

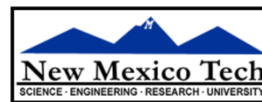
# Calibration



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21<sup>st</sup> Synthesis Imaging Workshop

28 May 2026



# References & Credits

- Interferometry and Synthesis in Radio Astronomy  
(3<sup>rd</sup> ed., Thompson, Moran, & Swenson)
- Synthesis Imaging in Radio Astronomy II  
(Editors: Taylor, Carilli, & Perley)
- Tutorials, telescope observing guides, software documentation
- Previous workshops (especially G. Moellenbrock's and G. Heald's previous talks on which parts of this talk are based)

# Idealized Visibilities

We wish to use our interferometer to obtain the visibility function:

$$V(u, v) = \int_{sky} I(l, m) e^{-i2\pi(ul+vm)} dl dm$$

which we intend to invert to obtain an image of the sky:

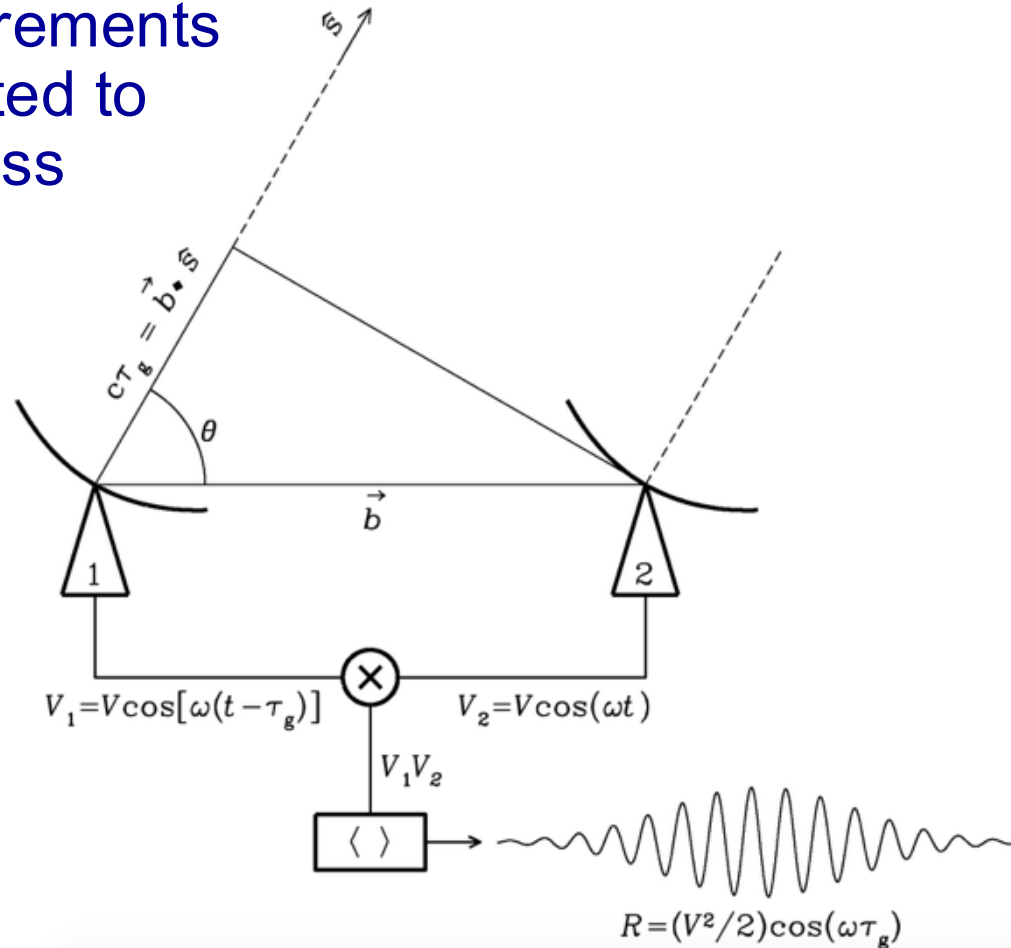
$$I(l, m) = \int_{uv} V(u, v) e^{i2\pi(ul+vm)} du dv$$

$V(u, v)$  describes the amplitude and phase of 2D sinusoids that add up to an image of the sky:

- Amplitude: “how bright and how compact is the source?”
- Phase: “where is the source?”

# Ideal Case

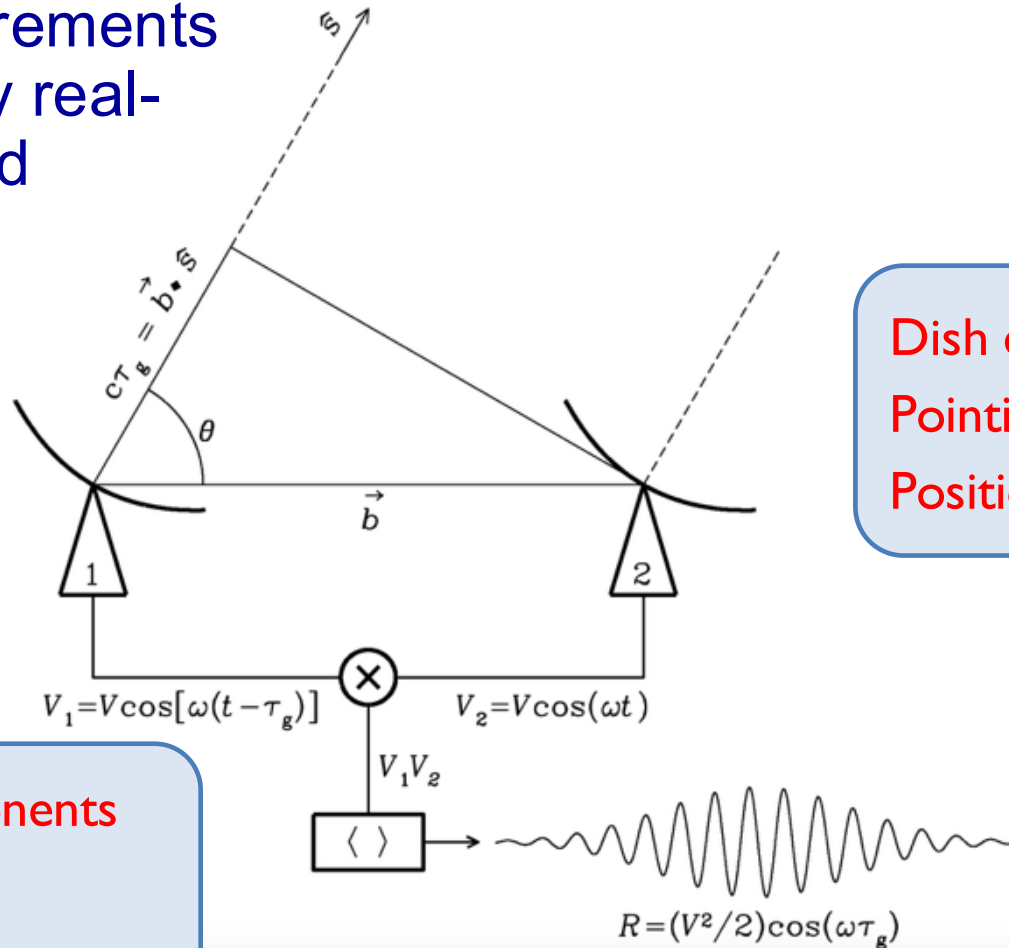
Visibility measurements are directly related to the sky brightness



