Wide-band Imaging

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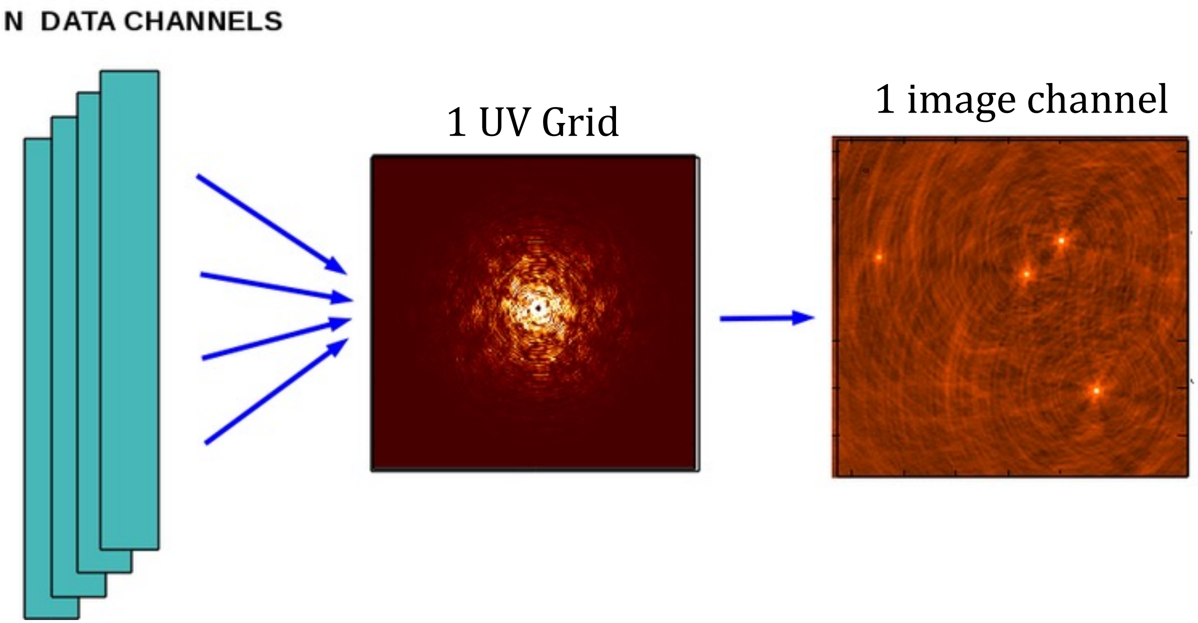
Introduction

This talk is about combining data over a range of frequencies in order to create a single ***continuum*** image.

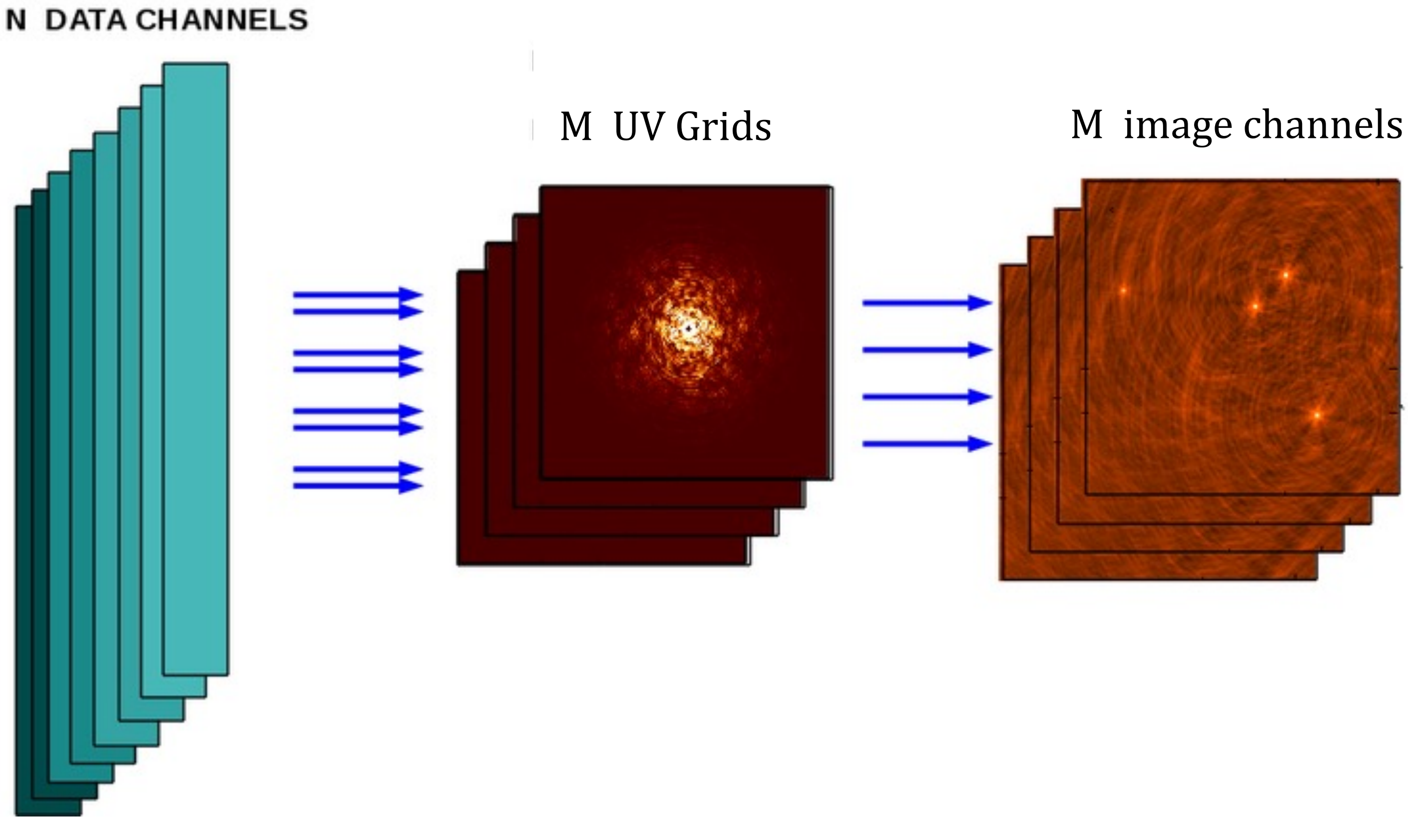


This process is called ***Multi-Frequency Synthesis*** (MFS)

(Figure adapted from casadocs .readthedocs.io)

Introduction

For data over a range of frequencies that is used to create a spectral image cube, see the 'Spectral Lines' talks on Tuesday by Ylva Pihlstrom



(Figure adapted from casadocs .readthedocs.io)

Wide Bandwidths

Some fundamental terminology:

		VLA L-band	VLA X-band
Frequency range	ν_{min}, ν_{max}	1 - 2 GHz	8 - 12 GHz
Bandwidth	$\nu_{max} - \nu_{min}$	1 GHz	4 GHz
Bandwidth Ratio	$\nu_{max} : \nu_{min}$	2 : 1	1.5 : 1
Fractional Bandwidth	$\frac{\nu_{max} - \nu_{min}}{\nu_o}$	66%	40%

Wide Bandwidths

New technologies are enabling even higher (instantaneous) fractional bandwidths, e.g.

- ATCA's 4cm receiver and and ngVLA Band 2 are both 4-12 GHz
- the BIGCAT upgrade for ATCA will provide 8 GHz of correlated bandwidth, and the ngVLA reference design has 20 GHz

→ 100% fractional bandwidth!

However, data over your full frequency range doesn't need to be observed simultaneously- what fundamentally matters to wide-band MFS imaging is only what you are ***combining*** to make your single continuum image.

