Widefield Imaging

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20th Synthesis Imaging Workshop

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Recap

- Rick Introduced Geometric Errors
- Josh introduced vCZ and Imaging
- Ian introduced direction dependent errors and self-calibration in addition to one possible solution facetting with direction dependence

I am here to make the case that do more **before you get to an image**

The data domain is ripe for fixing errors.

If any of material or the possibility of algorithm development appeals to you come find us and work with us

Outline

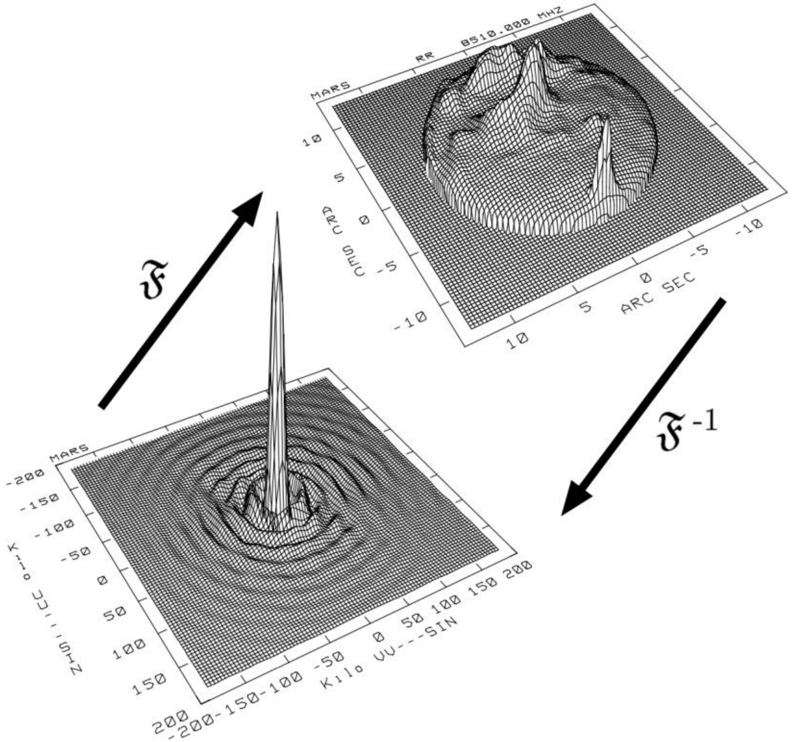
- vCZ Theorem
- Radio Inteferometric Measurement Equation
- Direction Dependent Effects
- Projection algorithms in theory
- Cost of imaging algorithms and their implications
- Widefield imaging in practice – a quick overview

van Cittert – Zernike Theorem (Josh's Talk)

An interferometer directly measures components of the Fourier transform of the intensity distribution

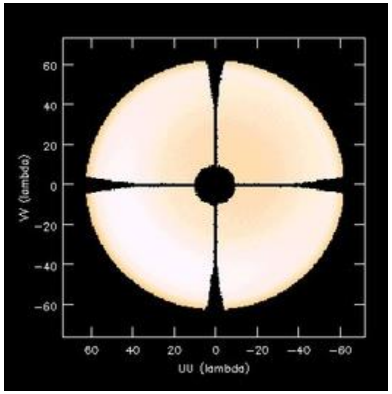
$$V_\nu(u,v) = \iint I_\nu(l,m) e^{-2\pi i(ul+vm)}\, dl\, dm$$

$$I_\nu(l,m) = \iint V_\nu(u,v) e^{2\pi i(ul+vm)}\, du\, dv$$

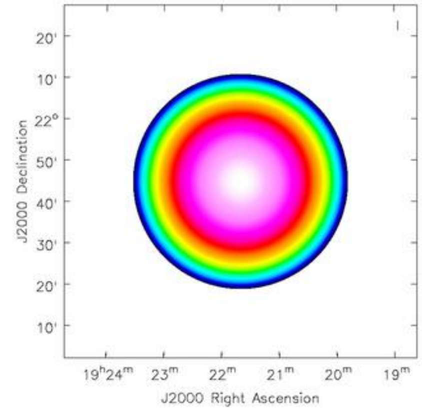


(Figure from Taylor, Carilli, & Perley)

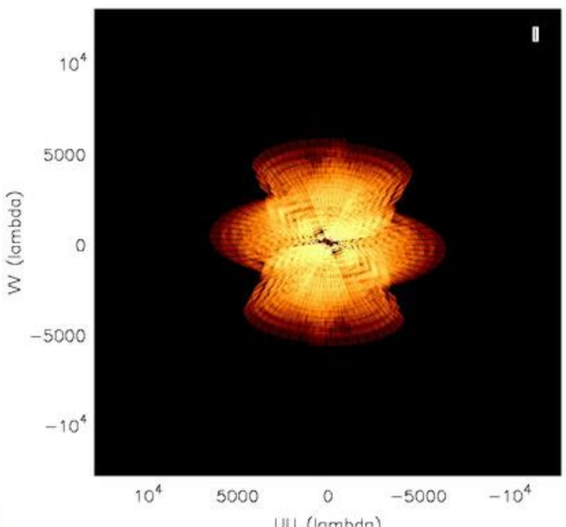
Useful Fourier Pairs



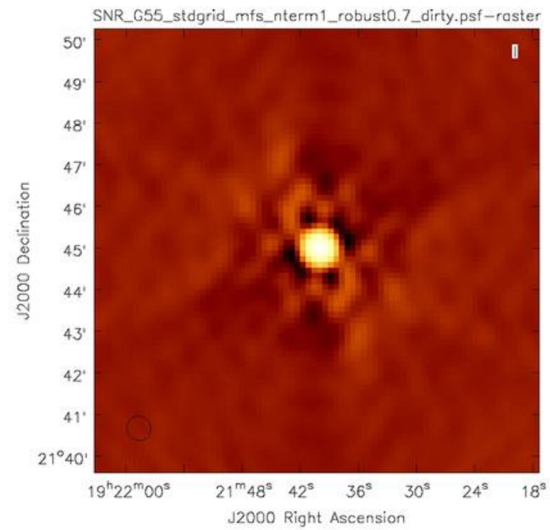
Aperture



Primary Beam



UV Coverage



Synthesized Beam - PSF

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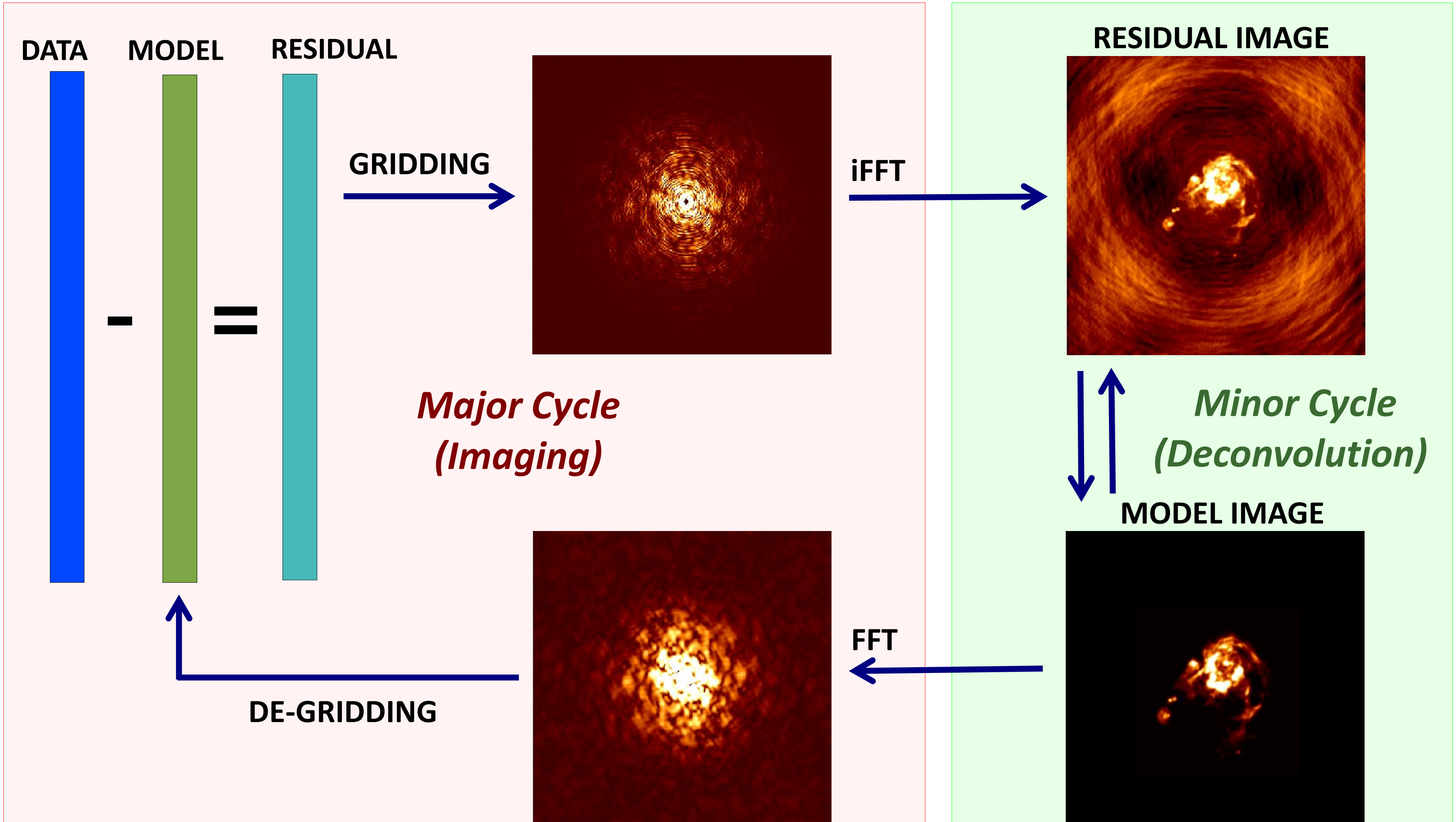
Recall Imaging