# Real Case

Visibility measurements are corrupted by real-world effects and include noise

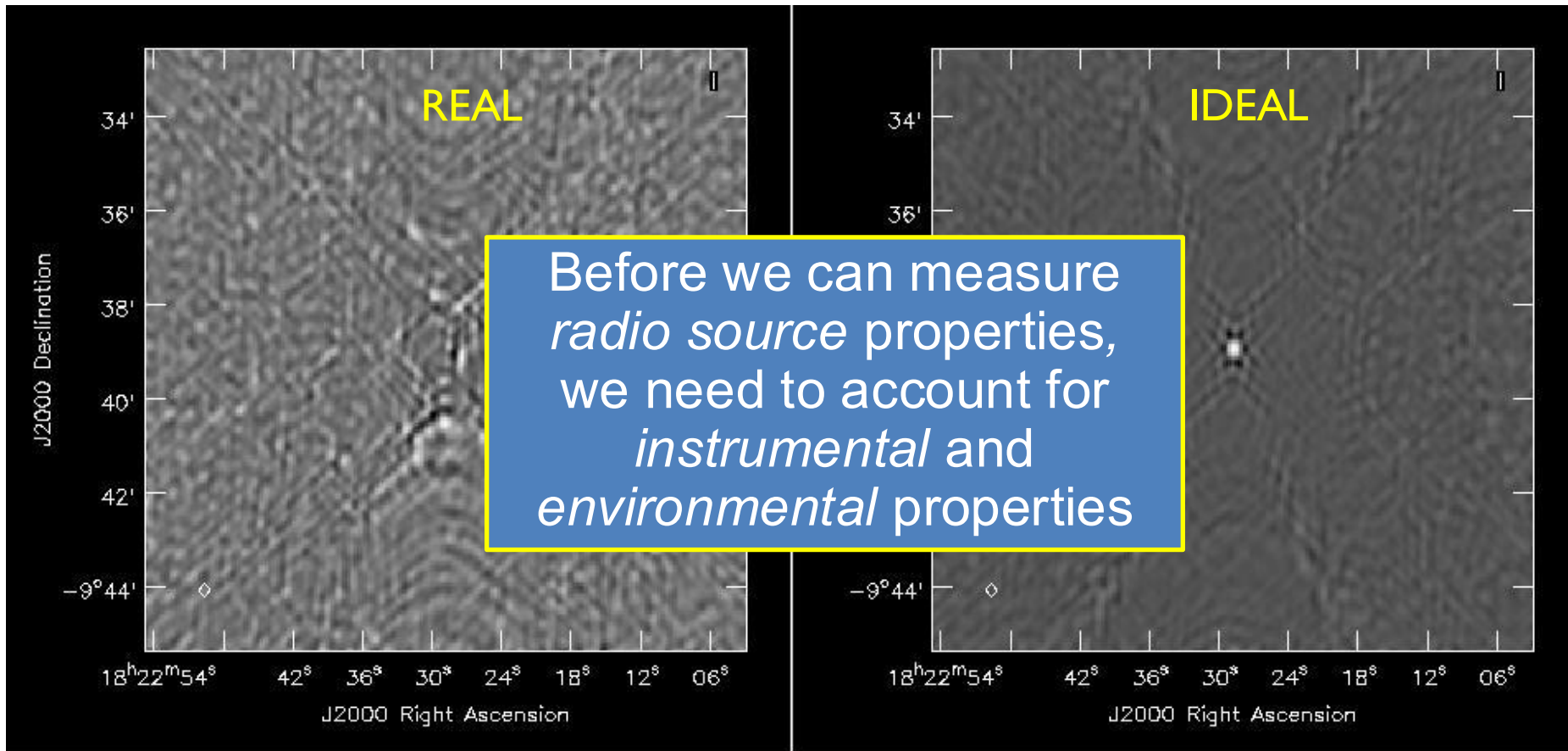
Ionosphere  
Troposphere



Dish deformation  
Pointing errors  
Position errors

Waveguide components  
Analog electronics  
Digital electronics

# Images of Real vs. Ideal Visibilities



# Realistic Visibilities

The real signal sampled by antenna  $i$ ,  $x_i(t)$ , is a combination of the desired signal,  $s_i(t,l,m)$ , corrupted by a factor  $J_i(t,l,m)$  and with added noise,  $n_i(t)$ :

$$x_i(t) = \int_{sky} \underline{J_i(t,l,m)} s_i(t,l,m) dl dm + n_i(t)$$

So we have an imperfect visibility measurement per antenna pair:

$$\begin{aligned} V_{ij}^{obs}(u, v) &= \langle x_i(t) \cdot x_j^*(t) \rangle_{\Delta t} \\ &= \underline{J_{ij}} V_{ij}^{true}(u, v) \quad (\text{for } J_i, J_j \text{ constant in } l, m) \end{aligned}$$

$J_{ij} = J_i J_j^*$  is a generalized operator characterizing the net effect of the observing process for antennas  $i$  and  $j$  on baseline  $ij$

# Solving for Calibration

Observe a celestial calibration source for which we have a **model**,

$$V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$$

define chi-squared:

$$\chi^2 = \sum_{\substack{i,j \\ i \neq j}} |V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod}|^2 w_{ij} \quad \left( w_{ij} = \frac{1}{\sigma_{ij}^2} \right)$$

and minimize chi-squared w.r.t. each  $J_i^*$   $\left( \frac{\partial \chi^2}{\partial J_i^*} = 0 \right)$

This problem is overconstrained: N antenna gains and N(N-1)/2 measured baselines  $\rightarrow$  redundancy allows for robust gain estimation and averaging

# Applying the Calibration

Correct by applying  $J$  to each visibility:

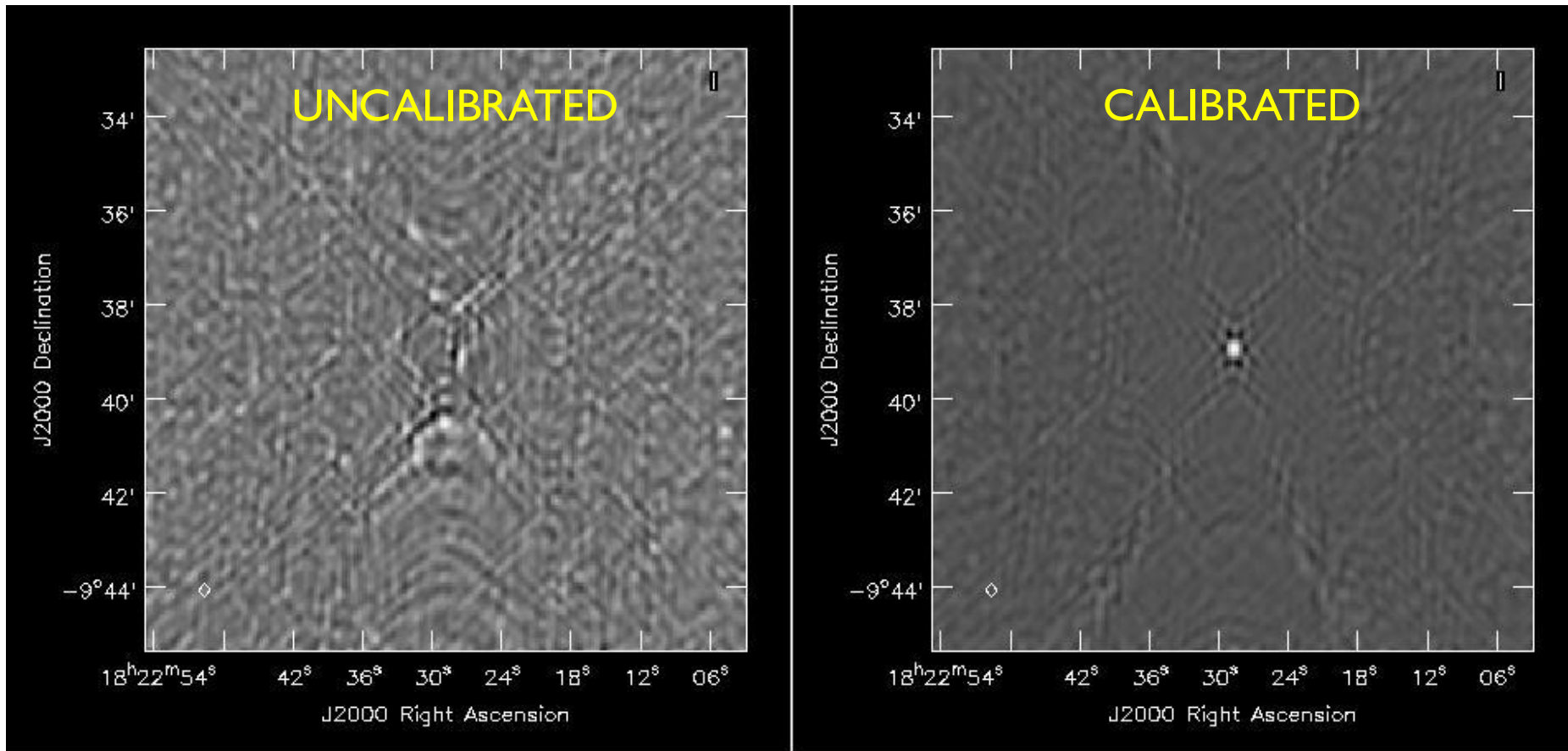
$$V_{ij}^{obs} = J_i J_j^* V_{ij}^{true} \quad \square \quad V_{ij}^{cor} = J_i^{-1} J_j^{*-1} V_{ij}^{obs}$$

Visibilities are corrected by the difference in antenna phases and the product of gain amplitudes

Calibration solutions can be applied to other targets (science fields)

