# Interferometry of Solar System Objects


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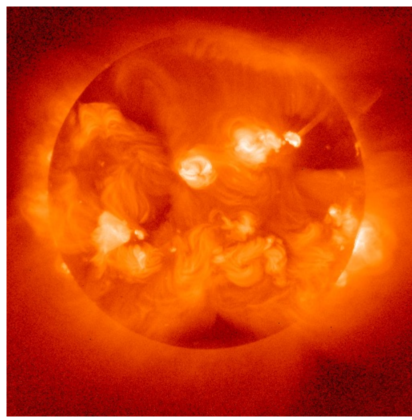
Atacama Large Millimeter/submillimeter Array
Expanded Very Large Array
Robert C. Byrd Green Bank Telescope
Very Long Baseline Array

NRAO

# Solar System Bodies

- Sun
- IPM
- Giant planets
- Terrestrial planets
- Moons
- Small bodies



# Why Interferometry?

resolution, resolution, resolution!

maximum angular extent of some bodies:

Sun & Moon - 0.5$^{o}$
Venus – 60”
Jupiter – 50”
Mars – 25”
Saturn – 20”

Mercury – 12”
Uranus – 4”
Neptune - 2.4”
Galilean Satellites - 1-2”
Titan – 1”

Triton - 0.1”
Pluto - 0.1”
MBA - .05 - .5”
NEA, KBO - 0.005 - 0.05”

(interferometry also helps with confusion!)

## Solar System Oddities

Radio interferometric observations of solar system bodies are similar in many ways to other observations, including the data collection, calibration, reduction, etc…

So why am I here talking to you? In fact, there are some differences which are significant (and serve to illustrate some fundamentals of interferometry).

# Differences

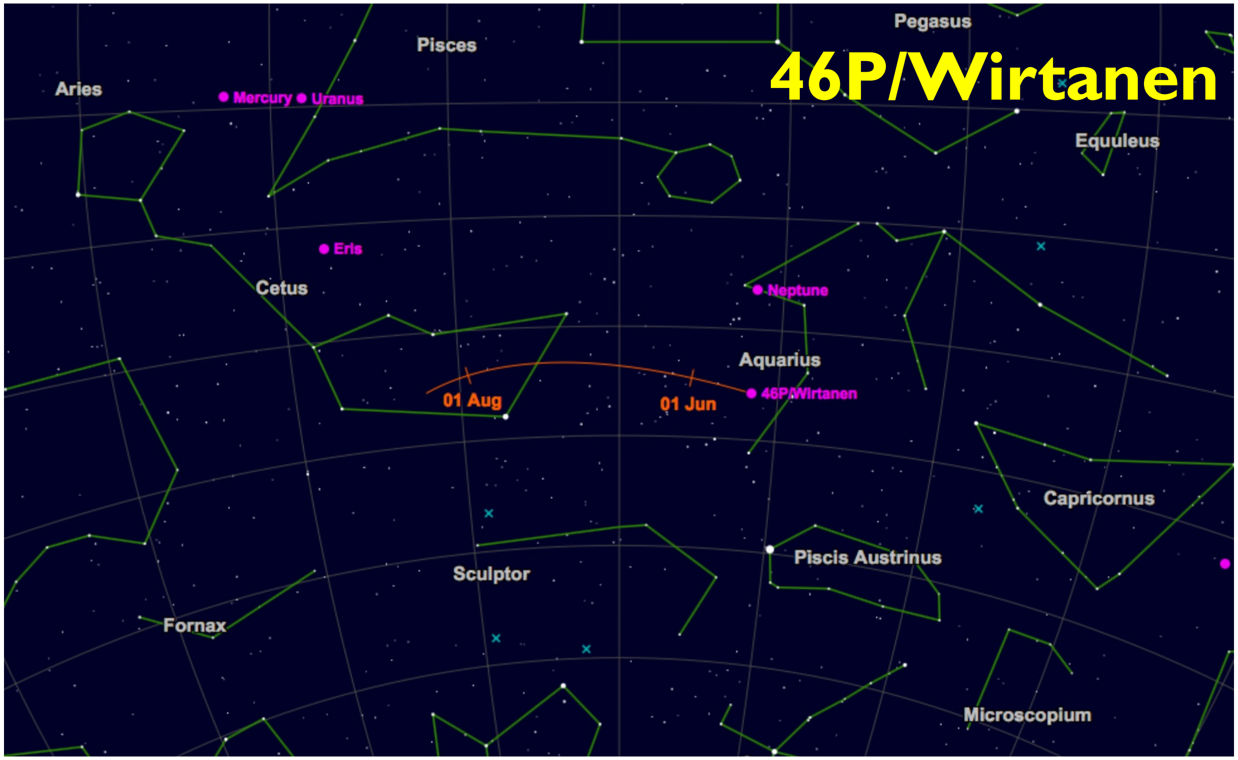
- Object motion
- Time variability
- Confusion
- Scheduling complexities
- Source strength
- Coherence
- Source distance
- Knowledge of source
- Optical depth

# Object Motion

All solar system bodies move against the (relatively fixed) background sources on the celestial sphere.  This motion has two components:

- "Horizontal Parallax" - caused by rotation of the observatory around the Earth.
- "Orbital Motions" - caused by motion of the Earth and the observed body around the Sun.

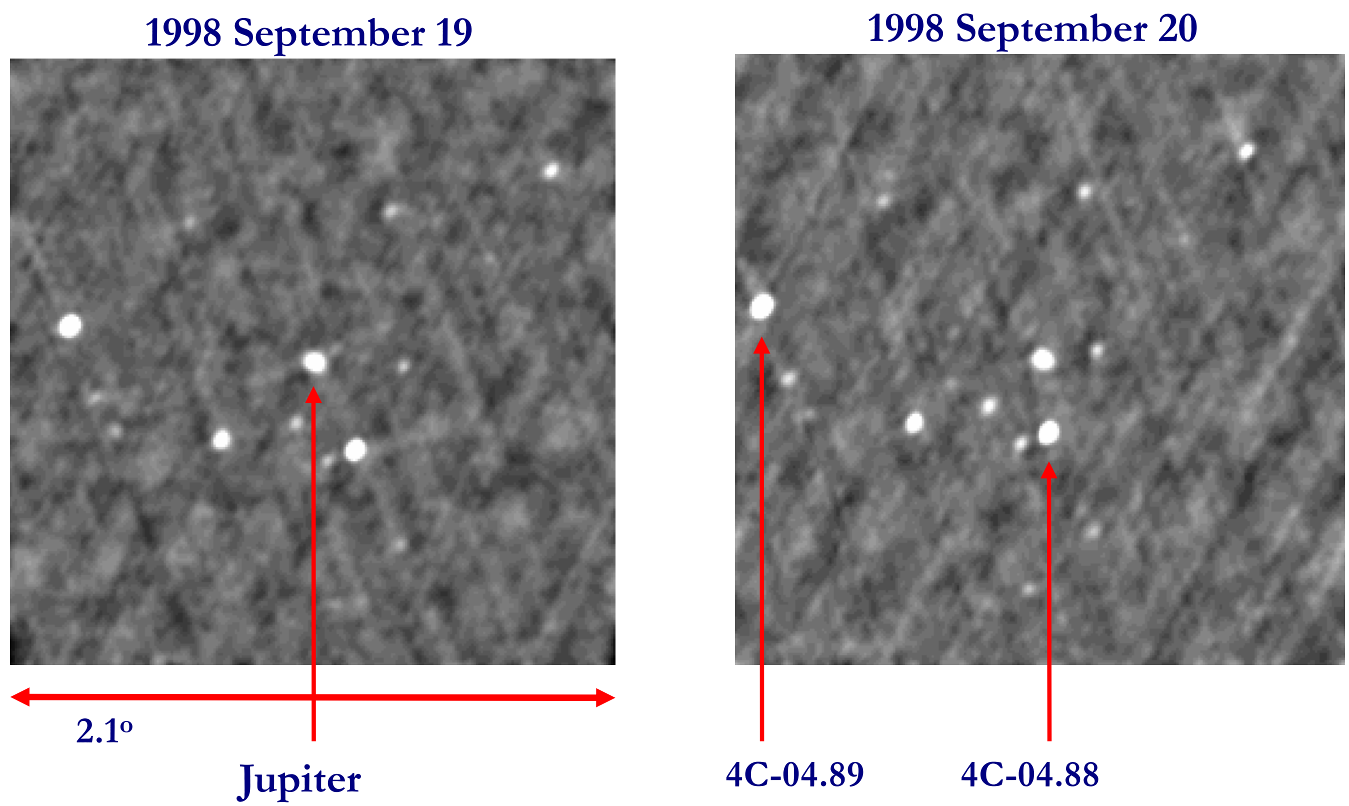
# Object Motion - an example



# Object Motion - an example

**Credit: CometWatch (youtube)**

# Object Motion - a practical example




de Pater & Butler 2003

# Time Variability

Time variability is a significant problem in solar system observations:

- Sun - very fast fluctuations (< 1 sec)
- Jupiter, Venus (others?) – lightning (< 1 sec)
- Others - rotation (hours to days), plus other intrinsic variability (clouds, seasons, etc.)
- Distance may change appreciably (need "common" distance measurements)

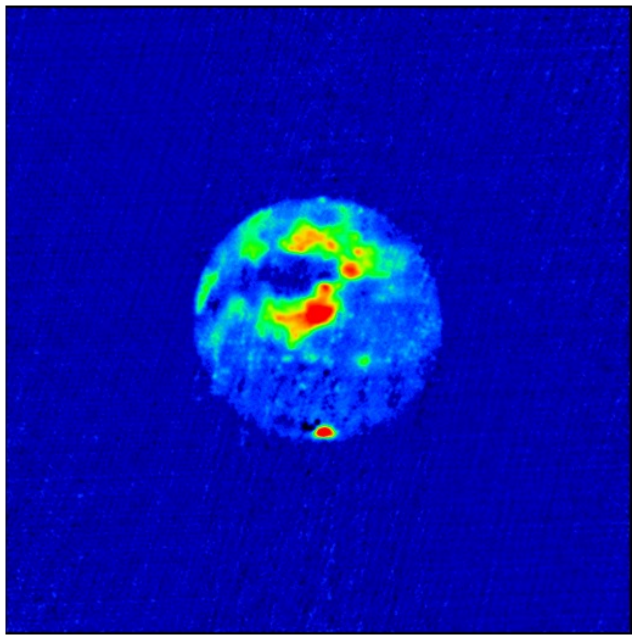
These must be dealt with.

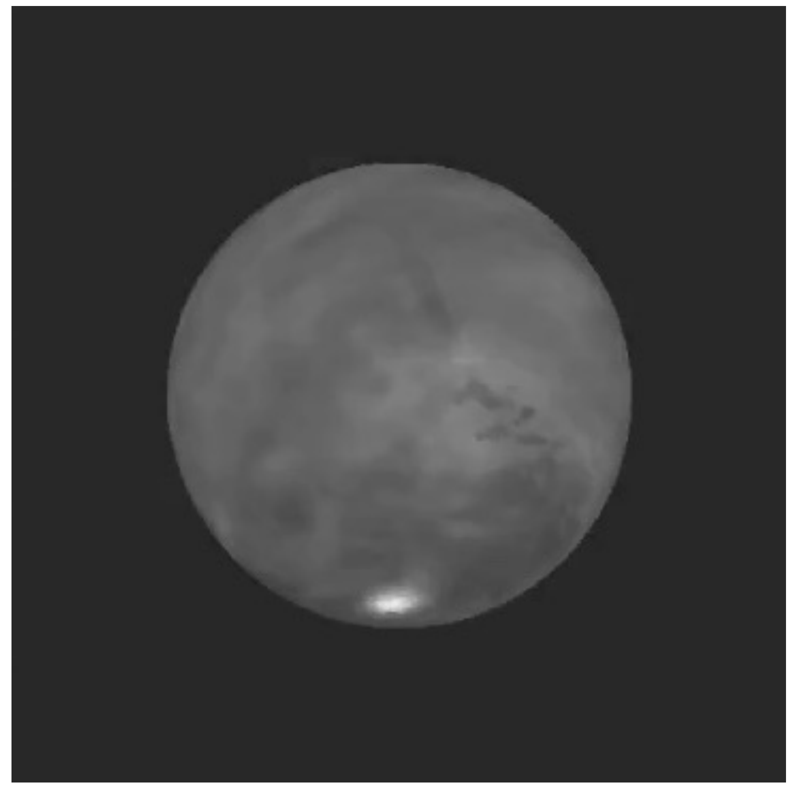
# Time Variability - an example

Mars radar

snapshots made every 10 mins

Butler, Muhleman & Slade 1994





# Implications

- Often can't use same calibrators
- Often can't easily add together data from different days
- Solar confusion
- Other confusion sources move in the beam
- Antenna and phase center pointing must be tracked (must have accurate ephemeris)
- Scheduling/planning - need a good match of source apparent size and interferometer spacings

# Source Strength & Coherence

Some solar system bodies are very bright. They can be so bright that they raise the antenna temperature:

- Sun ~ 6000 K (or brighter)
- Moon ~ 200 K
- Venus, Jupiter, etc. ~ 10s-100s of K

In the case of the Sun, special hardware may be required. In other cases, special processing may be needed (e.g., Van Vleck correction). In all cases, the system temperature (the noise) is increased.

