Wide-Band and Wide-field Imaging : Concepts

Urvashi Rau

National Radio Astronomy Observatory, Socorro, NM, USA

18th NRAO Synthesis Imaging Workshop

(18-25 May 2022)

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed
visibilities
(Data)

Direction
Independent
Gains

UV sampling
pattern

Sky Brightness
(Image)

Fourier transform
kernel

Calibration

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i(ul+vm)} dl\, dm$$

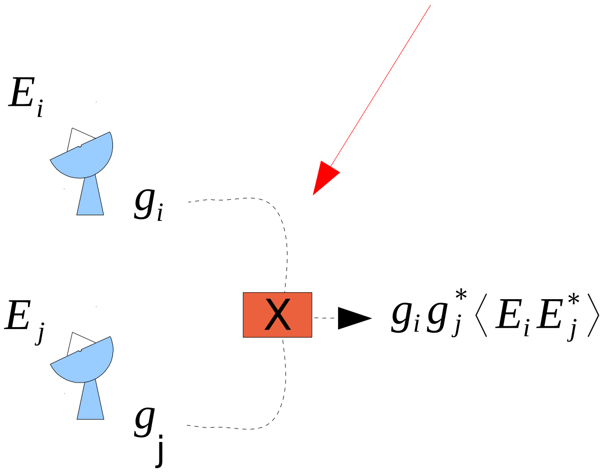
Observed visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel



Calibration

Solve for g_i and divide out $g_i g_j^*$

[Ref : J.Marvil Calibration lecture]

N antennas
N(N-1)/2 antenna-pairs (baselines)

Imaging & Deconvolution

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

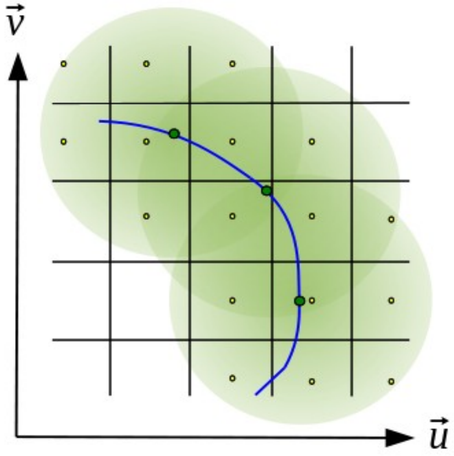
Sky Brightness (Image)

Fourier transform kernel

Imaging

Gridding
+
IFFT
+
Normalization

I^{obs}



Convolutional resampling

Accumulate measured visibilities onto a regular grid

[Ref : Imaging : D. Wilner]

Imaging & Deconvolution

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

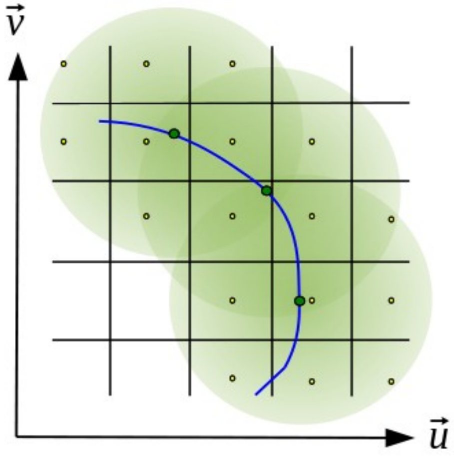
Sky Brightness (Image)

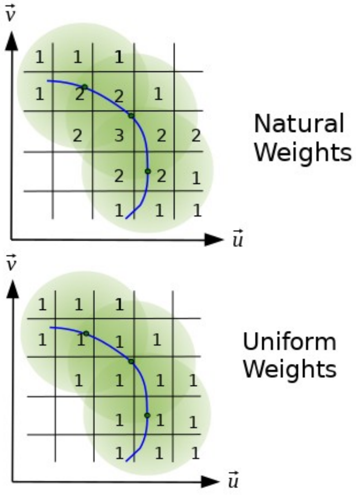
Fourier transform kernel

Imaging

Gridding
+
IFFT
+
Normalization

I^{obs}





Data weighting schemes during image formation

=> adjust sensitivity to different spatial scales

[Ref : Imaging : D. Wilner]

Imaging & Deconvolution

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

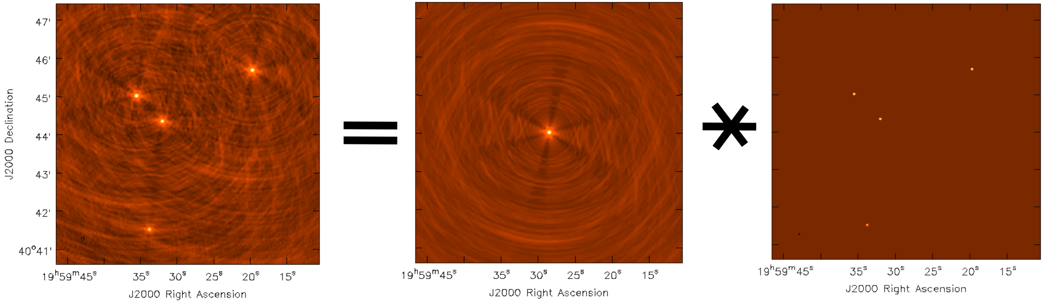
Fourier transform kernel

Gridding
+
IFFT
+
Normalization

I^{obs}

Deconvolution

$$I^{obs}(l,m) = I^{PSF}(l,m) * I^{sky}(l,m)$$

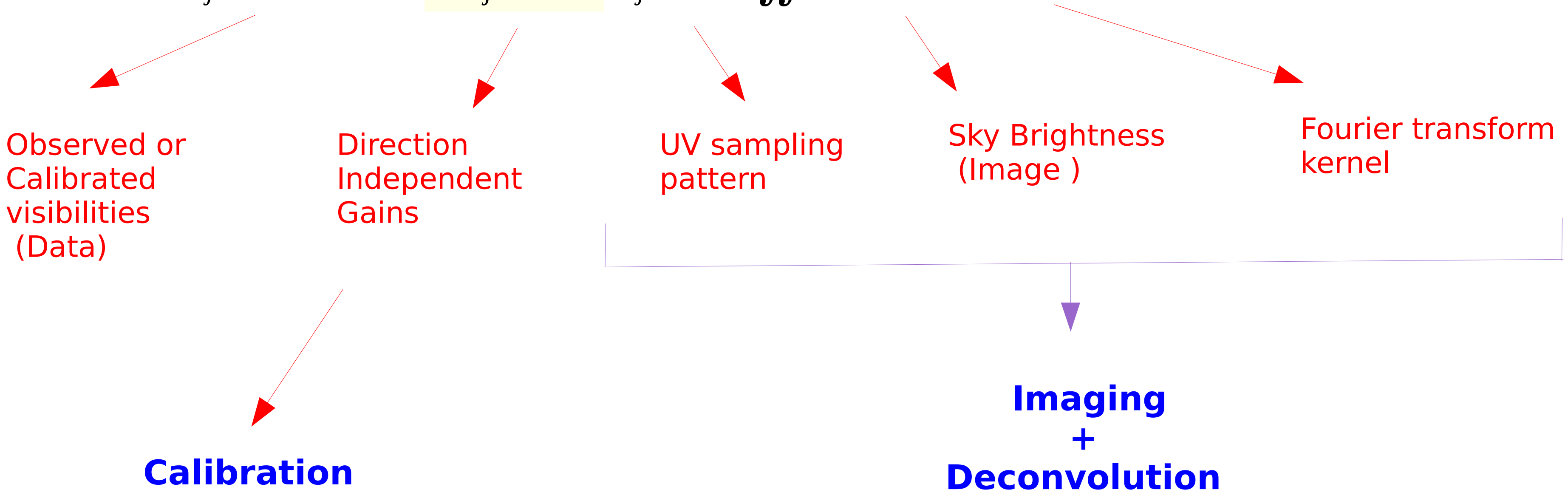


Algorithms :
Clean, MS-Clean, Asp, etc...

[Ref : Imaging : D. Wilner]

All together….

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$



Observed or
Calibrated
visibilities
(Data)

Direction
Independent
Gains

UV sampling
pattern

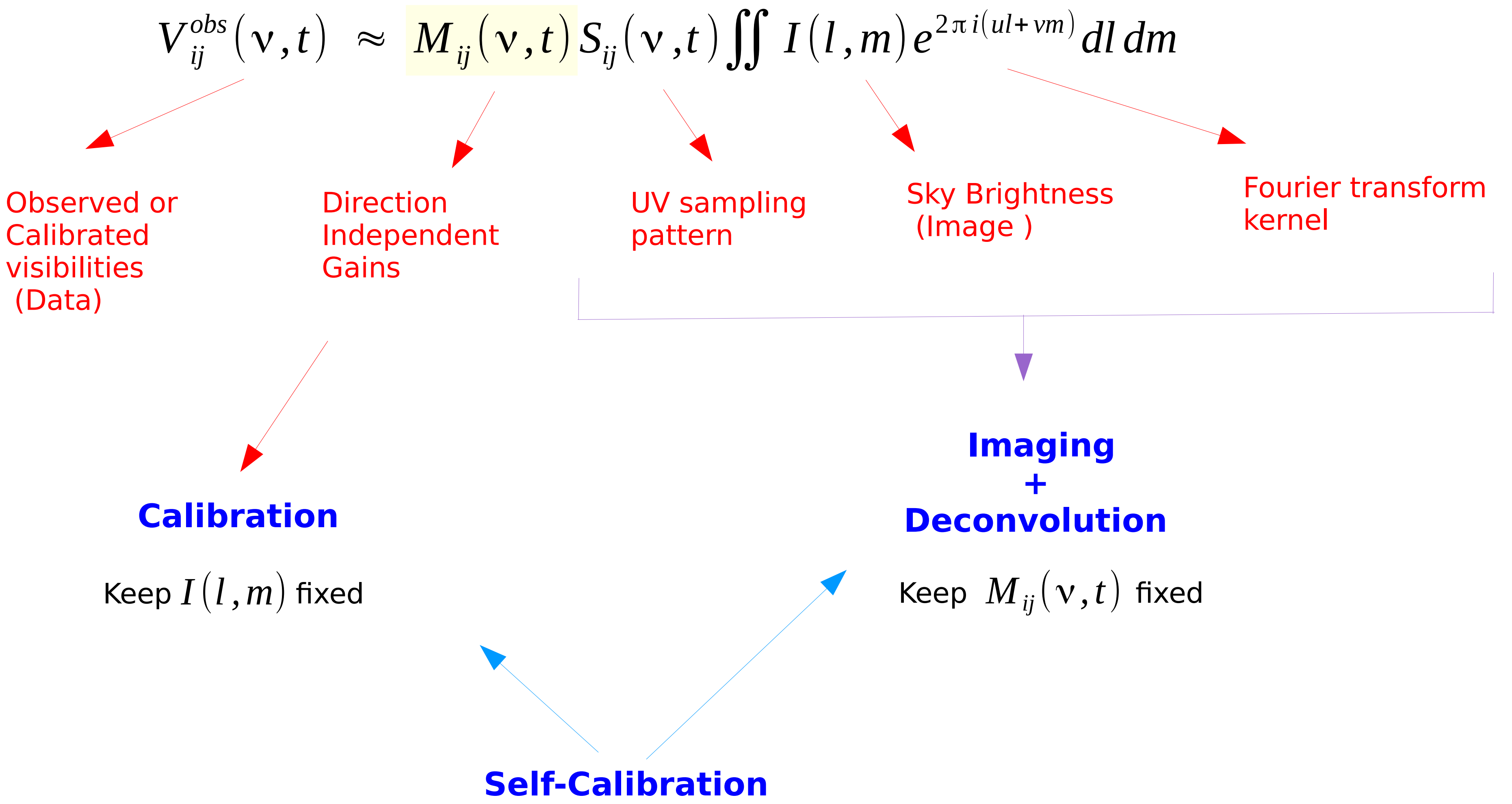
Sky Brightness
(Image)

Fourier transform
kernel

Calibration

Imaging
+
Deconvolution

Self-Calibration



$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed or Calibrated visibilities (Data)

Direction Independent Gains

UV sampling pattern

Sky Brightness (Image)

Fourier transform kernel

Calibration

Keep $I(l,m)$ fixed

Imaging + Deconvolution

Keep $M_{ij}(\nu,t)$ fixed

Self-Calibration

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

Observed or
Calibrated
visibilities
(Data)

Direction
Independent
Gains

UV sampling
pattern

Sky Brightness
(Image)

Fourier transform
kernel

This is only an approximation…..

