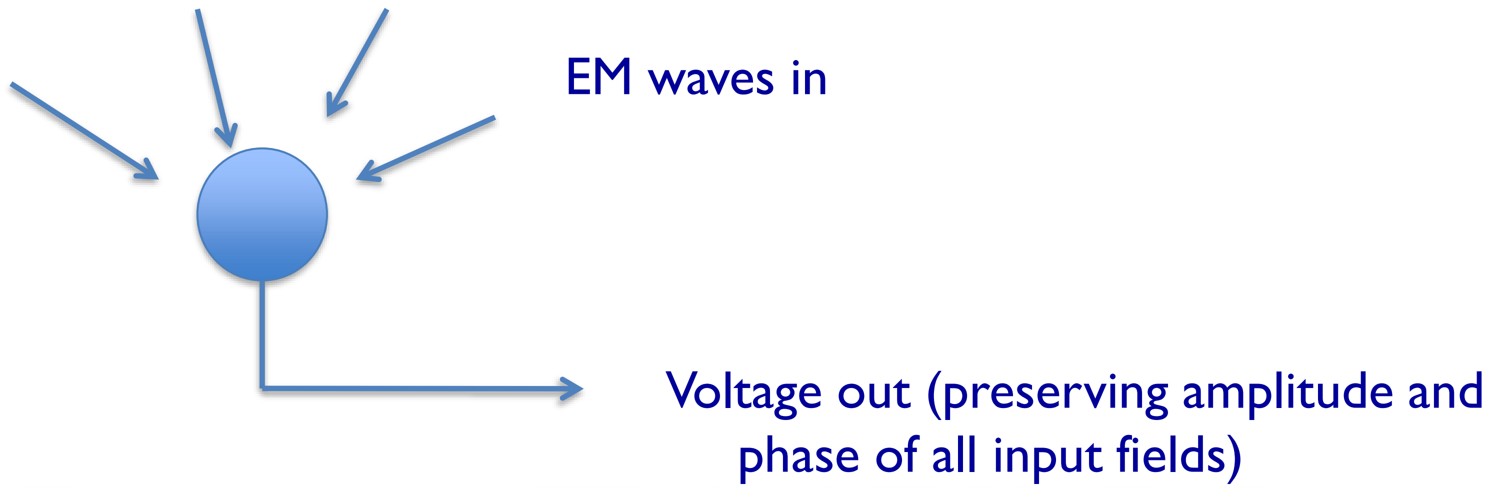
- Aperture Synthesis is an extension of this concept.



Signals summed here equivalent to those collected at prime focus

The Role of the Sensor (aka Antenna)

- Coherent interferometry is based on the ability to correlate the electric fields measured at spatially separated locations.
- To do this (without mirrors) requires conversion of the electric field E(**r**,ν,t) at some place (**r**) to a voltage V(ν,t) which can be conveyed to a central location for processing – without losing the phase information.
- For our purpose, the sensor (a.k.a. 'antenna') is simply a device which senses the electric fields at some place and converts this to a voltage which faithfully retains the amplitudes and phases of the electric fields.

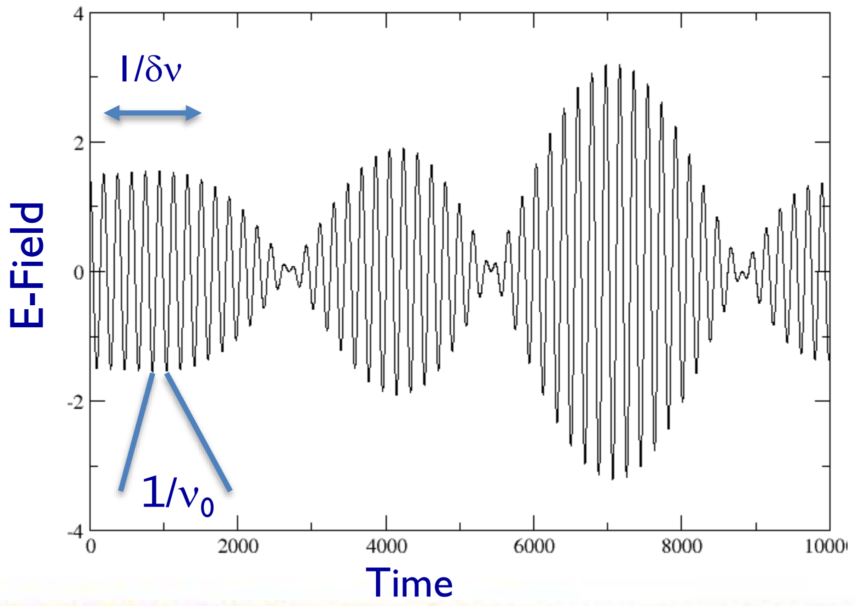


Quasi-Monochromatic Radiation

- Mathematical analysis of wideband noise is difficult.
- Analysis is simple if the fields are monochromatic – a single frequency of zero bandwidth – but such signals cannot exist in nature.
- So we consider instead 'quasi-monochromatic' radiation, where the bandwidth $\delta\nu$ is very small, but not zero.
- Then, for a time $dt \sim 1/\delta\nu$, the electric fields will be sinusoidal, with unchanging amplitude E and constant phase ϕ, described by

$$E_\upsilon(t) = E\cos(2\pi\upsilon t + \phi)$$

The figure shows an 'oscilloscope' trace of a narrow bandwidth noise signal. The period of the wave is $T_1 = 1/\nu_0$, the duration over which the signal is closely sinusoidal is $T_2 \sim 1/\delta\nu$. There are $N \sim T_2/T_1 \sim \nu_0/\delta\nu$ oscillations in a 'wave packet'.



Simplifying Assumptions

- We now consider the most basic interferometer, and seek a relation between the characteristics of the product of the voltages from two separated antennas and the distribution of the brightness of the originating source emission.
- To establish the basic relations, the following simplifications are introduced:
  - Isotropic sensors fixed in space – no rotation or motion
  - Quasi-monochromatic radiation
  - Point source in the far-field ($D >> B^2/\lambda$, $\theta << \lambda/B$)
  - No frequency conversions (an 'RF interferometer')
  - Single polarization
  - Propagation in vacuum, without distortions (no ionosphere, atmosphere …)
  - Idealized electronics (perfectly linear, perfectly uniform in frequency and direction, perfectly identical for both elements, no added noise.
- I will later relax most of these restrictions.

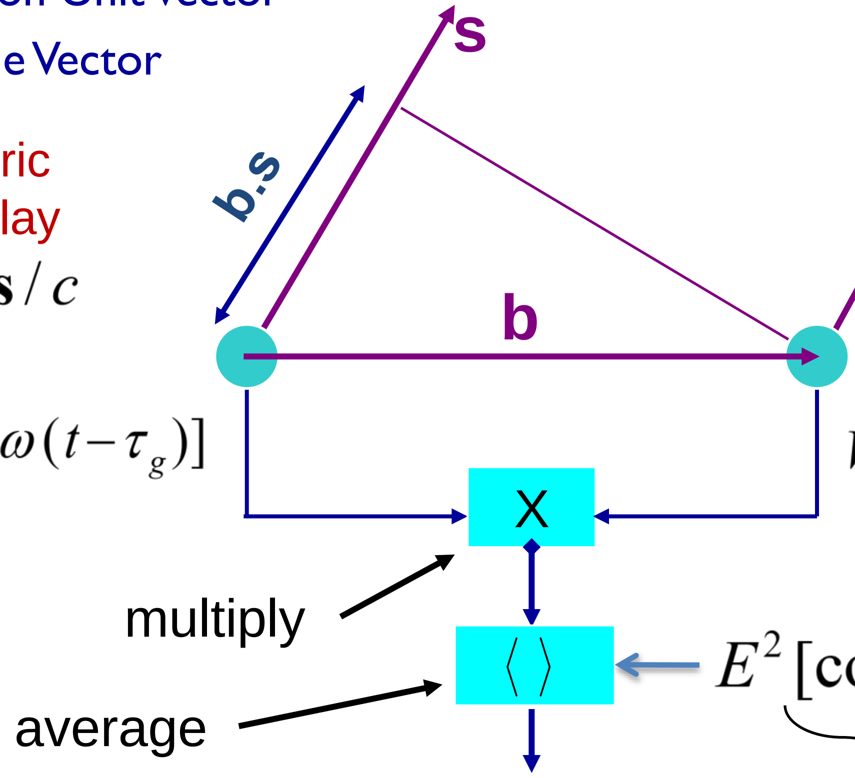
The Stationary, Quasi-Monochromatic Radio-Frequency Interferometer

s = Direction Unit Vector

b = Baseline Vector

Geometric Time Delay

$$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$$



$$V_1 = E\cos[\omega(t-\tau_g)]$$

$$V_2 = E\cos(\omega t)$$

The path lengths from sensors to multiplier are assumed equal!