Some types of emission from the Sun are coherent. In addition, reflection from planetary bodies in radar experiments is coherent (over at least part of the image). This complicates the interpretation of images made of these phenomena, and in fact violates one of the fundamental assumptions in radio interferometry.

# Source Distance - Wave Curvature

Objects wl … ear-field
of the inte …
complexity … to be a
plane wave … the
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Nowadays … ut this was
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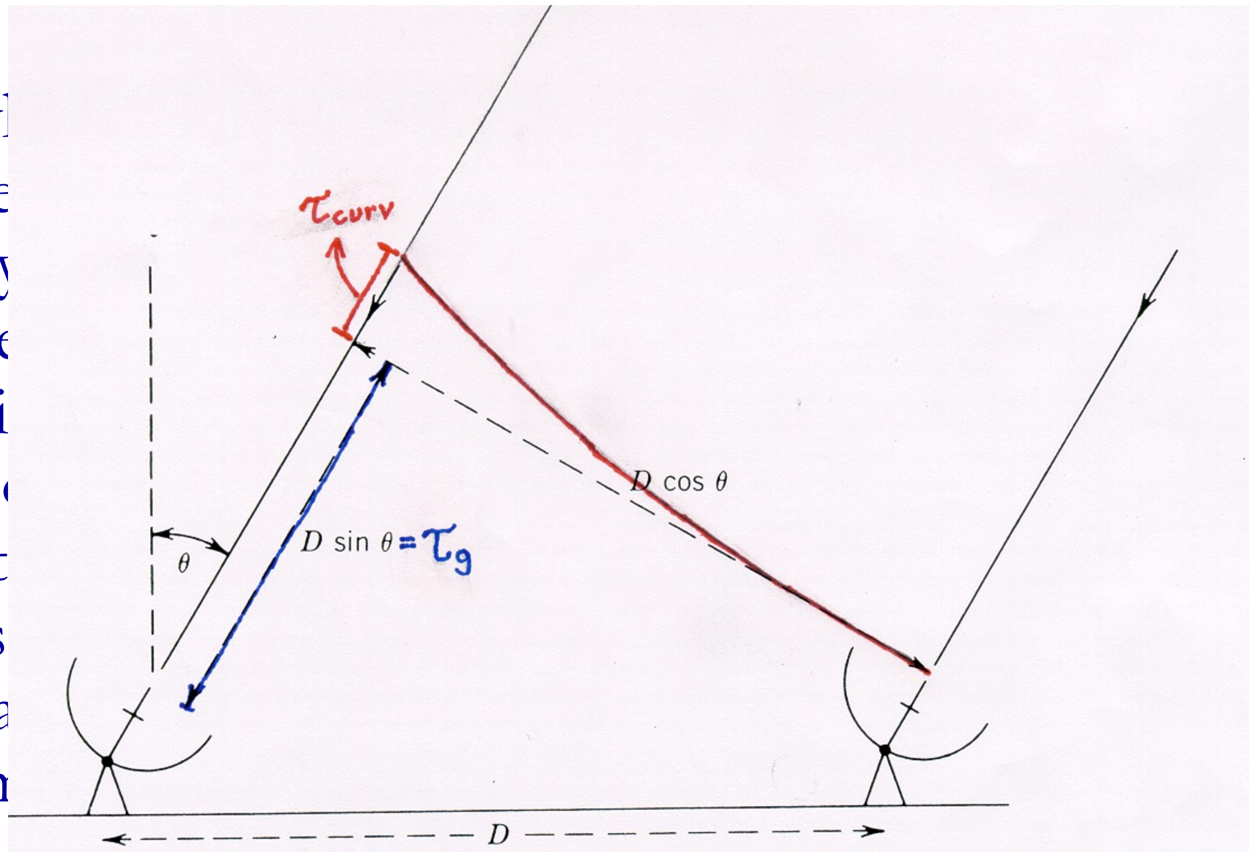


## Short Spacing Problem

As with other large, bright objects, there is usually a serious short spacing problem when observing the planets. This can produce a large negative "bowl" in images if care is not taken. This can usually be avoided with careful planning, and the use of appropriate models during imaging and deconvolution. More on this later.

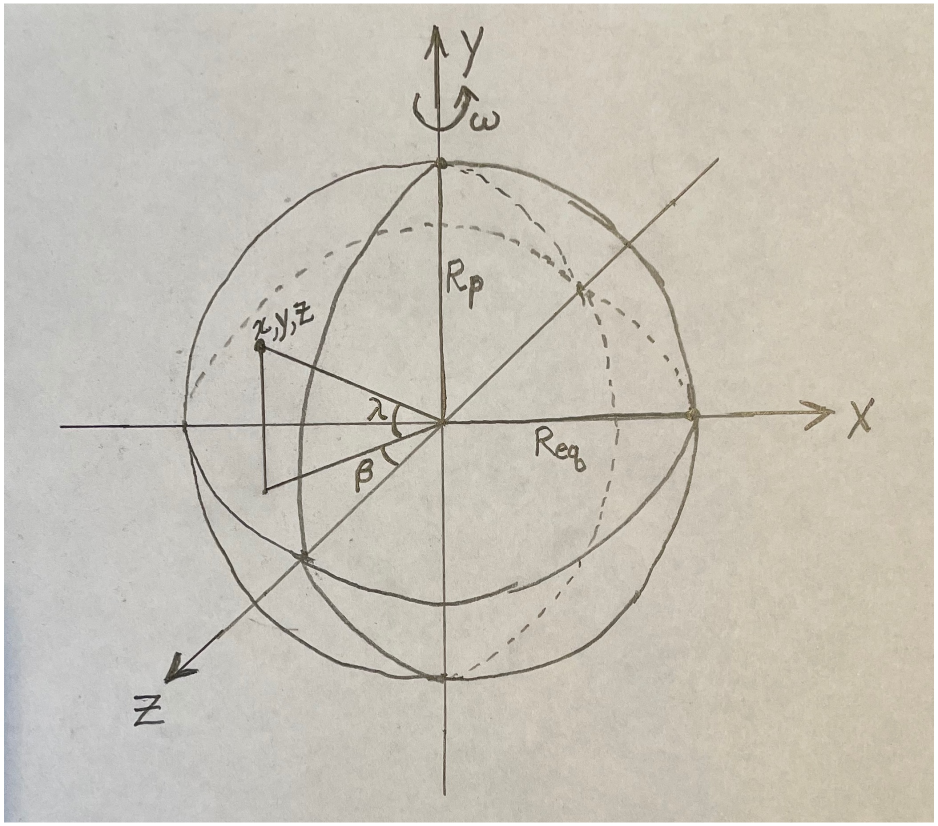
## Source Knowledge

There **is** an advantage in most solar system observations - we have a very good idea of what the general source characteristics are, including general expected flux densities and extent of emission. This can be used to great advantage in the imaging, deconvolution, and self-calibration stages of data reduction.

## Conversion of Coordinates

If we know the observed object's geometry well enough, then sky coordinates can be turned into planetographic surface coordinates - which is what we want for comparison, e.g., to optical images.

# Conversion of Coordinates




Longitude = $\beta$

Latitude = $\lambda$

Apparent radius = $R$

$$x = -R\cos\lambda\ \sin\beta$$
$$y = R\ \sin\lambda$$
$$z = R\ \cos\lambda\ \cos\beta$$

$$\boldsymbol{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

# Conversion of Coordinates

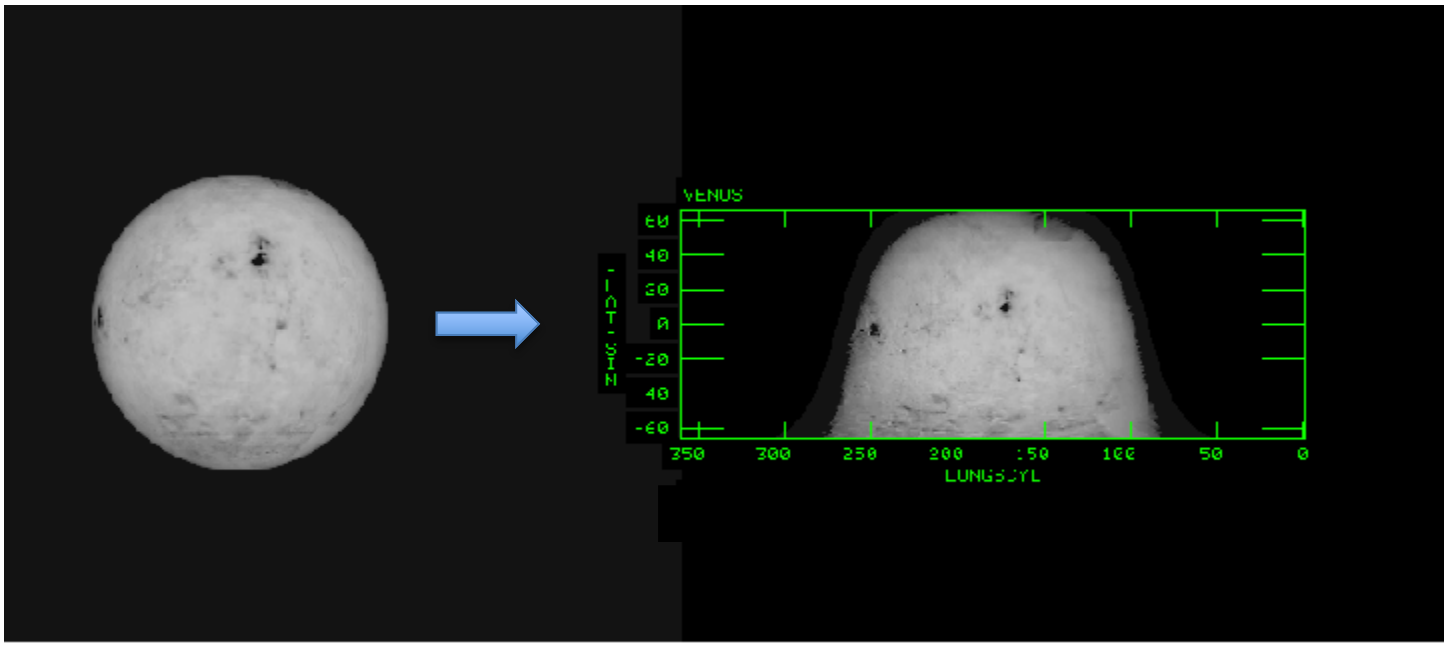
To find image coordinates for longitude $\beta$ and latitude $\lambda$, rotate $x$, $y$, and $z$ by the sub-Earth longitude $\beta_\oplus$ and latitude $\lambda_\oplus$, and by the North Pole Angle $\delta$ to find the image coordinates of a given latitude and longitude:

$$\boldsymbol{X}_i = \boldsymbol{R}_z(\delta)\,\boldsymbol{R}_y(\beta_\oplus)\,\boldsymbol{R}_x(\lambda_\oplus)\,\boldsymbol{X}$$