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iint I(l,m)\, e^{2\pi i(ul+vm)}\, dl\, dm$$

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

Sky-brightness varies
with frequency (time)

W-Term

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) \approx M_{ij}(\nu,t) S_{ij}(\nu,t) \iint I(l,m) e^{2\pi i (ul+vm)} \, dl \, dm$$

$$V_{ij}^{obs}(\nu,t) = \underbrace{M_{ij}(\nu,t)}_{\text{Direction Independent Gains}} \underbrace{S_{ij}(\nu,t)}_{\text{UV sampling function}} \iiint \underbrace{M_{ij}^{s}(l,m,\nu,t)}_{\text{Direction Dependent Effects}} I(l,m\underbrace{,\nu,t}_{\text{Sky-brightness varies with frequency (time)}}) e^{2\pi i (ul+vm+\overbrace{w(n-1)}^{\text{W-Term}})} \, dl \, dm \, dn$$

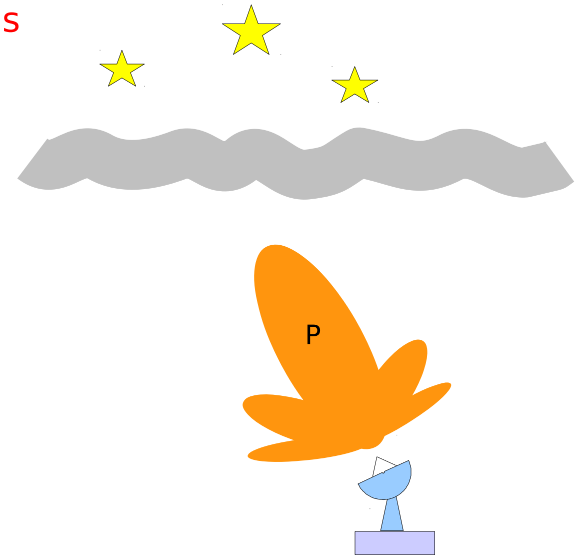
Primary beams and signal propagation effects...

Wide-Field Imaging : Direction dependent effects

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Independent
Gains

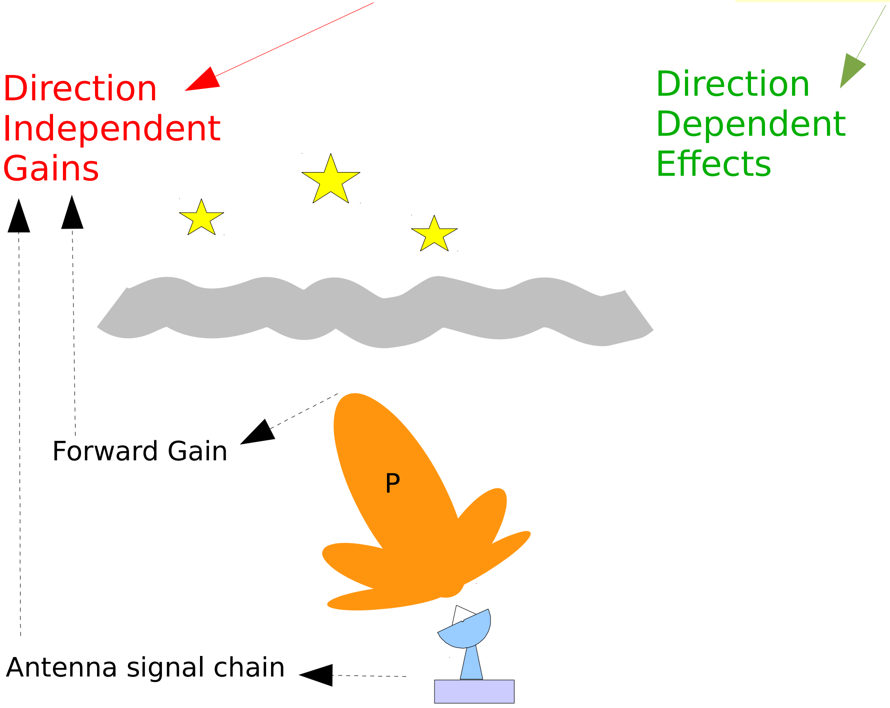
Direction
Dependent
Effects



[Ref : Antennas : R.Selina
Calibration : J.Marvil]

Wide-Field Imaging : Direction dependent effects

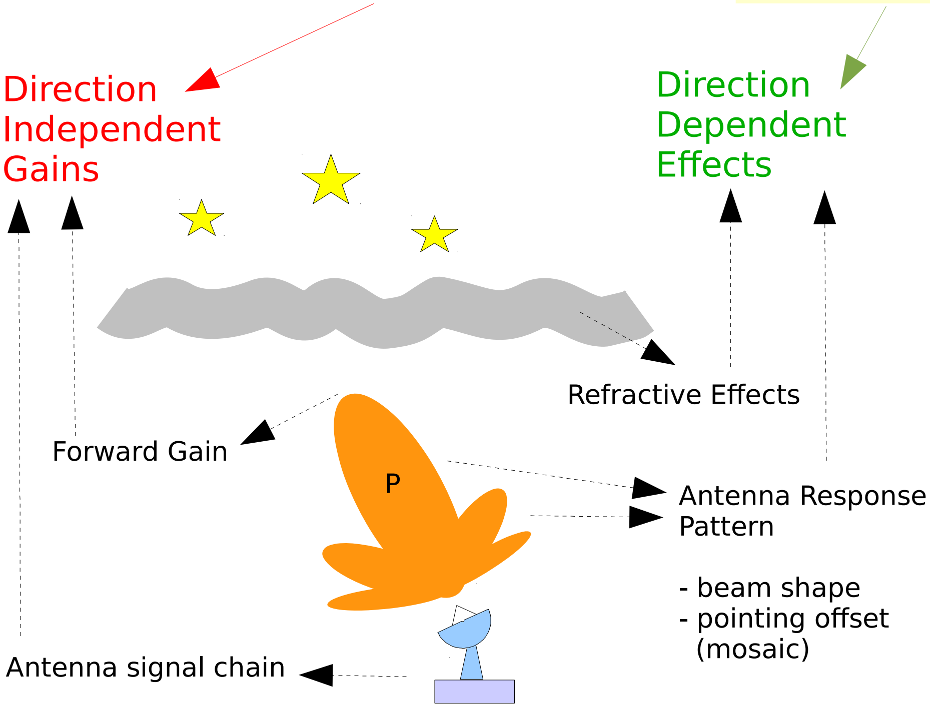
$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$



[Ref : Antennas : R.Selina
Calibration : J.Marvil]

Wide-Field Imaging : Direction dependent effects

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$



Direction
Independent
Gains

Direction
Dependent
Effects

Refractive Effects

Forward Gain

Antenna Response
Pattern

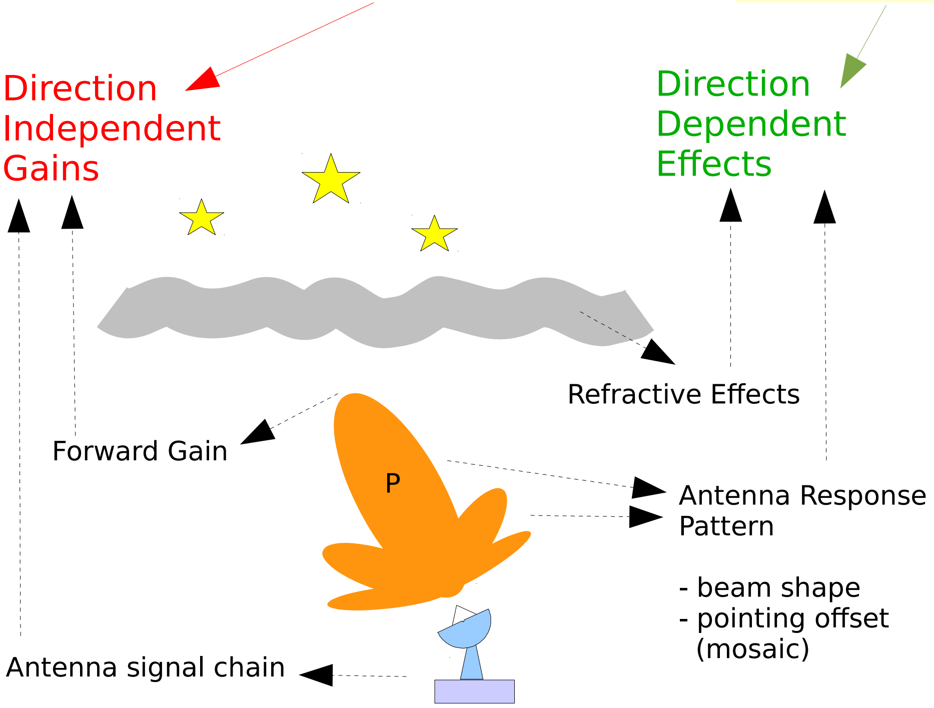
- beam shape
- pointing offset
  (mosaic)

Antenna signal chain

[Ref : Antennas : R.Selina
Calibration : J.Marvil]

Wide-Field Imaging : Calibration + Imaging

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$



Direction Independent Gains

Direction Dependent Effects

Refractive Effects

Forward Gain

P

Antenna Response Pattern

- beam shape
- pointing offset (mosaic)

Antenna signal chain

On-axis gains : **Calibration**

Off-axis effects : Use primary-beam or aperture models in **Imaging**

What models ?