$$E^2[\cos(\omega\tau_g) + \cos(2\omega t - \omega\tau_g)]$$

Unchanging — Rapidly varying

$$R_C = P\cos(\omega\tau_g)$$

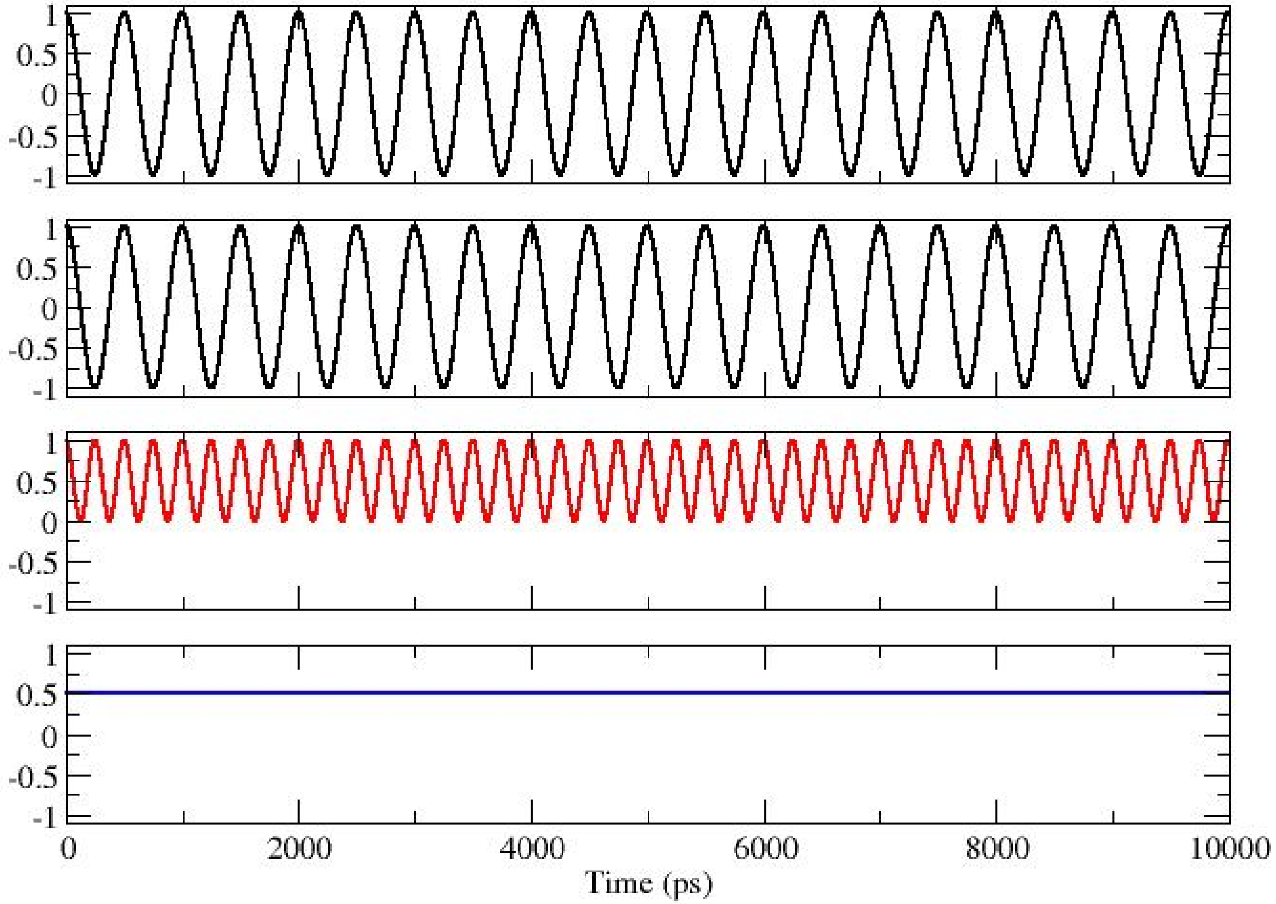
Where I replaced E^2 with P (power), and ignored factors of 2

Pictorial Example: Signals In Phase

2 GHz Frequency, with voltages in phase:

b.s = nλ, or $\tau_g = n/\nu$, so $\cos(\omega\tau) = \cos(2\pi n) = 1$

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average



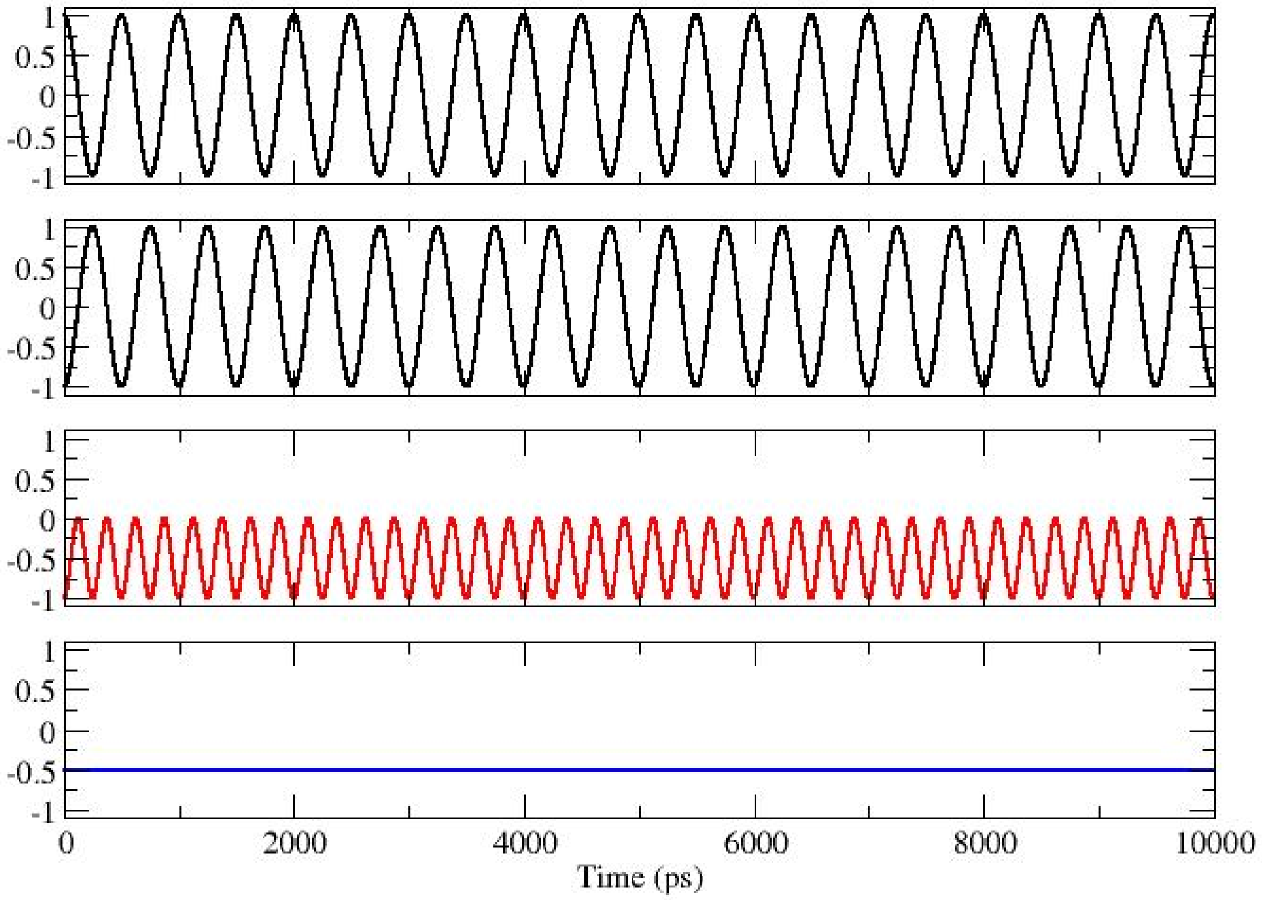
Positive constant

Pictorial Example: Signals out of Phase

2 GHz Frequency, with voltages out of phase:

b.s=(n +/- ½)λ τ_g = (n +/- ½)/ν so cos($\omega\tau+\pi$) = cos(π(2n+1)) = -1

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average



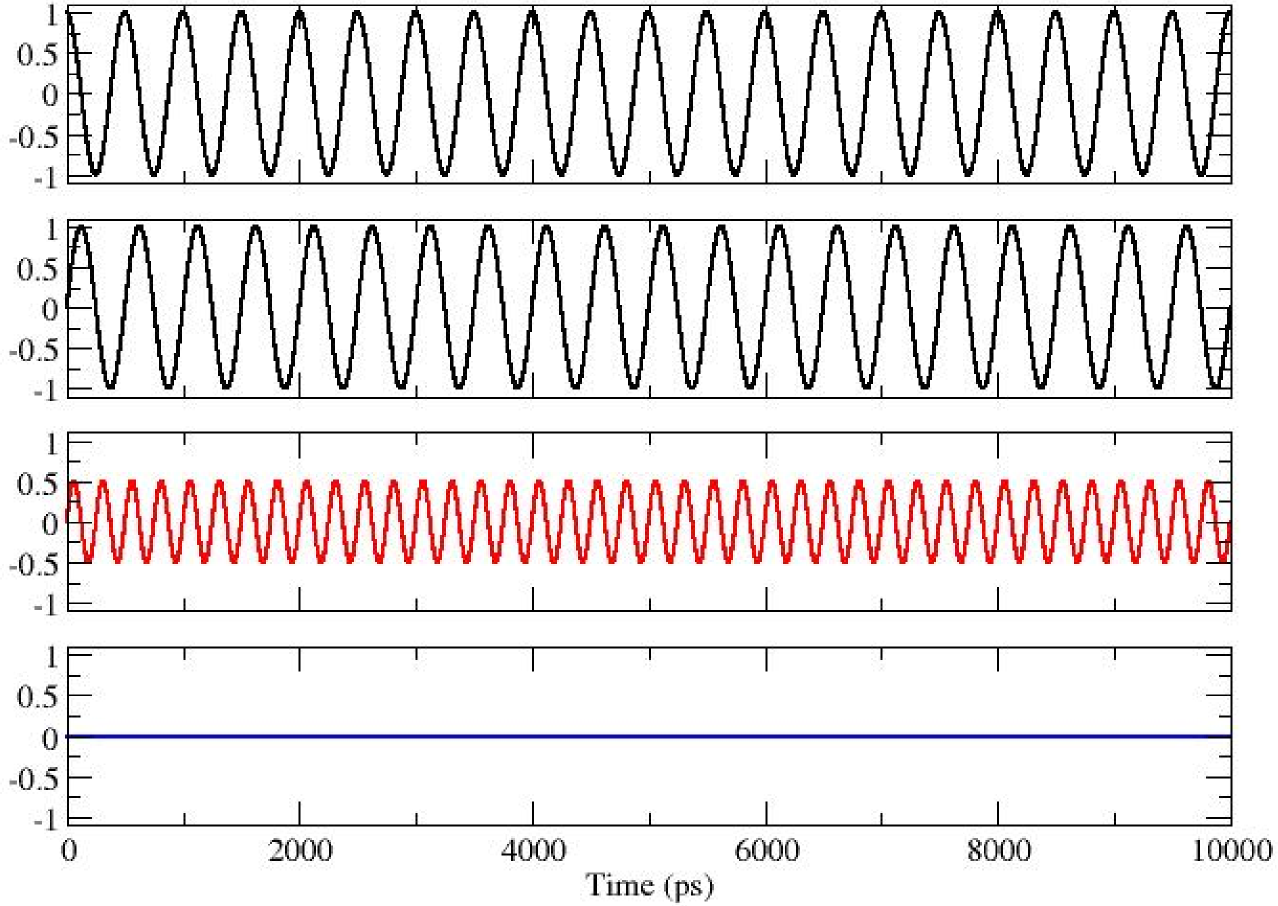
Negative constant

Pictorial Example: Signals in Quad Phase

2 GHz Frequency, with voltages in quadrature phase:

b.s=(n +/- ¼)λ, τ_g = (n +/- ¼)/ν so $\cos(\omega\tau+\pi/2) = \cos(\pi(2n+1/2)) = 0$

