Polarization in interferometry

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Reading list

- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)

Outline

EM wave physics

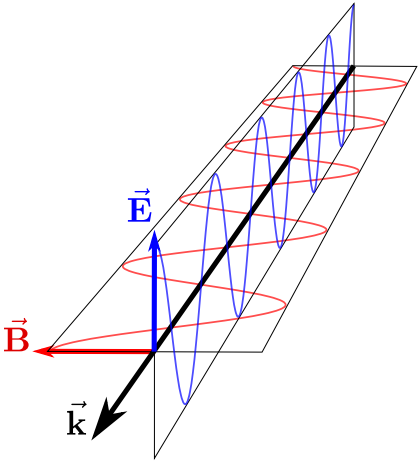
Astrophysics

Polarized EM-waves

Interferometric polarimetry

Messy reality

Electromagnetic (EM) wave



- Vector phenomenon
- From Maxwell's equations:
  $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k**
- Measure either **E** or **B**
- **E** is easier

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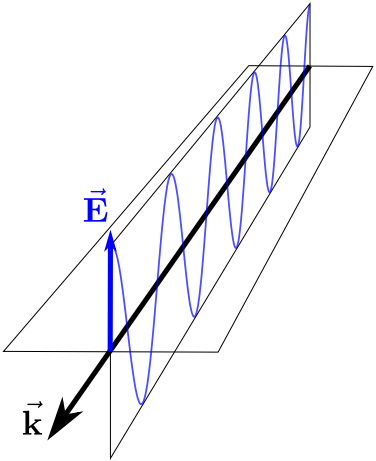
Electromagnetic (EM) wave



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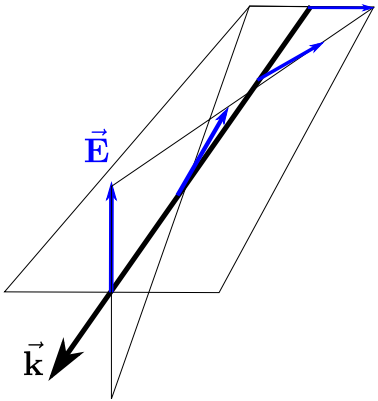
Electro-magnetic wave



- Vector phenomenon
- From Maxwell's equations: $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know **k**
- Measure either **E** or **B**
- **E** is easier
- But:
- E_x and E_y not equal
- **E** may rotate as function of **x** and t.
- **E** traces ellipse

"Polarization"

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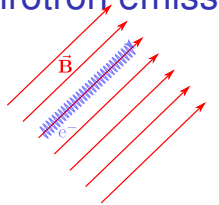
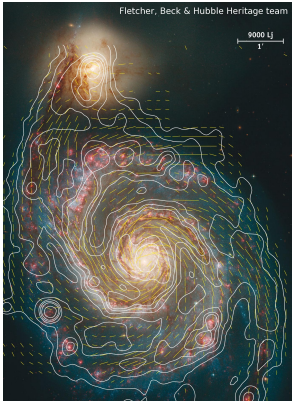
Synchrotron emission





Process

- Generates polarized emission
- Main emission mechanism at cm-m wavelength
- Up to 80% linearly polarized
- No circular
- $\langle \mathbf{E}_{\text{source}} \rangle \perp \mathbf{B}_{\text{source}}$

Polarimetry provides

- **B**-field direction
- Turbulence
- Indirectly: **B**-field strength

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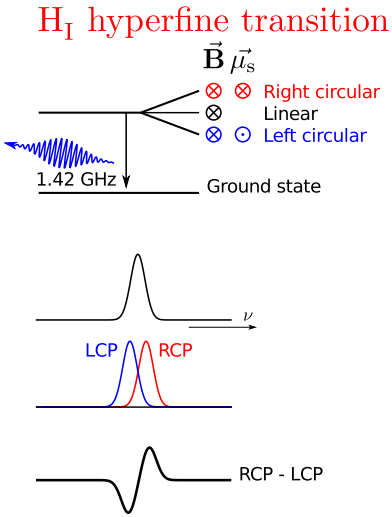
Zeeman splitting



Process

- Generates polarized emission
- Only in spectral lines
- If magnetic moment: e.g. H_I, OH, CN, H_2O
- **B**-field splits RCP and LCP
- Separation: 2.8 Hz mG^{-1}

Polarimetry provides

- **B**-field strength at source
- If detectable. . .

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Zeeman effect *Vlemmings et al. (2001)*

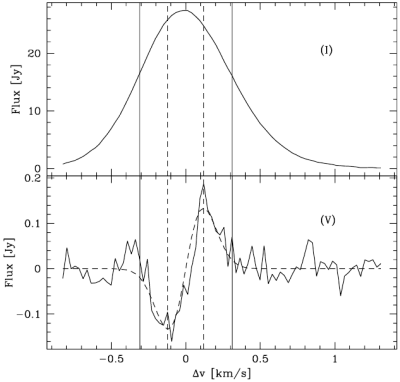

Fig. 4. Total power (I) and circular polarization (V) spectrum of the brightest H_2O maser feature around S Per. The dashed line is the fit of the synthetic V-spectrum to the observed spectrum. Also shown are the observed (dashed) and expected (solid) positions of the minimum and maximum of the V-spectrum.

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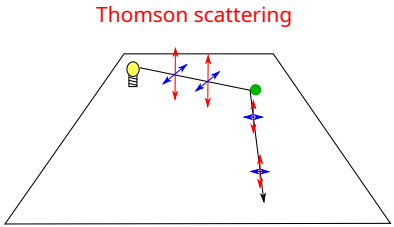
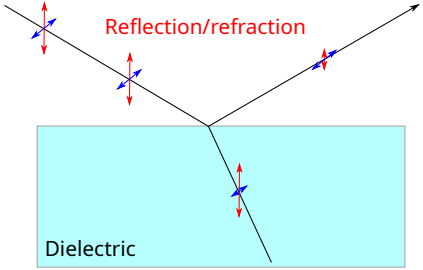
Scattering/reflection





Process

- Modifies polarization state
- Thomson scattering: no T dependence
- Planets / Moon: dielectric transition

Polarimetry provides

- Electron densities in cool gas
- Dust properties
- Lunar dielectric constant

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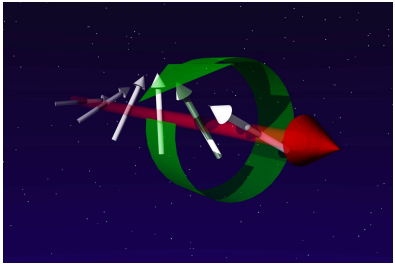
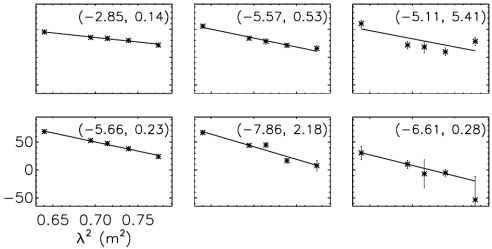
Faraday rotation



λ^2 law *Haverkorn et al. (2001)*



Process

- Modifies polarization state
- Delay between LCP and RCP
- Rotates linear pol angle
- $\Delta\chi = \chi_0 + \phi\lambda^2$

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot \mathrm{d}\mathbf{l}$$

Polarimetry provides

- Source plasma properties
- Intervening plasma properties
- Rare cases: 3D tomography

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Astrophysics

Polarized EM-waves

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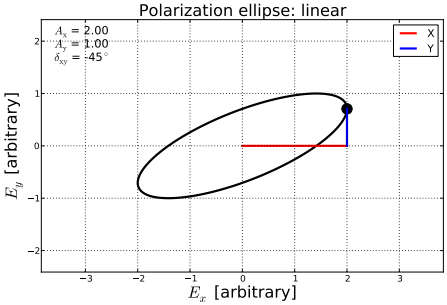
Messy reality

Polarization ellipse: cartesian basis

Geometry



Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

$$\begin{aligned} \mathbf{E} &= E_x \hat{\mathbf{e}}_x + E_y \hat{\mathbf{e}}_y \\ E_x &= A_x \cos(2\pi\nu t + \delta_x) \\ E_y &= A_y \cos(2\pi\nu t + \delta_y) \end{aligned}$$

- A_x = x-amplitude
- A_y = y-amplitude
- $\delta_{xy} = \delta_y - \delta_x$
- δ_{xy} = measure of ellipticity
- $\delta_{xy} > 0$: CW rotation $\Rightarrow$ LEP
- $\delta_{xy} = 0$: linear polarization
- $\delta_{xy} < 0$: CCW rotation $\Rightarrow$ REP

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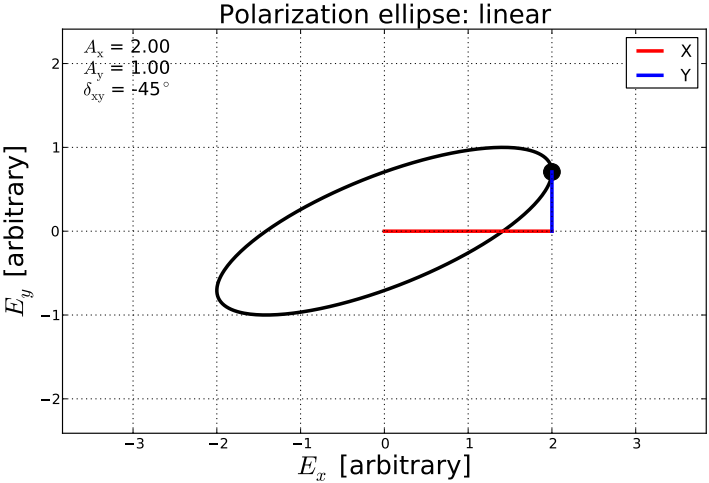
Polarization ellipse: *xy* 000



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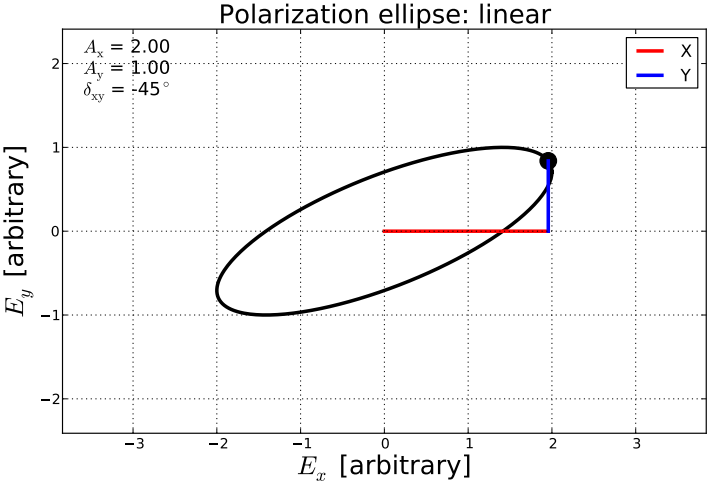
Polarization ellipse: *xy* 001



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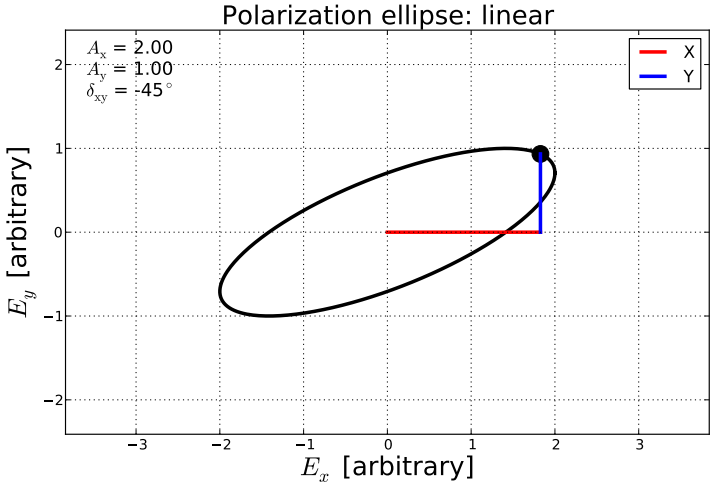
Polarization ellipse: *xy* 002



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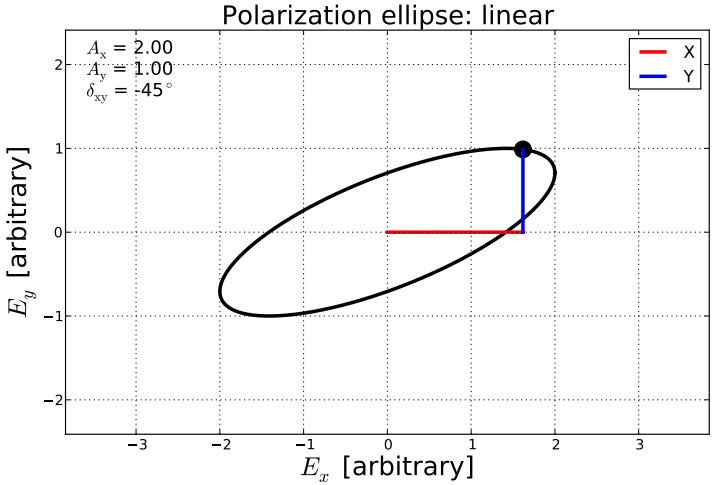
Polarization ellipse: *xy* 003



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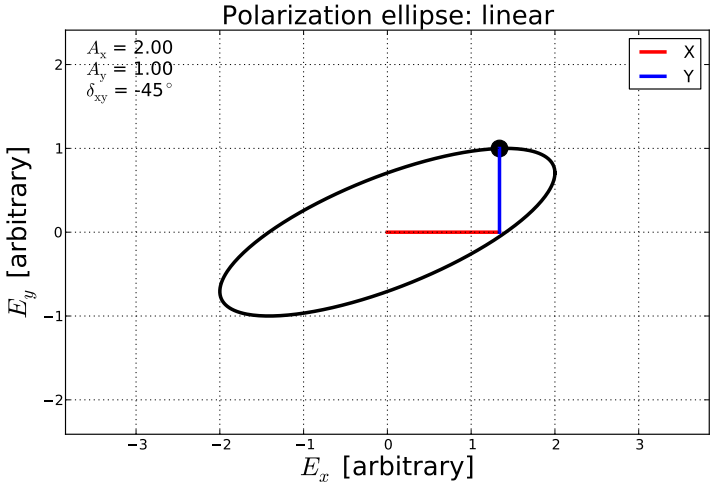
Polarization ellipse: *xy* 004



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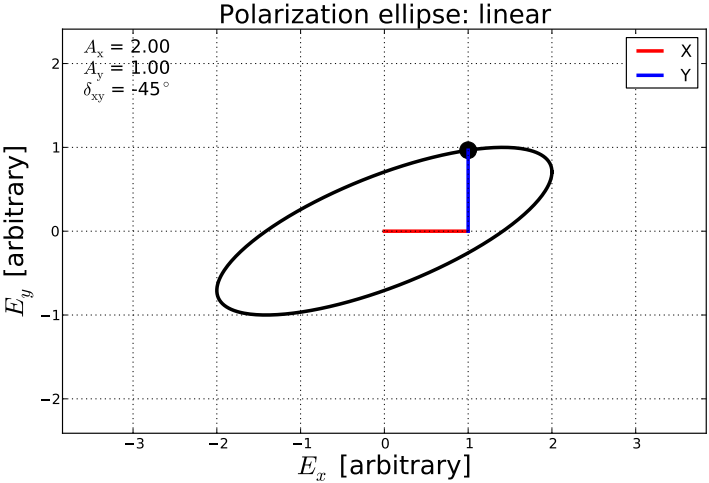
Polarization ellipse: *xy* 005



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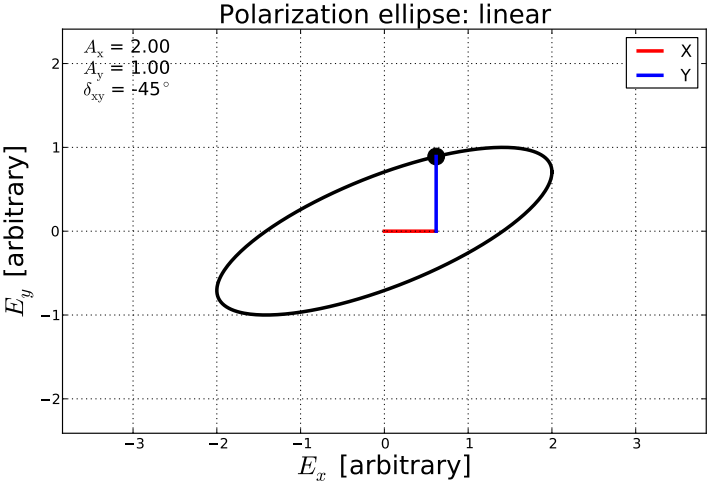
Polarization ellipse: *xy* 006



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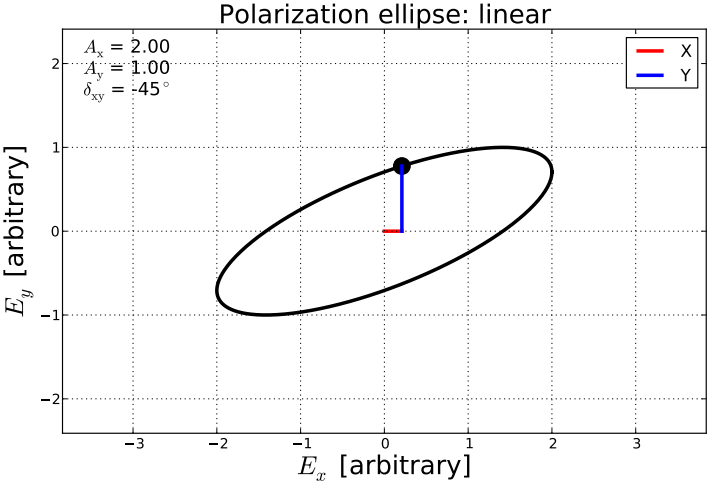
Polarization ellipse: *xy* 007



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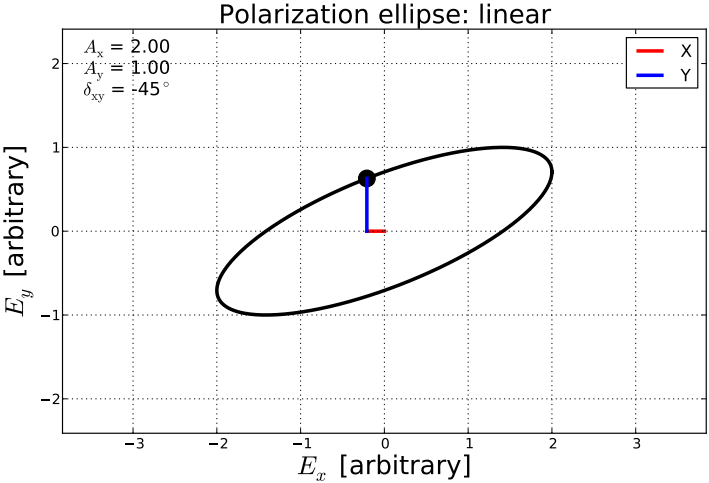
Polarization ellipse: *xy* 008



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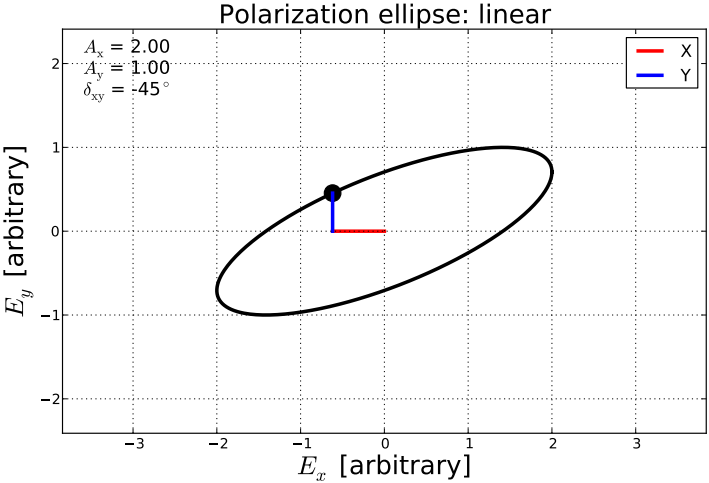
Polarization ellipse: *xy* 009



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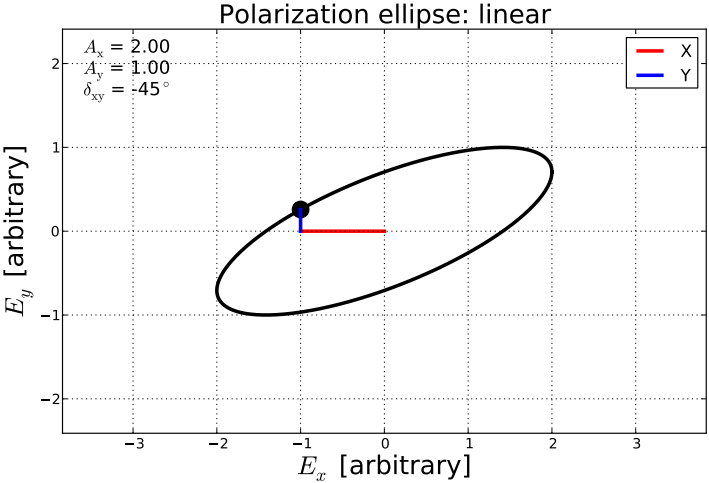
Polarization ellipse: *xy* 010



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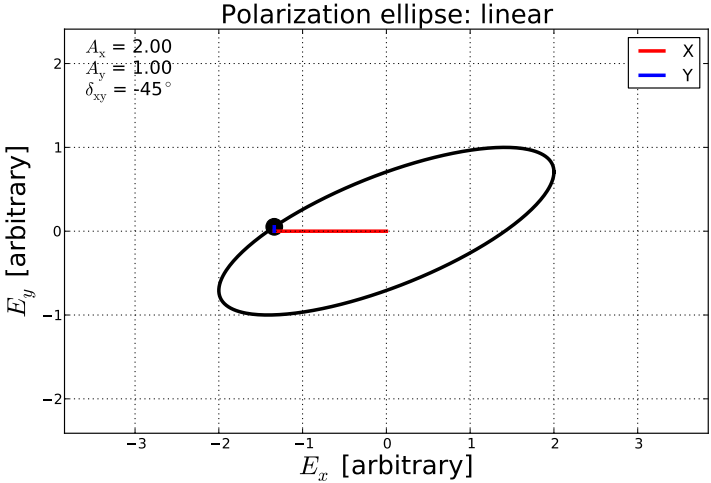
Polarization ellipse: *xy* 011



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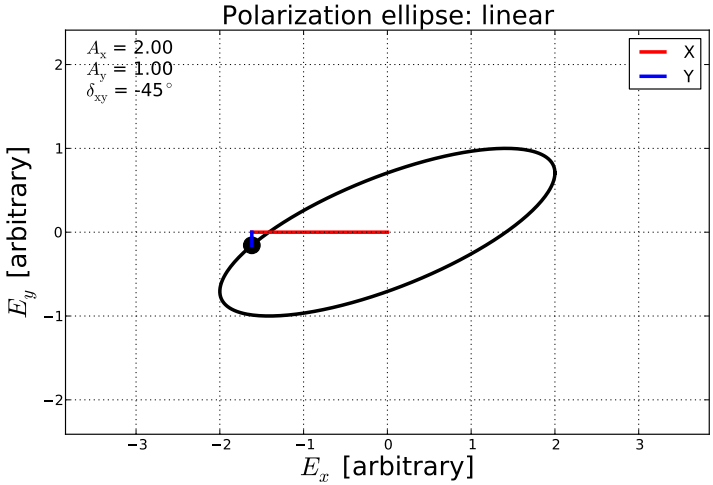
Polarization ellipse: *xy* 012



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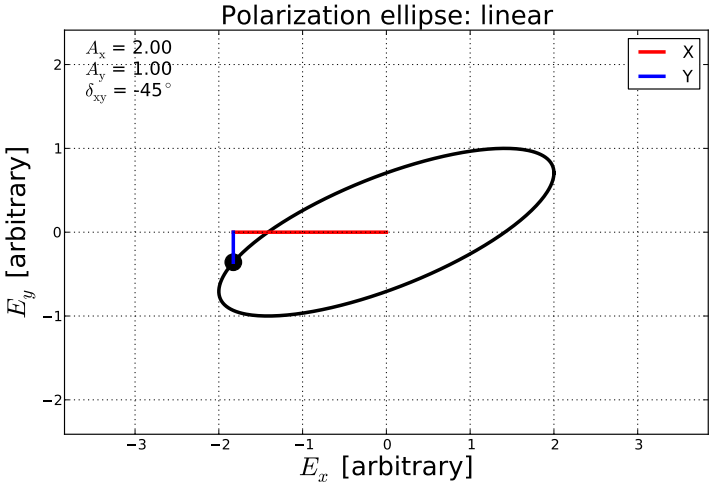
Polarization ellipse: *xy* 013



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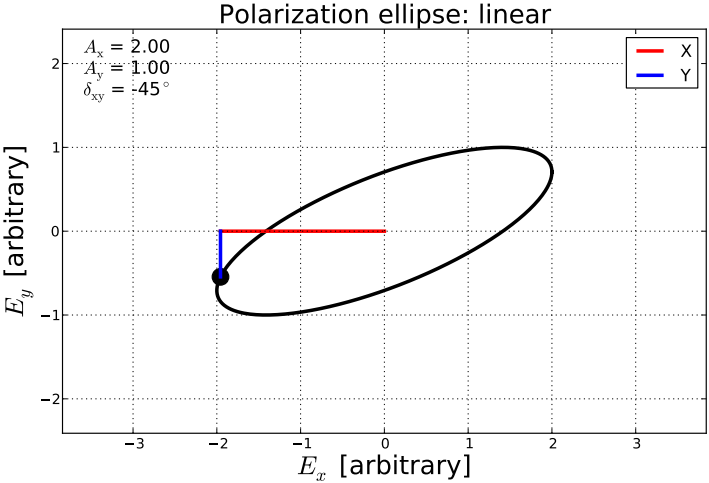
Polarization ellipse: *xy* 014