– Theoretical, or measured+fitted
(ray-traced, holography, ionosphere maps)

(or)

– Solve for DD gains or PB model params from the data themselves
(direction-dependent calibration)

[Ref : Antennas : R.Selina
Calibration : J.Marvil]

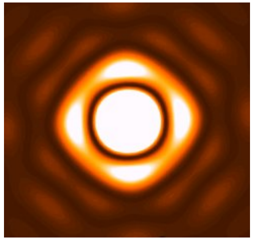
Primary Beams

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dl\,dm\,dn$$

Direction
Dependent
Effects

$$M_{ij}=P_{ij}$$

Primary Beam
for baseline ij



Primary Beams

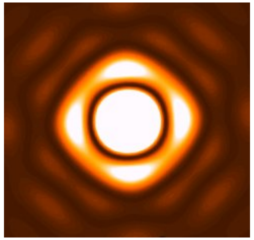
$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction
Dependent
Effects

$$M_{ij} = P_{ij}$$

The Sky is multiplied by a Primary Beam,
before being sampled by each baseline

Primary Beam
for baseline ij



$$I^{obs}(l,m) = \sum_{ij,t,\nu} I_{ij}^{PSF}(l,m,t,\nu) \; * \; \left[P_{ij}(l,m,t,\nu) \cdot I^{sky}(l,m) \right]$$

=> No longer a simple convolution equation

Image-domain Primary Beam Correction (pbcor)

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Dependent
Effects

Assume identical and invariant primary beams.

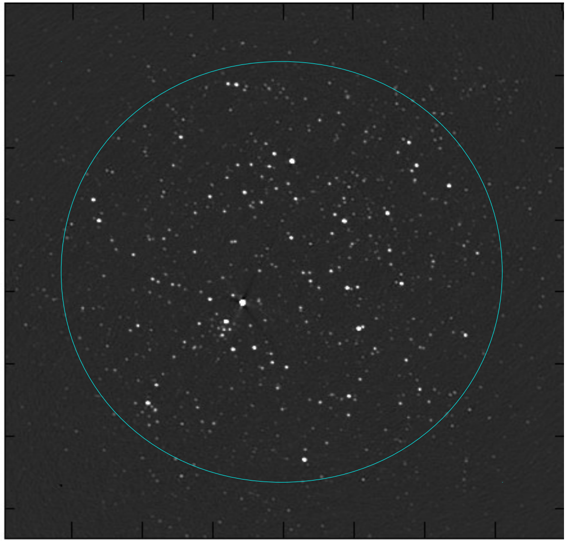
$$I^{obs}=I^{psf}*\left[P\,.\,I^{sky}\right]$$

Divide out an average primary beam model after deconvolution

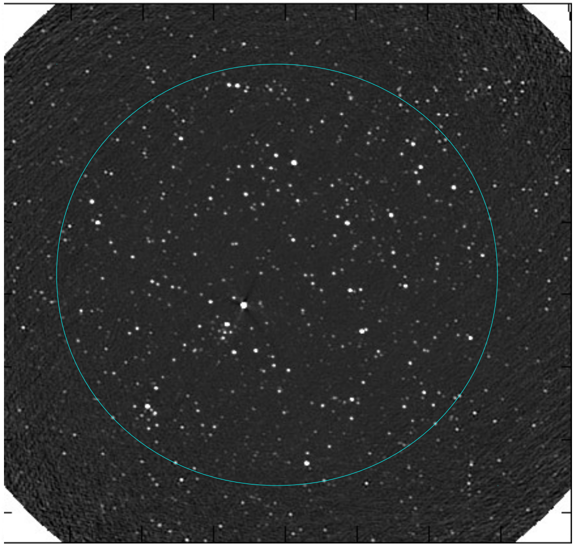
This is approximate

=> Dynamic range limits...

Output Image = PB x Sky



PB-corrected Image : Sky



A-Projection : Apply correction in UV-domain

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

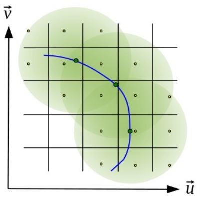
Direction
Dependent
Effects

When primary beams change across baseline, time, freq….

$$I_{ij}^{obs} = I_{ij}^{psf} * \left[P_{ij} . I^{sky}\right] \quad \longleftrightarrow \quad V_{ij}^{obs} = S_{ij} . \left[A_{ij} * V^{sky}\right]$$

For each visibility, apply $A_{ij}^{-1} \approx \dfrac{A_{ij}^{T}}{A_{ij}^{T} * A_{ij}}$

Use A_{ij}^{T} as the convolution function during gridding



A-Projection : Apply correction in UV-domain

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

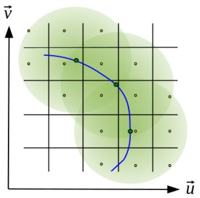
Direction
Dependent
Effects

When primary beams change across baseline, time, freq....

$$I_{ij}^{obs} = I_{ij}^{psf} * [P_{ij} . I^{sky}] \longleftrightarrow V_{ij}^{obs} = S_{ij} . [A_{ij} * V^{sky}]$$

For each visibility, apply $A_{ij}^{-1} \approx \frac{A_{ij}^{T}}{A_{ij}^{T} * A_{ij}}$

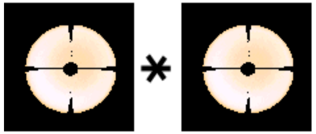
Use A_{ij}^{T} as the convolution function during gridding



Aperture Illumination for antennas i and j :

$$P_{ij} = E_i . E_j^{*} = FT[A_i * A_j^{*}] = FT[A_{ij}]$$

$A_{ij} =$



Ray-traced Model
(or) Zernicke fits to
holography data

A-Projection : Apply correction in UV-domain

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

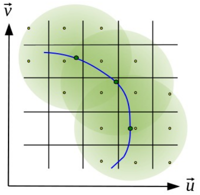
Direction
Dependent
Effects

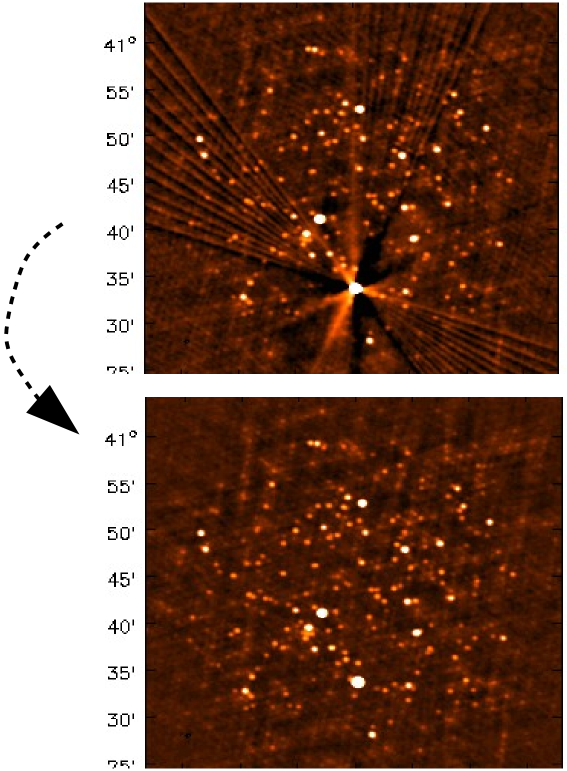
When primary beams change across baseline, time, freq....

$$I_{ij}^{obs} = I_{ij}^{psf} * \left[P_{ij} . I^{sky}\right] \quad \longleftrightarrow \quad V_{ij}^{obs} = S_{ij} . \left[A_{ij} * V^{sky}\right]$$

For each visibility, apply $A_{ij}^{-1} \approx \dfrac{A_{ij}^{T}}{A_{ij}^{T} * A_{ij}}$

Use A_{ij}^{T} as the convolution function during **gridding**



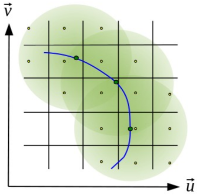


Mosaicking via A-Projection

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\,S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

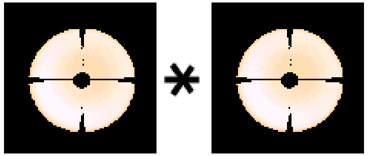
Direction
Dependent
Effects

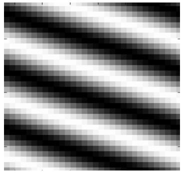
A-Projection : Use aperture illumination functions to construct gridding convolution functions.



Use A_{ij}^{T} as the convolution function

Add a phase gradient





FT => Primary Beam

FT => Shift the Primary Beam

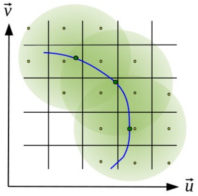
*[Ref : Mosaic : B.Mason *

Mosaicking via A-Projection

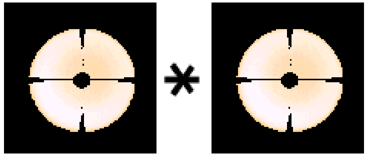
$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Dependent
Effects

A-Projection : Use aperture illumination functions to construct gridding convolution functions.

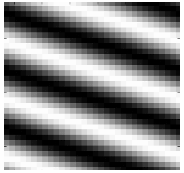


Use A_{ij}^{T} as the convolution function



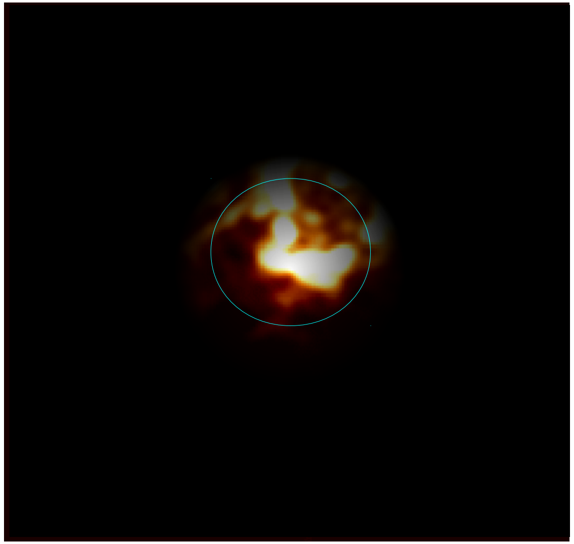
FT => Primary Beam

Add a phase gradient



FT => Shift the Primary Beam

Each pointing is gridded with a *different* phase gradient => shift the PB



*[Ref : Mosaic : B.Mason *

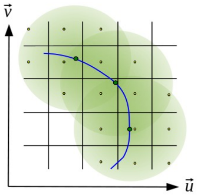
Mosaicking via A-Projection

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Dependent
Effects

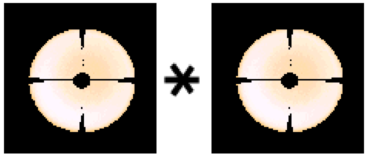
Each pointing is gridded with a *different* phase gradient => shift the PB

A-Projection : Use aperture illumination functions to construct gridding convolution functions.