(figure adapted from U. Rao)

Radio Interferometer Measurement Equation

$$V_{ij}^{obs}(\nu, t) \approx M_{ij}(\nu, t) S_{ij}(\nu, t) \int\int I(l, m) e^{2\pi i(ul+vm)} dl dm$$

Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Ima ge)

Fourier transform kernel

Radio Interferometer Measurement Equation

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int I(l,m)e^{2\pi i(ul+vm)}dldm$$

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int \boxed{M_{ij}^{S}(l,m,\nu,t)} I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dldmdn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects, PB, ionosphere

Sky-brightness varies with frequency (time)

W-Term

Widefield Effects - Antenna PB

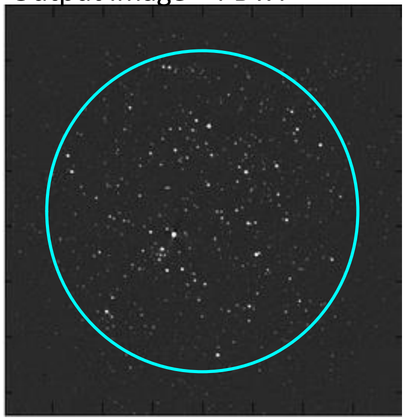
$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \int\int\int M_{ij}^{S}(l,m,\nu,t) I(l,m,\nu,t) e^{2\pi i(ul+vm+w(n-1))} dl dm dn$$

$$I^{obs}(l,m) = \sum_{ij,t,\nu} I_{ij}^{PSF}(l,m,t,\nu) * [P_{ij}(l,m,t,\nu) I_{ij}^{sky}(l,m,t,\nu)]$$

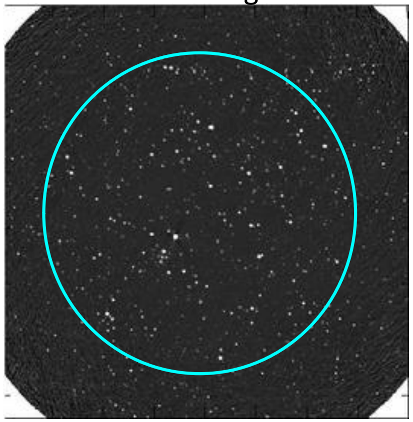
The Sky is multiplied by a Primary Beam, and is being sampled by each baseline

Normally we assume a singular non time varying model for the PB and divide it out of the equation

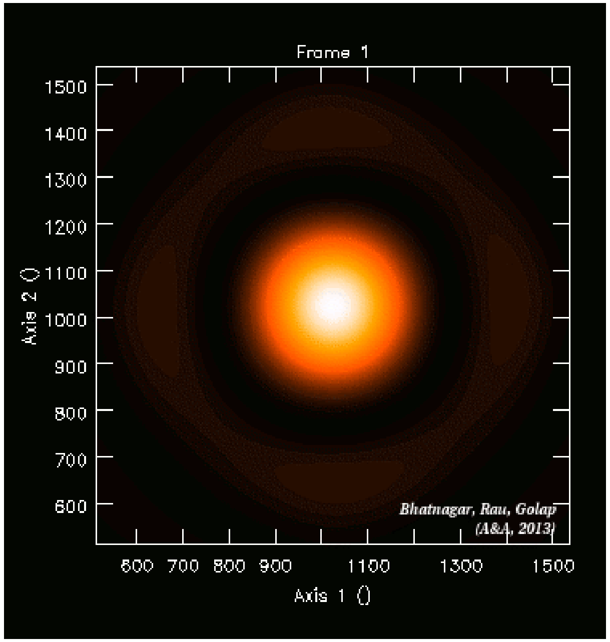
Output Image = PB x I

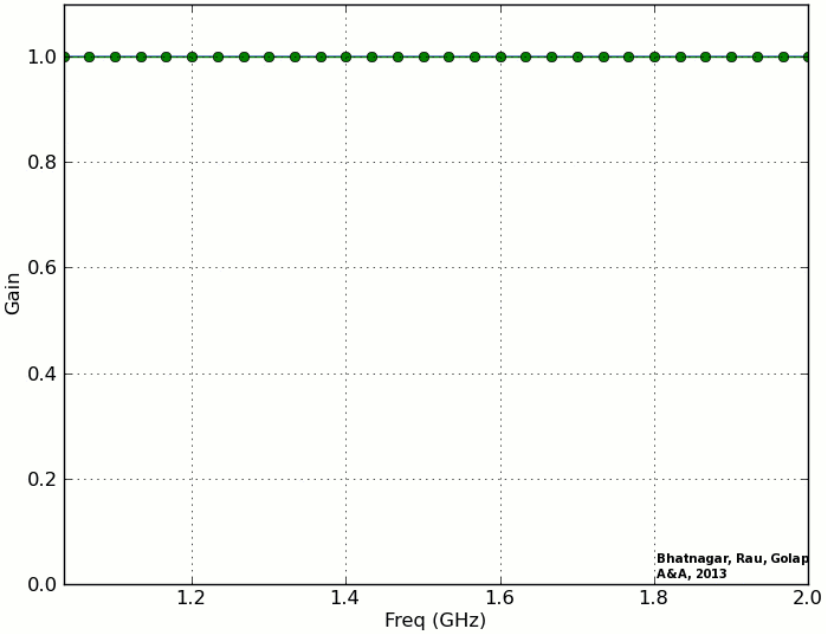


PB-corrected Image



Widefield Effects - Antenna PB

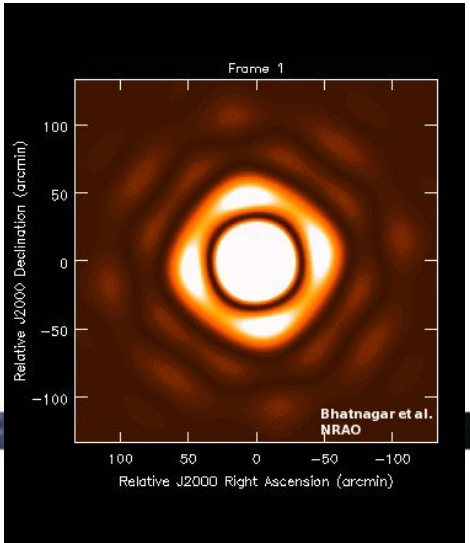




PBs change in time and frequency

They introduce a beam spectral index that introduces direction dependent effects into your image.

They can be measured and known apriori



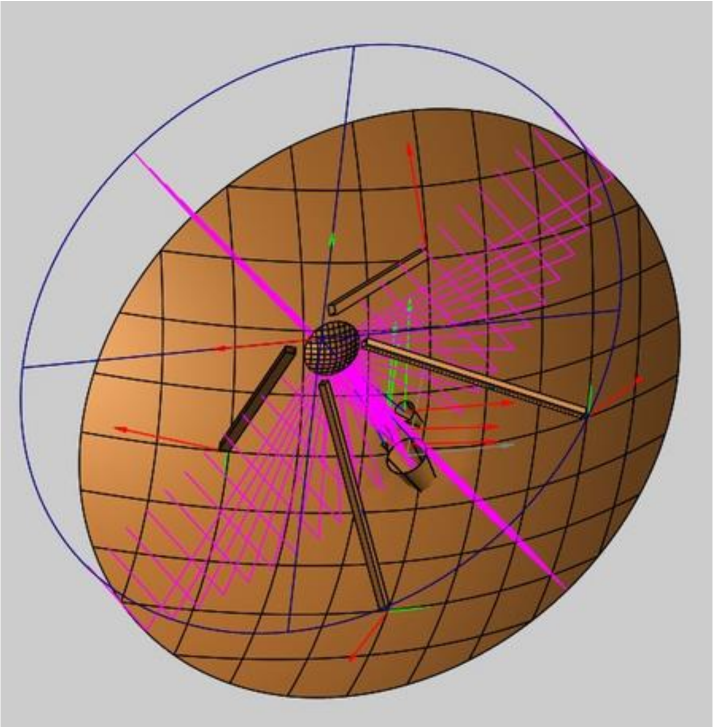
Measurement Equation - I

$$\vec{e_a} = J_i.\vec{\epsilon} \qquad \vec{e_b} = J_j.\vec{\epsilon}$$

$$\vec{I}_{ab} = \vec{e_a} \otimes \vec{e_b}$$

$$\vec{V}_{pq}^{obs} = \mathcal{F}_{pq} S^{ab} \vec{I}_{ab}$$

The measured power at the correlator is a cross correlation of the voltages received by two antennas. Note that the antennas can be of different types i,j The basis a,b are the sky stokes basis The matrix s performs the transfer of basis from stokes to an orthogonal feed basis p,q vCZ theorem relates the sky brightness distribution to the measured interferometric voltages.