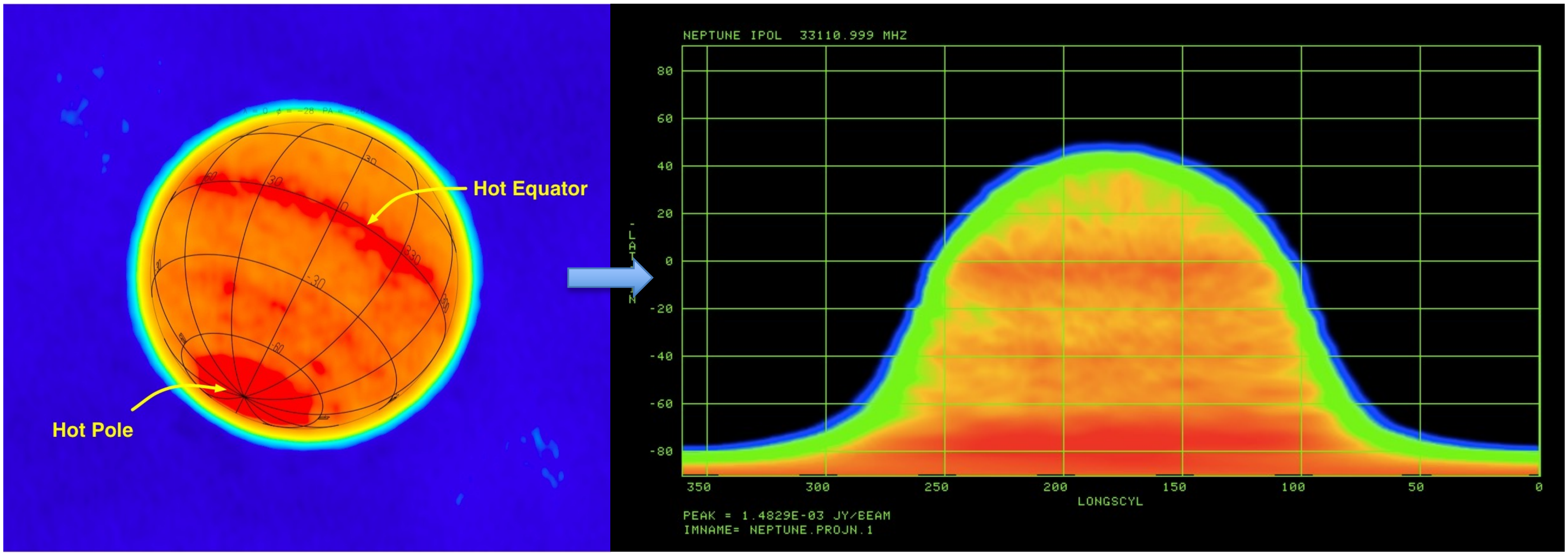
$$= \begin{bmatrix} \cos\delta & -\sin\delta & 0 \\ \sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\beta_\oplus & 0 & \sin\beta_\oplus \\ 0 & 1 & 0 \\ -\sin\beta_\oplus & 0 & \cos\beta_\oplus \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\lambda_\oplus & -\sin\lambda_\oplus \\ 0 & \sin\lambda_\oplus & \cos\lambda_\oplus \end{bmatrix} \boldsymbol{X}$$

$$= R \begin{bmatrix} \cos\delta\cos\lambda\sin(\beta_\oplus - \beta) + \sin\delta\,[\sin\lambda\cos\lambda_\oplus - \cos\lambda\sin\lambda_\oplus\cos(\beta - \beta_\oplus)] \\ -\sin\delta\cos\lambda\sin(\beta_\oplus - \beta) + \cos\delta\,[\sin\lambda\cos\lambda_\oplus - \cos\lambda\sin\lambda_\oplus\cos(\beta - \beta_\oplus)] \\ \sin\lambda\sin\lambda_\oplus + \cos\lambda\cos\lambda_\oplus\cos(\beta - \beta_\oplus) \end{bmatrix}$$

# Conversion of Coordinates



# Conversion of Coordinates

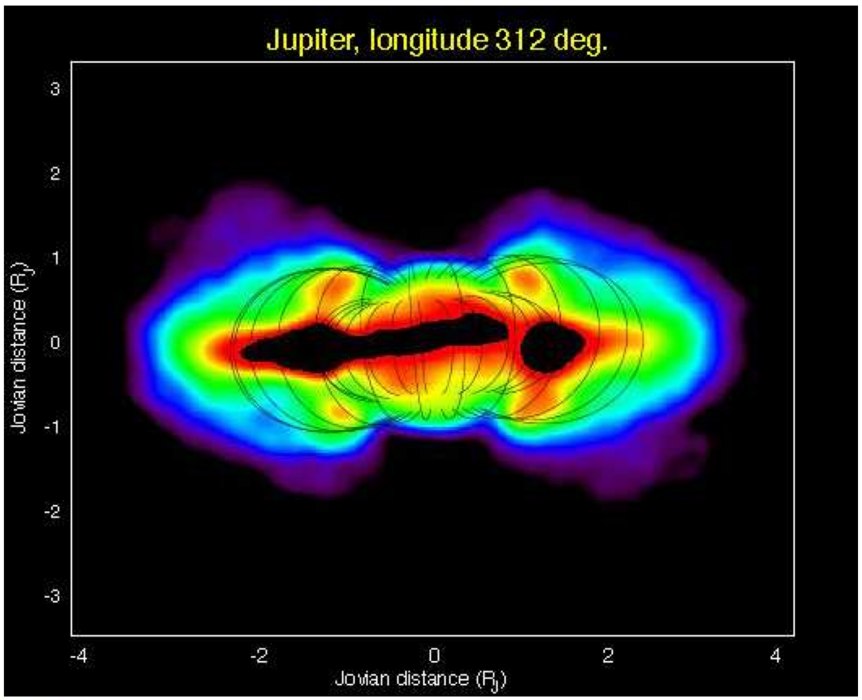


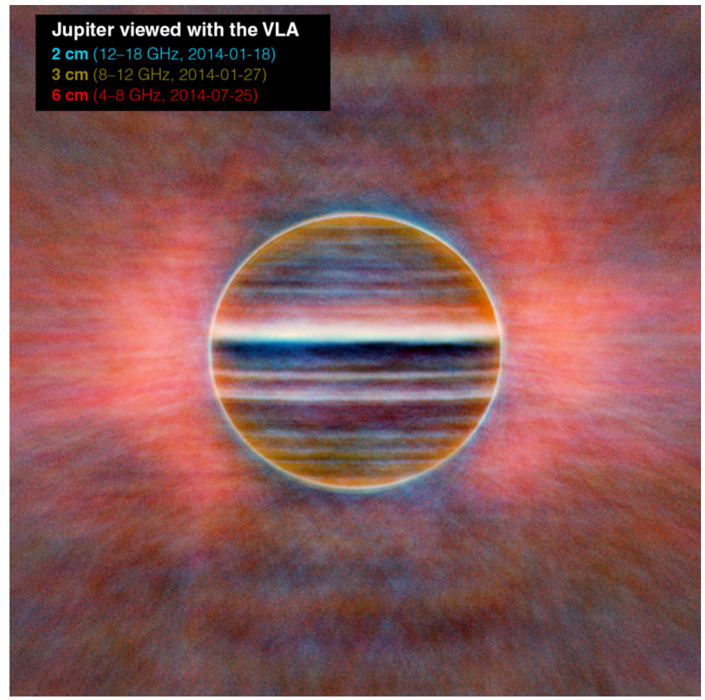
# Correcting for Rotation

If a planet rotates rapidly, we can either just live with the "smearing" in the final image (but note also that this violates our assumption about sources not varying), or try to make snapshots and use them separately (difficult in most cases because SNR is low). There are now two techniques to try to solve this problem; one for optically thin targets like Jupiter synchrotron radiation (Sault+ 1997; Leblanc+ 1997; de Pater & Sault 1998), one for optically thick targets (described in Sault+ 2004). This is possible because we know the viewing geometry and planetary cartographic systems precisely.

# Correcting for Rotation - Jupiter

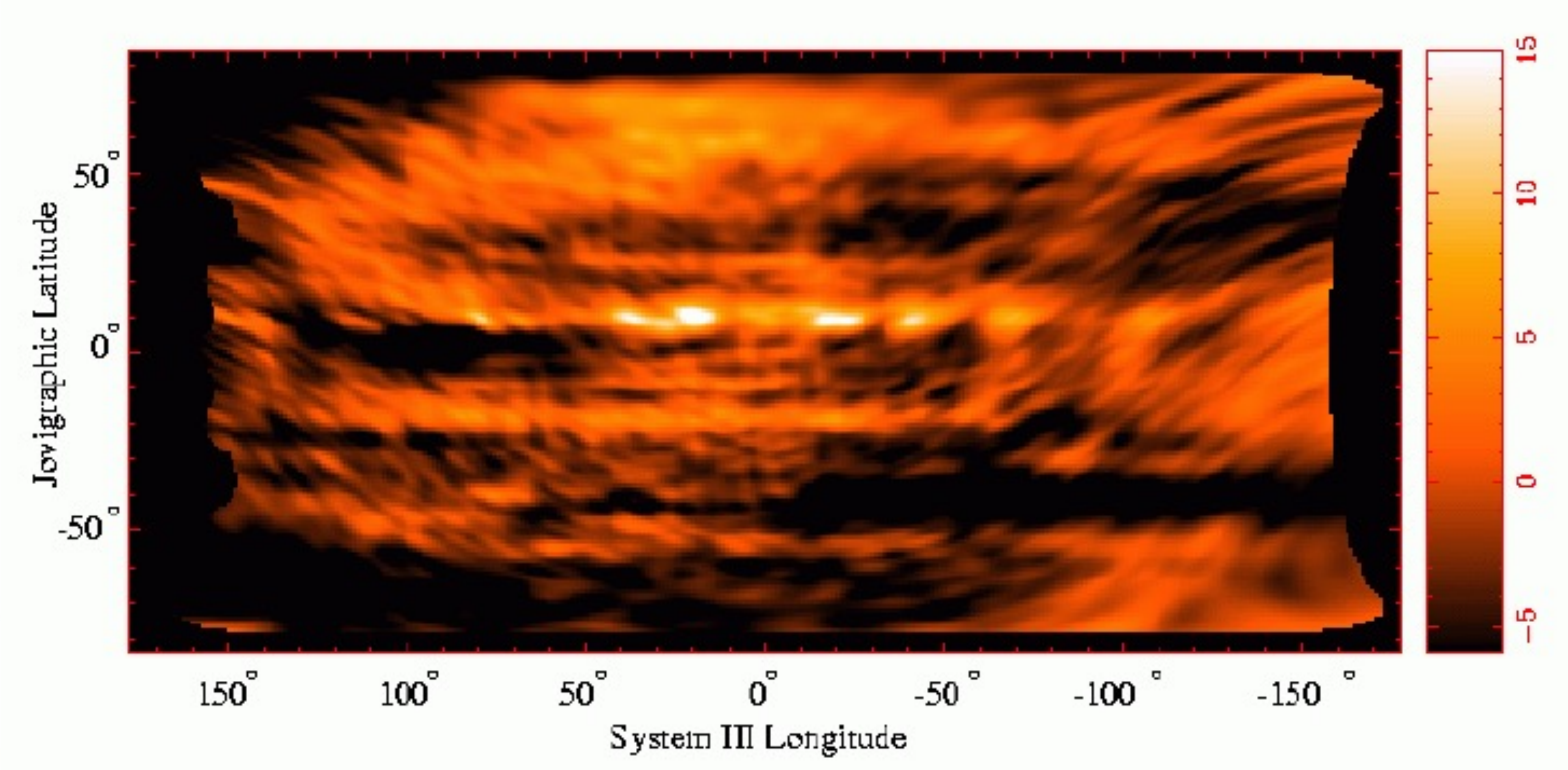
Jupiter at 20 cm (de Pater+ 1997) and 2-6 cm (de Pater+ 2016) averaged (smeared) over full track (period is ~10h):