- Antenna 1 Voltage
- Antenna 2 Voltage
- Product Voltage
- Average



Zero constant

General Comments on this Product

- The averaged product R_C is dependent on the received power, $P = E^2/2$ and geometric delay, τ_g, and hence on the baseline orientation and source direction. From slide 9, we have:

$$R_C = P\cos(\omega\tau_g) = P\cos\left(2\pi\frac{\mathbf{b}\cdot\mathbf{s}}{\lambda}\right)$$

- Note that R_C is not a function of:
  - The time of the observation -- provided the source itself is not variable!
  - The location of the baseline -- provided the emission is in the far-field.
  - The actual phase of the incoming signal – the distance to the source -- provided the source is in the far-field.

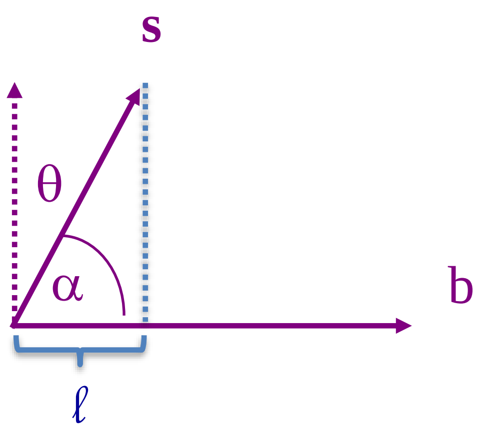
- The strength of the product is also dependent on practical factors such as the antenna sizes, electronic gains, bandwidth and time averaging – but these factors or ignored here (and can be calibrated for).

Define the Direction Cosine

- Consider a single baseline, and define the x-axis to extend along this baseline.

- Write $\mathbf{b} = u\hat{\mathbf{x}}$ where u =|**b**|/λ is the baseline length in wavelengths, and $\hat{\mathbf{x}}$ = the unit direction vector.
- Define the `direction cosine' as:
- $$l = \hat{\mathbf{x}} \bullet \mathbf{s} = \cos\alpha = \sin\theta$$
- Then:

$$\frac{\mathbf{b} \bullet \mathbf{s}}{\lambda} = u\cos\alpha = u\sin\theta = ul$$



- So the interferometer response is:

$$R_C = P\cos(\omega\tau_g) = P\cos\left(2\pi\frac{\mathbf{b}\cdot\mathbf{s}}{\lambda}\right) = P\cos(2\pi ul)$$

Two Illustrative Examples

- Consider the response R_c, as a function of angle, for two different baselines with u = b/λ = 10, and u = 25 wavelengths.
- Since

$$R_C = \cos(2\pi ul)$$

- We have, for u = 10:

$$R_C = \cos(20\pi\, l)$$

- And, for u = 25:

$$R_C = \cos(50\pi\, l)$$

- These are simple functions of angle on the sky.

Remember:

u = baseline length in wavelengths

$\ell = \sin\theta$,

θ = angular offset from plane perpendicular to the baseline

Whole-Sky Response

- Top:

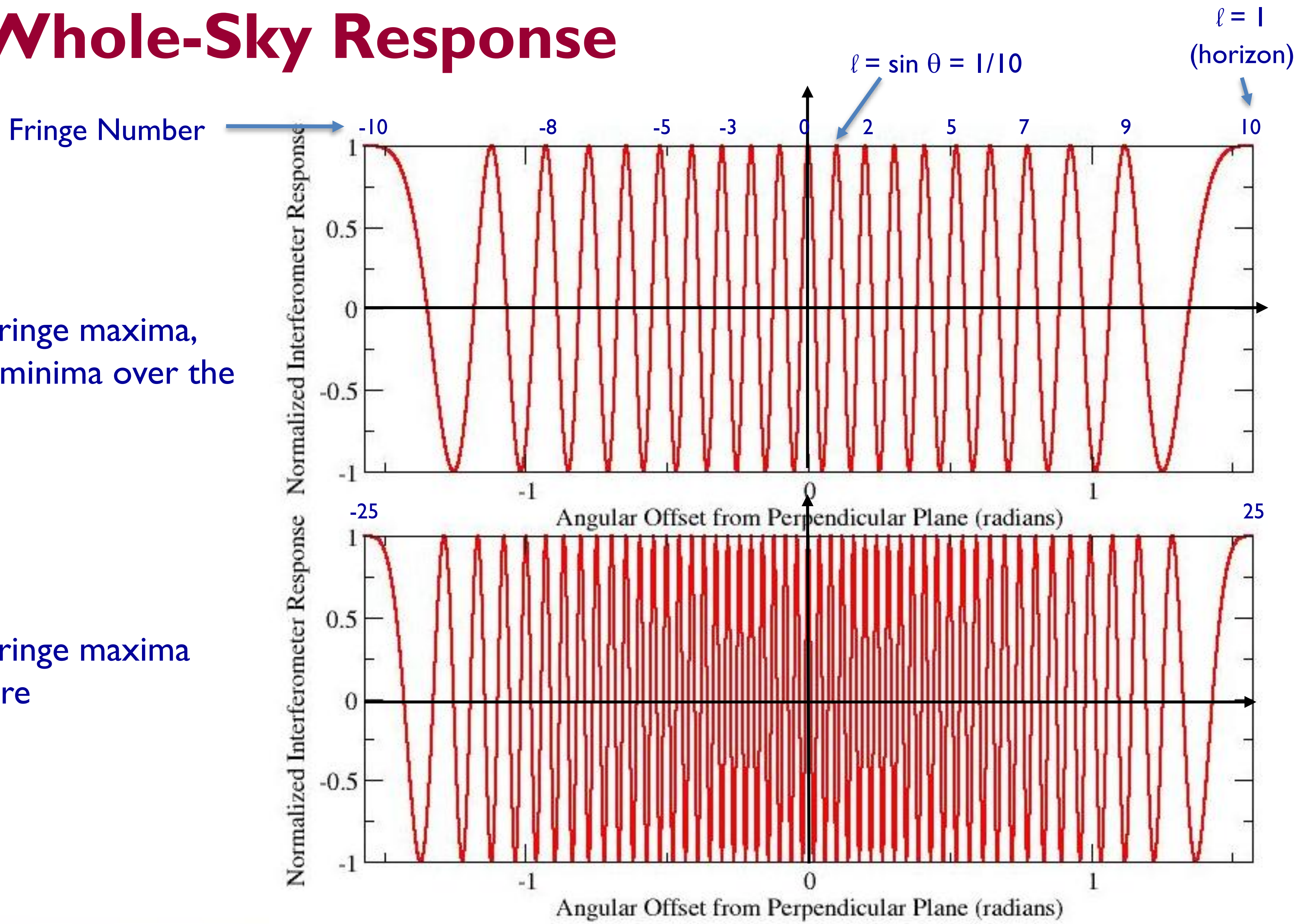
  u = 10

  There are 21 fringe maxima, and 20 fringe minima over the sphere.

- Bottom:

  u = 25

  There are 51 fringe maxima over the sphere



From an Angular Perspective

Top Panel:

The absolute value of the response for u = 10, as a function of angle.