This means you can build up larger bandwidths by:

→ observing with multiple frequency tunings

→ combining data from multiple receiver bands

Wide Bandwidths

Why is wide-bandwidth important for continuum imaging?

- Sensitivity

$$\sigma_S = \frac{2kT_s}{A_e[N(N-1)\,\Delta\nu\,\tau]^{1/2}}.$$

Increasing the bandwidth $\Delta\nu$ will decrease an image's thermal noise σ_S

Example: EVLA expansion project, 128 MHz → 8 GHz

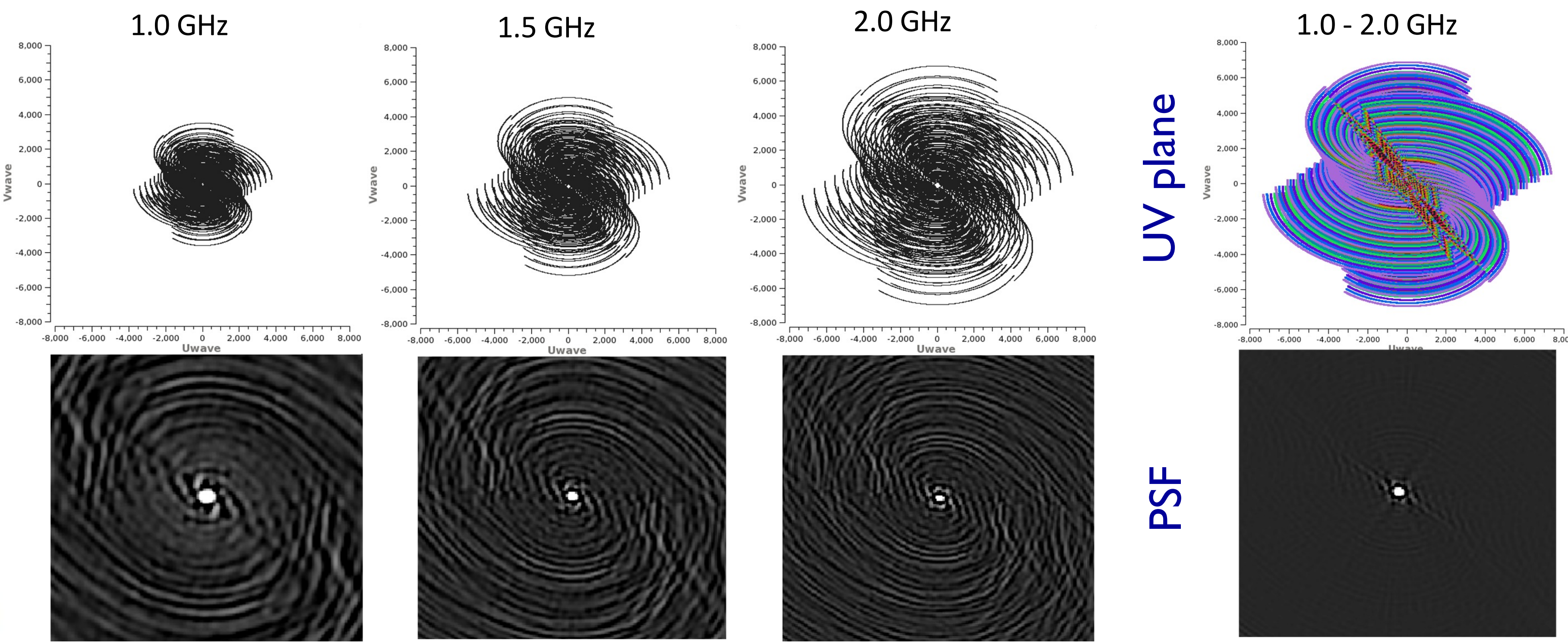
$$\sqrt{\frac{8\ \mathrm{GHz}}{128\ \mathrm{MHz}}} \approx 8$$ x better sensitivity

Wide Bandwidths

Why is wide-bandwidth important for continuum imaging?

- UV coverage: the baseline length in wavelengths

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{\lambda}\left[R(h,\theta)\right]\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$



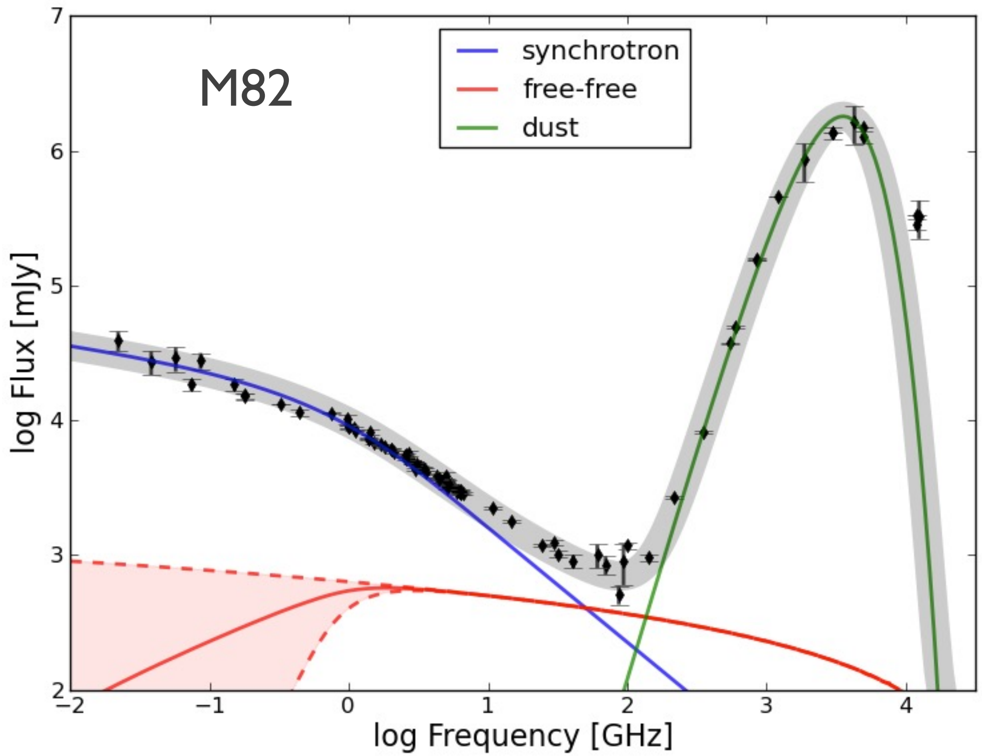
(Figure from casadocs.readthedocs.io)

Wide Bandwidths

Why is wide-bandwidth important for continuum imaging?

- Spectral information:

Wide bandwidths provide information about a source's spectral variation, from which we can deduce properties of the emission mechanism(s)



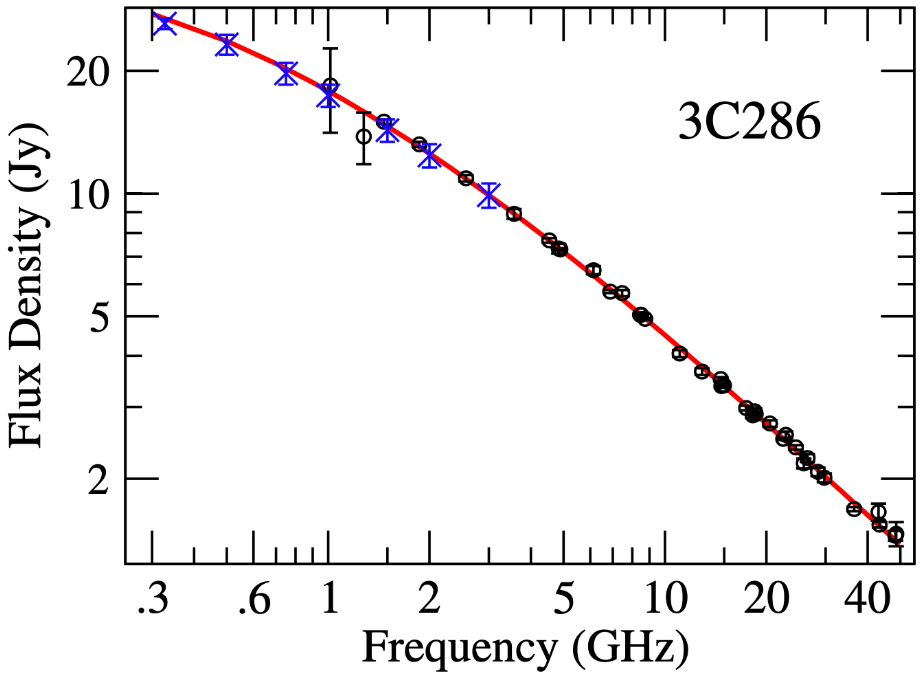
Spectral Variation

- ***Spectral variation*** is the primary challenge of wide-band MFS

- Two types of spectral variation:
  - ***intrinsic***: a property of the source itself
  - ***extrinsic:*** an instrumental effect of the telescope