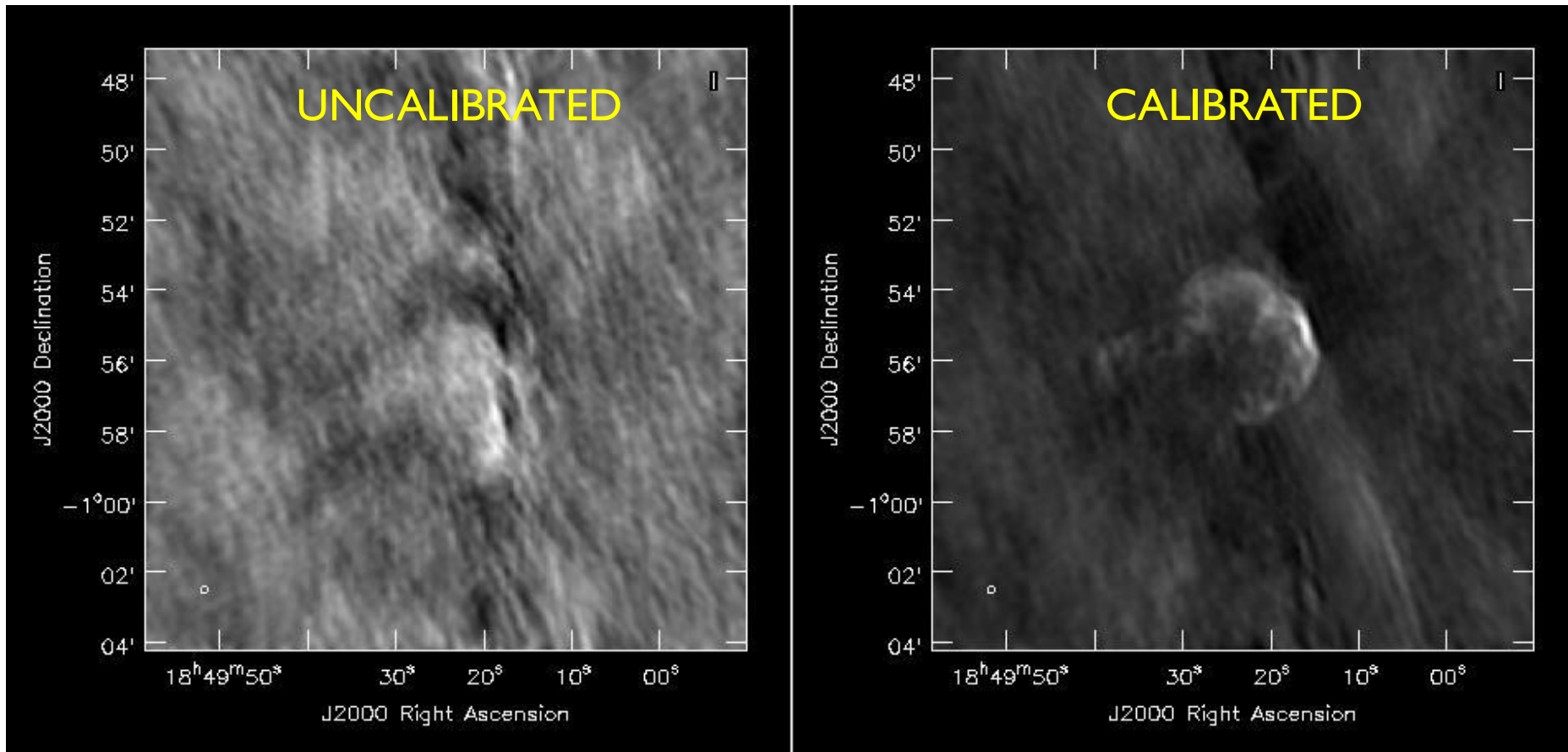
Caution: the inverse of un-modeled calibrator structure will be transferred when applying the solutions

# Calibration's Effect on Imaging



(Calibration applied to calibrator field)

# Calibration's Effect on Imaging



(Calibration applied to target field)

# Including Polarization

Denote polarization indices (p,q), typically linear (X,Y) or circular (R,L)

Each antenna produces two voltages

$$\begin{bmatrix} v_p \\ v_q \end{bmatrix} = \begin{bmatrix} J_{pp} & J_{pq} \\ J_{qp} & J_{qq} \end{bmatrix} \begin{bmatrix} e_p \\ e_q \end{bmatrix}$$

from  $J$  acting on the incident electric field  $e$

Each baseline then has four visibility correlations

$$\mathbf{V}_{ij} = \begin{bmatrix} V_{pp,ij} & V_{pq,ij} \\ V_{qp,ij} & V_{qq,ij} \end{bmatrix}$$

and the radio interferometric measurement equation becomes:

$$\mathbf{V}_{ij}^{\text{obs}} = \mathbf{J}_i \mathbf{V}_{ij}^{\text{true}} \mathbf{J}_j^H$$

# Solving for Calibration - Practical

- There typically is not one single calibrator which we can use to solve for all of the corrupting effects
- We don't want to solve for all effects at once
  - High quality solutions often require averaging
  - Averaging uncalibrated phases creates decorrelation
  - Some calibration terms should be determined in a similar direction as your science target(s)
  - Some calibration terms can be obtained from the observatory or from external databases
- Best approach is to decompose the net corruption into separate components that can be addressed individually

# Calibration Components

**F**  
**T**

$$\vec{J}_i = \vec{K}_i \vec{B}_i \vec{G}_i \vec{D}_i \vec{E}_i \vec{X}_i \vec{P}_i \vec{T}_i \vec{F}_i$$

Components of the **Jones matrix**  $J_i$ :

$F$  = ionospheric effects

$T$  = tropospheric effects

$P$  = parallactic angle

$X$  = linear polarization position angle

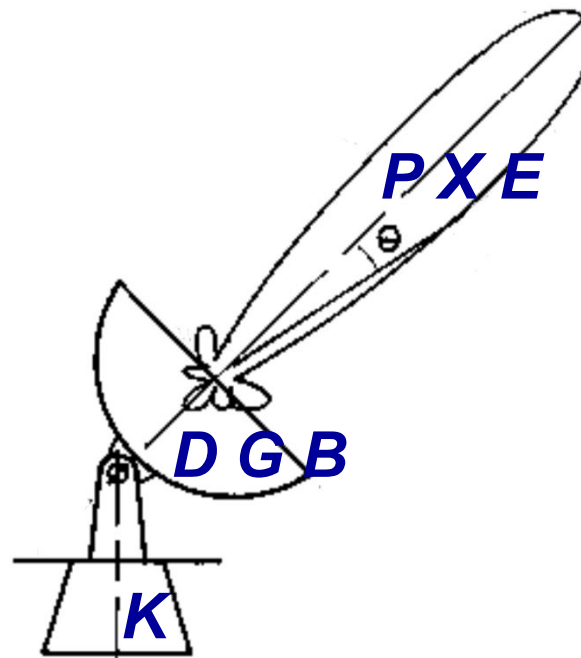
$E$  = antenna voltage pattern, gaincurve

$D$  = polarization leakage

$G$  = electronic gain

$B$  = bandpass response

$K$  = geometry



–  $J_i$  is a function of time, as are many of its components

# Ionospheric Effects, $F$

$$\vec{F}^{RL} = e^{i\Delta\phi} \begin{pmatrix} e^{-i\varepsilon} & 0 \\ 0 & e^{i\varepsilon} \end{pmatrix}; \quad \vec{F}^{XY} = e^{i\Delta\phi} \begin{pmatrix} \cos \varepsilon & \sin \varepsilon \\ -\sin \varepsilon & \cos \varepsilon \end{pmatrix}$$

The ionosphere introduces a dispersive path-length offset:

$$\Delta\phi \propto \frac{\int n_e dl}{\nu}$$

And also Faraday rotation:

$$\varepsilon \propto \frac{\int B_{\parallel} n_e dl}{\nu^2}$$

More important at lower frequencies ( $\lesssim 5$  GHz)

# Tropospheric Effects, $T$

$$\vec{T} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

## Polarization-independent effects due to the lower atmosphere

- Most important at  $\nu \gtrsim 15$  GHz where path length errors are a larger fraction of the wavelength, and water vapor and oxygen absorb/emit
- Zenith-angle-dependent opacity (higher air mass near horizon)
- One of the most highly time-variable calibration terms

# Parallactic Angle, $P$

$$\vec{P}^{RL} = \begin{pmatrix} e^{-i\chi} & 0 \\ 0 & e^{i\chi} \end{pmatrix}; \quad \vec{P}^{XY} = \begin{pmatrix} \cos \chi & \sin \chi \\ -\sin \chi & \cos \chi \end{pmatrix}$$

Accounts for changing orientation of sky in telescope's frame

- Constant for equatorial telescopes
- Varies for alt-az-mounted telescopes

$$\chi(t) = \arctan \left( \frac{\cos l \sin h(t)}{\sin l \cos \delta - \cos l \sin \delta \cos h(t)} \right)$$

$l$  = latitude,  $h(t)$  = hour angle,  $\delta$  = declination

- Analytically known based on geometry
- Rotates the position angle of linearly polarized radiation

# Linear Polarization Position Angle, $X$

$$\vec{X}^{RL} = \begin{pmatrix} e^{-i\Delta\chi} & 0 \\ 0 & e^{i\Delta\chi} \end{pmatrix}; \quad \vec{X}^{XY} = \begin{pmatrix} \cos \Delta\chi & \sin \Delta\chi \\ -\sin \Delta\chi & \cos \Delta\chi \end{pmatrix}$$

- Configuration of optics and electronics (and use of a refant) causes a net linear polarization position angle offset
- Can be treated as an offset to the parallactic angle, P
- For circular feeds, this is a phase difference between the R and L polarizations, which is frequency-dependent (a R-L phase bandpass)
- For linear feeds, this is the orientation of the dipoles (in the frame of the telescope) projected onto sky coordinates

# Antenna Voltage Pattern, $E$

$$\vec{E}^{pq} = \begin{pmatrix} E^p(l, m) & 0 \\ 0 & E^q(l, m) \end{pmatrix}$$

- Antennas of all designs have direction-dependent gain (primary beam)
- Antenna forward gain may change with elevation: ‘gain curve’
- We typically only include direction-independent effects in  $E$ , and defer  $(l, m)$  effects to be handled during imaging

# Polarization Leakage, $D$

$$\vec{D} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

Orthogonal polarizations are not perfectly isolated

- Potential origins include alignment errors, EM induction, polarizer
- Well-designed systems have  $d \sim$  a few percent or less on-axis