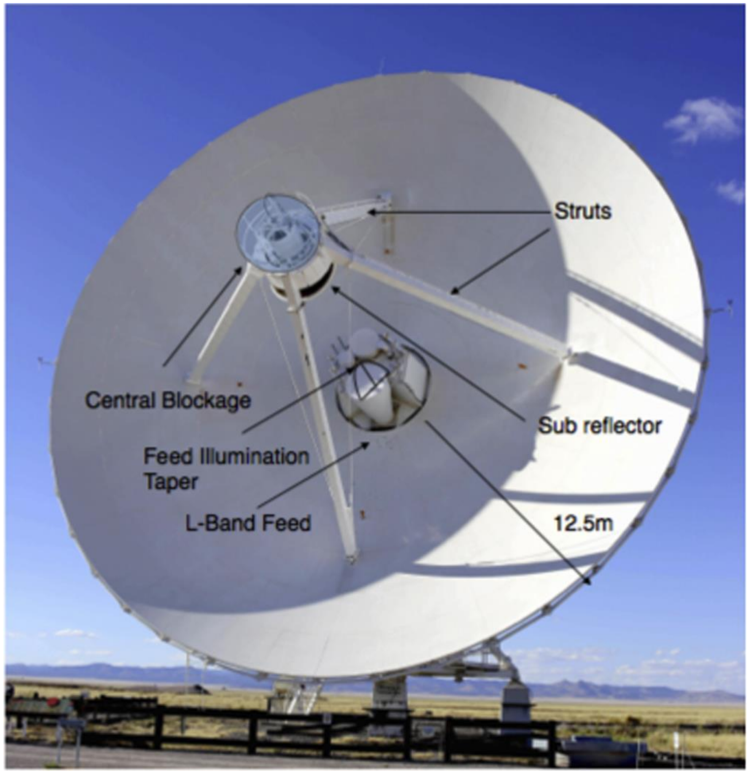
L-Band VLA Grasp 10 simulation .
Bruce Veidt DRAO.

Measurement Equation - II

$$\vec{I}_{ab} = (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}^*_b \vec{J}^*_j)$$

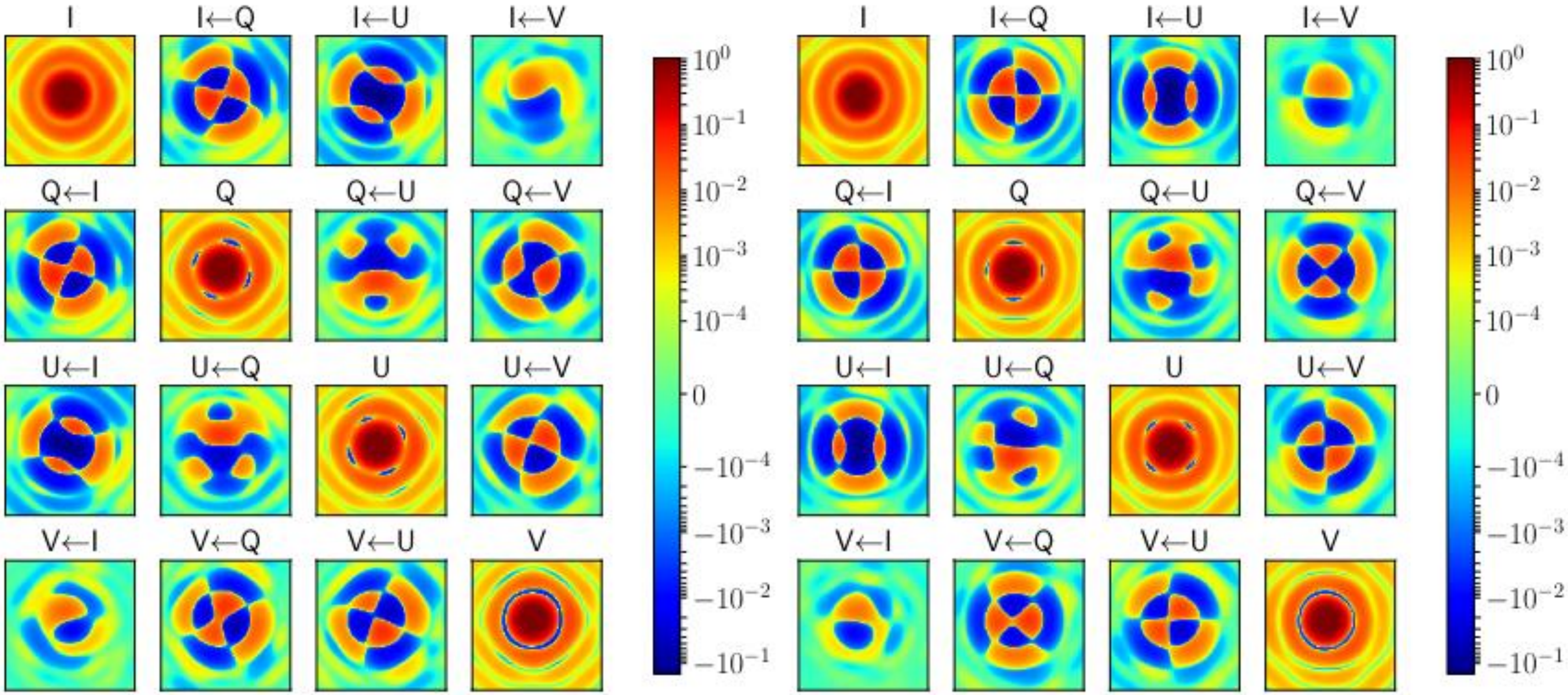
$$\vec{V}^{obs}_{pq} = \mathcal{F}_{pq} S^{ab} (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}^*_b \vec{J}^*_j)$$

$$\vec{V}^{obs}_{pq} =_{pq} S^{ab} (\vec{A}_i \circledast \vec{A}^*_j) \star \vec{V}_{ab}$$



Our goal then is to be able to reconstruct true sky brightness or the true sky coherence function