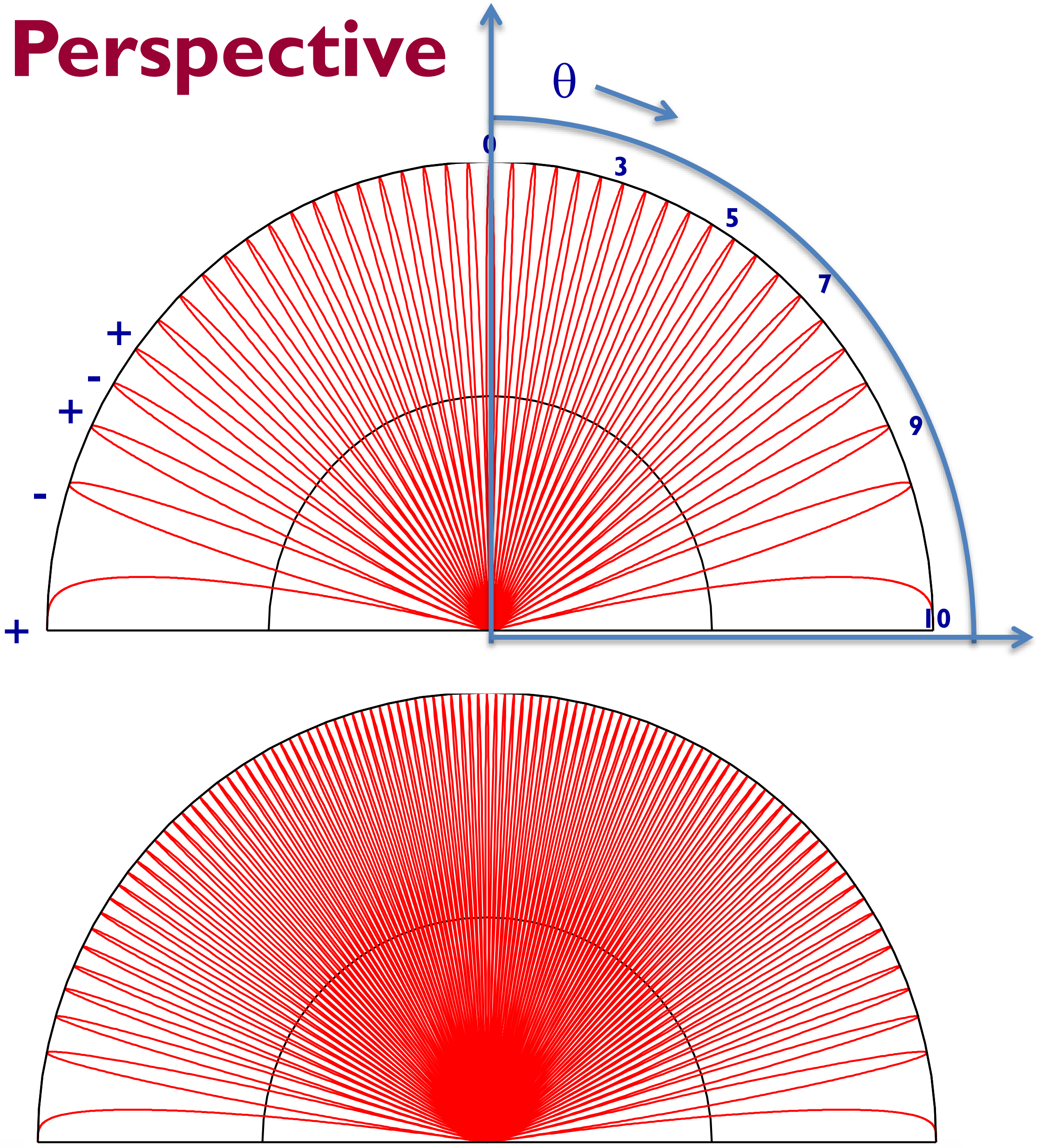
The 'lobes' of the response pattern alternate in sign.

Bottom Panel:

The same, but for u = 25.

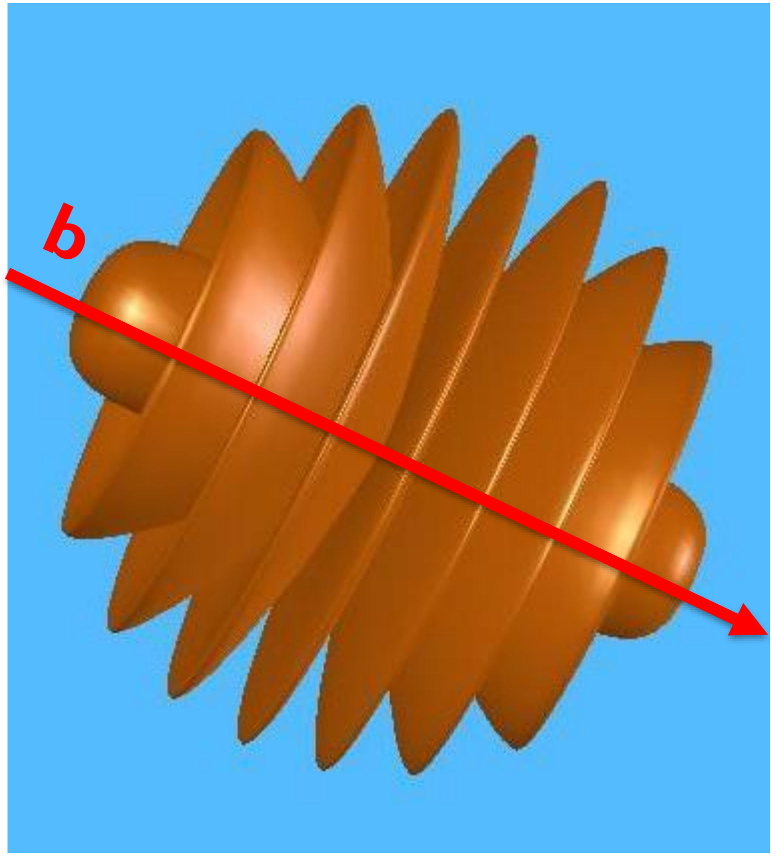
Angular separation between lobes (of the same sign) is

$\delta\theta \sim 1/u = \lambda/b$ radians.



Full Sky Pattern

- The preceding plot is a meridional cut through the hemisphere, oriented along the baseline vector **b**.
- In the three-dimensional space, the fringe pattern consists of a series of coaxial cones, oriented along the baseline vector.
- The figure is a two-dimensional representation when u = 4.
- As viewed along the baseline vector, the fringes show a 'bulls-eye' pattern – concentric circles.

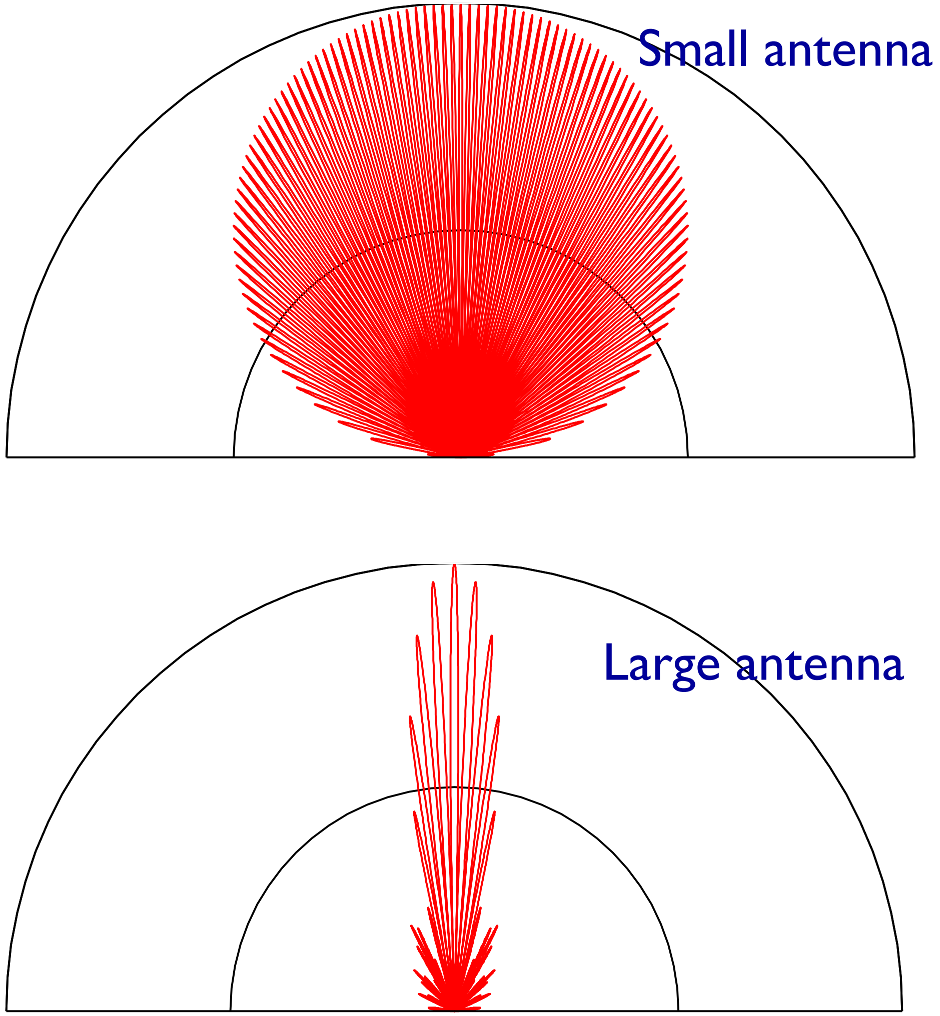


The Effect of the Sensor (aka Antenna)

- The patterns shown presume the sensor has isotropic response.
- This is a convenient assumption, but doesn't represent reality.
- Real sensors impose their own patterns, which modulate the amplitude and phase of the interferometer response.
- Large sensors have good angular resolution and high gain
  - very useful for some applications like imaging individual objects.
- Small sensors have poor angular resolution and low gain
  - useful for wide-angle surveys.
- Emphasis: The fringe pattern is a function of the baseline length (in wavelengths) and orientation, and is not affected by the antenna patterns.

The Effect of Sensor Patterns

- Sensors (or antennas) are not isotropic and have their own responses – both amplitude and phase.
- **Top Panel:** The interferometer pattern with a cos(θ)-like sensor response.
- **Bottom Panel:** A multiple-wavelength aperture antenna has a narrow beam, but also sidelobes.
- **Emphasis:** The fringe pattern is a function of the array geometry. The angular modulation is from the sensor.



From Point Sources to Extended Emission

- Everything I stated earlier holds for 'point sources'. What are these?
- Point Source: Emission from an object whose angular extent is much, much smaller than the interferometer fringe scale.
- That is: $$\theta_{rad} \ll \lambda / B$$

  where θ = angular size, λ = wavelength, B = baseline
- What happens when the source angular size is comparable to, or larger than, the fringe scale?
- Short answer: Under nearly all conditions, you just 'add up' the individual responses from the components of the extended source.