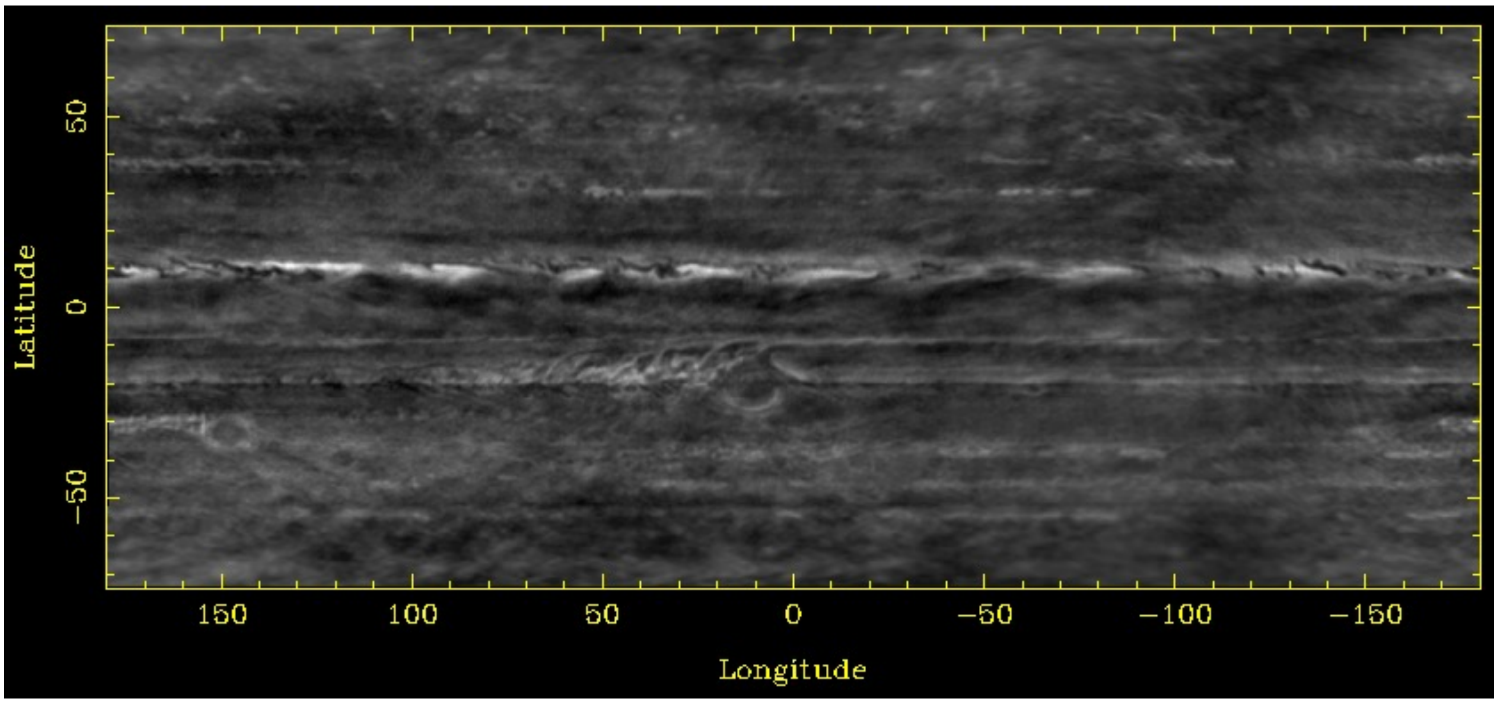
# Correcting for Rotation - Jupiter

Jupiter at 2cm from several tracks – Sault+ 2004:



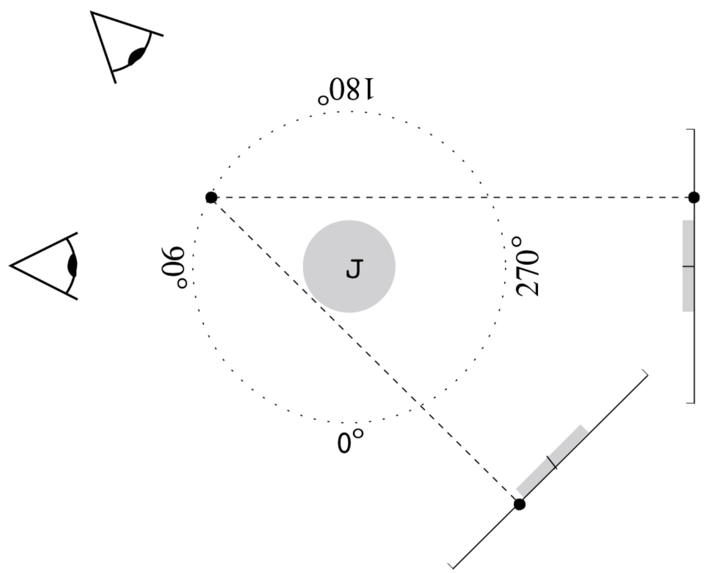
# Correcting for Rotation - Jupiter

Jupiter at 1.3cm from a more recent observation – de Pater+ 2016 – showing the dramatic increase in sensitivity of the VLA:

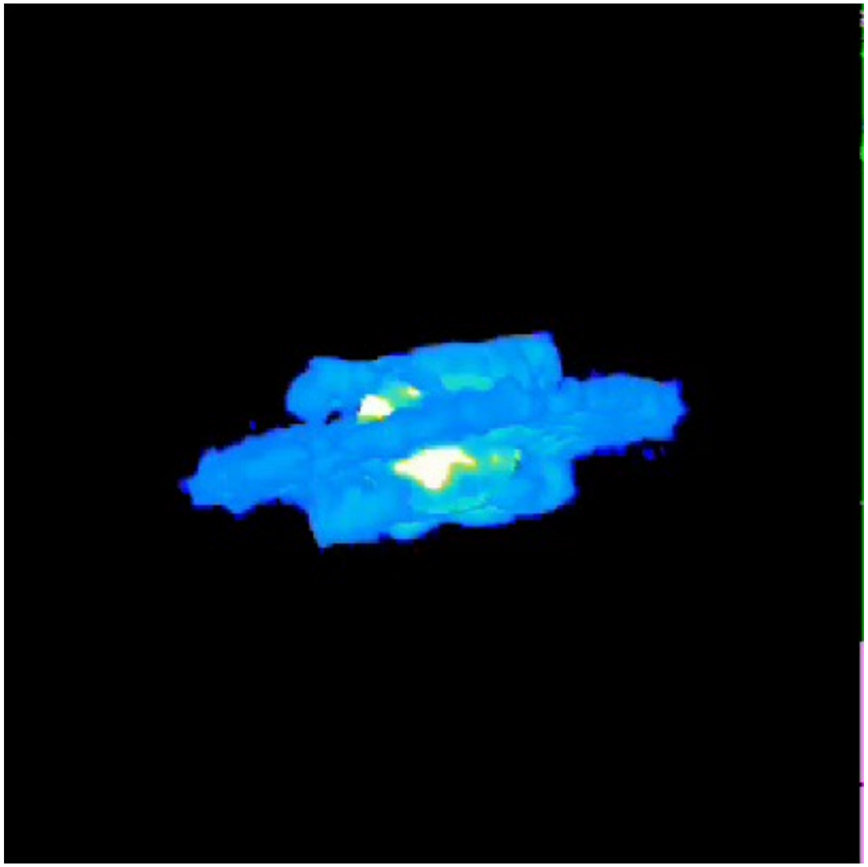


# Correcting for Rotation - Jupiter

If the emission mechanism is optically thin (this is only the case for the synchrotron emission), then we can make a full 3-D reconstruction of the emission:

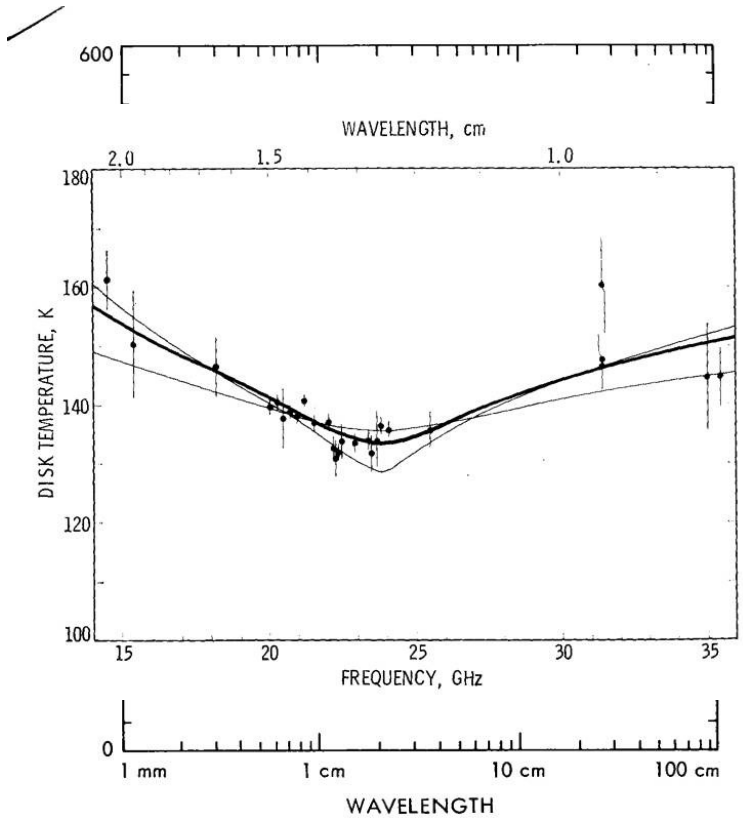


# Correcting for Rotation - Jupiter



# Spectral Lines

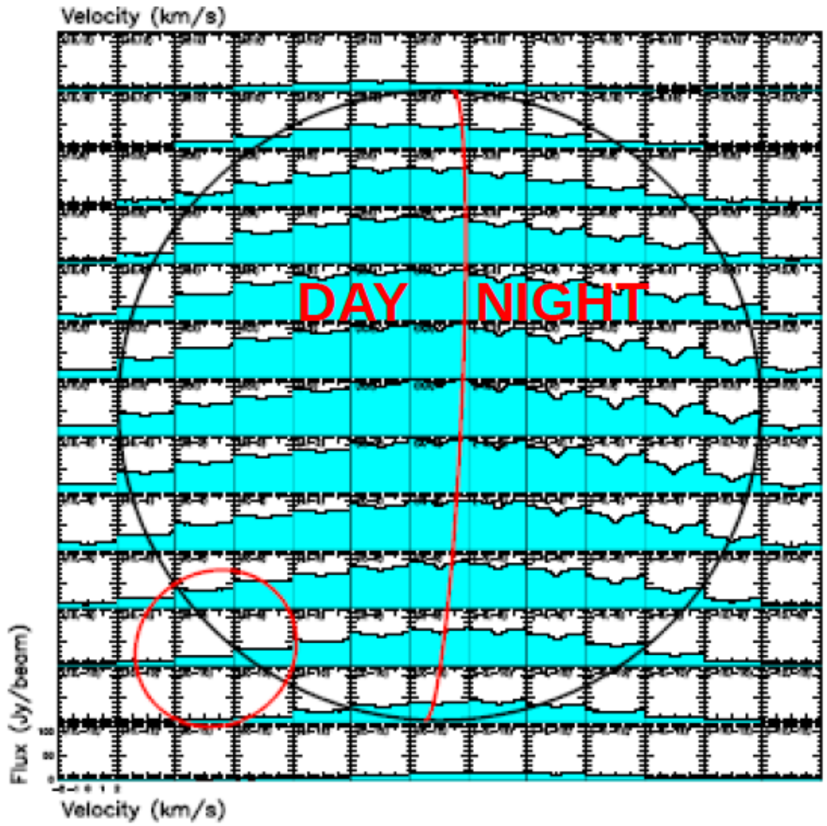
Species in the atmosp' es, just as they do in other astronomical so proportional to the molecular abundance.




$\gamma \sim p * 3$ MHz/mbar, are probing down to at least 1 bar pressure,

# Spectral Lines – Venus Example

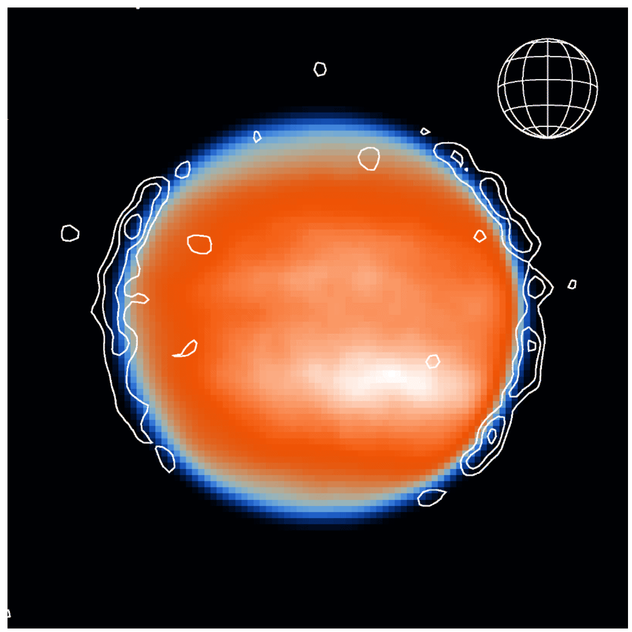
By knowing the geometry, we can get both abundances and winds as a function of latitude, or time of day, for Venus.