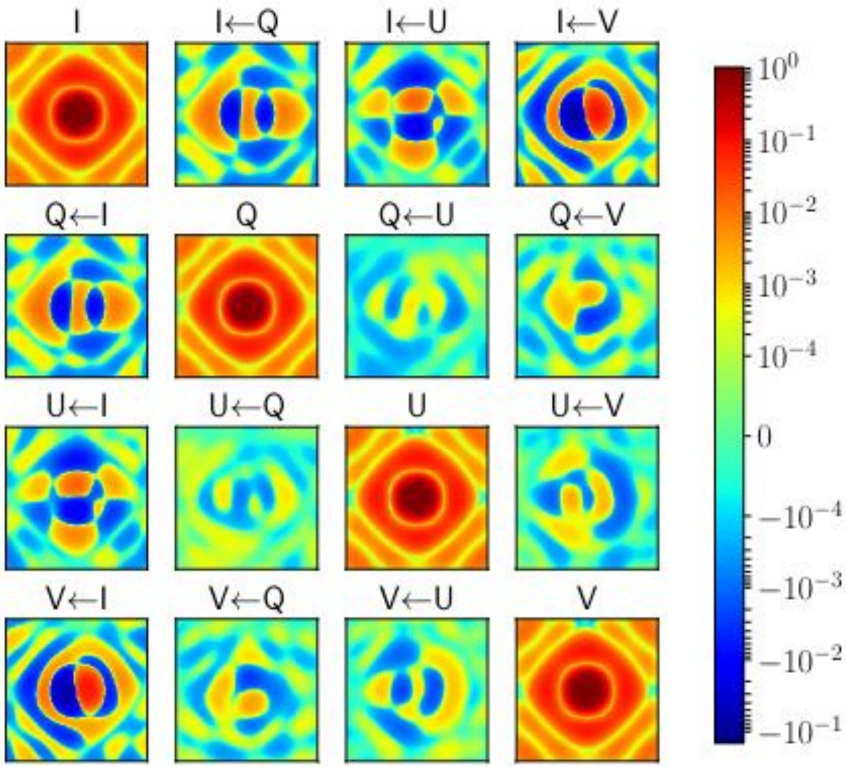
Widefield Effects - Antenna PB



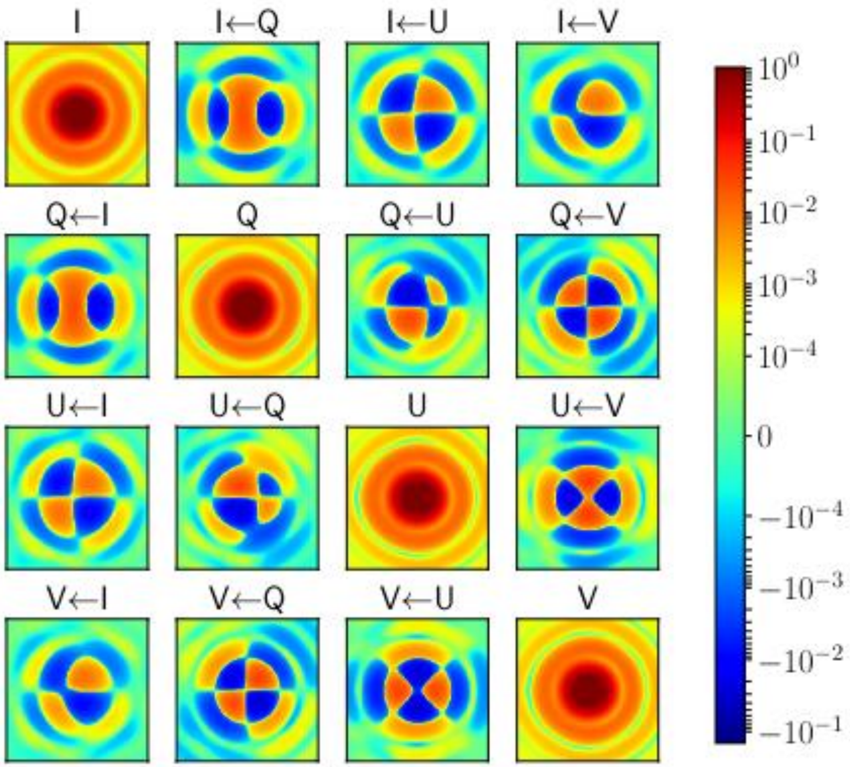
(c) ALMA DV

(d) ALMA DA

Widefield Effects - Antenna PB



(a) EVLA



(b) MeerKAT

Sekhar et al 2021

A-Projection

If the antenna A term were (approximately hermitian) we could consider an inversion operation of the form

$$(\vec{A}_i \circledast \vec{A}_j)^{M^\dagger} \star \vec{V}_{ab}^{obs} = |Aij|^2 \star \vec{V}_{ab}$$

This term that is applied on the left is the kernel for A-projection at the time of gridding. The term in the modulus is the square of the antenna PB (forward gain) in the image plane so we divide it out after we take the FT to go from data to the image.

$$\frac{\mathcal{F}^\dagger (\vec{A}_i \circledast \vec{A}_j)^{M^\dagger} \star \vec{V}_{ab}^{obs}}{|Mij|} = PB.I_{ab} \qquad \mathcal{F}^\dagger(\vec{A}_{ij}) = \vec{M}_{ij}$$

Bhatnagar et al 2008, 2013

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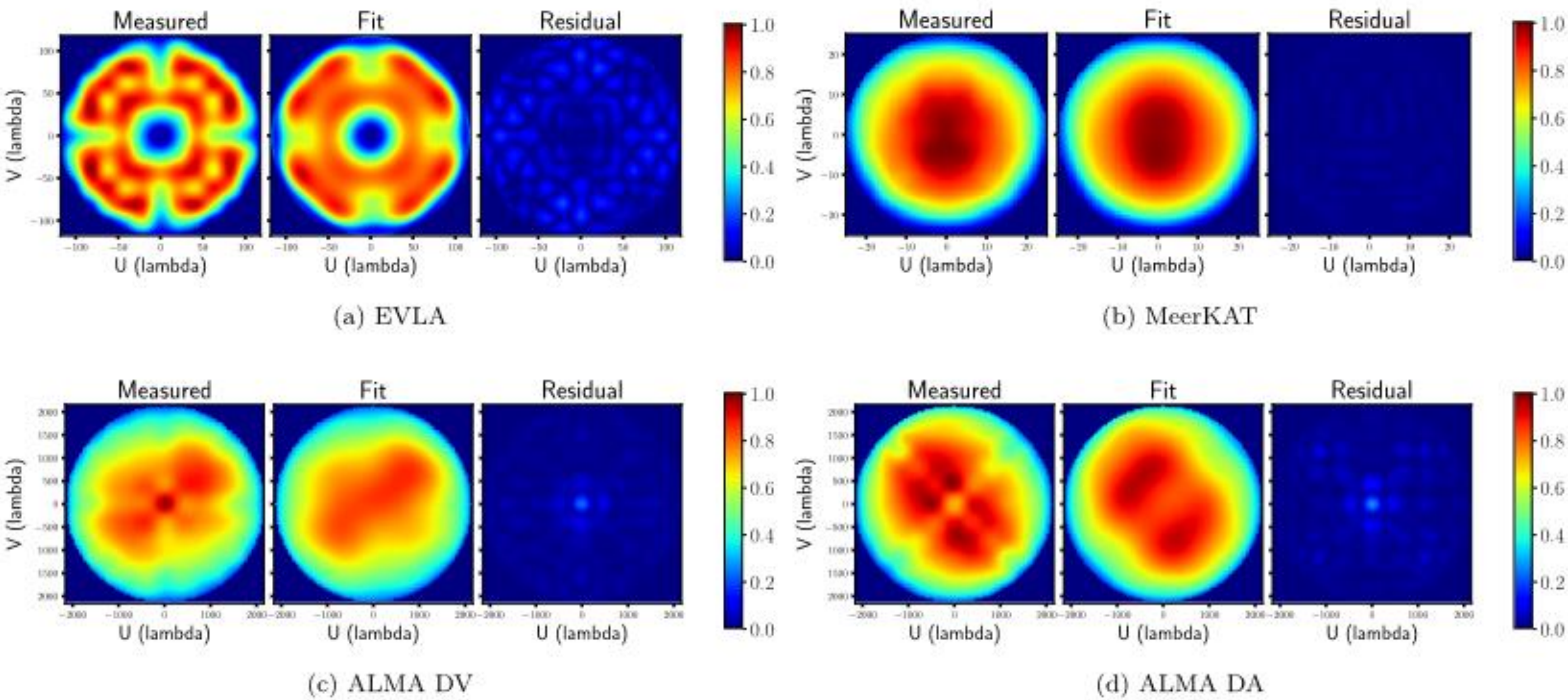
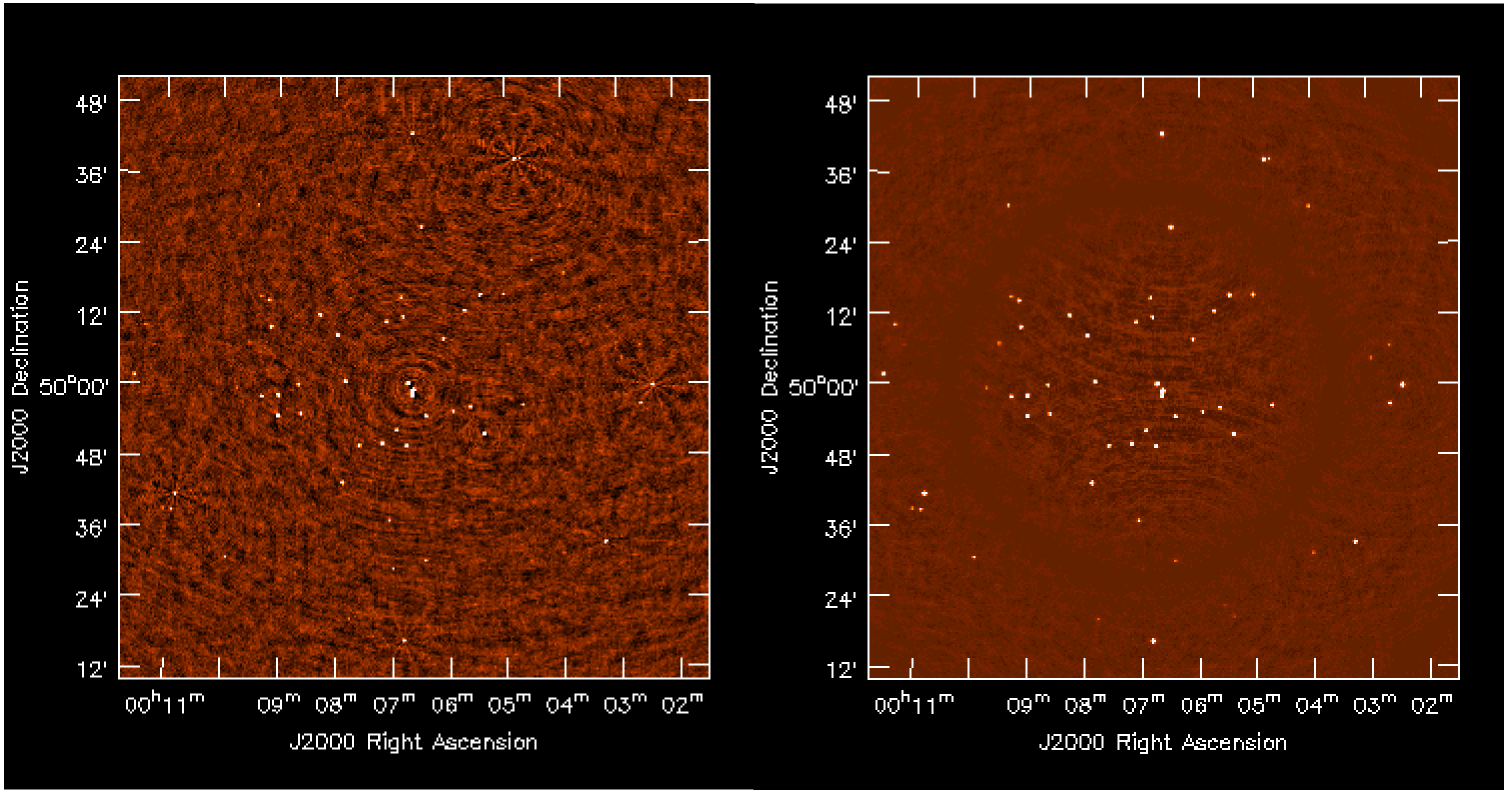
Widefield Effects - Antenna Apertures