The Response from an Extended Source

- For an extended source, the voltage output from a single antenna is the spatial integration of the E-field emission over the primary beam: $V = \iint E(\mathbf{s})d\Omega$
- So the correlator response becomes $R_C = \left\langle \iint E_1(\mathbf{s})d\Omega_1 \times \iint E_2(\mathbf{s})d\Omega_2 \right\rangle$
- The averaging and integrals can be interchanged and, **providing the emission is spatially incoherent**, we get

$$R_C = \iint I_\nu(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

- The response is the sum of the spatial integration of the brightness I_ν modulated by the cosinusoidal interferometer pattern.
- This expression links what we want: the brightness on the sky, $I_\nu(\mathbf{s})$, to something we can measure - R_C, the interferometer response.
- Can we recover $I_\nu(\mathbf{s})$ from observations of R_C?

A picture is worth 1000 words …

- As stated earlier, these concepts are not difficult but are unfamiliar. We need to think in new ways, to get a deeper understanding of how all this works.
- To aid, I have generated images of interferometer fringes, of various baseline lengths and orientations.
- I then 'observe' a real source (Cygnus A, of course), to show what the interferometer actually measures.
- For all these, the 'observations' are made at 2052 MHz. The Cygnus A image is taken from real VLA data.
- To keep things simple, all simulations are done at meridian transit.

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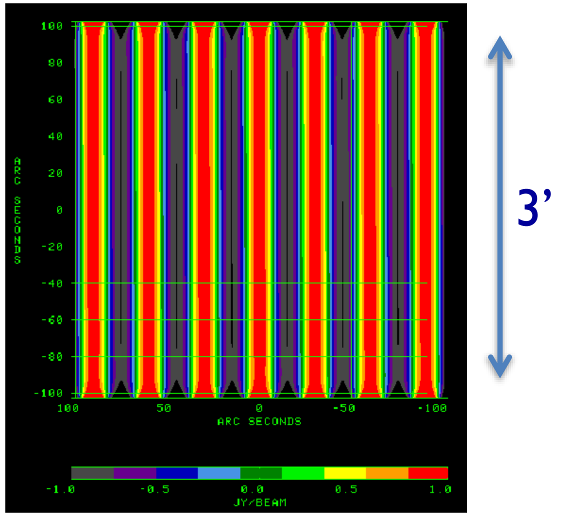
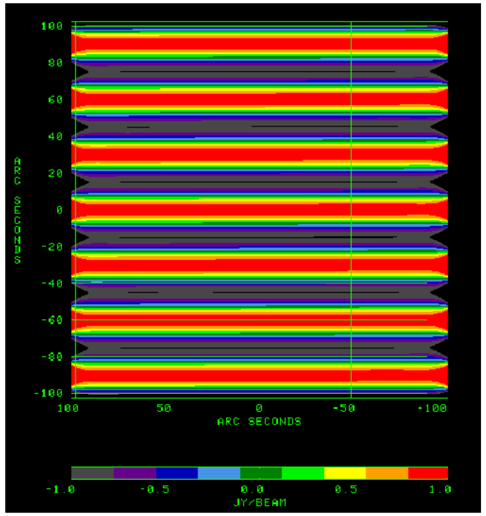
‘Real’ Fringes … 1Km Baseline at 2052 MHz

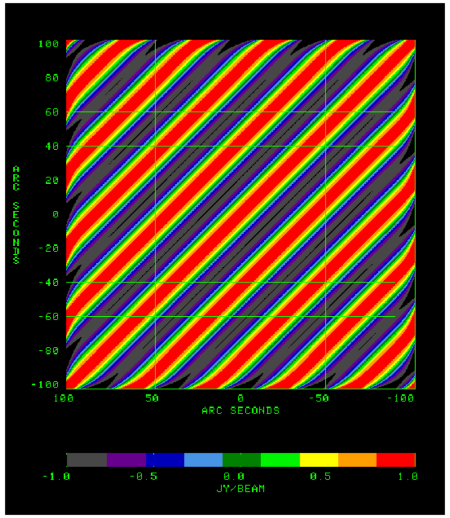
- The fringe separation given by baseline length in wavelengths, the orientation given by the orientation of the baseline.



East-West baseline makes vertical fringes



North-South baseline makes horizontal fringes



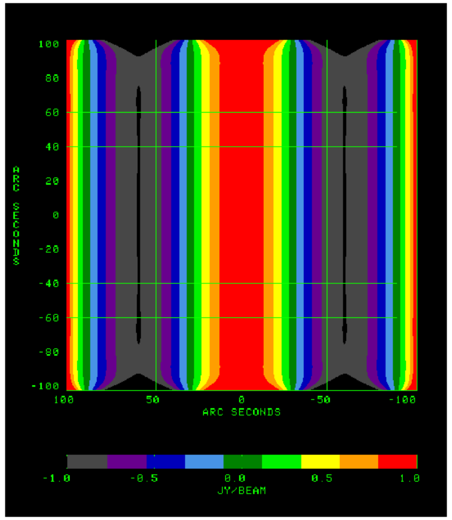
Rotated baseline makes rotated fringes

- Red = positive maximum. Black = negative maximum. Green = zero
- Fringe angular spacing given by baseline length in wavelengths:

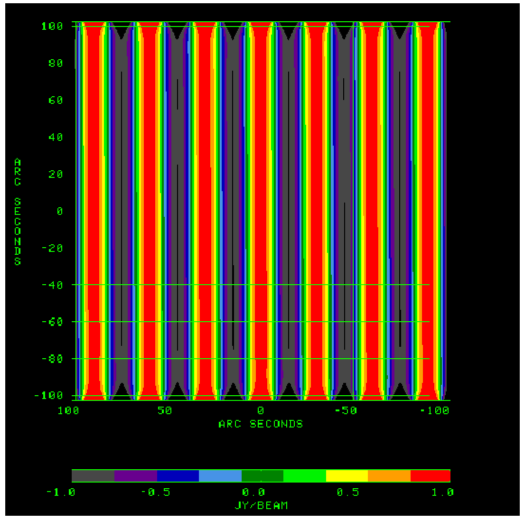
$$\Delta\theta = \lambda / B = 30.2''$$

Longer Baselines => Smaller Fringe Spacing

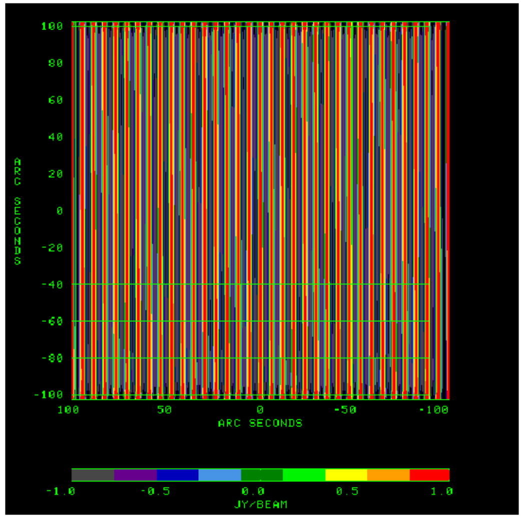
- Longer baselines make finer fringes, shorter baselines make larger fringes:



250 meter baseline
120 arcsecond fringe



1000 meter baseline
30 arcsecond fringe

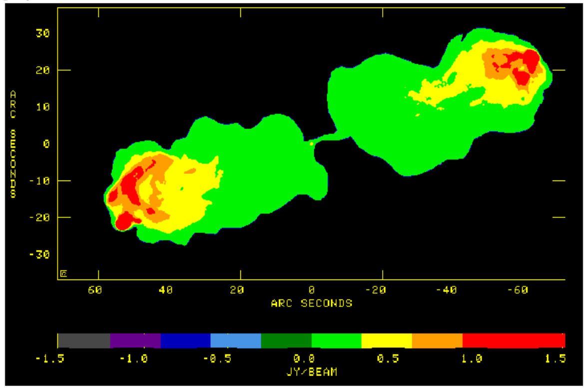


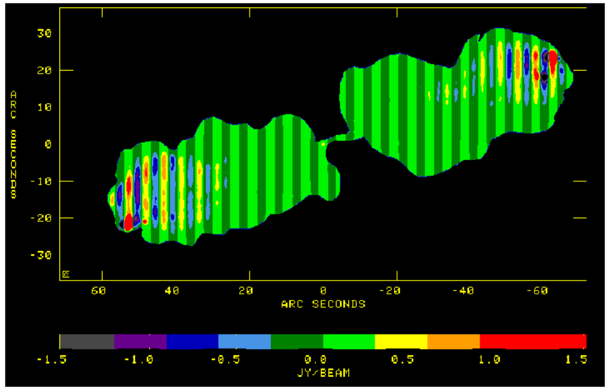
5000 meter baseline
6 arcsecond fringe

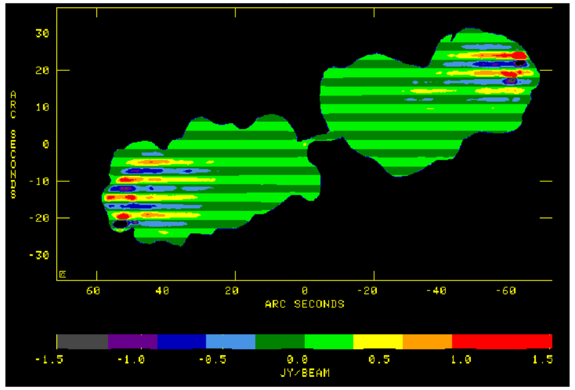
- What the interferometer measures is the integral (sum) of the product of this pattern with the actual brightness.

For a Real Source (Cygnus A = 3C405)

- Cygnus A is a powerful, nearby radio galaxy.
- The left panel shows the color-coded brightness at 2052 MHz. (2" res^{n})
- The other two panels show how 6.9 km EW and NS interferometers 'see' the source.







Zero-Spacing Image
Sum = 1029 Jy

6.3 km EW spacing
Sum = 70 Jy

6.3 km NS spacing
Sum = -19 Jy

- Dark green, blue, and purples => Negative correlation
- Light green, yellow, and reds => Positive correlation
- Don't be alarmed by the `negative flux' in the third panel.