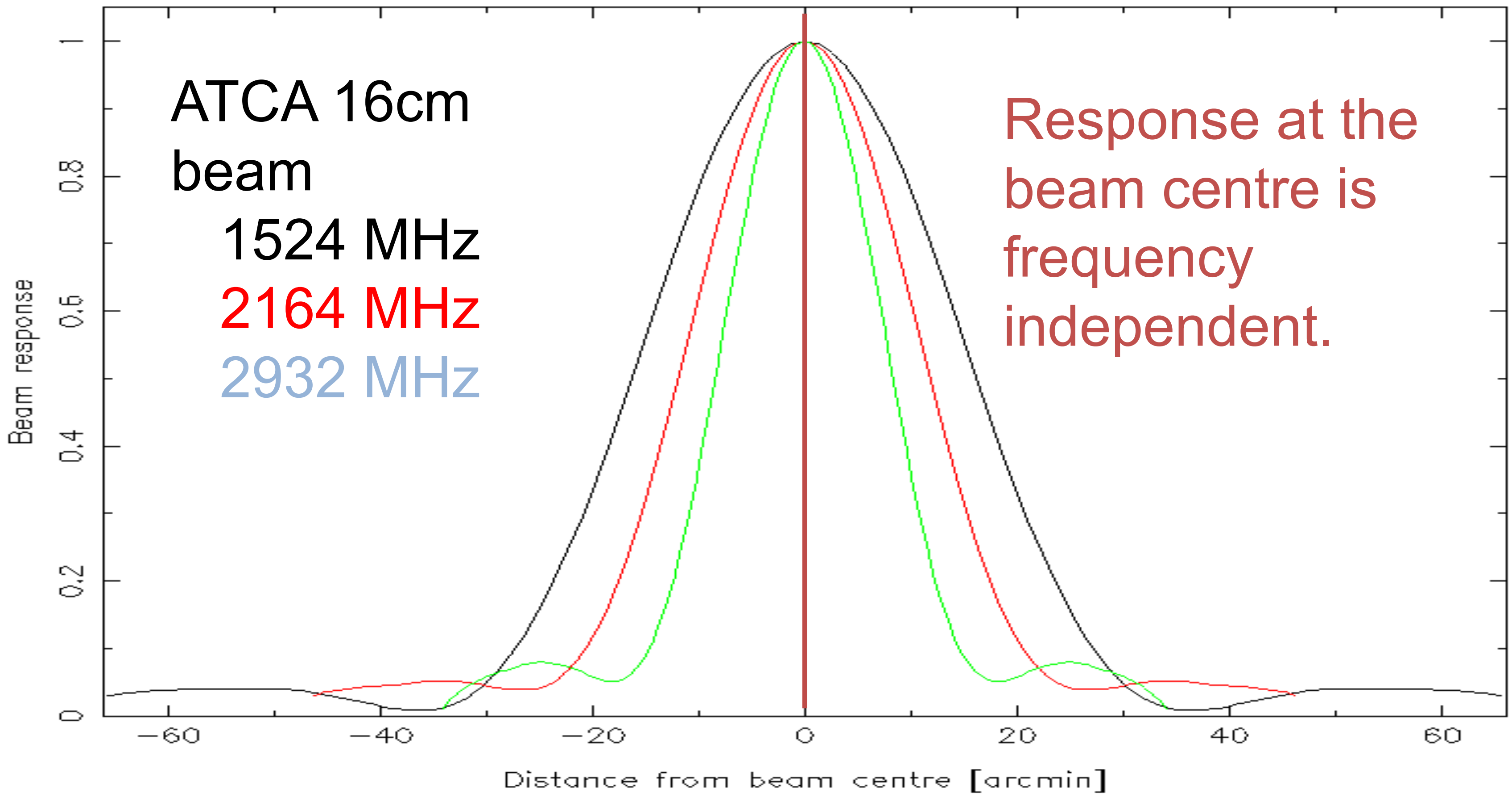
- Together, these give the source an ***apparent*** spectral variation, and it is this net effect that is present in our visibilities

400% flux variation over
160% fractional bandwidth
if we observed from 1-10 GHz →
(Perley & Butler 2013)



Spectral Variation

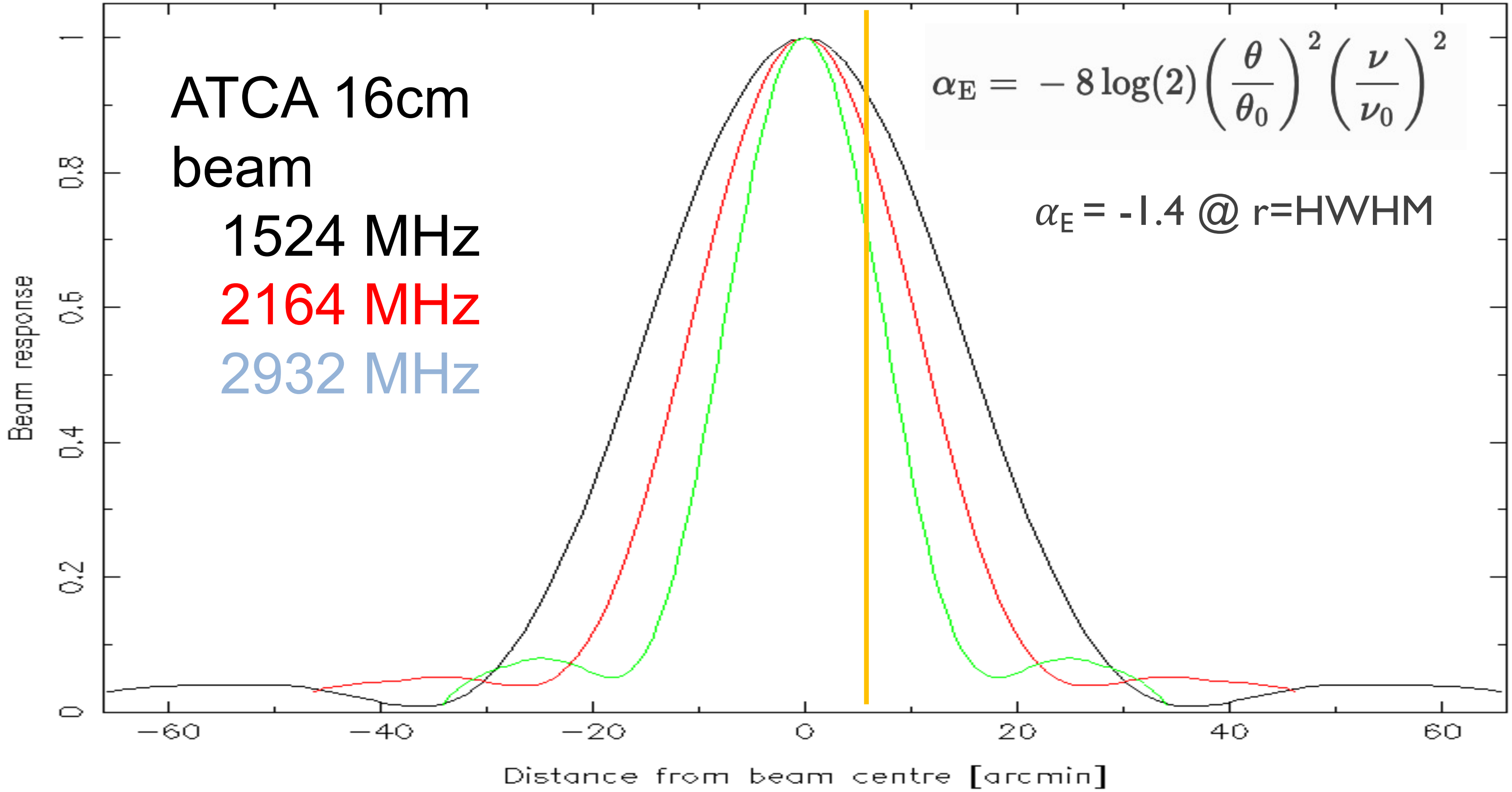
- ***extrinsic***: the primary beam size scales with frequency