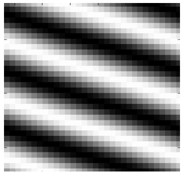


Use A_{ij}^{T} as the convolution function

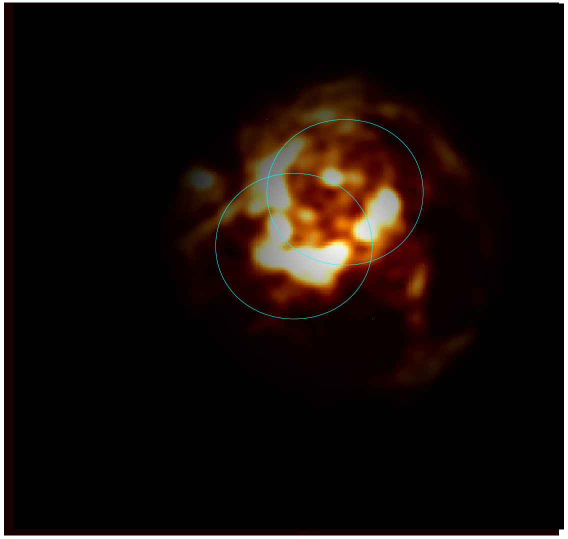
Add a phase gradient



FT => Primary Beam



FT => Shift the Primary Beam



[Ref : Mosaic : B.Mason \

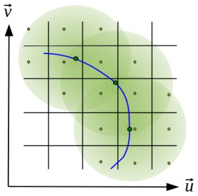
Mosaicking via A-Projection

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction
Dependent
Effects

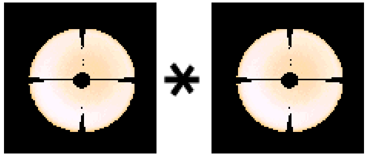
Each pointing is gridded with a *different* phase gradient => shift the PB

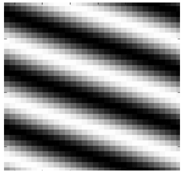
A-Projection : Use aperture illumination functions to construct gridding convolution functions.

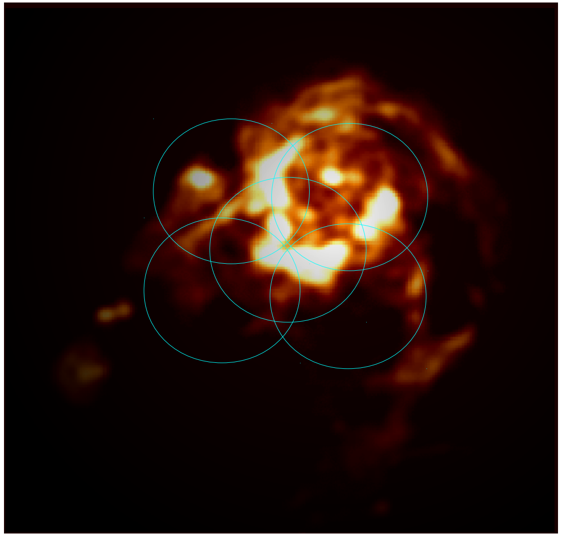


Use A_{ij}^{T} as the convolution function

Add a phase gradient







FT => Primary Beam

FT => Shift the Primary Beam

[Ref : Mosaic : B.Mason \

Polarization : Full-Mueller A-Projection

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

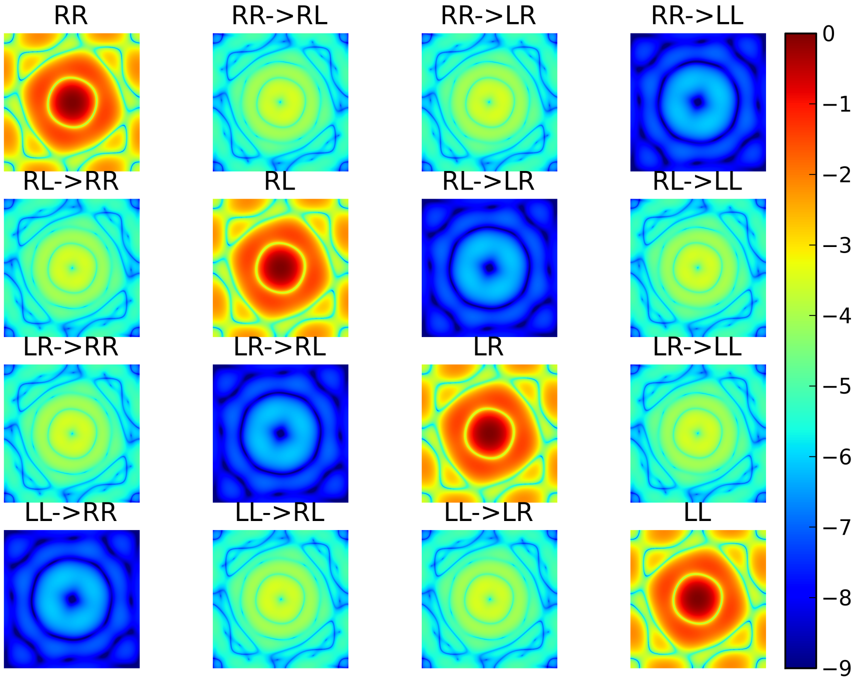
Direction
Dependent
Effects

Full polarization primary beams

- Needed for IQUV imaging over the full PB
- Account for direction-dependent Polarization leakage.

Full-Mueller A-Projection applies the conjugate transpose of the full matrix

=> A_{ij}^{T} is a 4x4 matrix application



Images from P.Jagannathan

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

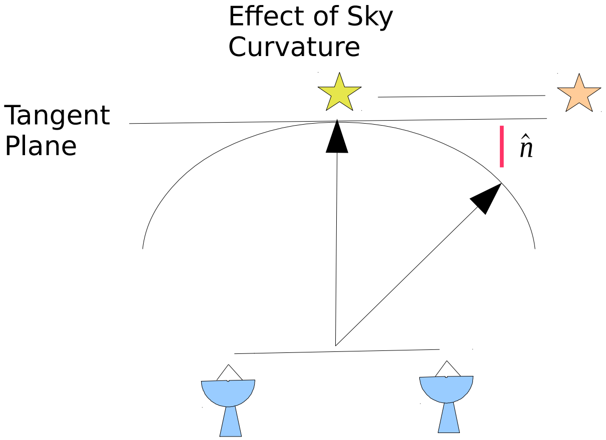
W-Term

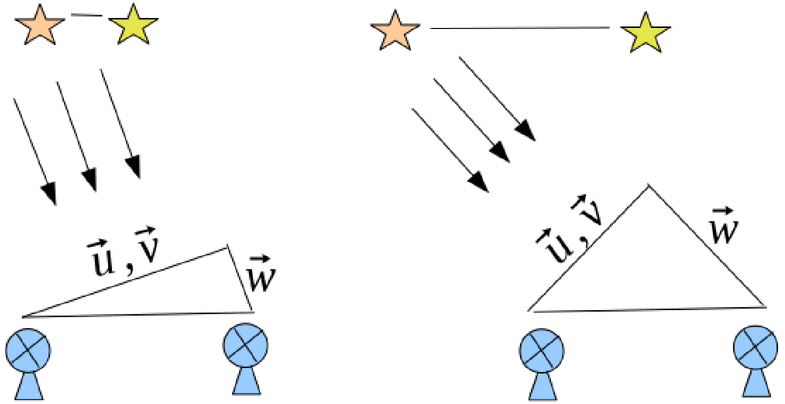
W-Term effect...

Wide-Field Imaging : Non-coplanar baselines / W-term

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

W-Term





A known geometric effect

Algorithms

3D Imaging,
W-stacking,
Faceting,
W-Projection

For a field-of-view given by the Primary Beam of an antenna of diameter D, at wavelength λ and with a maximum baseline length of B....

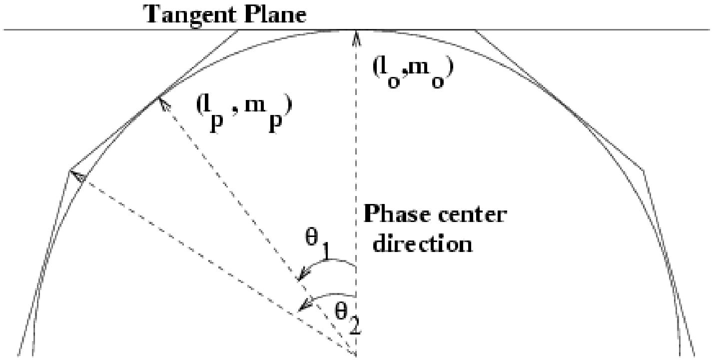
The w-term becomes relevant if $\frac{\lambda B}{D^2} > 1$

See R.Perley's talks

W-term : Faceting

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

W-Term



- Approximate the celestial sphere by a set of tangent planes (facets) such that 2D geometry is valid per facet

- Image each facet with its own phase reference center and re-project to the tangent plane

Algorithm Variants:

Deconvolve facets separately before re-projecting and stitching

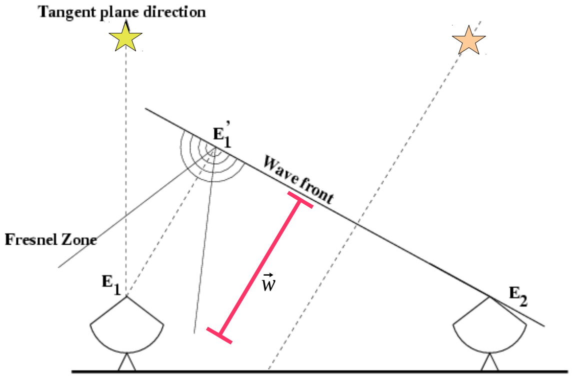
(or)

Image all facets onto the same tangent plane grid and perform a joint deconvolution.