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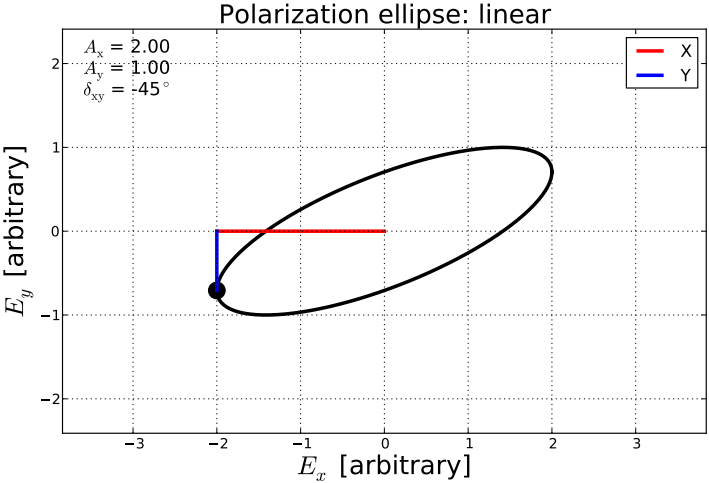
Polarization ellipse: *xy* 015



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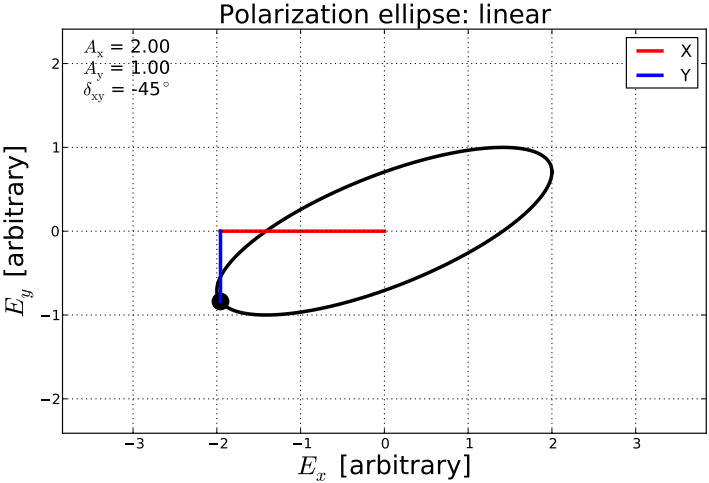
Polarization ellipse: *xy* 016



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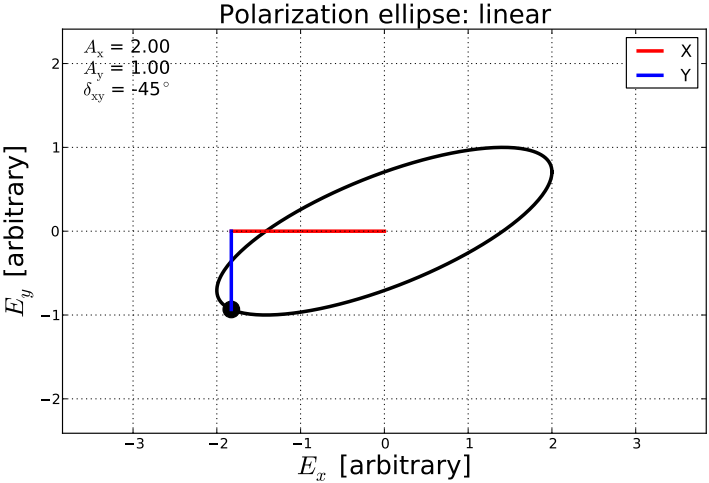
Polarization ellipse: *xy* 017



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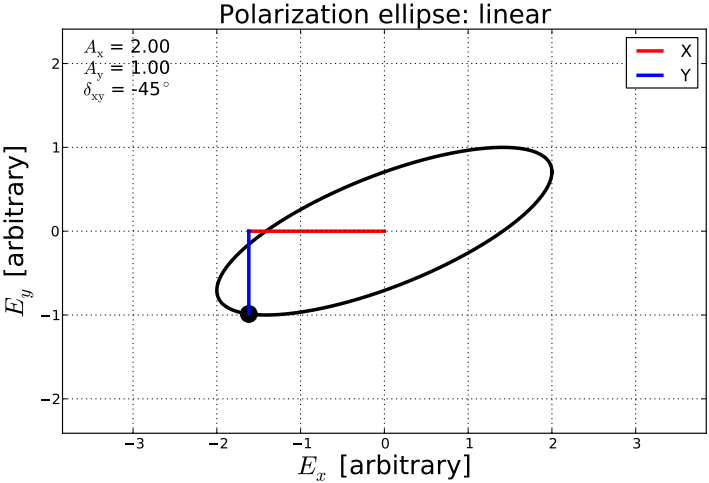
Polarization ellipse: *xy* 018



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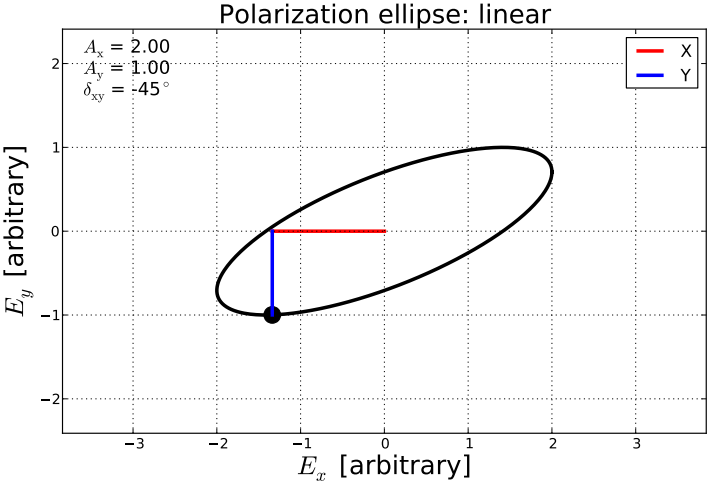
Polarization ellipse: *xy* 019



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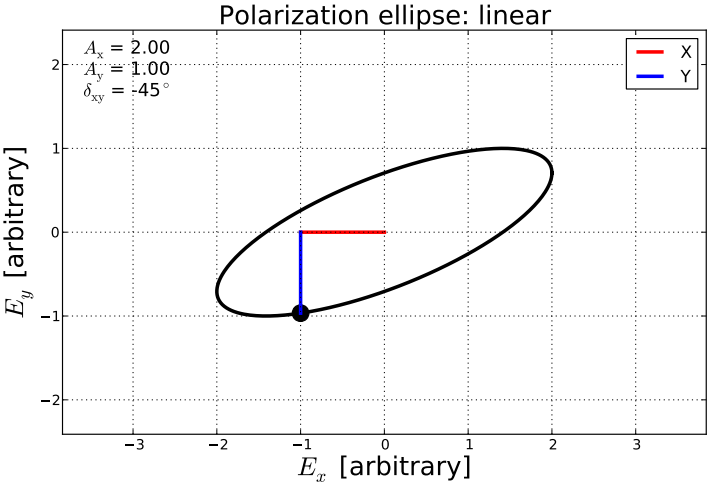
Polarization ellipse: *xy* 020



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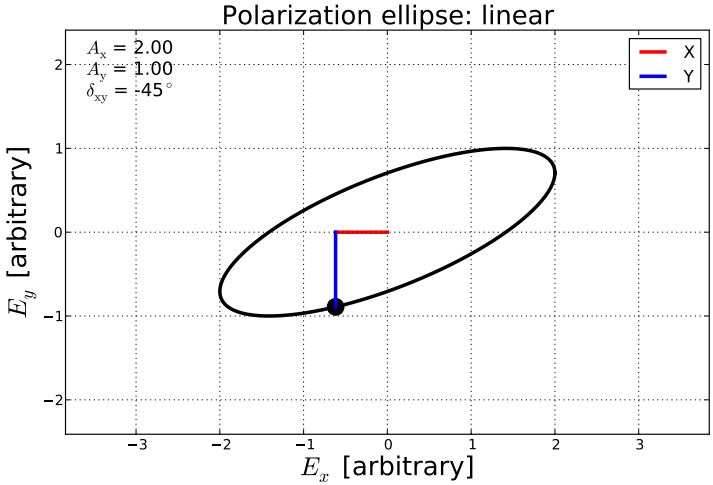
Polarization ellipse: *xy* 021



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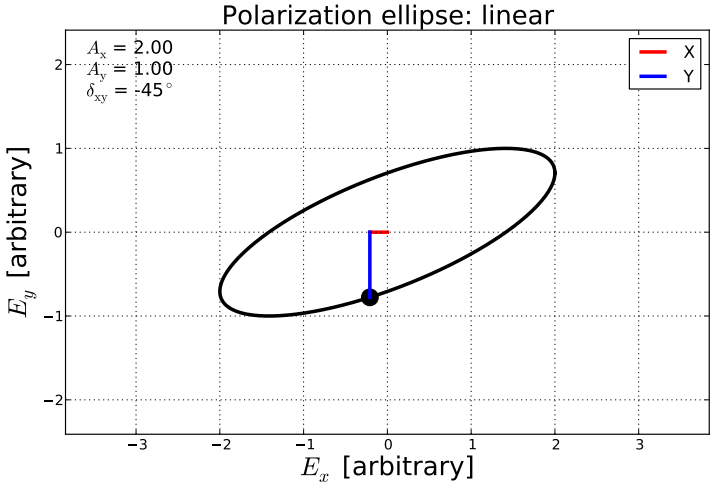
Polarization ellipse: *xy* 022



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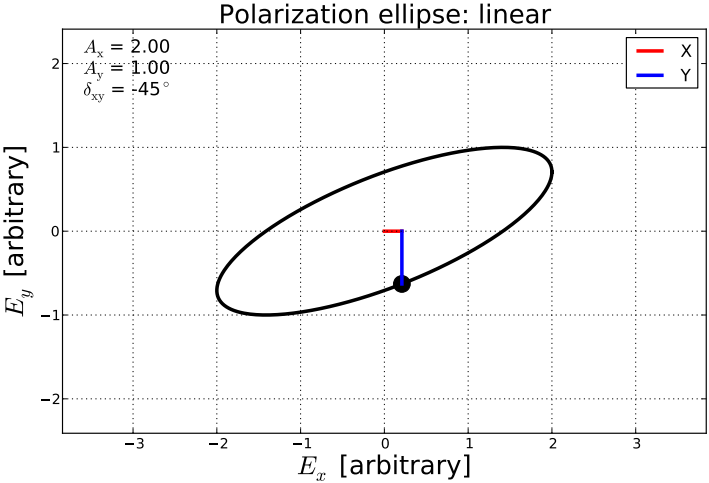
Polarization ellipse: *xy* 023



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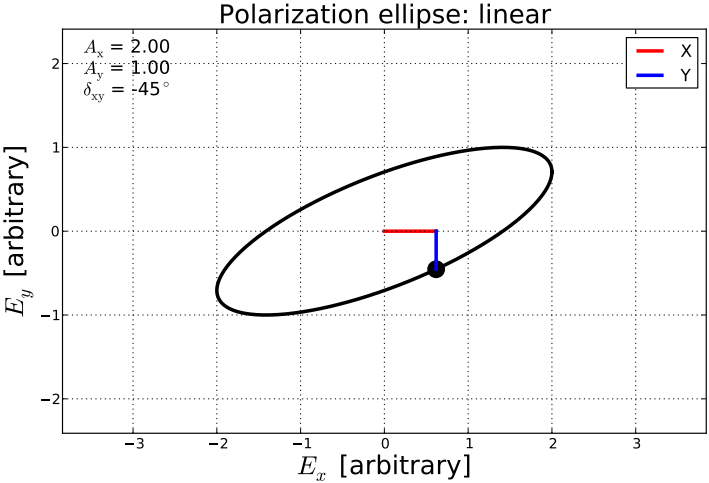
Polarization ellipse: *xy* 024



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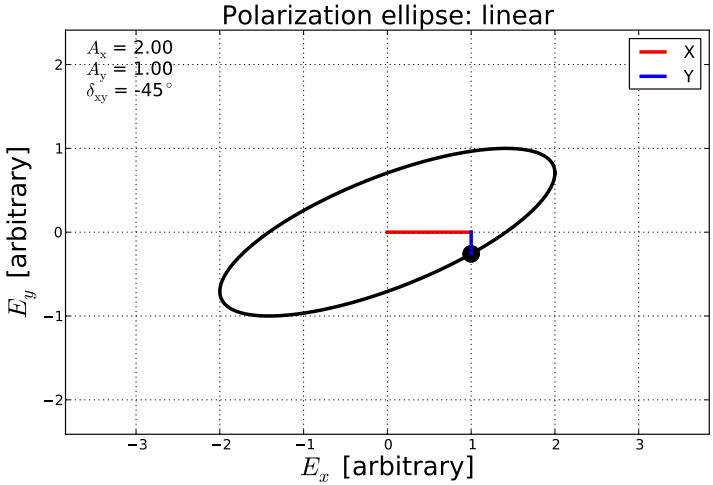
Polarization ellipse: *xy* 025



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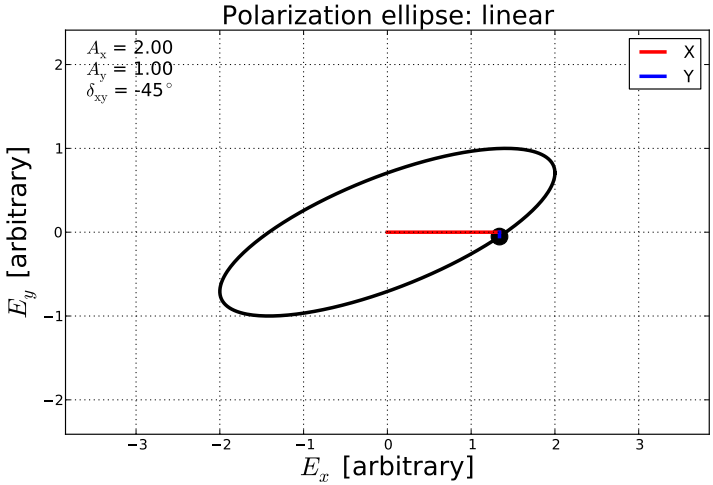
Polarization ellipse: *xy* 026



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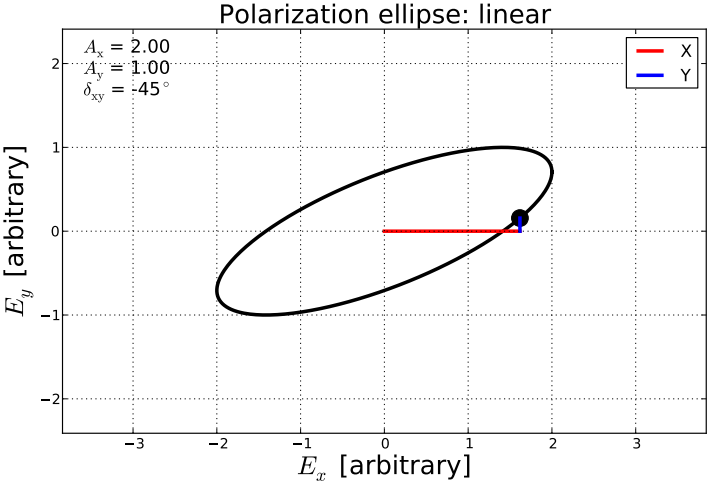
Polarization ellipse: *xy* 027



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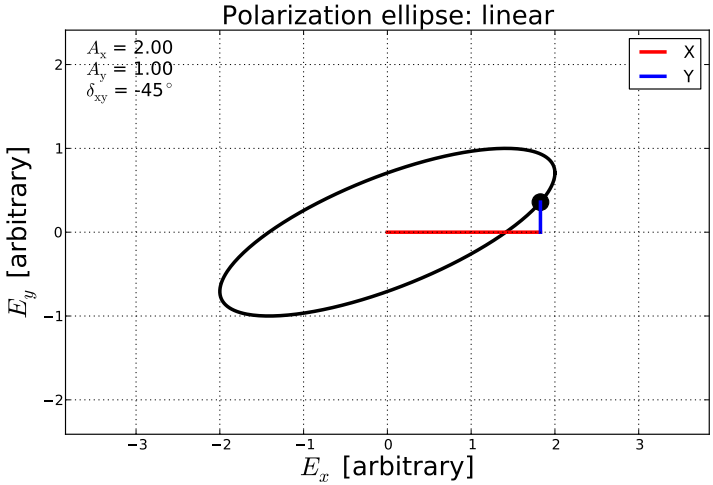
Polarization ellipse: *xy* 028



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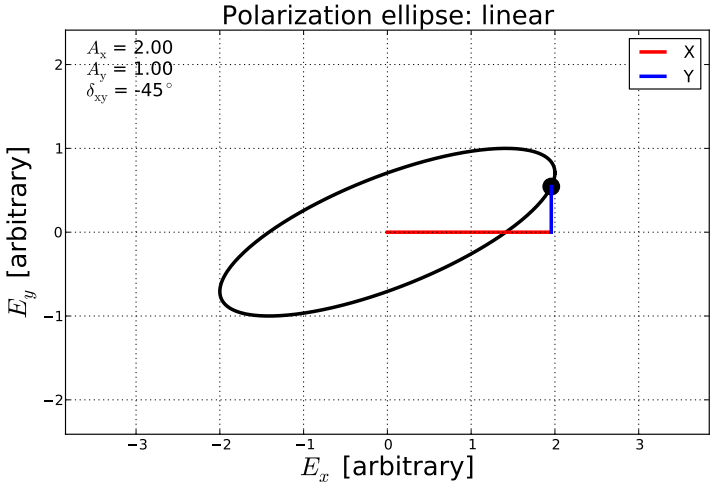
Polarization ellipse: *xy* 029



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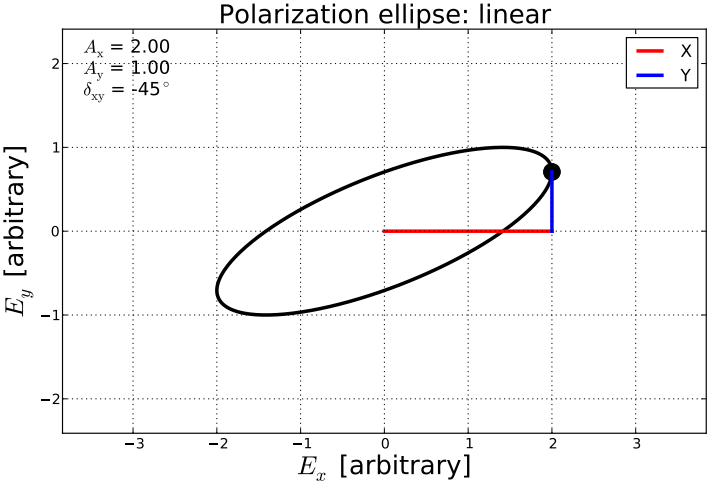
Polarization ellipse: *xy* 030



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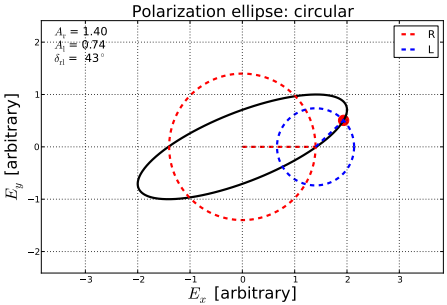
Polarization ellipse: circular basis

Geometry



Viewing from antenna towards source, watching orientation and length of **E** vector on a plane at a fixed location in space.

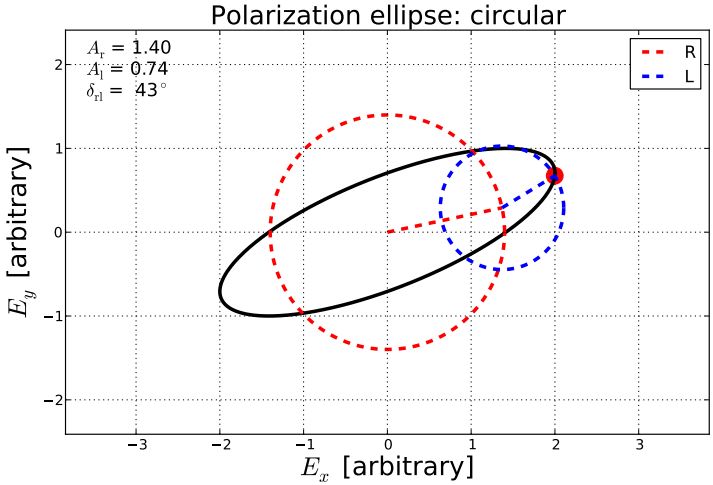
$$\mathbf{E} = A_r\hat{\mathbf{e}}_r + A_l\hat{\mathbf{e}}_l$$

$$\hat{\mathbf{e}}_r = \begin{pmatrix} \cos(2\pi\nu t + \delta_r) \\ \sin(2\pi\nu t + \delta_r) \end{pmatrix}$$

$$\hat{\mathbf{e}}_l = \begin{pmatrix} \cos(2\pi\nu t + \delta_l) \\ -\sin(2\pi\nu t + \delta_l) \end{pmatrix}$$

- $A_r + A_l$ = semi-major axis
- $\|A_r - A_l\|$ = semi-minor axis
- $\delta_{rl} = \delta_l - \delta_r$
- $-\frac{1}{2}\delta_{rl}$ = position angle of MA
- $\delta_{rl} > 0$: MA rotated CW
- $\delta_{rl} = 0$: MA along x-axis
- $\delta_{rl} < 0$: MA rotated CCW

Polarization ellipse: circular 001



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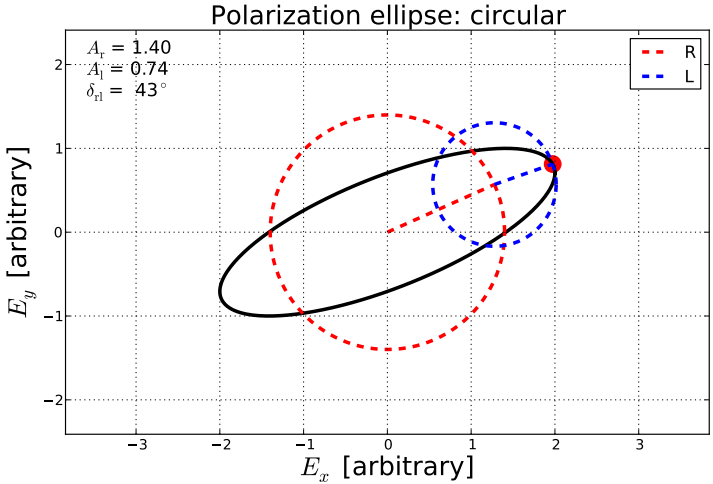
Polarization ellipse: circular 002



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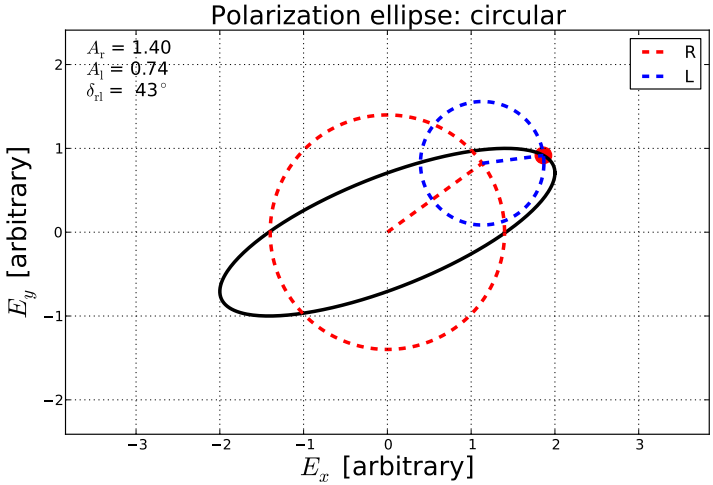
Polarization ellipse: circular 003



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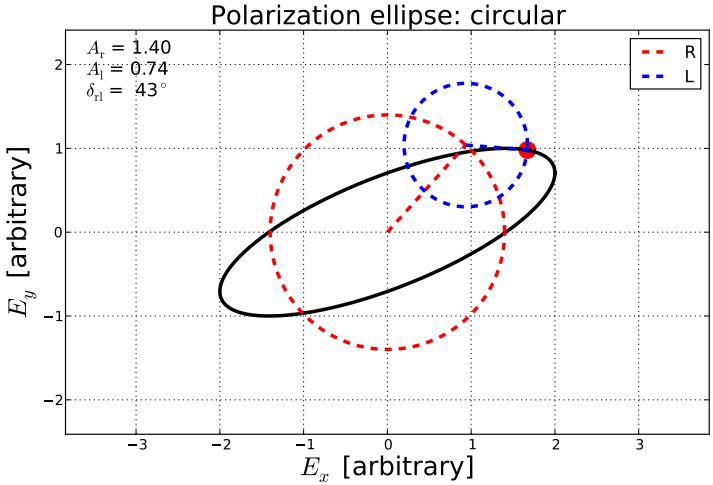
Polarization ellipse: circular 004



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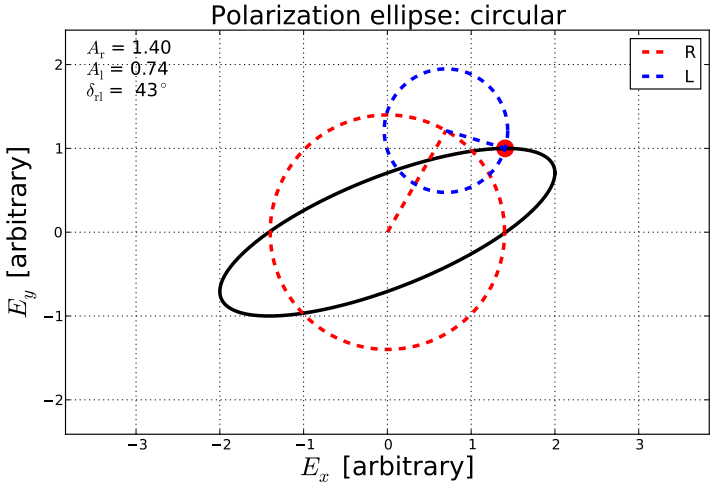
Polarization ellipse: circular 005



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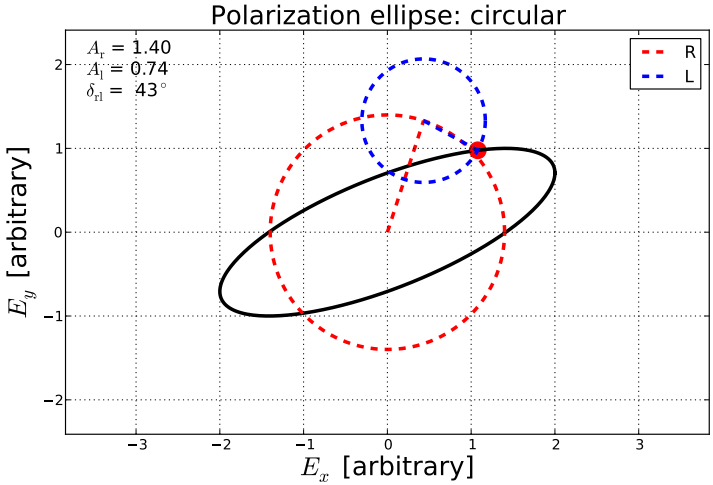
Polarization ellipse: circular 006