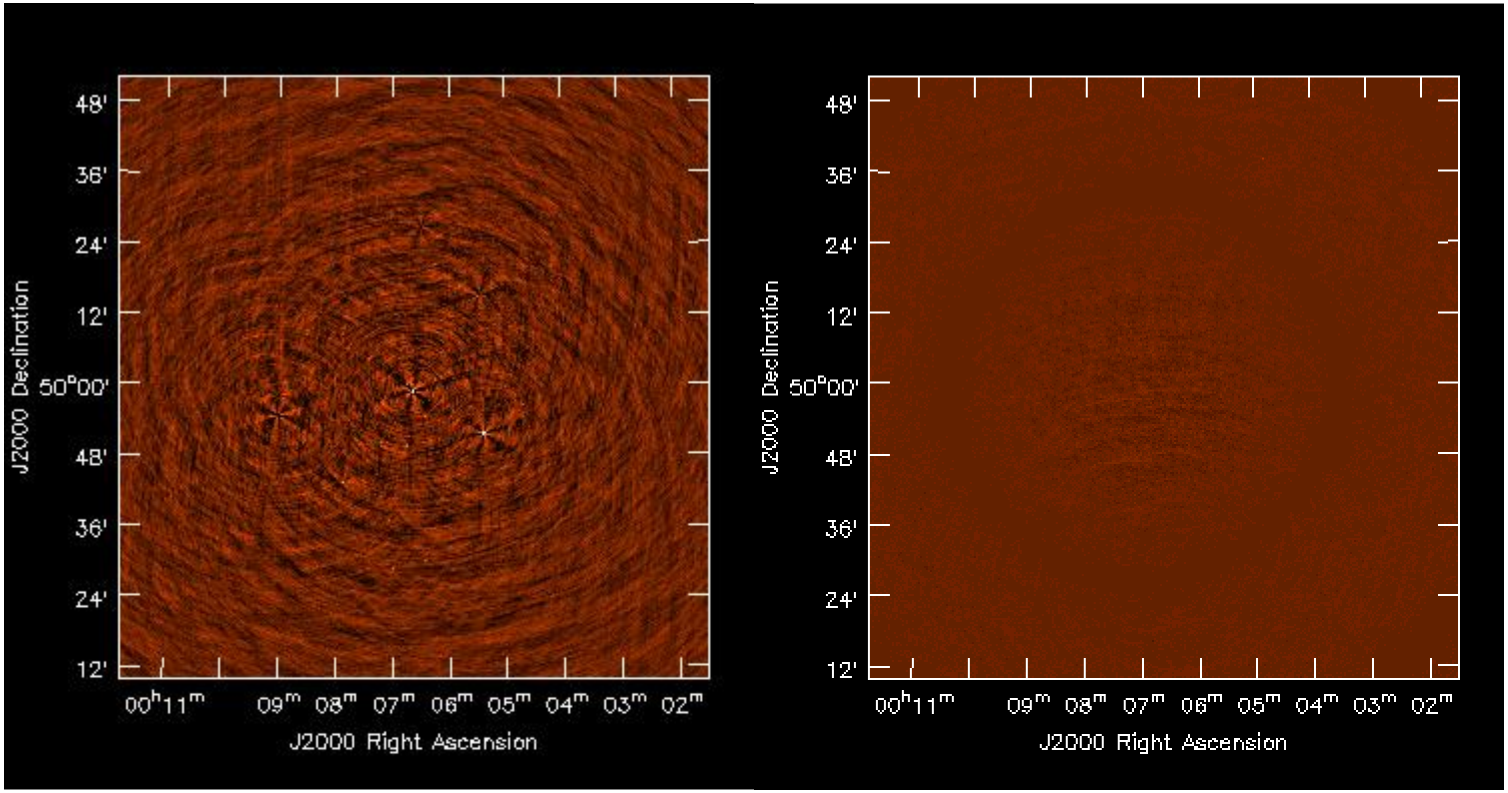


A-Projection in action



Stokes I

A-Projection in action



Stokes V

Widefield Effects - Geometry

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int I(l,m)e^{2\pi i(ul+vm)}dldm$$

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)\boxed{e^{2\pi i(ul+vm+w(n-1))}}dldmdn$$

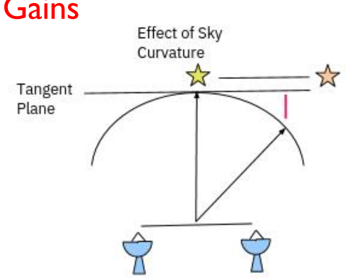
Direction Independent Gains

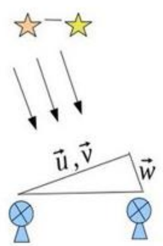
UV sampling function

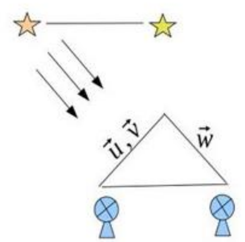
Direction Dependent Effects, PB, ionosphere

Sky-brightness varies with frequency (time)

W-Term





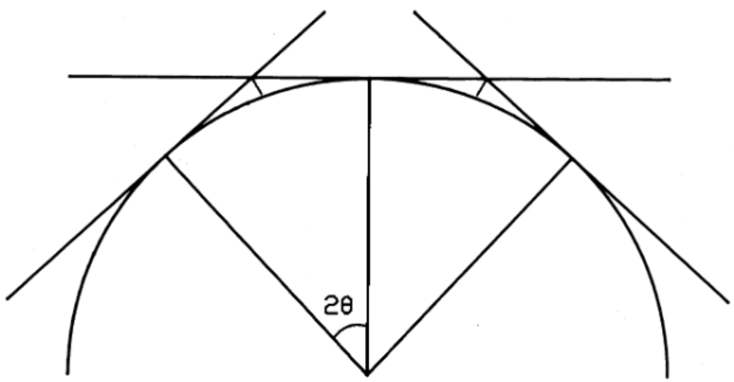


The w-term becomes relevant if λB/(D*D) > 1

For a field-of-view given by the Primary Beam of an antenna of diameter D, at wavelength λ and with a maximum baseline length of B

Widefield Effects – Facet Imaging

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dldmdn$$



Deconvolve facets separately before re- projecting and stitching

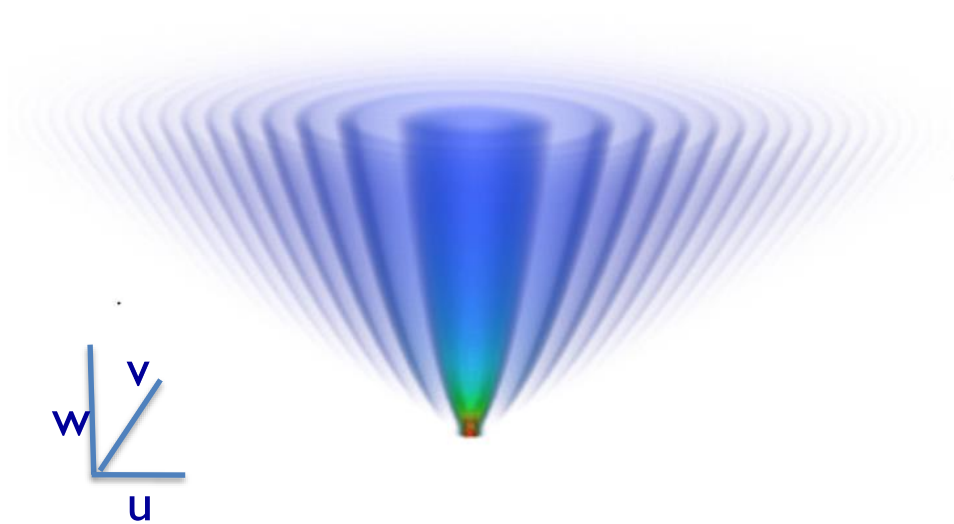
Image all facets onto the same tangent plane grid and perform a joint deconvolution.