W-term : W-Projection

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}dl\,dm\,dn$$

W-Term



For ideal 2D imaging we need to measure E_1
Instead, we measure E_1'

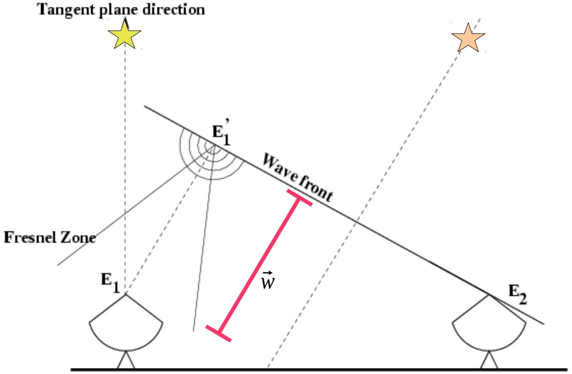
E_1' and E_1 are related by a Fresnel diffraction/propagation kernel.

$$G(u,v,w)=FT\left[e^{2\pi\iota w\sqrt{1-l^2-m^2}}\right]$$

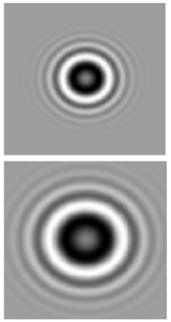
W-term : W-Projection

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

W-Term



W-term appears as a convolution in the UV-domain



$$V^{o}(u,v,w)=V(u,v,w=0)*G(u,v,w)$$

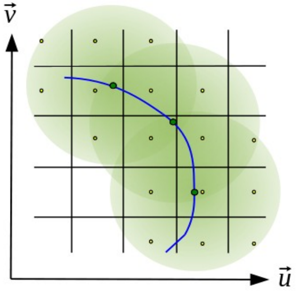
For ideal 2D imaging we need to measure E_1
Instead, we measure E_1'

E_1' and E_1 are related by a Fresnel diffraction/propagation kernel.

$$G(u,v,w)=FT\left[e^{2\pi i w\sqrt{1-l^2-m^2}}\right]$$

=> Correct it by another convolution with the inverse/conjugate kernel (during the gridding step)

=> Use different kernels for different W values (appropriately quantized)

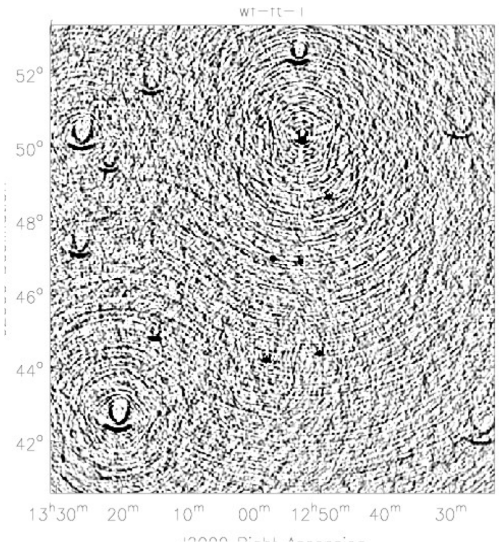


W-Term : Algorithm Comparison

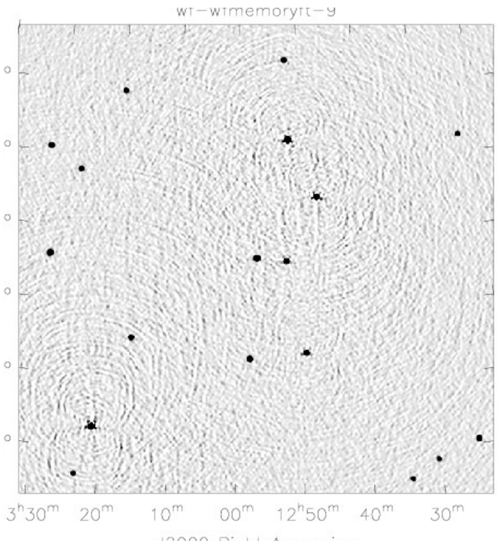
$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)I(l,m,\nu,t)e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

W-Term

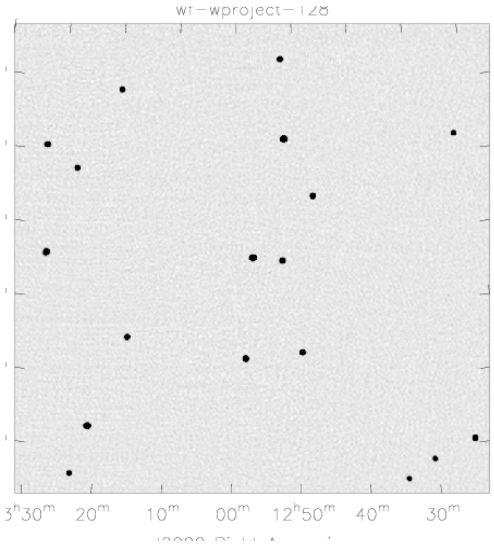
2D Imaging



Facet Imaging



W-Projection

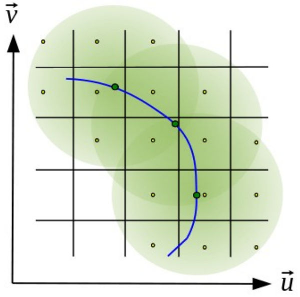


Correcting all Direction Dependent effects together……

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction Dependent Effects

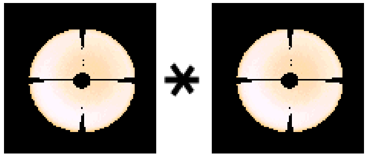
W-Term



A-W-Projection : Correct multiple Direction Dependent effects together, during gridding

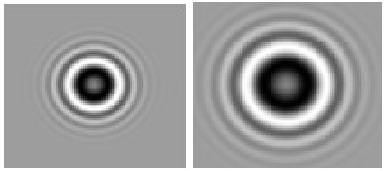
The gridding convolution function (per visibility) is a combination of the following……

Aperture illumination functions (ij, time, freq, pol)



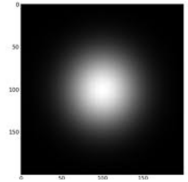
Primary Beams

FT of Fresnel kernels



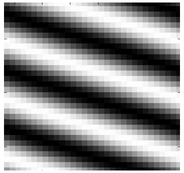
W-term effects

Prolate Spheroidal



Anti-Aliasing Filter

Phase gradient



Mosaic

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=\underbrace{M_{ij}(\nu,t)}_{\text{Direction Independent Gains}}\underbrace{S_{ij}(\nu,t)}_{\text{UV sampling function}}\iiint\underbrace{M_{ij}^{s}(l,m,\nu,t)}_{\text{Direction Dependent Effects}}I(l,m,\underbrace{\nu,t}_{\text{Sky-brightness varies with frequency and time}})e^{2\pi i(ul+vm+\underbrace{w(n-1)}_{\text{W-Term}})}\,dl\,dm\,dn$$

Wide-Band Effects.....

Wide-Band Imaging

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

UV sampling function

Primary Beam varies with frequency

Sky-brightness varies with frequency

Frequency Range :	(1 – 2 GHz)	(4 – 8 GHz)	(8 – 12 GHz)
Bandwidth : $\nu_{max}-\nu_{min}$	1 GHz	4 GHz	4 GHz
Bandwidth Ratio : $\nu_{max}:\nu_{min}$	2 : 1	2 : 1	1.5 : 1
Fractional Bandwidth : $(\nu_{max}-\nu_{min})/\nu_{mid}$	66%	66%	40%

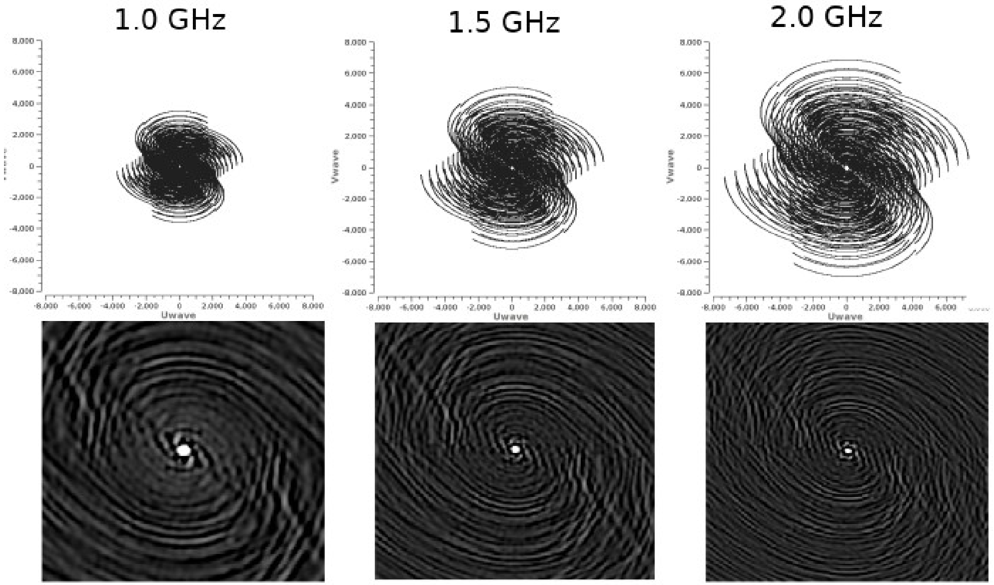
\+ Imaging sensitivity improves with increased bandwidth

– Frequency dependent effects (sky and instrument) are stronger with increased bandwidth

Multi-Frequency Synthesis

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

UV sampling function

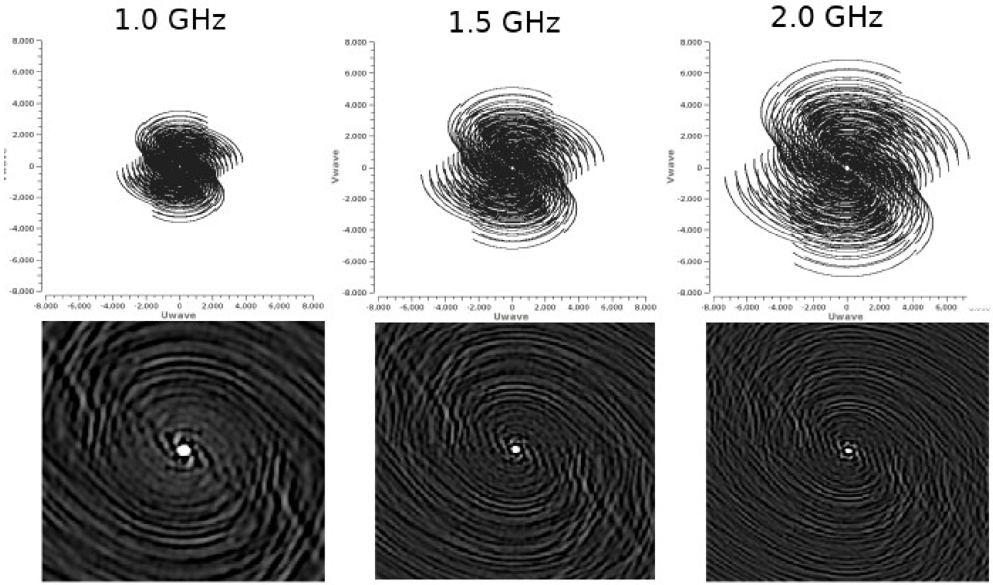


Observed image : $I_{\nu}^{obs}=I_{\nu}^{sky}*PSF_{\nu}$

Multi-Frequency Synthesis

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i (ul+vm+w(n-1))}\, dl\, dm\, dn$$