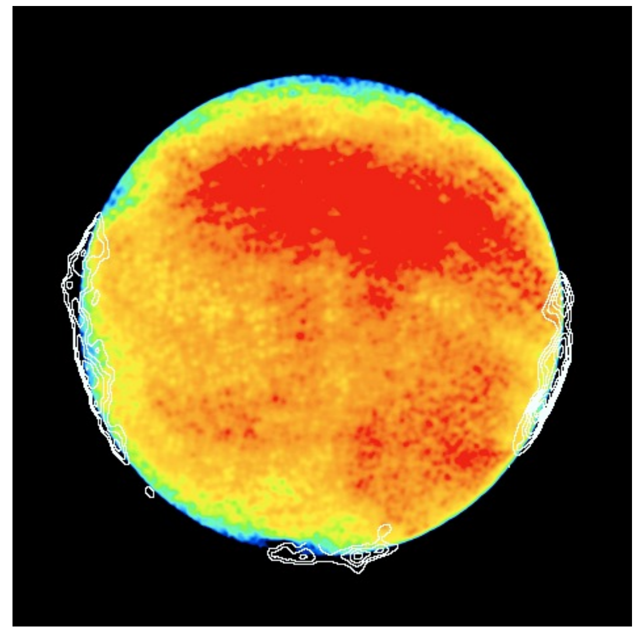

Moullet+ 2012

# Spectral Lines – Mars Example

Similarly, we can get water vapor abundances on Mars as a function of latitude and time of day, or season.



Clancy+ 1992



Butler+ 2005

## Lack of Source Knowledge

If the true source position is not where the phase center of the instrument was pointed, then a phase error is induced in the visibilities.

If you don't think that you knew the positions beforehand, then the phases can be "fixed." If you think you knew the positions beforehand, then the phases may be used to derive an offset.

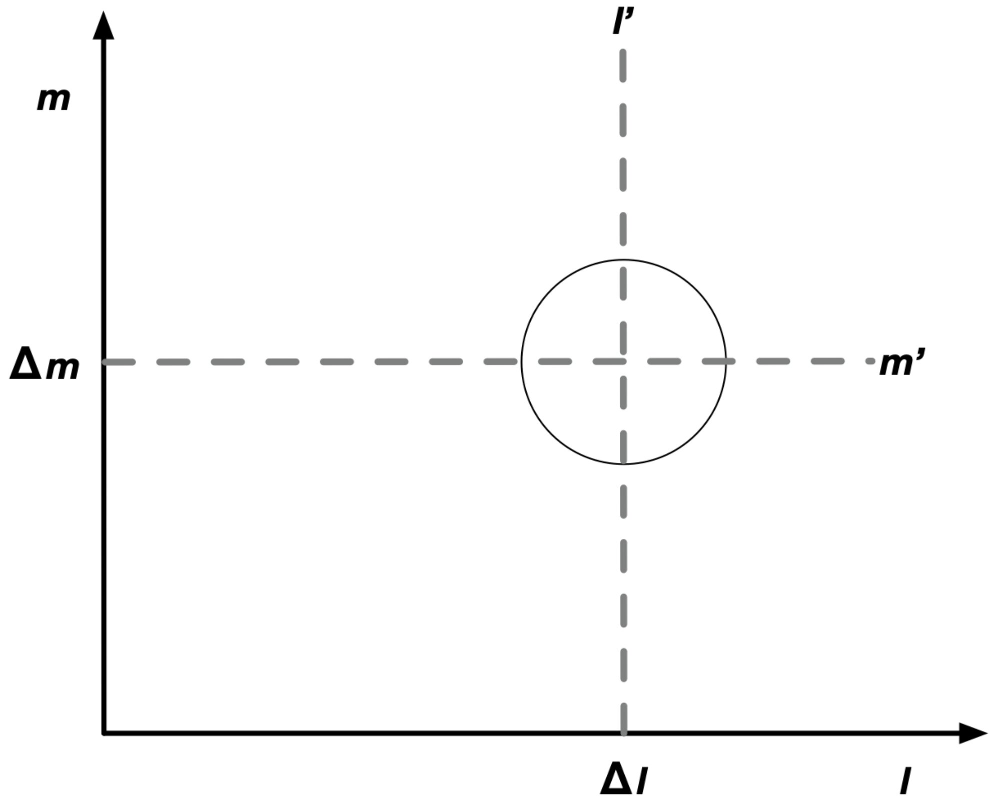
# Position Shifts

- Consider a situation where we observed at some sky position $l, m$ but where the actual position of the source is at coordinates some (small, in terms of spherical projection) distance away $\Delta l, \Delta m$.
- Any change of sky position can be generalized this way…
- Our measured visibilities are (ignoring primary beam):

$$V(u,v) = \iint I(l,m)\, e^{-2\pi i(ul+vm)}\, dl\, dm$$

- Define a new coordinate system $l', m'$ with the appropriate shifts.

# Position Shifts



# Position Shifts

- Substitute $l = l' + \Delta l$ and $m = m' + \Delta m$ into the measured visibility equation to get (eventually):

$$V(u,v) = \iint I(l',m') e^{-2\pi i(u\Delta l + v\Delta m)} e^{-2\pi i(ul' + vm')}\, dl'\, dm'$$

$$V(u,v) = e^{-2\pi i(u\Delta l + v\Delta m)}\ V'(u,v)$$

$$V'(u,v) = e^{2\pi i(u\Delta l + v\Delta m)}\ V(u,v)$$

$$V'(u,v) = G\ \ V(u,v)$$

- Where $G$ is just a complex gain multiplier:

$$G = e^{2\pi i(u\Delta l + v\Delta m)} = \cos\bigl(2\pi(u\Delta l + v\Delta m)\bigr) + i\sin\bigl(2\pi(u\Delta l + v\Delta m)\bigr)$$

which can change over time. This is what MODPO and fixvis() do. (UVFIX if the position is not changing with time.)

# Position Shifts

You may have a reason to shift the phase center in your data:

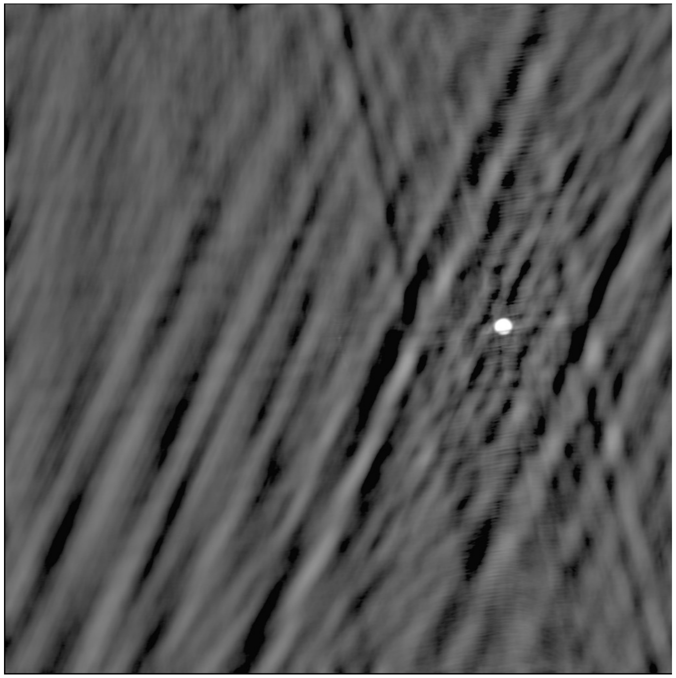
- Observations at low frequencies must account for background sources
- Observations where positions were not well known at the time, but improved later (comets and asteroids)
- Observations of satellites (moons) of planets

## Position Shifts – an example

An example of observations of Titan – Saturn's largest moon. Observations were done in 1992 (Grossman & Muhleman) – 17 different orbital positions, at X-band, in the CnB- and C-configurations. Titan was tracked, and even though the separation from Saturn is relatively large in many of the observations, confusion from Saturn is a major problem, so must be dealt with.

# Position Shifts – an example

If you just make an image of the data with Titan tracked, doing nothing particularly special, you end up with something like this:



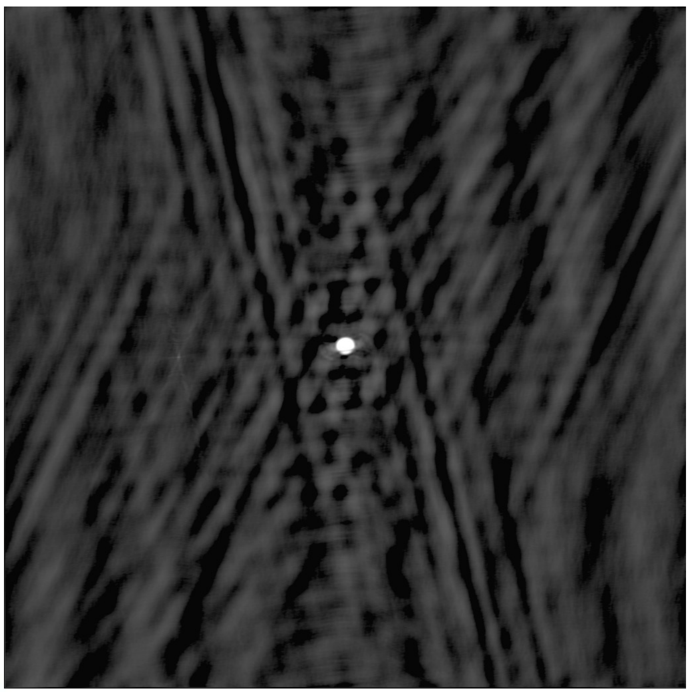
(you can actually just barely see Titan at the center of the image, hiding in all that noise!)

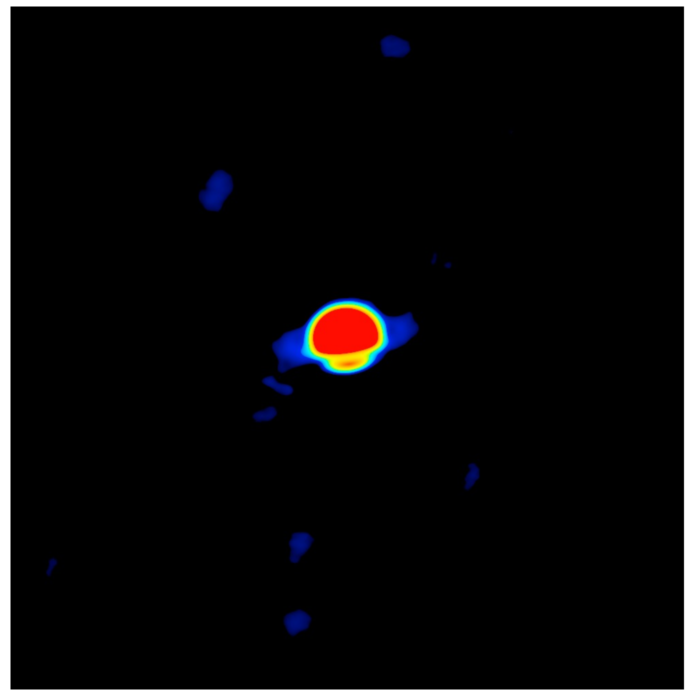
# Position Shifts – an example

- Titan is too weak to self-calibrate in old VLA data (~1.5 mJy).
- Saturn has plenty of flux density to self-calibrate, but if you try to self-calibrate the original data using Saturn, you get large errors, because Saturn is actually moving in the image during the observation.
- Using the ephemeris information for Saturn and Titan, however, we can shift Saturn to the phase center, and then use it to self-calibrate, then subtract it, and then shift Titan back to the phase center and image it.

# Position Shifts – an example

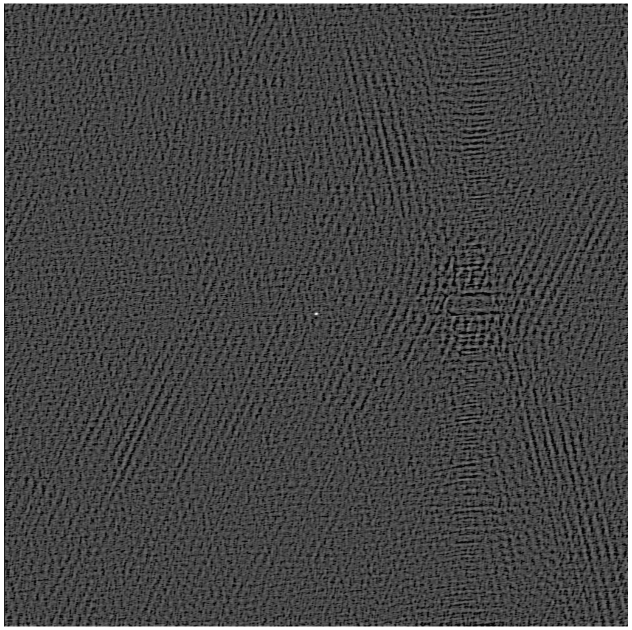
Best image of Saturn after shifting and self-calibration.  Still lots of artifacts, but Titan is much more clearly evident.

