Cross correlators for radio astronomy

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What is a correlator?

- Incident radiation
- Delay $\tau(t)$
- Wave $\rightarrow$ voltage
- Downconversion
- Sampling, Quantization
- $\tau_1$
- $\tau_2$
- Visibilities

Compensate for this… before/while splitting by frequency and multiplying.
Why correlators matter to YOU
Correlators and interferometry
Correlators and Interferometry
Correlators and Interferometry

Sky brightness at frequency $v_0$

Visibilities (real component shown, unit is $\lambda_0 = c / v_0$)
Monochromatic == problematic

\[ V(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(l, m) I(l, m) e^{-2\pi i (ul + vm)} \, dl \, dm. \]

- \( u \times l + v \times m \) is supposed to be constant, but both \( u \) and \( v \) depend on frequency.
- No truly monochromatic radiation!
- Fortunately, “fairly narrow” band of \( \Delta \nu \) (\textit{quasi-monochromatic}) can suffice:
  - Real world viewpoint: different frequency components stay “in phase” as wavefront propagates from one antenna to the next.
Monochromatic == problematic

\[ l \quad s_0 \]

\[ u \]
Monochromatic == problematic

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- \( u \times l + v \times m \) is supposed be constant, but both \( u \) and \( v \) depend on frequency
- No truly monochromatic radiation!
- Fortunately, “fairly narrow” band of \( \Delta \nu \) (quasi-monochromatic) can suffice:
  - if \( \Delta u \times l \ll 1 \) and \( \Delta v \times m \ll 1 \) then the different frequency components stay in phase and we’re ok
  - Correlator needs to slice at least this finely
Correlators and Interferometry

Sky brightness at frequency $\nu_0$

Visibilities (real component shown, unit is $\lambda_0 = c / \nu_0$)
Correlators and Interferometry

Sky brightness at frequency $\nu' = \nu_0 + \delta\nu$

Visibilities (real component shown, unit is $\lambda' = c / \nu'$)
A “dumb” correlator

- Use many analog filters to make many narrow channels; correlate each one separately with a standard complex correlator:
A “dumb” correlator

- Use many analog filters to make many narrow channels; correlate each one separately with a standard complex correlator:

\[ \frac{1}{T} \int_0^T (\cdot) \, dt \]

\[ X_R(\tau_0) \]

\[ X_I(\tau_0) \]
The output

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The output

B' metres
Making it feasible

- Analog filters are costly & finnicky; this would be expensive and temperamental
Making it feasible

- Analog filters are costly & finnicky; this would be expensive and temperamental
- Fortunately, we can (and do) digitize the signal – meaning we can use a digital substitute: **digital filterbank**
The “FX” correlator
The “FX” correlator

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The “FX” correlator
The “FX” correlator

- Since this architecture consists of a Fourier transform (F) followed by cross-multiplication (X), we dub this the “FX” correlator.
Righting the wrongs

Wave -> voltage
Downconversion
Sampling, Quantization
\( \tau_1 \)

Wave -> voltage
Downconversion
Sampling, Quantization
\( \tau_2 \)

Visibilities

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Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every \( \Delta t \) sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies \( \geq 1 / (2 \Delta t) \) Hz (\textit{band-limited})

Adequately sampled

Undersampled, cannot be reconstructed
Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every $\Delta t$ sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies $\geq 1 / (2 \Delta t)$ Hz ("band-limited")

Spectral power

```
Frequency
-3/(2\Delta t) -1/(\Delta t) -1/(2\Delta t) 0 1/(2\Delta t) 1/\Delta t 3/(2\Delta t)
Spectral power
```

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Sampling

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Frequency

Spectral power

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Quantization

• When correlation is low (almost always) even very coarse quantization is ok!
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![Graph showing voltage vs time sample with different lines for combined signal + RFI, quantised signal, and quantisation error.](image)

until the headroom runs out...
Quantization

• When correlation is low (almost always) even very coarse quantization is ok!

• Sensitivity loss due to quantisation:
  • 8 bit: 0.1%
  • 4 bit: 1.3%
  • 2 bit: 12%
  • 1 bit: 36%

• Correct visibility amplitudes for this sensitivity loss
Righting the wrongs

Wave -> voltage
Downconversion
Sampling, Quantization

Wave -> voltage
Downconversion
Sampling, Quantization

τ₁
τ₂

Visibilities

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Delay compensation

• Delay to the nearest sample is easy:

\[ \text{level/binary code} \]

\[ \text{sample\# (time)} \]
Delay compensation

• Delay to the nearest sample is easy:

\[ FL1_n = \text{level/binary code} \]
Delay compensation

- In practice, delay all to common reference

\[ \tau_1 \]

\[ \tau_2 \]
Fractional-sample correction

• Sampling prevents perfect alignment of data streams; always a small error
Fractional-sample correction

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Fractional-sample correction

- Sampling prevents perfect alignment of datastreams; always a small error

\[ \phi = -v \times \epsilon \]

Correction applied with a complex multiplication to rotate phase
Righting the wrongs