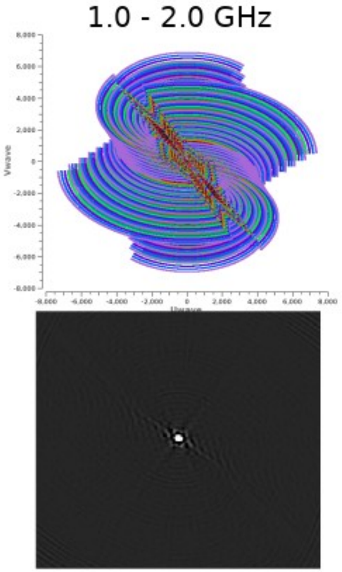
UV sampling function



Multi-Frequency Synthesis

Combine data from multiple channels

- Improve PSF
- Improve SNR



$$I_{wb}^{obs} = \sum_{\nu} \left[I_{\nu}^{sky} * PSF_{\nu} \right]$$

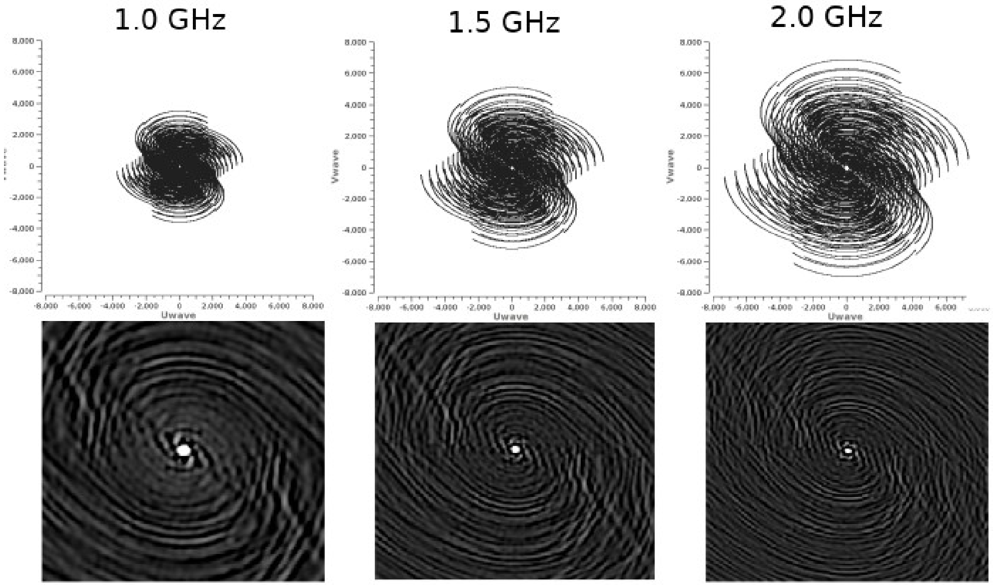
Observed image : $I_{\nu}^{obs} = I_{\nu}^{sky} * PSF_{\nu}$

Multi-Frequency Synthesis

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

UV sampling function

Sky-brightness varies with frequency

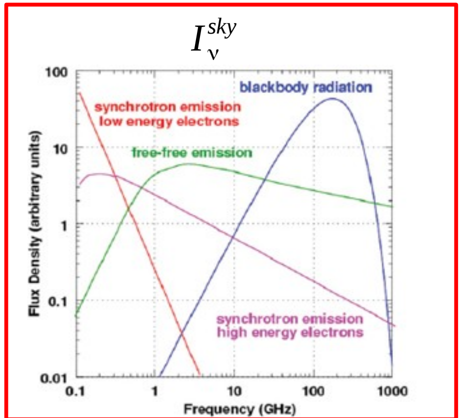


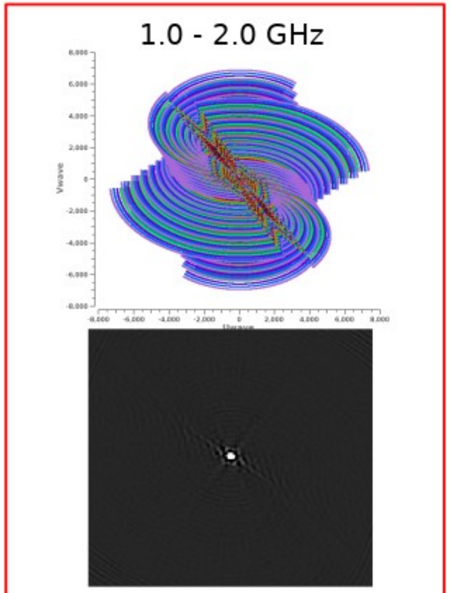
Observed image : $I_{\nu}^{obs}=I_{\nu}^{sky}*PSF_{\nu}$

Multi-Frequency Synthesis

Combine data from multiple channels

- Improve PSF
- Improve SNR





$$I_{wb}^{obs}=\sum_{\nu}\left[I_{\nu}^{sky}*PSF_{\nu}\right]$$

Wide-band Deconvolution

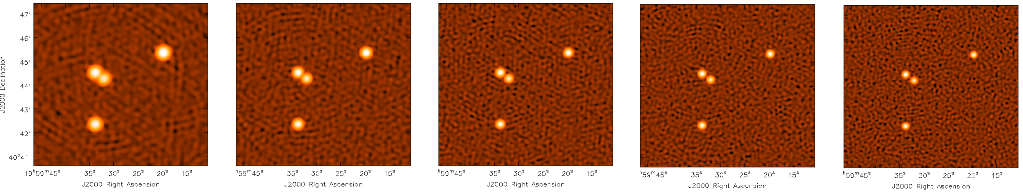
$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

UV sampling function

Sky-brightness varies with frequency

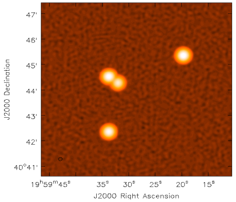
Cube Imaging :

(1) Reconstruct each chan/spw separately
(2) Smooth to the lowest available resolution
(3) Combine to calculate continuum and spectra



Continuum image

(sum of smoothed channel maps)



Wide-band Deconvolution

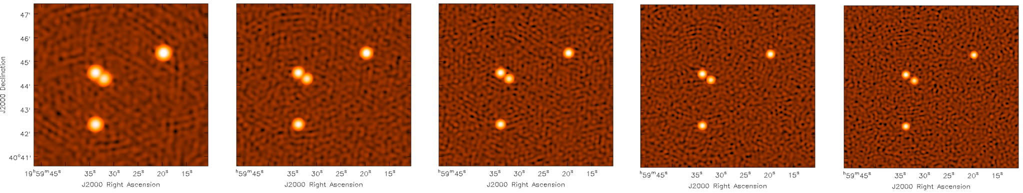
$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

UV sampling function

Sky-brightness varies with frequency

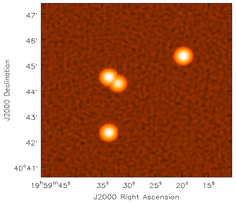
Cube Imaging :

(1) Reconstruct each chan/spw separately
(2) Smooth to the lowest available resolution
(3) Combine to calculate continuum and spectra



Continuum image

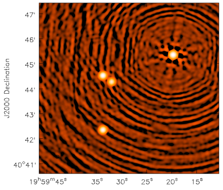
(sum of smoothed channel maps)



Multi-Frequency-Synthesis (MFS) :

Combine data from all frequencies and do a joint reconstruction

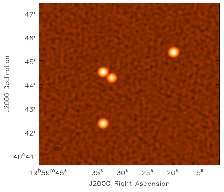
Flat-spectrum assumption



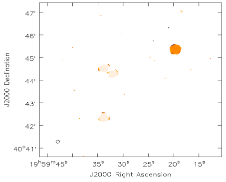
Intensity

Model intensity and spectrum together :

$$I_\nu^{sky} = \sum_t I_t^m \left(\frac{\nu-\nu_0}{\nu_0}\right)^t$$



Intensity



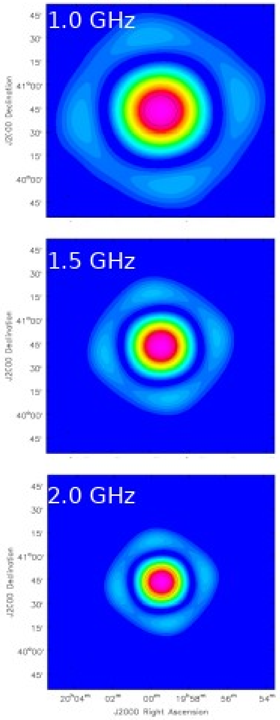
Spectral Index

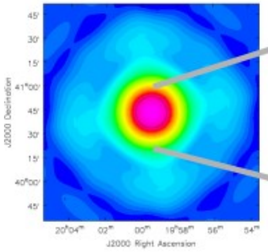
Wide Band + Wide Field : Frequency-dependent Primary Beams

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i\left(ul+vm+w(n-1)\right)}\,dl\,dm\,dn$$

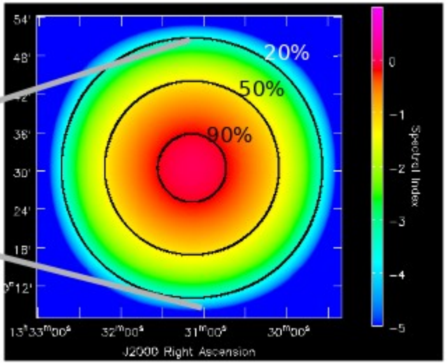
Primary Beam varies
with frequency

VLA PBs





Average PBs



Spectral Index of PB

(About -1.4 at the HPBW)

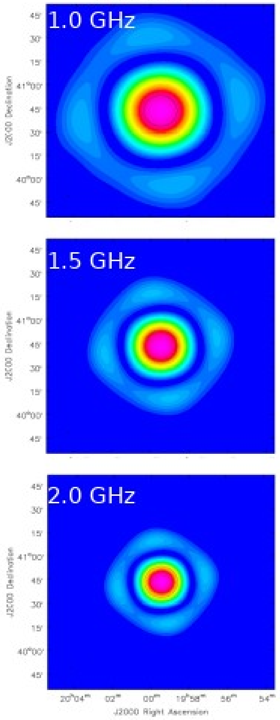
$$I^{obs}_{wf,wb}=\sum_{\nu}\left[\left(P_{\nu}\cdot I^{sky}_{\nu}\right)*PSF_{\nu}\right]$$

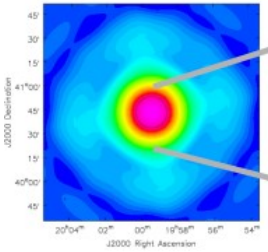
Wide Band + Wide Field : Frequency-dependent Primary Beams

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\,S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

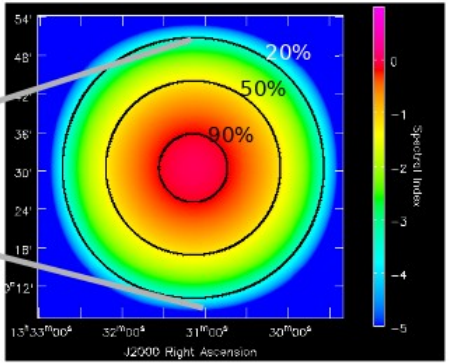
Primary Beam varies
with frequency

VLA PBs





Average PBs



Spectral Index of PB

(About -1.4 at the HPBW)

$$I_{wf,wb}^{obs} = \sum_{\nu} \left[\left(P_\nu \cdot I_\nu^{sky} \right) * PSF_\nu \right]$$

Cube Imaging

-- Sky model represents $I(\nu)P(\nu)$
-- Divide the output image at each frequency by $P(\nu)$