$D$  does not include off-axis leakage, a separate issue for polarimetry

# “Electronic” Gain, $G$

$$\vec{G}^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

A catch-all for most time-dependent amplitude and phase effects introduced by antenna electronics and other generic effects

- Includes scaling from engineering units to radio astronomy units (Jy)
- Includes any internal system monitoring such as a noise source
- Often includes residual tropospheric and ionospheric effects

# Bandpass Response, $B$

$$\vec{B}^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

Like  $G$  but as a function of frequency

- Filters used to select frequency passband not square
- Optical and electronic reflections introduce ripples
- Often assumed time-independent, but not necessarily so
- Typically (but not necessarily) normalized

# Geometry, $K$

$$\vec{K}^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

- $K$  is a clock- & geometry-parameterized version of  $G$
- Typical contributions to  $K$  are antenna positions and delay refinements
- Must have correct geometry for Fourier Transform relation to work

# Decoupling Calibration Components

$$\vec{J}_i = \vec{K}_i \vec{B}_i \vec{G}_i \vec{D}_i \vec{E}_i \vec{X}_i \vec{P}_i \vec{T}_i \vec{F}_i$$

- Make an informed decision about which terms to ignore, based on your science requirements, calibration strategy, telescope and conditions
- Use external (*a priori*) information: **F**, **T**, **P**, **E** and parts of **G**, **K**
- Parameterize terms appropriately (e.g. zenith angle vs. time)
- Other effects need to be solved for by observing calibrator sources

# Observing Calibrator Sources

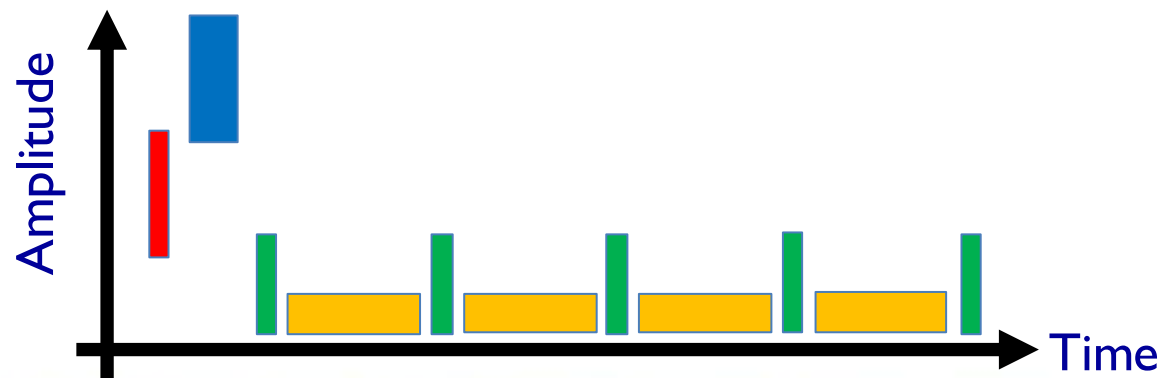
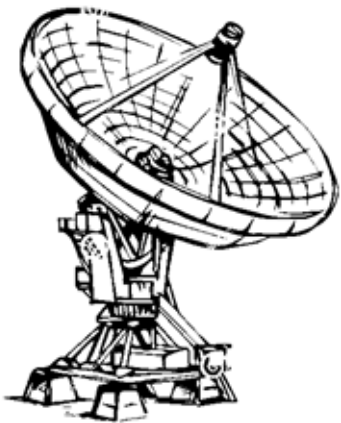
**Flux Density:** 'standard candle' with known structure and spectral energy distribution. Typically observe once per observation.

**Delay, Bandpass:** very bright, preferably unresolved. Typically observe once per observation.

**Gain:** bright, preferably unresolved, accurate position. Observe before and after the **target source** more frequently than the coherence time.

**Pol Angle:** polarized source with full-Stokes model

**Pol Leakage:** unpolarized, preferably unresolved or multiple scans over a range of parallactic angles



# Flux-density Calibration

- Bootstrap the flux density scale by enforcing gain amplitude consistency over all calibrators:

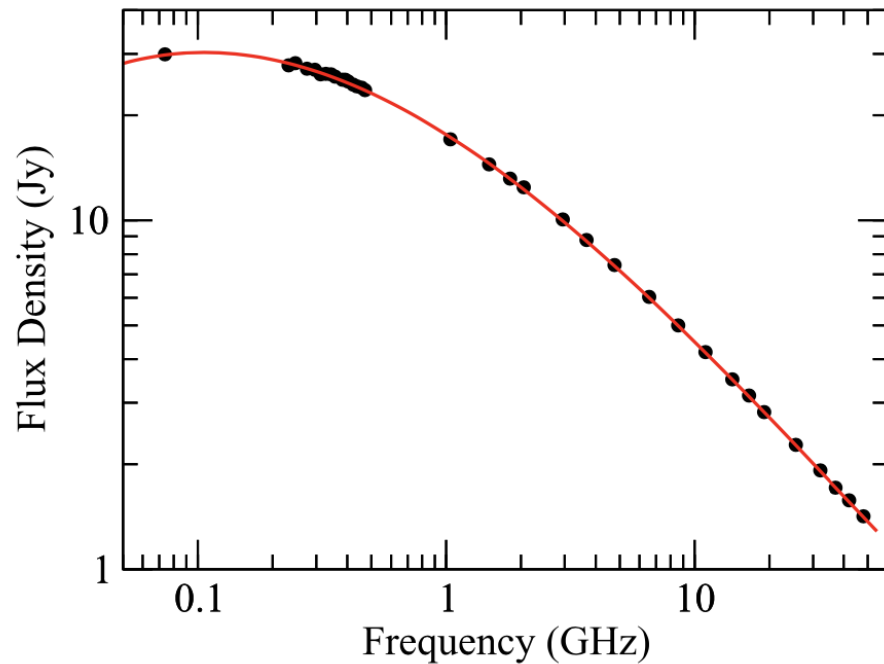
$$\left\langle \frac{|G_i|}{|G_i(fd\ cal)|} \right\rangle_{time, antennas} = 1.0$$

- Important to have previously removed other known gain effects, e.g. elevation-dependence
- I.e., assume the system is linear and time-independent:

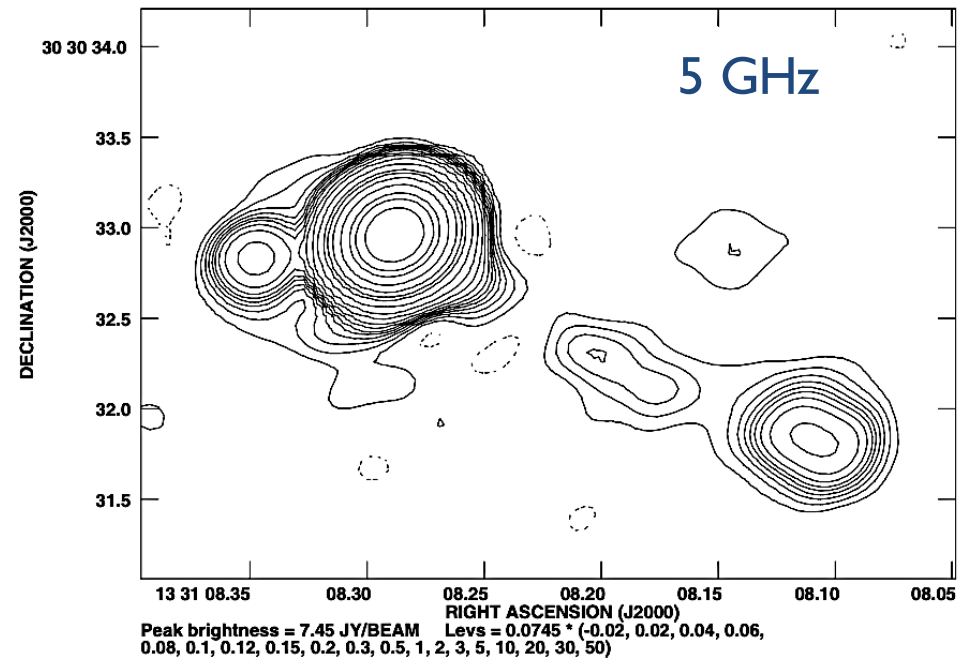
$$\frac{|V_{u,v \rightarrow 0}(fd\ cal)|}{F(fd\ cal)} = \frac{|V_{u,v \rightarrow 0}(other\ cal)|}{F(other\ cal)}$$

# Flux-density Calibration

- Use observatory-provided flux calibrator models, e.g. 3C 286



(Perley & Butler 2017)



(Perley & Butler 2013)

# Reference Antenna

- Phase solutions are typically referred to a specific antenna, the *refant*, which is assumed to have constant (zero) phase
  - The *refant's* phase variations are distributed to the solutions of all other antennas
  - Asserts phase continuity and a cross-hand phase frame
  - Assists in the inspection and interpolation of solutions
- An ideal *refant* should be:
  - Available over the entire observation (time, frequency)
  - Stable and generally 'well behaved'

# Examination and Editing

- Calibration is very susceptible to 'bad data' and radio interference
  - Initial data examination and 'flagging' are very important
  - $J_i$  will try to make the bad data match the model!

