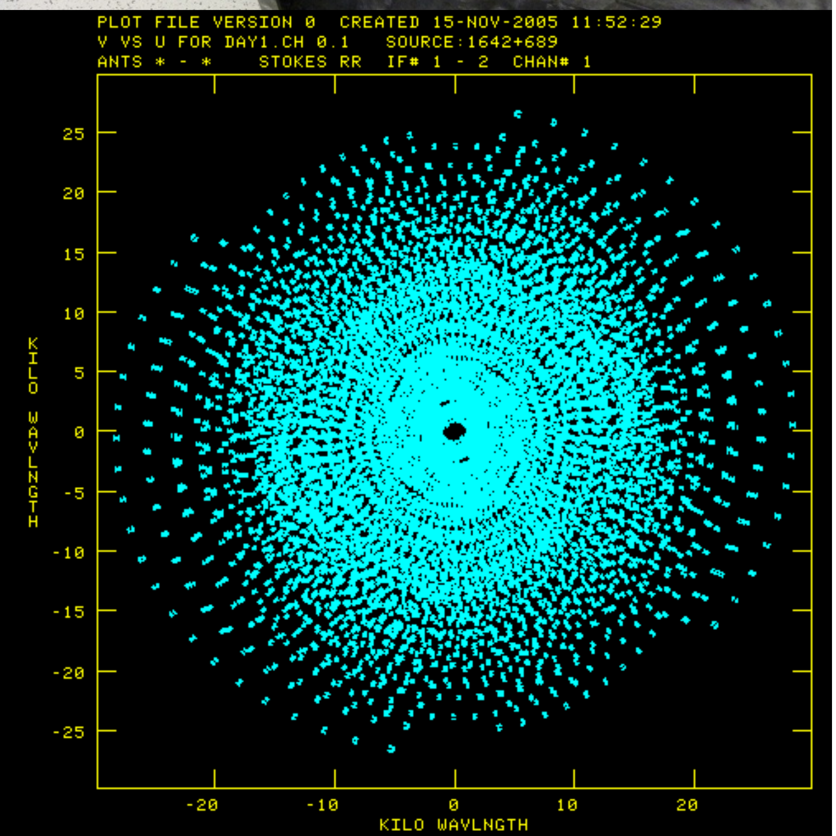


Cross correlators

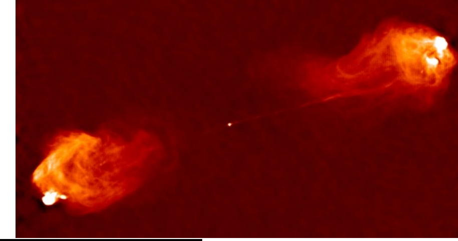
for radio astronomy

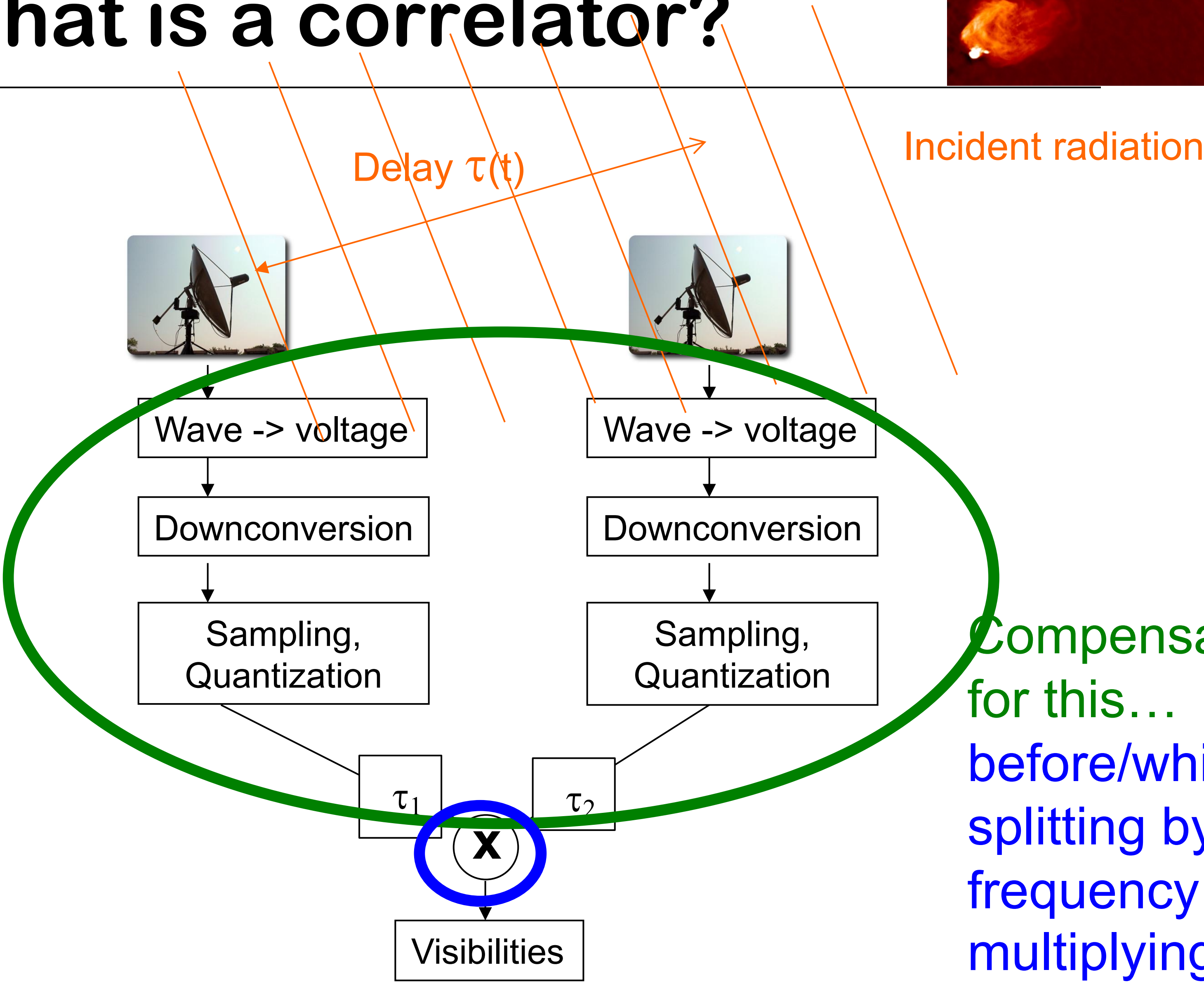
Adam Deller

May 19, 2022

SWINBURNE UNIVERSITY OF TECHNOLOGY

What is a correlator?

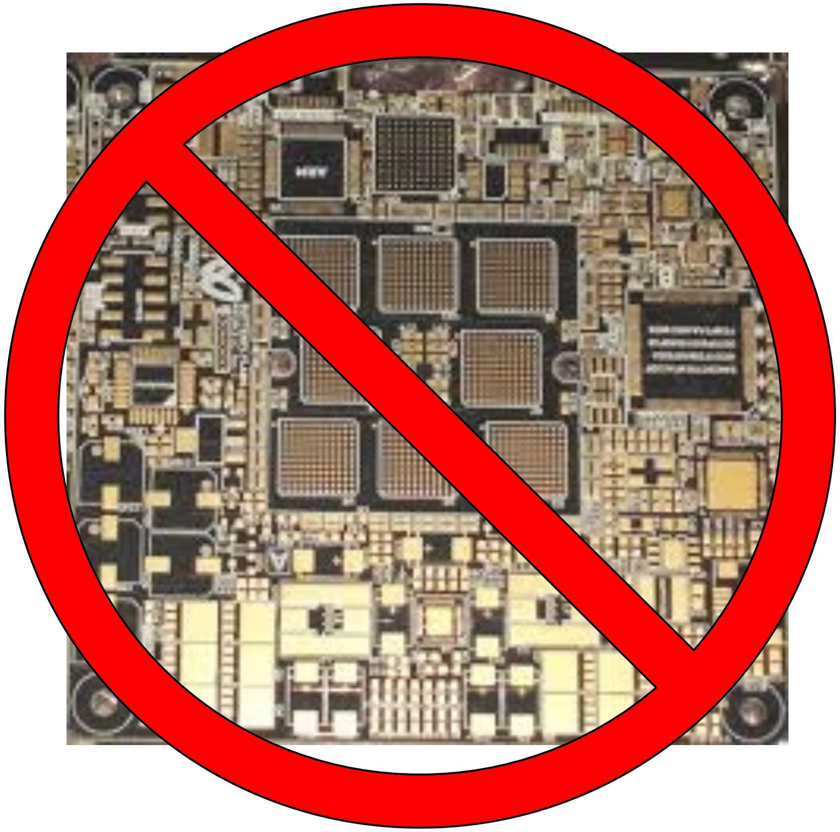


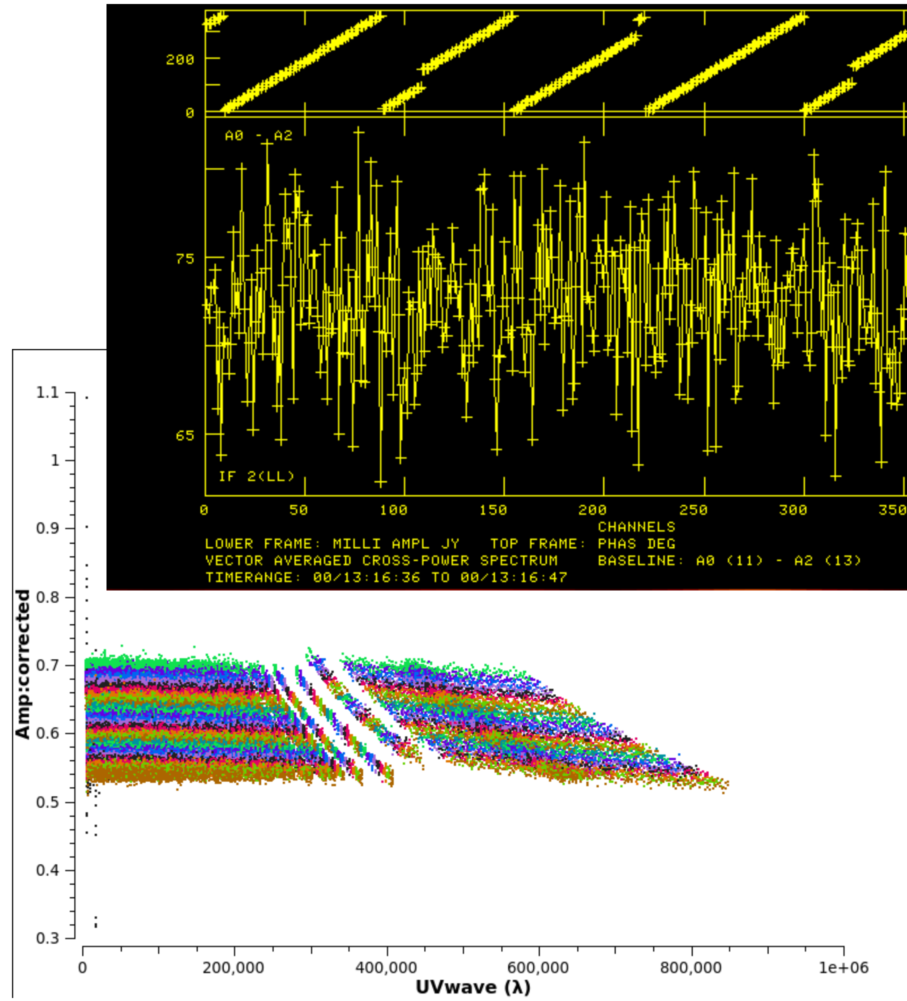


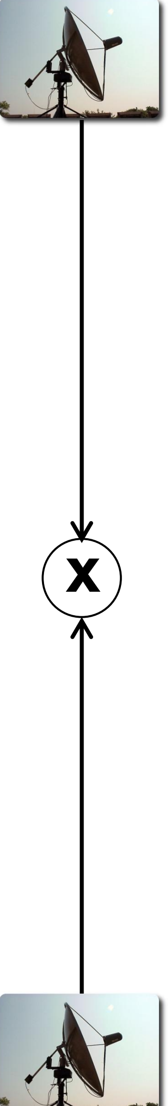
Compensate for this…

before/while splitting by frequency and multiplying

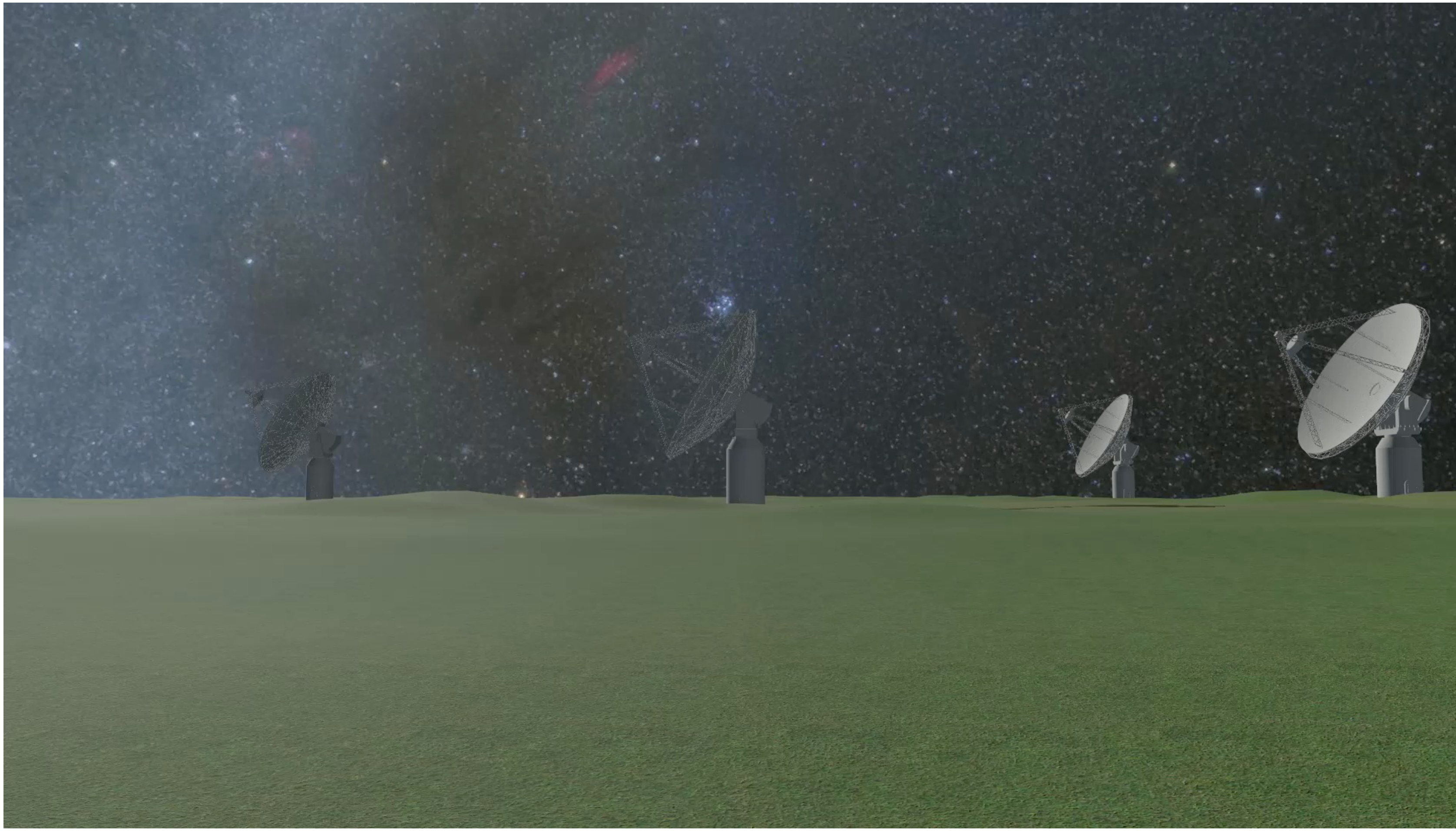
Why correlators matter to YOU





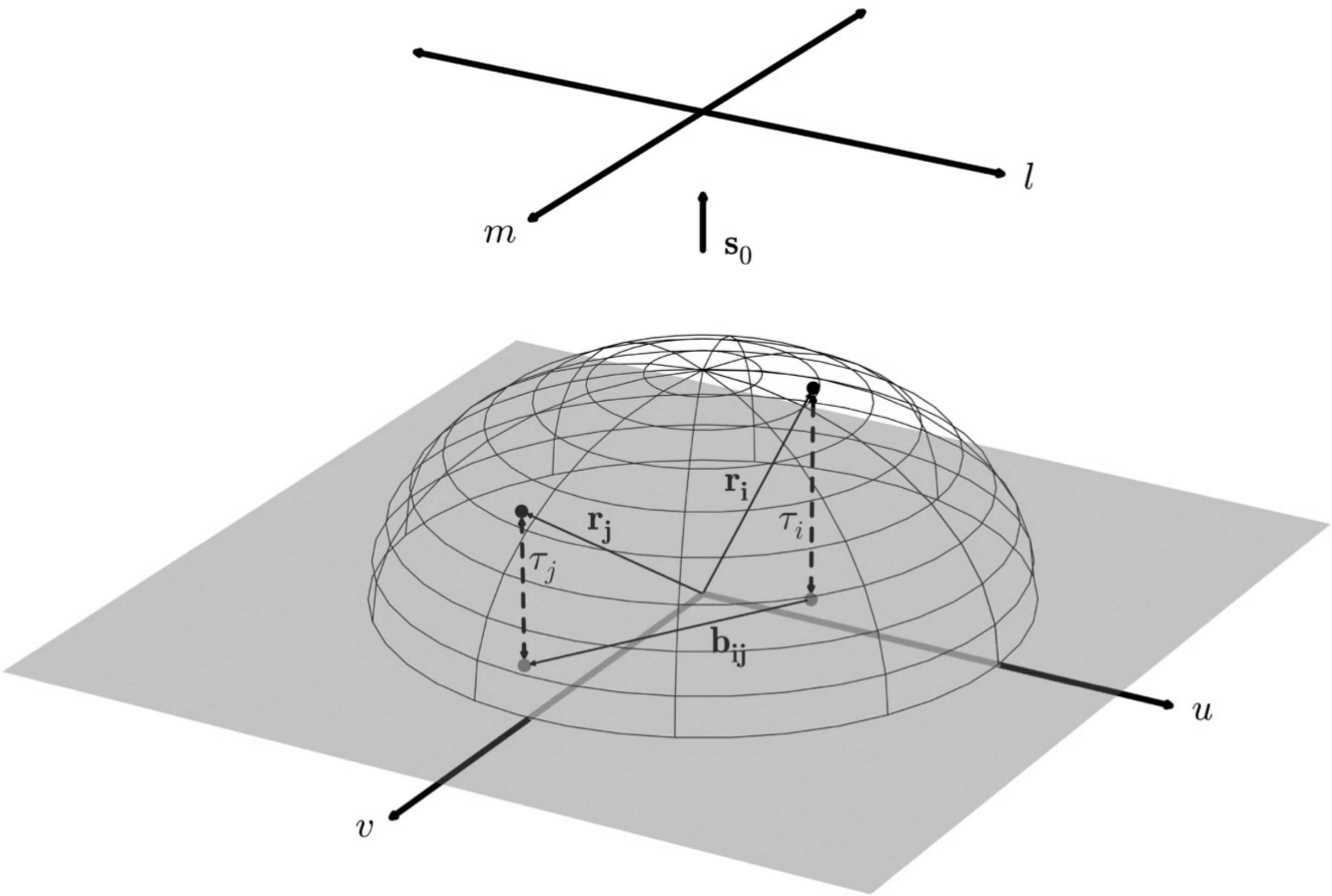


Correlators and interferometry

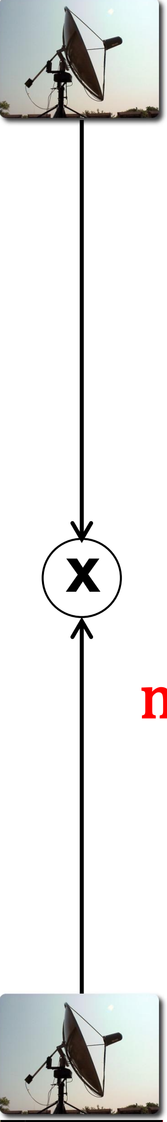


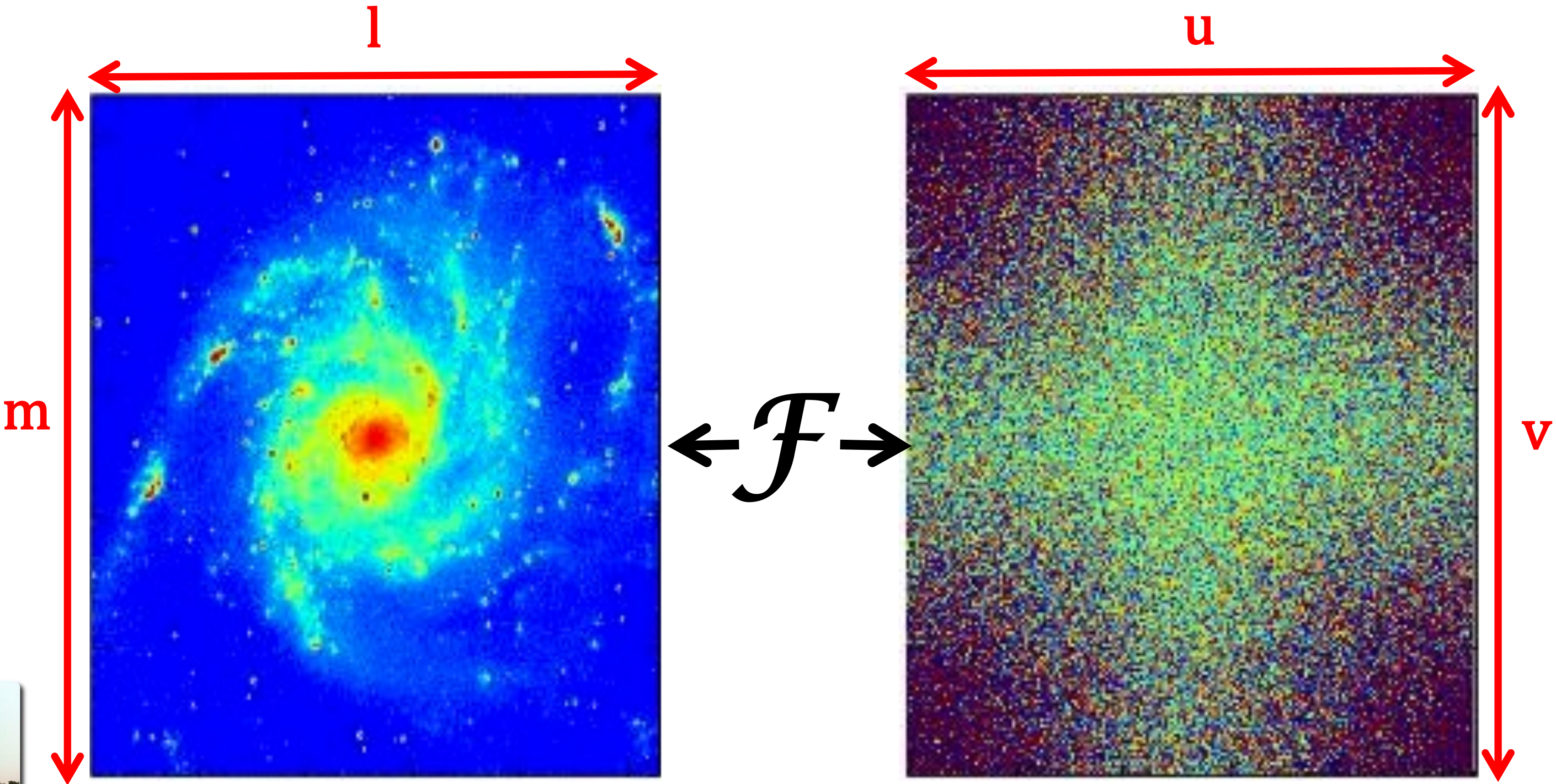
SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlators and Interferometry



Correlators and Interferometry





Sky brightness at frequency ν_0

Visibilities (real component shown, unit is $\lambda_0 = c / \nu_0$)