(Figure by J. Stevens)

Spectral Variation

- ***extrinsic***: spectral index depends on distance from the beam centre



(Figure by J. Stevens)

Spectral Variation

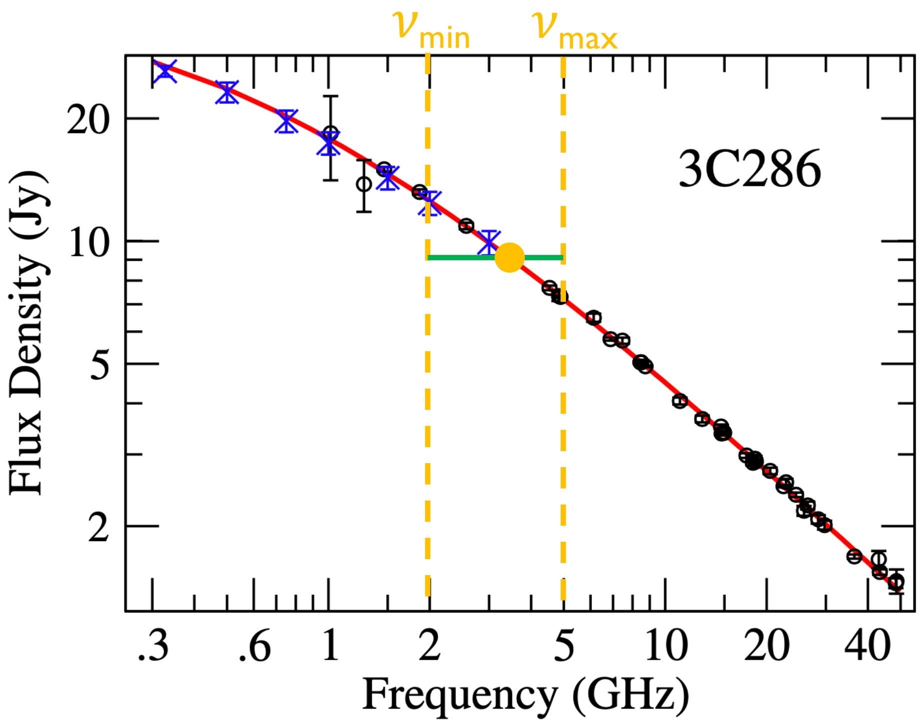
Why is spectral variation a problem?

The standard CLEAN model has no frequency dependence!

During the major cycle, the ***clean model*** is subtracted from the measured visibilities; we call the result of this the ***residual data***.

The model will match the data near the center frequency but deviate at the edges.

Flux remaining in the residual data will be convolved by the dirty beam, leading to artifacts in the image

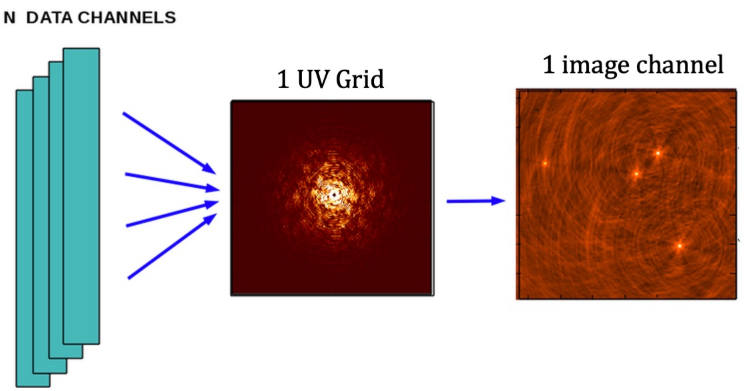


Spectral Variation

Why is spectral variation a problem?

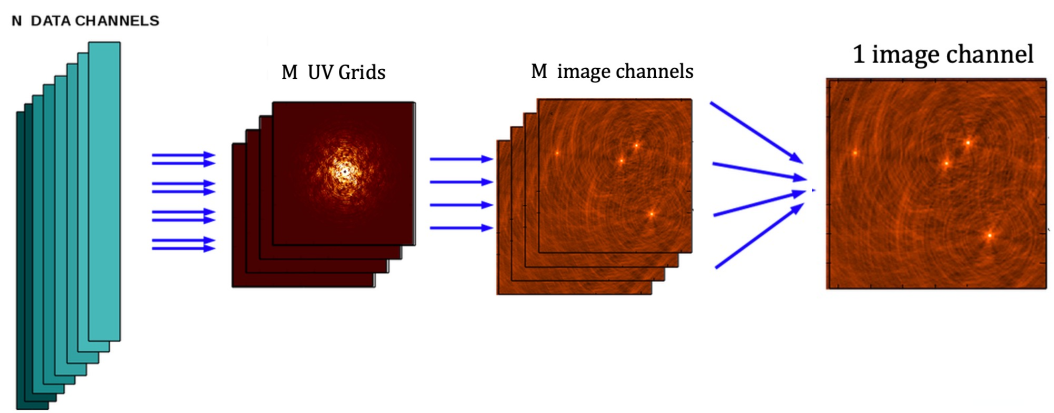
Convolutional artifacts in the dirty image are dependent on each source's spectral variation → ***each source has a unique PSF***

The PSF used by CLEAN will not perfectly subtract from these artifacts, leaving some convolutional pattern behind



We can average channels in either the UV domain or in the image domain

The additive property of the FT tells us these two scenarios are equivalent



Spectral Variation

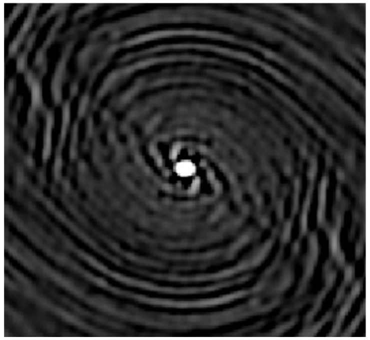
Why is spectral variation a problem?