So … What Next?

- The interferometer casts a cosinusoidal pattern on the sky, with the result that we obtain a response which is some function of the source brightness and the fringe separation and orientation.
- By definition, 'Cosine' fringes have a peak passing through the central position of the source ('phase center').
- But in fact, something is missing. 'Cosine' fringes are not sufficient to allow recovery of the sky brightness.
- To answer why …
- Time for some mathematics…. Starting with a seeming digression about odd and even functions.
- (All will be clear shortly…)

A Short Mathematics Digression – Odd and Even Functions

- Any real function, I(x,y), can be expressed as the sum of two real functions which have specific symmetries:

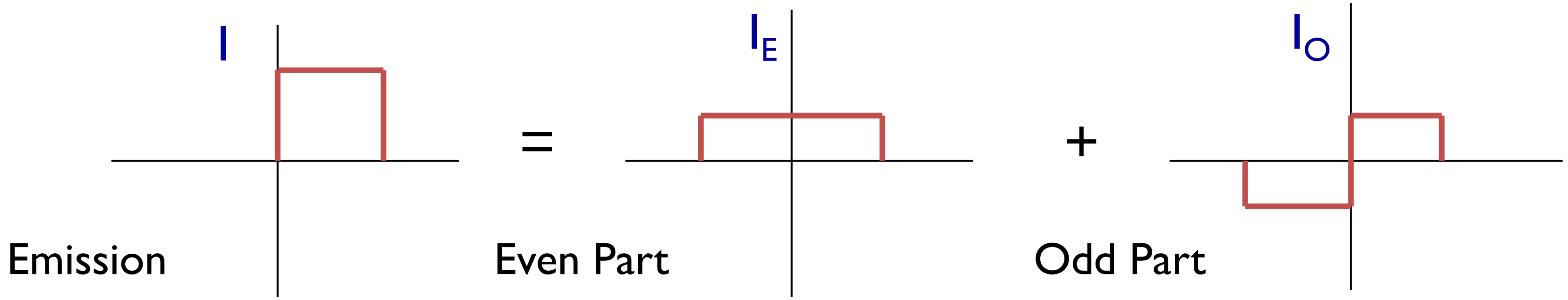
$$I(x,y) = I_E(x,y) + I_O(x,y)$$

An even part:

$$I_E(x,y) = \frac{I(x,y) + I(-x,-y)}{2} = I_E(-x,-y)$$

An odd part:

$$I_O(x,y) = \frac{I(x,y) - I(-x,-y)}{2} = -I_O(-x,-y)$$



The Cosine Correlator is Blind to Odd Structure

- Suppose that the source of emission has a component with odd symmetry, for which

$$I_o(x) = -I_o(-x)$$

- Since the cosine fringe pattern is even, the response of our interferometer to the odd brightness distribution is 0!

$$R_c = \iint I_o(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega = 0$$

- Thus, the cosine correlator response R_c:

$$R_C = \iint \left(I(\mathbf{s}) \right)\cos(2\pi\nu\,\mathbf{b}\cdot\mathbf{s}/c)d\Omega = \iint \left(I_E(\mathbf{s}) \right)\cos(2\pi\nu\,\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

is sensitive to only the even component of the source structure.

Hence, we need more information if we are to completely recover the source brightness.

Thus: More Information is Needed

- The integration of the cosine response, R_c, over the source brightness is sensitive to only the even part of the brightness:

$$R_C = \iint I(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega = \iint I_E(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

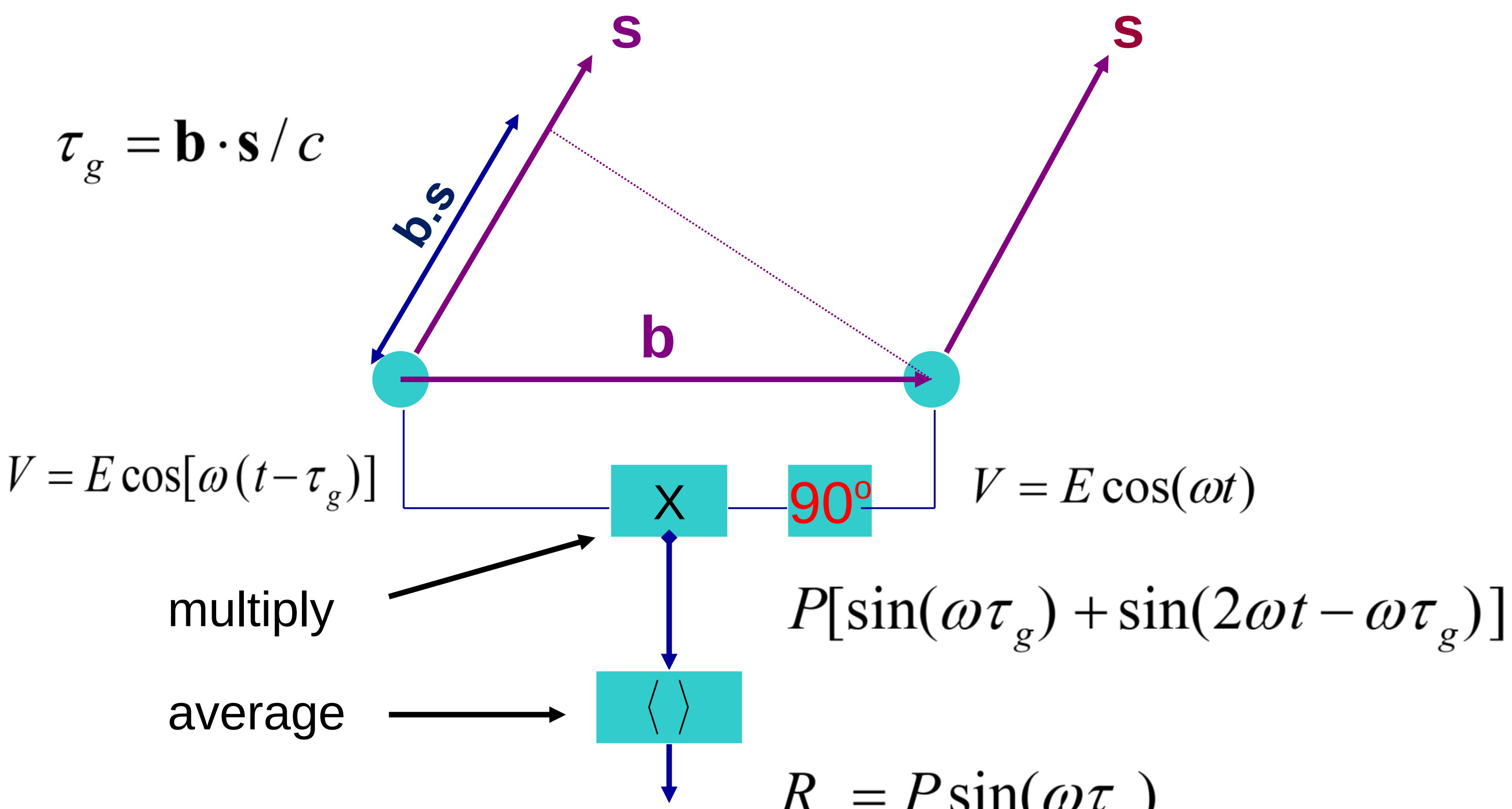
  since the integral of an odd function (I_O) with an even function (cos x) is zero.
- To recover the 'odd' part of the brightness, I_O, we need an 'odd' fringe pattern.
- Let us replace the 'cos' with 'sin' in the integral, to get

$$R_s = \iint I(\mathbf{s})\sin(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega = \iint I_o(\mathrm{s})\sin(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

  since the integral of an even times an odd function is zero.
- These `sine' fringes see only the `odd' component.
- To obtain this necessary component, we must make a 'sine' pattern.

Making a SIN Correlator

- We generate the 'sine' pattern by inserting a 90 degree phase shift in one of the signal paths.



Define the Complex Visibility

- We now DEFINE a complex function, the complex visibility, V, from the two independent (real) correlator outputs R_C and R_S:

$$V = R_C - iR_S = Ae^{-i\phi}$$

where

$$A = \sqrt{R_C^2 + R_S^2}$$

$$\phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

- This gives us a beautiful and useful relationship between the source brightness, and the response of an interferometer:

$$V_\nu(\mathbf{b}) = R_C - iR_S = \iint I_\nu(\mathbf{s})\, e^{-2\pi i \nu \mathbf{b}\cdot\mathbf{s}/c}\, d\Omega$$

- Under some circumstances, this is a 2-D Fourier transform, giving us a well established way to recover $I(\mathbf{s})$ from $V(\mathbf{b})$.