SWINBURNE UNIVERSITY OF TECHNOLOGY

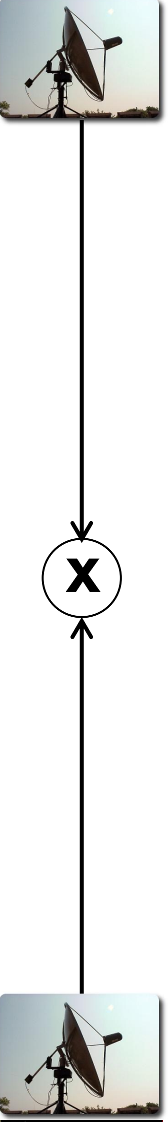
Monochromatic == problematic

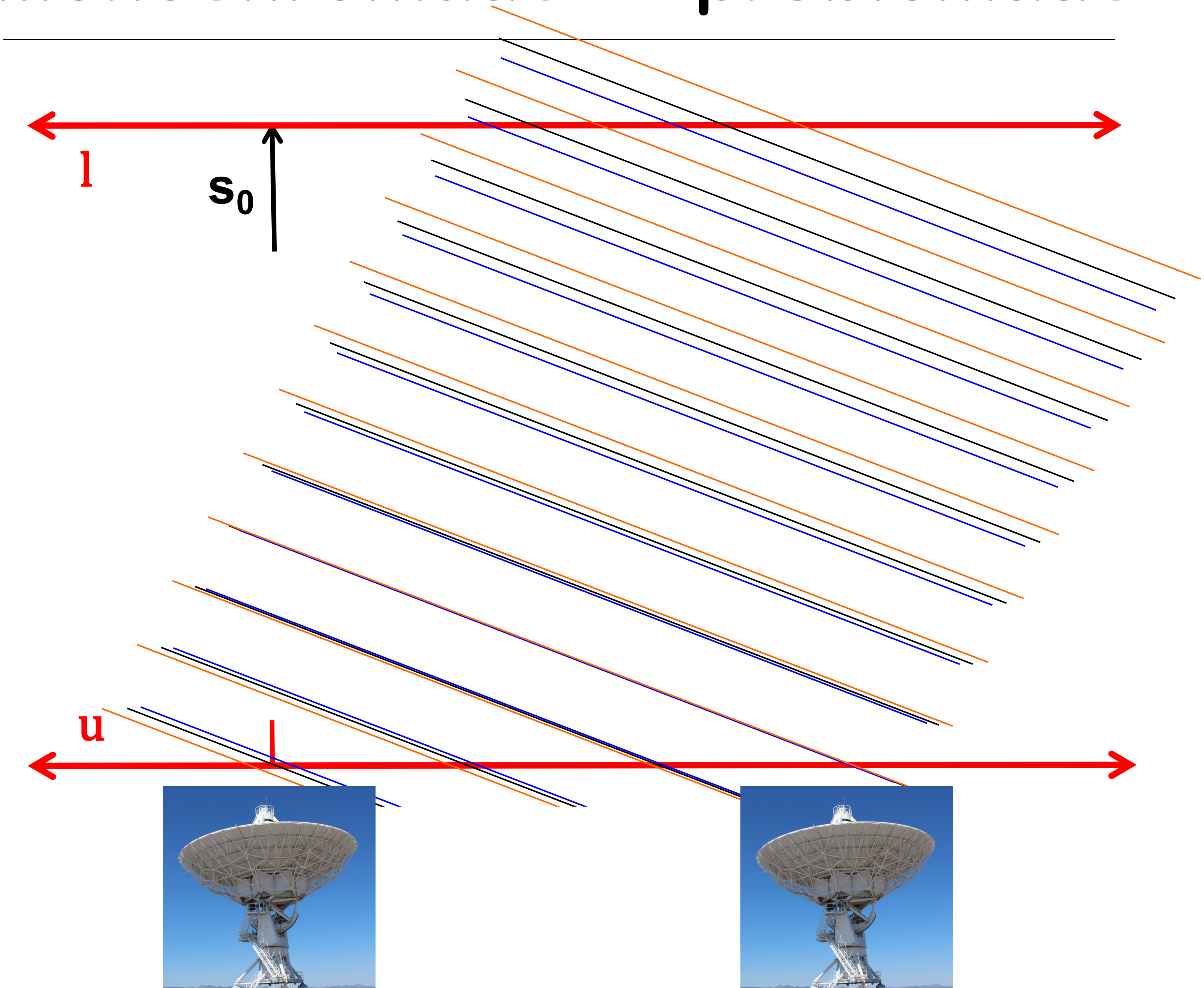
$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{A}(l,m) I(l,m) e^{-2\pi i(ul+vm)}\, dl\, dm\,.$$

- $\mathrm{u} \times \mathrm{l} + \mathrm{v} \times \mathrm{m}$ is supposed be constant, but both u and v depend on frequency
- No truly monochromatic radiation!
- Fortunately, "fairly narrow" band of $\Delta\nu$ (*quasi-monochromatic*) can suffice:
  - Real world viewpoint: different frequency components stay "in phase" as wavefront propagates from one antenna to the next

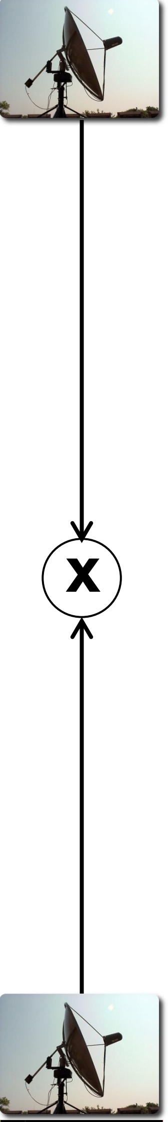
SWINBURNE UNIVERSITY OF TECHNOLOGY

Monochromatic == problematic





SWINBURNE UNIVERSITY OF TECHNOLOGY



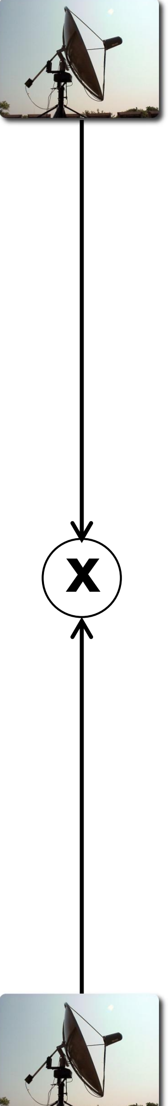
Monochromatic == problematic

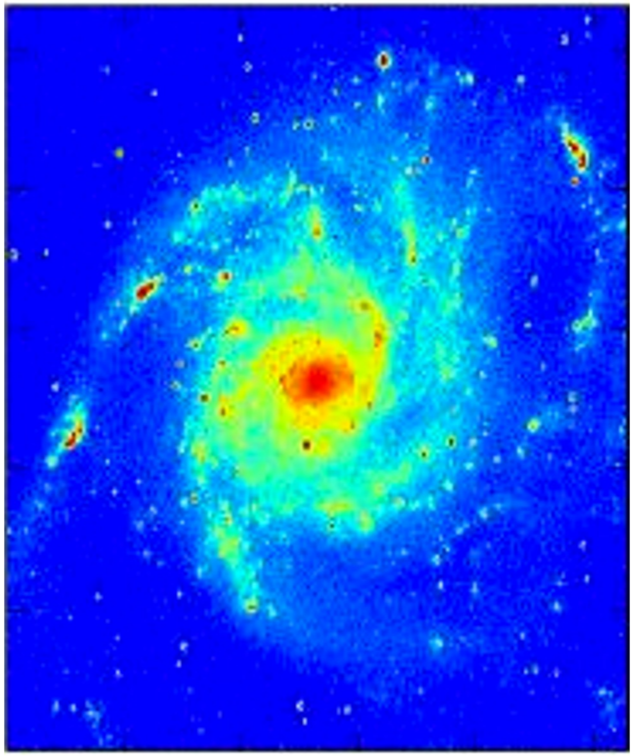
$$V(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathcal{A}(l,m) I(l,m) e^{-2\pi i(ul+vm)}\, dl\, dm\,.$$

- $\mathrm{u} \times \mathrm{l} + \mathrm{v} \times \mathrm{m}$ is supposed be constant, but both u and v depend on frequency
- No truly monochromatic radiation!
- Fortunately, “fairly narrow” band of $\Delta\nu$ (*quasi-monochromatic*) can suffice:
  - if $\Delta\mathrm{u} \times \mathrm{l} \ll 1$ and $\Delta\mathrm{v} \times \mathrm{m} \ll 1$ then the different frequency components stay in phase and we’re ok
  - Correlator needs to slice at least this finely

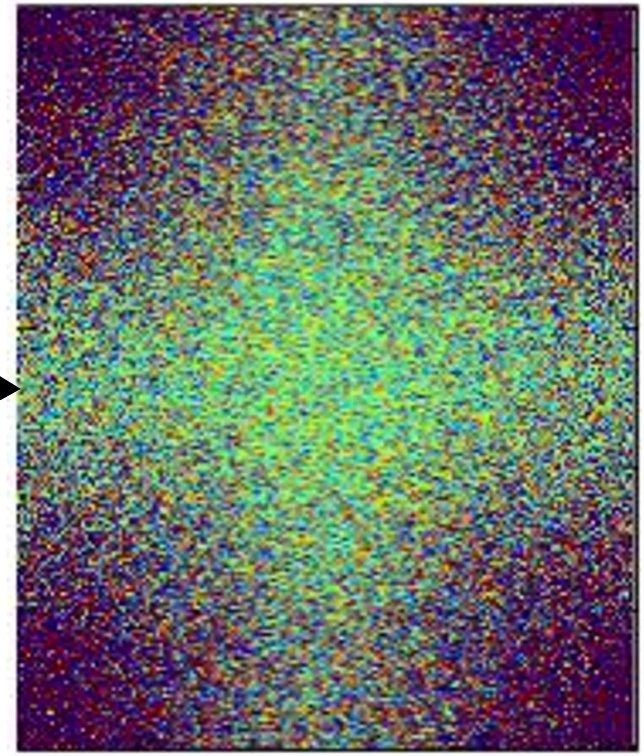
SWINBURNE
SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlators and Interferometry





$\leftarrow \mathcal{F} \rightarrow$

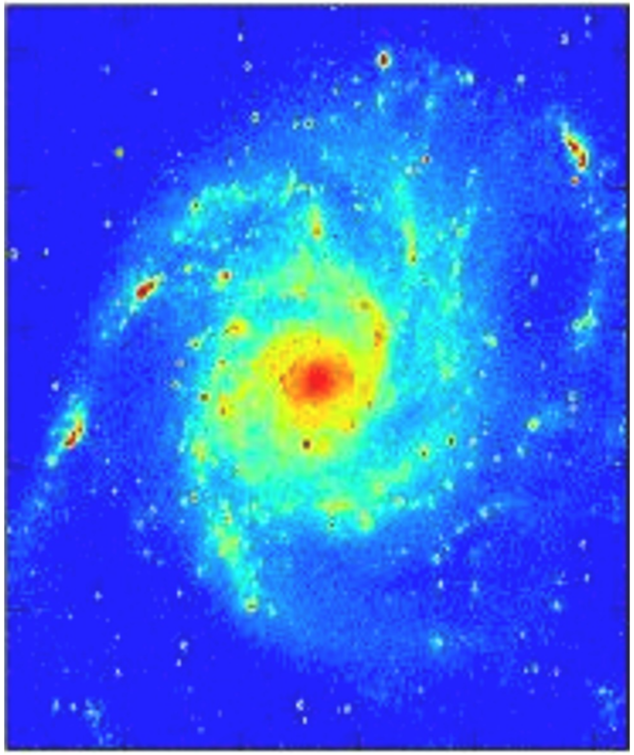


Sky brightness at frequency ν_0

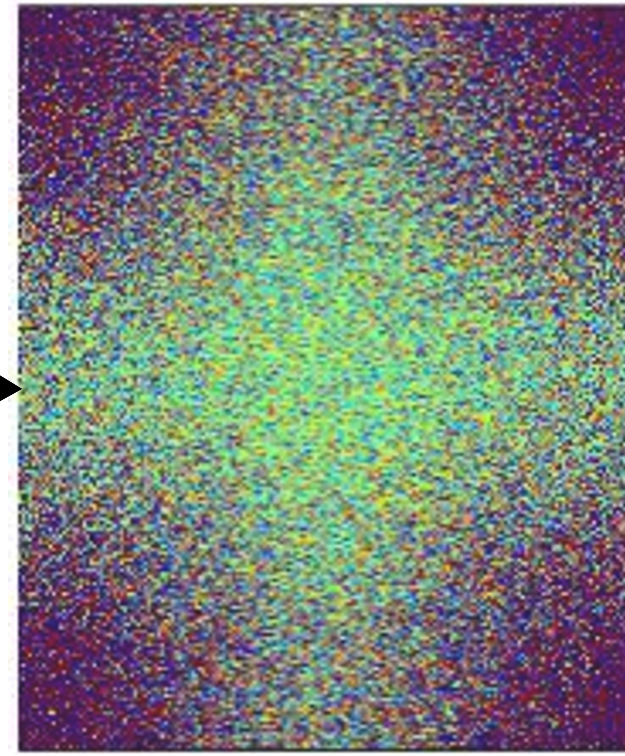
Visibilities (real component shown, unit is $\lambda_0 = c / \nu_0$)

SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlators and Interferometry



$\leftarrow \mathcal{F} \rightarrow$

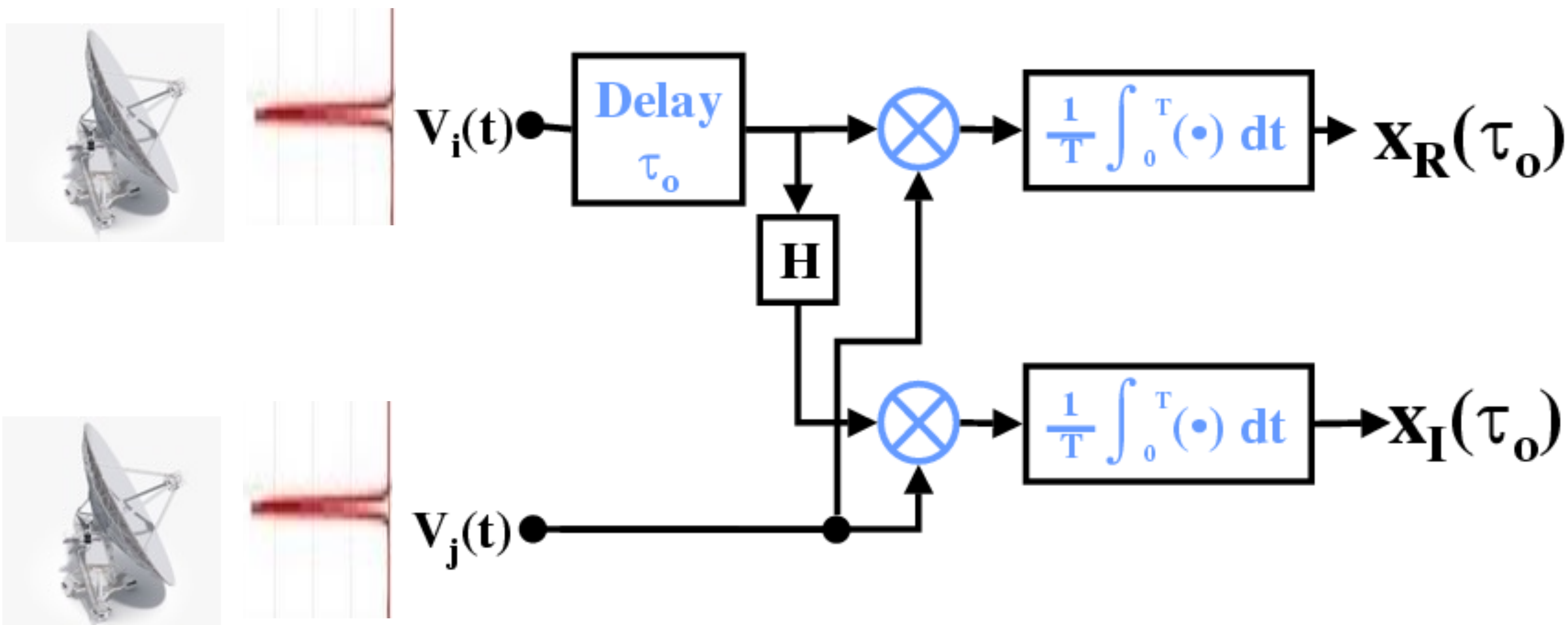


Sky brightness at frequency $\nu' = \nu_0 + \delta\nu$

Visibilities (real component shown, unit is $\lambda' = c / \nu'$)

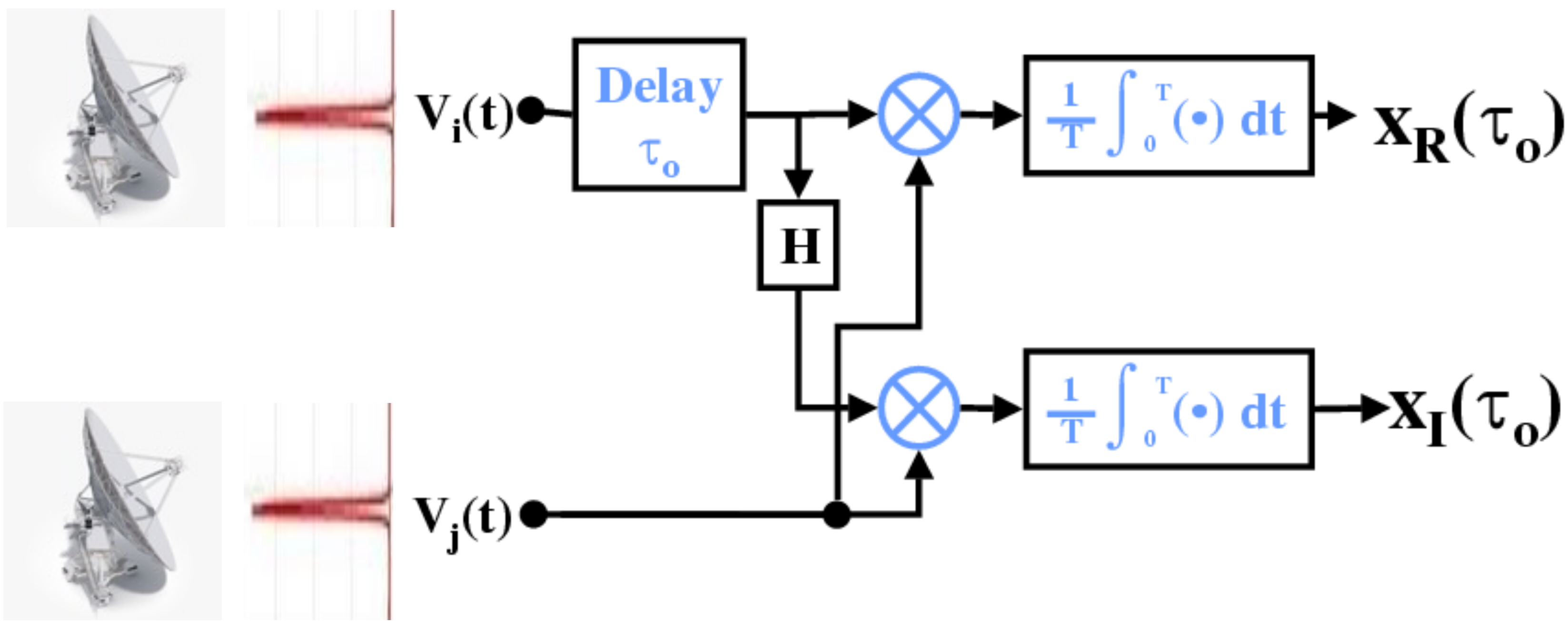
A “dumb” correlator

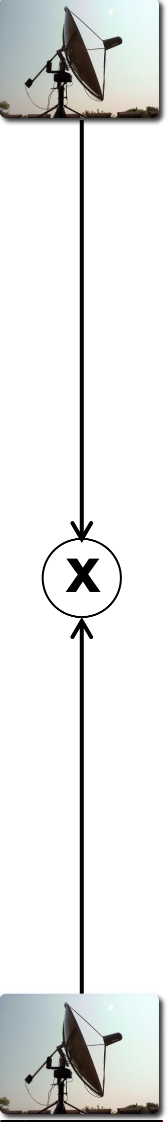
- Use many analog filters to make many narrow channels; correlate each one separately with a standard complex correlator:



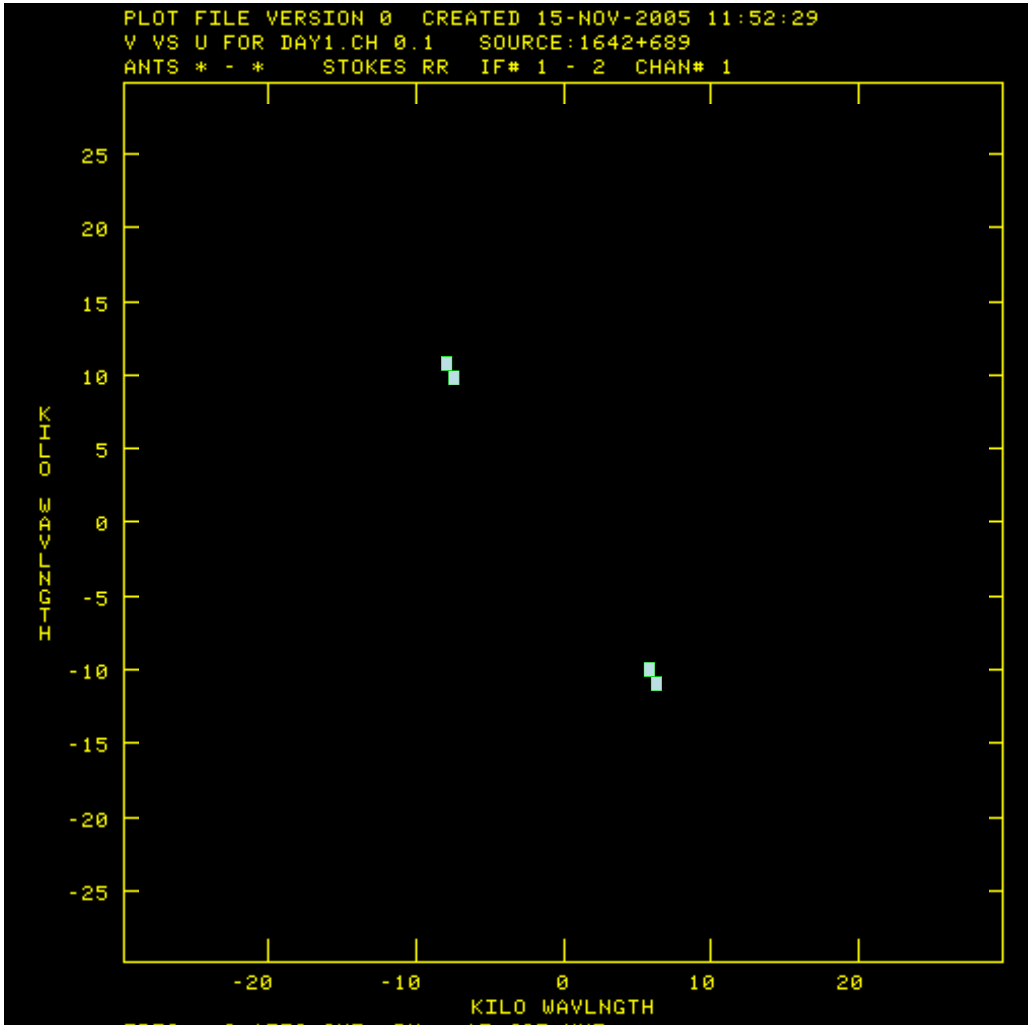
A "dumb" correlator

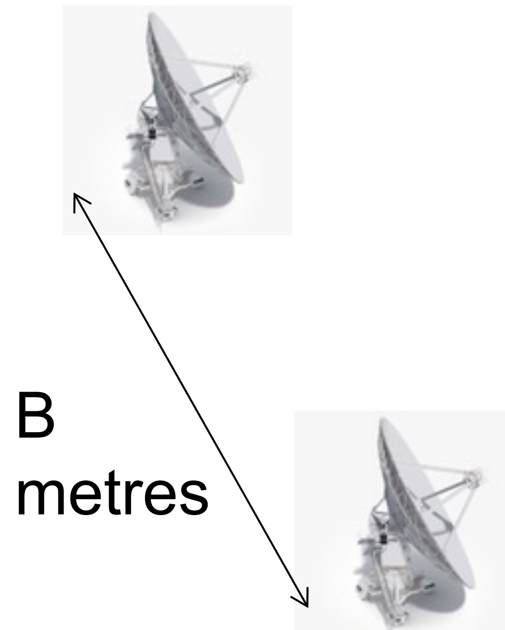
- Use many analog filters to make many narrow channels; correlate each one separately with a standard complex correlator:

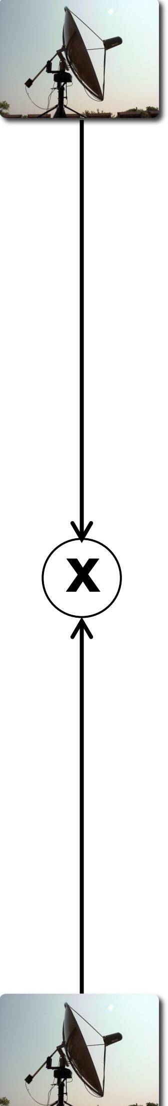



X

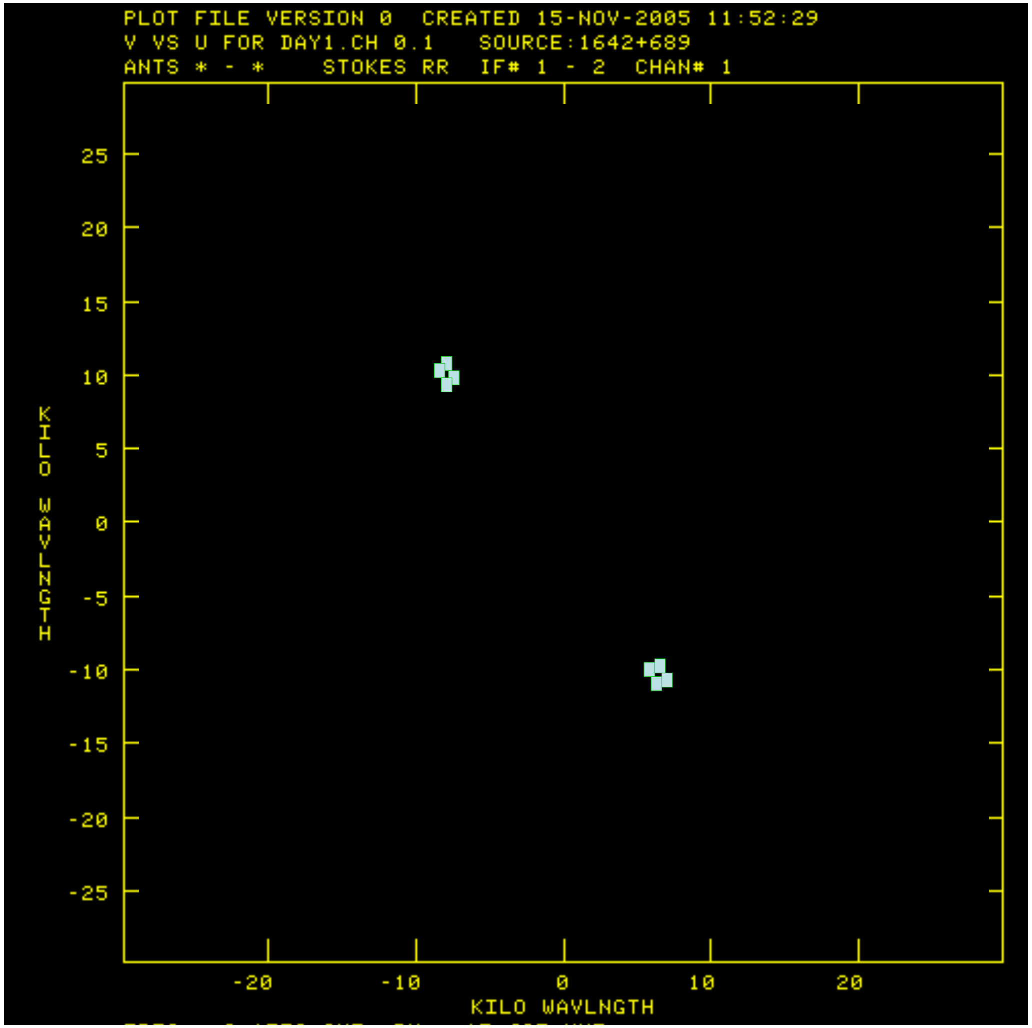
The output

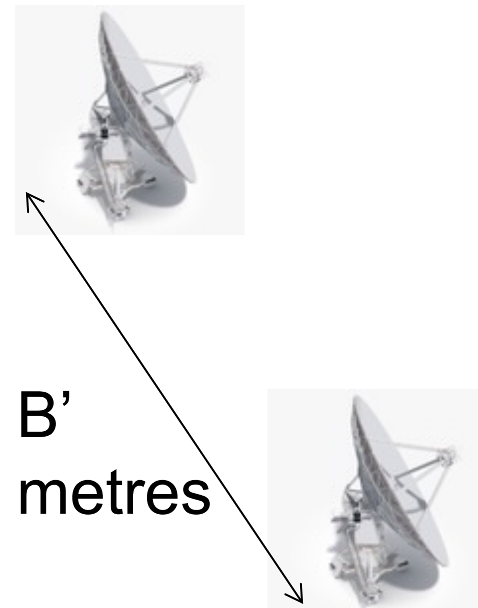

PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1 SOURCE:1642+689
ANTS * - * STOKES RR IF# 1 - 2 CHAN# 1
KILO WAVLNGTH
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-10
0
10
20
KILO WAVLNGTH


B
metres



The output

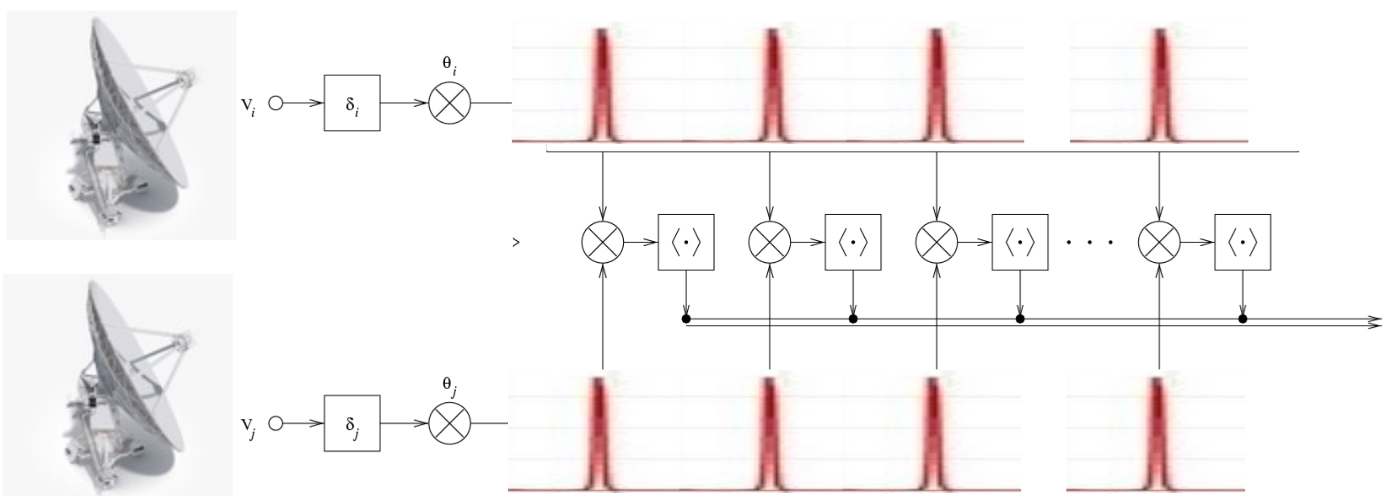




SWINBURNE UNIVERSITY OF TECHNOLOGY

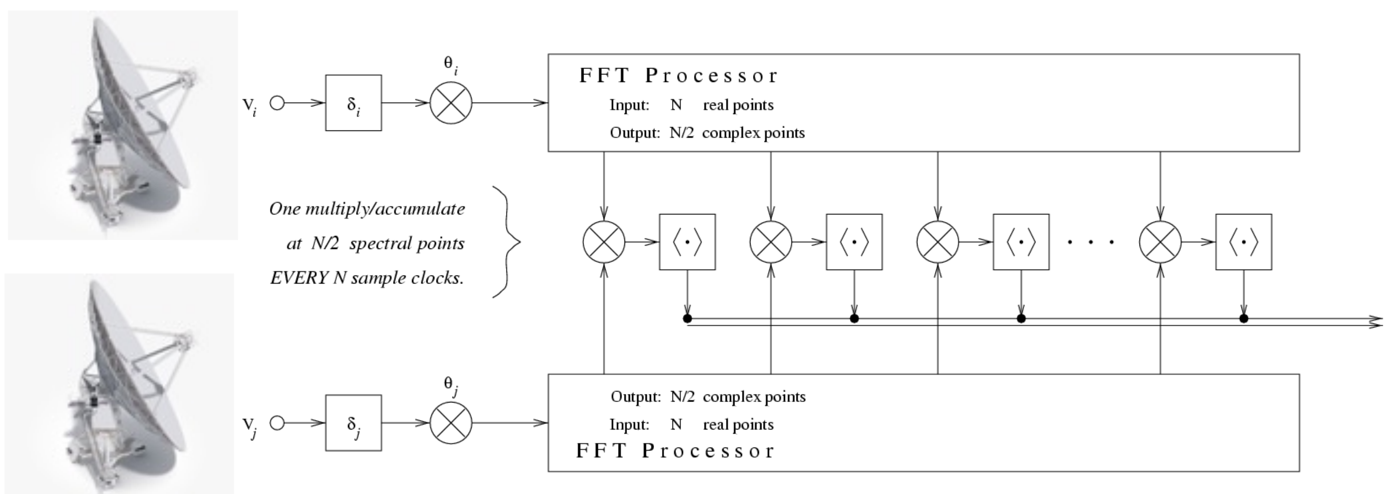
Making it feasible

- Analog filters are costly & finnicky; this would be expensive and temperamental

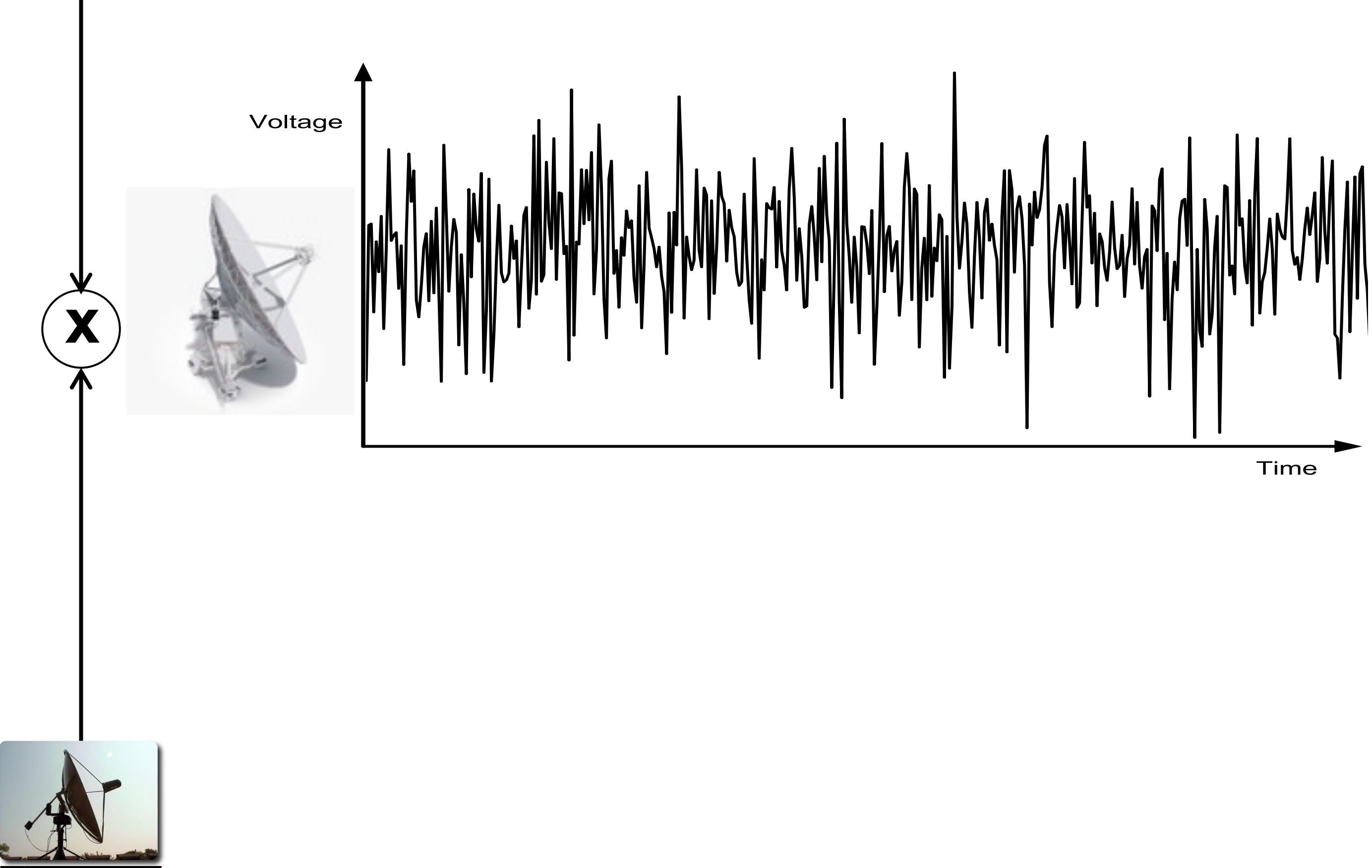


Making it feasible

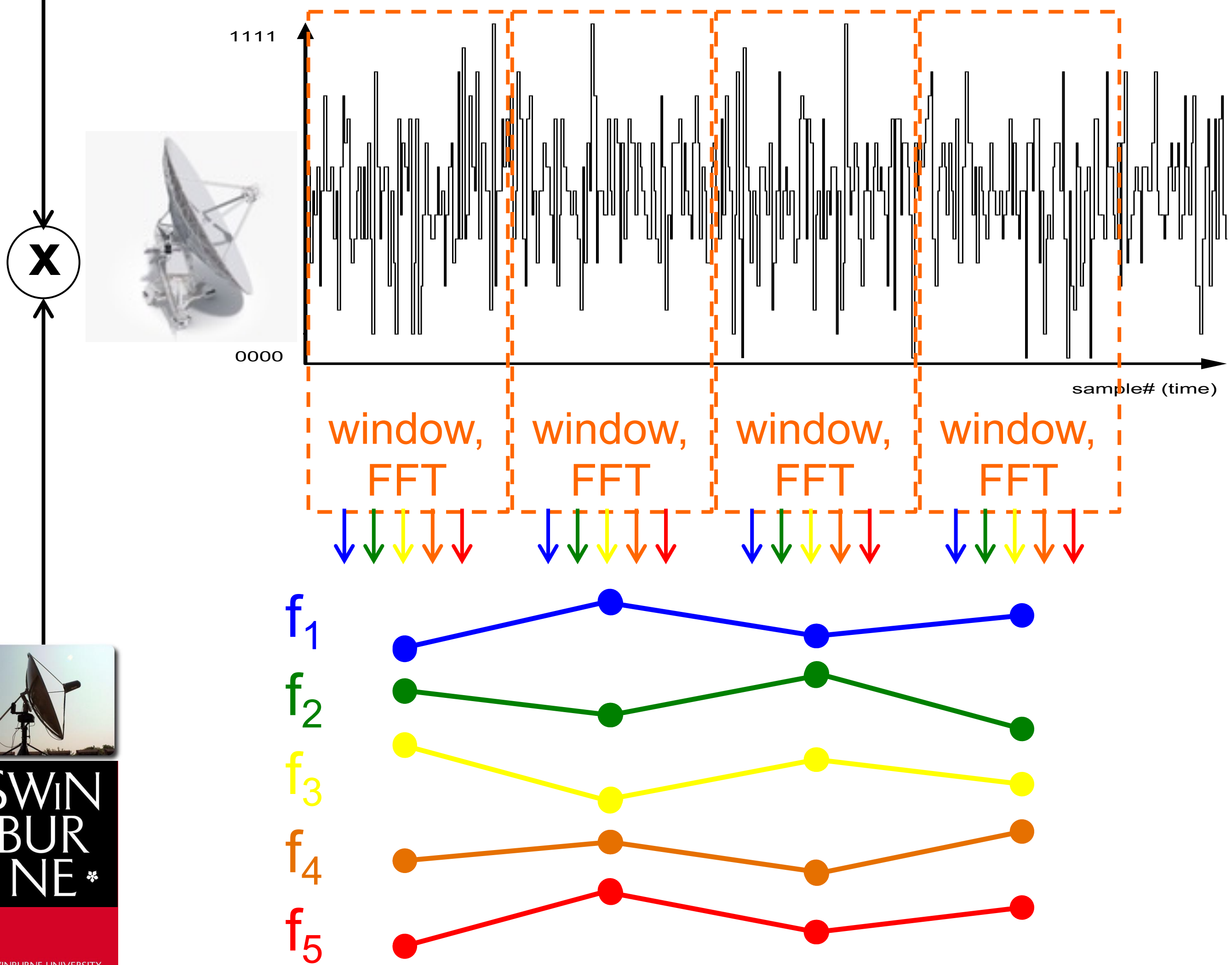
- Analog filters are costly & finnicky; this would be expensive and temperamental
- Fortunately, we can (and do) digitize the signal – meaning we can use a digital substitute: **digital filterbank**