Approximate the celestial sphere by a set of tangent planes (facets) such that 2D geometry is valid per facet - Image each facet with its own phase reference center and re-project to the tangent plane

Cornwell & Perley 1998

Widefield Effects – W Term & W Projection

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dldmdn$$



Fresnel kernel

Cornwell et al 2008, 2012

The w-term can arise from non-coplanar baselines

FoV > sqrt(resolution) - widefield definition

W-projection is the inversion of the fresnel kernel

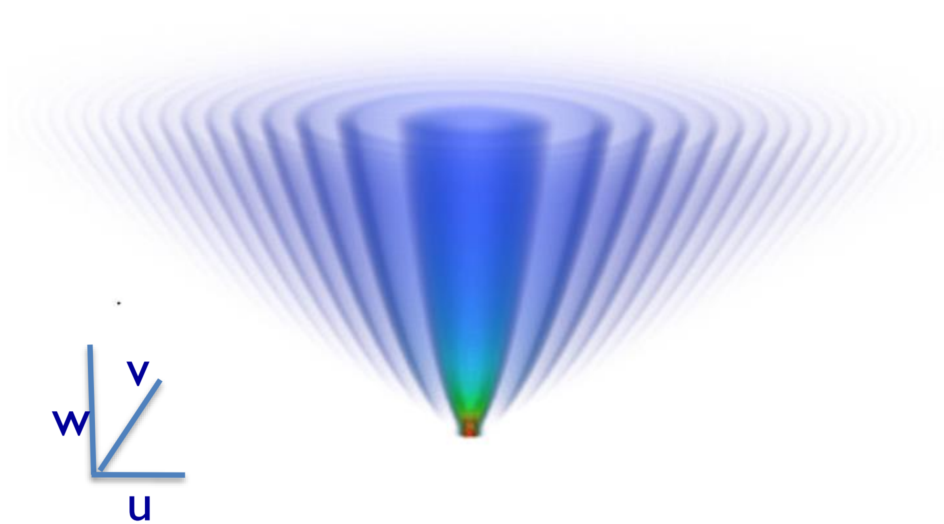
$$V(u,v,w) = \tilde{G}(u,v,w) \otimes V(u,v)$$

$$\tilde{G}(u,v,w) = \int \frac{e^{\,j2\pi w(\sqrt{1-l^2-m^2}-1)}}{\sqrt{1-l^2-m^2}}\, e^{\,j2\pi(ul+vm)}\, dldm$$

$$\tilde{G}(u,v,w) \approx \frac{e^{\,j\pi\frac{u^2+v^2}{w}}}{jw}$$

Widefield Effects – W Term & W snapshot

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)S_{ij}(\nu,t)\int\int\int M_{ij}^{S}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dldmdn$$



Fresnel kernel

$$w = au + bv$$

$$a = \tan Z \sin\chi$$
$$b = -\tan Z \cos\chi$$

$$l' = l + a\left(\sqrt{1-l^2-m^2}-1\right)$$
$$m' = m + b\left(\sqrt{1-l^2-m^2}-1\right)$$

A snapshot of a coplanar array when ignoring w will result in a coord shift.

So if you can take many snapshots and regrid them you get back your w corrected image.

Caveat – cost of regrid for each snapshot > facetting or w-projection

Find an optimum duration to bin into a single snapshot.

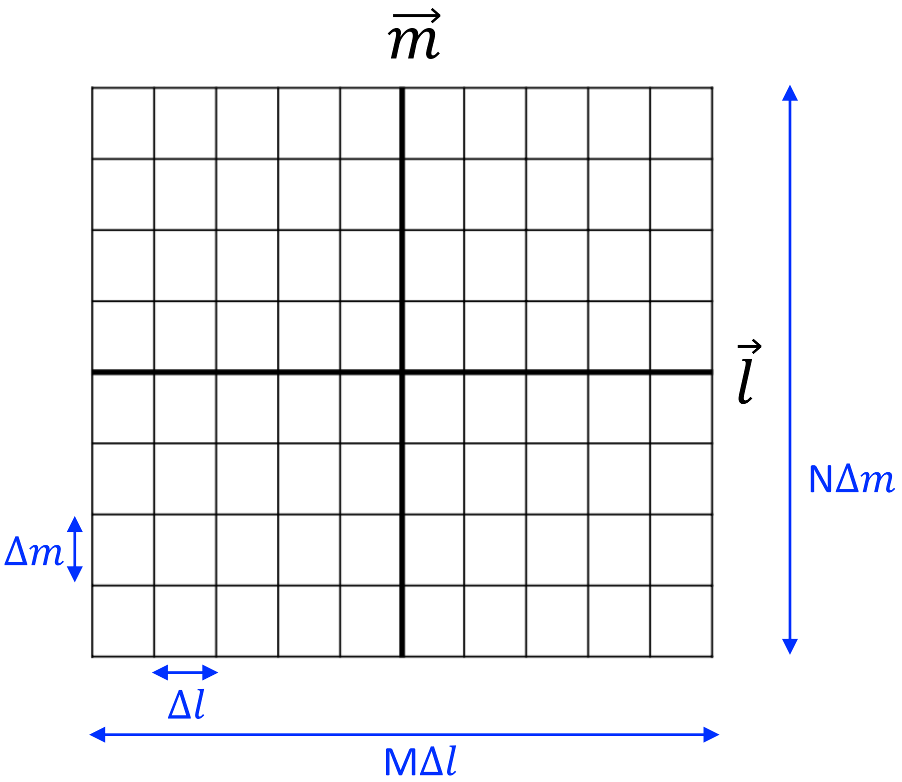
Cornwell et al 2008, 2012

Offringa et al 2014

Gridding

Practical to think first about the image grid:

- predict your angular resolution from diffraction: $\theta_{syn} \approx \dfrac{\lambda}{b_{max}}\ (radians)$
- choose an image cell size Δl that oversamples θ_{syn} by a factor of 4~5
- choose an image size MxN (pixels) based on your desired field of view (FOV)
- FOV will be M Δl x N Δm
- typically want Δl=Δm, M=N



Gridding

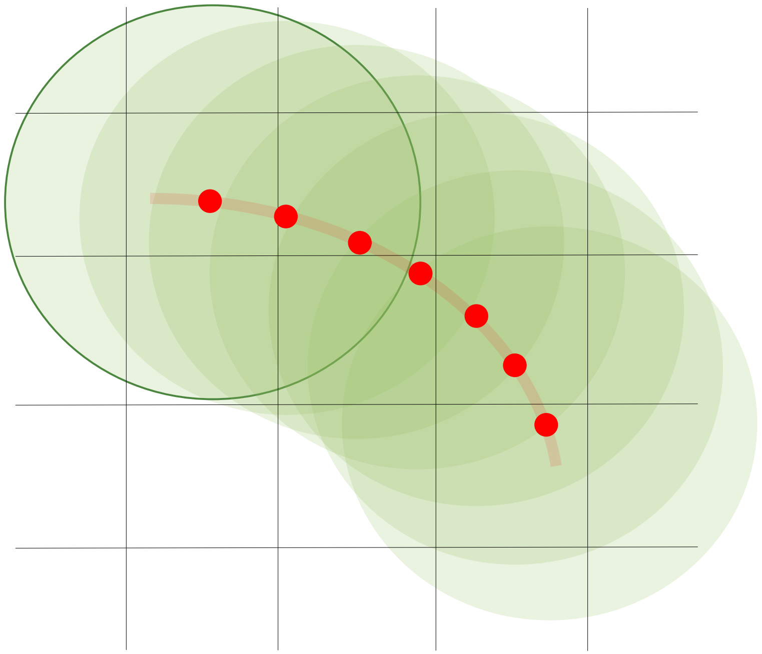
We can improve the gridding process through ***convolutional resampling***

Each visibility is convolved by a ***gridding kernel*** that distributes the visibility across multiple cells

We can oversample the kernel to reduce interpolation errors

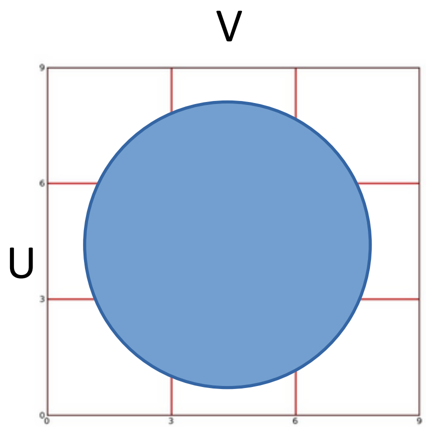
Specific kernels can address other issues including:

- Aliasing – prolate spheroidal
- w-term – Fresnel kernel
- Mosaicking – Phase Gradient



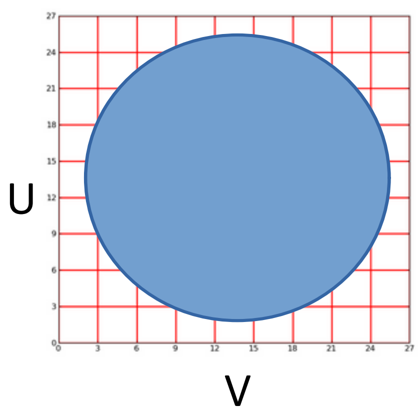
The effect of the kernel can be divided out in the image plane