Multi-Term MFS + Wideband-PBcor

-- Taylor coefficients represent $I(\nu)P(\nu)$

-- Polynomial division
by PB Taylor coefficients $\frac{(I_0^m, I_1^m I_2^m, \ldots)}{(P_0, P_1, P_2, \ldots)} = (I_0^{sky}, I_1^{sky} I_2^{sky} \ldots)$

Wideband A-Projection

-- Remove $P(\nu)$ during gridding (before model fitting)

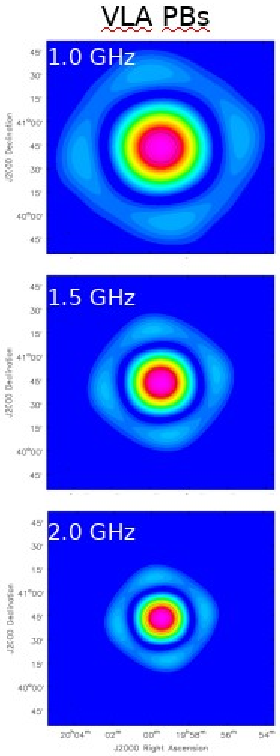
$$A_\nu^{-1} \approx \frac{A_{\nu_c}^T}{A_{\nu_c}^T * A_\nu} \quad \text{where} \quad P_\nu . P_{\nu_c} \approx P_{\nu_{mid}}^2$$

-- Output spectral index image represents only the sky

Wide Band Mosaics

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

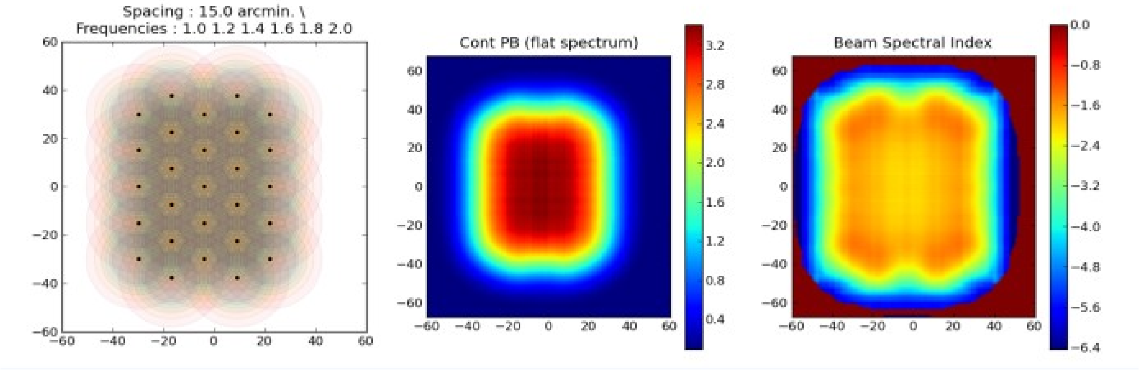
Primary Beam varies
with frequency


VLA PBs

1.0 GHz

1.5 GHz

2.0 GHz


Spacing : 15.0 arcmin. \
Frequencies : 1.0 1.2 1.4 1.6 1.8 2.0

Cont PB (flat spectrum)

Beam Spectral Index

The mosaic primary beam has an artificial spectral index all over the F-O-V

=> There is no part of the mosaic that is free from wideband primary-beam effects.

Algorithms :

Combinations of.....

- Stitched vs Joint Mosaics
- Cube vs MFS
- PBCor vs A-Projection

Different numerical accuracies and compute costs.

Depends on the sky.

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t) = M_{ij}(\nu,t)\, S_{ij}(\nu,t) \iiint M_{ij}^{s}(l,m,\nu,t)\, I(l,m,\nu,t)\, e^{2\pi i(ul+vm+w(n-1))}\, dl\, dm\, dn$$

Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

Sky-brightness varies
with frequency and time

W-Term

Put it all together…..

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

Sky-brightness varies
with frequency and time

W-Term

Calibration

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

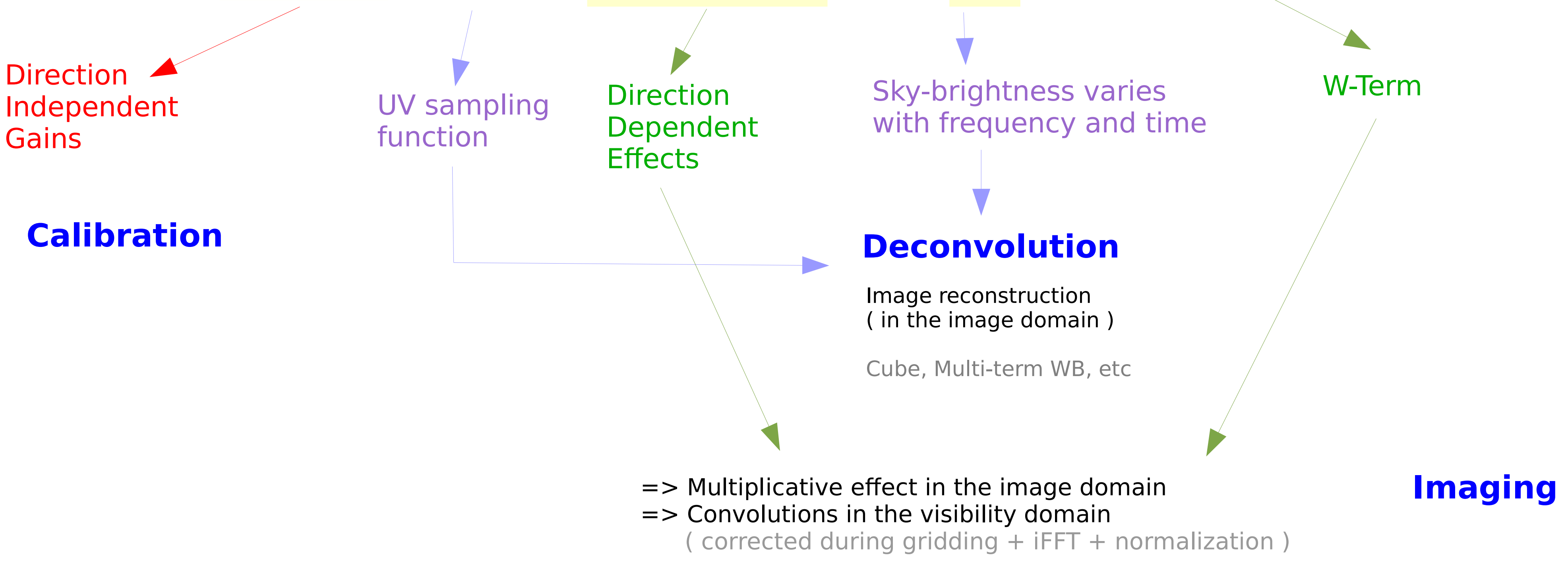
Calibration

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Imaging

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$



Direction
Independent
Gains

UV sampling
function

Direction
Dependent
Effects

Sky-brightness varies
with frequency and time

W-Term

Calibration

Deconvolution

Image reconstruction
(in the image domain)

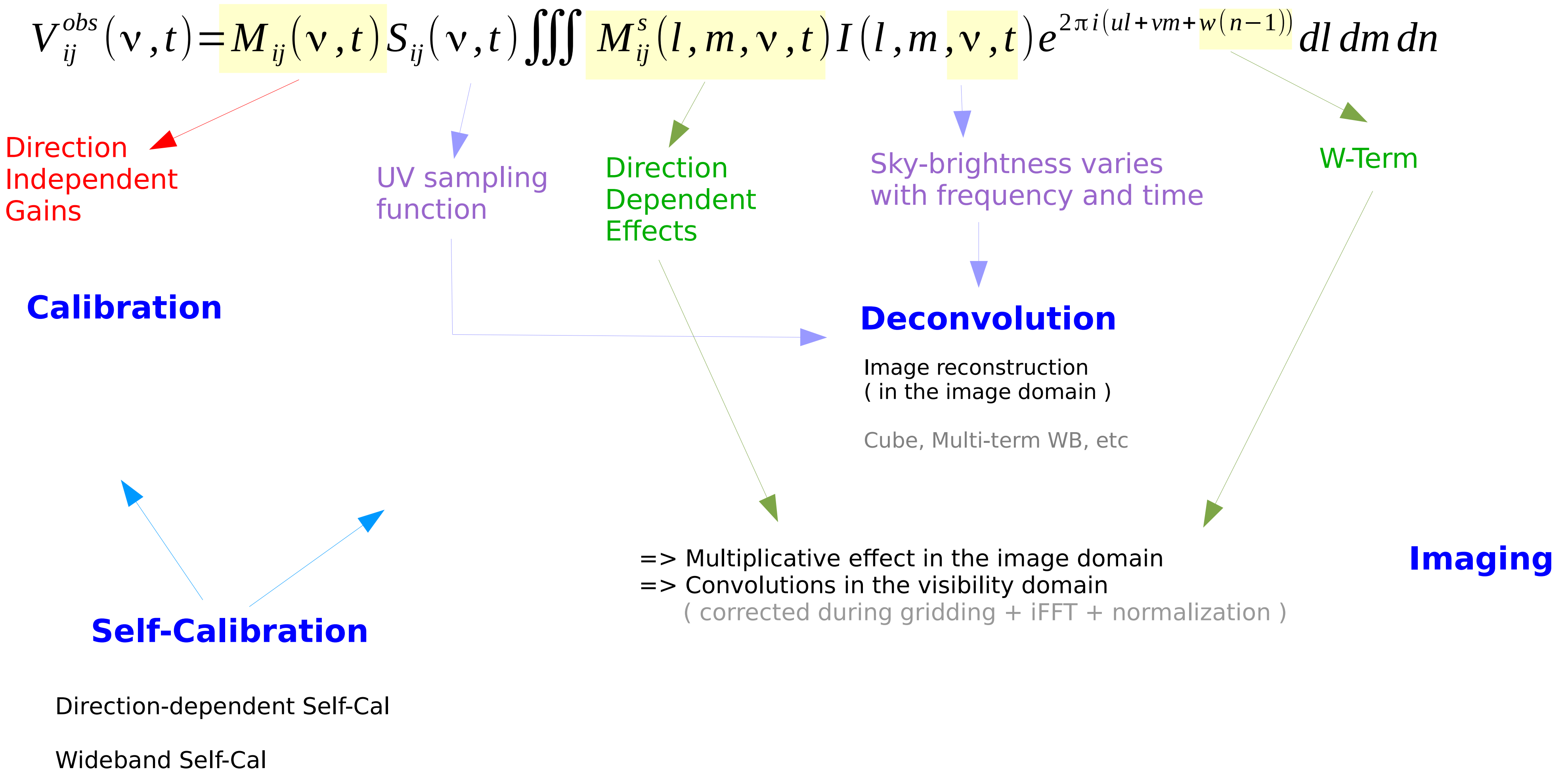
Cube, Multi-term WB, etc

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Imaging

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$



Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Calibration

Deconvolution

Image reconstruction
(in the image domain)

Cube, Multi-term WB, etc

=> Multiplicative effect in the image domain
=> Convolutions in the visibility domain
(corrected during gridding + iFFT + normalization)

Imaging

Self-Calibration

Direction-dependent Self-Cal

Wideband Self-Cal

Direction-dependent Self-Calibration

$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)\,S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

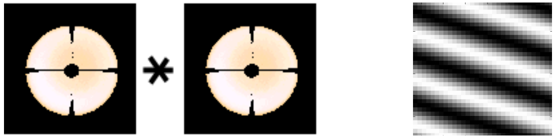
Sky-brightness varies with frequency and time

W-Term

Model the sky and instrument separately

Parameterize the aperture illumination function

- Solve for pointing offsets
- Solve for ionospheric propagation effects



Algorithms : Pointing Self-Cal (with A-Proj)

Model the sky and instrument together

Joint solutions for gains in multiple directions

- Does not use a known PB model
- Difficult to get accurate source flux
- Good approach for source-subtraction

Algorithms : DD-facets, etc....