The Complex Correlator and Complex Notation

- A correlator which produces both 'Real' and 'Imaginary' parts – or the Cosine and Sine fringes, is called a 'Complex Correlator'
  - For a complex correlator, think of two independent sets of projected sinusoids, separated by 1/4 fringe spacing.
  - In our scenario, both components are necessary, because we have assumed there is no motion – the 'fringes' are fixed on the source emission, which is itself stationary.
- The complex output of the complex correlator also means we can use complex analysis throughout: Let:

$$V_1 = A\cos(\omega t) \rightarrow Ae^{-i\omega t}$$

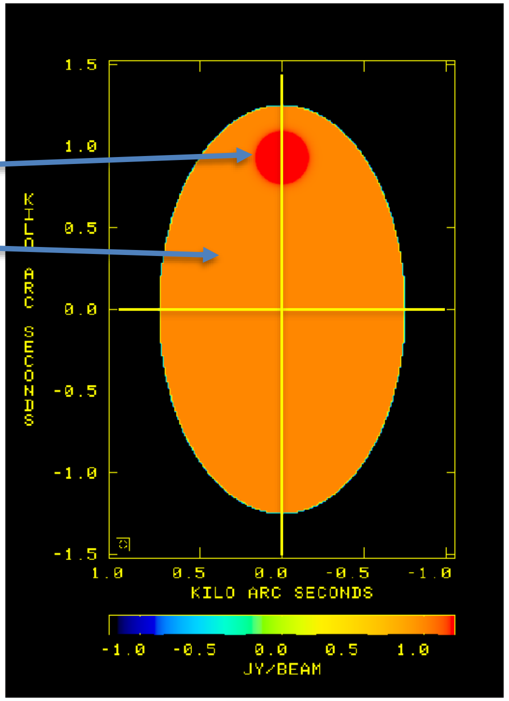
$$V_2 = A\cos[\omega(t - \mathbf{b} \bullet \mathbf{s} / c)] \rightarrow Ae^{-i\omega(t - b \bullet s / c)}$$

- Then:

$$P_{corr} = \langle V_1 V_2^* \rangle = A^2 e^{-i\omega \mathbf{b} \bullet \mathbf{s} / c}$$

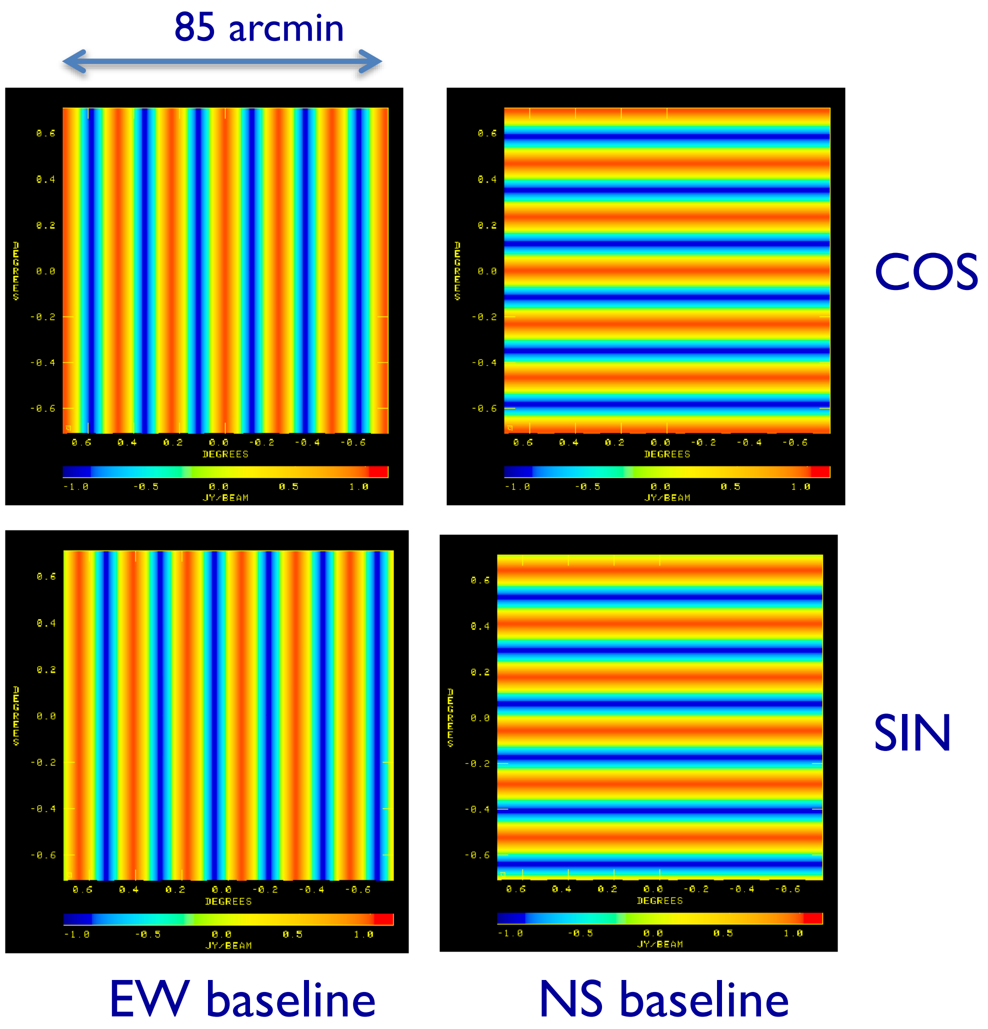
Visualizing Visibilities

- An intimate familiarity with interferometer visibilities is essential in understanding how interferometers work.
- Fortunately, the concepts can be easily grasped through pictures.
- To illustrate, I have generated a mock source, consisting of a flat elliptical disk, (1500" x 2500") and a bright, circular gaussian 'spot' of width 150" near one end.
    - Flux of disk: 1283 Jy.
    - Peak brightness of hotspot: 6 Jy/beam
    - Brightness of disk: 1 Jy/beam
    - Flux of hotspot: 56 Jy
    - Beamsize = 45 arcseconds
    - The odd color wedge is chosen to illustrate the main points
        - Color wedge runs from -1.25 Jy/beam (black) to +1.25 Jy/beam (red).



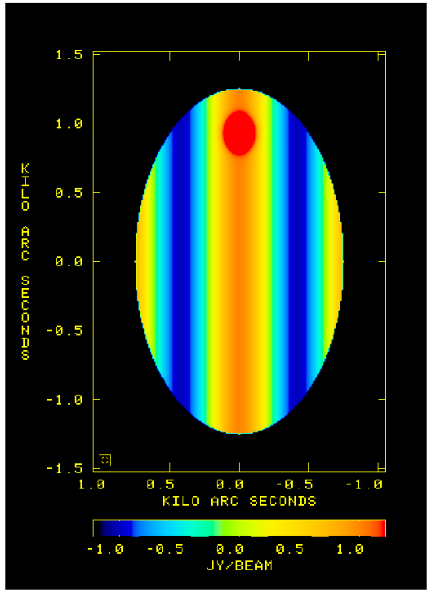
Our Mock Interferometer Response

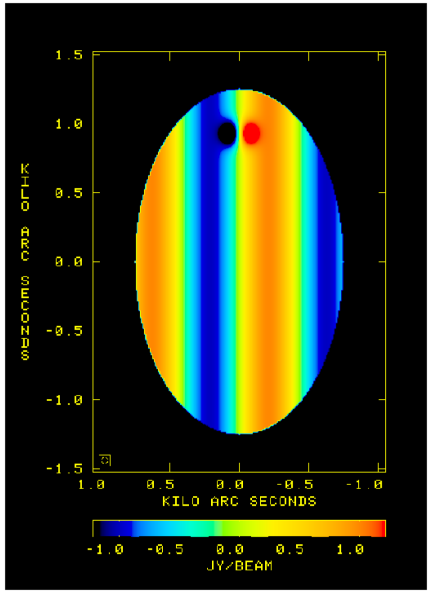
- We now `observe' this source with an interferometer, with seven baselines running from 12 through 525 meters, at a frequency of 1471 MHz.
  - These numbers are appropriate for the MeerKAT interferometer, but can be scaled to any other.
- The mock interferometer has only EW and NS spacings.
- Shown here are the two fringe patterns (Cos and Sin) for the two 50-meter baselines.
- Fringe spacing = 841”
- =300*206265/(1471*50)



What does the interferometer 'do'? E-W baseline

- Recall that the interferometer multiplies the actual brightness by the fringe pattern (both COS and SIN), and integrates (adds) over the field of view.
- The complex visibility is made from these products as:
  - COS => Real Part
  - SIN => Imaginary Part
- Shown are the COS and SIN products for the 50-m EW baseline.





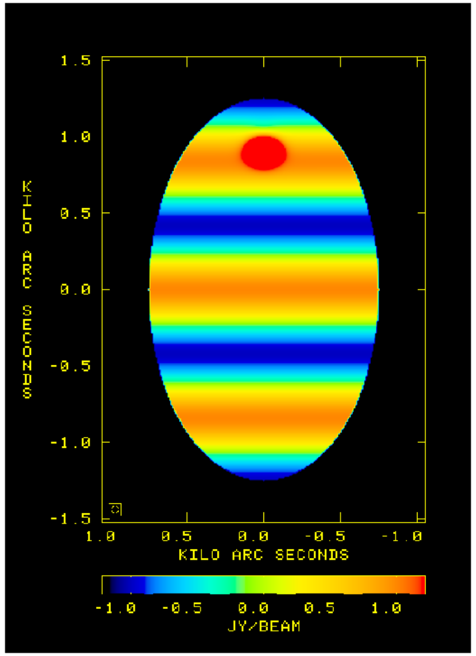
Cos Σ = -104 Jy   Sin Σ = -0.14 Jy

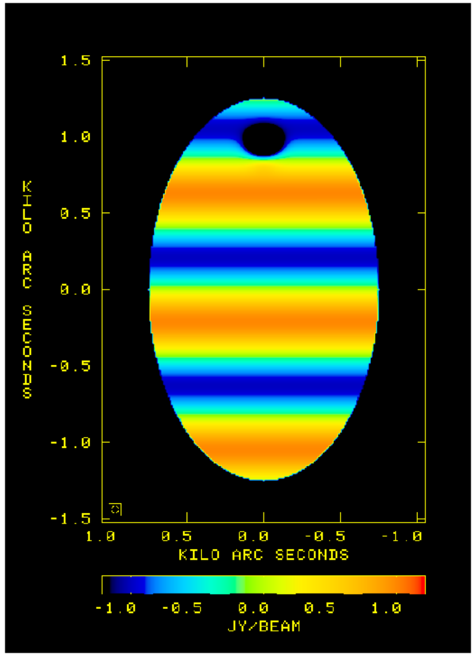
Thus: A = 104 Jy, ϕ = 180 degrees.

NB: The EW even symmetry requires the SIN integral = 0, so the phase must be either 0 or 180.