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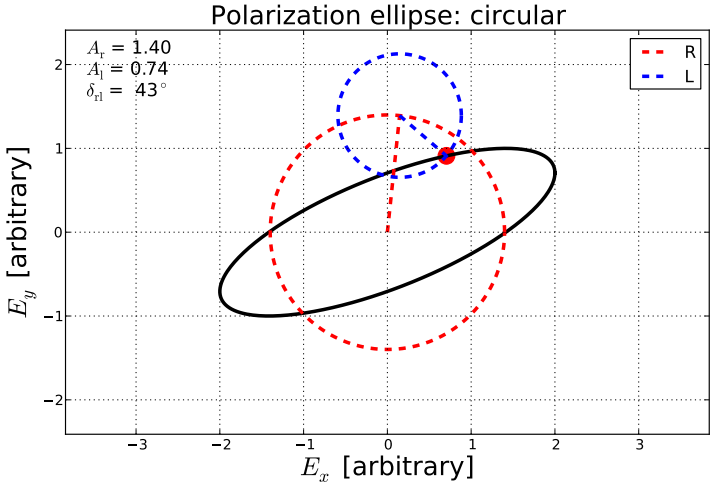
Polarization ellipse: circular 007



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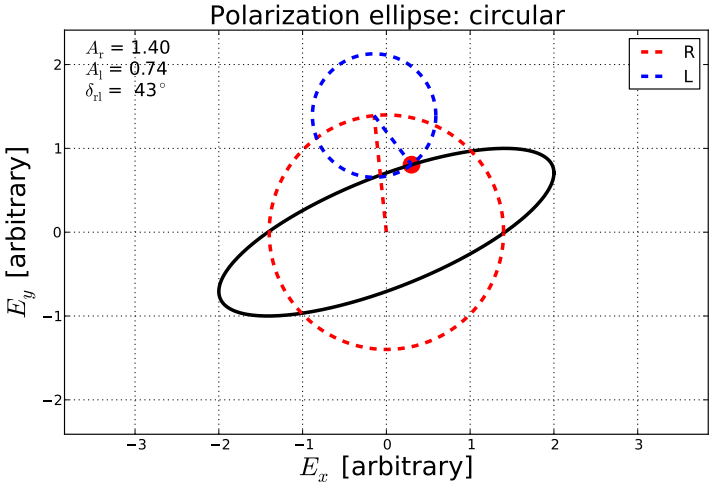
Polarization ellipse: circular 008



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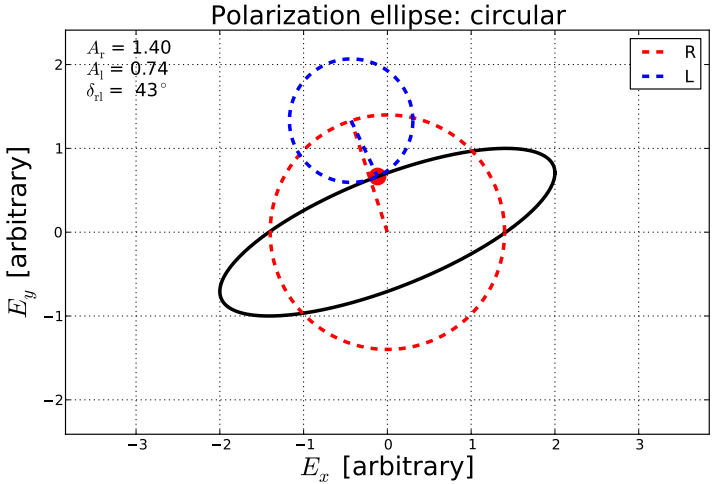
Polarization ellipse: circular 009



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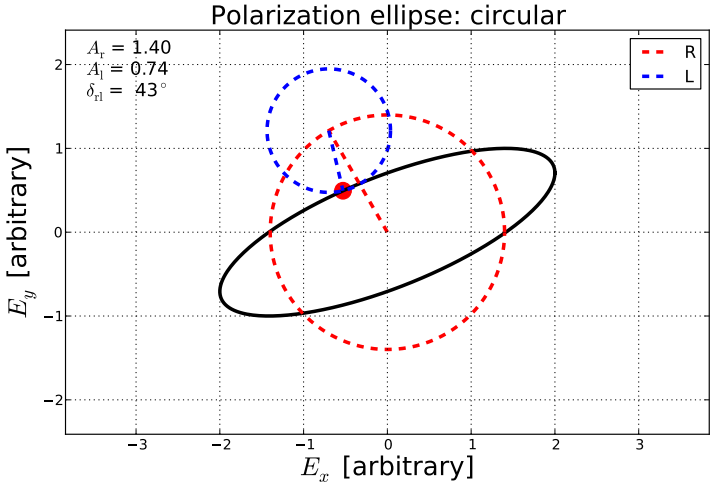
Polarization ellipse: circular 010



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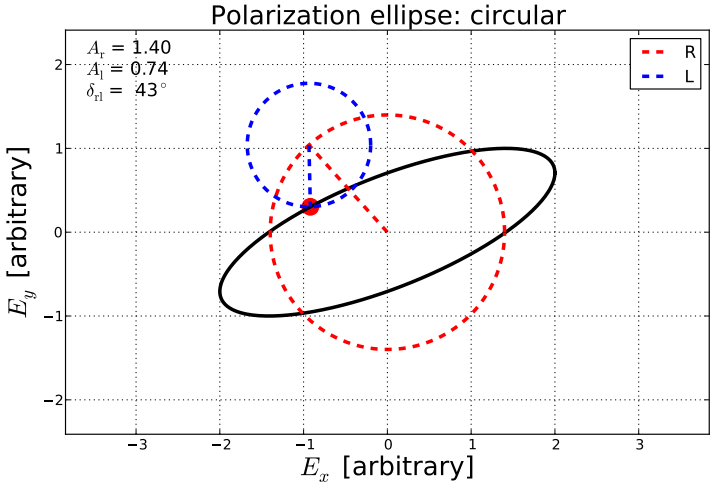
Polarization ellipse: circular 011



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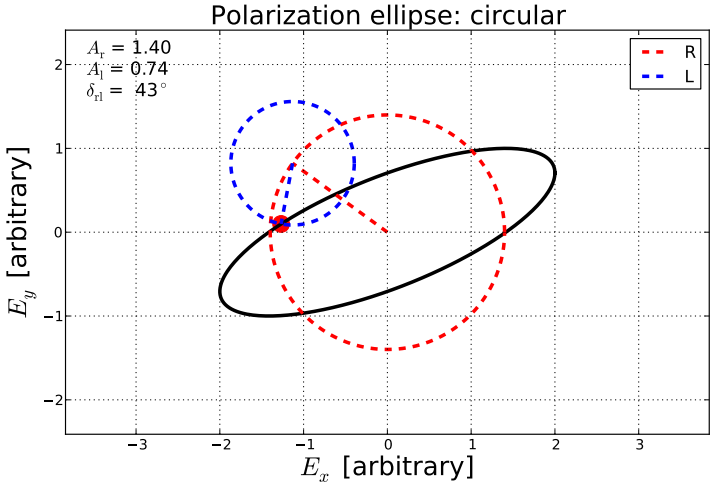
Polarization ellipse: circular 012



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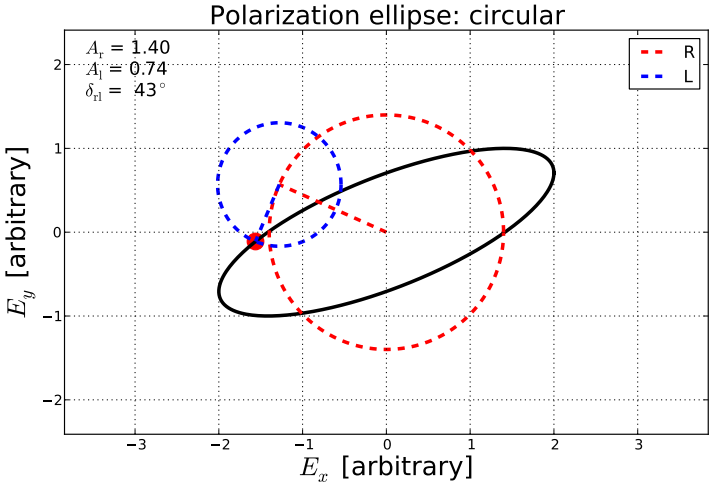
Polarization ellipse: circular 013



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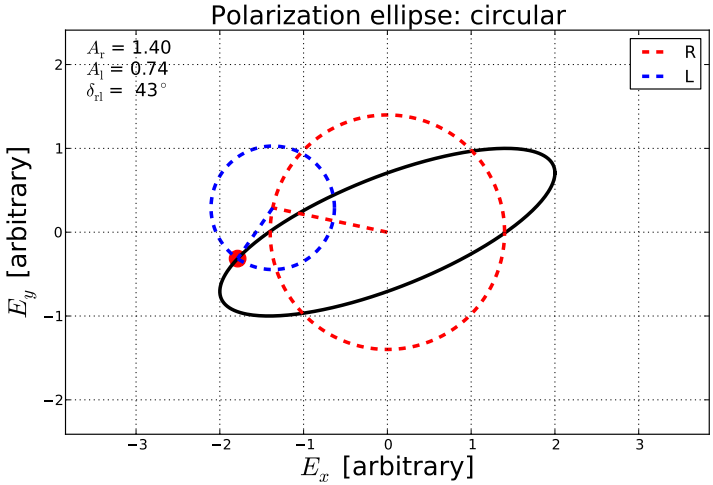
Polarization ellipse: circular 014



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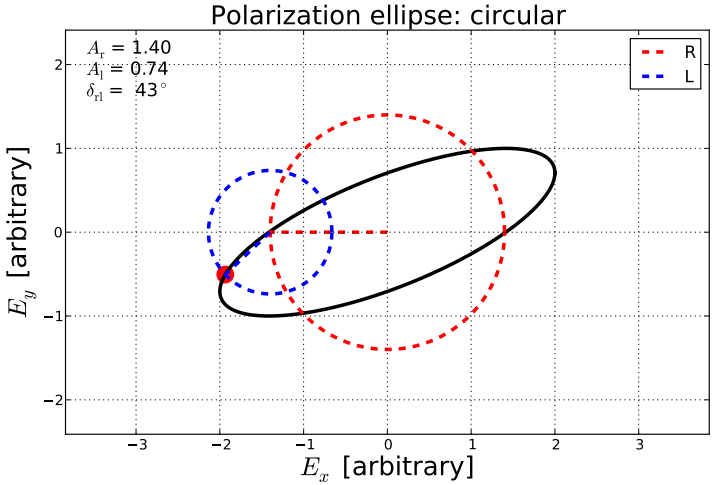
Polarization ellipse: circular 015



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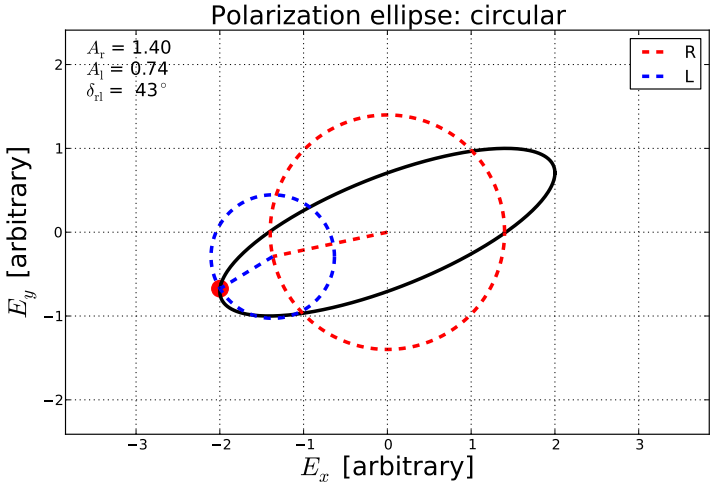
Polarization ellipse: circular 016



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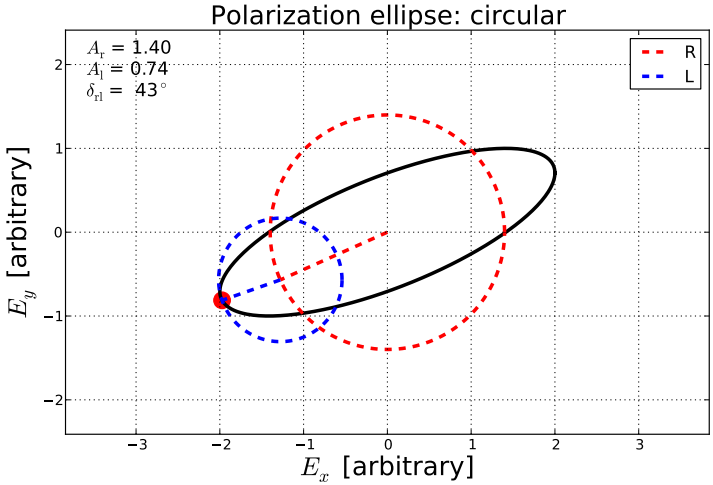
Polarization ellipse: circular 017



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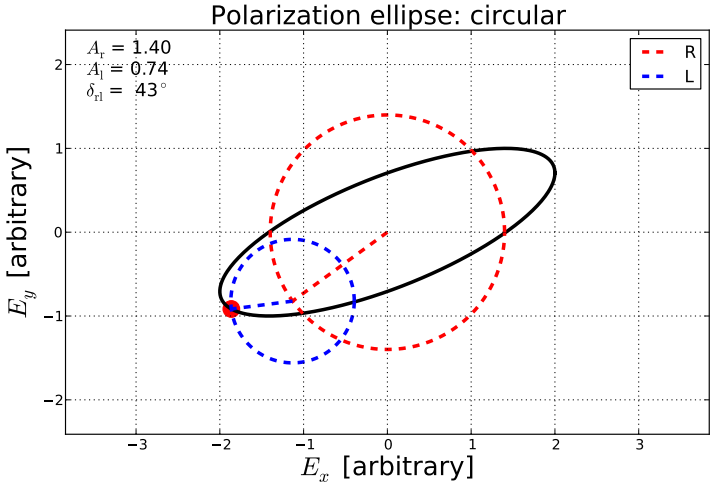
Polarization ellipse: circular 018



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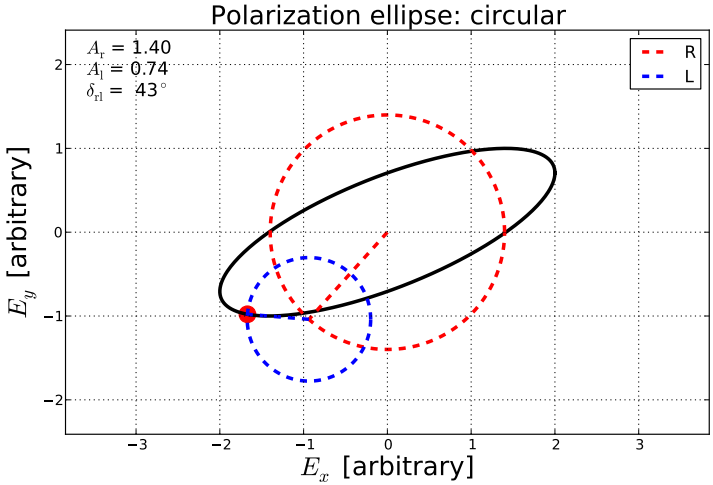
Polarization ellipse: circular 019



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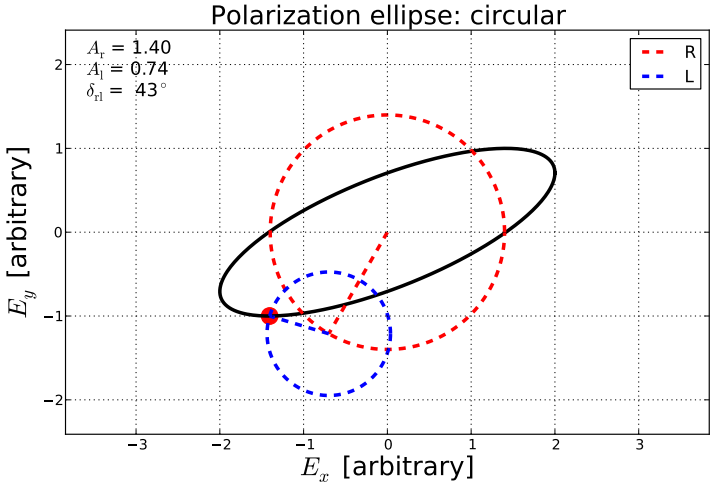
Polarization ellipse: circular 020



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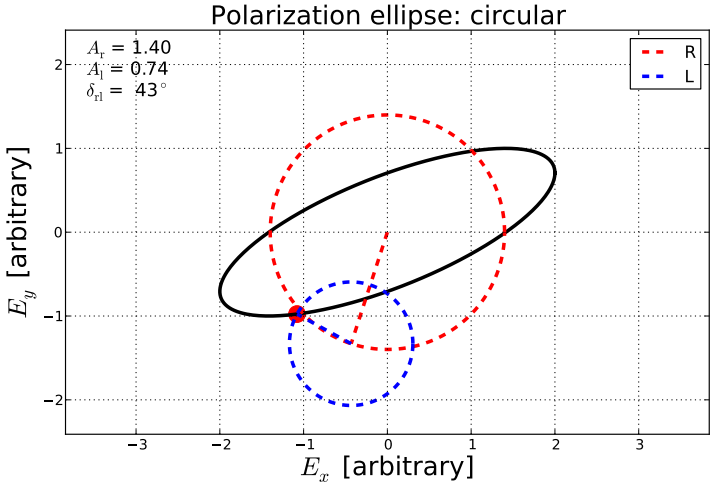
Polarization ellipse: circular 021



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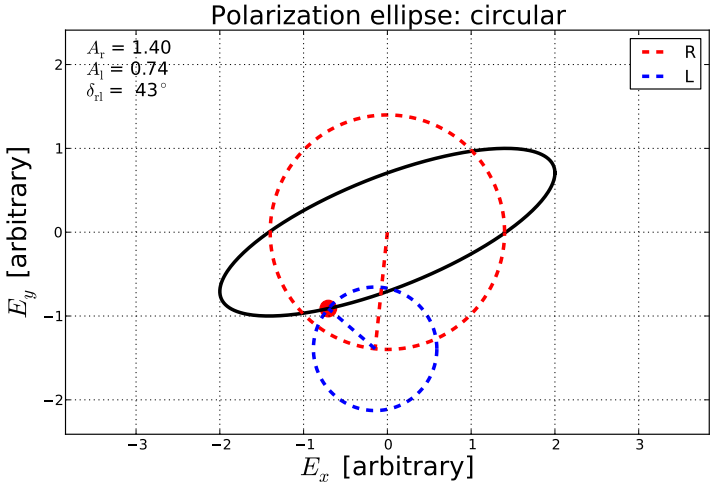
Polarization ellipse: circular 022



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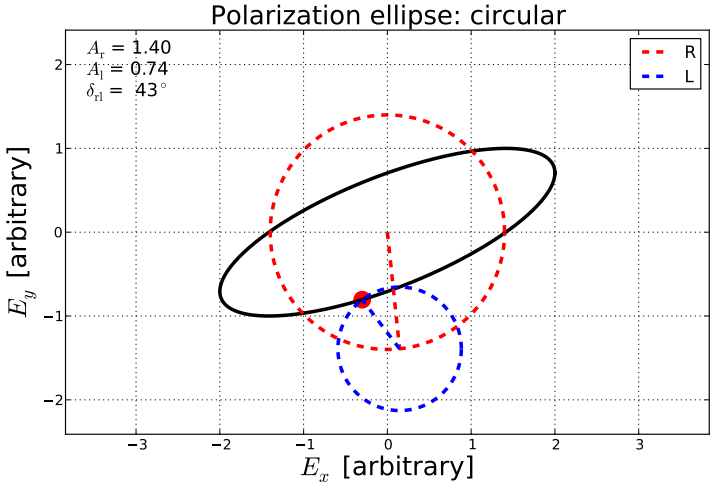
Polarization ellipse: circular 023



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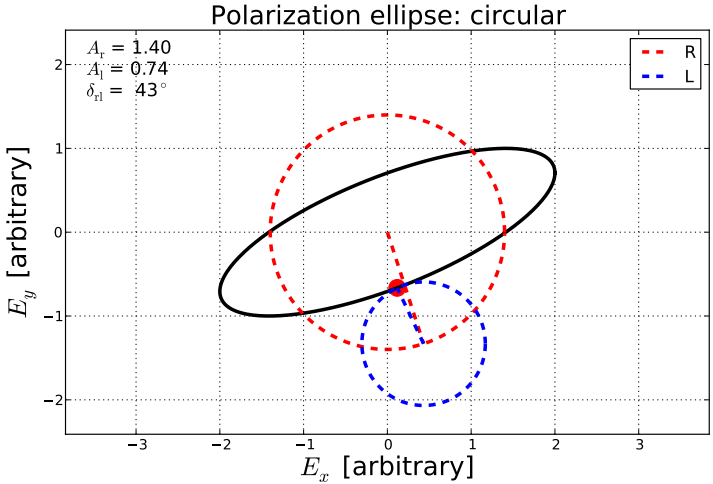
Polarization ellipse: circular 024



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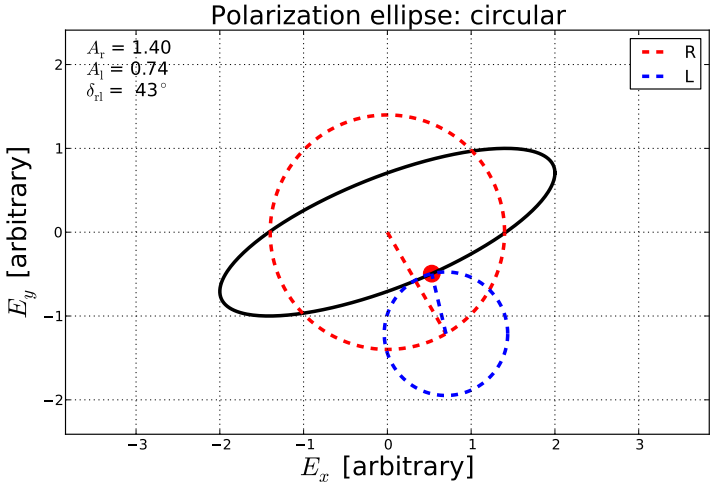
Polarization ellipse: circular 025



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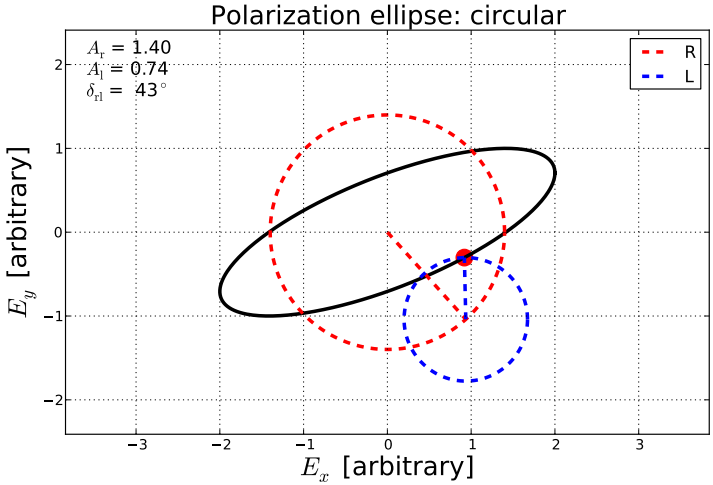
Polarization ellipse: circular 026



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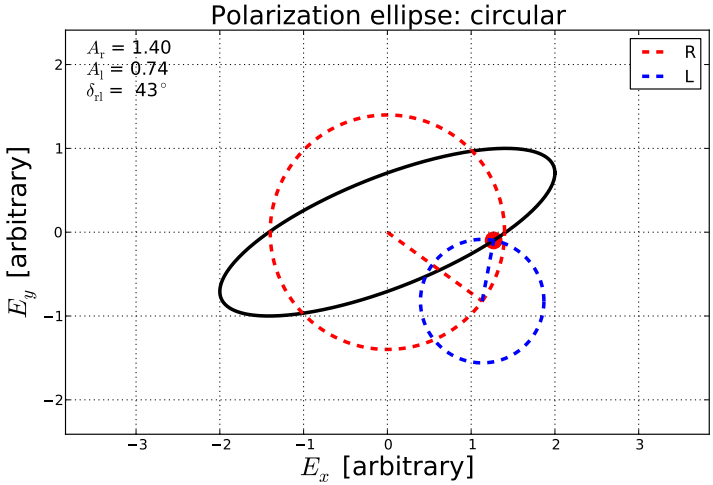
Polarization ellipse: circular 027



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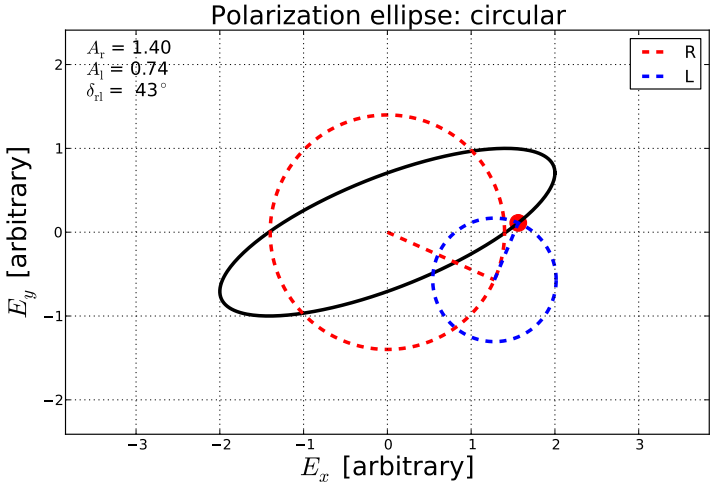
Polarization ellipse: circular 028