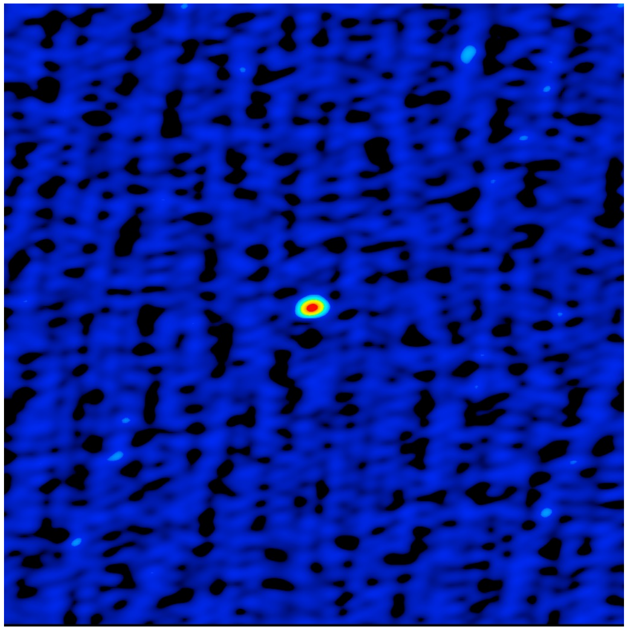




# Position Shifts – an example

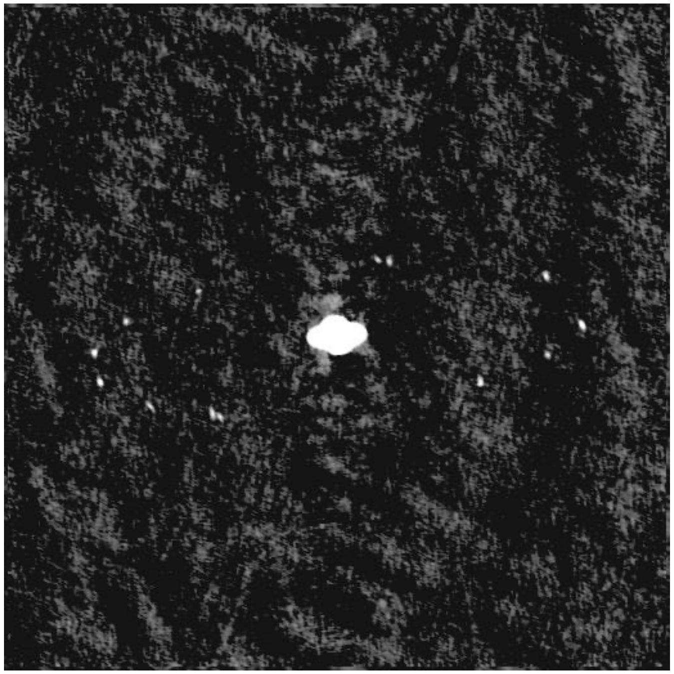
After subtracting Saturn, shifting Titan back to the phase center, and imaging using only longer baselines (use only baselines > 20 k$\lambda$), we get a nice image of Titan.





# Position Shifts – an example

Then you can do neat things like adding all 17 observations together after shifting Saturn to the phase center.



# Expected Flux Density

For optically thick objects with small brightness temperature (most solar system bodies), we must take into account the fact that the body blocks out the background radiation. The expected flux density is:

$$S = (B_p - B_{bg})\Omega_p$$

For body brightness $B_p$ and solid angle $\Omega_p$. $B_{bg}$ is the brightness of the background, which is mostly the CMB and galactic emission. Since the magnitude of the background brightness (at least that of the 2.725 K CMB) is a significant fraction of the brightness of most solar system objects, this effect must be accounted for when deriving brightness temperature (the interesting physical quantity) from the measured flux density.

# Expected Flux Density

$$V(u,v) = \iint_{-\infty}^{\infty} A(l,m)\, I(l,m)\, e^{-2\pi i(ul+vm)}\, dl\, dm$$

Ignore primary beam and break into on-planet and off-planet components:

$$V(u,v) = \int_{disk} I_p\, e^{-2\pi i(ul+vm)}\, dl\, dm + \int_{off-disk} I_{bg}\, e^{-2\pi i(ul+vm)}\, dl\, dm$$

$$\int_{off-disk} I_{bg}\, e^{-2\pi i(ul+vm)}\, dl\, dm = \int_{-\infty}^{\infty} I_{bg}\, e^{-2\pi i(ul+vm)}\, dl\, dm - \int_{disk} I_{bg}\, e^{-2\pi i(ul+vm)}\, dl\, dm$$

$$V(u,v) = \int_{disk} (I_p - I_{bg})\, e^{-2\pi i(ul+vm)}\, dl\, dm + \int_{-\infty}^{\infty} I_{bg}\, e^{-2\pi i(ul+vm)}\, dl\, dm$$

$$V(u,v) = \int_{disk} (I_p - I_{bg})\, e^{-2\pi i(ul+vm)}\, dl\, dm + \delta(0,0) = \int_{disk} (I_p - I_{bg})\, e^{-2\pi i(ul+vm)}\, dl\, dm$$

# Real Data - what to expect

But …

# Real Data - what to expect

If the sky brightness is circularly symmetric, then the 2-D Fourier relationship between sky brightness and visibility reduces to a 1-D Hankel transform:

$$V(q) = 2\pi R \int_0^R A(r) I(r) J_0(2\pi r q) r\, dr$$

For a "uniform disk" of total flux density $F$, this reduces to:

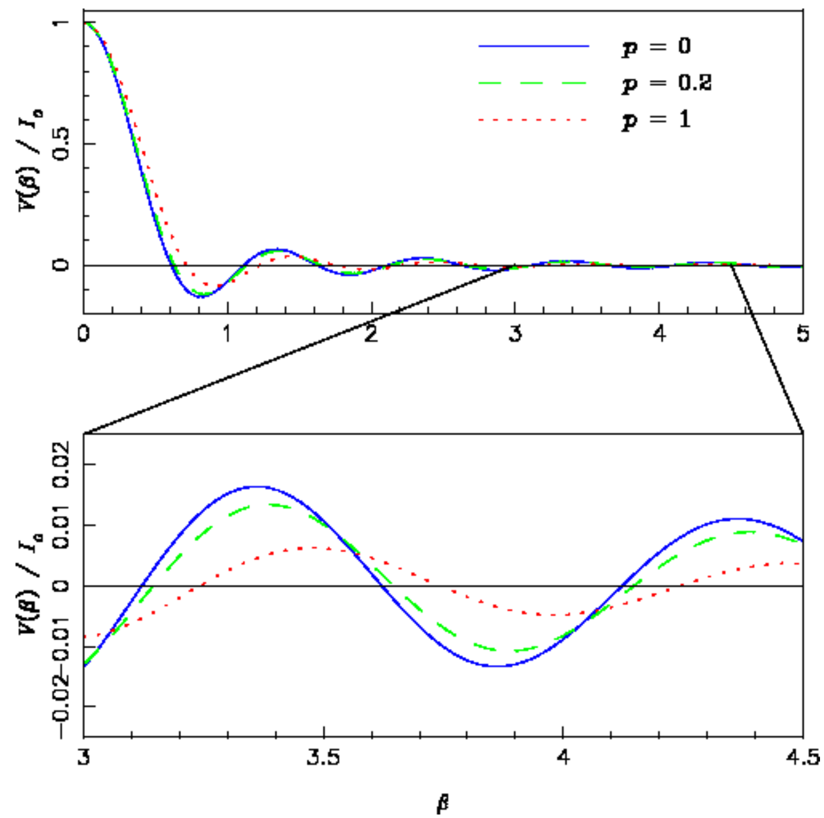
$$V(\beta) = F\pi R^2 \frac{J_1(2\pi\beta)}{\pi\beta}$$

and for a "limb-darkened disk" (of a particular form), this reduces to:

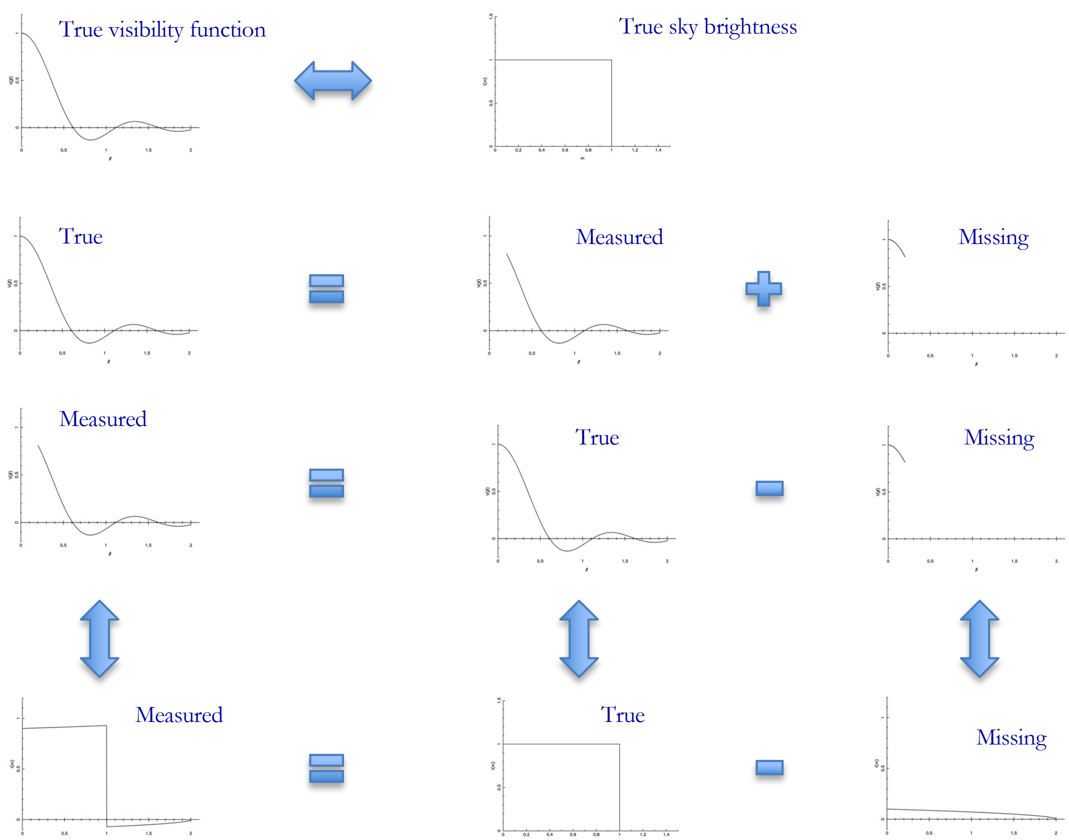
$$V(\beta) = F\pi R^2 \Lambda_q(2\pi\beta)$$

# Real Data - what to expect

Theoretical visibility functions for a circularly symmetric “uniform disk” and 2 limb-darkened disks.