Convolutional artifacts in the dirty image are dependent on each source's spectral variation → ***each source has a unique PSF***

The wide-band PSF can be thought of as the average of each narrow-band PSF

Each source's ***dirty beam*** is the average of each narrow-band PSF ***weighted by the source's spectrum***

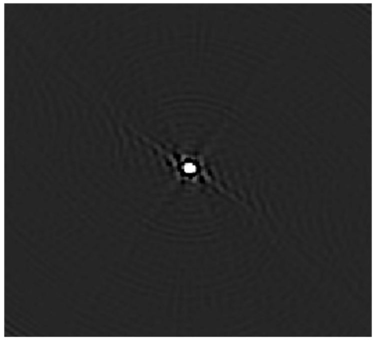
1.0 GHz



1.5 GHz

2.0 GHz

1.0 - 2.0 GHz



Dynamic Range

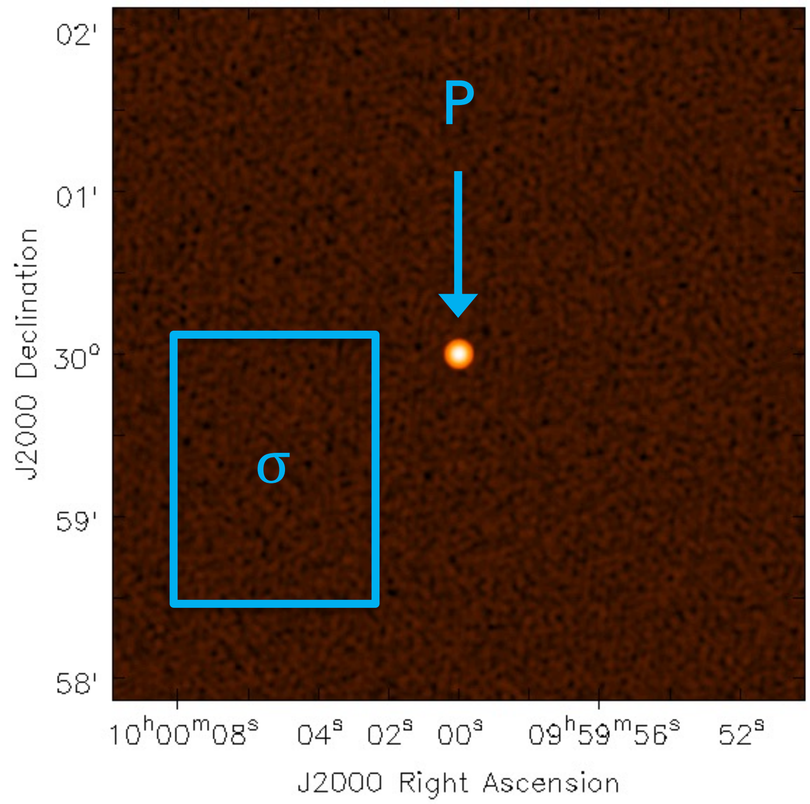
Adopt a simple definition using the peak flux S_{peak} and the off-source image RMS σ:

$$DR = \frac{S_{peak}}{\sigma}$$

S_{peak} = 0.6 Jy

σ = 2.5 uJy

DR = 240,000



Dynamic Range

Image artifacts can prevent you from reaching the theoretical noise limit:

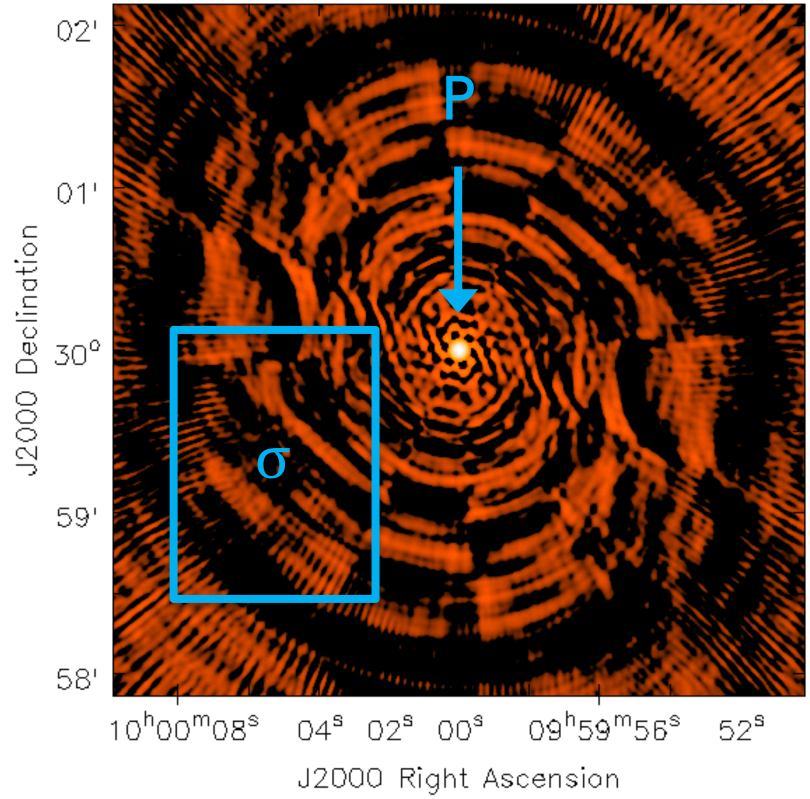
$$\mathrm{DR} = \frac{S_{peak}}{\sigma}$$

$S_{peak} = 0.6$ Jy

$\sigma = 81$ uJy

DR = 7,400

Image artifact due to unmodeled spectral structure →



Dynamic Range

When the dynamic range is lower, the noise is less likely to be dominated by artifacts