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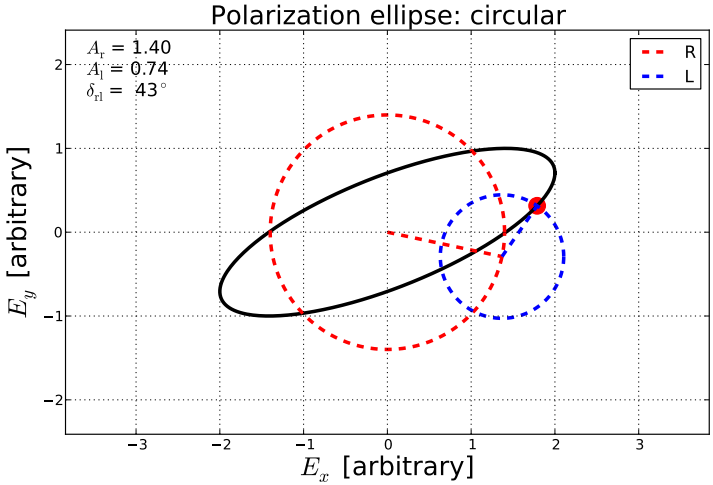
Polarization ellipse: circular 029



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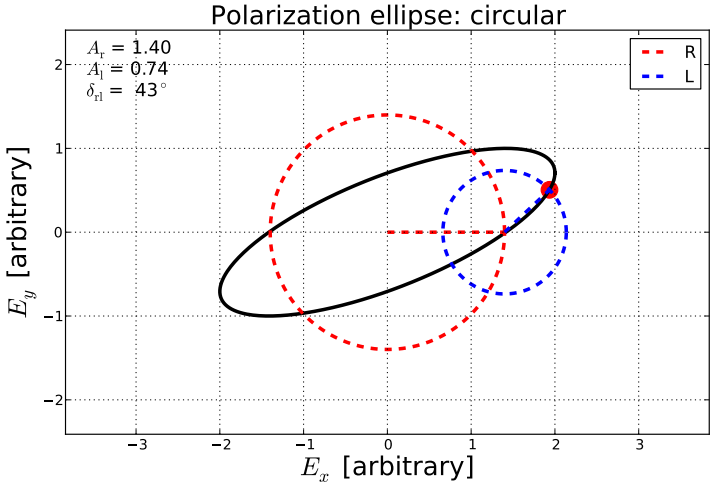
Polarization ellipse: circular 030



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Relation between cartesian and circular

$$
\begin{aligned}
A_{\mathrm{r}} &= \frac{1}{2}\sqrt{A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 - 2A_{\mathrm{x}}A_{\mathrm{y}}\sin\delta_{\mathrm{xy}}} \\
A_{\mathrm{l}} &= \frac{1}{2}\sqrt{A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 + 2A_{\mathrm{x}}A_{\mathrm{y}}\sin\delta_{\mathrm{xy}}} \\
\tan\delta_{\mathrm{rl}} &= \frac{2A_{\mathrm{x}}A_{\mathrm{y}}\cos\delta_{\mathrm{xy}}}{A_{\mathrm{x}}^2 - A_{\mathrm{y}}^2}
\end{aligned}
$$

Stokes parameters

- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1852)
- Chandrasekhar introduced them to astronomy (1946)

$$
\begin{aligned}
I &= A_{\mathrm{x}}^2 + A_{\mathrm{y}}^2 \\
Q &= A_{\mathrm{x}}^2 - A_{\mathrm{y}}^2 \\
U &= 2A_{\mathrm{x}}A_{\mathrm{y}} \cos \delta_{\mathrm{xy}} \\
V &= -2A_{\mathrm{x}}A_{\mathrm{y}} \sin \delta_{\mathrm{xy}}
\end{aligned}
\qquad
\begin{aligned}
I &= A_{\mathrm{r}}^2 + A_{\mathrm{l}}^2 \\
Q &= 2A_{\mathrm{r}}A_{\mathrm{l}} \cos \delta_{\mathrm{rl}} \\
U &= -2A_{\mathrm{r}}A_{\mathrm{l}} \sin \delta_{\mathrm{rl}} \\
V &= A_{\mathrm{r}}^2 - A_{\mathrm{l}}^2
\end{aligned}
$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$

Stokes parameters

- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1852) *ABCD*
- Chandrasekhar introduced them to astronomy (1946) $I_l I_r UV$

$$I = A_x^2 + A_y^2$$
$$Q = A_x^2 - A_y^2$$
$$U = 2A_x A_y \cos\delta_{xy}$$
$$V = -2A_x A_y \sin\delta_{xy}$$

$$I = A_r^2 + A_l^2$$
$$Q = 2A_r A_l \cos\delta_{rl}$$
$$U = -2A_r A_l \sin\delta_{rl}$$
$$V = A_r^2 - A_l^2$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$

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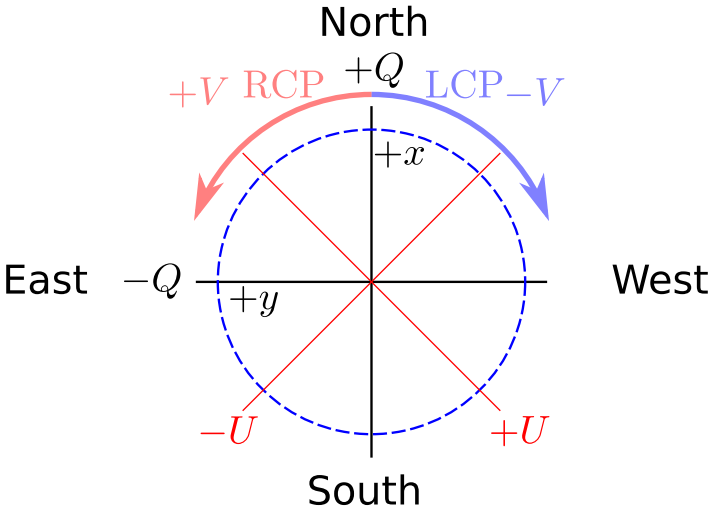
IAU conventions



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Public angry letter from IAU (Dec 2015)

International Astronomical Union
Union Astronomique Internationale
98bis bd Arago, F - 75014 Paris, France ■ www.iau.org

Piero Benvenuti – General Secretary
e-mail: iau-generalsecretary@iap.fr

To Whom It May Concern

The issue

Scientists working on the polarization of [redacted] use a convention for the polarization angle (PA) which is opposite to the IAU approved standard. This may cause confusion and misunderstandings.

Backgroud

The convention astronomers follow for the PA (Polarization Angle) goes back to the 19th century and it has been in use for observations going from radio to gamma rays: the PA increases counter-clockwise when looking at the source. This convention is consistent with the one used for the Position Angle and it has been enforced by the IAU with a Resolution by Commissions 25 and 40 at the IAU XVth General Assembly in Sydney in 1973 (see Transactions of the IAU, Vol. XVB, pg. 166).

Recently, the scientists investigating the polarization of [redacted] have unfortunately adopted the opposite convention (PA increasing clockwise when looking at the source). This corresponds to a change of sign of the U Stokes parameter and is causing confusion and misunderstandings, in particular in the case of polarization data coming from experiments and satellites which are used by [redacted] and by other astronomers.

Recommendation

The IAU recommends that all astronomers, including those [redacted] follow the IAU Resolution for the Polarization Angle in all their publications.

Paris, December 8th, 2015

Piero Benvenuti
IAU General Secretary

Pietro Ubertini
President, Division B

Saul J. Adelman
President, Commission B6

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Stokes parameters revisited

Quasi-monochromatic approximation

- Monochromatic radiation does not exist
- Finite bandwidth $\Delta\nu$; averaging time $\tau \gg \Delta\nu^{-1}$

$$
\begin{aligned}
I &= \langle A_{\mathrm{x}}^2 \rangle + \langle A_{\mathrm{y}}^2 \rangle \\
Q &= \langle A_{\mathrm{x}}^2 \rangle - \langle A_{\mathrm{y}}^2 \rangle \\
U &= \langle 2 A_{\mathrm{x}} A_{\mathrm{y}} \cos \delta_{\mathrm{xy}} \rangle \\
V &= \langle -2 A_{\mathrm{x}} A_{\mathrm{y}} \sin \delta_{\mathrm{xy}} \rangle
\end{aligned}
\qquad
\begin{aligned}
I &= \langle A_{\mathrm{r}}^2 \rangle + \langle A_{\mathrm{l}}^2 \rangle \\
Q &= \langle 2 A_{\mathrm{r}} A_{\mathrm{l}} \cos \delta_{\mathrm{rl}} \rangle \\
U &= \langle -2 A_{\mathrm{r}} A_{\mathrm{l}} \sin \delta_{\mathrm{rl}} \rangle \\
V &= \langle A_{\mathrm{r}}^2 \rangle - \langle A_{\mathrm{l}}^2 \rangle
\end{aligned}
$$

$$I^2 \geq Q^2 + U^2 + V^2$$

- Fractional linear pol: $p = \sqrt{Q^2 + U^2}/I \leq 1$
- Fractional circular pol: $v = \|V\|/I \leq 1$

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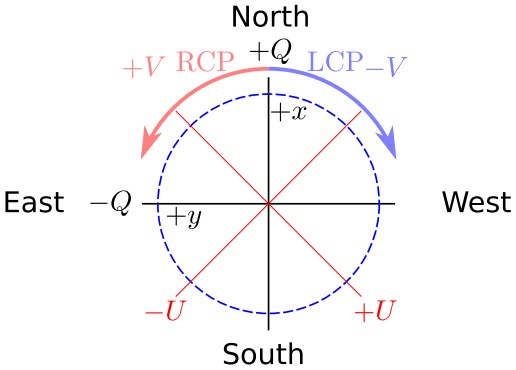
Polarization orientation and Stokes parameters

(Vector length $\propto$ flux, not E-field;
Unit circle represents total flux I)

Polarization	I (Jy)	Q (Jy)	U (Jy)	V (Jy)
	1	0	0	0
	1	-1	0	0
	1	0	+1	0
	1	+1	0	0
	1	$+\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	0
	1	0	0	$-\frac{1}{2}$



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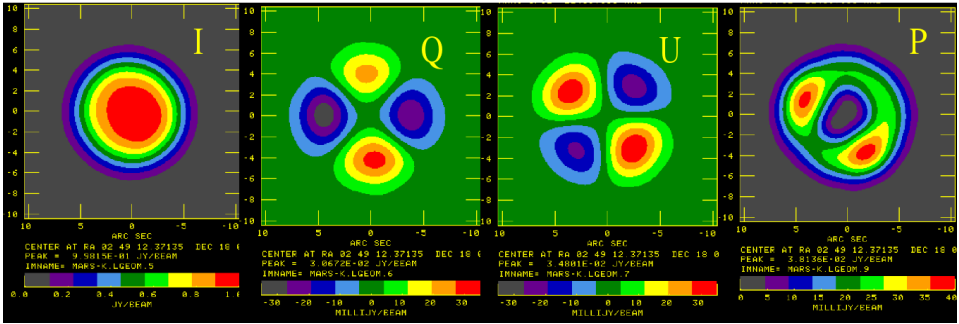
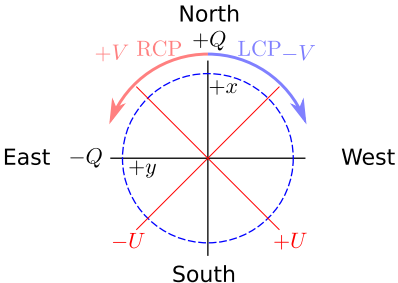
TEST: Mars with VLA at 23.4 GHz *Perley*

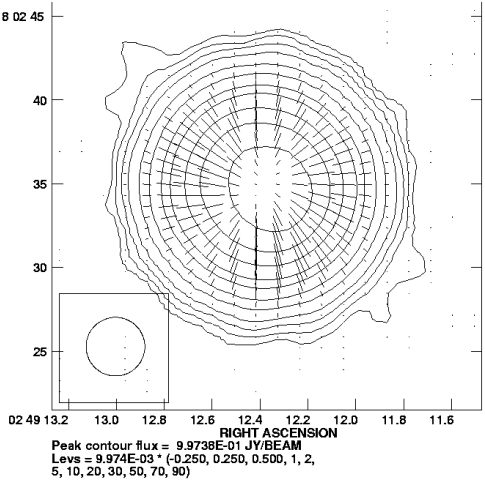


- This is thermal emission
- 1) Draw a map of the pol vectors
- 2) Why is it even polarized?



Wait for it. . .

Answer: Mars polarization vectors *Perley*



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Outline

EM wave physics

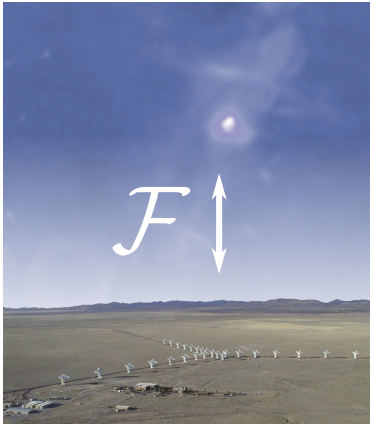
Astrophysics

Polarized EM-waves

Interferometric polarimetry

Messy reality

Stokes visibilities



$$
\begin{aligned}
\mathcal{I}(u,v) &= \mathcal{F}^{+}(I(l,m)) \\
\mathcal{Q}(u,v) &= \mathcal{F}^{+}(Q(l,m)) \\
\mathcal{U}(u,v) &= \mathcal{F}^{+}(U(l,m)) \\
\mathcal{V}(u,v) &= \mathcal{F}^{+}(V(l,m)),
\end{aligned}
$$

where

$$
\mathcal{F}^{+}(f) = \int_{lm} f \mathrm{e}^{+2\pi \mathrm{i} \nu (ul+vm)/c} \mathrm{d}l\mathrm{d}m
$$

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Stokes parameters in complex notation

Cartesian

$$E_\mathrm{x} = \Re\left\{A_\mathrm{x}\mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$
$$E_\mathrm{y} = \Re\left\{A_\mathrm{y}\mathrm{e}^{\mathrm{i}\delta_\mathrm{xy}}\mathrm{e}^{2\pi\mathrm{i}\nu t}\right\}$$

$$\begin{aligned}
I &= \langle A_\mathrm{x}^2\rangle + \langle A_\mathrm{y}^2\rangle &&= \langle E_\mathrm{x}E_\mathrm{x}^*\rangle + \langle E_\mathrm{y}E_\mathrm{y}^*\rangle \\
Q &= \langle A_\mathrm{x}^2\rangle - \langle A_\mathrm{y}^2\rangle &&= \langle E_\mathrm{x}E_\mathrm{x}^*\rangle - \langle E_\mathrm{y}E_\mathrm{y}^*\rangle \\
U &= \langle 2A_\mathrm{x}A_\mathrm{y}\cos\delta_\mathrm{xy}\rangle &&= \langle E_\mathrm{x}E_\mathrm{y}^*\rangle + \langle E_\mathrm{y}E_\mathrm{x}^*\rangle \\
V &= \langle -2A_\mathrm{x}A_\mathrm{y}\sin\delta_\mathrm{xy}\rangle &&= -\mathrm{i}\left(\langle E_\mathrm{x}E_\mathrm{y}^*\rangle - \langle E_\mathrm{y}E_\mathrm{x}^*\rangle\right)
\end{aligned}$$

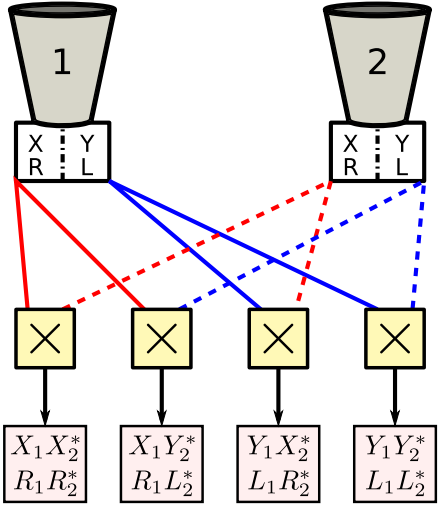
Stokes parameters in complex notation

Circular

$$E_\mathrm{r} = A_\mathrm{r} \mathrm{e}^{2\pi \mathrm{i} \nu t}$$
$$E_\mathrm{l} = A_\mathrm{l} \mathrm{e}^{-\mathrm{i}\delta_\mathrm{rl}} \mathrm{e}^{-2\pi \mathrm{i} \nu t}$$

$$\begin{aligned}
I &= \langle A_\mathrm{r}^2 \rangle + \langle A_\mathrm{l}^2 \rangle &&= \langle E_\mathrm{r} E_\mathrm{r}^* \rangle + \langle E_\mathrm{l} E_\mathrm{l}^* \rangle \\
Q &= \langle 2 A_\mathrm{r} A_\mathrm{l} \cos \delta_\mathrm{rl} \rangle &&= \langle E_\mathrm{r} E_\mathrm{l}^* \rangle + \langle E_\mathrm{l} E_\mathrm{r}^* \rangle \\
U &= \langle -2 A_\mathrm{r} A_\mathrm{l} \sin \delta_\mathrm{rl} \rangle &&= \mathrm{i} \left(\langle E_\mathrm{r} E_\mathrm{l}^* \rangle - \langle E_\mathrm{l} E_\mathrm{r}^* \rangle \right) \\
V &= \langle A_\mathrm{r}^2 \rangle - \langle A_\mathrm{l}^2 \rangle &&= \langle E_\mathrm{r} E_\mathrm{r}^* \rangle - \langle E_\mathrm{l} E_\mathrm{l}^* \rangle
\end{aligned}$$

Correlating interferometer



Antenna

Polarizer
(XY *or* RL)

Complex
correlators

Output

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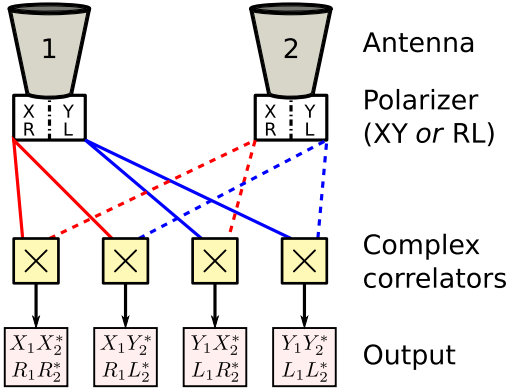
Stokes visibilities from correlator outputs



Cartesian

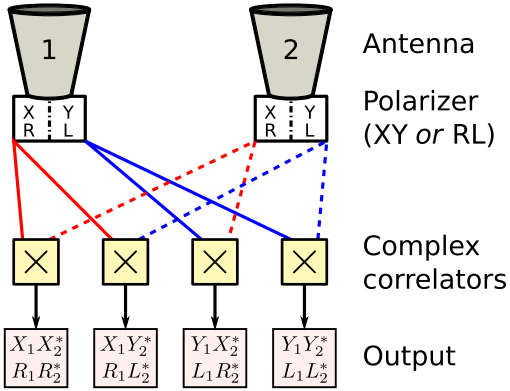
$$\mathcal{I} = x_1 x_2^* + y_1 y_2^*$$
$$\mathcal{Q} = x_1 x_2^* - y_1 y_2^*$$
$$\mathcal{U} = x_1 y_2^* + y_1 x_2^*$$
$$\mathcal{V} = -\mathrm{i}\,(x_1 y_2^* - y_1 x_2^*)$$

Circular

$$\mathcal{I} = r_1 r_2^* + l_1 l_2^*$$
$$\mathcal{Q} = r_1 l_2^* + l_1 r_2^*$$
$$\mathcal{U} = \mathrm{i}\,(r_1 l_2^* - l_1 r_2^*)$$
$$\mathcal{V} = r_1 r_2^* - l_1 l_2^*$$

- From here on, $\langle\cdot\rangle$ is implied for correlator outputs.

Coherency matrix



From here on, p and q designate either x and y, or r and l.

- Polarizers produce vector:

$$\mathbf{e}_i = \begin{pmatrix} p_i \\ q_i \end{pmatrix}$$

- Correlator multiplies:

$$\mathbf{E}_{ij} = \mathbf{e}_i \mathbf{e}_j^\dagger = \begin{pmatrix} p_i \\ q_i \end{pmatrix} \begin{pmatrix} p_j^* & q_j^* \end{pmatrix}$$

$$\mathbf{E}_{ij} = \begin{pmatrix} p_i p_j^* & p_i q_j^* \\ q_i p_j^* & q_i q_j^* \end{pmatrix}$$

- $\mathbf{E}_{ij}$ is the coherency matrix

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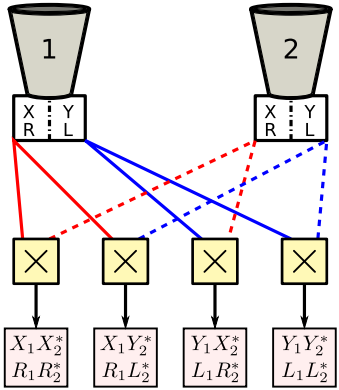
Outline

Jones calculus



Antenna

Polarizer (XY *or* RL)

Complex correlators

Output

Until now...

- Assumed all systems perfect

From now...

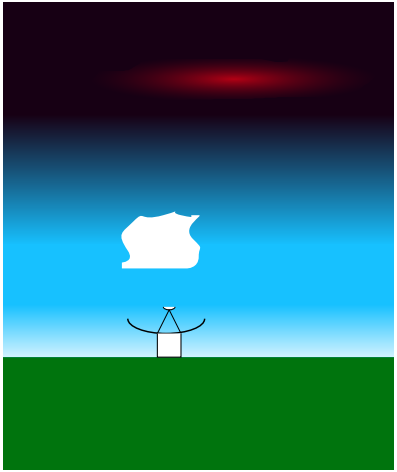
- Assume all systems linear:

$$\mathbf{e}'_i = \mathbf{J}_i \mathbf{e}_i$$

- $\mathbf{J}_i$ (2×2) is Jones matrix
- Cross correlation:

$$\begin{aligned}
\mathbf{E}'_{ij} &= \mathbf{e}'_i \mathbf{e}'^{\dagger}_j \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{e}_i \left(\mathbf{J}_j \mathbf{e}_j\right)^{\dagger} \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{e}_i \mathbf{e}^{\dagger}_j \mathbf{J}^{\dagger}_j \\
\mathbf{E}'_{ij} &= \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}^{\dagger}_j
\end{aligned}$$

Measurement equation



- The measurement equation:

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j^{\dagger}$$

- Invertible!

$$\mathbf{E}_{ij} = \mathbf{J}_i^{-1} \mathbf{E}'_{ij} \mathbf{J}_j^{\dagger -1},$$

- where

$$\mathbf{J} = \mathbf{RPDOWFT}$$

- . . . riiiiight. . .

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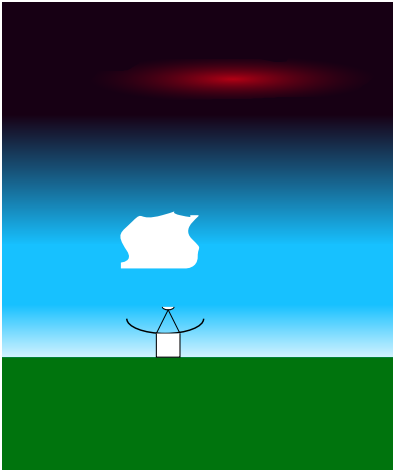
Measurement equation: examples



- Perfect instrument:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

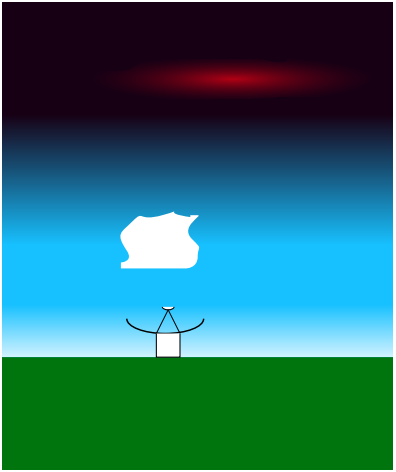
- Time delay:

$$\mathbf{J} = \begin{pmatrix} e^{2\pi i \nu \tau_p} & 0 \\ 0 & e^{2\pi i \nu \tau_q} \end{pmatrix}$$

- Receiver gain:

$$\mathbf{J} = \begin{pmatrix} g_p & 0 \\ 0 & g_q \end{pmatrix}$$

Measurement equation: examples



- Polarization leakage:

$$\mathbf{J} = \begin{pmatrix} g_{\mathrm{p}} & d_{\mathrm{q}\to\mathrm{p}} \\ d_{\mathrm{p}\to\mathrm{q}} & g_{\mathrm{q}} \end{pmatrix}$$

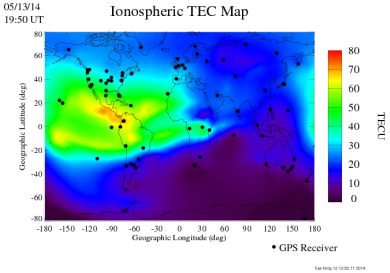
- Parallactic angle or feed rotation XY:

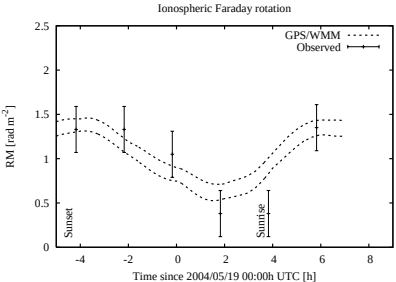
$$\mathbf{J} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

- Parallactic angle or feed rotation RL:

$$\mathbf{J} = \begin{pmatrix} \mathrm{e}^{+\mathrm{i}\theta} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\theta} \end{pmatrix}$$

Ionospheric Faraday rotation





- Remember: $\Delta\chi = \chi_0 + \phi\lambda^2$
- Faraday depth

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot d\mathbf{l}$$

- ionosphere: plasma within Earth's magnetic field
- $\phi \approx$ -10 – +10 rad m^{-2}
- *Very* significant below 1 GHz
- Use TEC/IGRF models for correction, check with pulsar.

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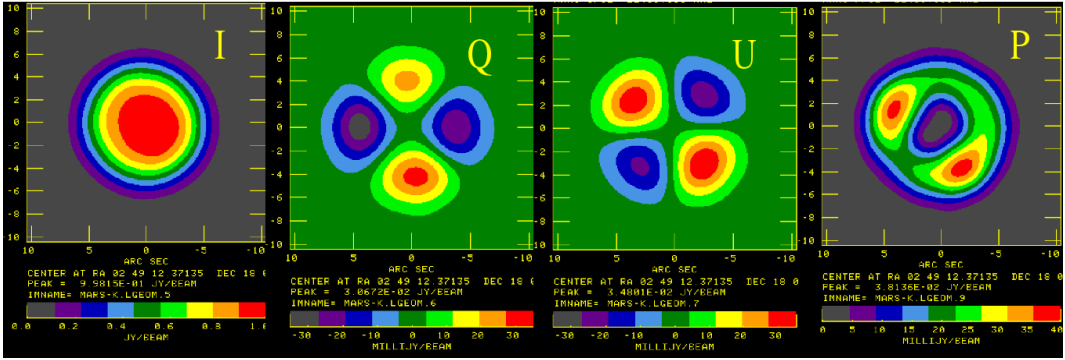
Take away this...