Wave -> voltage

Downconversion

Sampling, Quantization

τ₁

x

Wave -> voltage

Downconversion

Sampling, Quantization

τ₂

x

Visibilities

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Fringe rotation

Signal at sky frequency ~GHz

Downconversion

Signal at baseband ~0 Hz
Fringe rotation

• Implementation: rotate phase using complex multiplier

• $\Delta \phi = 2\pi \nu_{lo} \tau_g$  \[ \nu_{lo} = \text{local oscillator frequency,} \]
  \[ \tau_g = \text{applied delay} \]

• Update rate of $\Delta \phi$ depends on how fast $\tau_g$ changes:
  – If $\tau_g$ is changing fast, correct every recorded sample individually (before the FFT)
  – For shorter baseline / low frequency instruments, can do post-channelisation or even post-accumulation
Alternate implementation

• We have shown how to build a practical FX correlator, which first Fourier transforms and then multiplies

• Convolution theorem: **Multiplication** in the frequency domain is equivalent to **convolution** in the time domain

• It is mathematically equivalent to convolve the two signals in the time domain and then Fourier transform
An equivalent “XF” correlator
An equivalent “XF” correlator

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An equivalent “XF” correlator

Multiply & accum.

visibility amplitude

sample# (time)
An equivalent “XF” correlator

Multiply & accum.

visibility amplitude

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An equivalent “XF” correlator

Multiply & accum.
An equivalent “XF” correlator
An equivalent “XF” correlator
A realistic XF correlator
A realistic XF correlator

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**XF vs FX**

- Different windowing in time domain gives different spectral response

![Graph showing spectral response comparison between XF and FX]

- 22% sidelobes! Reduce with Hanning smoothing
- 5% sidelobes
- Lag weighting
XF vs FX: which is better?

- Desire for reduced artifacts favours FX
  - Main advantage of XF: can use very efficient low-precision integer multipliers up-front
  - But FX many fewer operations overall, unaffected by trend to higher bit depth
  - FX also: access to frequency domain at short timescale allows neat tricks and higher precision correction of delay effects
  - Modern correlators mostly FX-style, and often have multiple cascaded filter steps (~GHz recorded band chopped into ~100 MGz chunks and correlated separately)
The full package

Frequency conversion

Sampling @ 8 Gsps

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The full package

Wide sampled band

Frequency (GHz)

32-channel digital filterbank

Each of these 128 MHz chunks can then be treated by separate FX style correlator in parallel: fringe rotation, channelization, delay compensation, and cross-multiplication.
Correlator platforms

Voltages \times \text{Something that can multiply and add} \rightarrow \text{Visibilities}
Correlators on CPUs

status = vectorFFT_CtoC_cf32(complexunpacked, fftd, pFFTSpecC, fftbuffer);
if(status != vecNoErr)
    csevere << startl << "Error doing the FFT!!" << endl;

... status = vectorAddProduct_cf32(vis1, vis2, &(scratchspace->threadcrosscorrs[result]])
Correlators on CPUs

• Many positive points:
  – Can implement in “normal” code (e.g., C++); maintainable, many skilled coders
  – Development effort transferrable across generations of hardware
  – Incremental development is trivial
  – Natively good at floating point (good for FX), no cost to do high precision

• One major disadvantage:
  – CPUs not optimised for correlation; big system like ngEHT would take many CPUs.
Correlators on CPUs

The Very Long Baseline Array, 10 stations

The European VLBI Network, ~30 stations

The Long Baseline Array, Australia, ~6 stations

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Correlators on GPUs

Like CPUs, GPUs are mounted on a standard motherboard
Correlators on GPUs

• Advantages:
  – More powerful and more efficient than CPUs
  – Also good at floating point

• Disadvantages:
  – Writing code is more difficult (GPUs are more specialized, less flexible: need to carefully manage data transfers)
  – Fewer expert GPU programmers available
  – Transfer-ability of code across hardware generations harder (capabilities change faster, need new code to use)
Correlators on GPUs

The Low Frequency Array (LOFAR), 76 stations

GMRT, India, 30 stations

Now underway: adding GPU acceleration to “general purpose” software correlators

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Correlators on FPGAs
Correlators on FPGAs

• Advantages:
  – More efficient than CPUs or GPUs, particularly for integer multiplication – big power savings

• Disadvantages:
  – Programming is harder again (especially debugging), yet fewer experts
  – Transfer-ability across hardware generations even more limited
  – Synchronous (clocked) system, less robust to perturbations c.f. CPUs/GPUs
Correlators on FPGAs

“Roach” reconfigurable FPGA board used for correlation

MeerKAT, 64 dishes

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Correlators on ASICs

As with FPGAs, ASICs are mounted on boards
Correlators on ASICs

• Advantages:
  – Highest possible efficiency, low per-unit cost

• Disadvantages:
  – Highest development cost (time and manufacturing setup)
  – Specialized knowledge required
  – Can’t be changed / very difficult to upgrade during lifetime
Correlators on ASICs

The Atacama Large Millimetre Array, Chile

The Very Large Array, New Mexico
Correlator platform overview

Development effort required

Reuse-ability

Correlator capacity per hardware $$

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The end