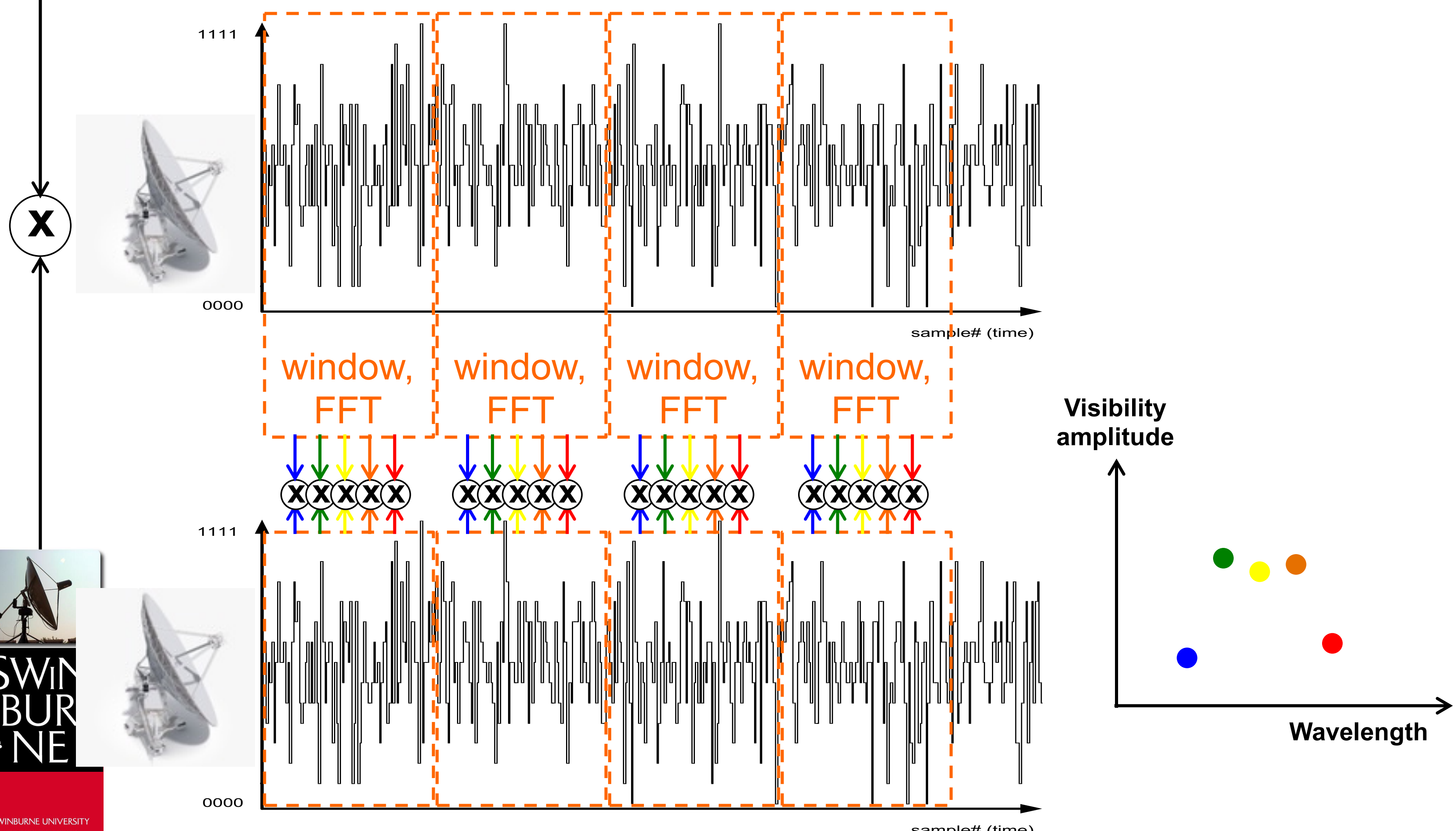
The “FX” correlator



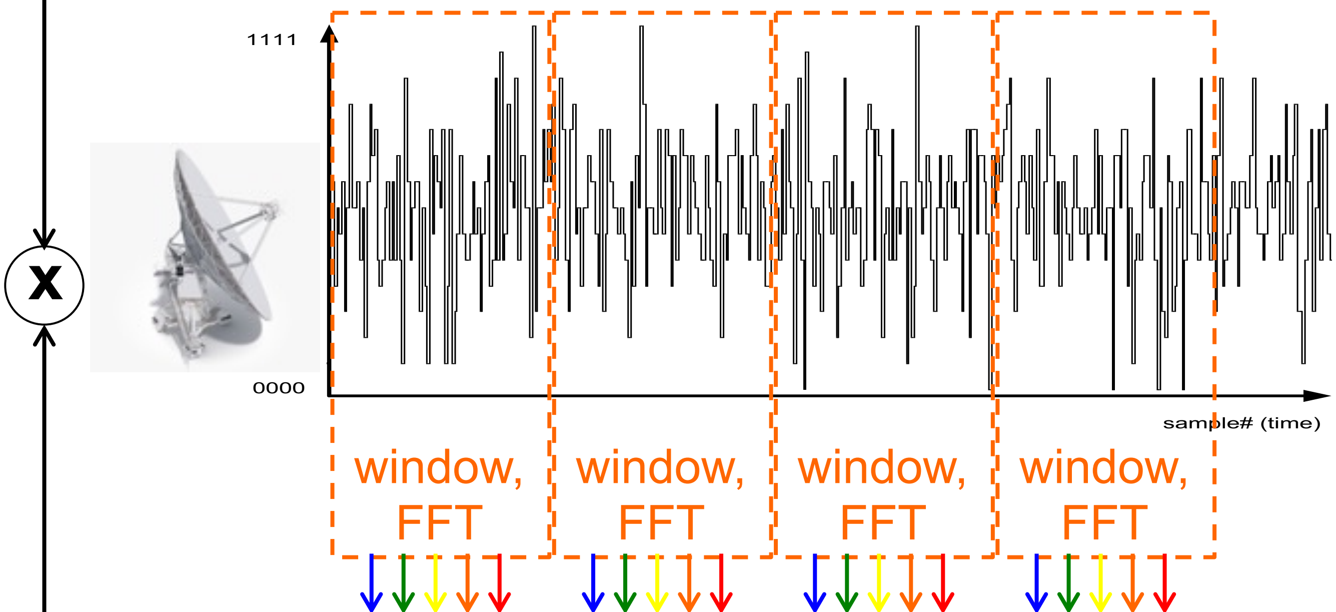
The “FX” correlator



The "FX" correlator

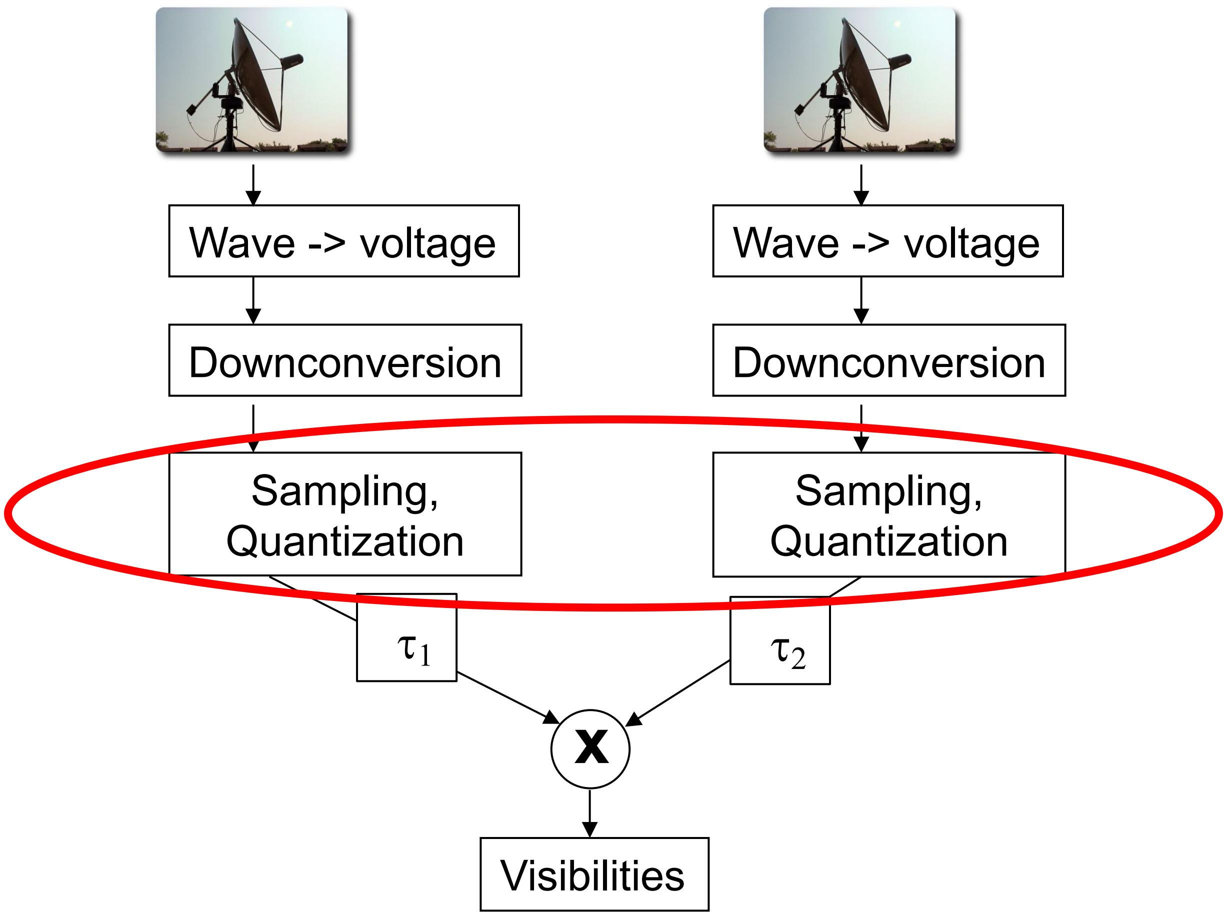


The “FX” correlator



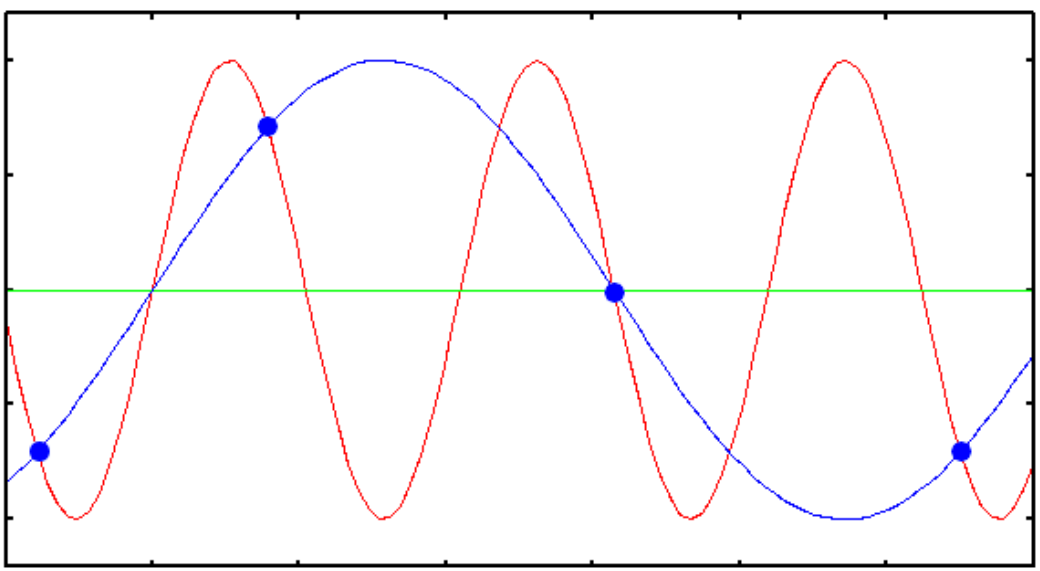
- Since this architecture consists of a Fourier transform (F) followed by cross-multiplication (X), we dub this the “FX” correlator

Righting the wrongs



Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every Δt sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies ≥ 1 / (2 Δt) Hz (*band-limited*)

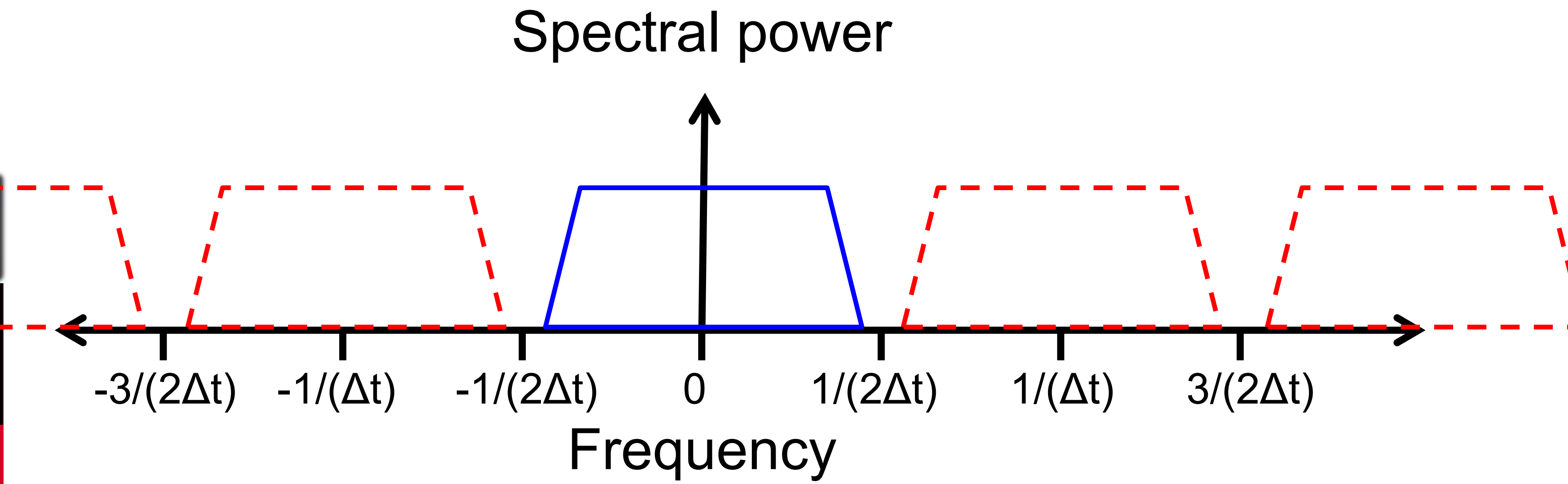


Adequately sampled

Undersampled, cannot be reconstructed

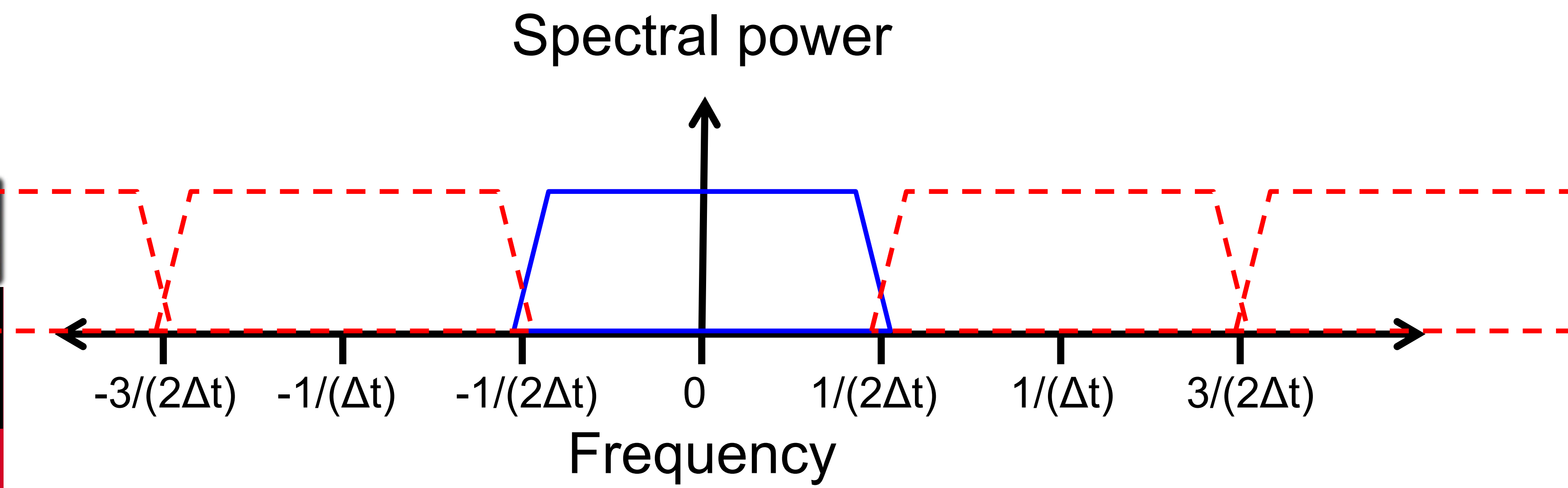
Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every Δt sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies ≥ 1 / (2 Δt) Hz (*band-limited*)



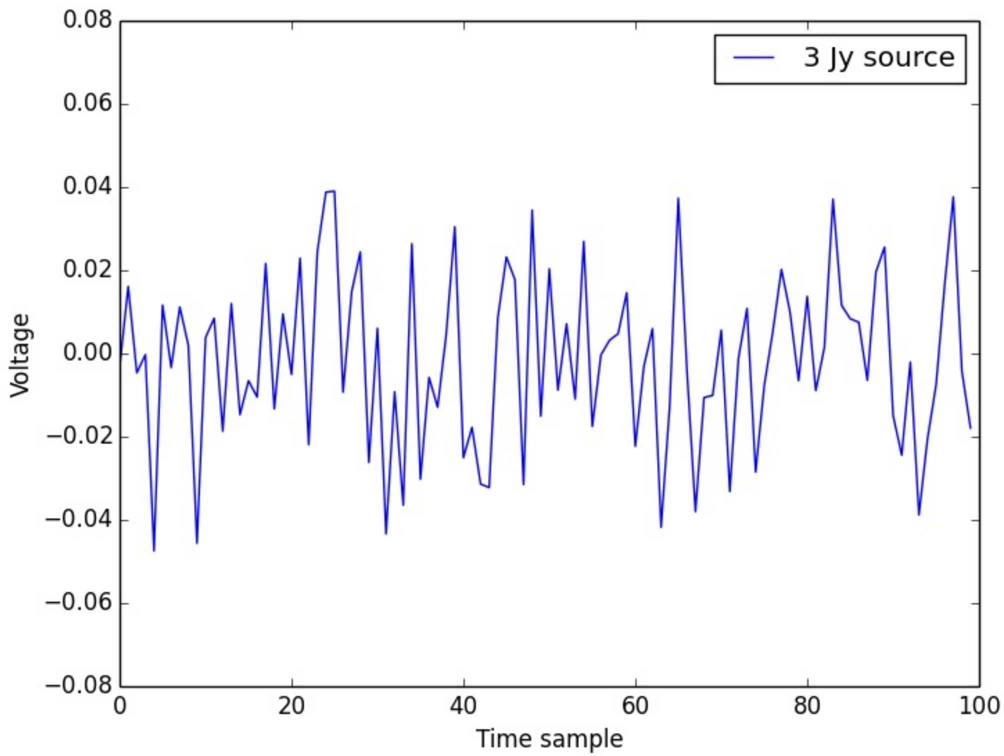
Sampling

- Nyquist-Shannon sampling theorem:
  - real-valued signal is sampled every Δt sec
  - Original signal can be reconstructed perfectly so long as contains no power at frequencies $\geq 1 / (2\,\Delta t)$ Hz (*band-limited*)



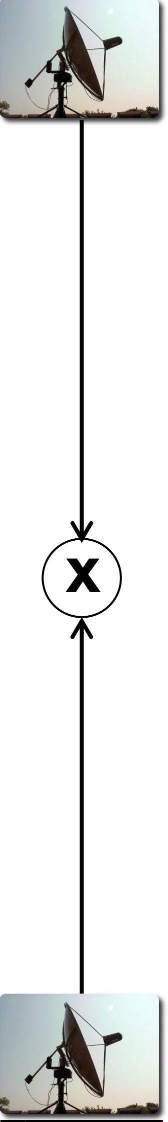
Quantization

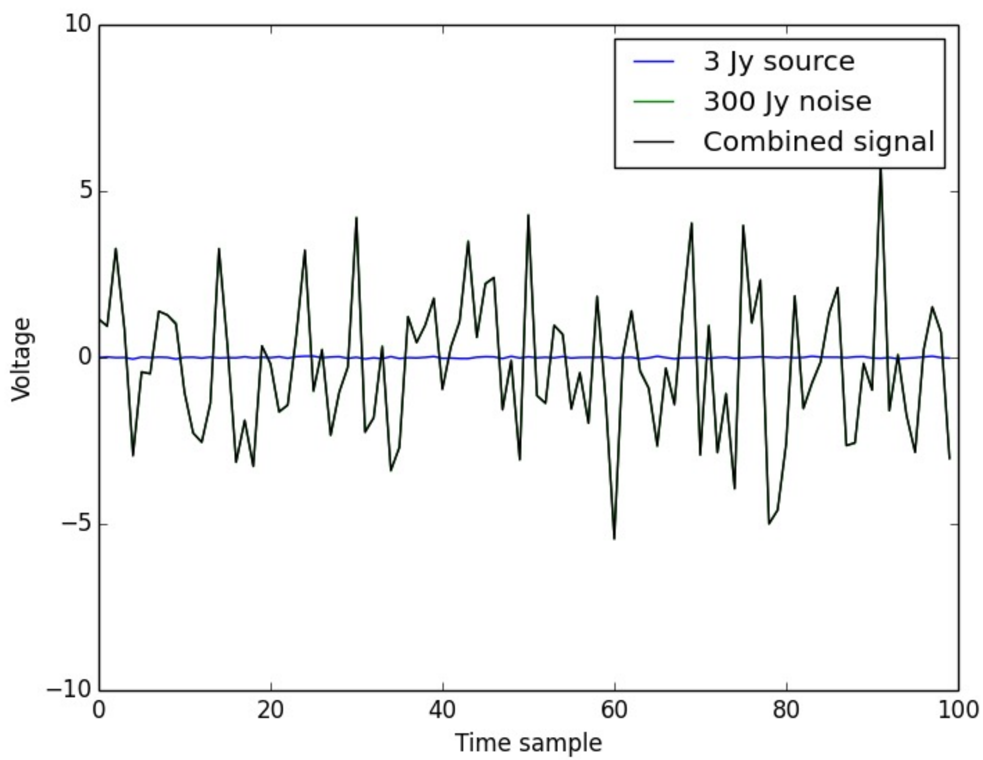
- When correlation is low (almost always) even very coarse quantization is ok!



Quantization

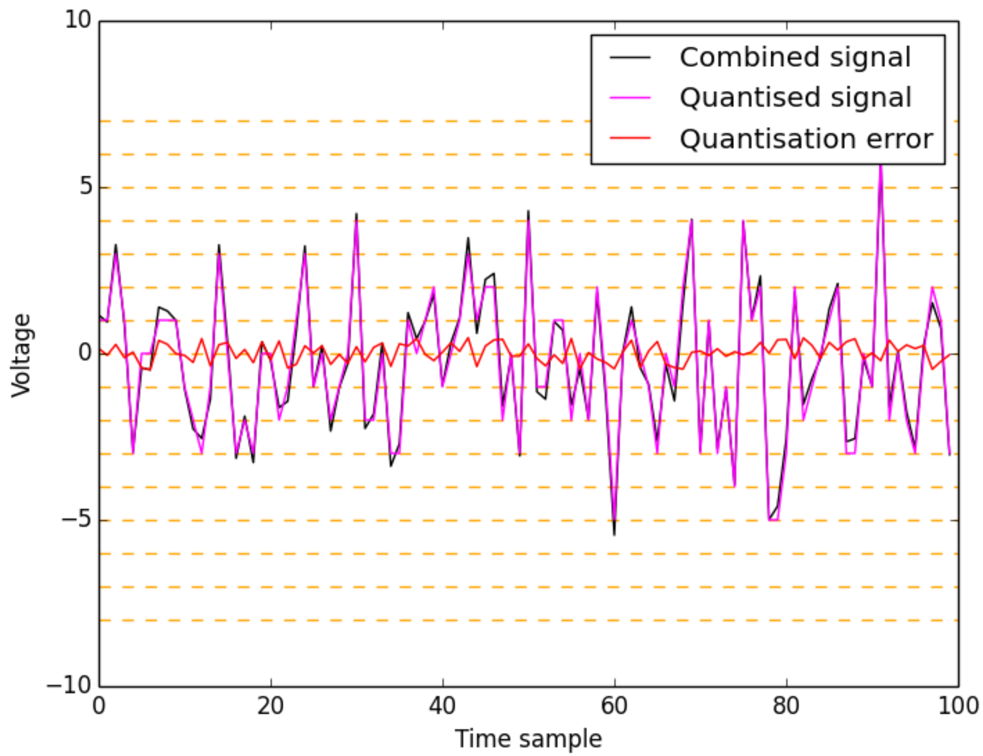
- When correlation is low (almost always) even very coarse quantization is ok!





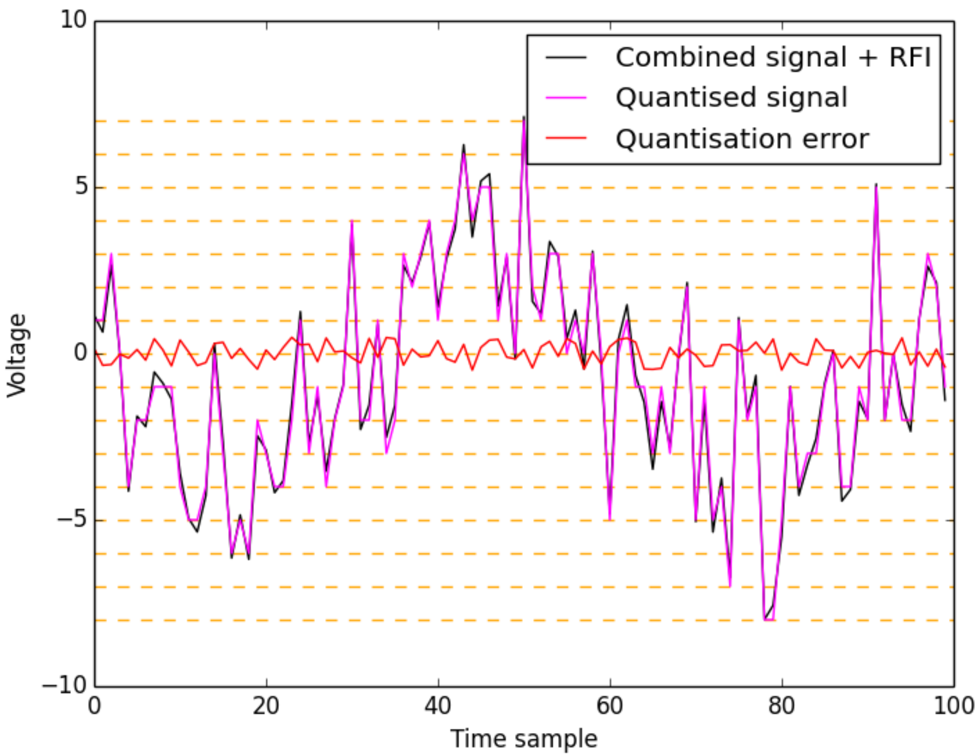
Quantization

- When correlation is low (almost always) even very coarse quantization is ok!



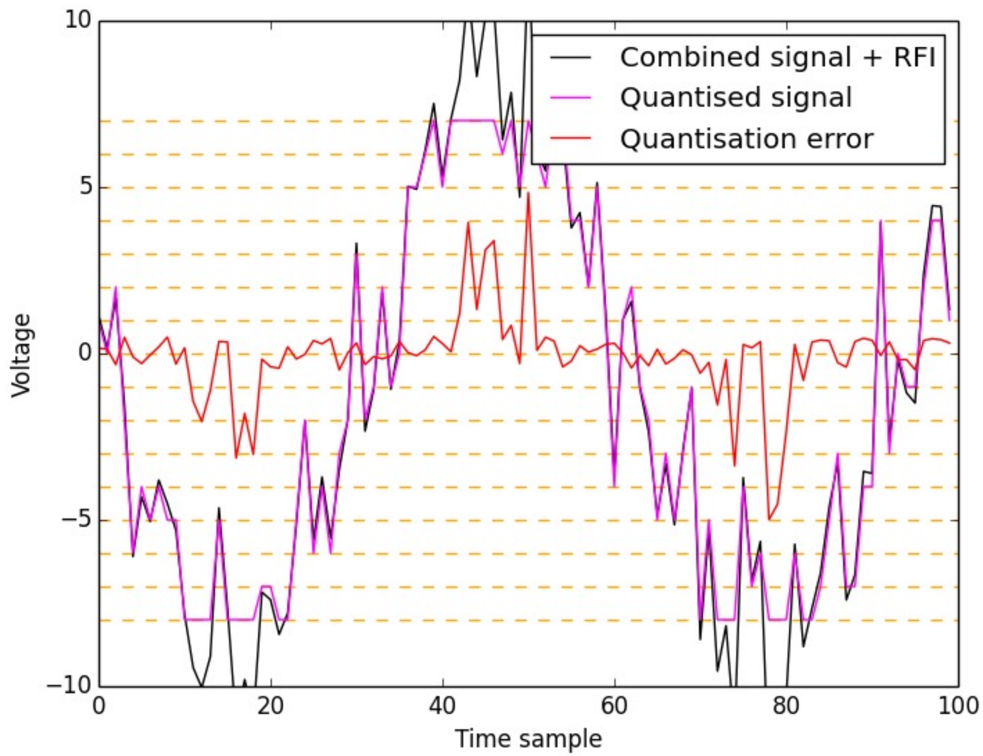
Quantization

- When correlation is low (almost always) even very coarse quantization is ok!



Quantization

- When correlation is low (almost always) even very coarse quantization is ok!