- Radio antennas are fundamentally polarized
- Polarimetry required for certain astrophysical observations
- Linear systems make for fairly straightforward calibration
- Understanding polarimetry improves your unpolarized calibration and imaging

Reading list

- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)

TEST: Mars with VLA at 23.4 GHz *Perley*



- This is thermal emission
- 1) Draw a map of the polarization vectors.
- 2) Why is it even polarized?

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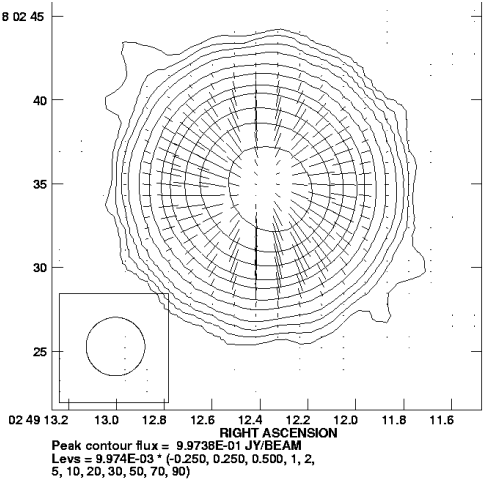
Answer: Mars polarization vectors *Perley*



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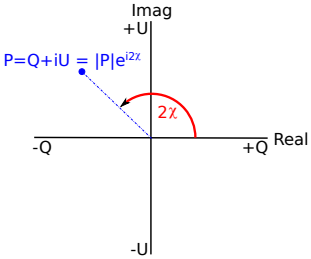
Faraday rotation measure synthesis



$$
\begin{aligned}
P(\lambda_c^2) &= \int_{\lambda^{2\prime}} \int_{\Omega} \int_{-\infty}^{\infty} W(\Omega, \lambda_c^2 + \lambda^{2\prime}) f(\Omega, \phi, \lambda_c^2 + \lambda^{2\prime}) e^{2i\phi(\lambda_c^2 + \lambda^{2\prime})} \, d\phi \, d\Omega \, d\lambda^{2\prime} \\
P(\lambda^2) &= W(\lambda^2) \int_{-\infty}^{\infty} f(\phi) e^{2i\phi\lambda^2} \, d\phi \\
f(\phi) * R(\phi) &= \int_{-\infty}^{\infty} P(\lambda^2) e^{-2i\phi\lambda^2} \, d\lambda^2
\end{aligned}
$$

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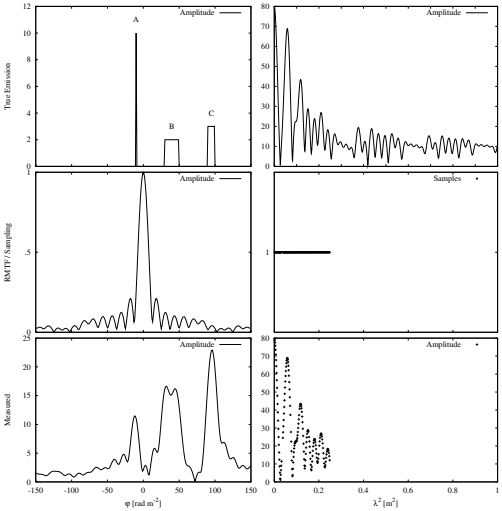
Short wavelength



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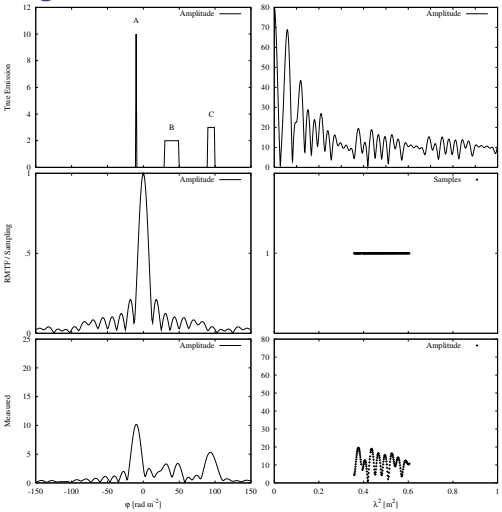
Intermediate wavelength



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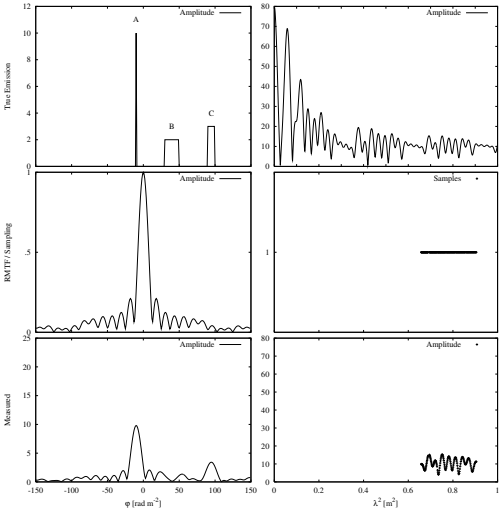
Long wavelength



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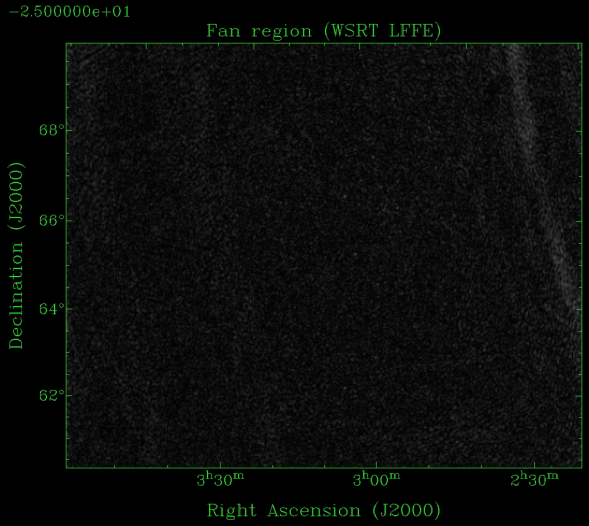
Fan region *G. Bernardi*



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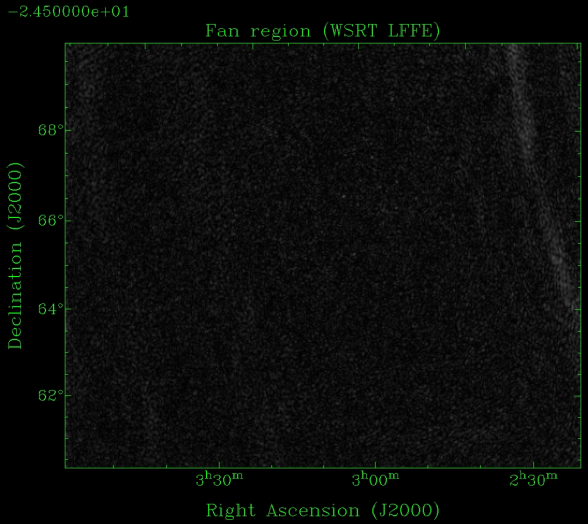

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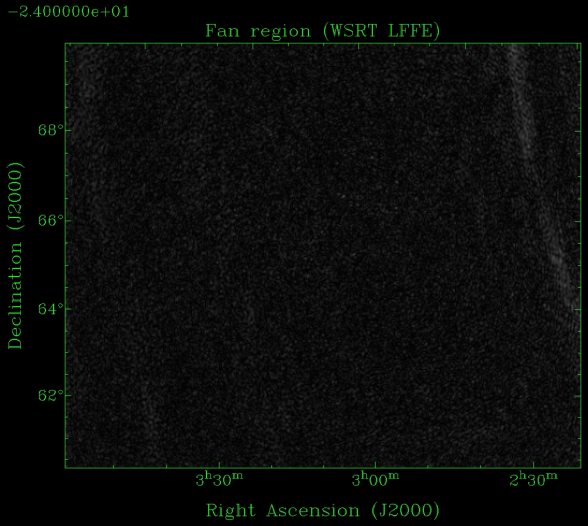

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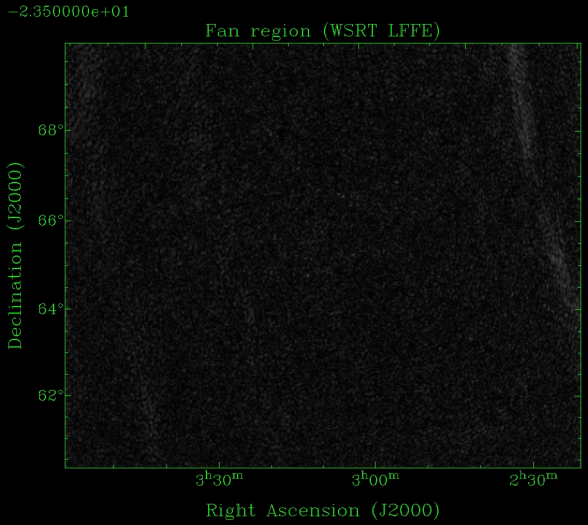

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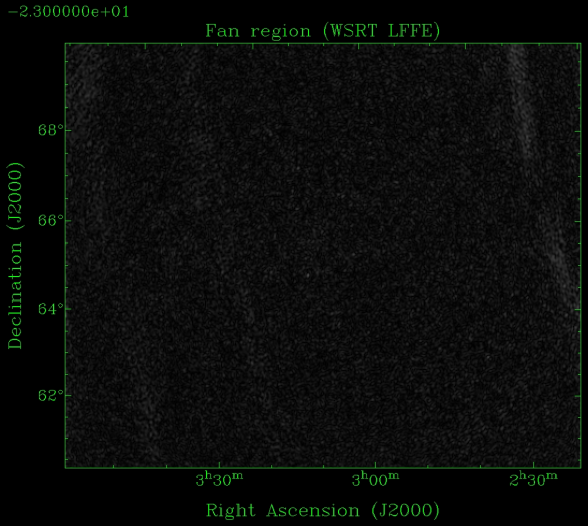

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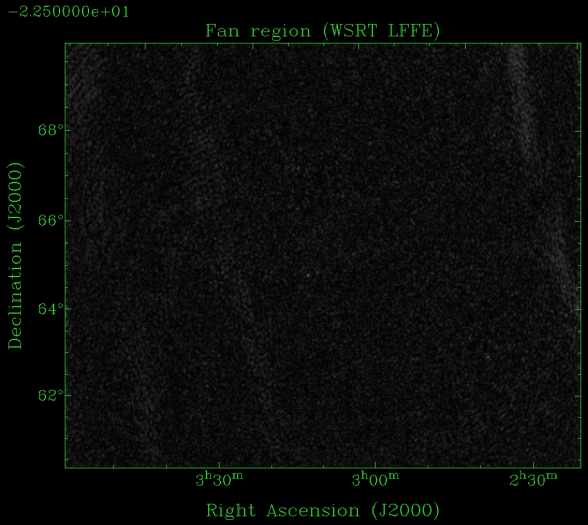

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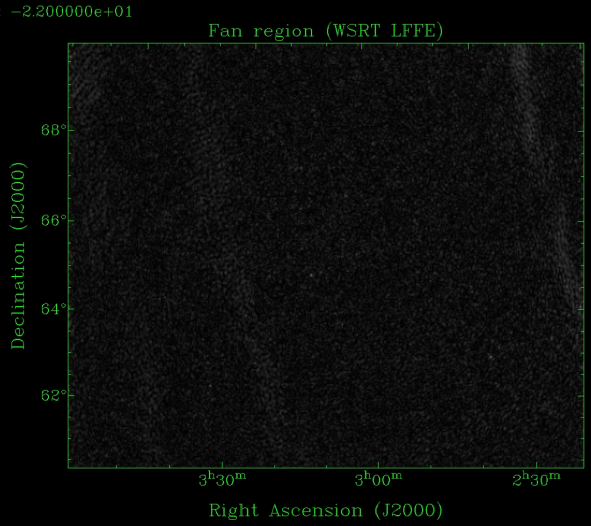

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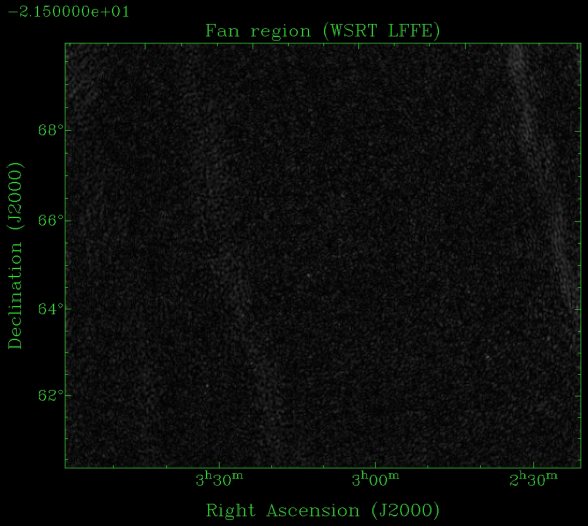

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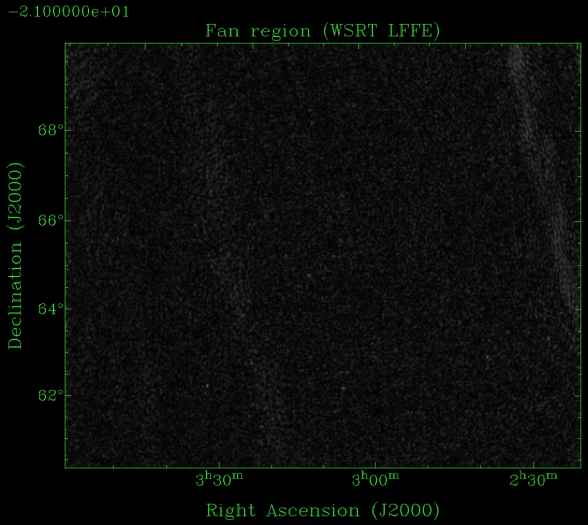

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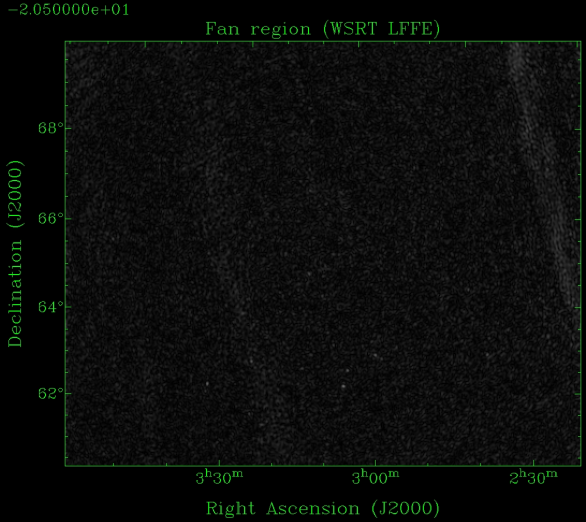

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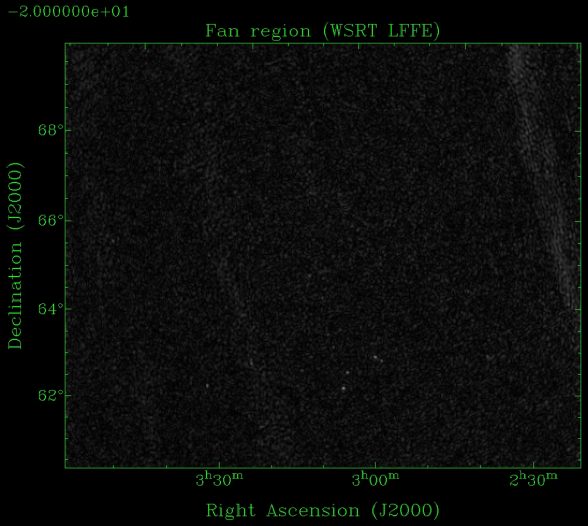

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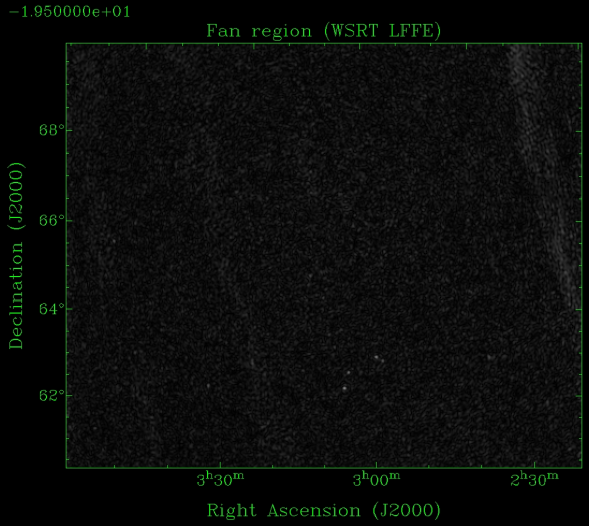

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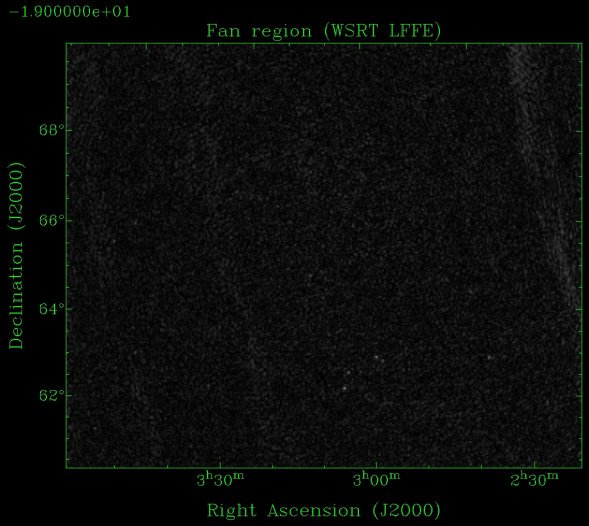

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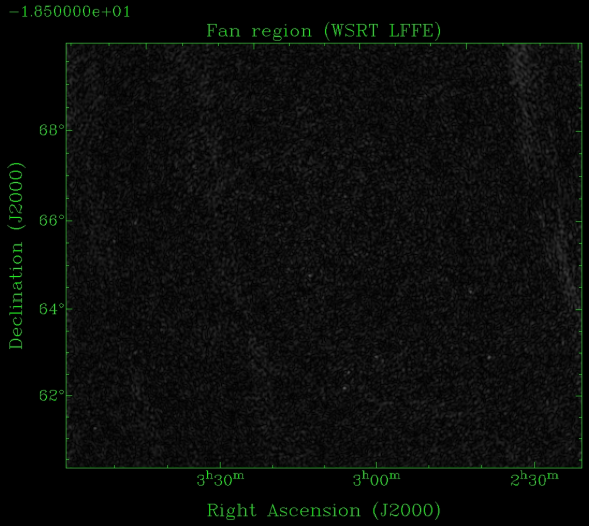

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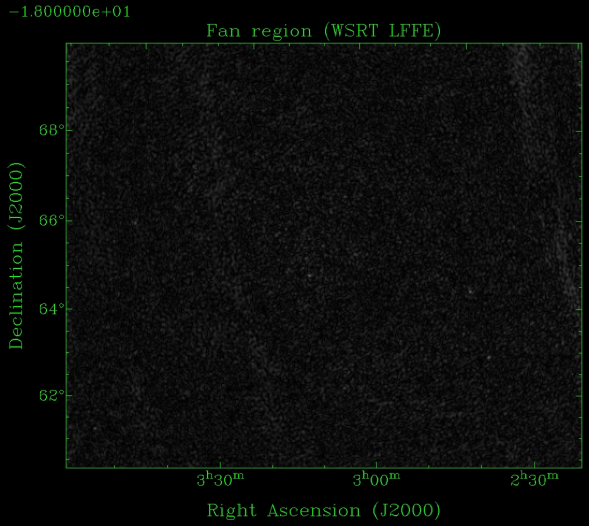

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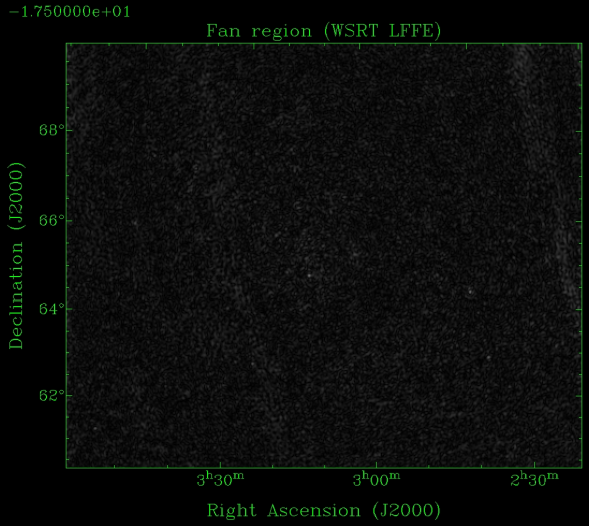

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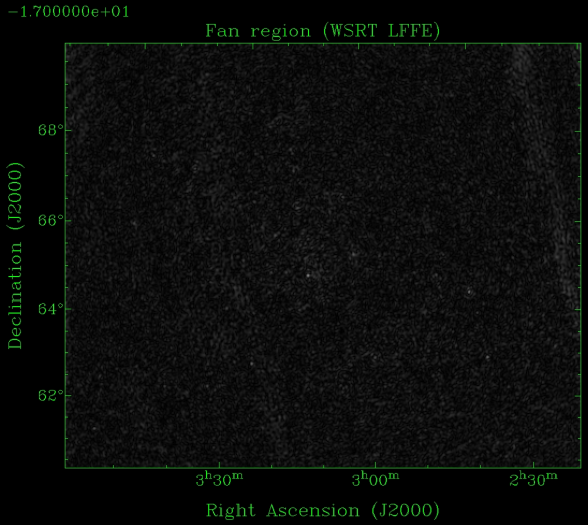

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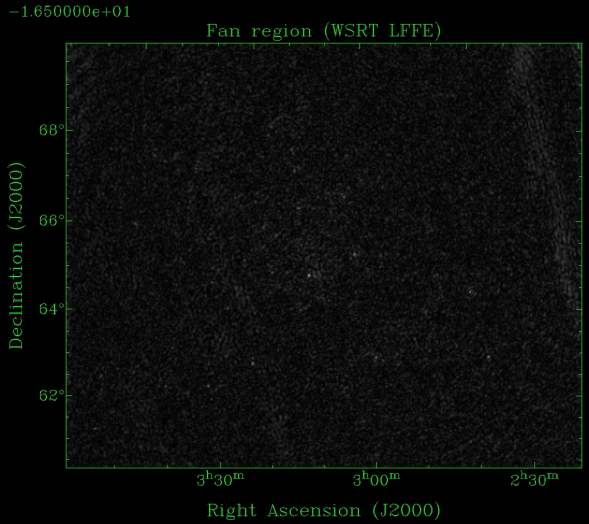

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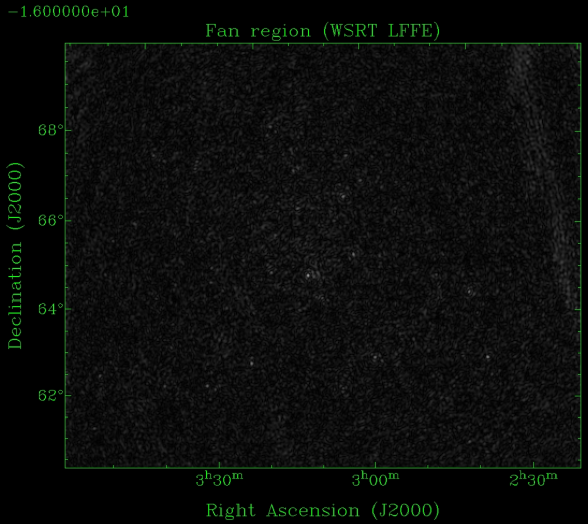

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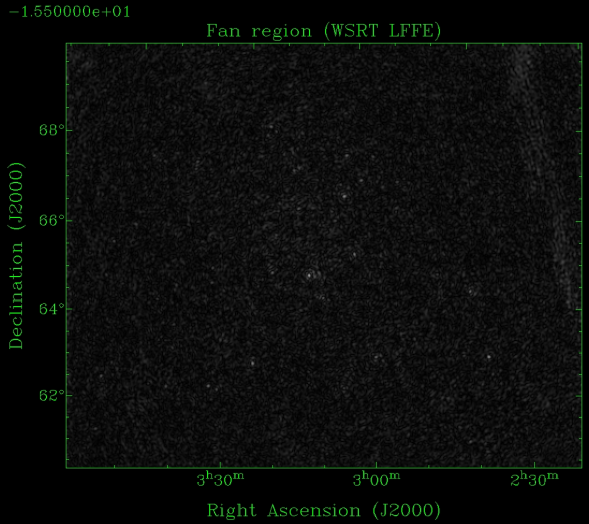

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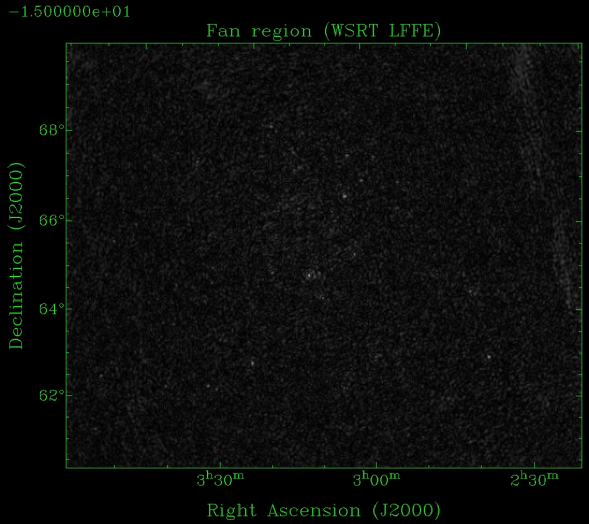