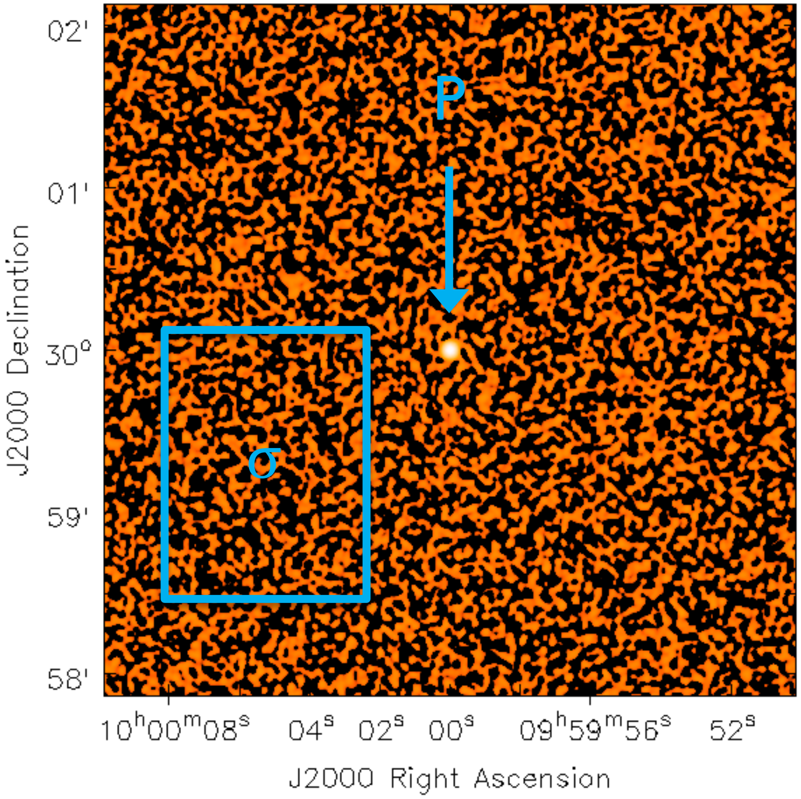
$$\text{DR} = \frac{S_{peak}}{\sigma}$$

$S_{peak} = 0.6$ Jy

$\sigma = 1.4$ mJy

$\text{DR} = 430$

The same wide-band image artifact is still in this image →

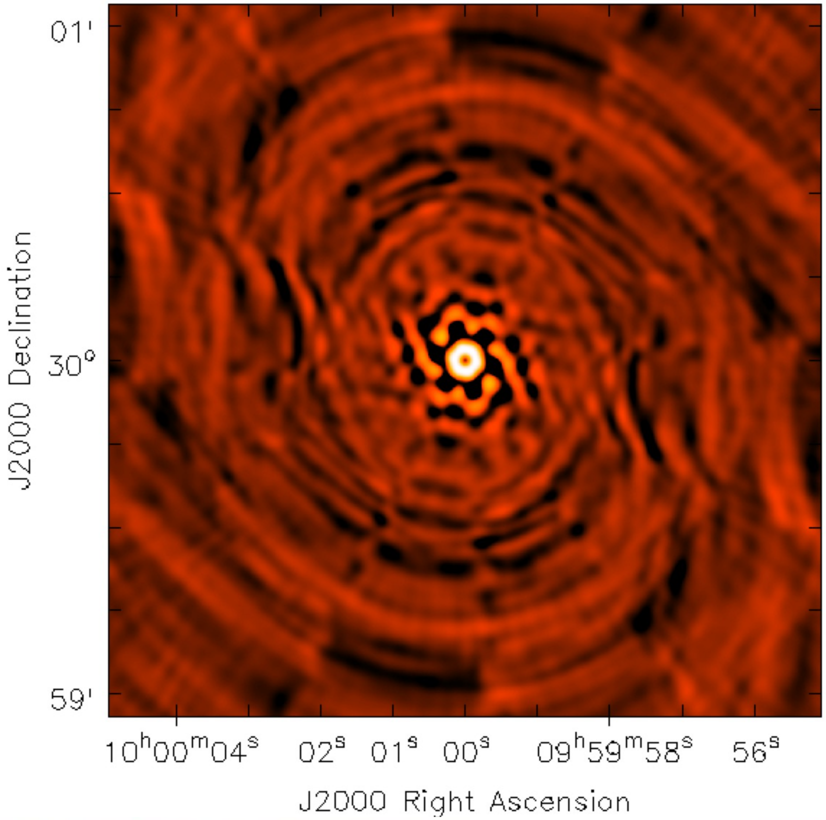


Dynamic Range

How do we prevent convolutional artifacts from limiting dynamic range?

- Understand their nature → Tuesday's Error Recognition talk
- If it is a wide-band problem:
  - create a cube of narrow-band images
    - shallow deconvolution
    - loss of resolution
  - **use a wide-band deconvolution algorithm**

Example residual image showing a wide-band artifact →



Wide-band Algorithms

Some different wide-band deconvolution algorithms include:

CASA:

- ***Multi-term Multi-frequency Synthesis (MT-MFS)***
  Rau and Cornwell (2011)

- ***MTMFS via Cube (MVC)***
  Bhatnagar, Rau and Golap (2013)

WSClean:

- ***Joined-channel Deconvolution***
  Offringa and Smirnov (2017)

These algorithms are also compatible with ***multi-scale*** deconvolution

Wide-band Algorithms

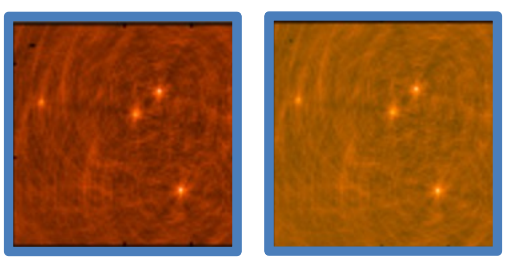
Multi-term Multi-frequency Synthesis (MT-MFS)

Models the spectrum of each clean component by a Taylor series

$$\vec{I}^{m}_{\nu} = \sum_{t=0}^{N_t-1} w^t_\nu \vec{I}^{sky}_t \quad \text{where} \quad w^t_\nu = \left(\frac{\nu - \nu_0}{\nu_0}\right)^t$$

where I_t^{sky} is the Taylor coefficient image and N_t is the order of the Taylor series expansion, i.e. a polynomial in ***linear flux*** vs. ***linear frequency***

N_t Model and Residual Images



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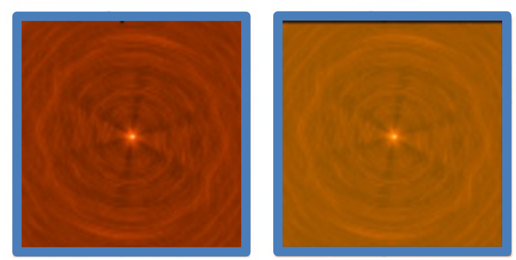
N_t PSFs



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Wide-band Algorithms

Multi-term Multi-frequency Synthesis (MT-MFS)

The ***major cycle*** produces a set of N_t Taylor coefficient images and PSFs by gridding with the applied weighting factor $w_v{}^t$

The ***minor cycle*** proceeds iteratively, constructing a frequency-dependent model image and subtracting PSFs from the residual images.

The ***restored*** Taylor coefficient images can be related to a power law:

$$I_{\nu}^{sky} = I_{\nu_0}^{sky} \left(\frac{\nu}{\nu_0}\right)^{\alpha+\beta\, log(\nu/\nu_0)}$$

$$\begin{aligned} \boldsymbol{I}^{\mathrm{m}}_{\nu_0} &= \boldsymbol{I}^{\mathrm{m}}_{0} \\ \boldsymbol{I}^{\mathrm{m}}_{\alpha} &= \boldsymbol{I}^{\mathrm{m}}_{1} / \boldsymbol{I}^{\mathrm{m}}_{0} \\ \boldsymbol{I}^{\mathrm{m}}_{\beta} &= \left[\boldsymbol{I}^{\mathrm{m}}_{2} / \boldsymbol{I}^{\mathrm{m}}_{0}\right] - \left[\boldsymbol{I}^{\mathrm{m}}_{\alpha}(\boldsymbol{I}^{\mathrm{m}}_{\alpha} - 1)/2\right] \end{aligned}$$

The primary beam's effect on $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ will need to be estimated and removed

Wide-band Algorithms

MT-MFS via Cube (MVC)

The wide bandwidth is partitioned into a small number (~5-10) of frequency intervals

The ***major cycle*** produces one image per interval, resulting in a ***coarse cube*** of PSFs and images, using standard narrow-band techniques

Each channel of the image cube is corrected for the primary beam

Wide-band Algorithms

MT-MFS via Cube (MVC)

The frequency weighting factor w_ν^t is applied to each channel of the coarse cubes, then channels are averaged over to produce N_t Taylor coefficient images and PSFs.

The ***minor cycle*** and ***restore*** steps proceed in the same way as MT-MFS

No further correction is needed for the primary beam's effect on α and β

Wide-band Algorithms

Joined-channel Deconvolution

The ***major cycle*** creates a coarse cube of PSFs and images, and also collapses the cube to form a single image at the reference frequency.

The ***minor cycle*** finds the brightest peak in the single image and does a model fit to the cube at that peak position. Polynomial and power-law models are supported.

Each channel is deconvolved with its own PSF which is scaled to the fitted model at that channel's frequency and multiplied by a gain factor $\boldsymbol{\gamma}$. The new residual cube is then collapsed to form a new single image at the reference frequency.