While for the NS baseline…

- The N-S asymmetry of the model (due to the `hotspot') means the NS baselines 'see' a wider range in the SIN (imaginary) component:





Cos Σ = 91.9 Jy

Sin Σ = -30.6 Jy

Thus: A = 96.9 Jy, ϕ = -18.4 degrees.

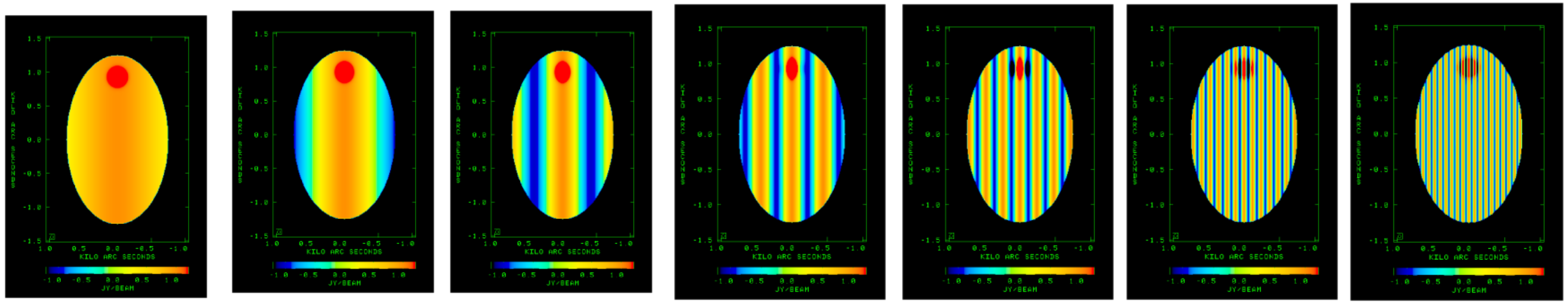
NB: There are no symmetries here, so phases can be any value.

Visibilities as a function of baseline length

- Showing a sequence as the E-W baselines gets longer:

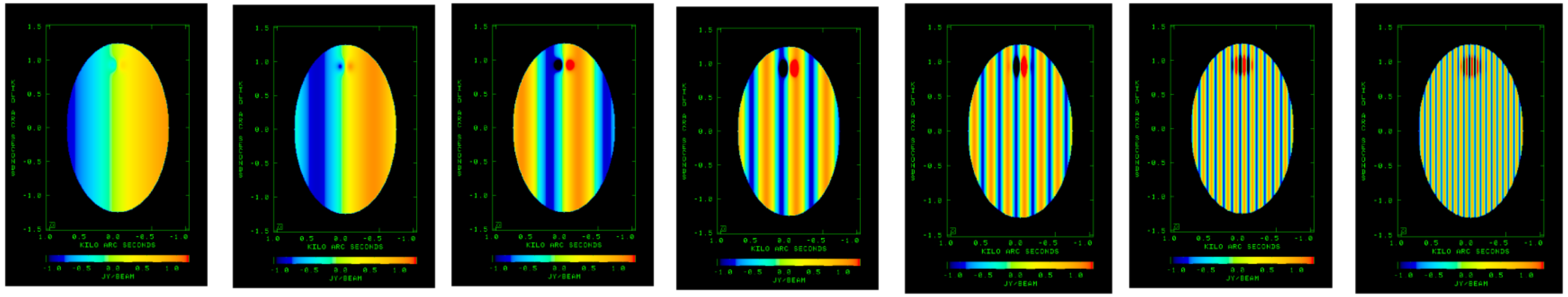
Baseline:	12.5m	25m	50m	95m	175m	350m	525m

COS



Σ	1069	429	-104	8.6	13.3	6.0	5.15
A	1069	429	104	8.6	13.3	6.0	5.15
ϕ	-0.3	0.1	180	0.0	0.0	0.0	0.0
Σ	-6.3	0.6	-0.1	0.0	0.1	0.0	0.1

SIN



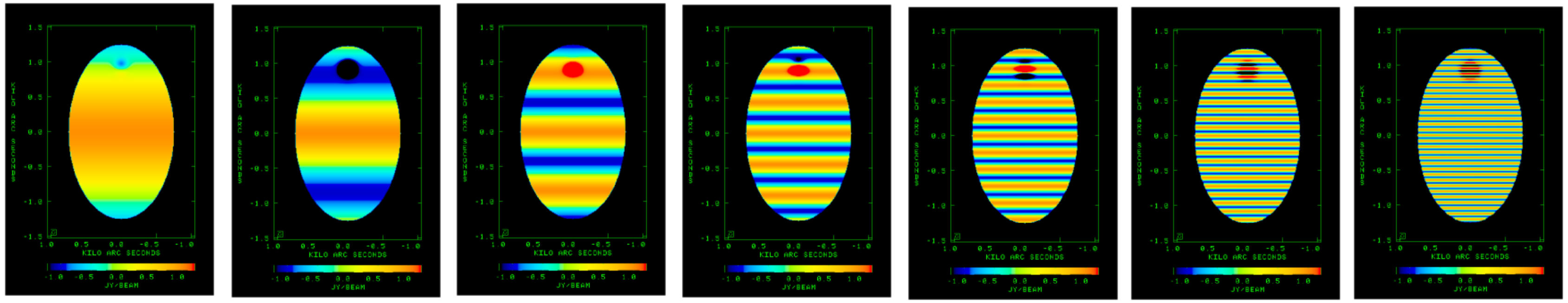
- Note that the ‘SIN’ fluxes are all very low, so the visibility phases are ~zero or 180 degrees.

As a function of N-S baseline length…

- Showing a sequence as the baseline gets longer:

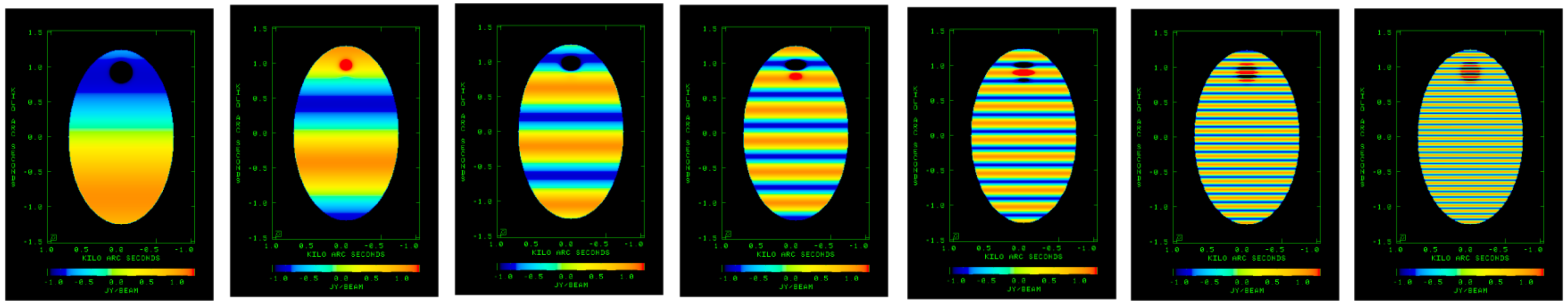
Baseline:	12.5m	25m	50m	95m	175m	350m	525m

COS



Σ	624	-201	91.9	4.2	14.9	3.5	6.8
A	627	202	96.9	21.2	17.7	3.5	6.8
ϕ	-5.4	175	-18	-78	34	3.6	0
Σ	-59	17.4	-30.6	-20.8	10.0	0.2	0.0

SIN



- Note that the 'SIN' fluxes are now much larger, and the visibility phases far from zero or 180.

Some Take-Aways…

- In general, as the baseline gets longer, the visibility amplitude declines.
- The decline need not be smooth – complicated emission will often have an oscillating, and decaying, visibility function.
- The visibility from a 'zero-spacing' interferometer (aka 'a single dish') equals the total flux density of the source.
  - For Stokes 'I', this cannot be less than all visibilities at longer spacings.
- The visibilities for a source with even symmetry about some axis must be real (phase 0 or 180) for fringes parallel to that axis.
  - An extreme case of this is a smooth circular source: The emission is even about all axes (through the center), so all visibilities are real.
- The visibilities for a source with odd symmetry about some axis must be imaginary (phase 90 or -90) for fringes parallel to that axis.
  - This is not possible for Stokes 'I', but is possible for Q, U, or V.

Some Thoughts to Ponder

- The complex visibility **amplitude** is independent of the source location*, and linearly related to source flux density.
- The complex visibility **phase** is a function of source location, and independent of source flux density.
- These two statements, restated, are:
  - 1) Doubling the source brightness doubles the visibility amplitude, but doesn't change the visibility phase
  - 2) Shifting the source position changes the phase, but does not change the visibility amplitude.*
- Reversing the elements of an interferometer negates the phase of the complex visibility, and leaves the amplitude unchanged.
- For those of you familiar with Fourier transforms, the equivalent statement is that:
  - 'As the source brightness is a real function, its Fourier transform is Hermitian'.

* Ignoring any attenuation due to the primary beam…

Visibilities Lead to Images …

- The Visibility is a unique function of the source brightness.
- The two functions are related through a Fourier transform.

$$V_\upsilon(u,v) \Leftrightarrow I(l,m)$$

- How we go from visibilities to images is the subject of a later lecture.
- An interferometer, at any one time, makes one measure of the visibility, at baseline coordinate (u,v).
- `A sufficient number of measures' of the visibility function (as derived from an interferometer) will provide us a `reasonable estimate' of the source brightness.
- How many is 'sufficient', and how good is 'reasonable'?
- These simple questions do not have easy answers…

Final Comments …

- The formalism presented here presumes much … including that there is no motion between source and interferometer.
- Real interferometers:
  - Are on a rotating platform
  - Utilize real sensors (antennas)
  - Use wide bandwidths
  - Average data over time
  - Employ frequency downconversion
  - Have to deal with corruptions due to many causes…
- How we manage these issues are the subjects of my next lecture, and by following lectures.