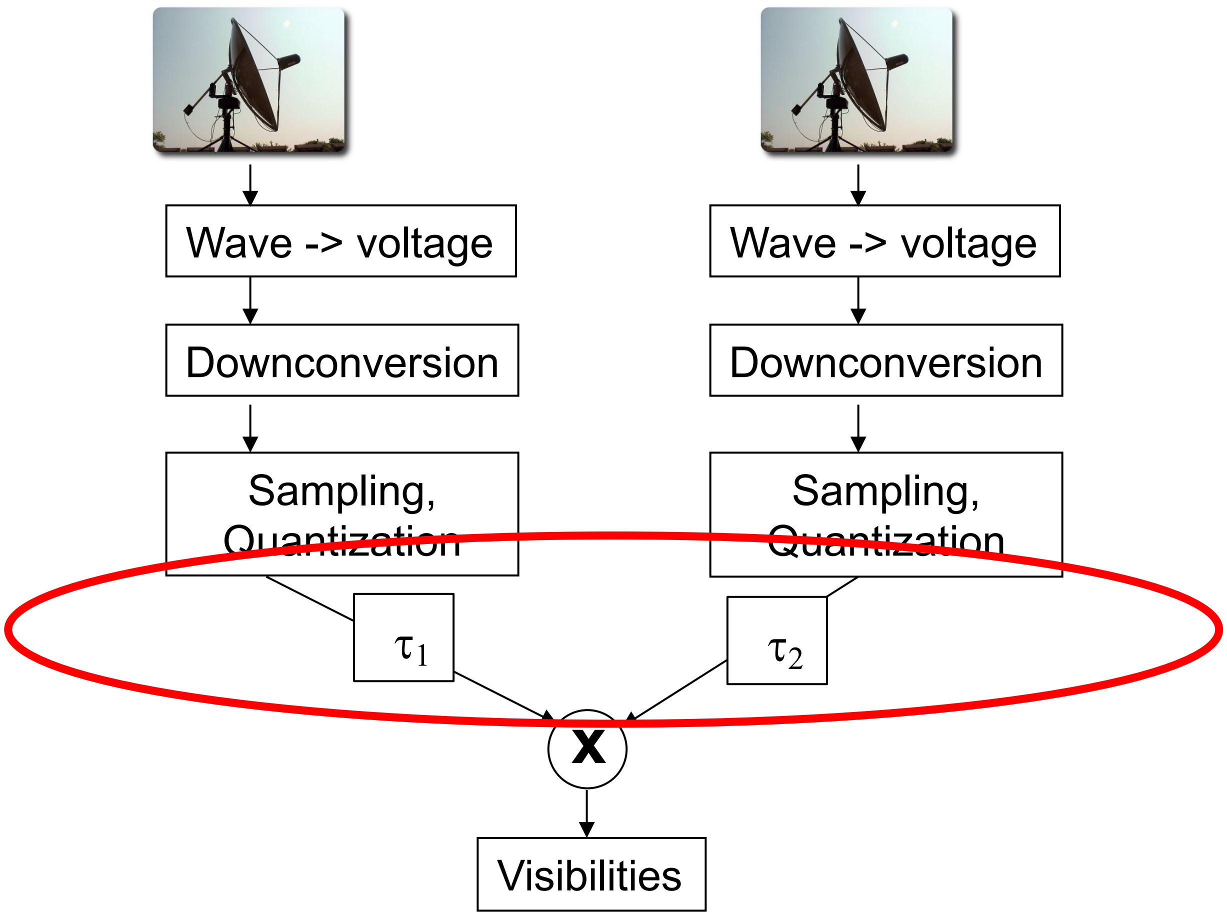
until the headroom runs out…

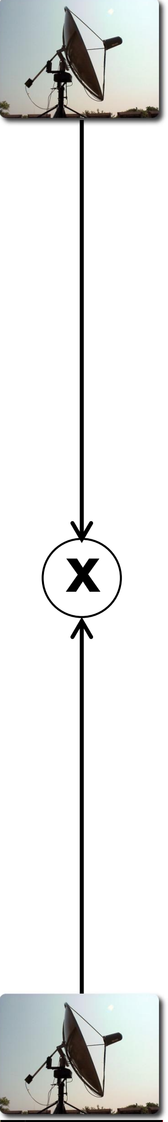
SWINBURNE UNIVERSITY OF TECHNOLOGY

Quantization

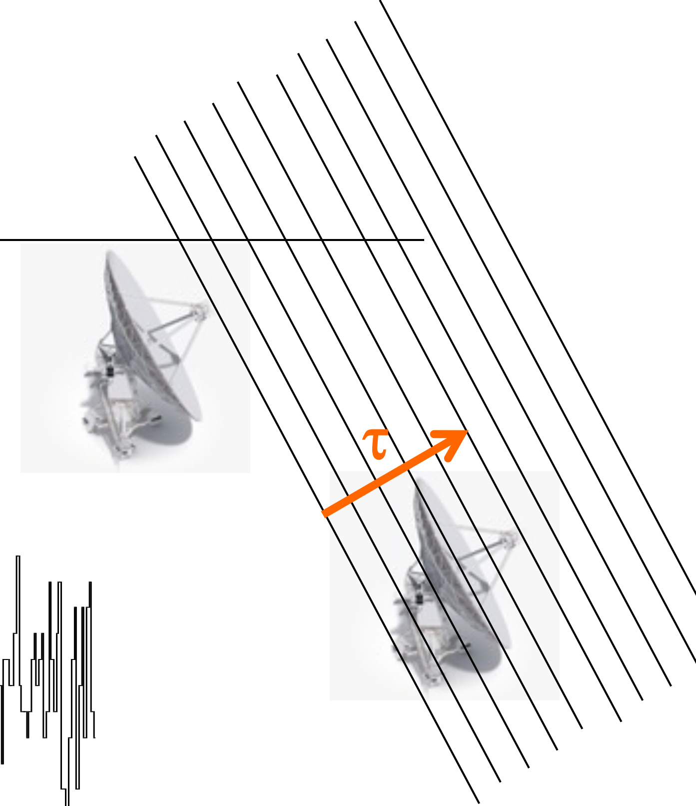
- When correlation is low (almost always) even very coarse quantization is ok!
- Sensitivity loss due to quantisation:
  - 8 bit: 0.1%
  - 4 bit: 1.3%
  - 2 bit: 12%
  - 1 bit: 36%
- Correct visibility amplitudes for this sensitivity loss

Righting the wrongs

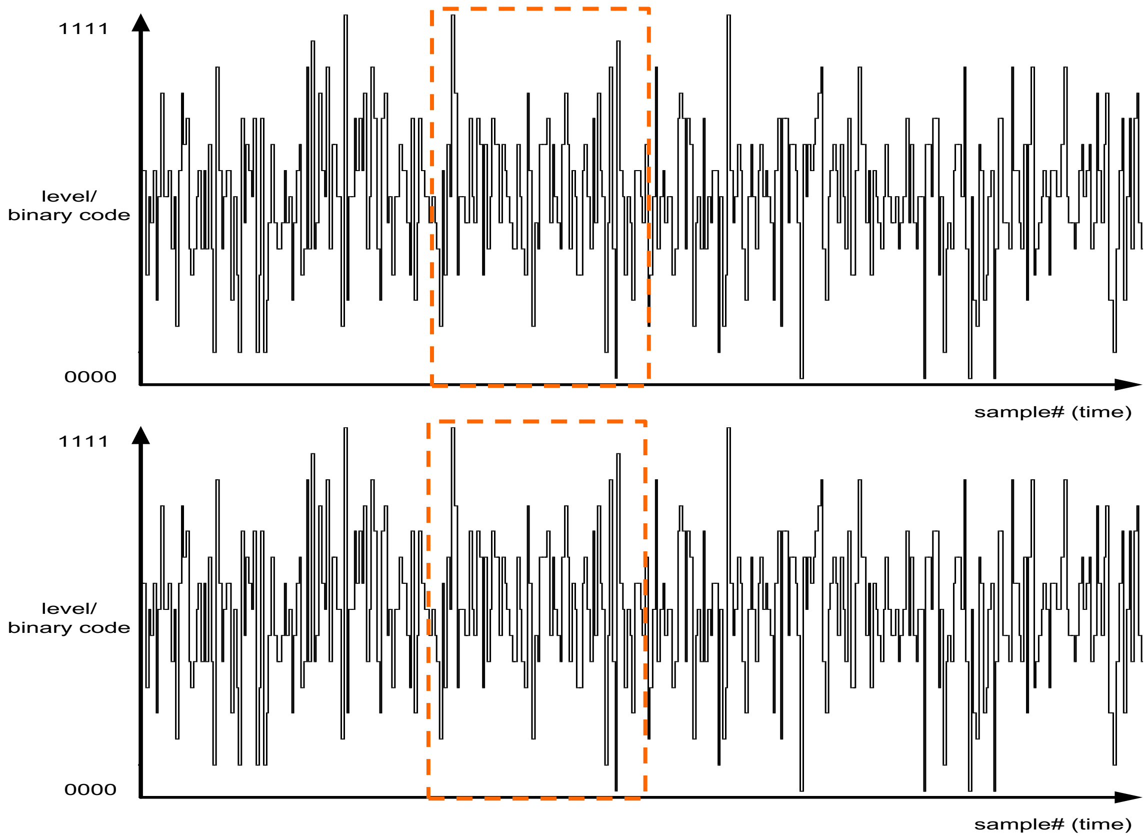




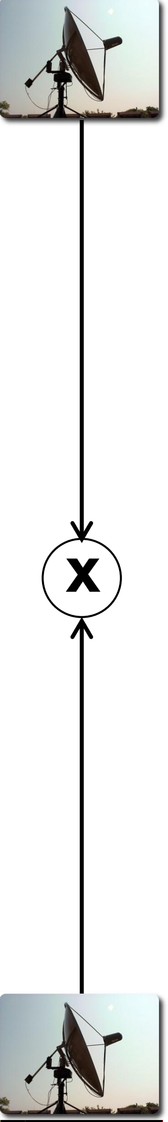
Delay compensation



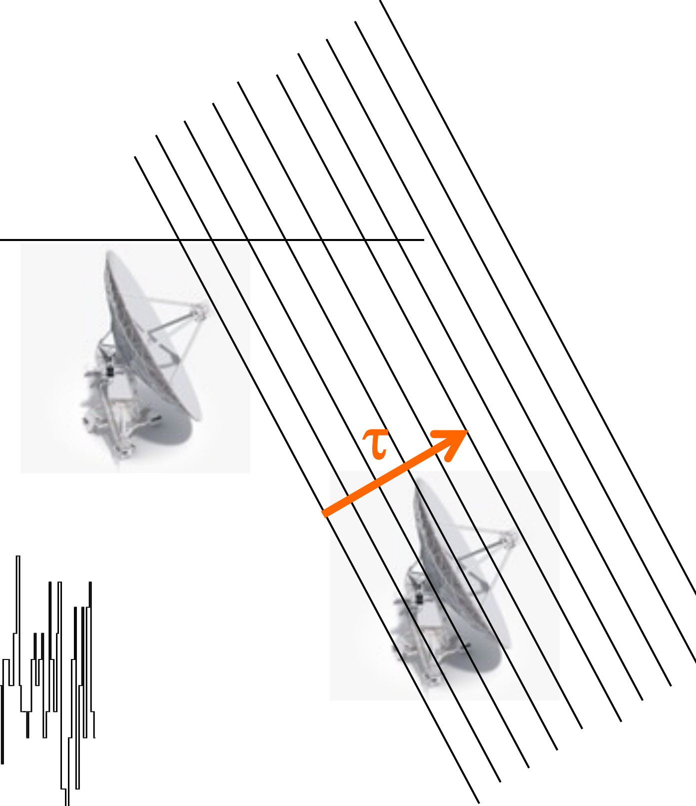
- Delay to the nearest sample is easy:



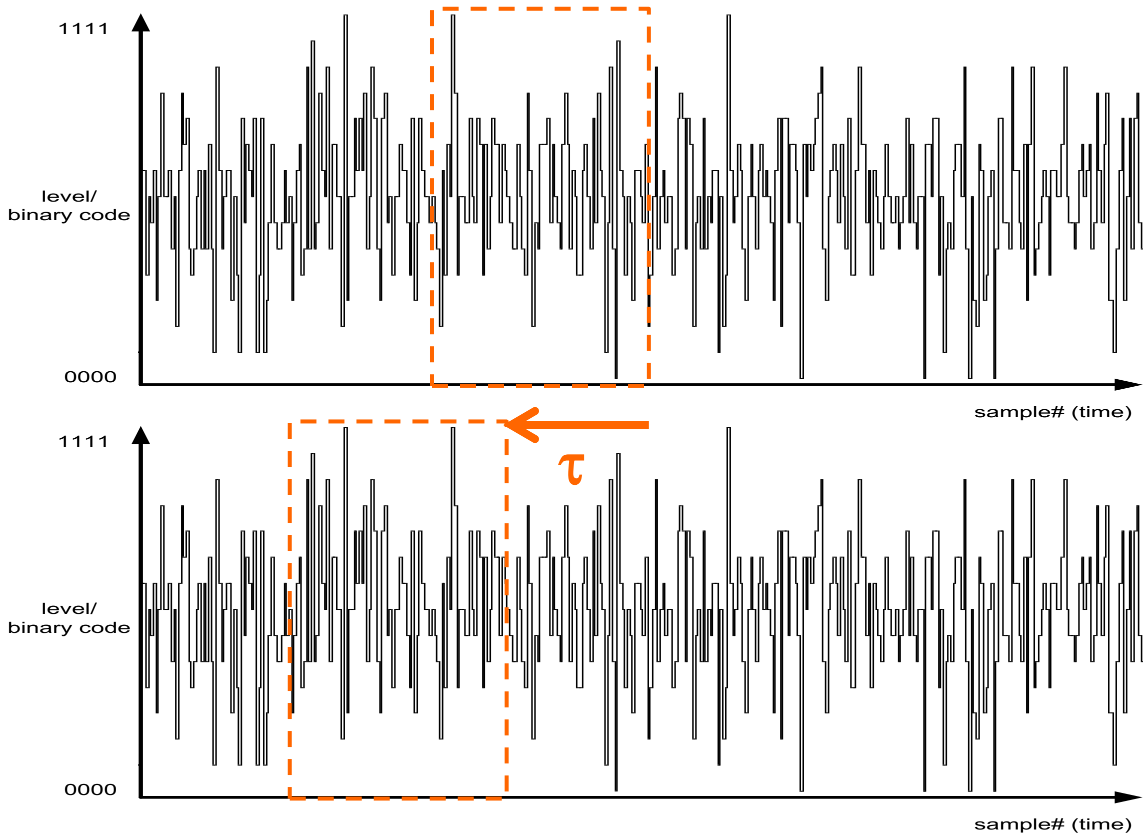
SWINBURNE UNIVERSITY OF TECHNOLOGY



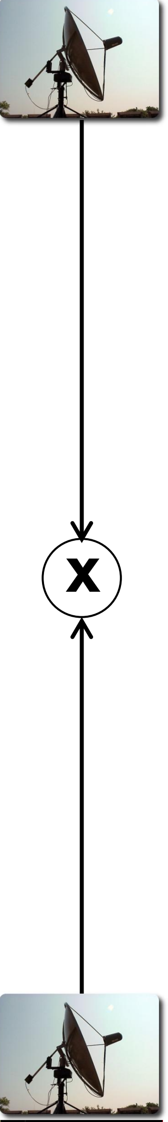
Delay compensation



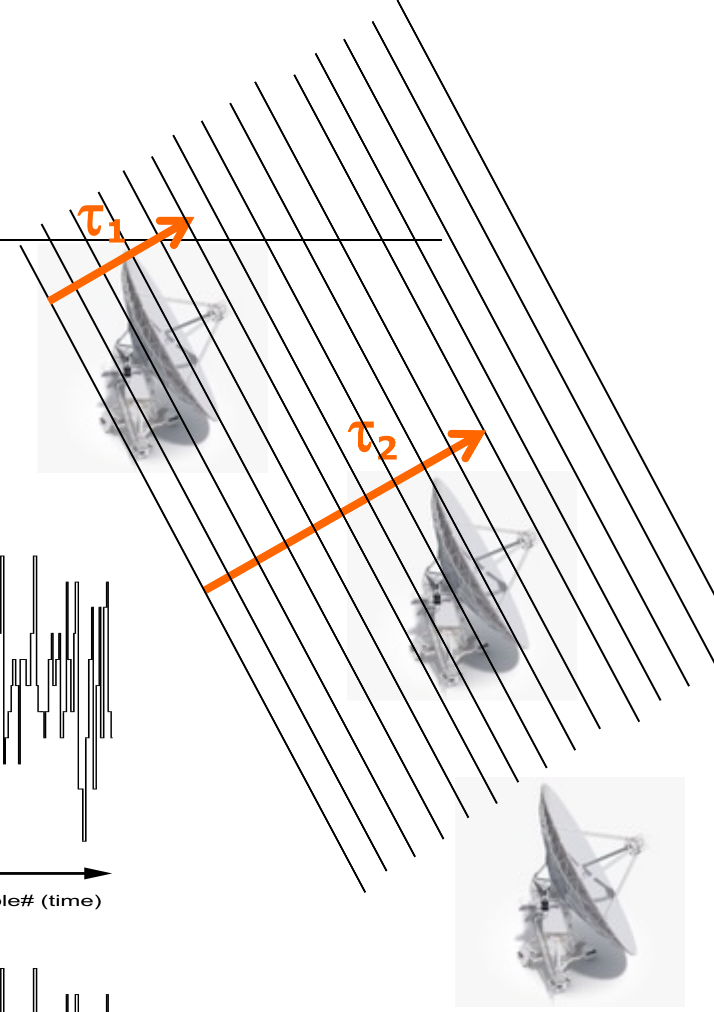
- Delay to the nearest sample is easy:

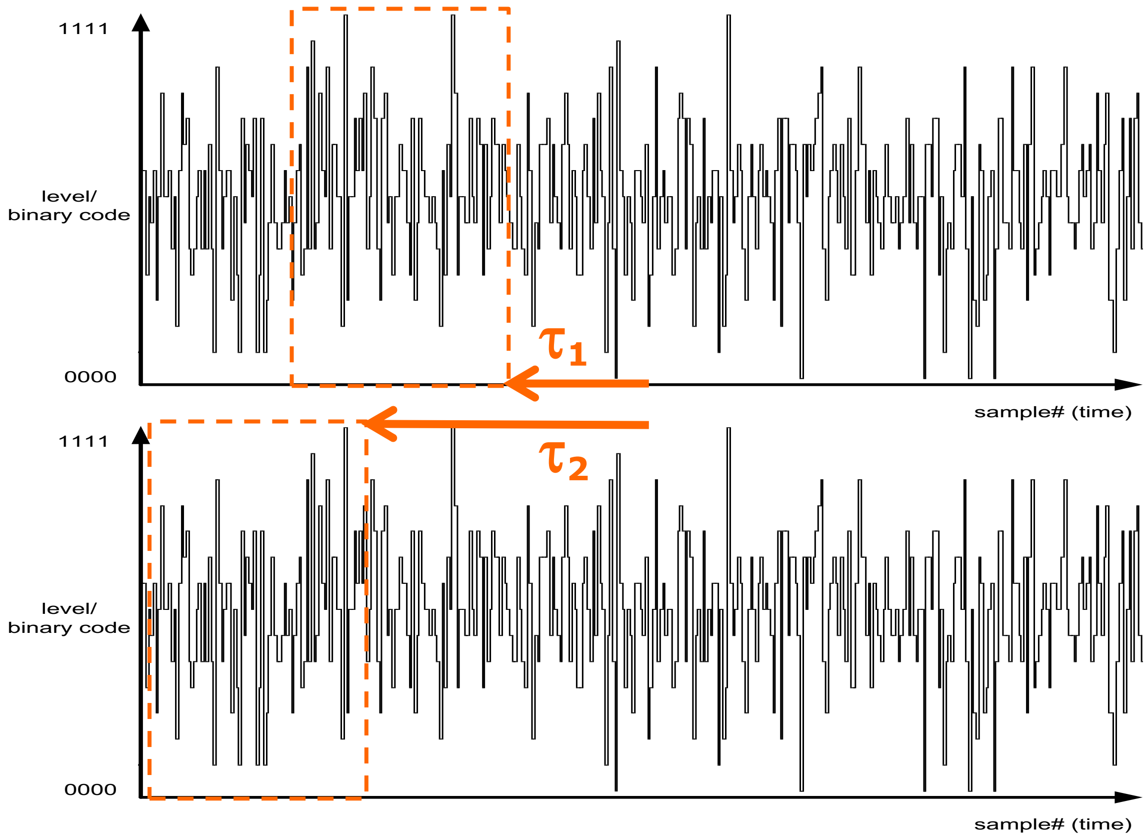


Delay compensation



- In practise, delay all to common reference

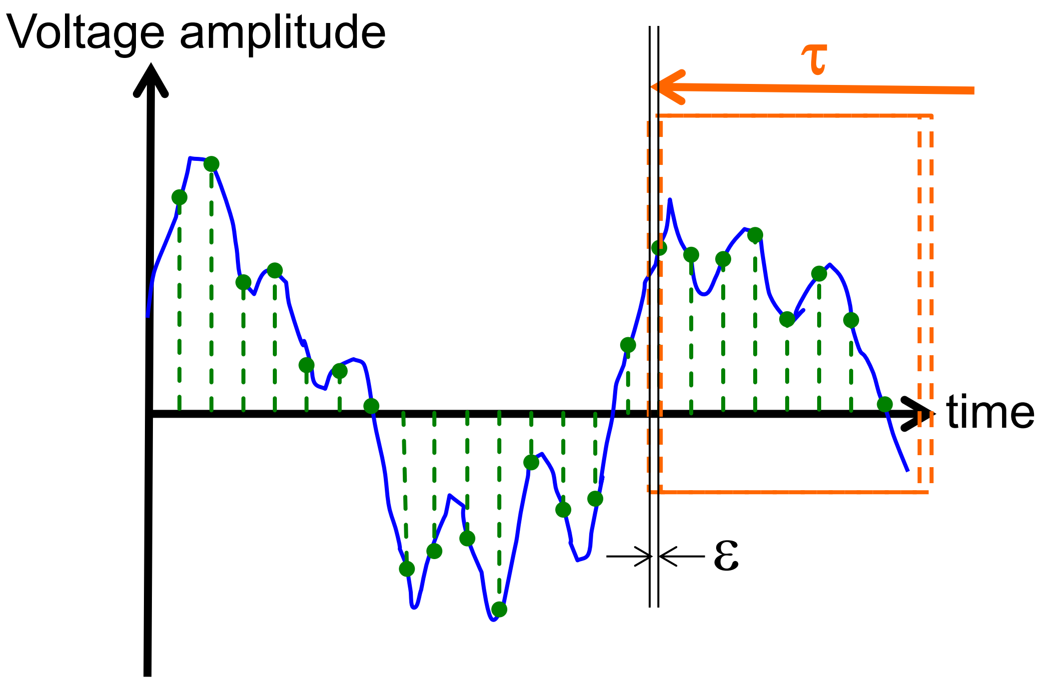


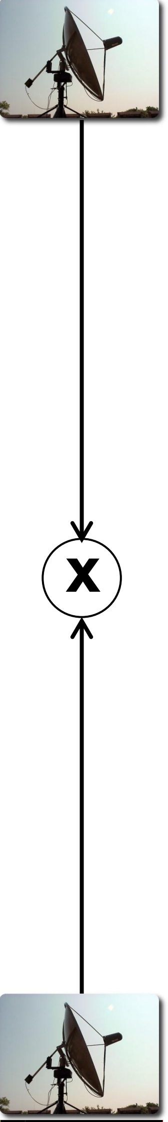


SWINBURNE UNIVERSITY OF TECHNOLOGY

Fractional-sample correction

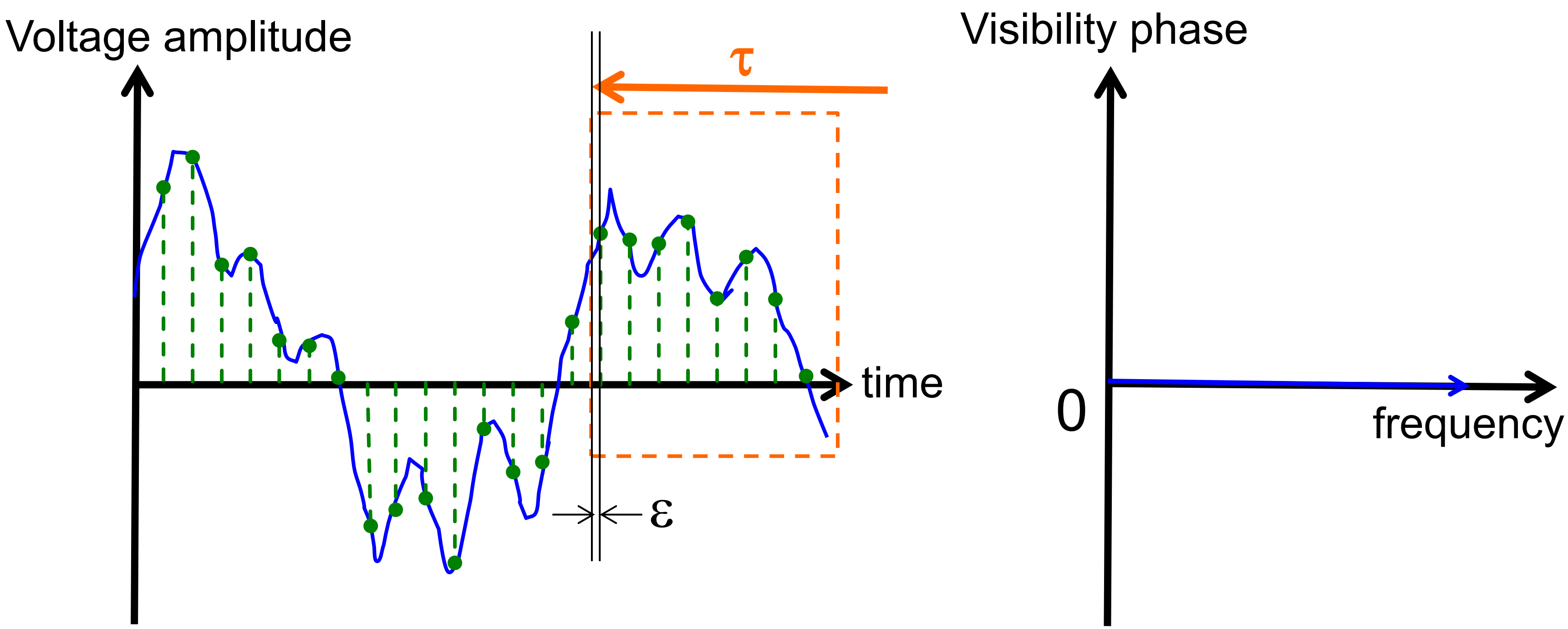
- Sampling prevents perfect alignment of datastreams; always a small error