Gridding kernels and FoV - A-Projection

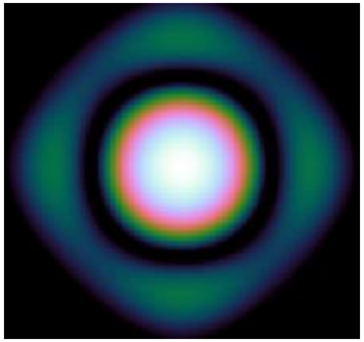


Support size 3x3

Imaging FoV out to 20% point



Support size 9x9



Imaging FoV out to 1st side-lobe

Computational Cost - Imaging

Gridding cost scales directly as $N_{vis} \times N_{sup}^2$

For an interferometer like the VLA no. Of vis ~ 10^10

Prolate Spheroidal Term support ~3x3,

A Term support ~ 9x9

W Term support ~ 200x200

Facetting support ~ 3x3x num facets

Gridding is consequently 80% of your imaging cost

Wide-field Imaging - I

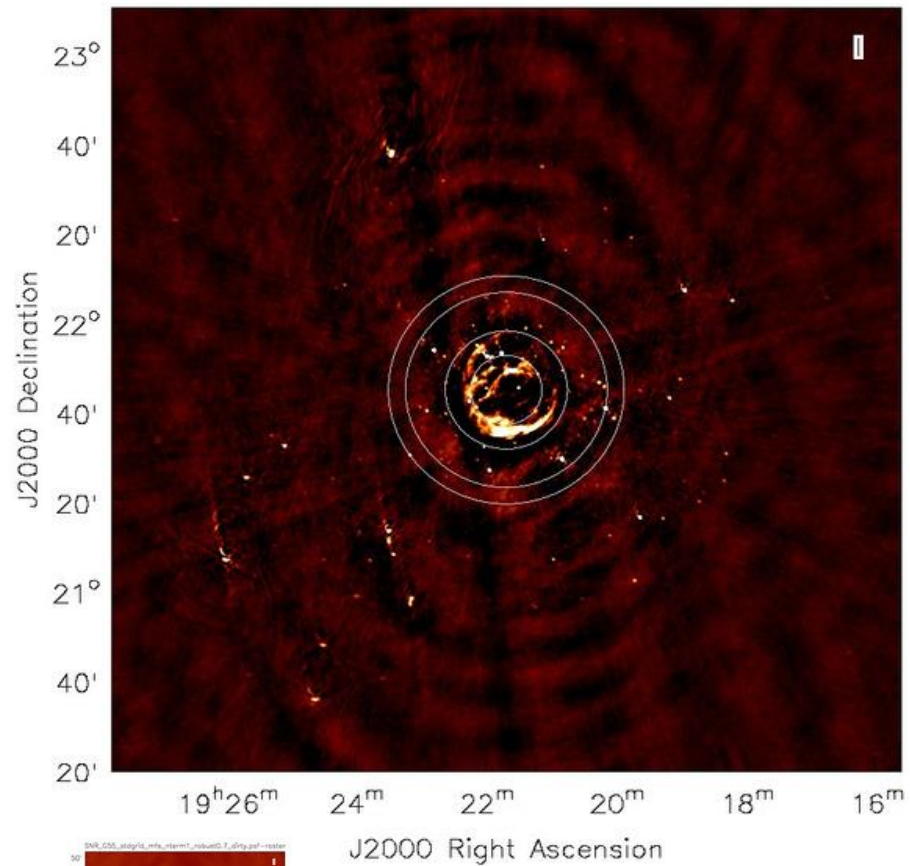
Fourier Transform of the gridded visibilities. Gridding kernel used is a prolate spheroidal function.

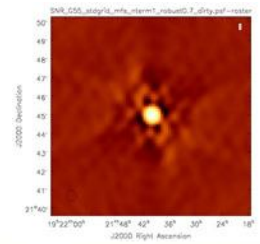
The imprint of the sampling function(psf) is clearly present.

Source well outside PB mainlobe.

Note bright sources far from the object of interest.

Field has point sources and extended emission.





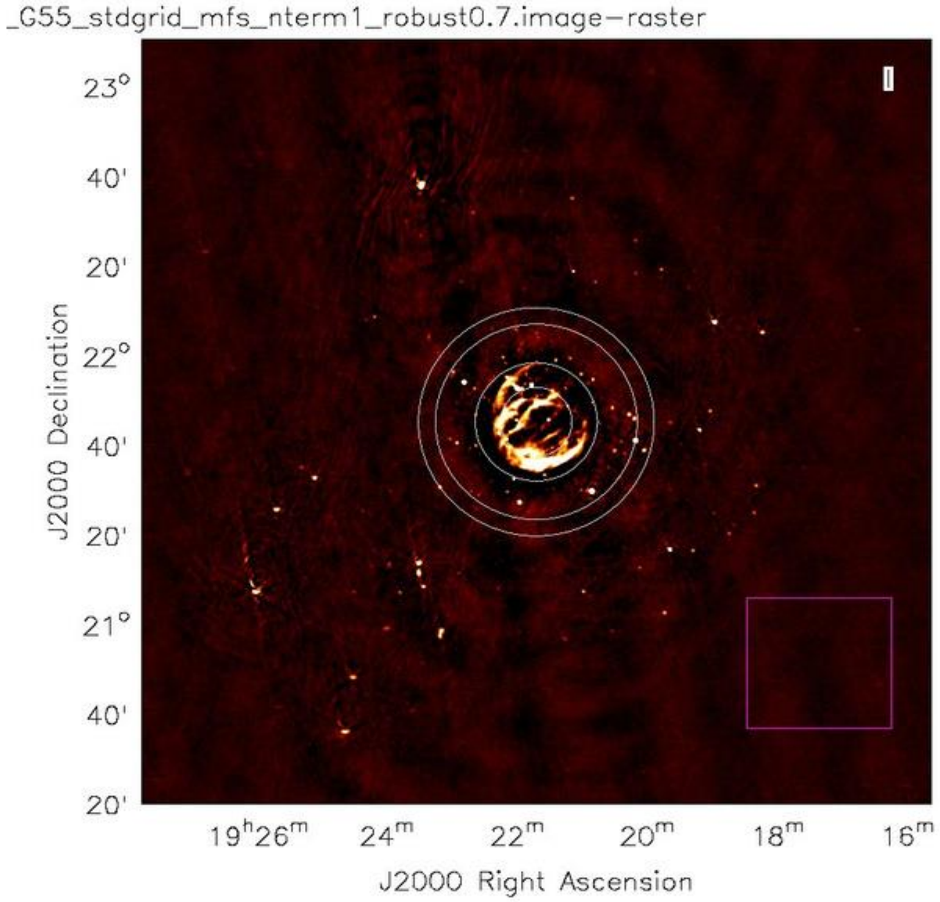
RMS : 21 microJy

Wide-field Imaging - II

Deconvolution clearly improved the image.

The distortions around the bright sources in the edges are showing wide-field effects.

How do we mitigate the widefield effects?



RMS : 14 microJy

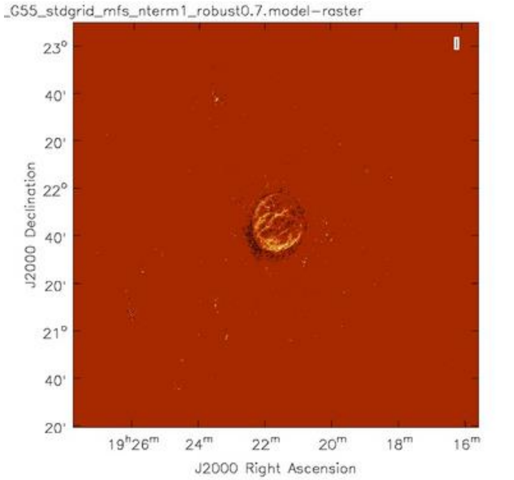
Wide-field Imaging - III

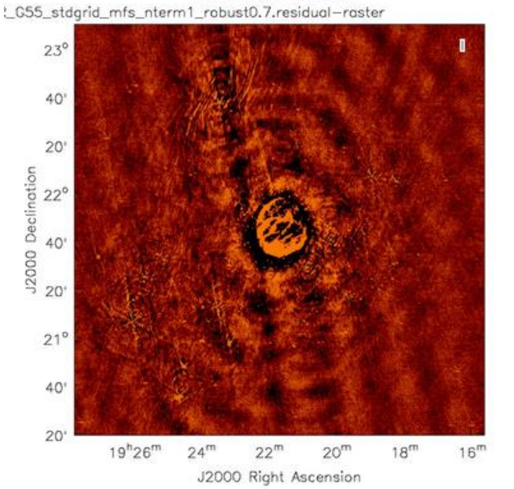
Model image shows that you need a lot of point sources to model extended emission.

Need a better set of basis to model the emission from the sky.

The residuals are not truly noise-like so we have not hit thermal noise yet.

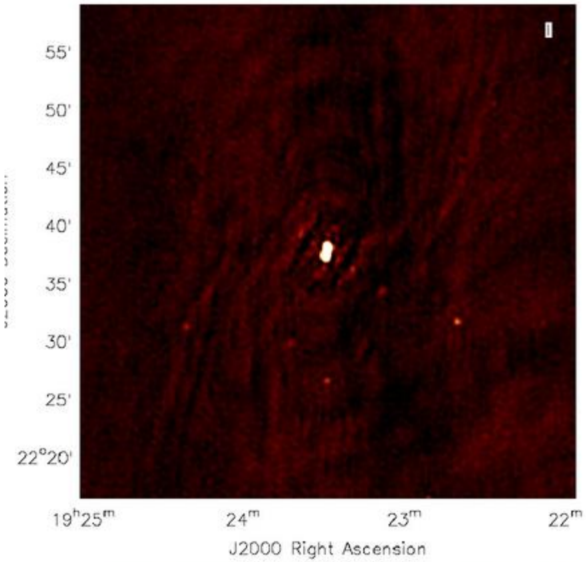
The negatives around the source arise from missing zero spacings.