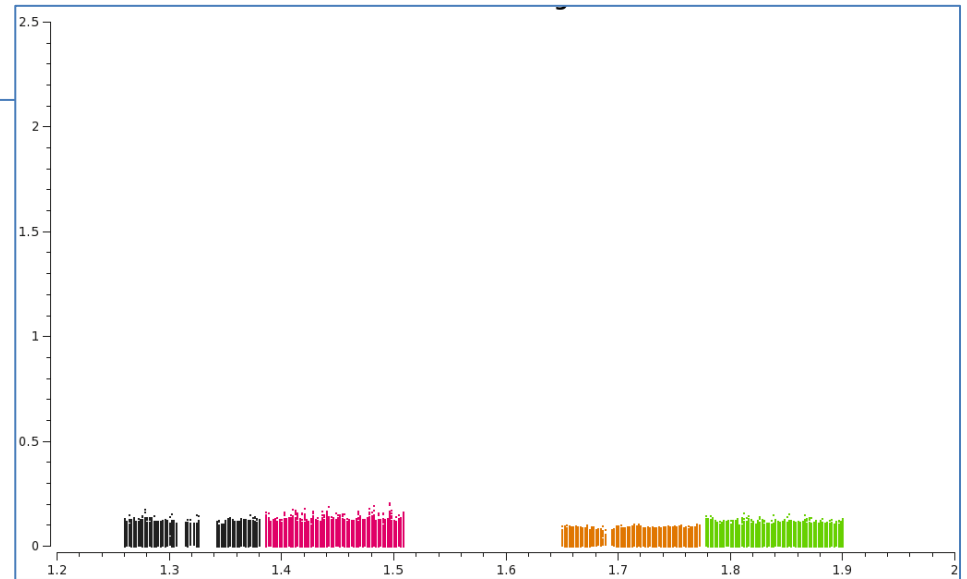
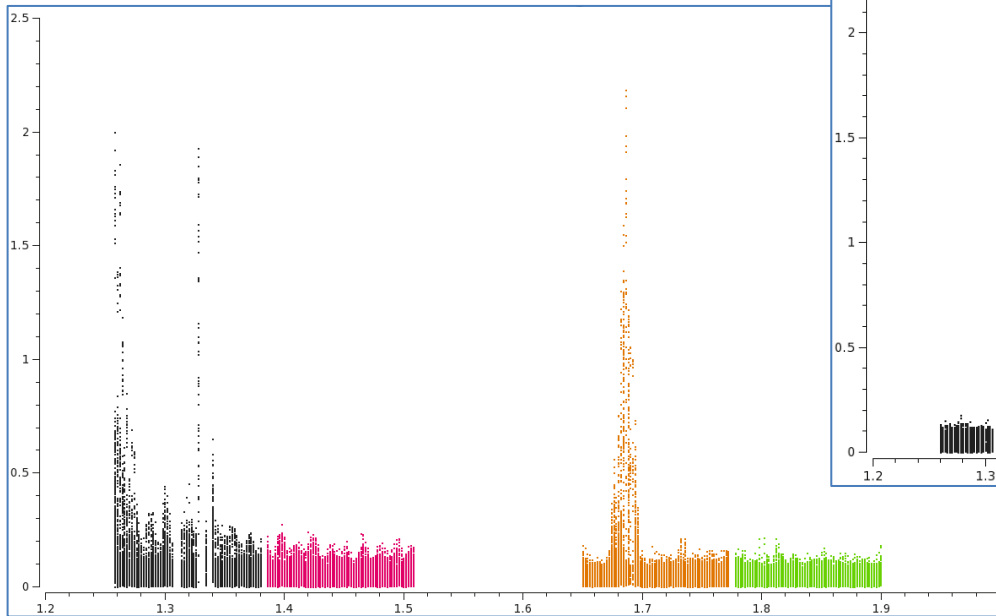
$$V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$$

- Evaluate calibration tables— are solutions continuous and sensible?
- Iterate— provisional calibration can make bad data easier to identify

# Examination and Editing

Visibility amplitude vs. frequency, colored by spectral window.

**Before flagging**



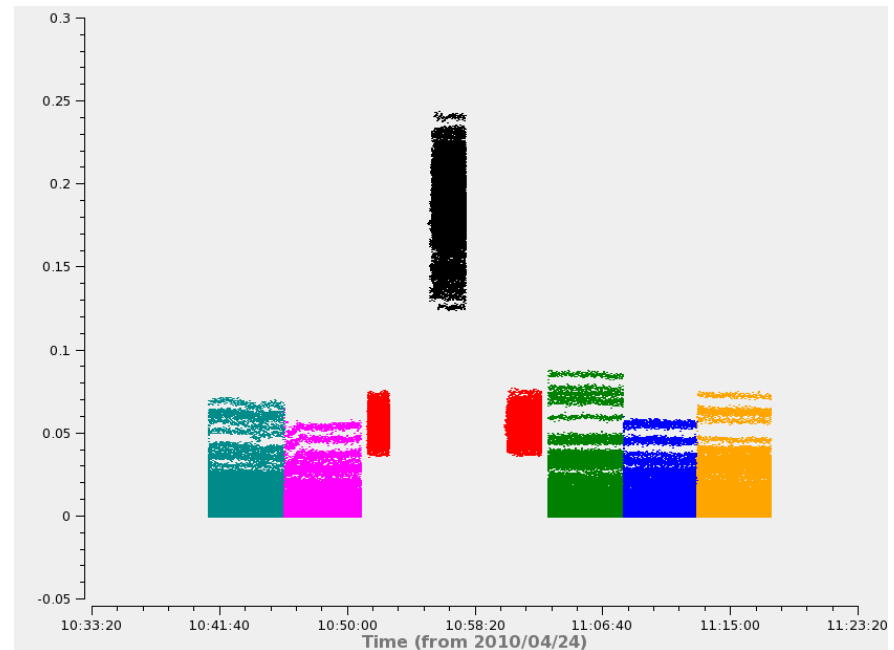
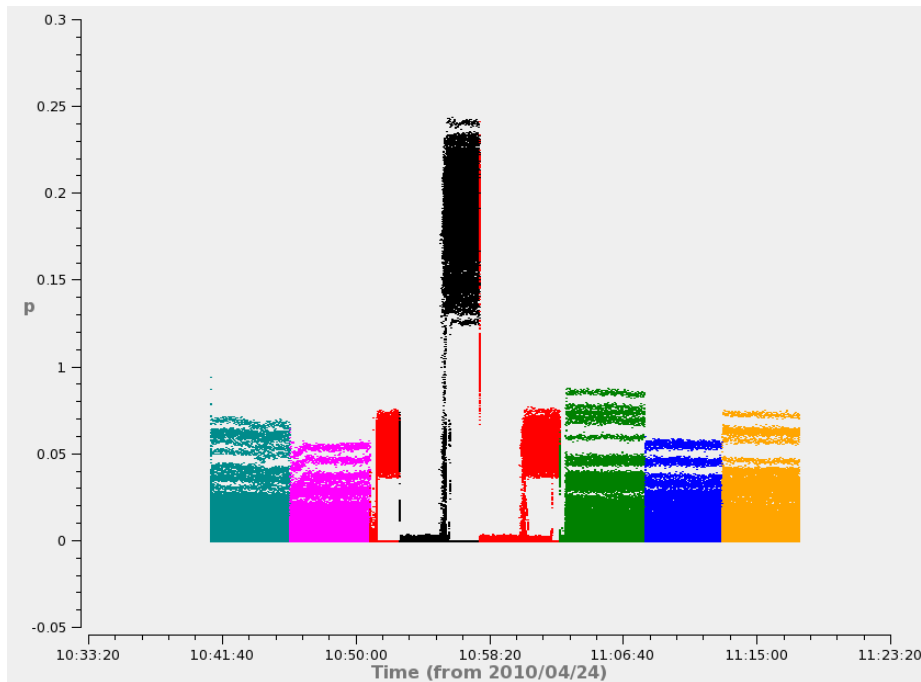
**After flagging**

(figures from VLA Flagging casaguide)

# Examination and Editing

Visibility amplitude vs. time, colored by scan.