Fractional-sample correction

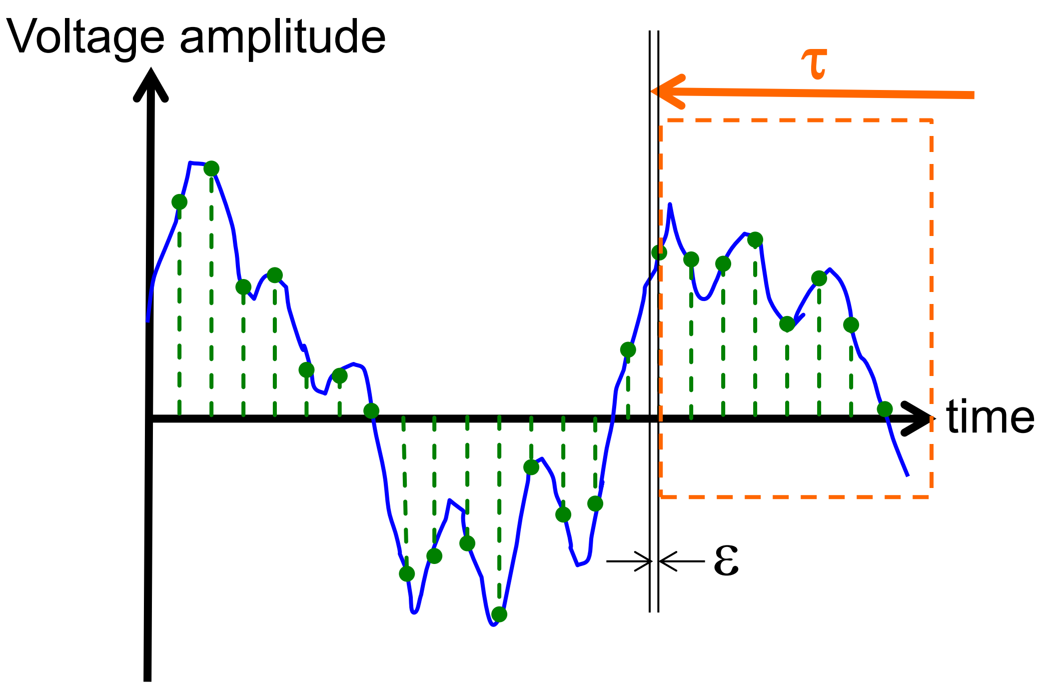
- Sampling prevents perfect alignment of datastreams; always a small error

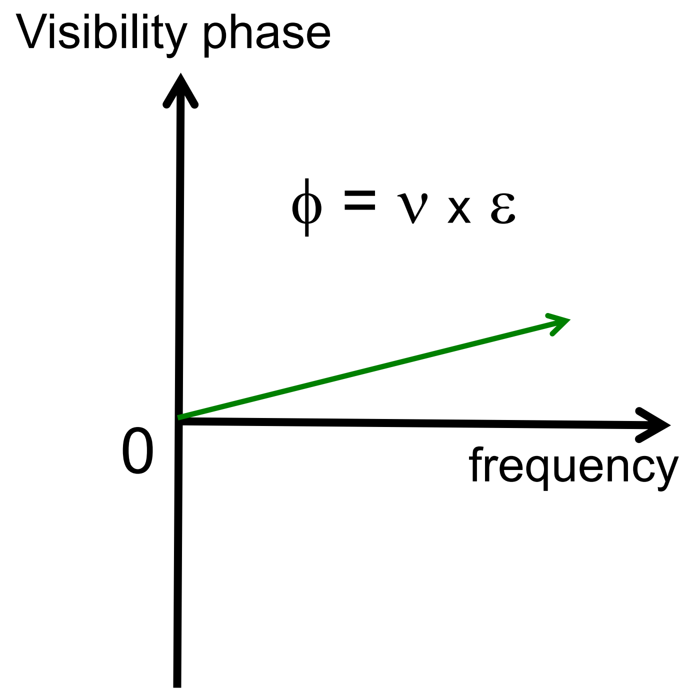


SWINBURNE UNIVERSITY OF TECHNOLOGY

Fractional-sample correction

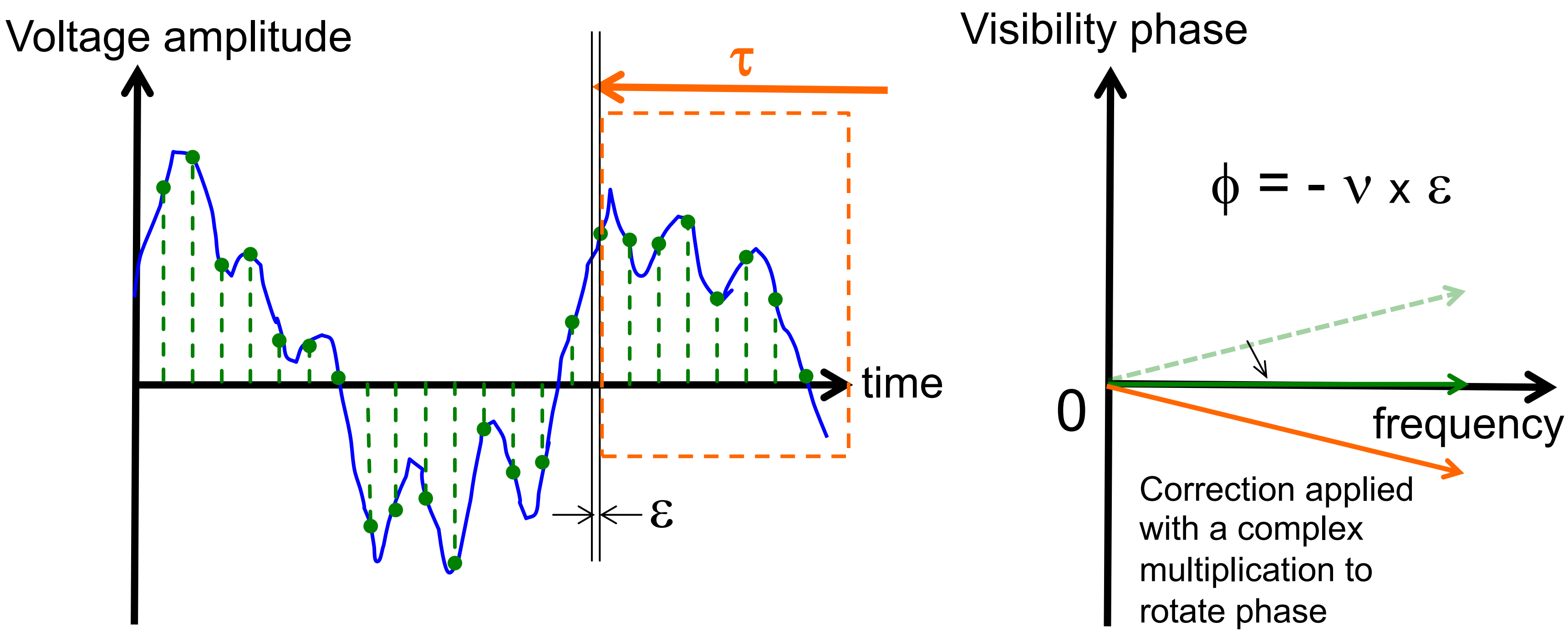
- Sampling prevents perfect alignment of datastreams; always a small error

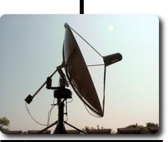




Fractional-sample correction

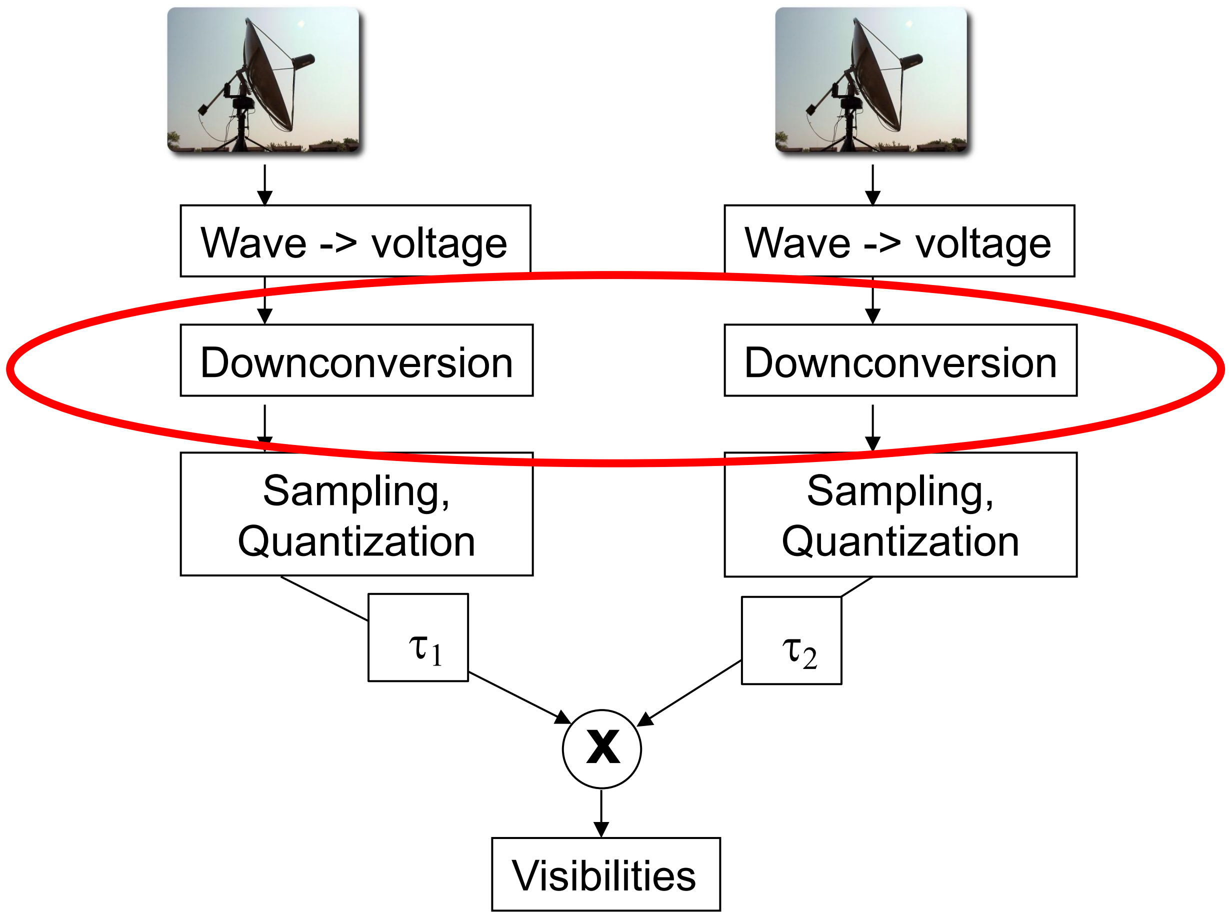
- Sampling prevents perfect alignment of datastreams; always a small error



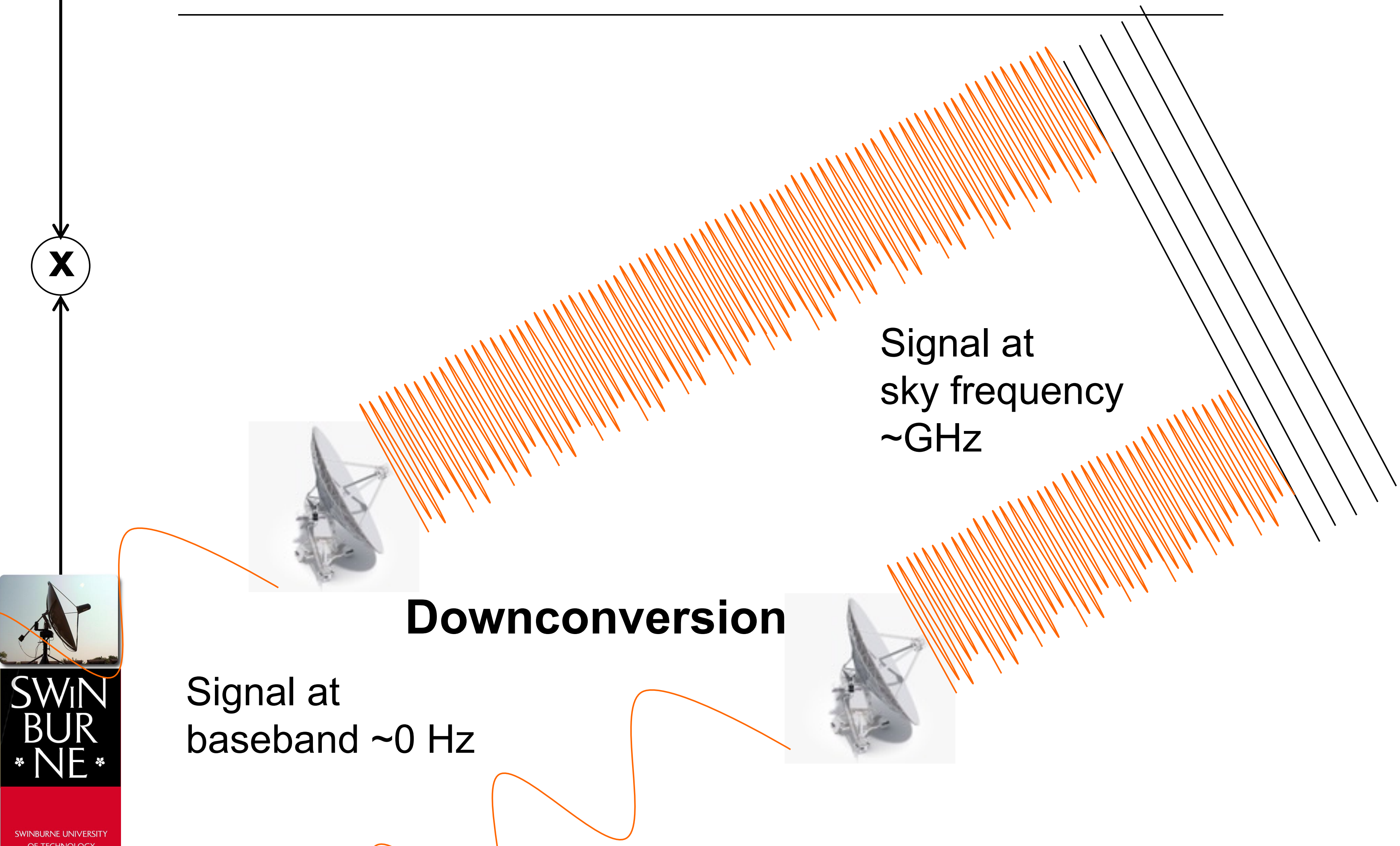


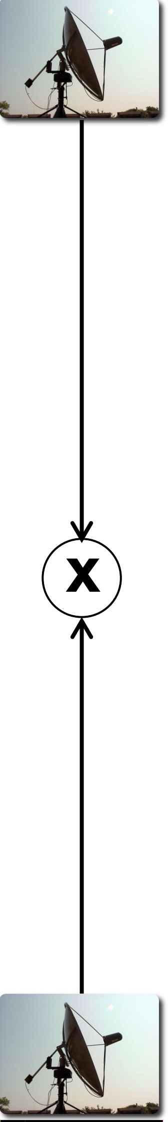
SWINBURNE UNIVERSITY OF TECHNOLOGY

Righting the wrongs



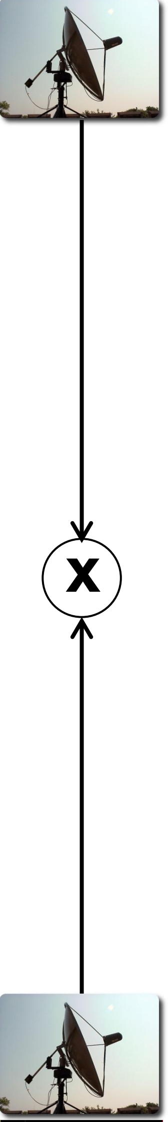
Fringe rotation





Fringe rotation

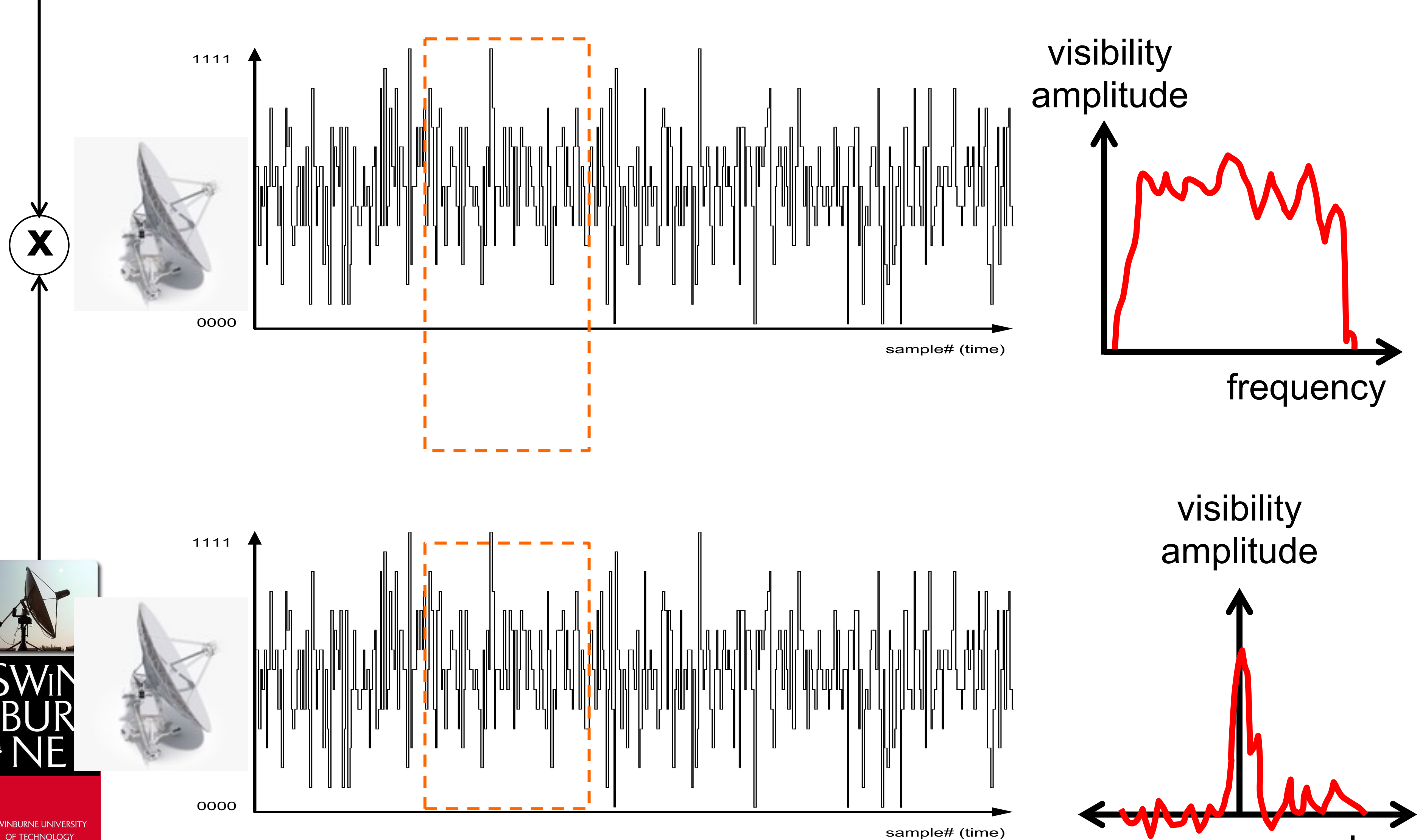
- Implementation: rotate phase using complex multiplier
- $\Delta\phi = 2\pi \nu_{lo} \tau_g$ ν_{lo} = local oscillator frequency, τ_g = applied delay
- Update rate of $\Delta\phi$ depends on how fast τ_g changes:
  - If τ_g is changing fast, correct every recorded sample individually (before the FFT)
  - For shorter baseline / low frequency instruments, can do post-channelisation or even post-accumulation