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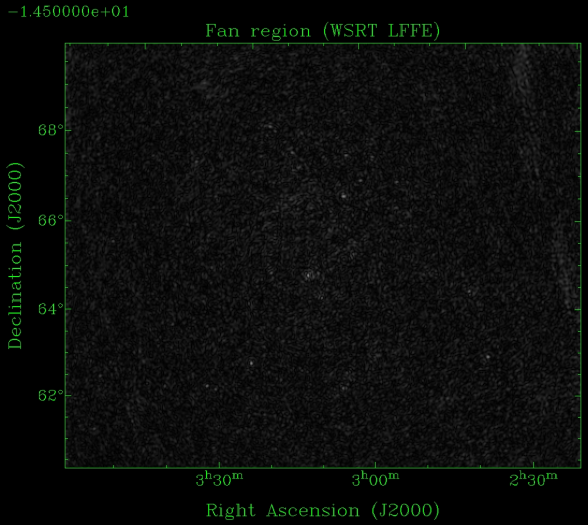

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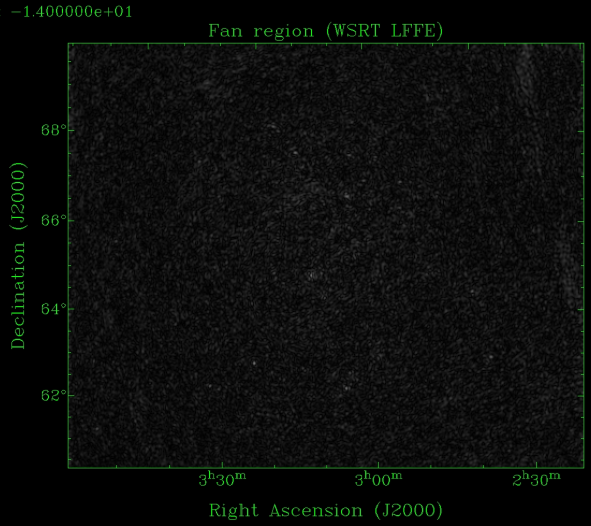

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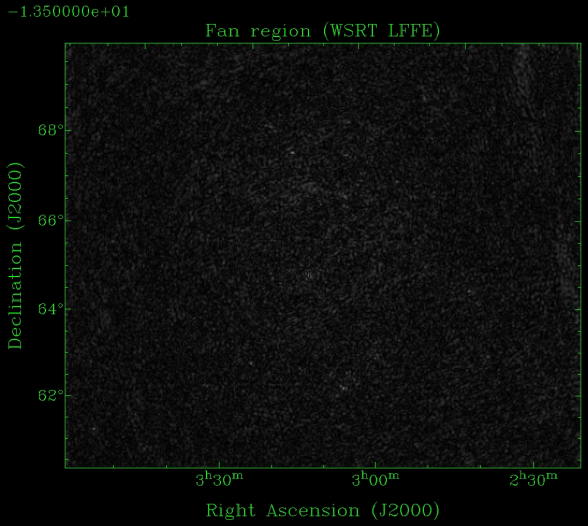

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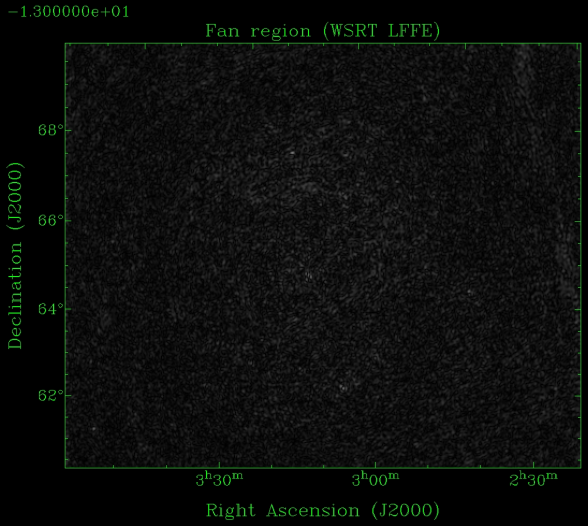

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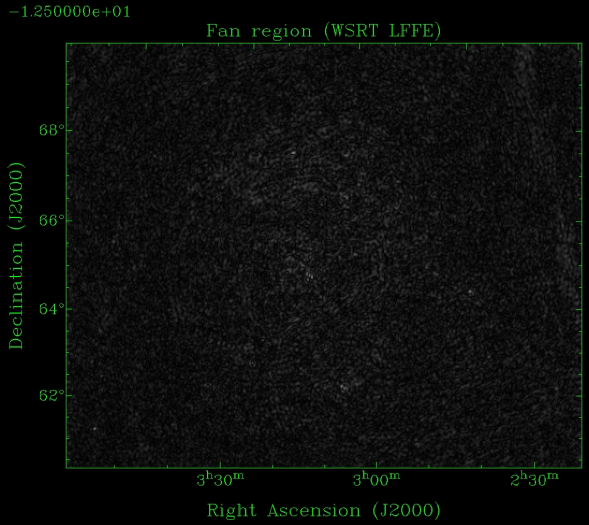

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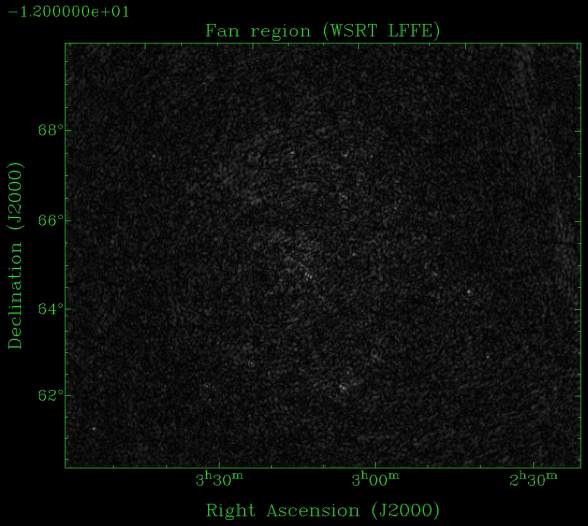

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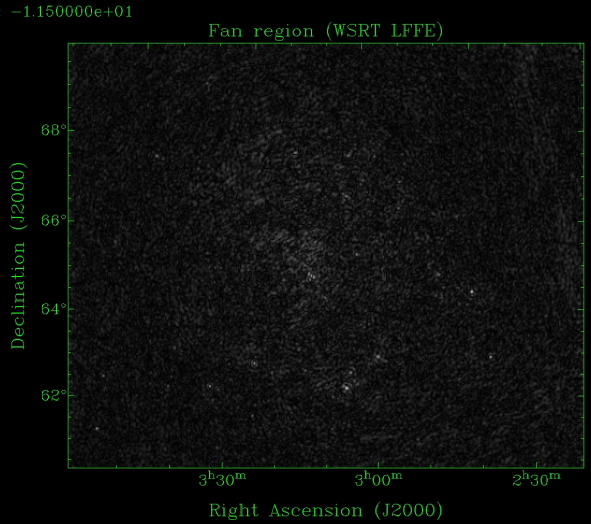

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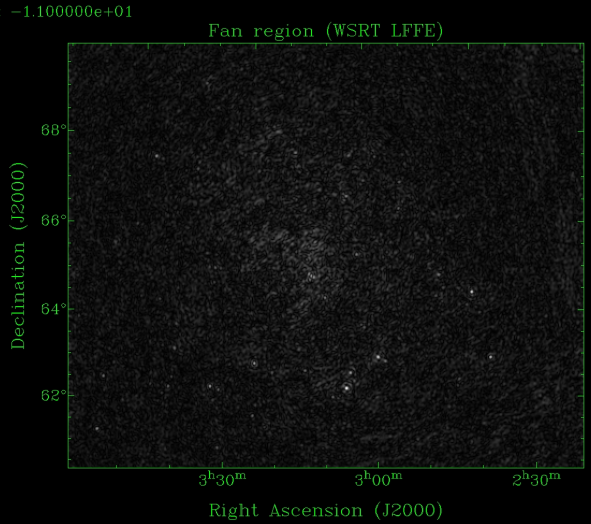

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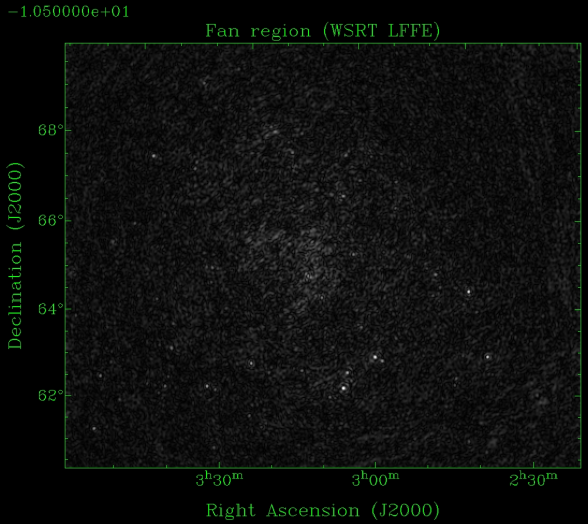

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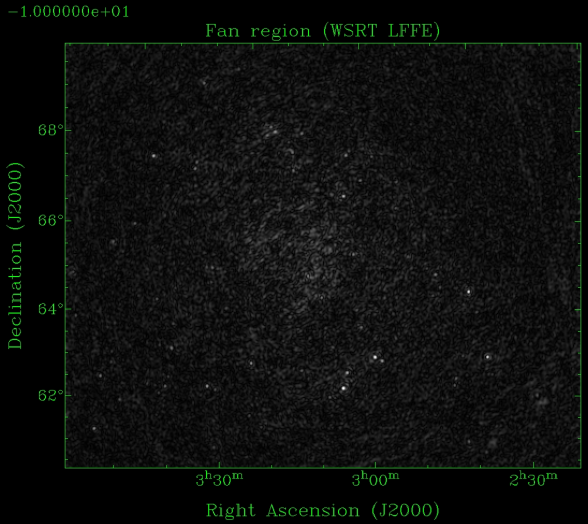

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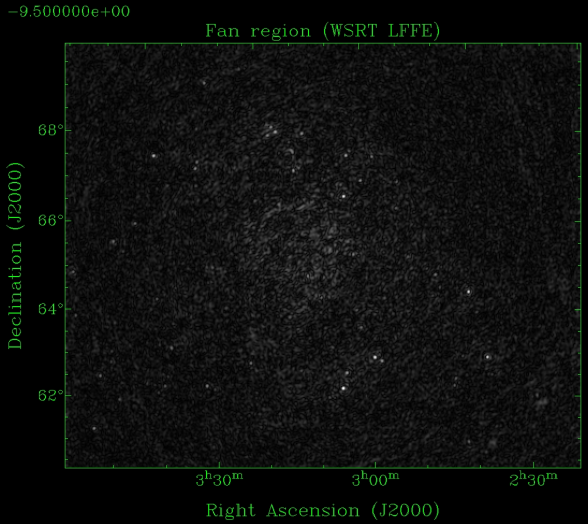

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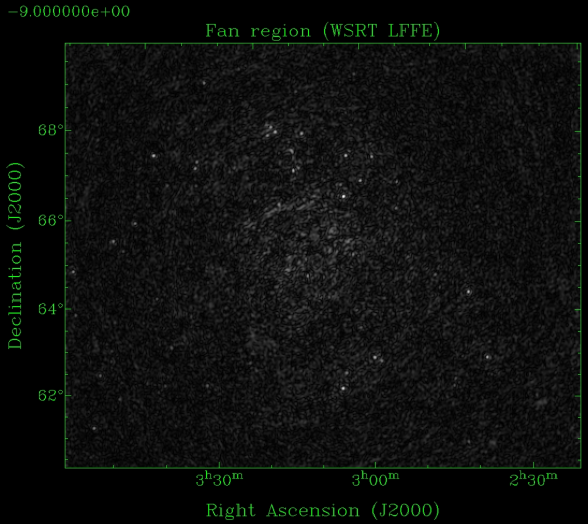

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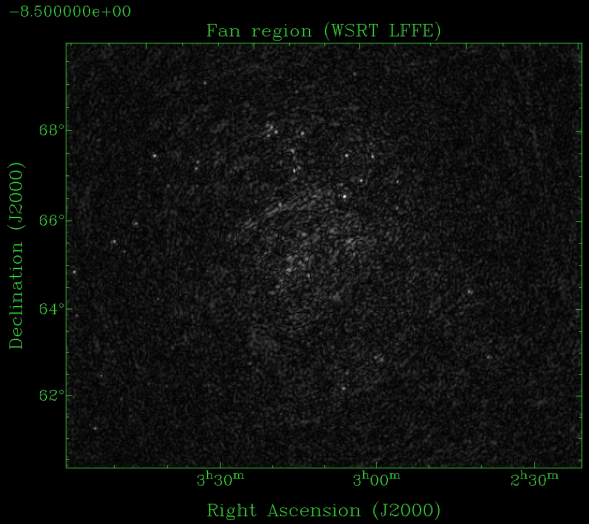

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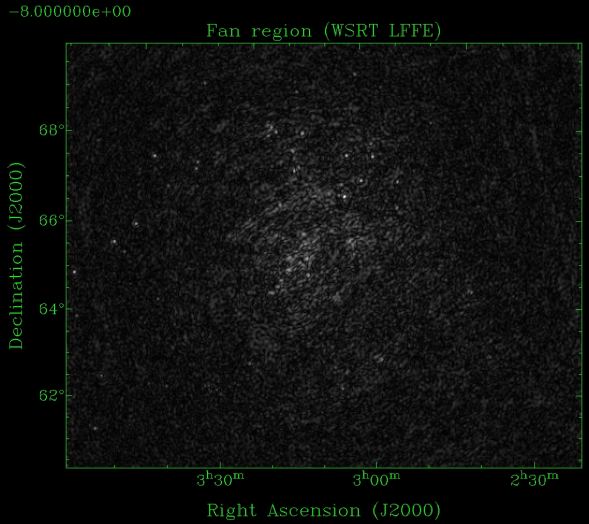

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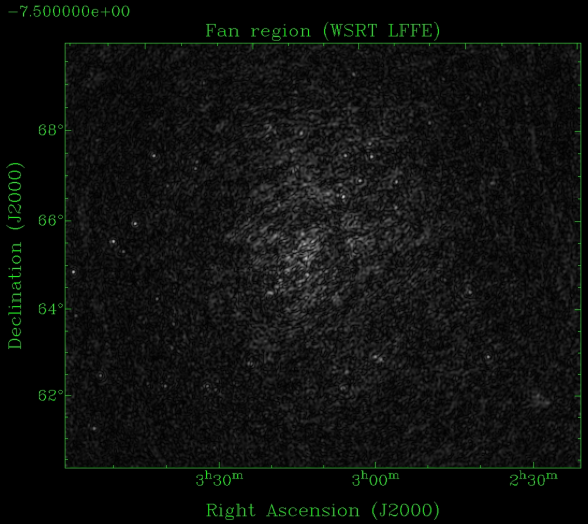

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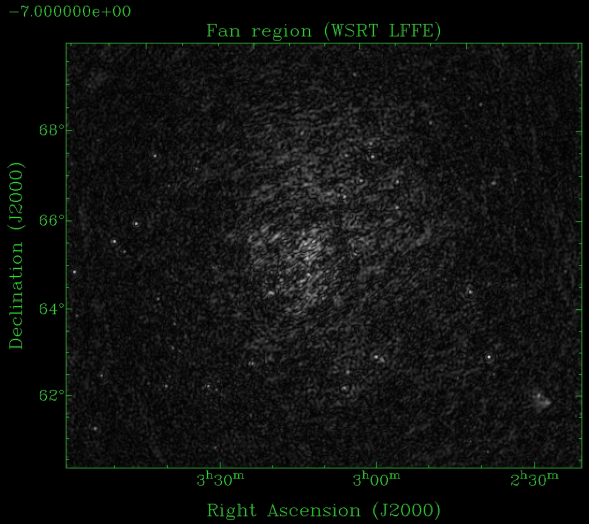

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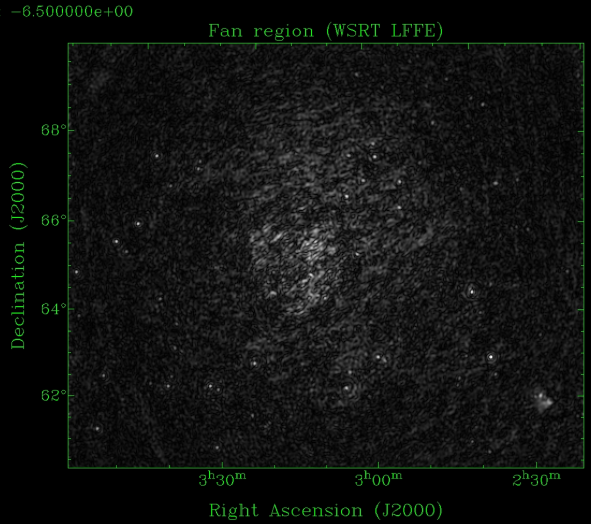

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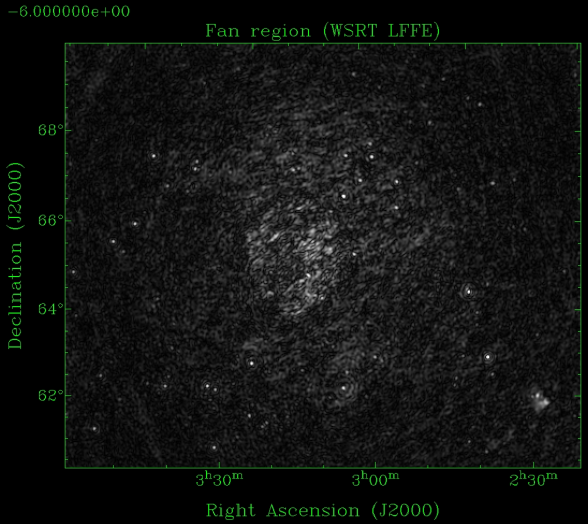

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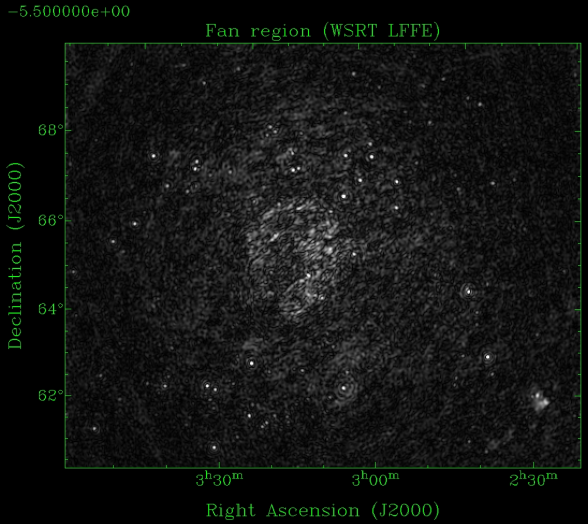

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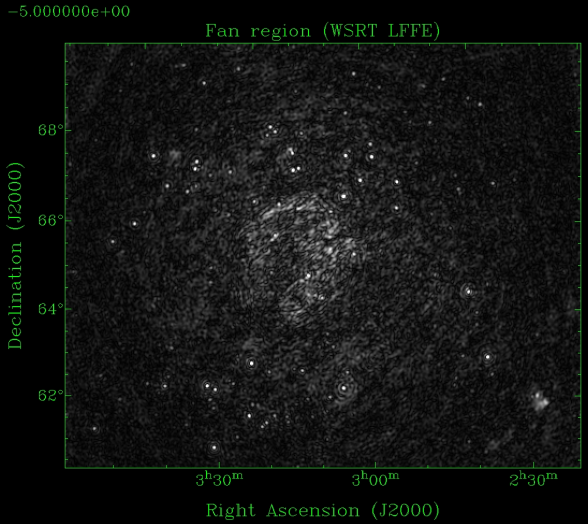

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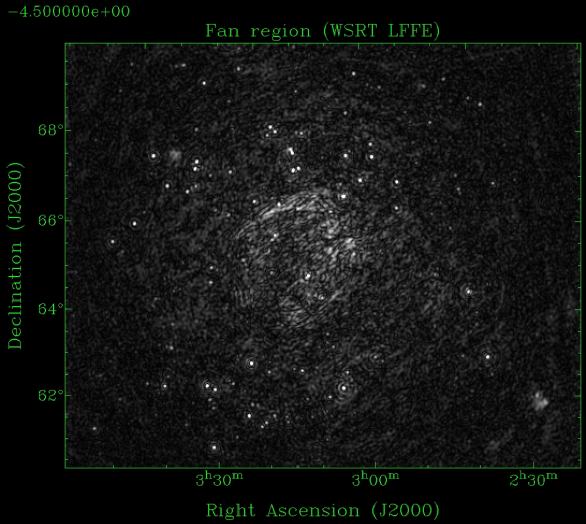

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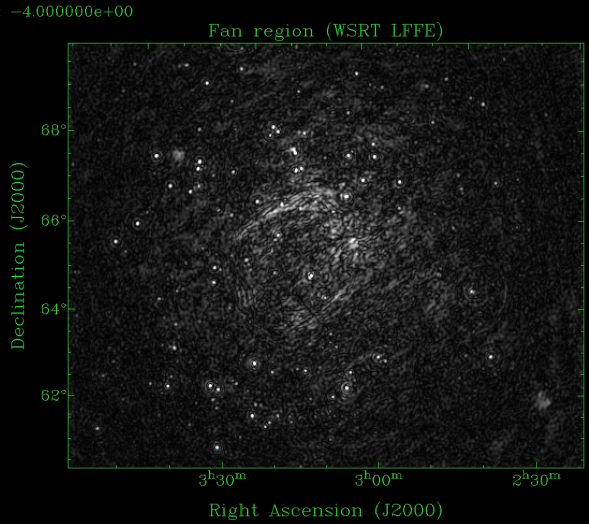

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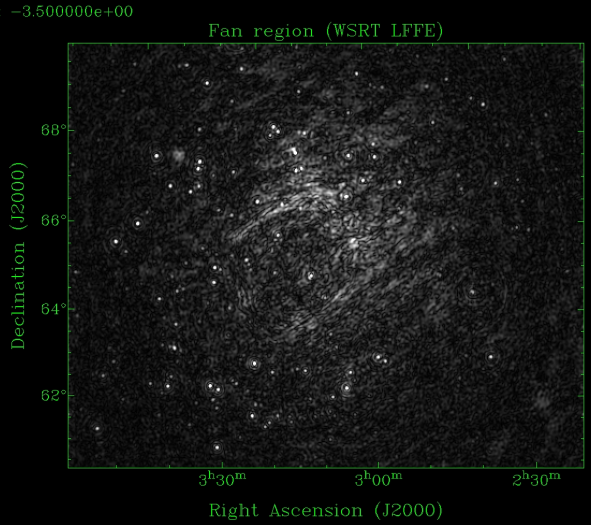

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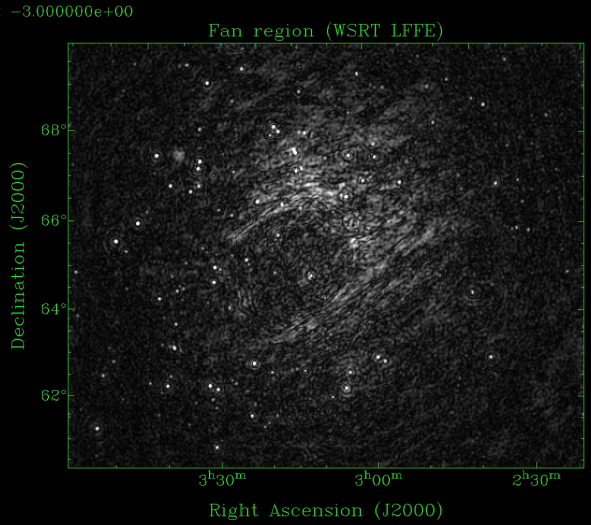

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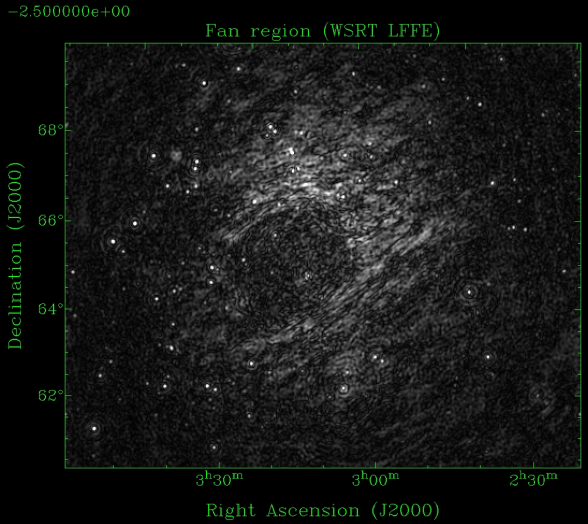

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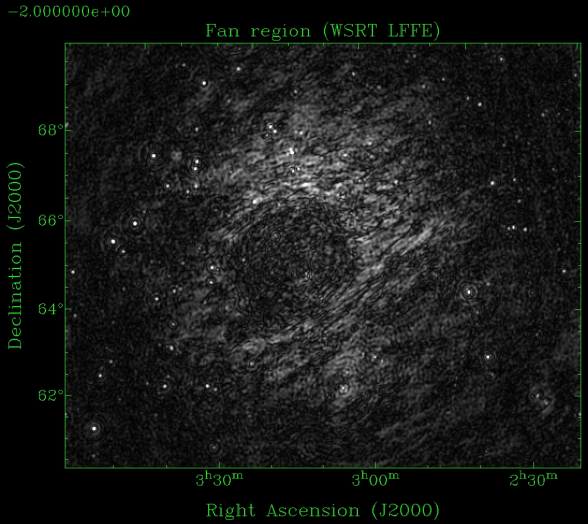

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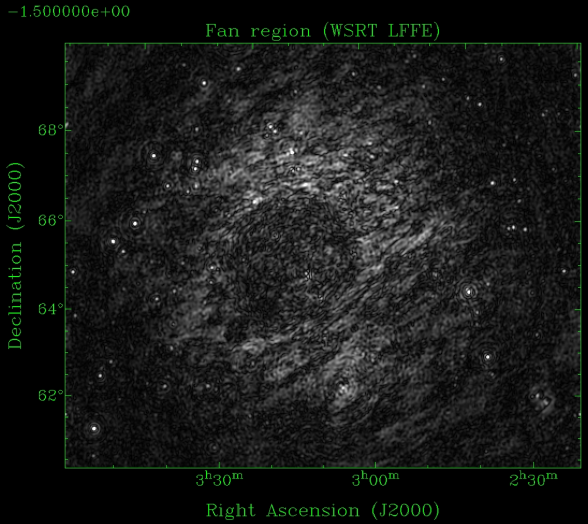