Radio Interferometry – Measurement Equations

$$V_{ij}^{obs}(\nu,t)=\underbrace{M_{ij}(\nu,t)}_{\text{Direction Independent Gains}}\underbrace{S_{ij}(\nu,t)}_{\text{UV sampling function}}\iiint \underbrace{M_{ij}^{s}(l,m,\nu,t)}_{\text{Direction Dependent Effects}}I(l,m,\underbrace{\nu,t}_{\text{Sky-brightness varies with frequency and time}})e^{2\pi i(ul+vm+\overbrace{w(n-1)}^{\text{W-Term}})}\,dl\,dm\,dn$$

How are these algorithms realized in software...... ?

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

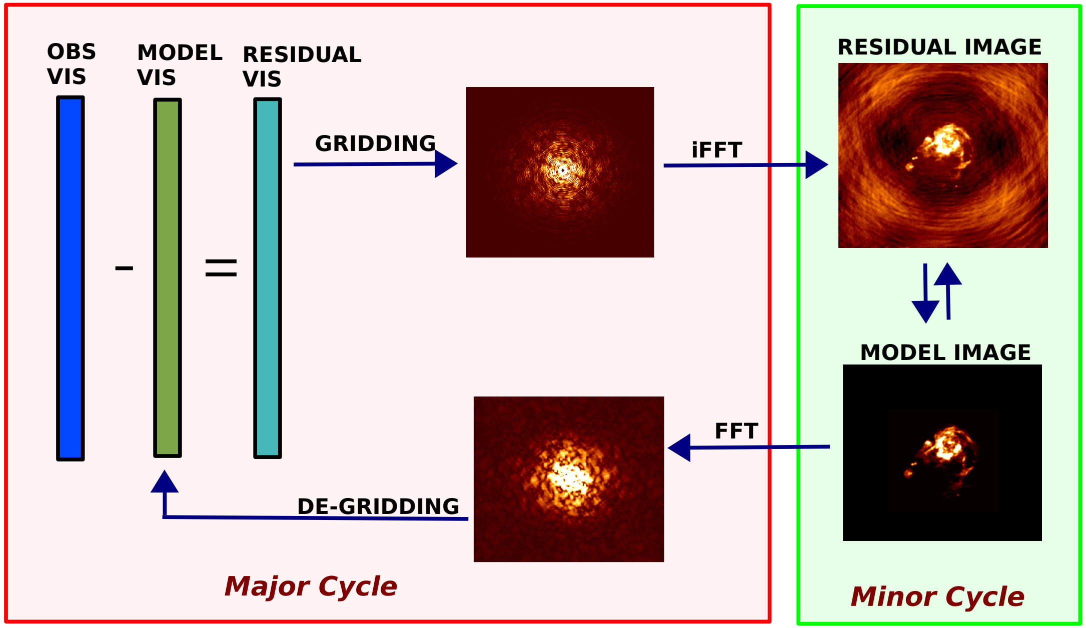
The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Forward and Reverse transforms

Calc $\frac{\delta \chi^2}{\delta I^m}$



OBS VIS - **MODEL VIS** = **RESIDUAL VIS**

GRIDDING

iFFT

RESIDUAL IMAGE

MODEL IMAGE

FFT

DE-GRIDDING

Major Cycle

Minor Cycle

Image Reconstruction

Deconvolution Algorithms

Iterative Image Reconstruction

The generalized forward problem $V^{obs} = [A] I^m + n$

The generalized inverse problem $I^m = [A]^{-1} V^{obs}$

L2 data regularization

+ Sky model (multiscale, wideband, timevar)
+ Solver/Optimizer with constraints/biases

Forward and Reverse transforms

Calc $\dfrac{\delta \chi^2}{\delta I^m}$

Gridding Options

- Primary-Beams
  - Wideband
  - Full Pol

- W-Term
- Mosaic

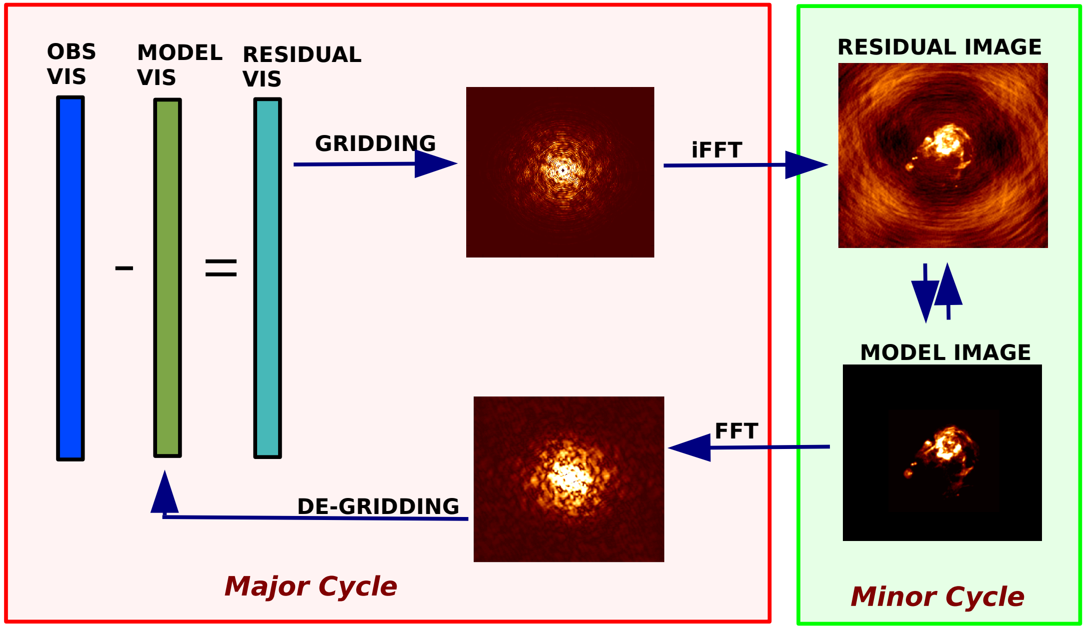


Image Reconstruction

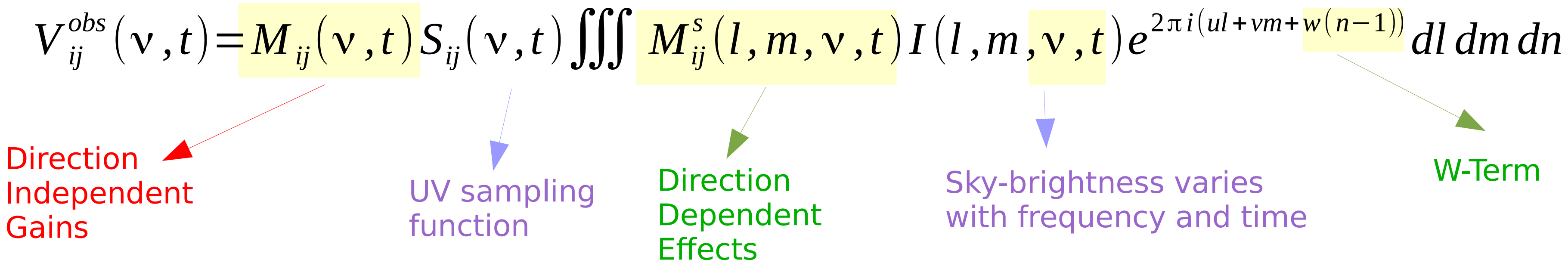
Deconvolution Algorithms

Sky models
- Delta function
- Gaussians
- Wideband

Algorithms
- Clean (greedy)
- Many other compressed sensing ideas

Compute Cost : Data volume, Image size, N_channels
Size of gridding convolution function
Deconvolution algorithm
Sky brightness (intensity and structure)

Some References….



$$V_{ij}^{obs}(\nu,t)=M_{ij}(\nu,t)S_{ij}(\nu,t)\iiint M_{ij}^{s}(l,m,\nu,t)\,I(l,m,\nu,t)\,e^{2\pi i(ul+vm+w(n-1))}\,dl\,dm\,dn$$

Direction Independent Gains

UV sampling function

Direction Dependent Effects

Sky-brightness varies with frequency and time

W-Term

Primary Beam (A-Proj) : Bhatnagar, S., Cornwell, T. J., Golap, K., and Uson, J. M., “Correcting direction-dependent gains in the deconvolution of radio interferometric images,” Astron. & Astrophys. 487, 419–429 (Aug. 2008).

Full-pol A-Projection : Tasse, C., van der Tol, S., van Zwieten, J., van Diepen, G., and Bhatnagar, S., “Applying full polarization A-Projection to very wide field of view instruments: An imager for LOFAR,” AAP 553, A105 (May 2013).

Full-pol PB models : Jagannathan, P., Bhatnagar, S., Brisken, W., and Taylor, A. R., “Direction-dependent corrections in polarimetric radio imaging. ii. a-solver methodology: A low-order solver for the a-term of the a-projection algorithm,” The Astronomical Journal 155(1), 3 (2018).

Wideband Sky Model : Rau, U. and Cornwell, T. J., “A multi-scale multi-frequency deconvolution algorithm for synthesis imaging in radio interferometry,” AAP 532, A71 (Aug. 2011).

W-Term : Cornwell, T. J., Golap, K., and Bhatnagar, S., “The non-coplanar baselines effect in radio interferometry: The w-projection algorithm,” IEEE Journal of Selected Topics in Sig. Proc. 2, 647–657 (Oct 2008).

Mosaicing : Sault, R. J., Staveley-Smith, L., and Brouw, W. N., “An approach to interferometric mosaicing.,” AAPS 120,375–384 (Dec. 1996).

Pointing Self-Cal : Bhatnagar, S. and Cornwell, T. J., “The pointing self-calibration algorithm for aperture synthesis radio telescopes,” The Astronomical Journal 154(5), 197 (2017).

DD-cal + wideband : Tasse, C., Hugo, B., Mirmont, M., Smirnov, O., Atemkeng, M., Bester, L., Hardcastle, M. J., Lakhoo,R., Perkins, S., and Shimwell, T., “Faceting for direction-dependent spectral deconvolution,” AAP 611, A87 (Apr. 2018).