## Before flagging



## After flagging

(figures by G. Moellenbrock)

# Advanced Techniques

- Make use of ancillary hardware, when available:
  - Ionospheric instrumentation to measure TEC
  - Water vapor radiometer to infer tropospheric phase
  - Tipping scans to fit atmospheric opacity
  - Pulse cal to track radiometer gain, X-Y phase
  - Paddle scans to calibrate flux density
- Self-calibration: create and use source model to refine  $J_i$
- Direction-dependent (wide-field) calibration

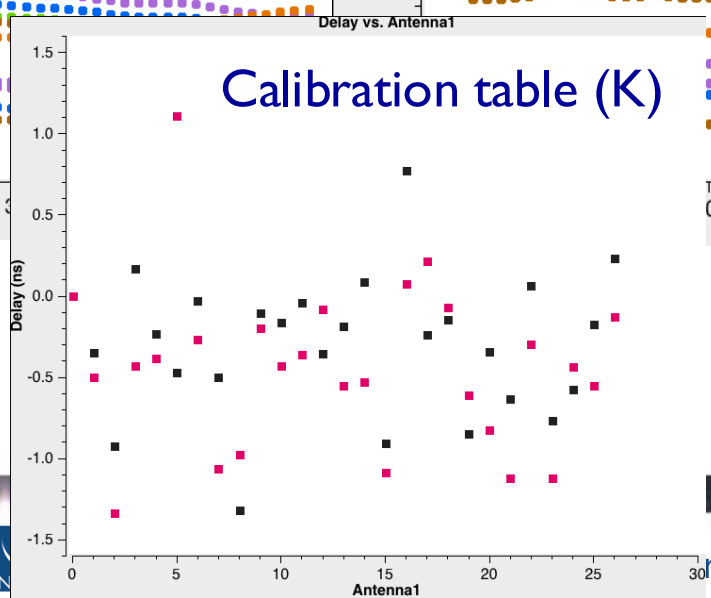
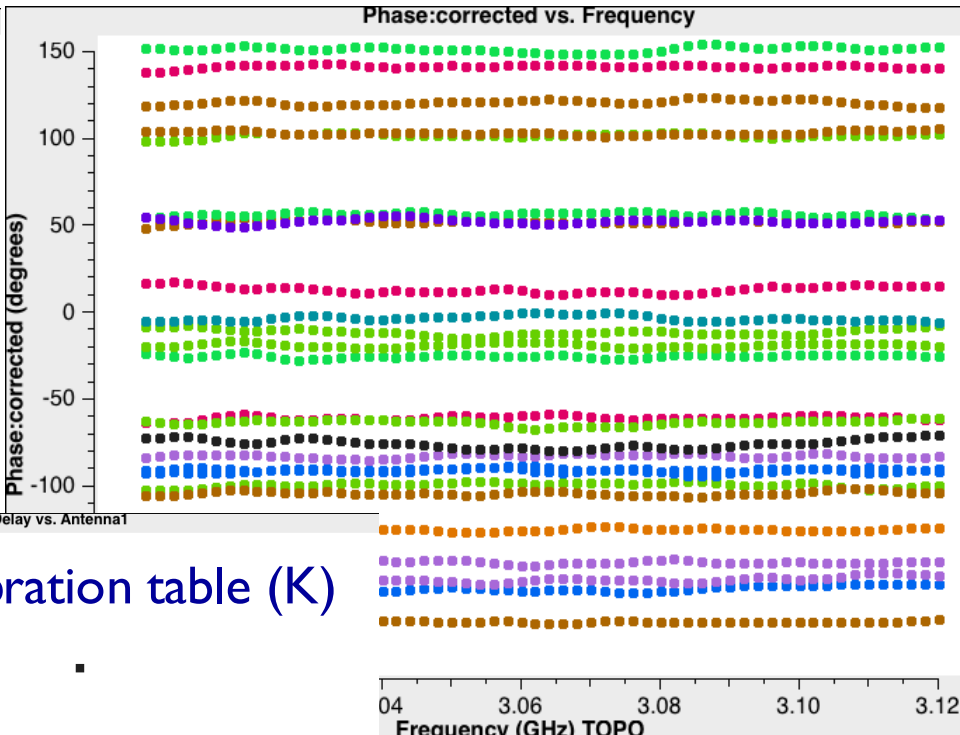
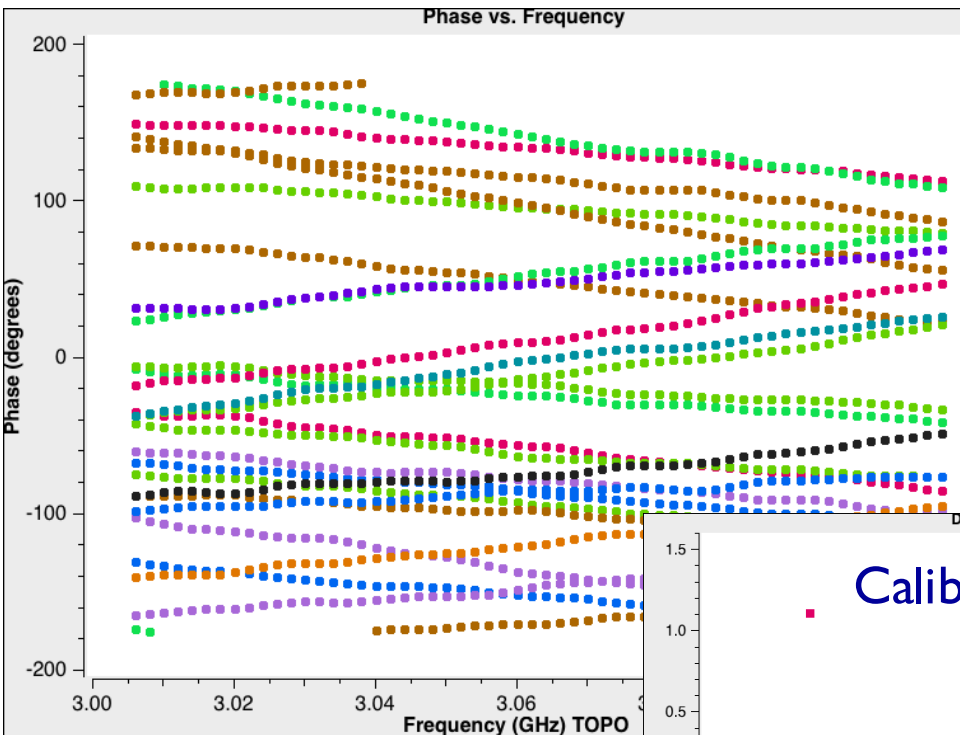
# A Practical Demonstration

- Example calibrator scan from VLA S-band ( $\sim 3$  GHz)
  - 27 antennas
  - 351 baselines
  - 64 frequency channels
- Assume point source of 1 Jy
- Solve for delay, bandpass and complex gain (amplitude and phase)
- Correct the visibilities by applying the calibration solutions