# Short Spacing Problem Revisited



# Real Data - polarization

For emission from solid surfaces on planetary bodies, the relationship between sky brightness and *polarized* visibility becomes (again assuming circular symmetry) a different Hankel transform (order 2):

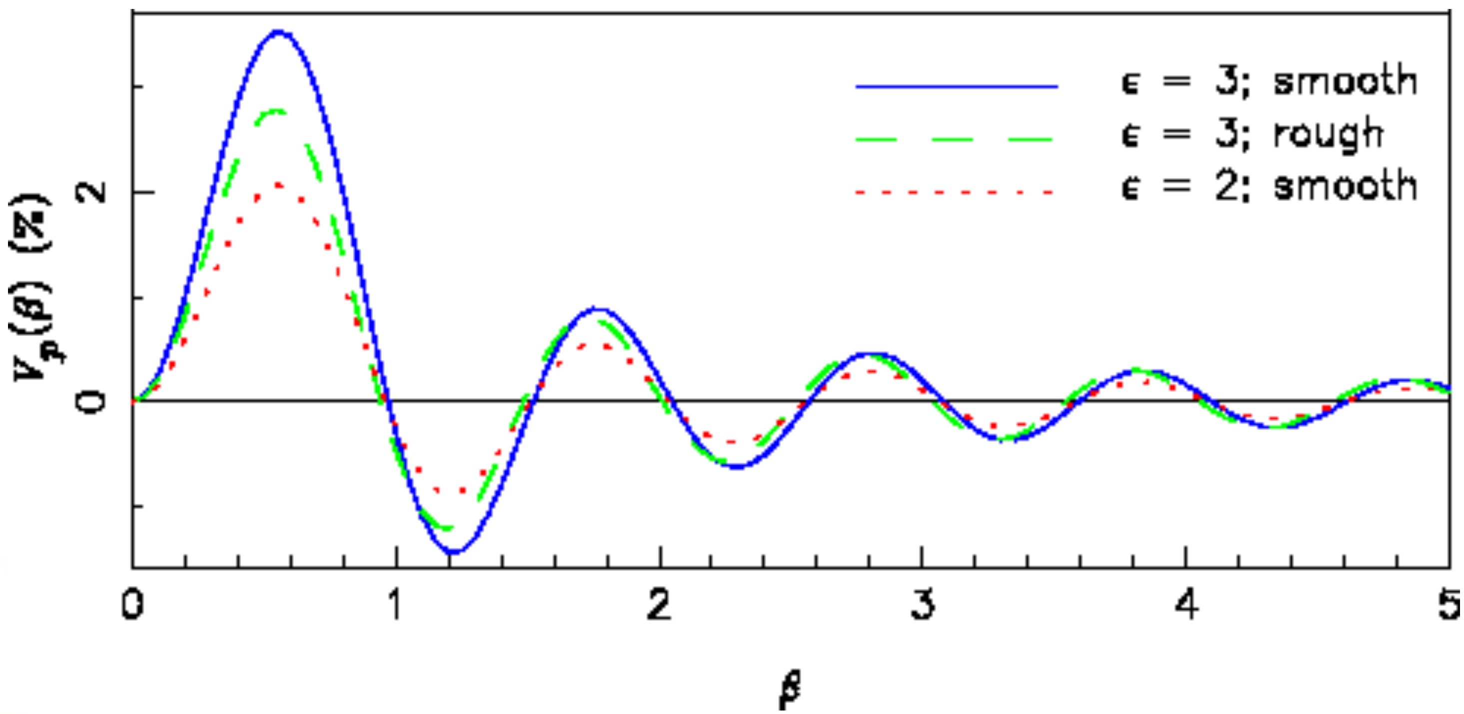
$$V_p(\beta) = \int_0^1 A(\rho)(R_{\parallel} - R_{\perp})J_2(2\pi\rho\beta)\rho\, d\rho$$

this cannot be solved analytically. Note that roughness of the surface is a confusion (it modifies the effective Fresnel reflectivities). For circular measured polarization, this visibility is formed via:

$$V_p = \frac{\Re\{V_{RL} + V_{LR}\}\cos 2\psi + \Im\{V_{RL} - V_{LR}\}\sin 2\psi}{V_0}$$

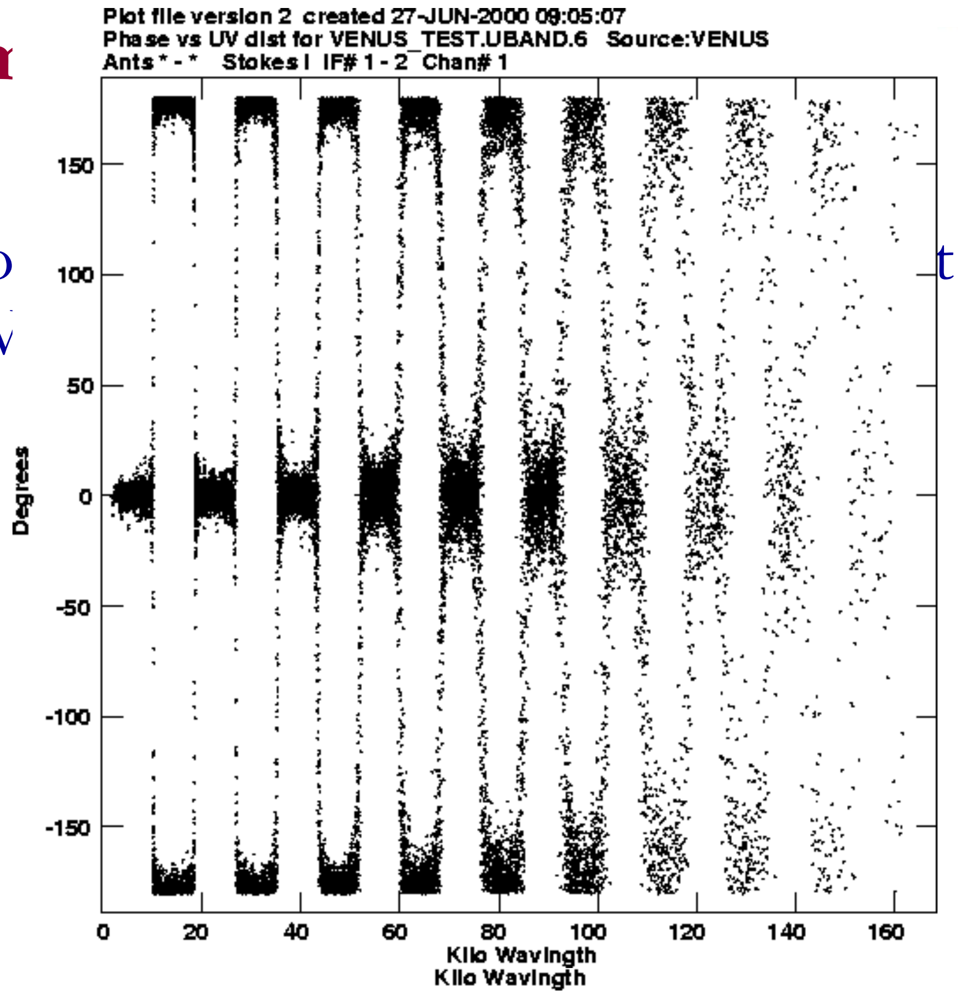
# Real Data - polarization

Examples of expected polarization response:



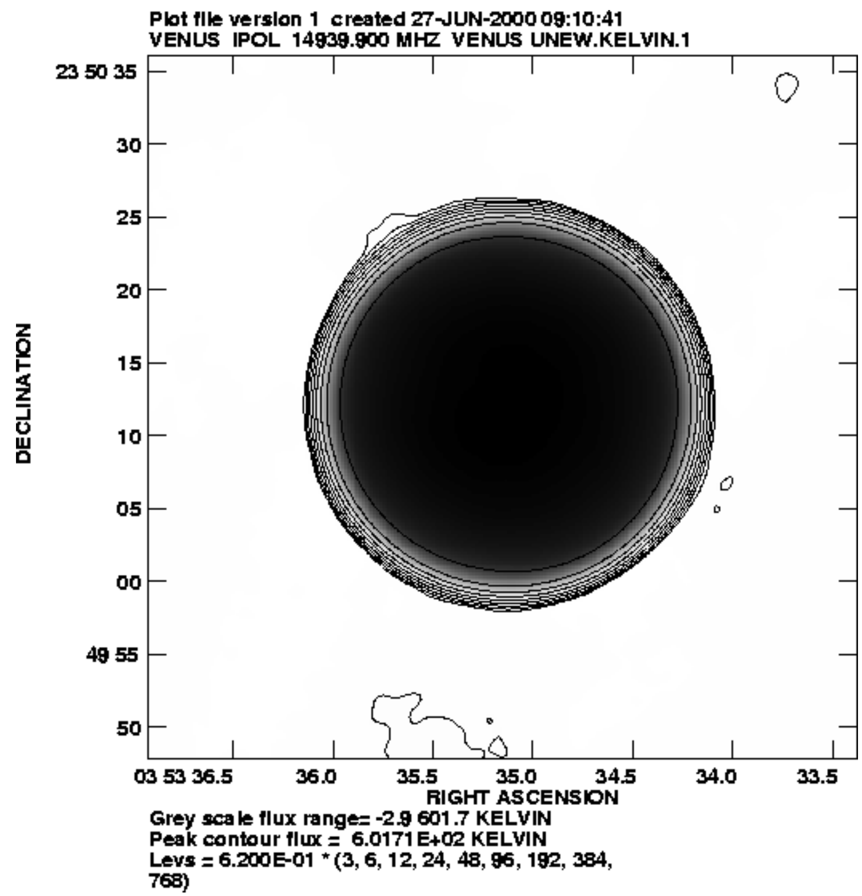
# Real Data - n

Visibility data fo t 0.674 AU
distance in the V



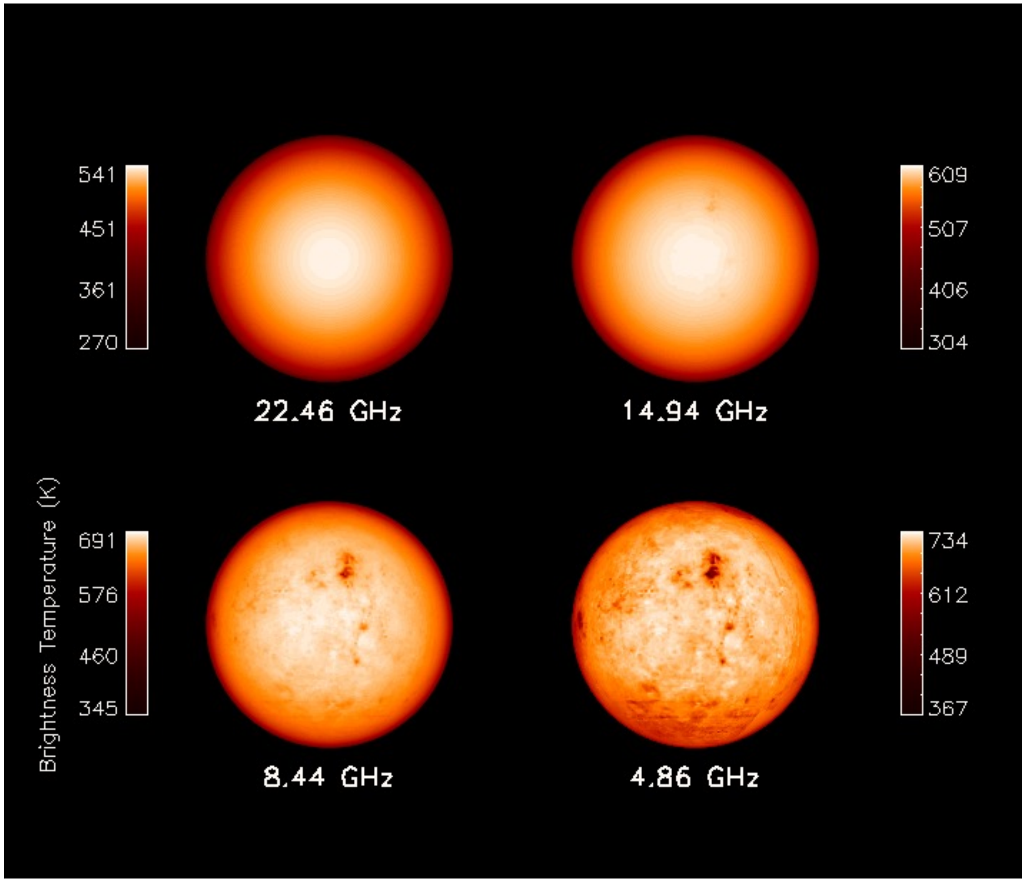
# Real Data - an example

The resultant image:



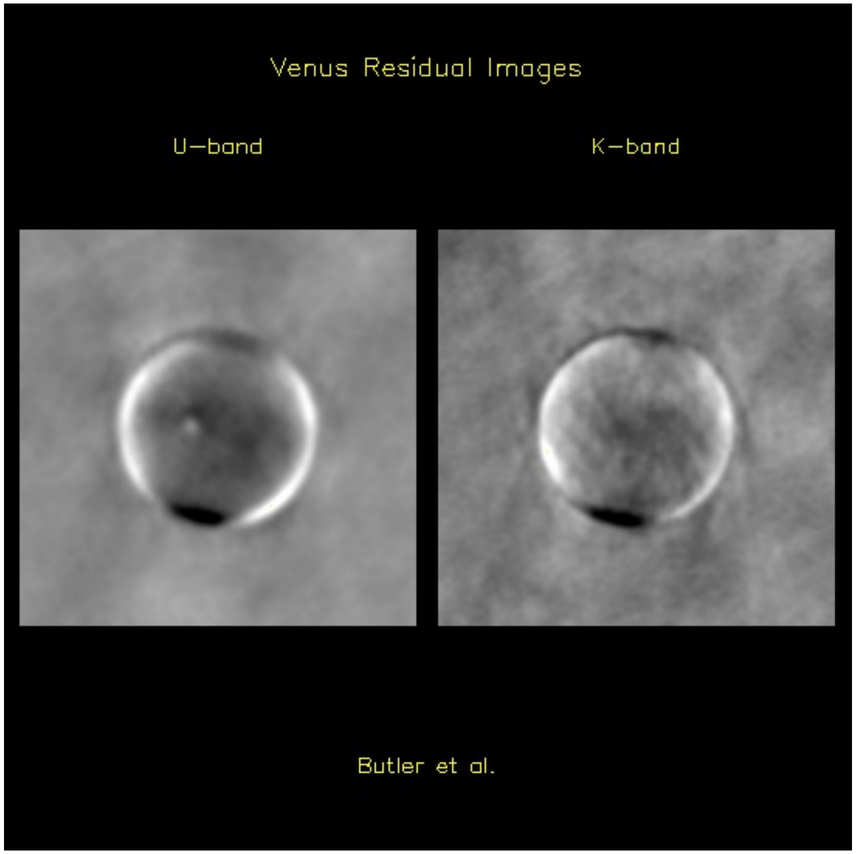
# Real Data - an example

Venus models at C, X, Ku, and K-bands:



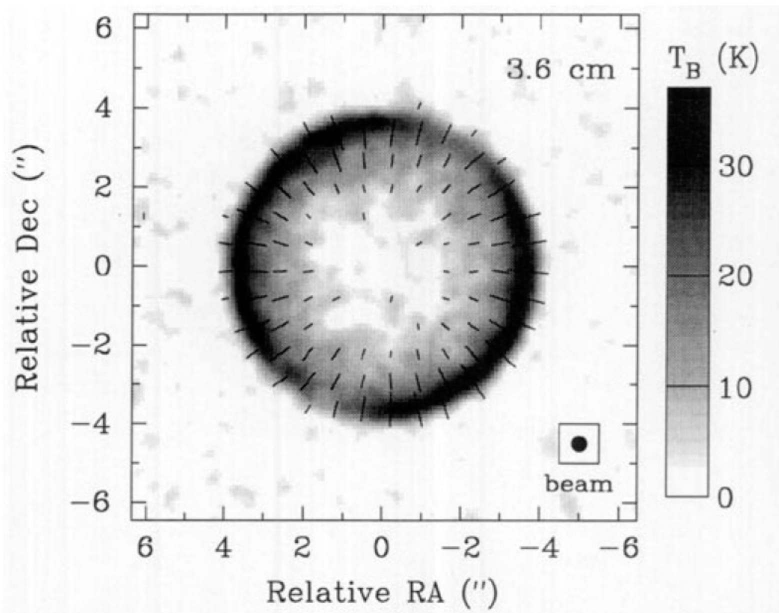
# Real Data - an example

Venus residual images at Ku- and K-bands:



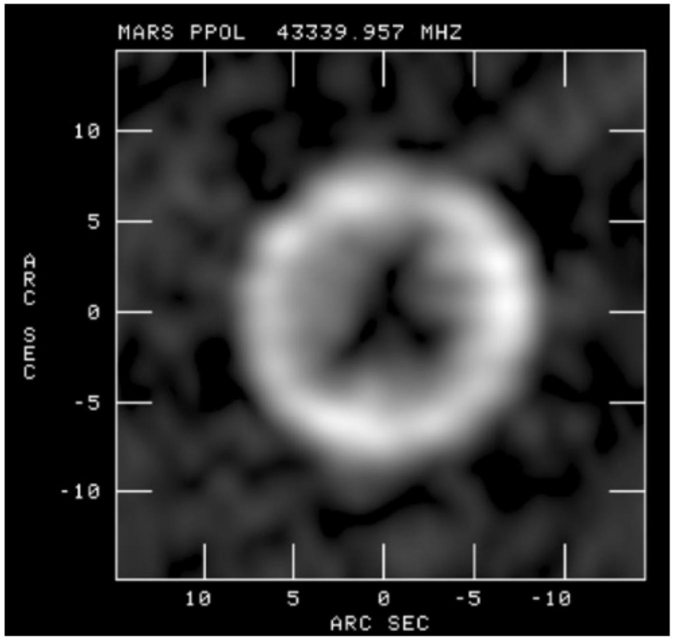
# Real Data - a polarization example

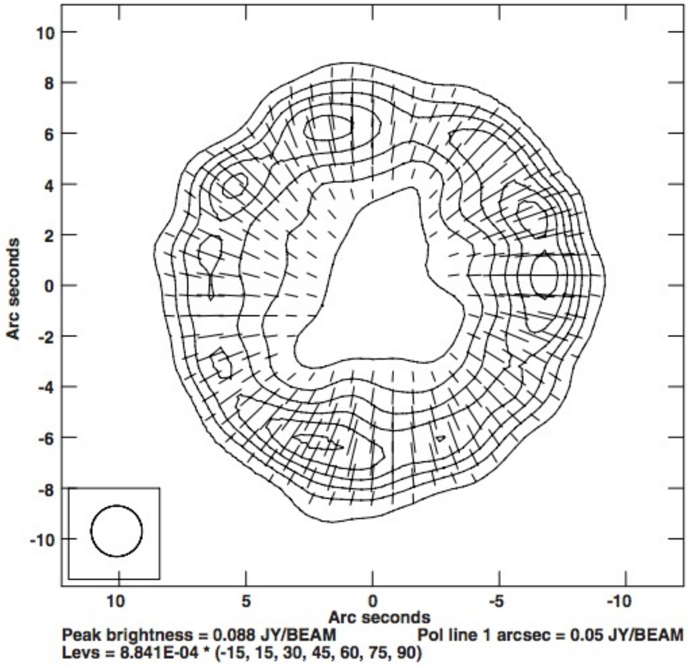
Mitchell & de Pater (1994) observations of Mercury showing the polarization pattern on the sky:



# Real Data - a polarization example

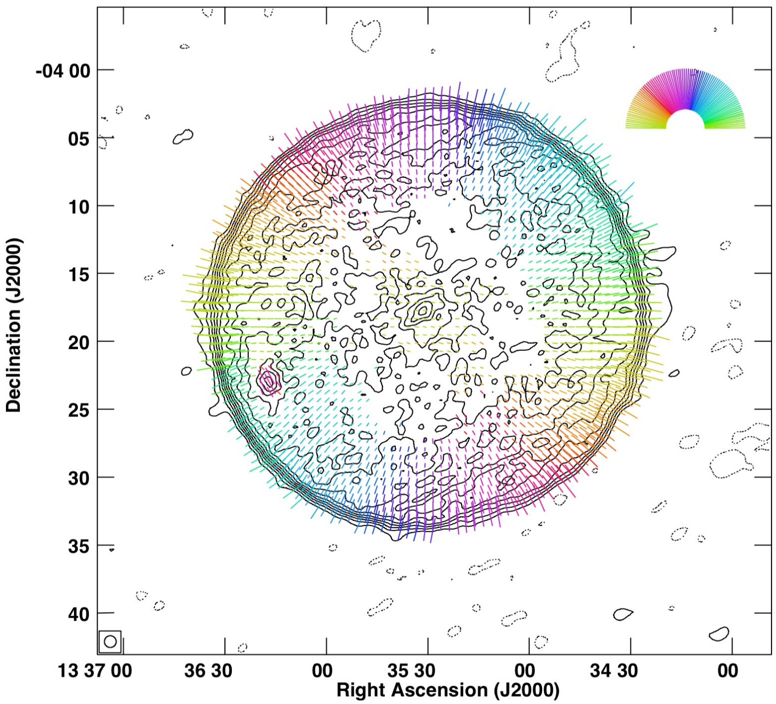
Similarly for Mars (Perley & Butler 2013):

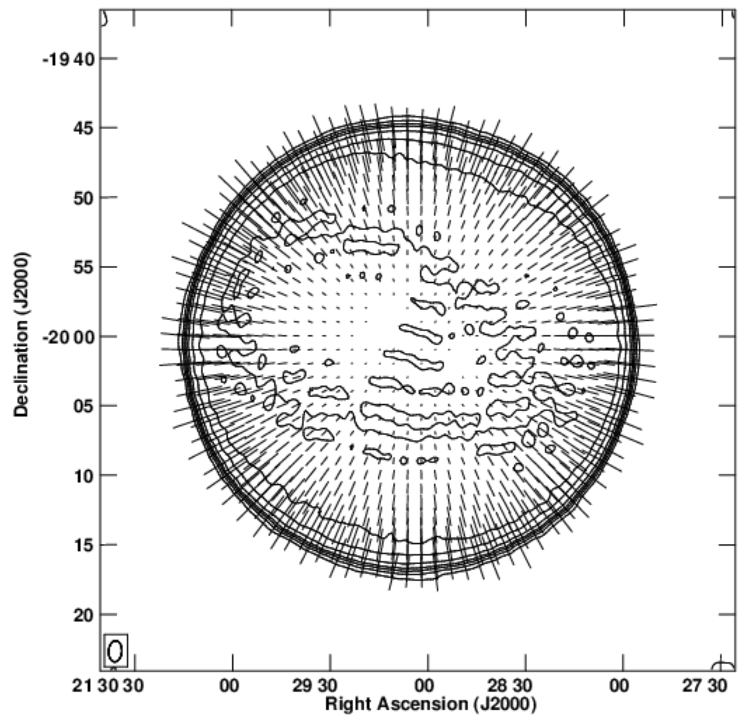


# Real Data - a polarization example

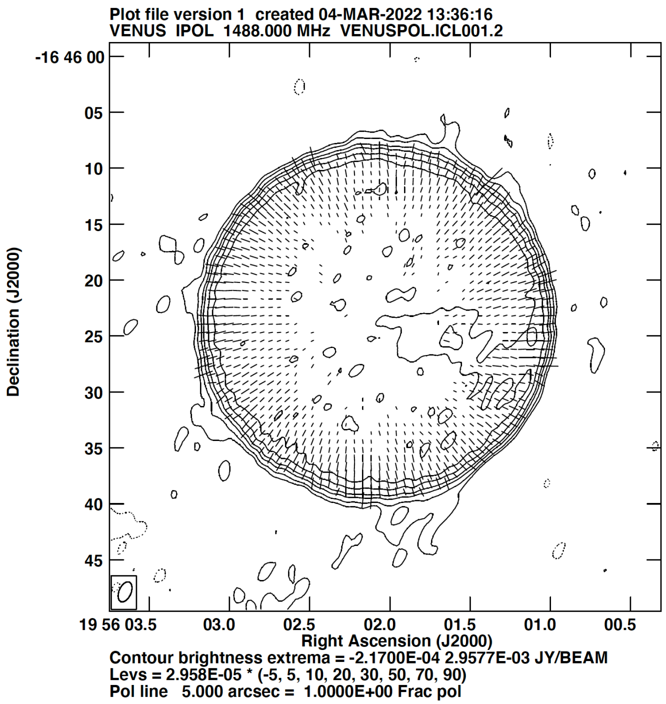
Similarly for the Moon (Siegler+ in prep; Perley+ in prep)

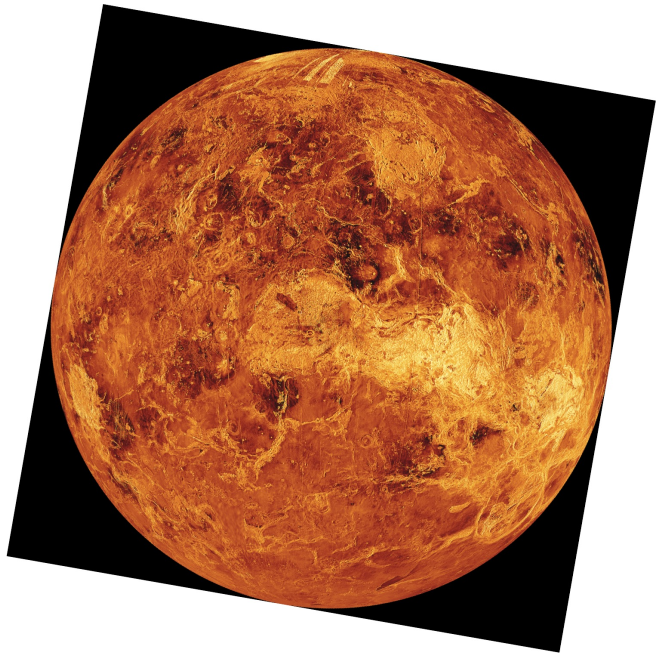


# Real Data - a polarization example

Similarly for Venus (Perley+ in prep)



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