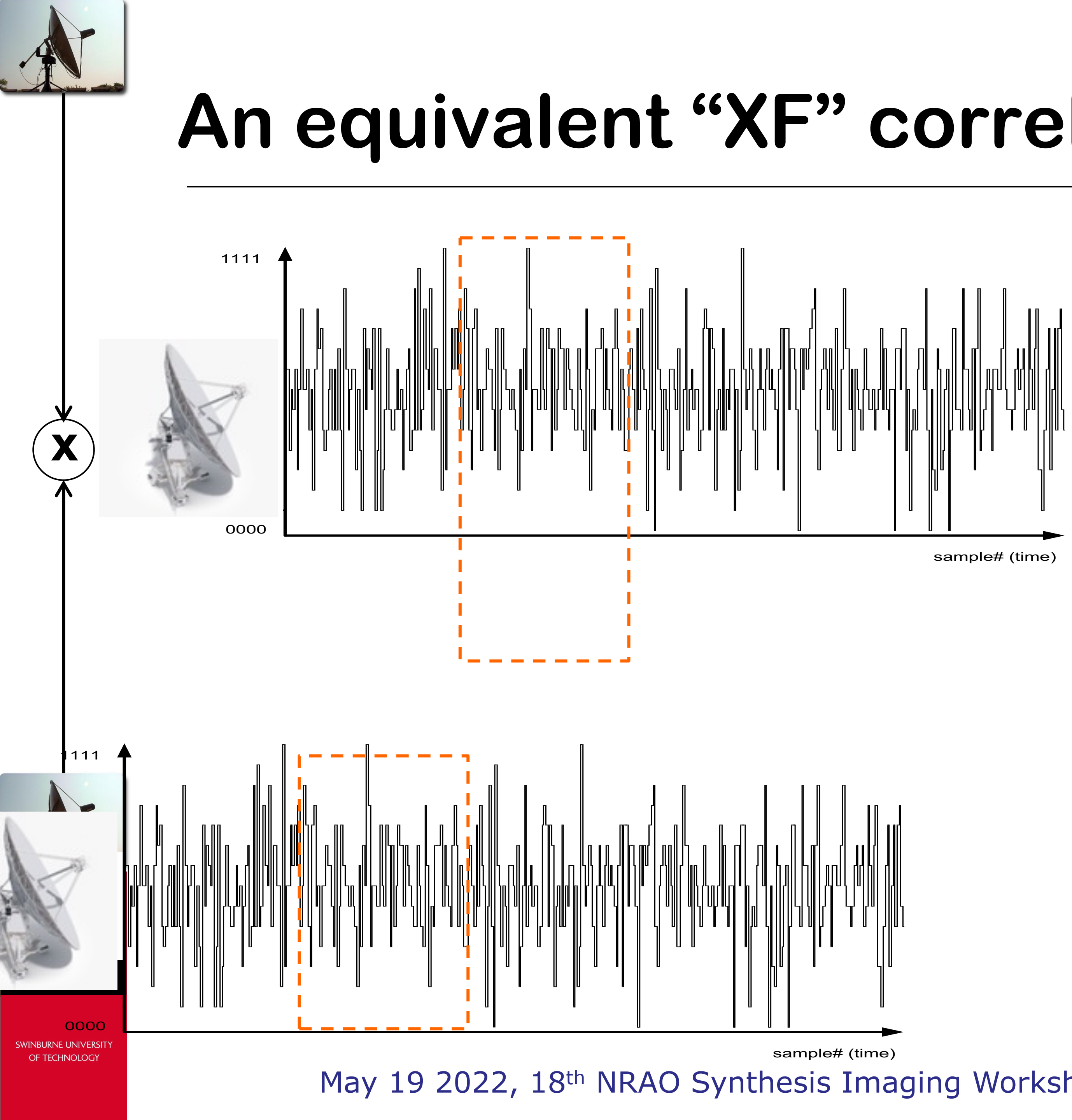
Alternate implementation

- We have shown how to build a practical FX correlator, which first Fourier transforms and then multiplies
- Convolution theorem: **Multiplication** in the frequency domain is equivalent to **convolution** in the time domain
- It is mathematically equivalent to convolve the two signals in the time domain and then Fourier transform