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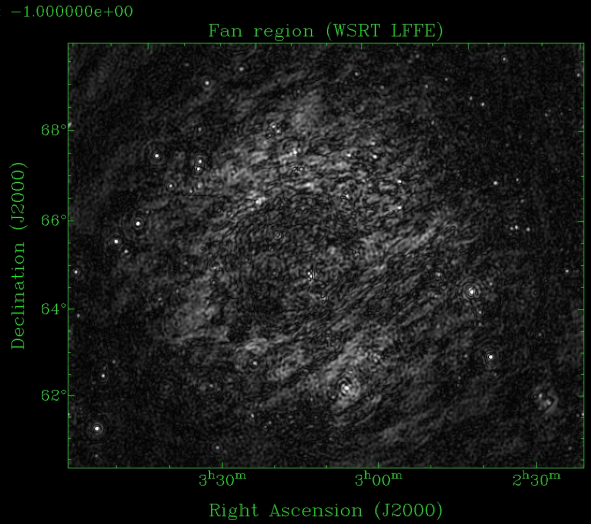

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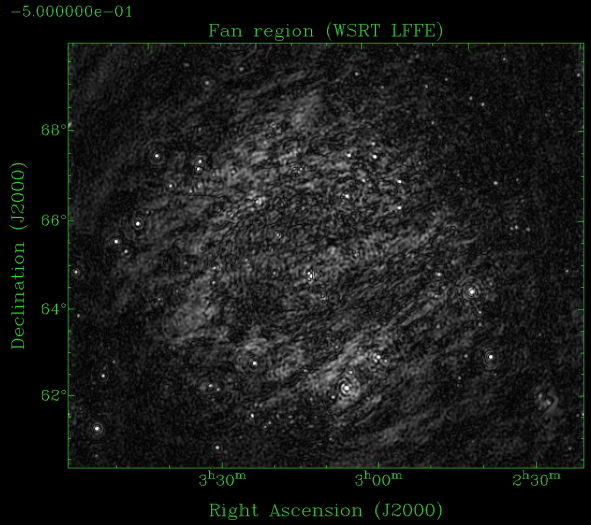

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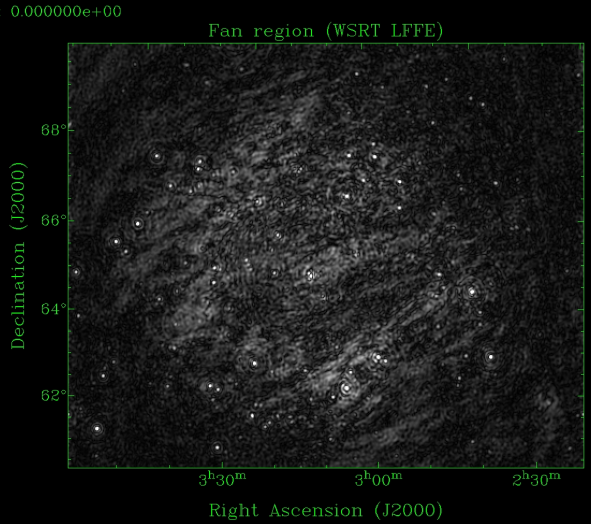

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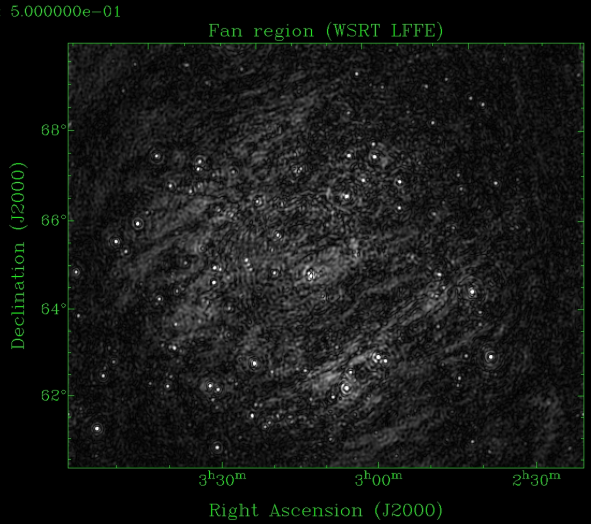

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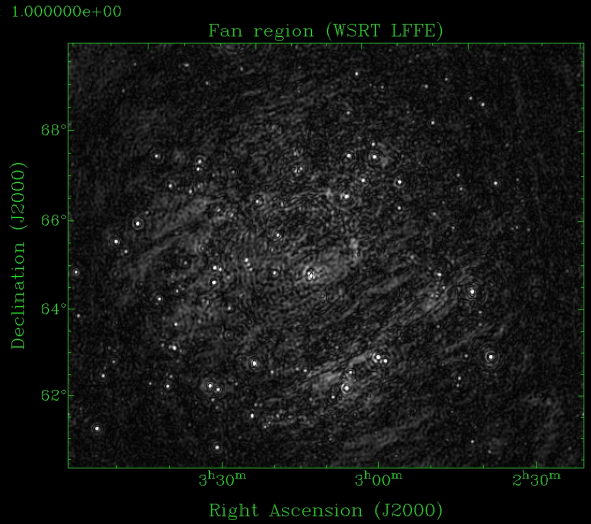

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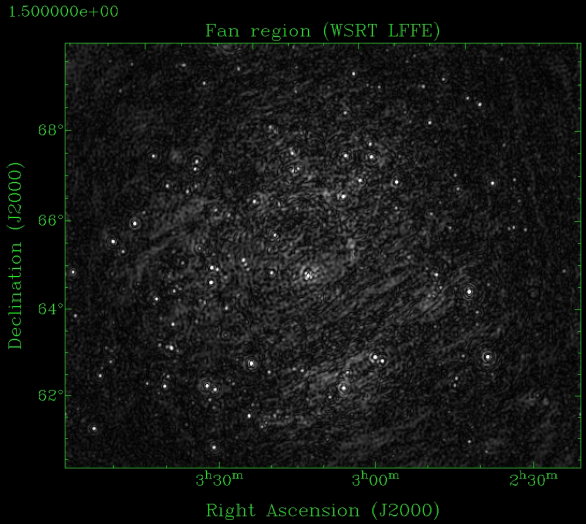

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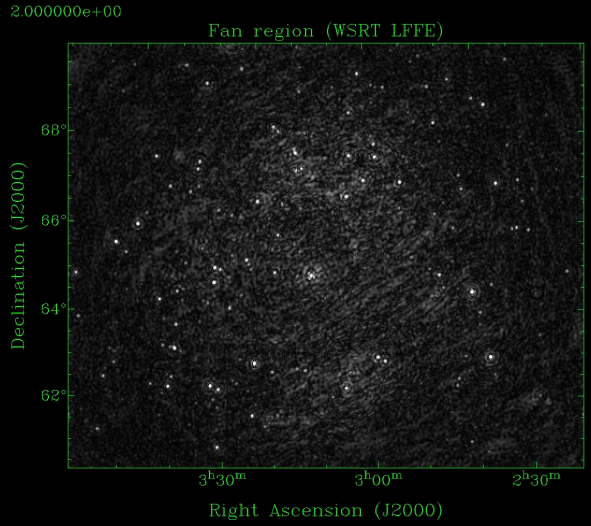

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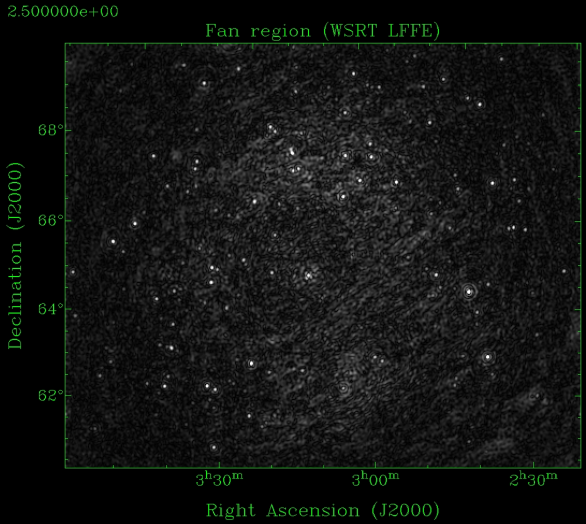

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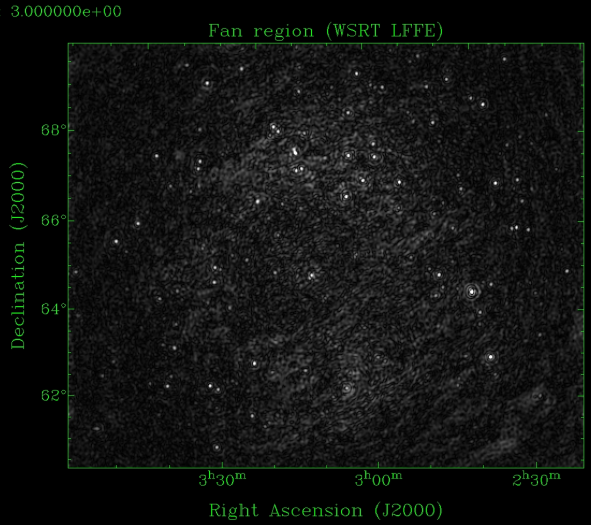

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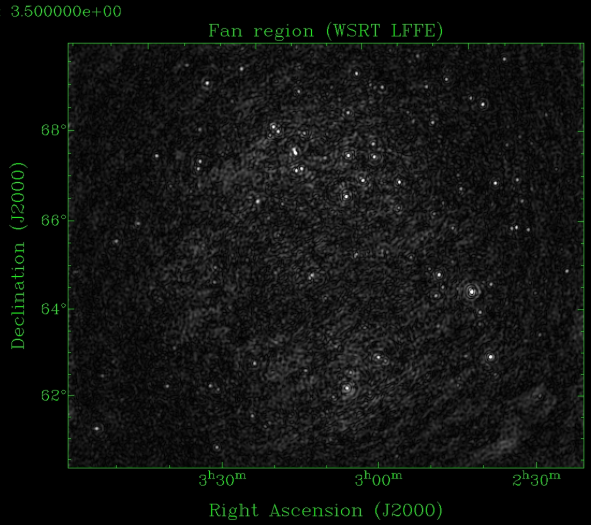

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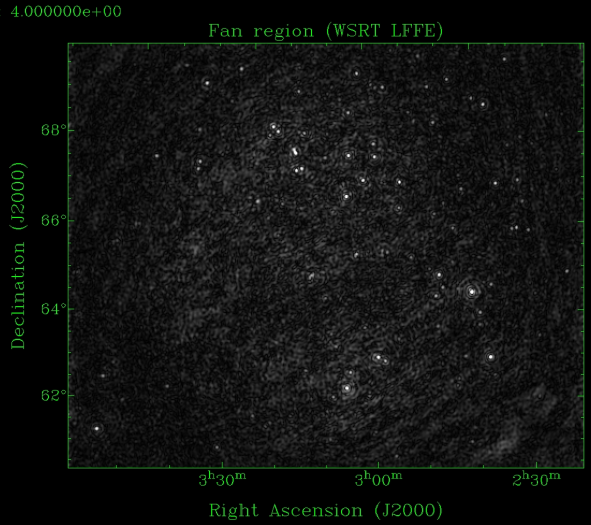

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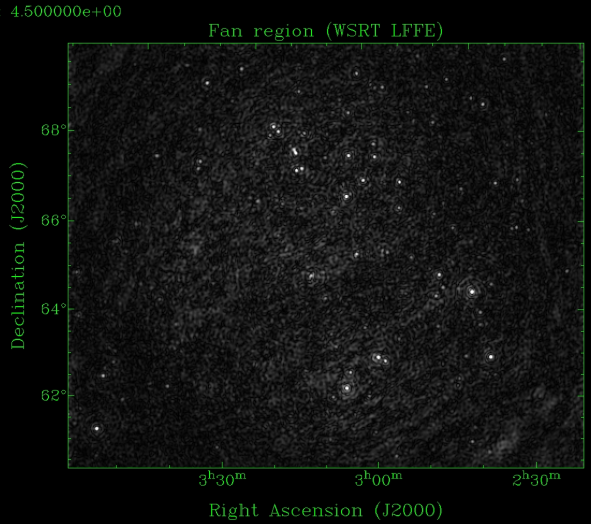

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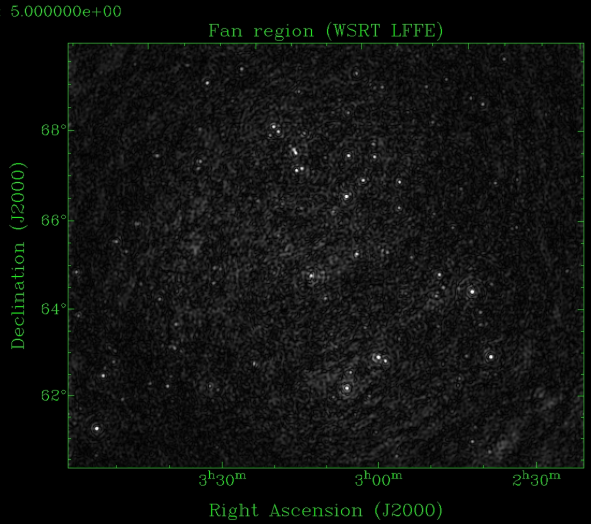

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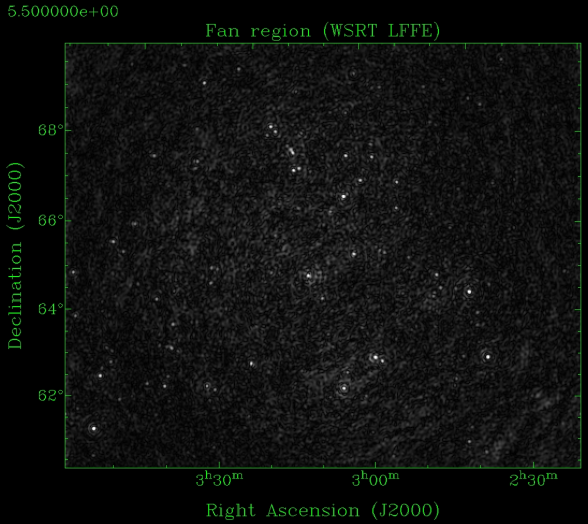

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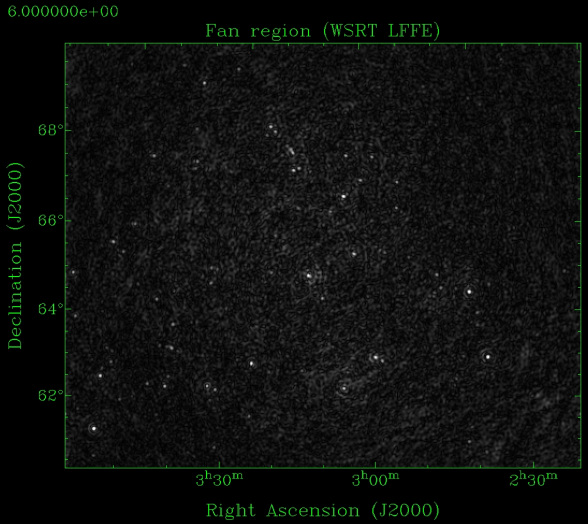

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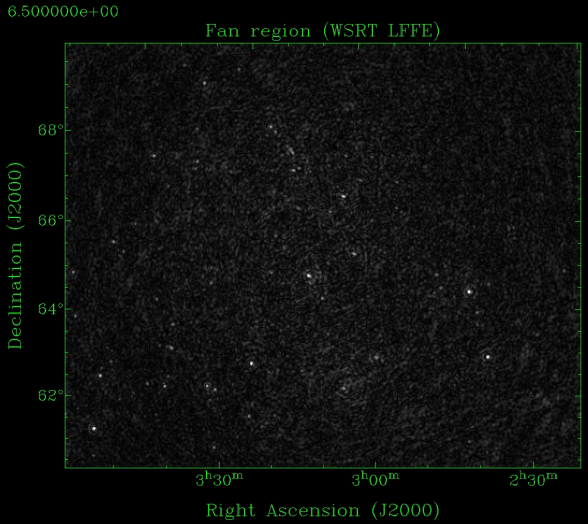

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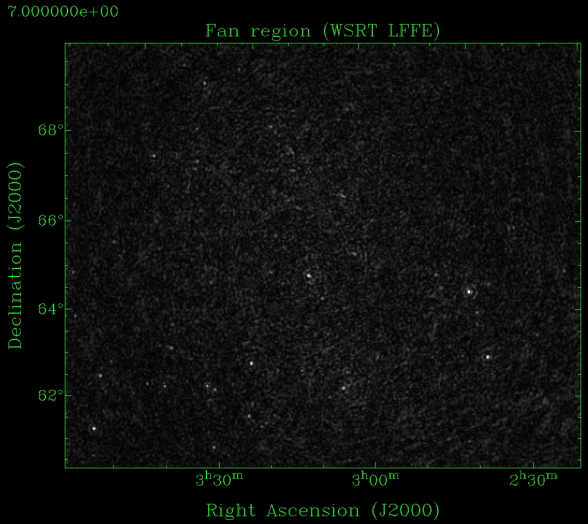

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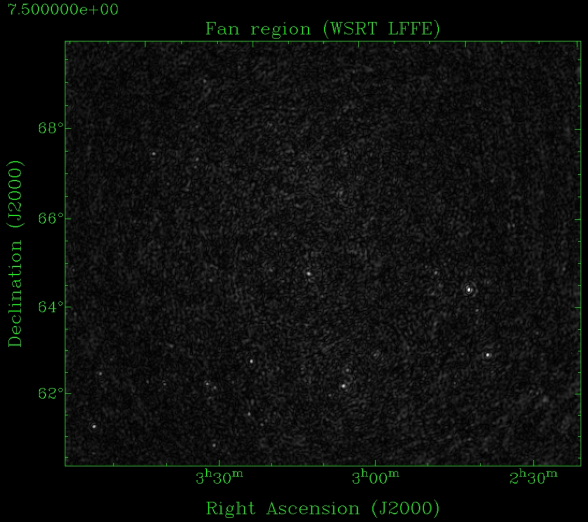

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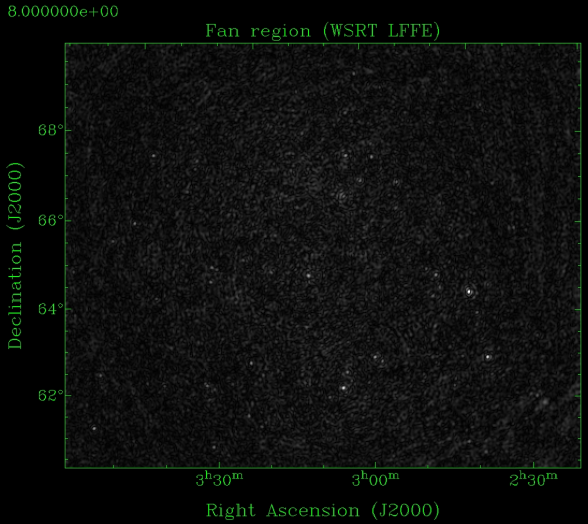

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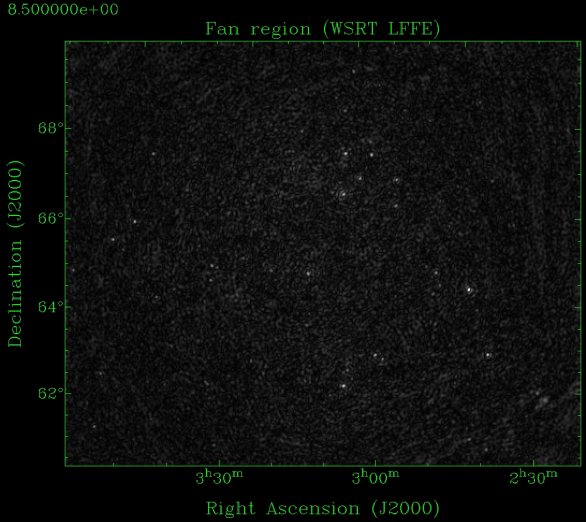

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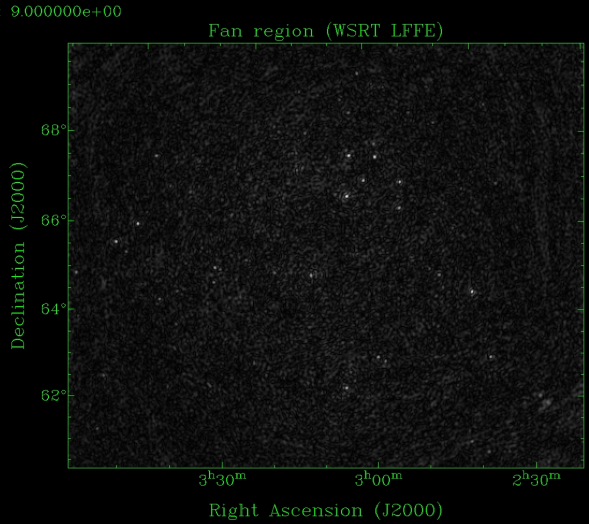

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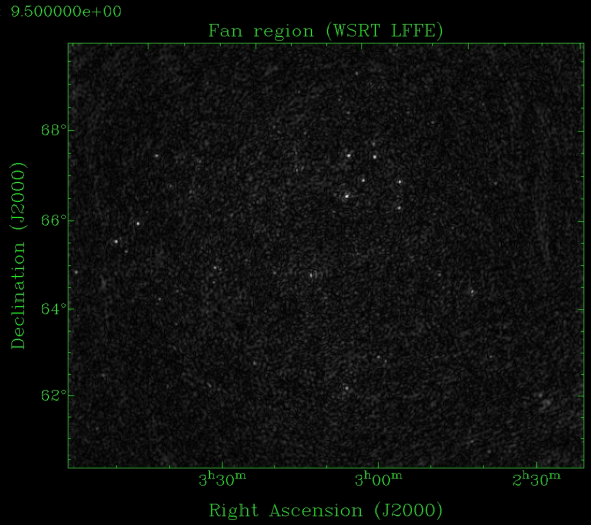

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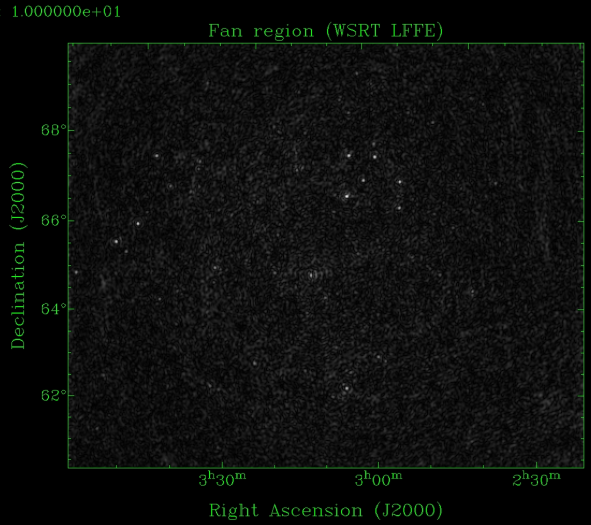

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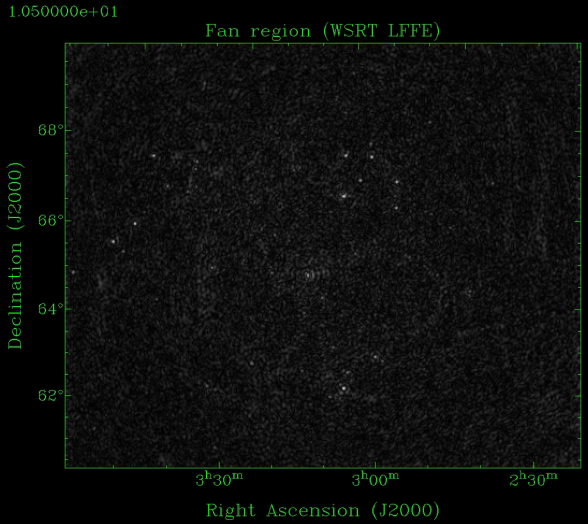

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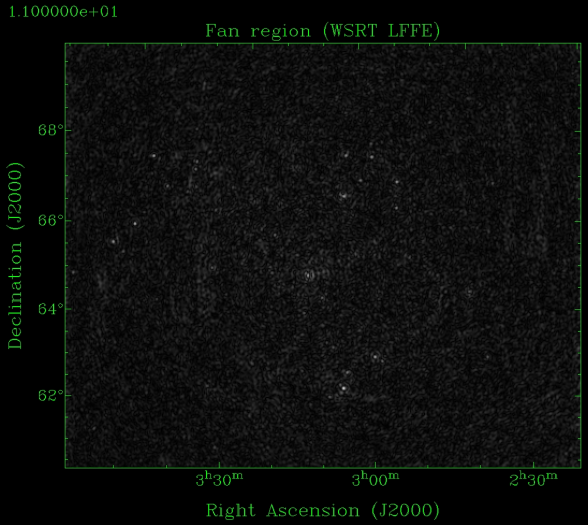

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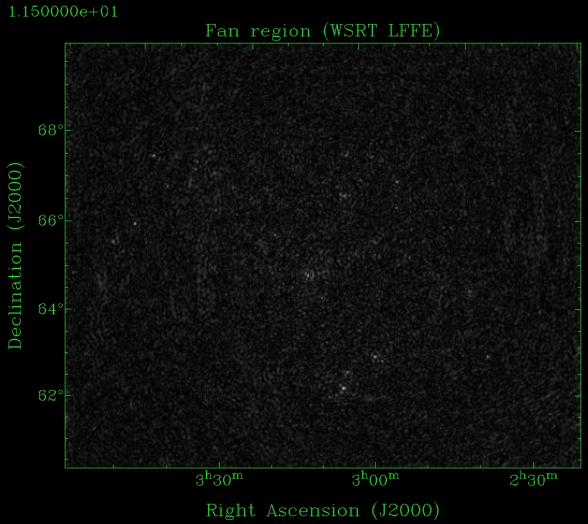

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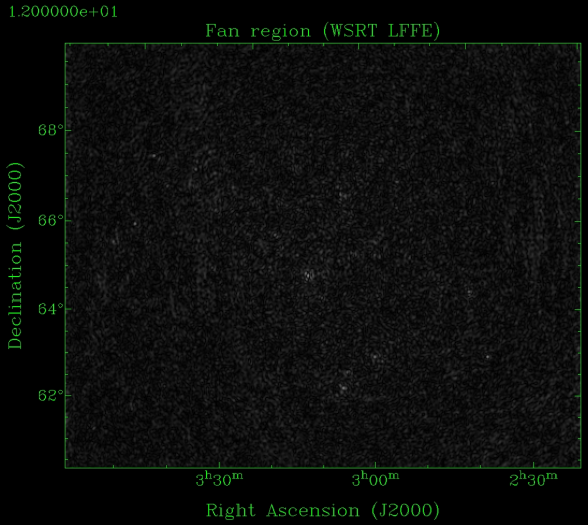