# A Practical Demonstration

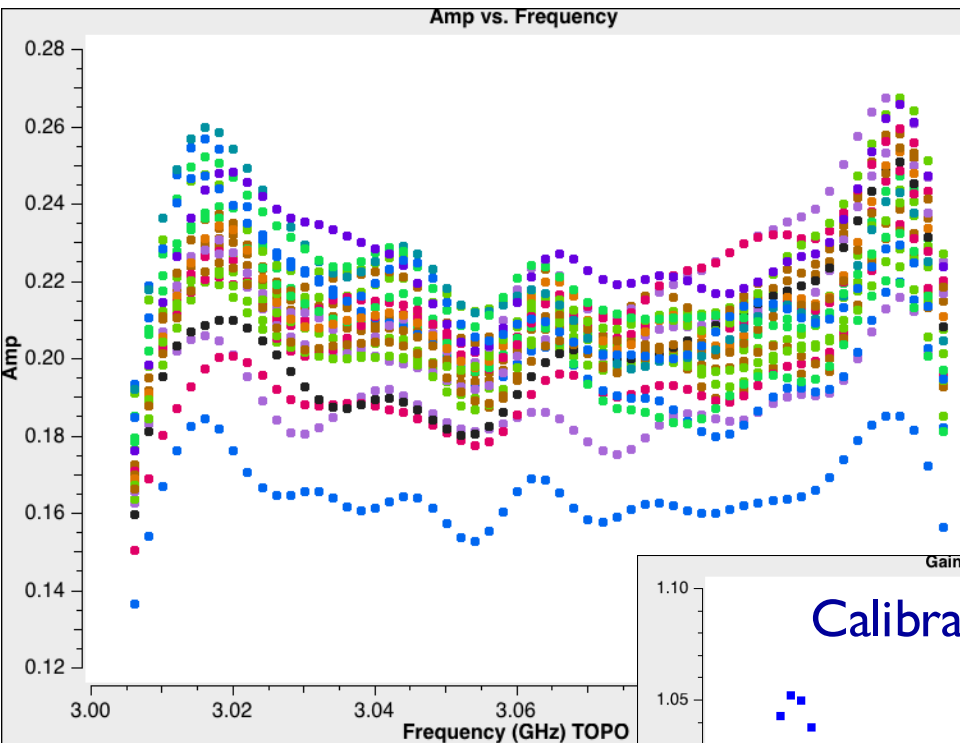
Uncalibrated

Calibrated

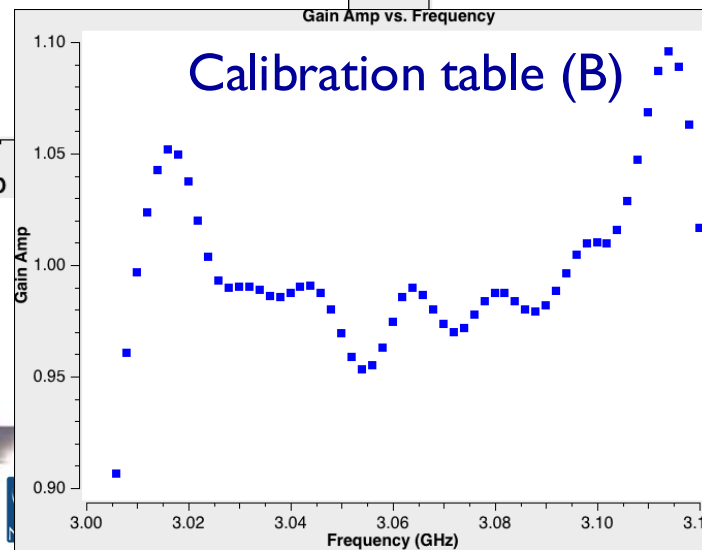
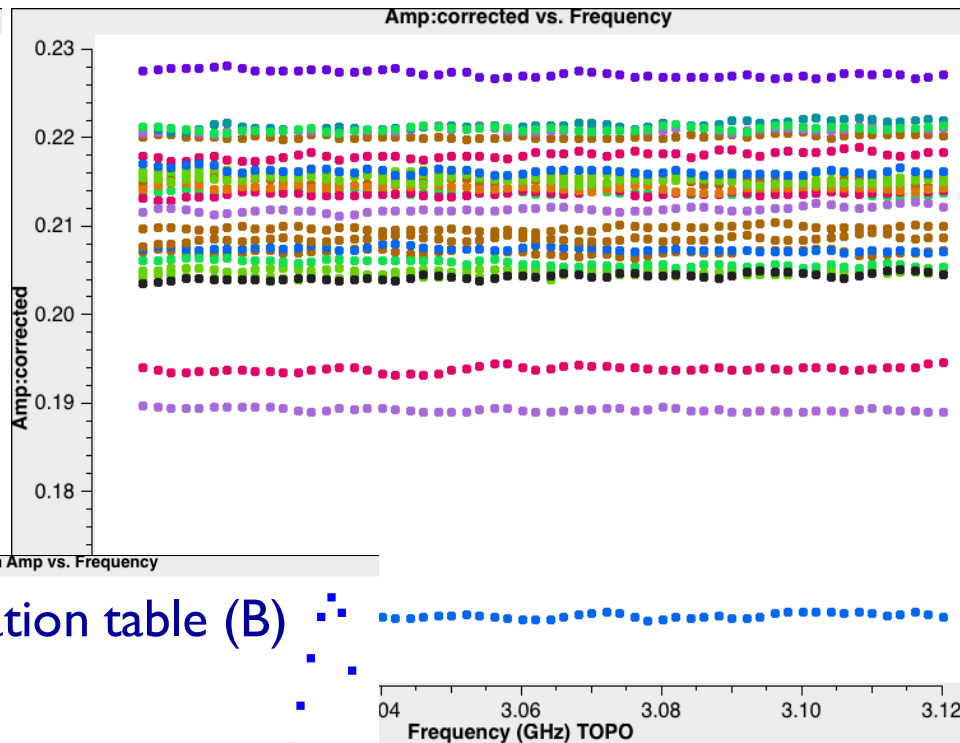


# A Practical Demonstration

Uncalibrated



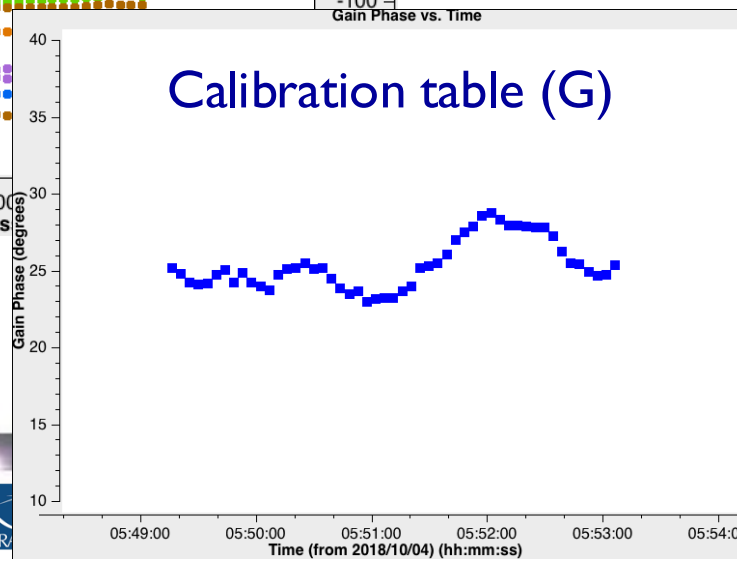
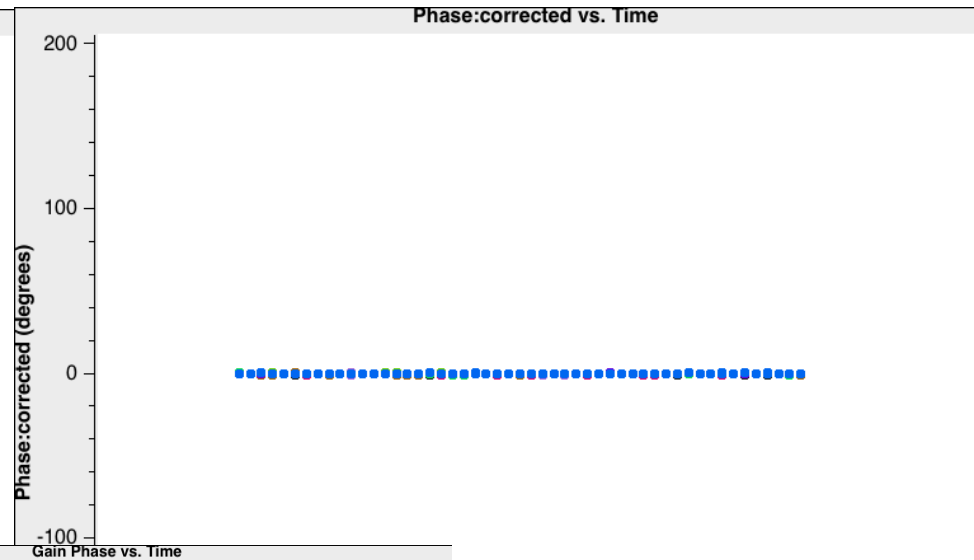
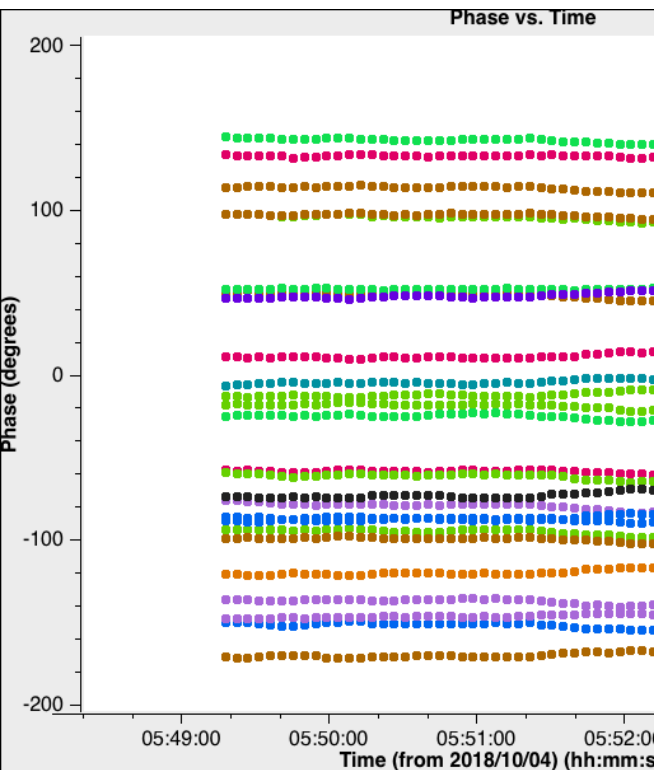
Calibrated