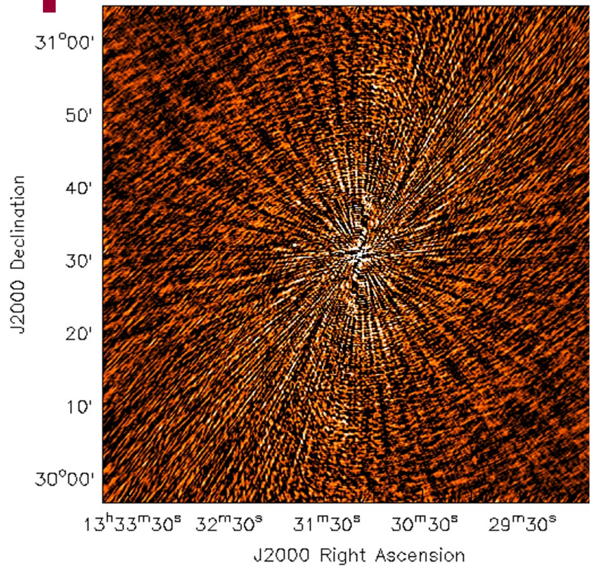
Examples

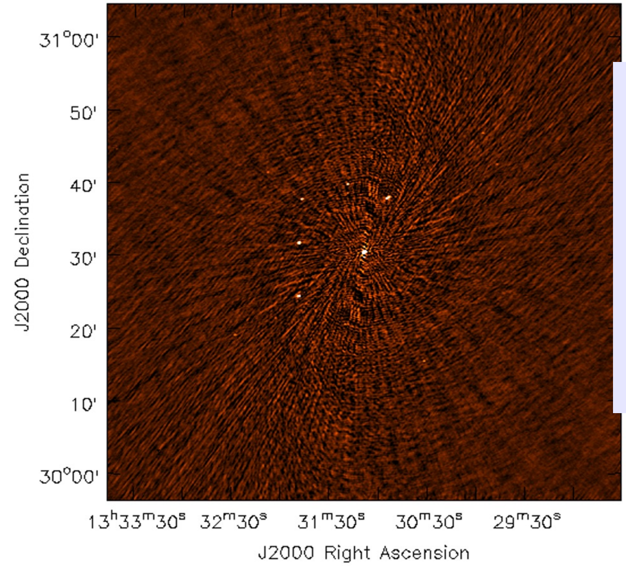
3C 286: 15 minutes of VLA L-Band data (1-2 GHz)

NTERMS = 1

Rms :
9 mJy -- 1 mJy

DR :
1600 - 13000





NTERMS = 2

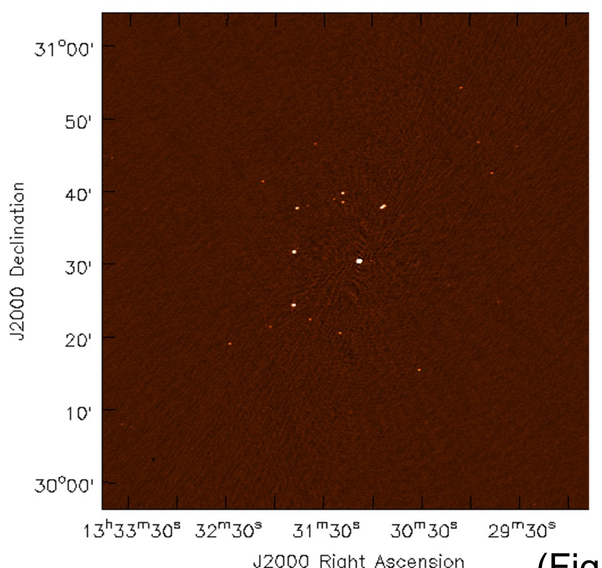
Rms :
1 mJy -- 0.2 mJy

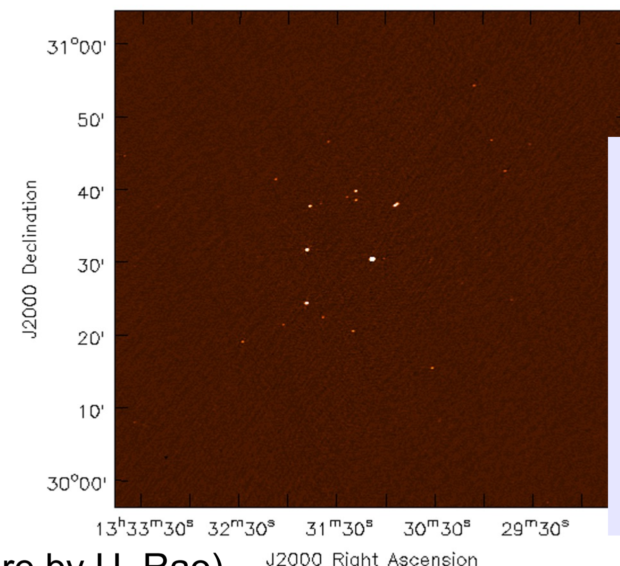
DR :
10,000 - 17,000

NTERMS = 3

Rms :
0.2 mJy -- 85 uJy

DR :
65,000 - 170,000





NTERMS = 4

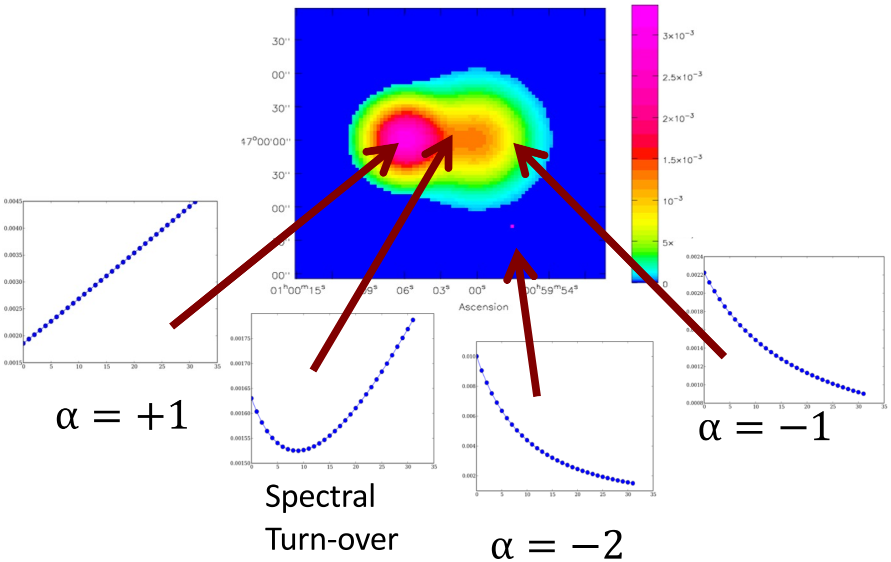
Rms
0.14 mJy -- 80 uJy

DR :
>110,000
- 180,000

(Figure by U. Rao)