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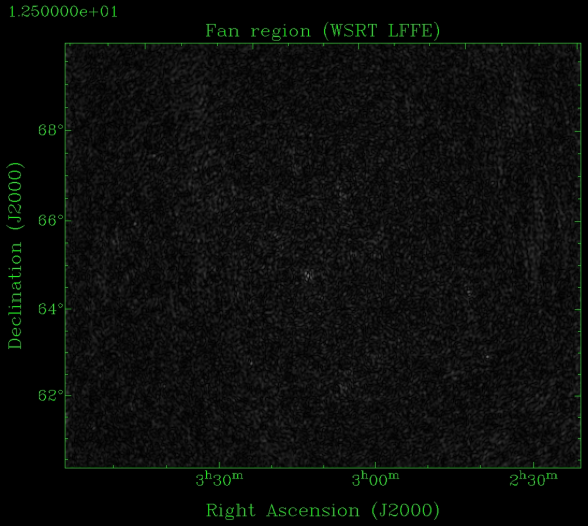

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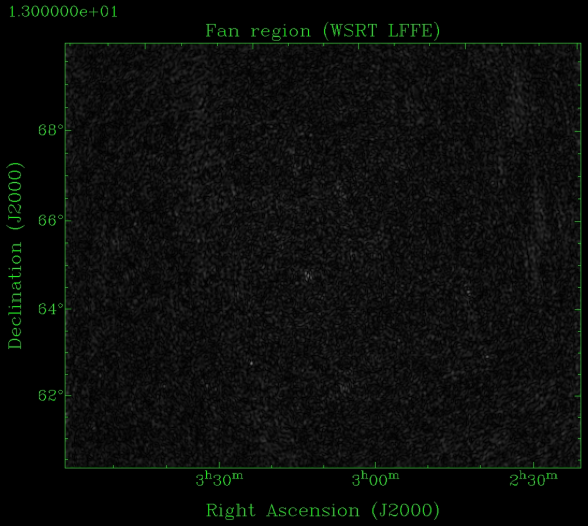

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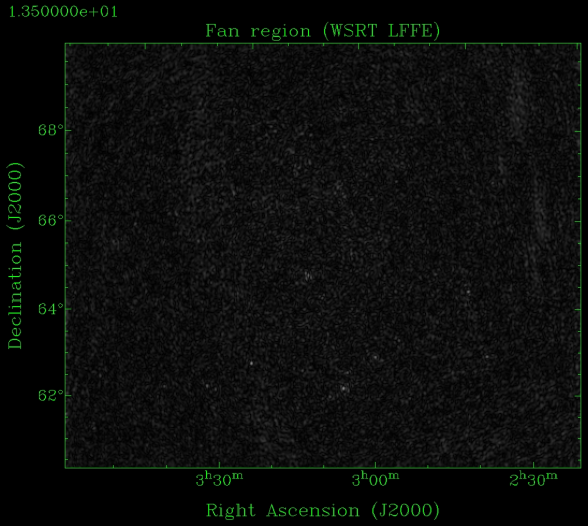

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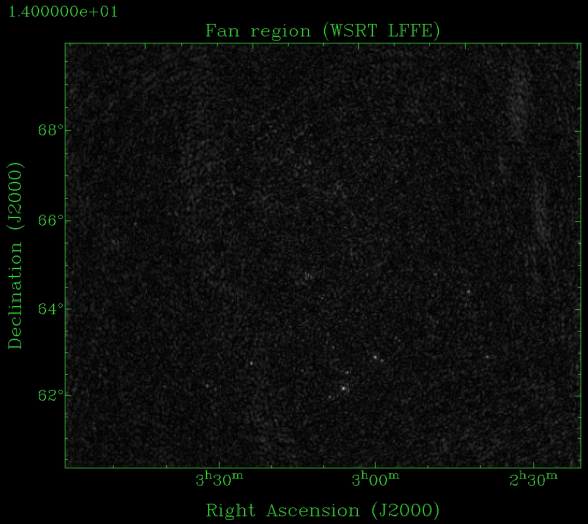

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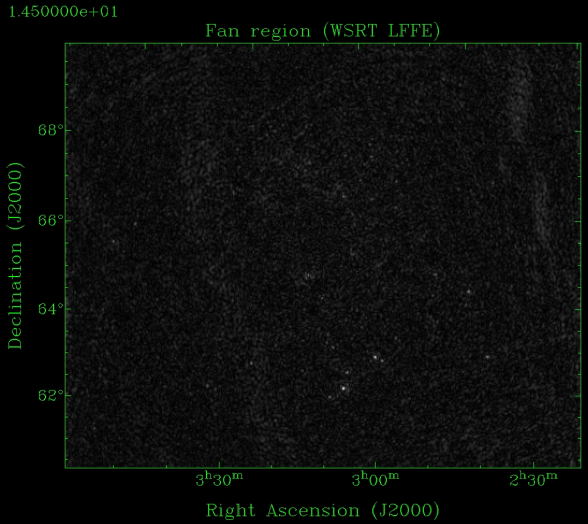

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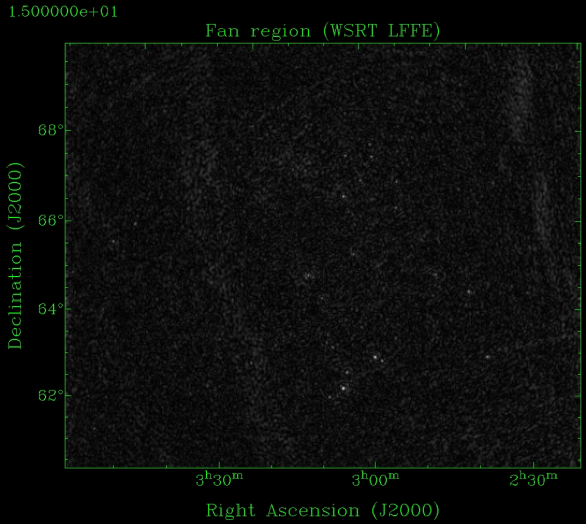
Fan region *G. Bernardi*



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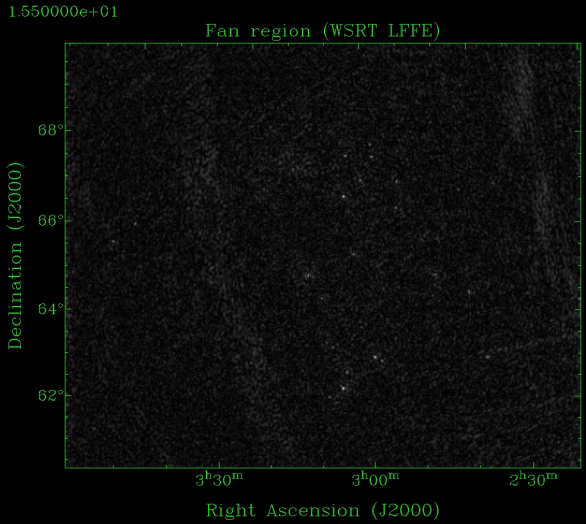
Fan region *G. Bernardi*



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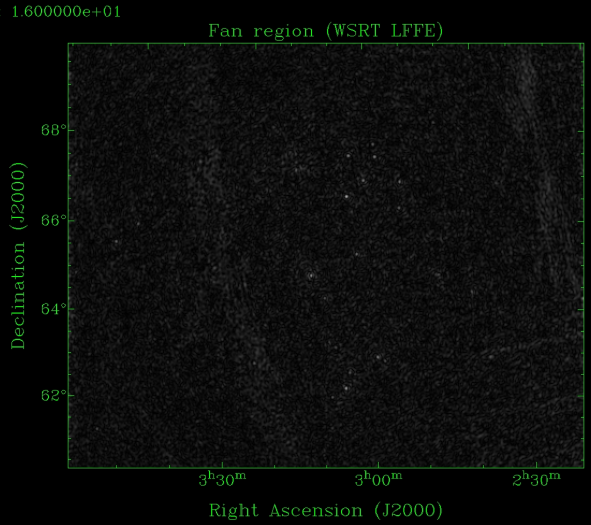
Fan region *G. Bernardi*



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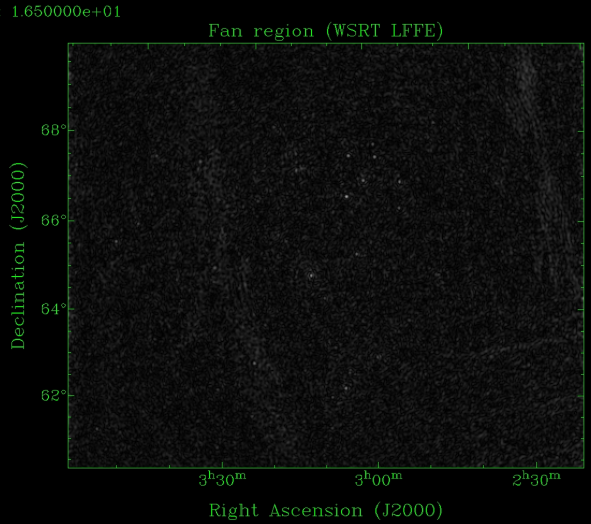
Fan region *G. Bernardi*



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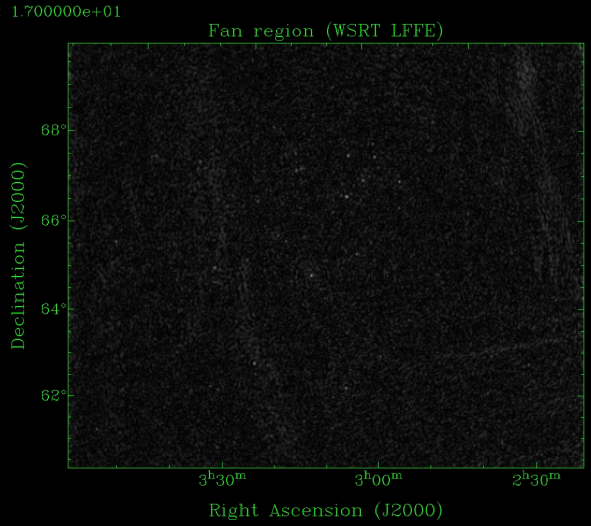
Fan region *G. Bernardi*



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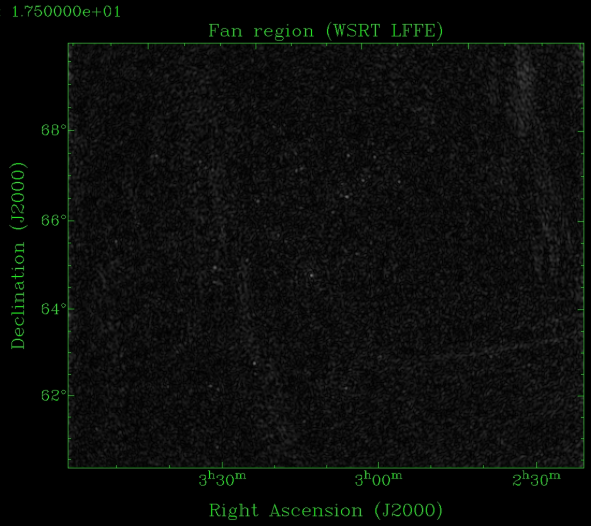
Fan region *G. Bernardi*



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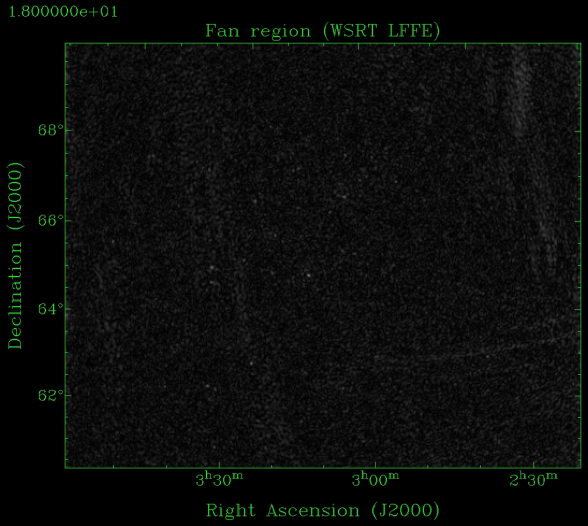
Fan region *G. Bernardi*



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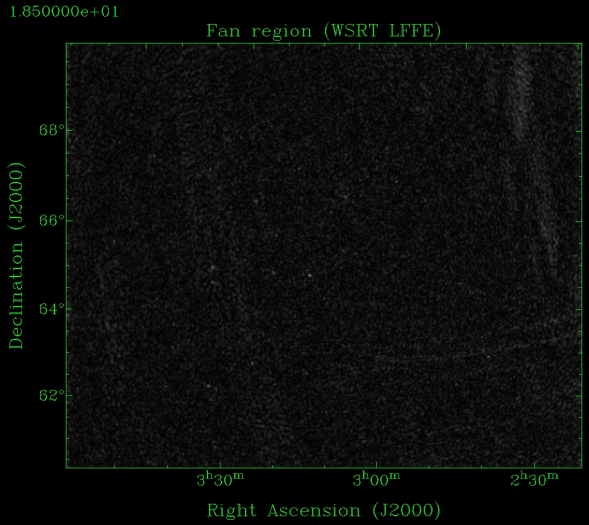
Fan region *G. Bernardi*



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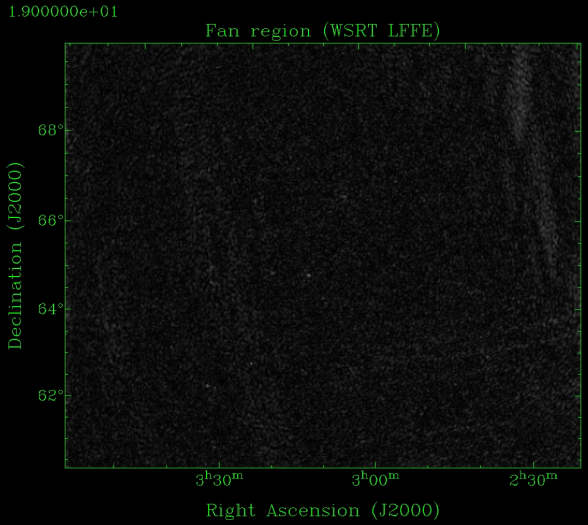
Fan region *G. Bernardi*



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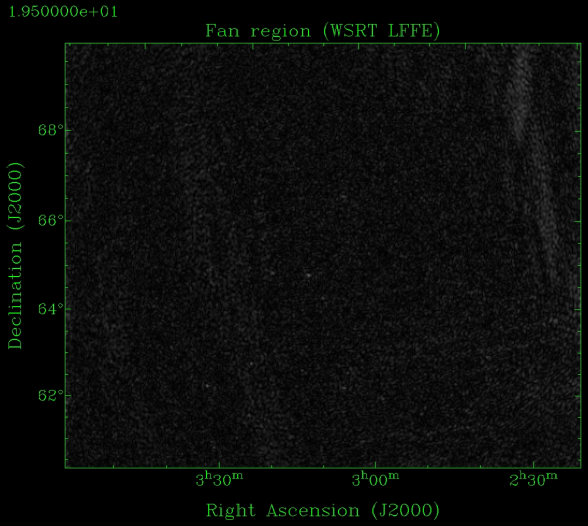
Fan region *G. Bernardi*



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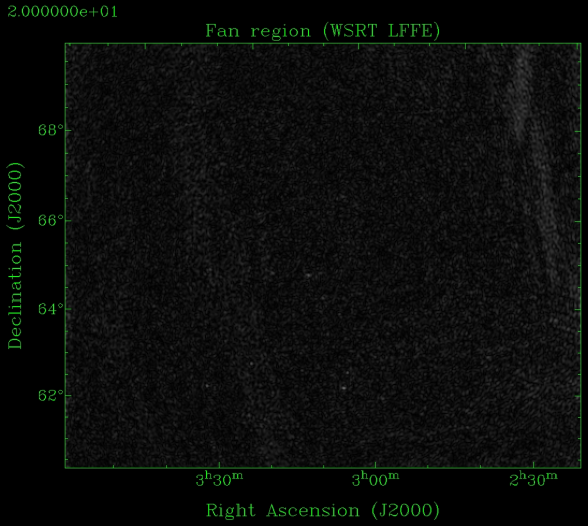
Fan region *G. Bernardi*



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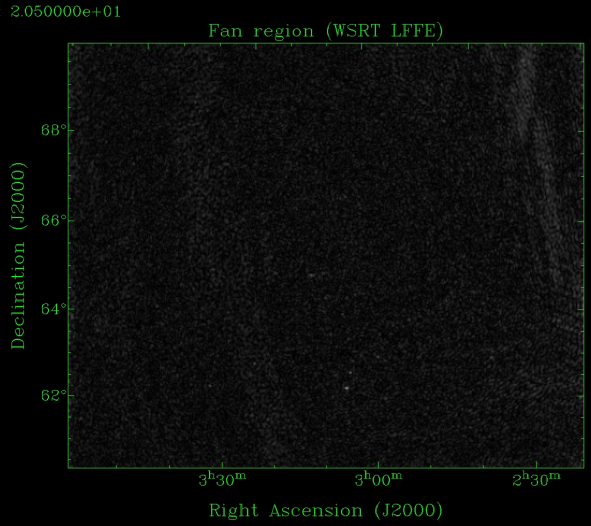
Fan region *G. Bernardi*



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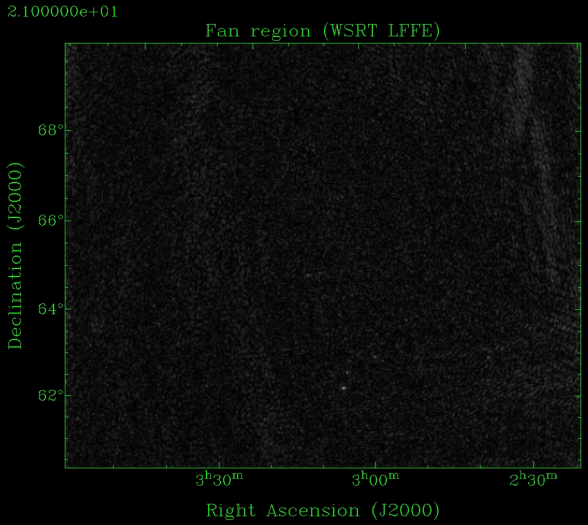
Fan region *G. Bernardi*



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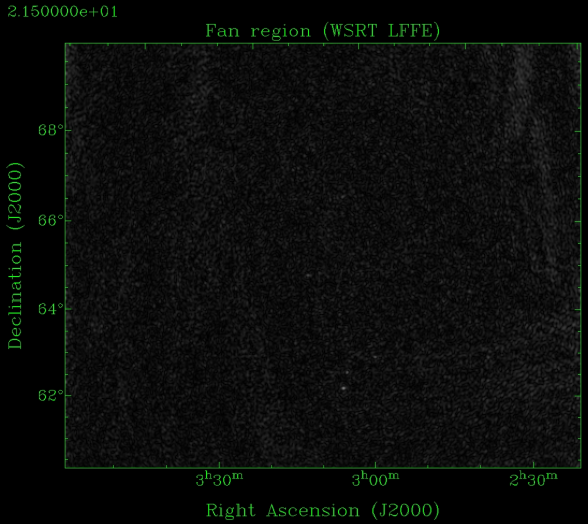
Fan region *G. Bernardi*



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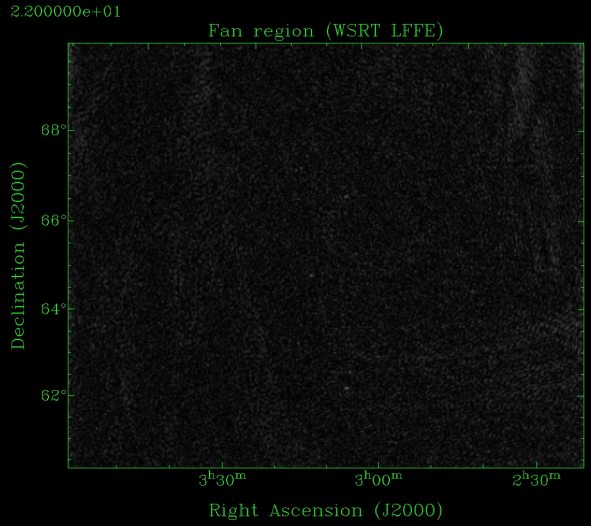
Fan region *G. Bernardi*



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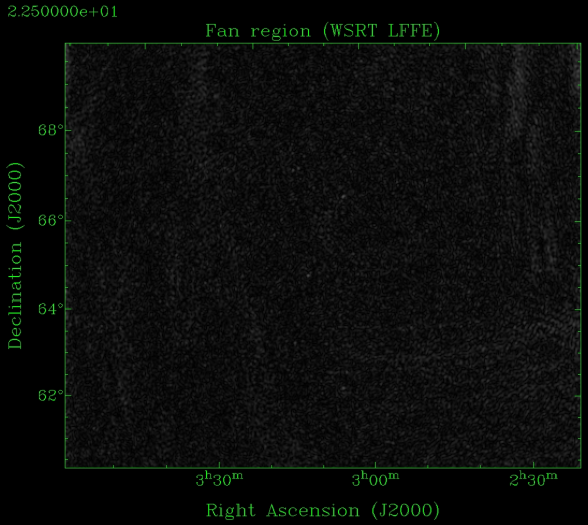
Fan region *G. Bernardi*



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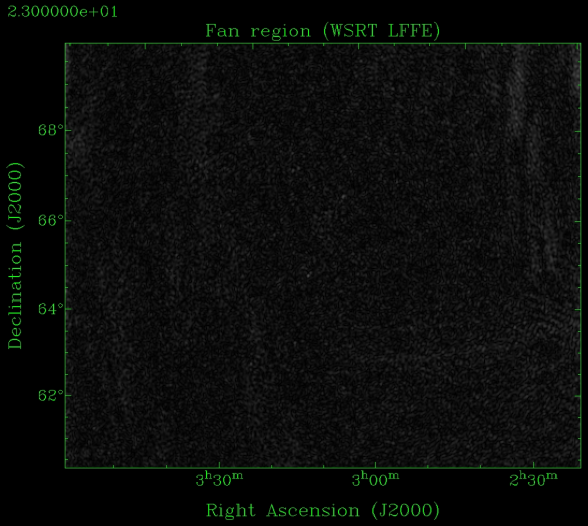
Fan region *G. Bernardi*



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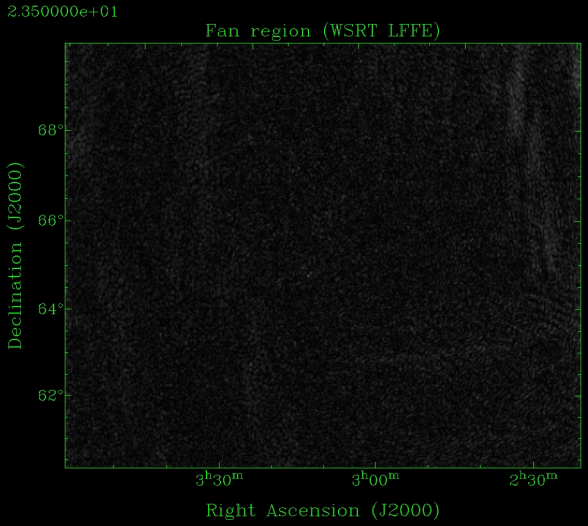
Fan region *G. Bernardi*



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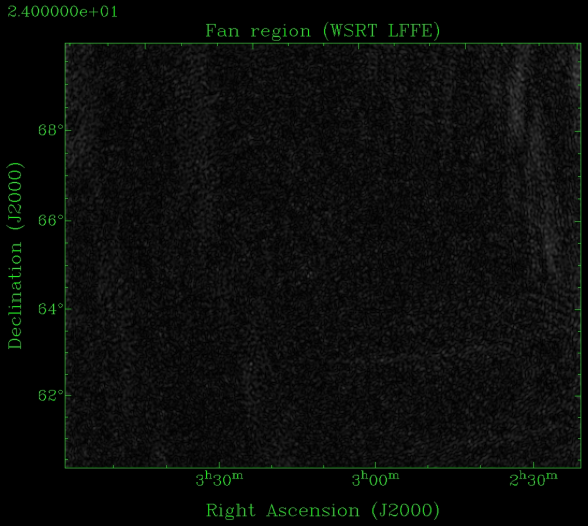
Fan region *G. Bernardi*



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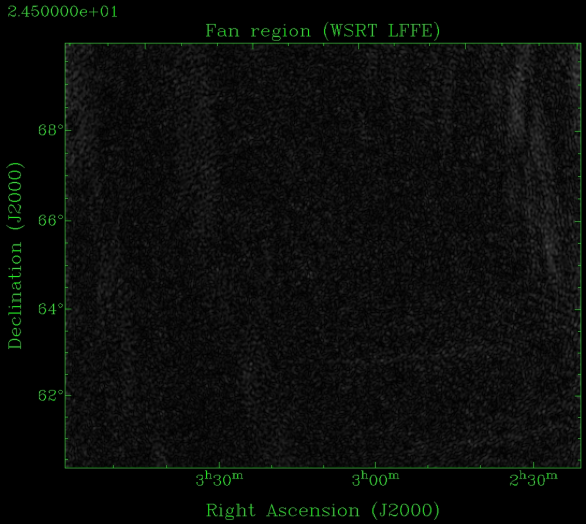
Fan region *G. Bernardi*



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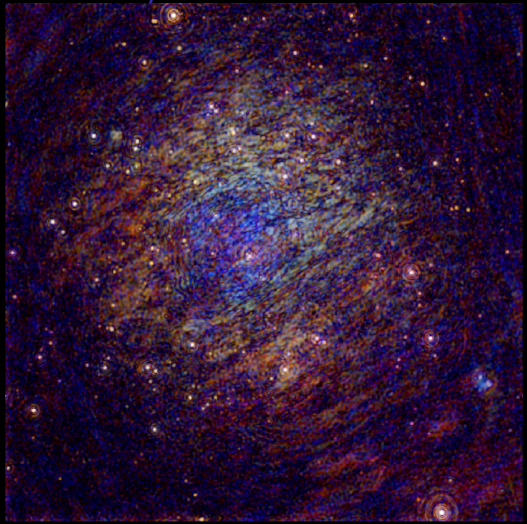
Fan region (combined RGB)



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