# A Practical Demonstration

Uncalibrated

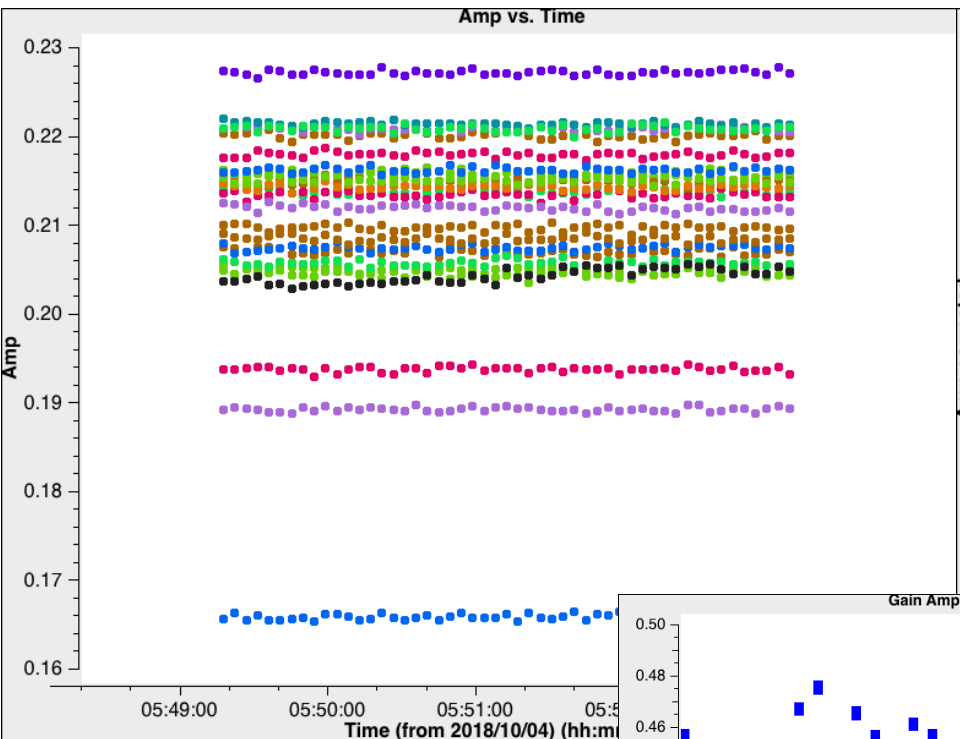
Calibrated



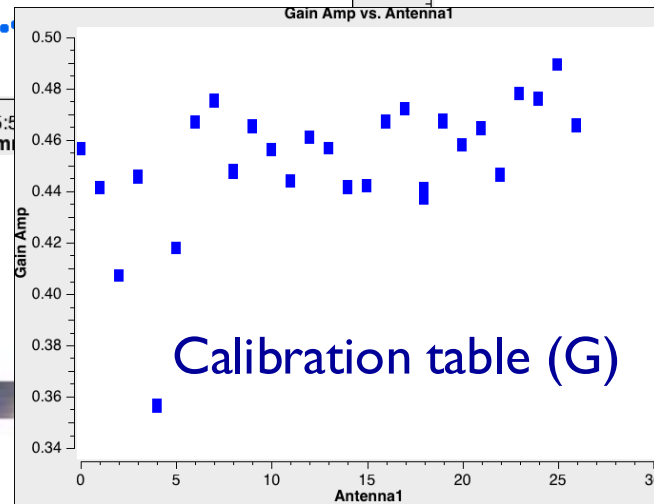
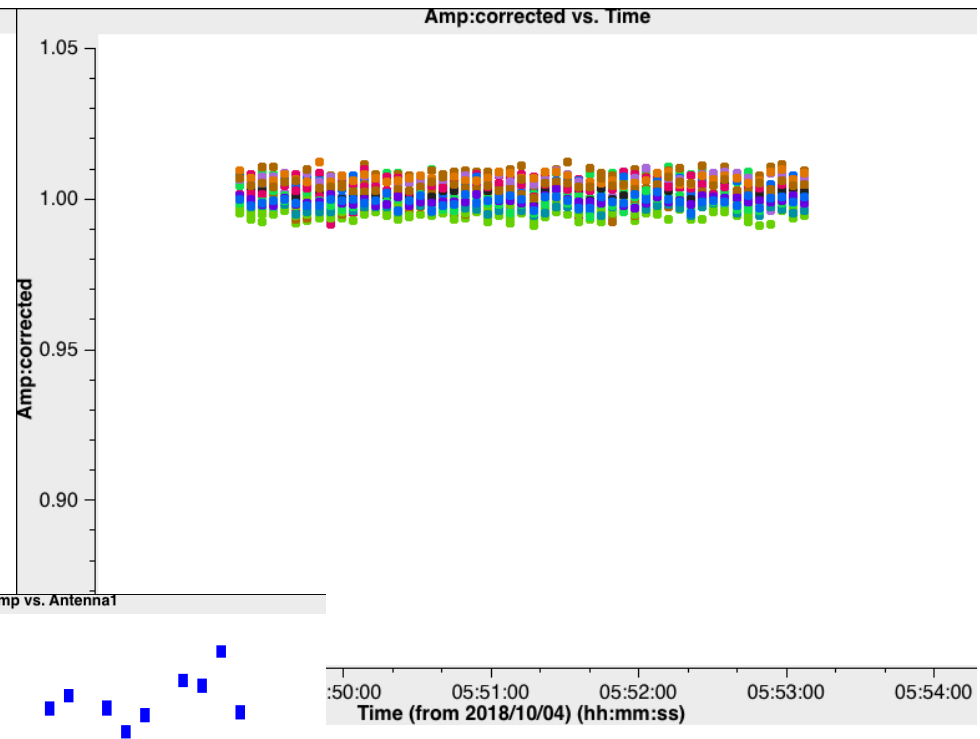
Calibration table (G)

# A Practical Demonstration

## Uncalibrated



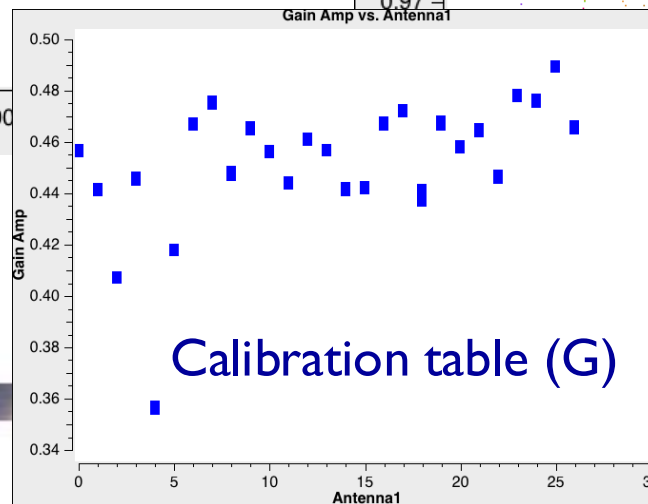
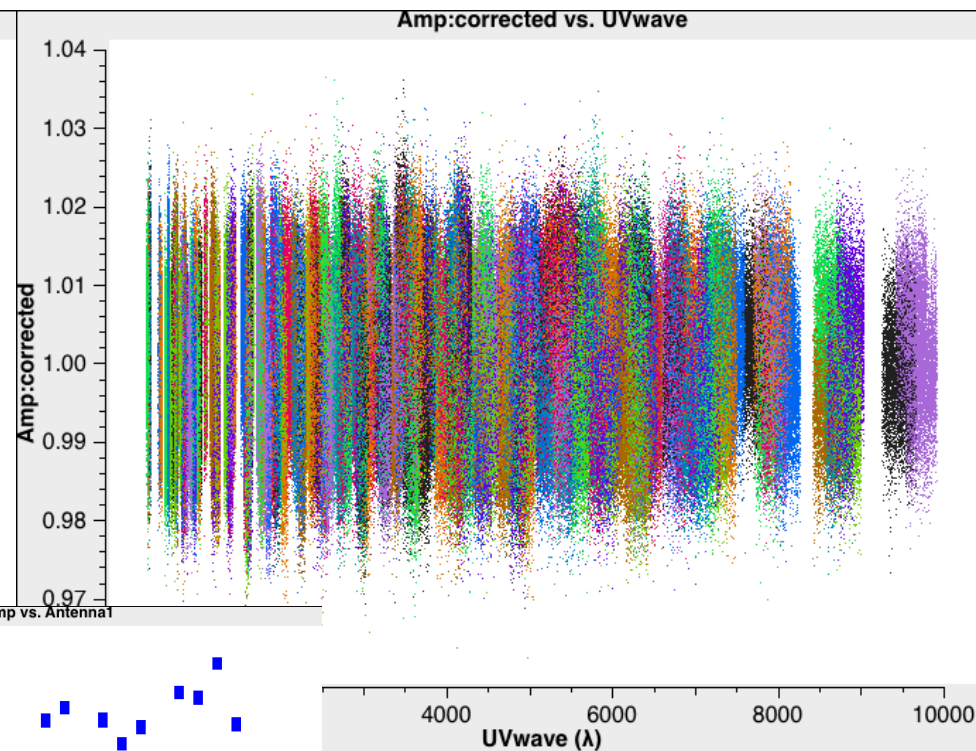
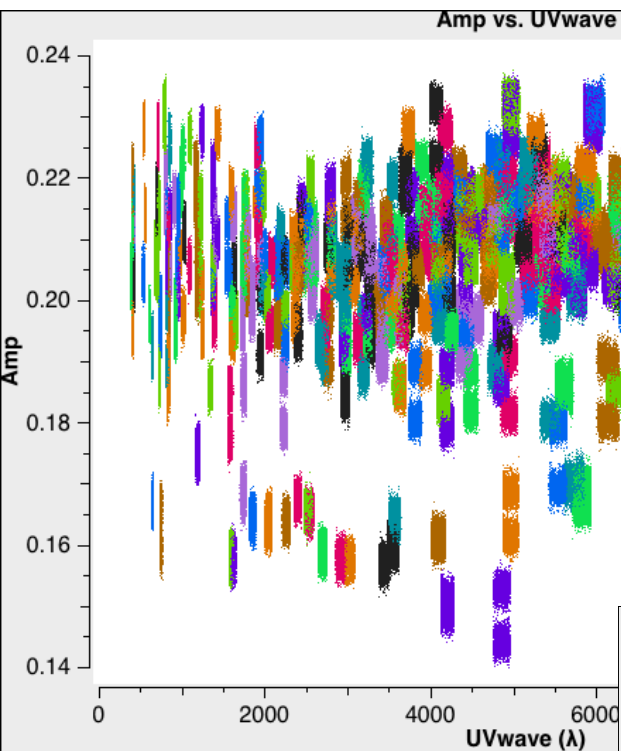
## Calibrated



# A Practical Demonstration

Uncalibrated

Calibrated



# Summary

- Calibration deals primarily with correcting the (baseline-based) visibilities for antenna-based effects
- Corrected visibilities are a prerequisite for imaging and analysis
- Accurate calibration relies on separating the corrupting effects
- Have a calibration strategy, not a recipe!



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