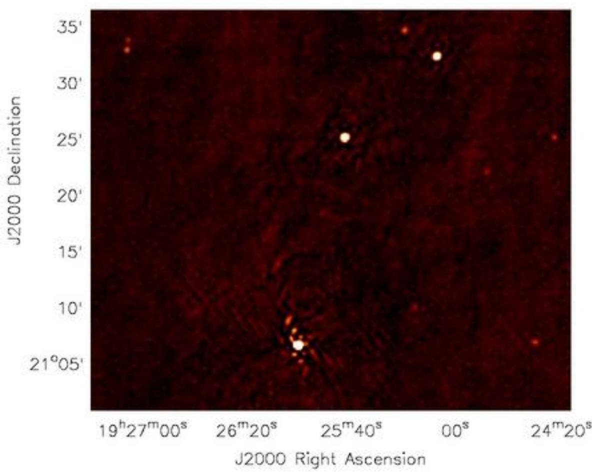


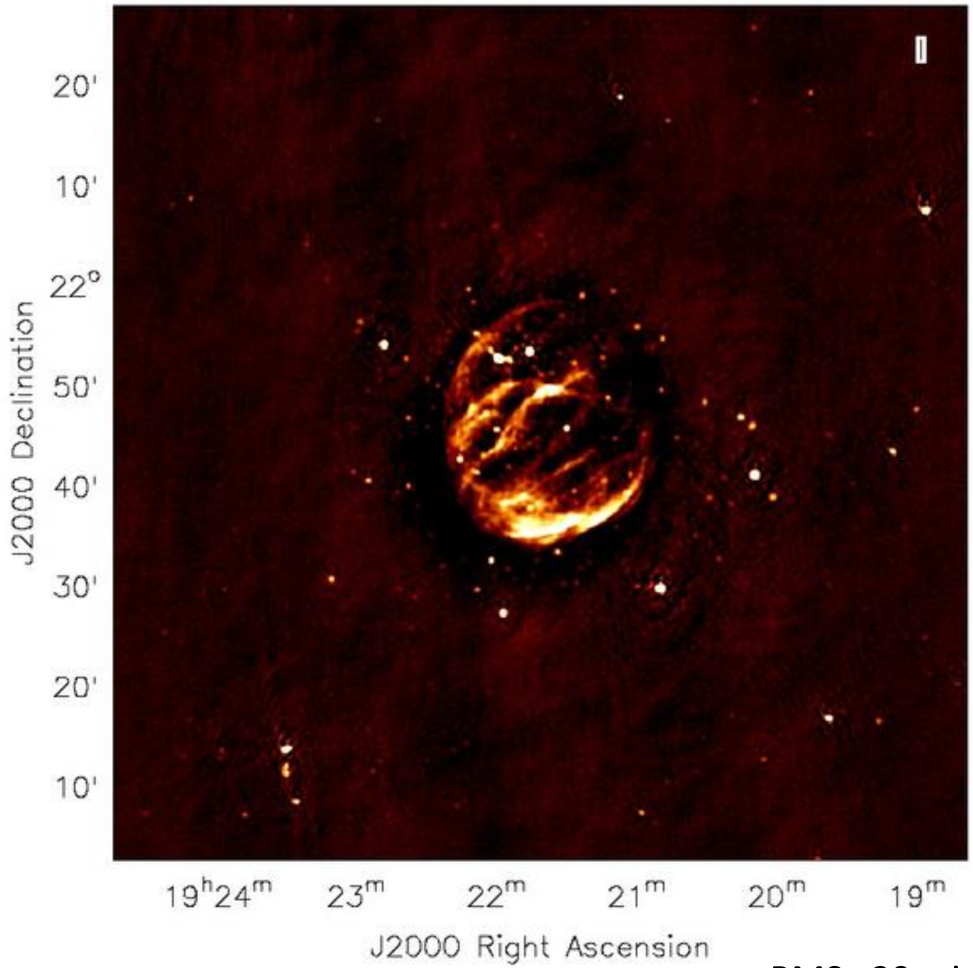


RMS : 14 microJy

Outlier Fields

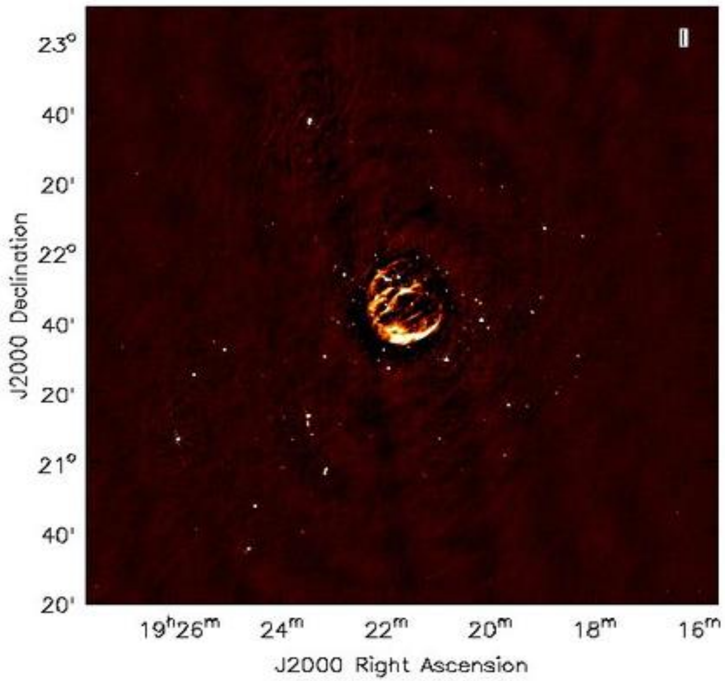




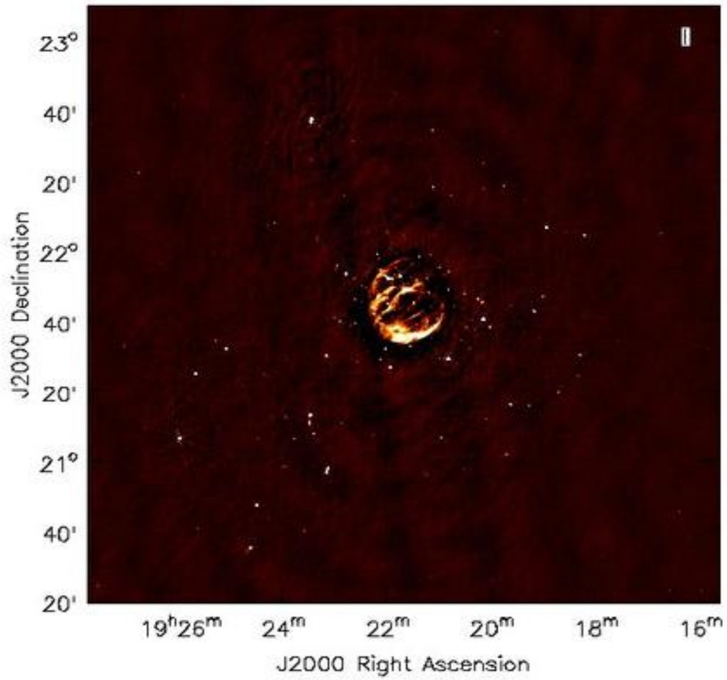


RMS : 20 microJy

Facetting vs W-Projection



9x9 facets. 13 microJy
Support kernel 9x9



128 w planes. 12.7 microJy
Max Support kernel 56x56

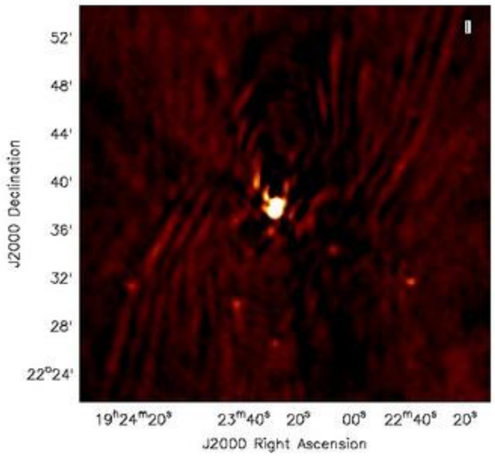
Wide-field Correction

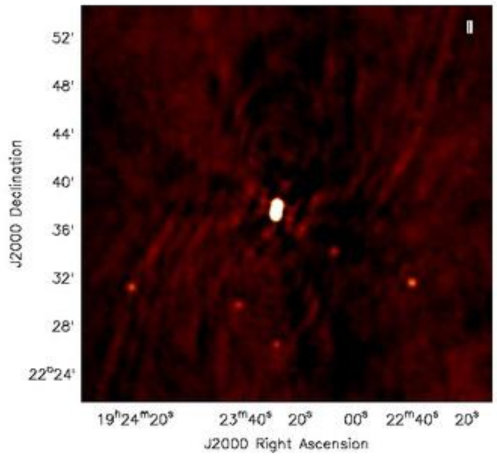
The source distortion from the W phase term is removed.

Both facetting and W-projection produce the same result.

W-Projection uses a fresnel kernel spanning 56x56 uv pixels

Facetting used the standard prolate spheroidal kernel in its 81 facets to grid them independently.





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Summary

Only choose the level of algorithmic complexity you need to achieve your science.

Every algorithm comes with its own compute cost – ask yourself do you really need that shiny fancy new algorithm

Corrections for non-coplanar array and large FoV can be carried out in multiple ways

- Snapshot Imaging
- Facetting
- W-projection
- Some combination of the above (as in wsclean)

If you can correct while you image it is a bonus so please do

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