Examples

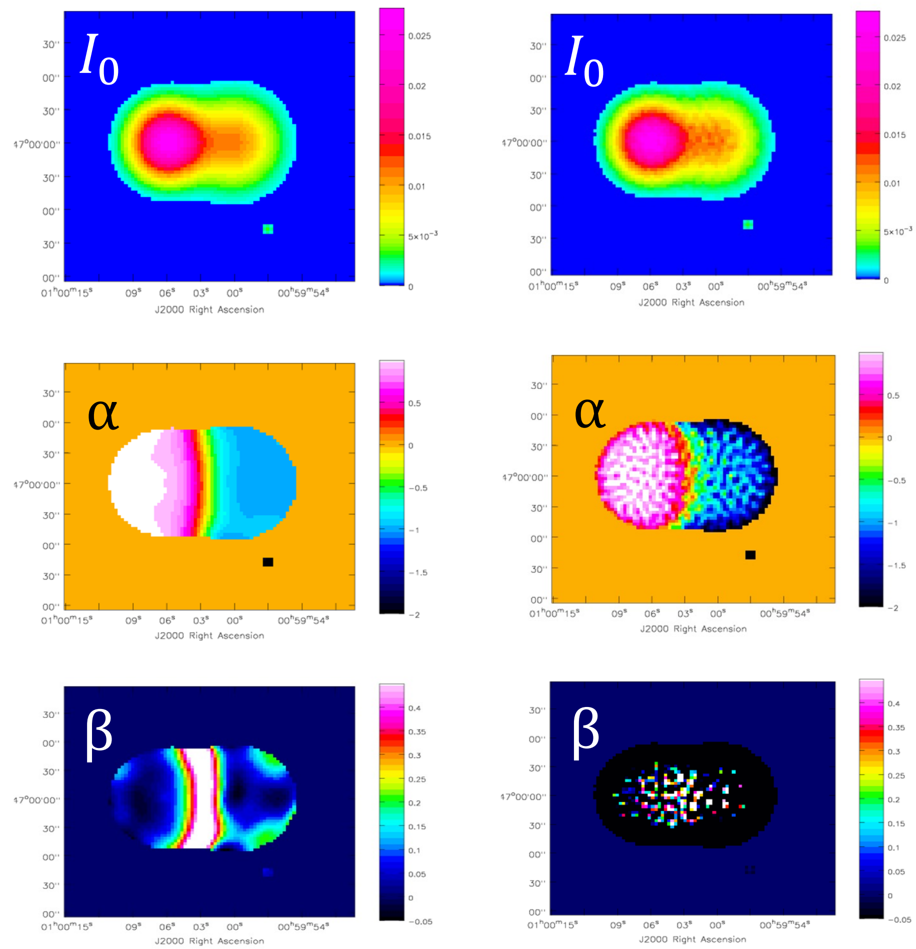
Model Intensity Image



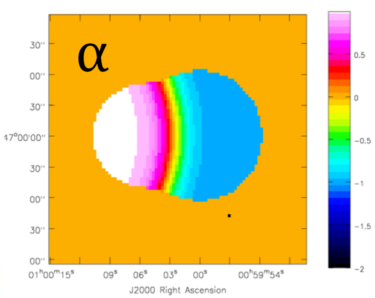
MT-MFS results for a 1-2 GHz VLA simulation (no noise)

multi-scale

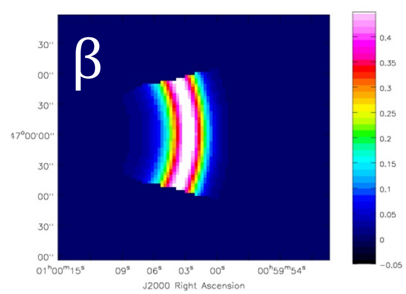
point-source



Model Spectral Index



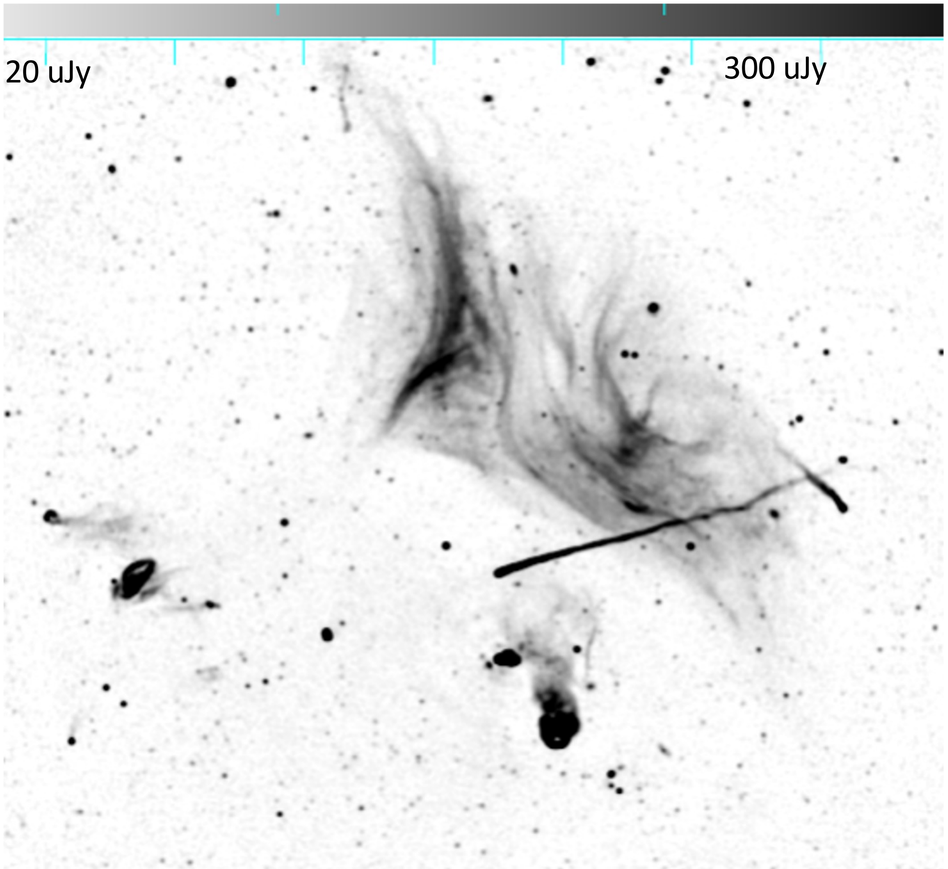
Model spectral curvature



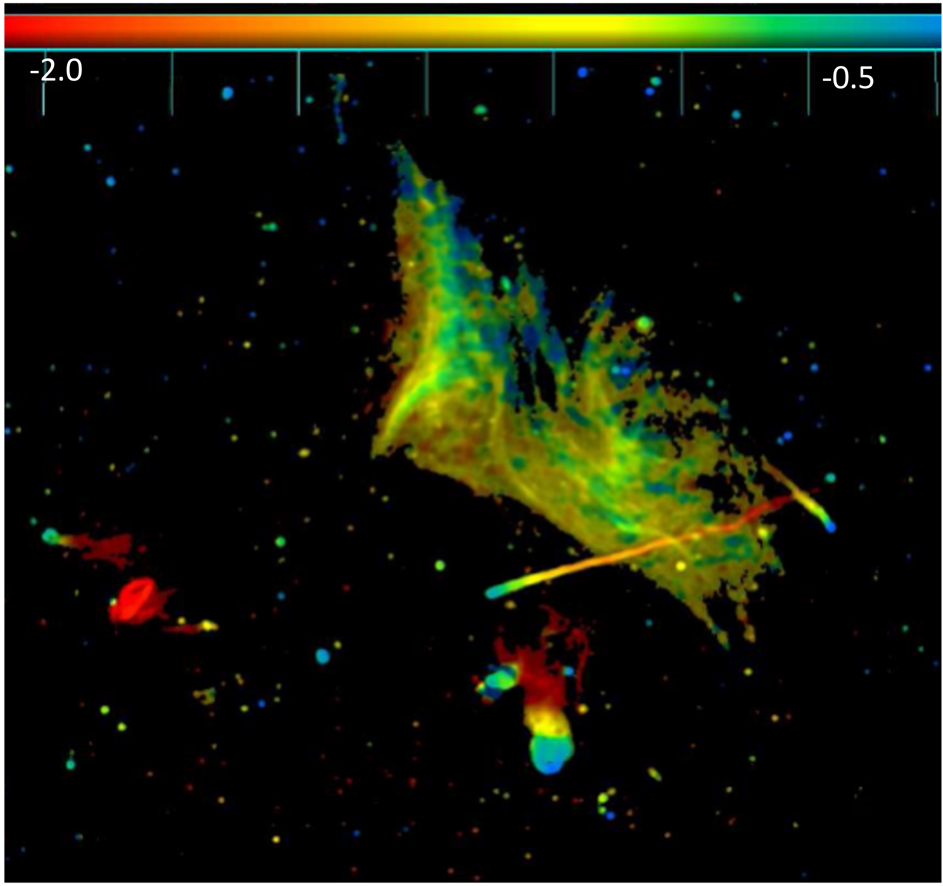
(Figure by U. Rao)

Examples

Abell 2256 (Owen et al 2014)
VLA L, S & C bands (1-2, 2-4, 4-8 GHz)



Intensity



Intensity weighted Spectral Index

Summary

- Wide-band imaging is a fundamental concern for high dynamic range continuum imaging

- Unmodeled spectral structure causes convolutional artifacts to persist in the final (cleaned) image

- Wide-band imaging and deconvolution algorithms account for each source's spectral variation, both intrinsic and apparent, in the model used for deconvolution

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