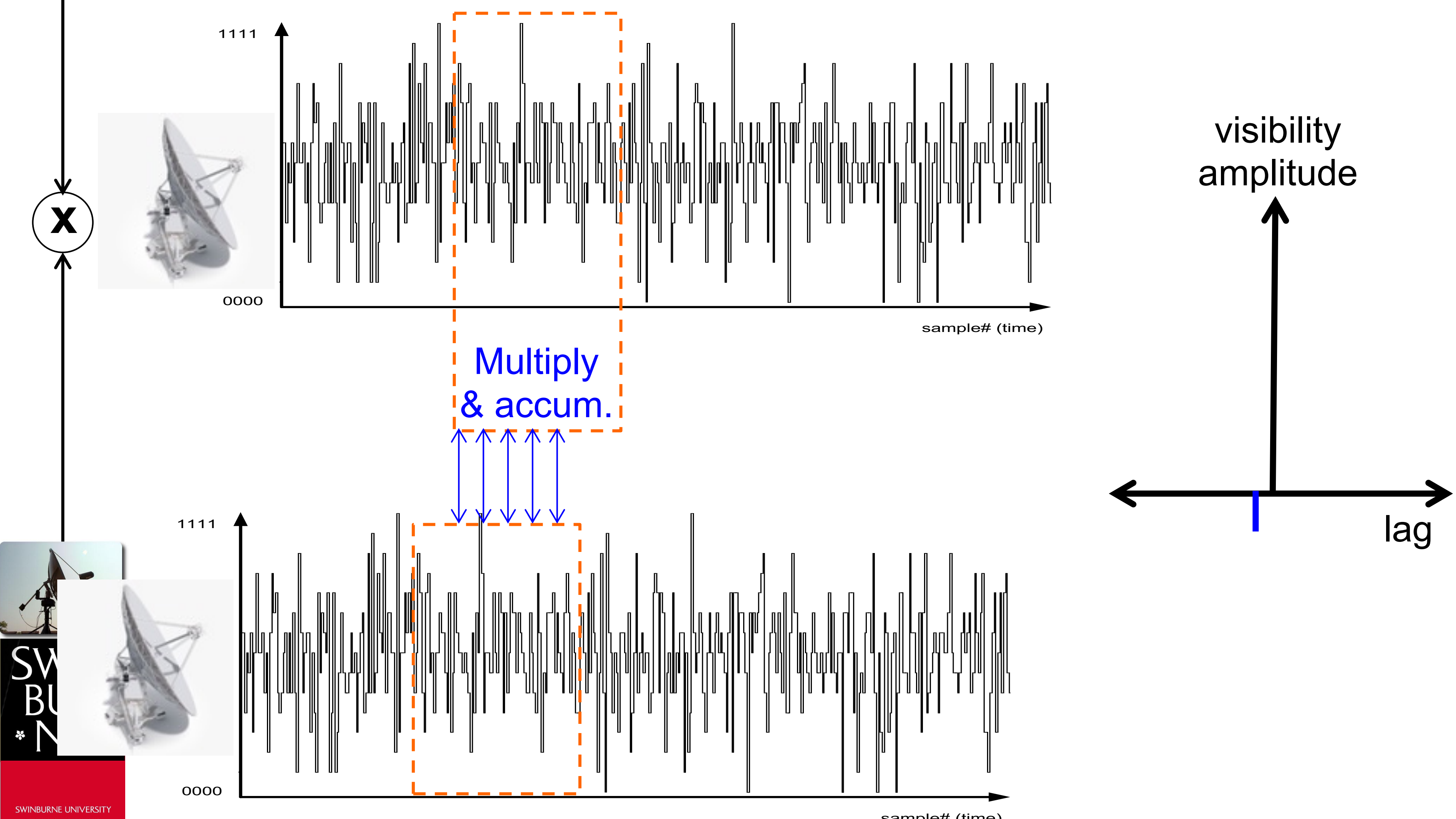
An equivalent "XF" correlator



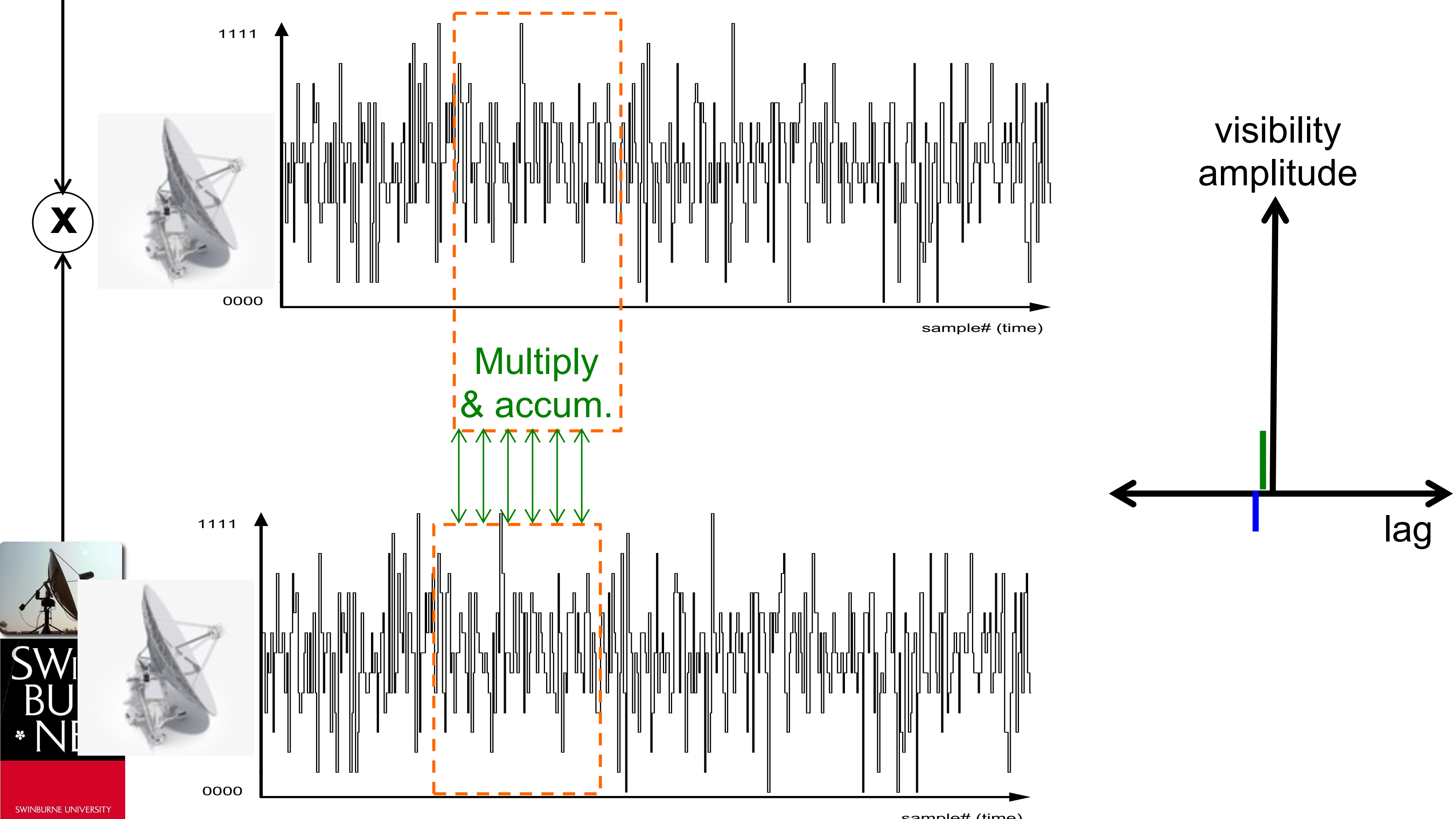
An equivalent “XF” correlator



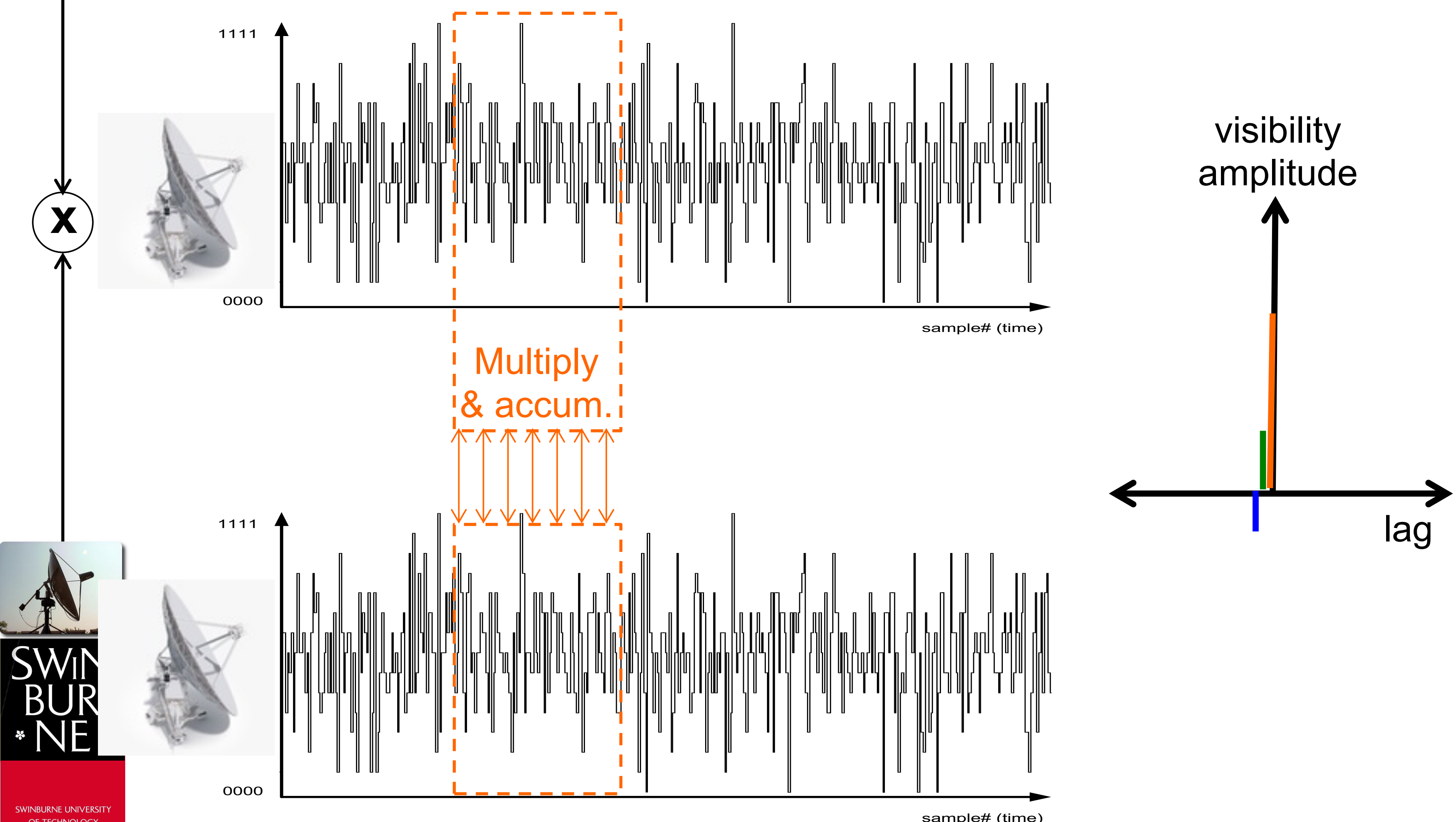
An equivalent “XF” correlator



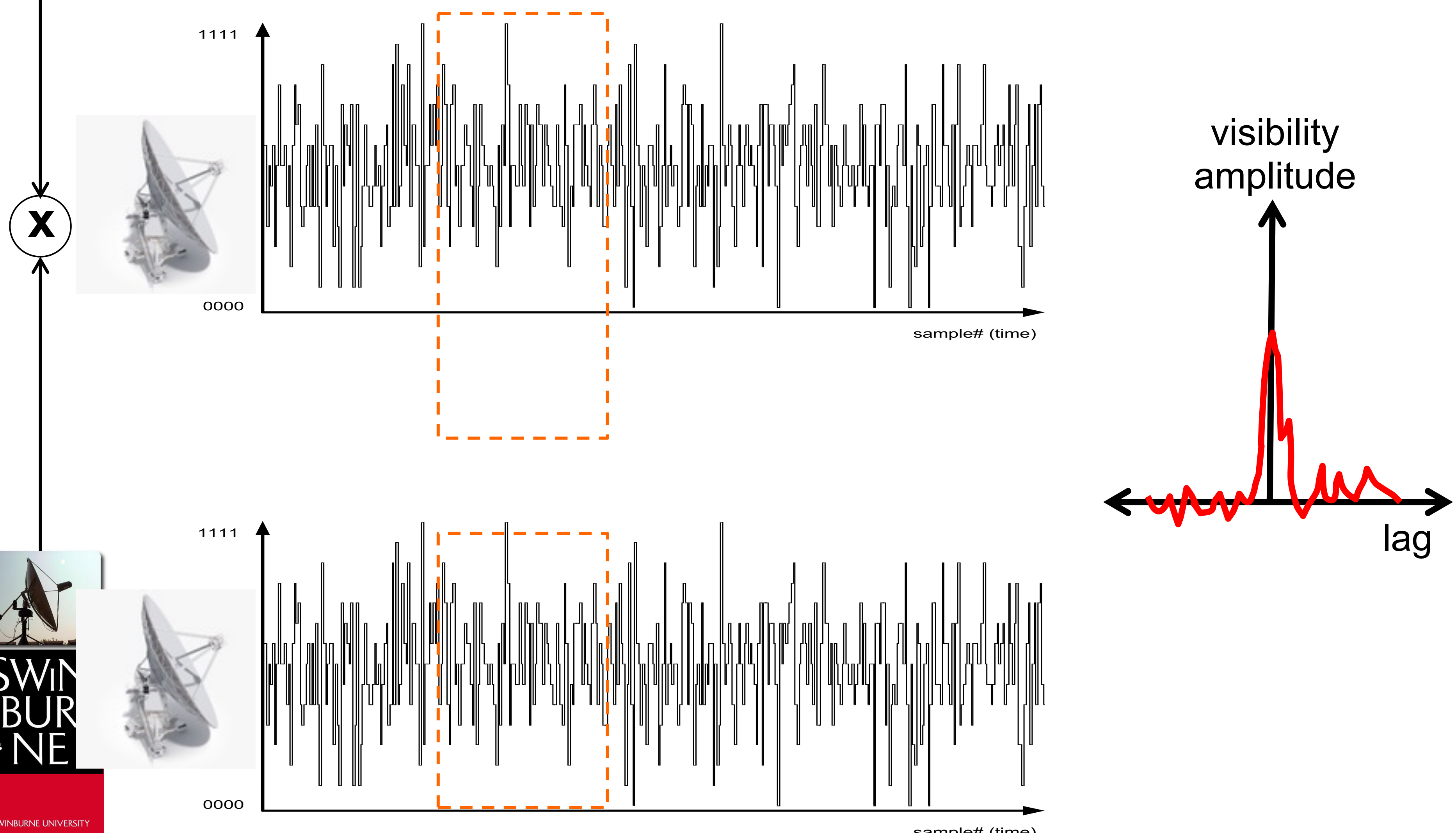
An equivalent "XF" correlator



An equivalent “XF” correlator

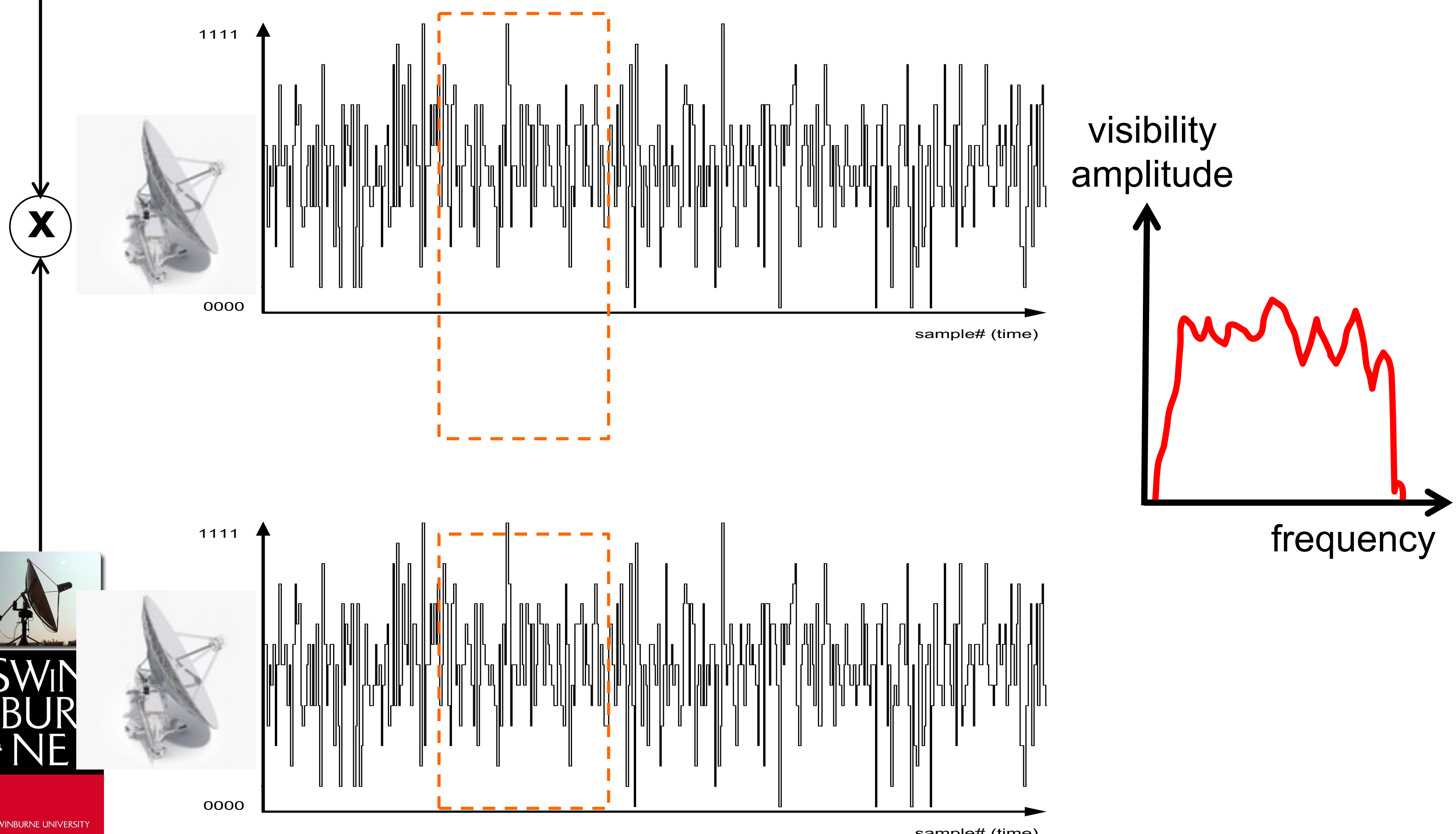


An equivalent "XF" correlator

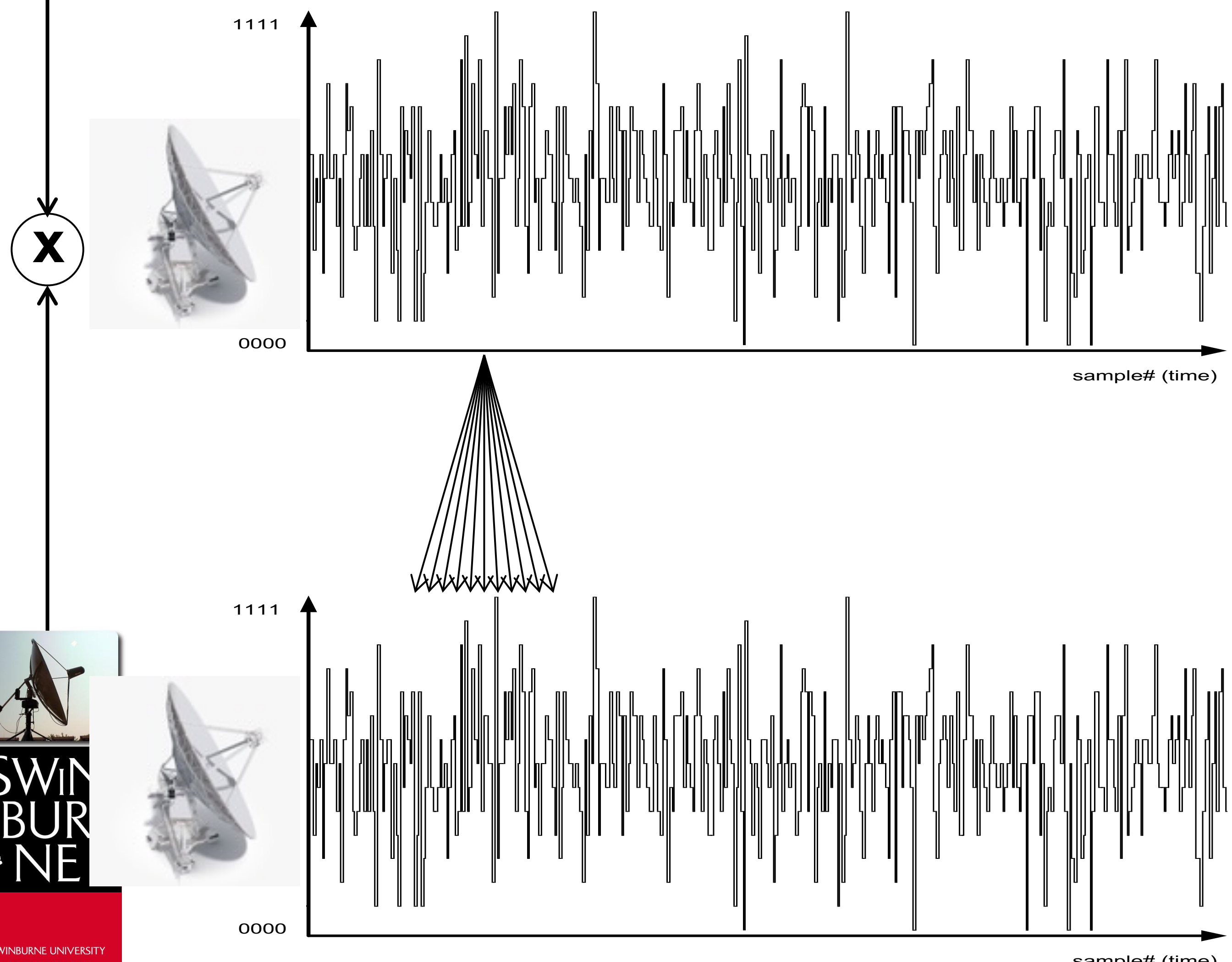


SWINBURNE UNIVERSITY OF TECHNOLOGY

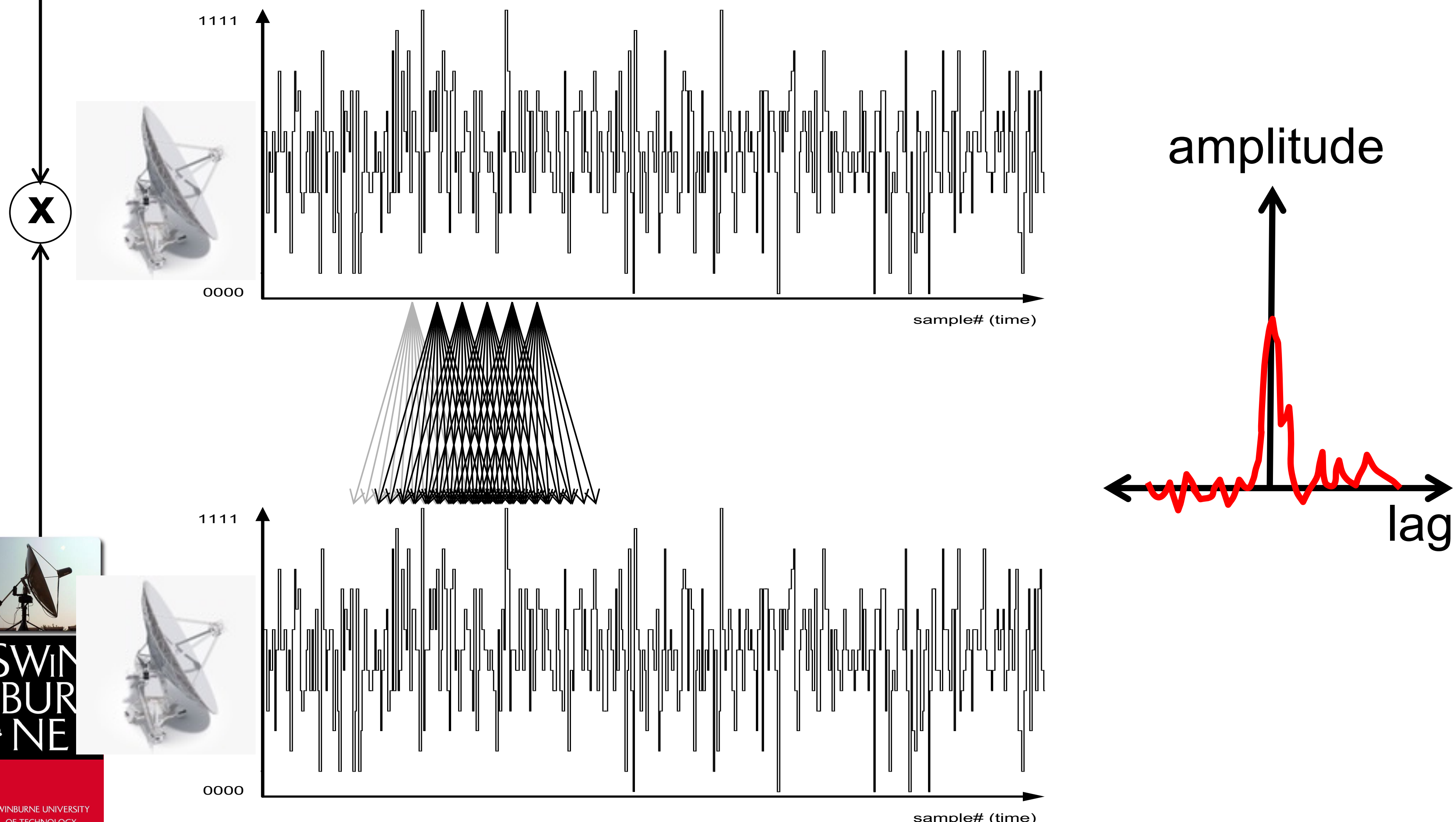
An equivalent "XF" correlator



A realistic XF correlator



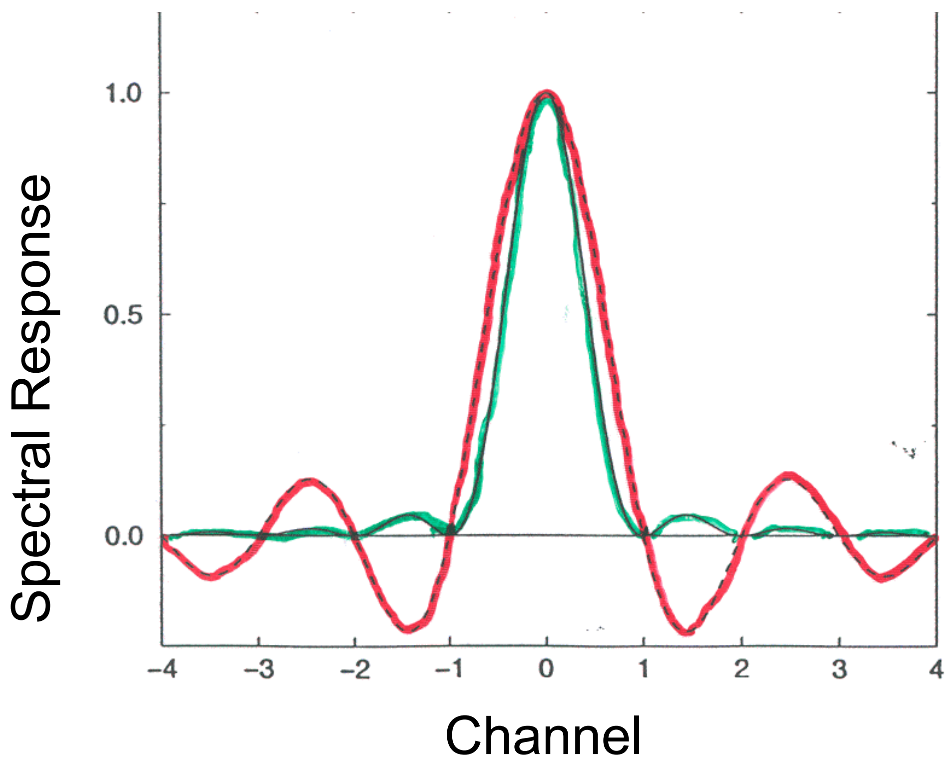
A realistic XF correlator



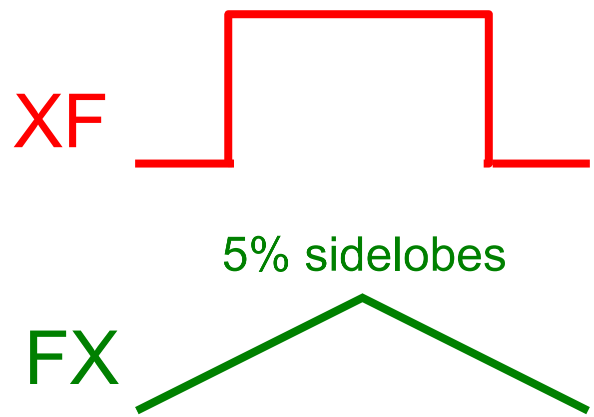
SWINBURNE UNIVERSITY OF TECHNOLOGY

XF vs FX

- Different windowing in time domain gives different spectral response



22% sidelobes! Reduce with Hanning smoothing

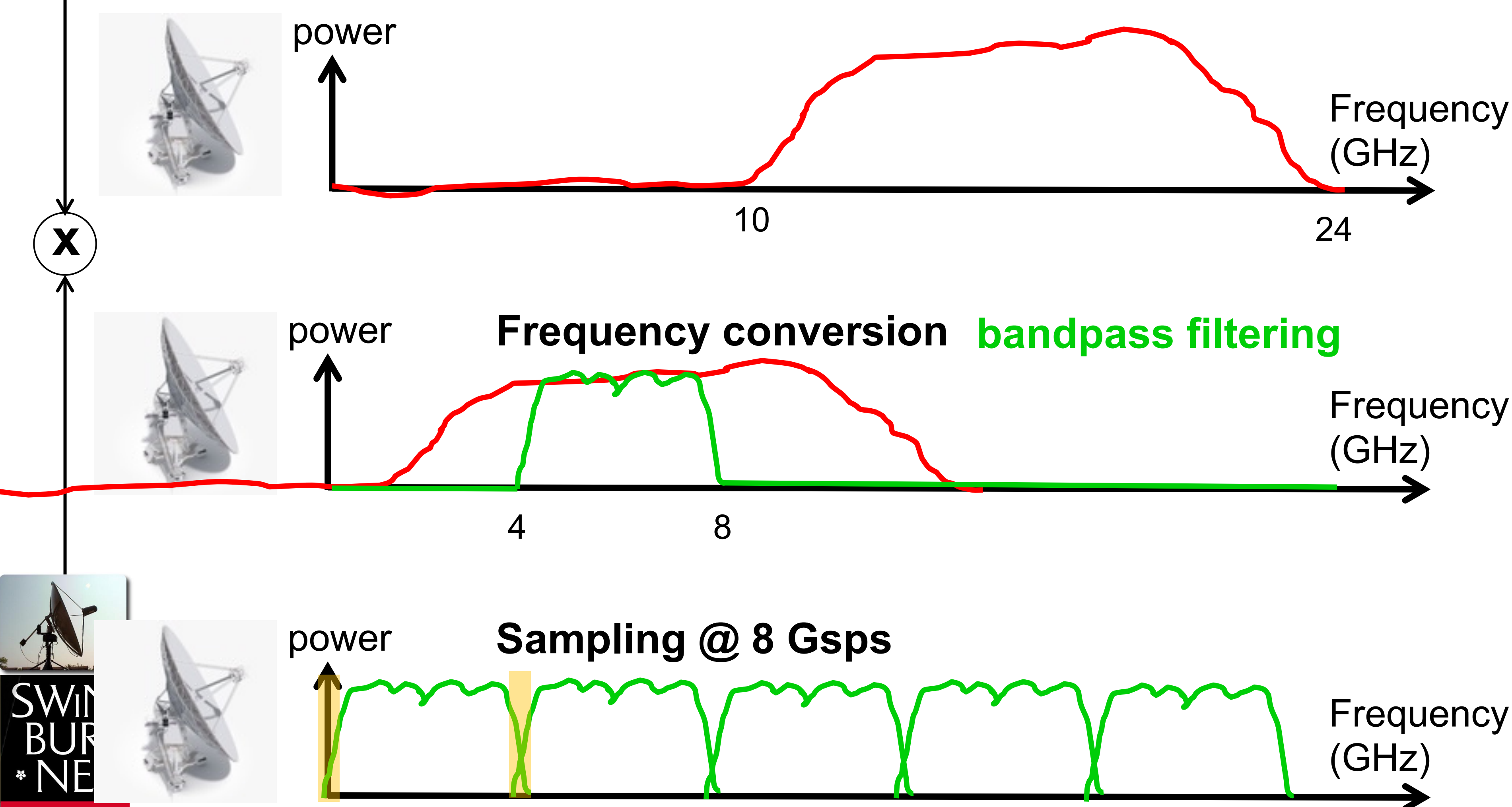


Lag weighting

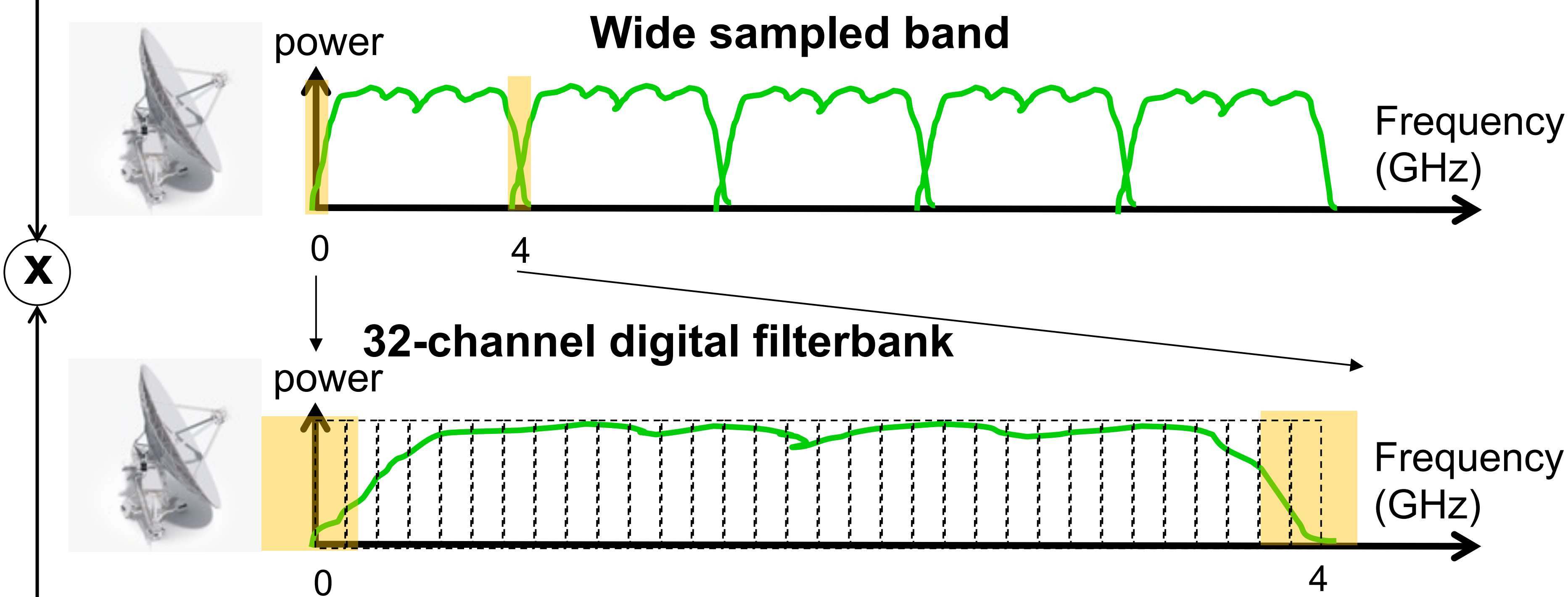
XF vs FX: which is better?

- Desire for reduced artifacts favours FX
  - Main advantage of XF: can use very efficient low-precision integer multipliers up-front
  - But FX many fewer operations overall, unaffected by trend to higher bit depth
  - FX also: access to frequency domain at short timescale allows neat tricks and higher precision correction of delay effects
  - Modern correlators mostly FX-style, and often have multiple cascaded filter steps (~GHz recorded band chopped into ~100 MGz chunks and correlated separately)

The full package



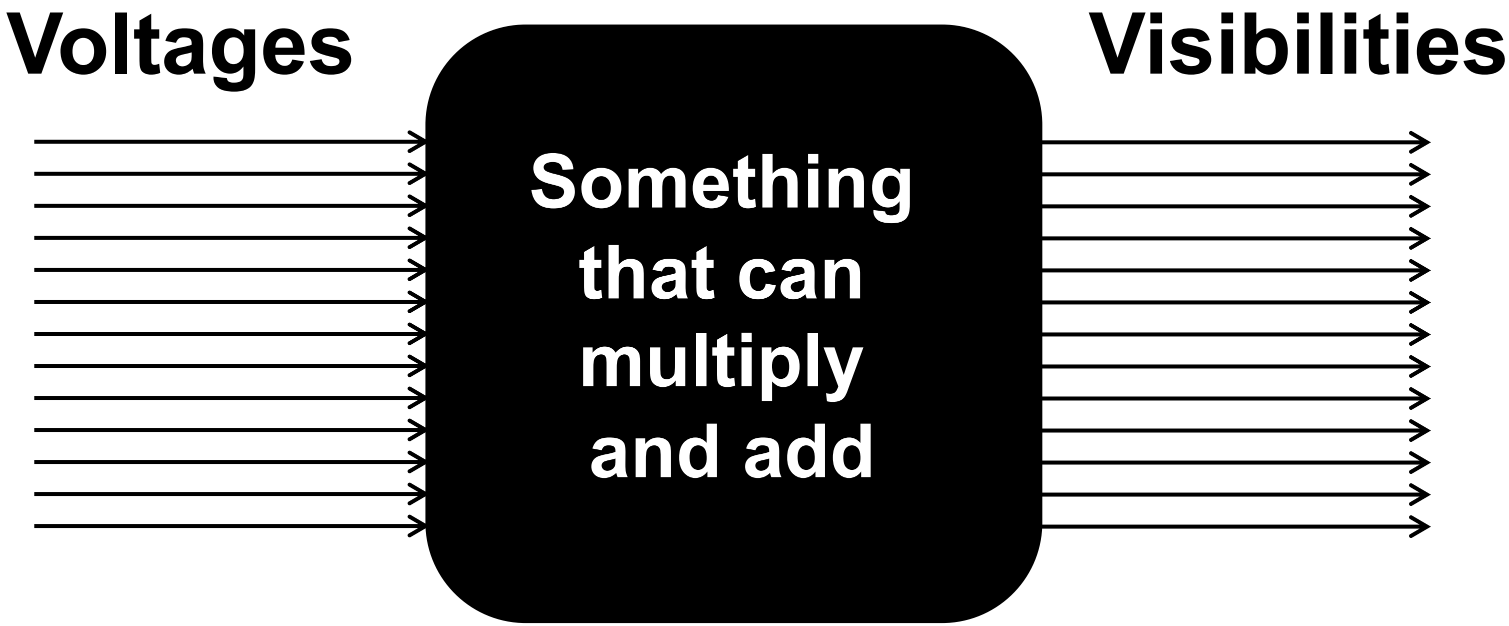
The full package



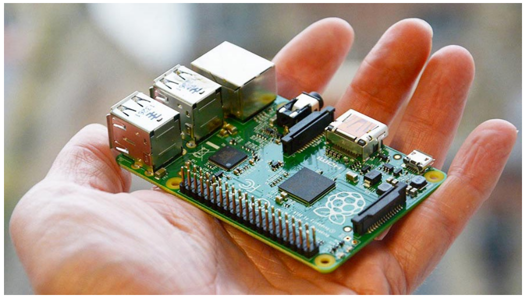
Each of these 128 MHz chunks can then be treated by separate FX style correlator in parallel: fringe rotation, channelization, delay compensation, and cross-multiplication

SWINBURNE
SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlator platforms



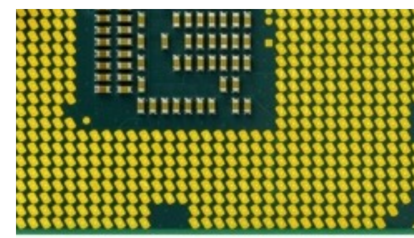
Correlators on CPUs




```
status = vectorFFT_CtoC_cf32(complexunpacked, fftd, pFFTSpecC, fftbuffer);
if(status != vecNoErr)
  csevere << startl << "Error doing the FFT!!!" << endl;

     ...

status = vectorAddProduct_cf32(vis1, vis2, &(scratchspace->threadcrosscorrs[resul
```



SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlators on CPUs

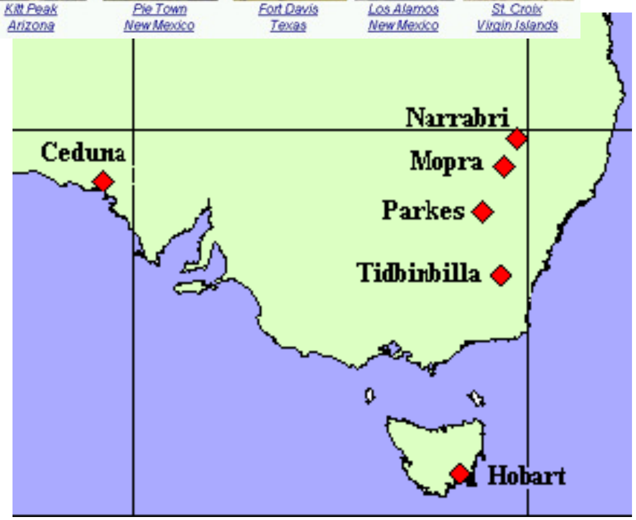
- Many positive points:
  - Can implement in “normal” code (e.g., C++); maintainable, many skilled coders
  - Development effort transferrable across generations of hardware
  - Incremental development is trivial
  - Natively good at floating point (good for FX), no cost to do high precision
- One major disadvantage:
  - CPUs not optimised for correlation; big system like ngEHT would take **many** CPUs.

SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlators on CPUs

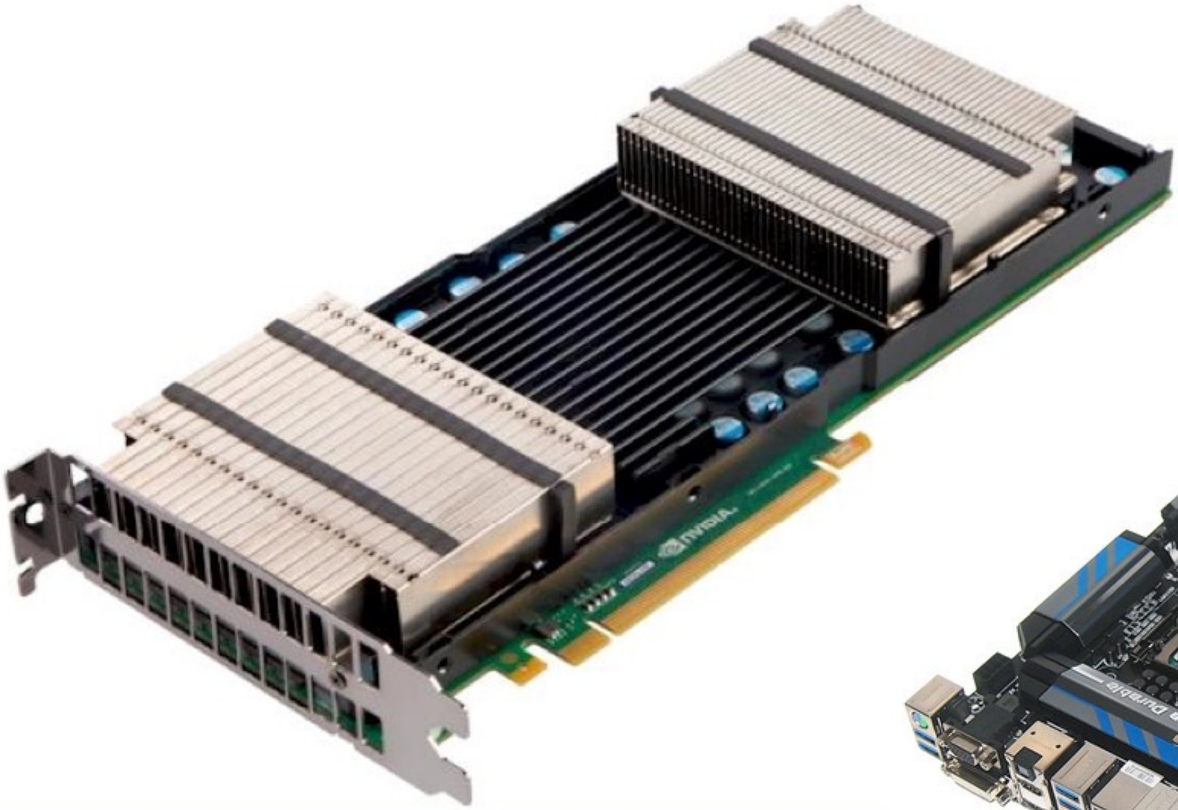
The Very Long Baseline Array, 10 stations

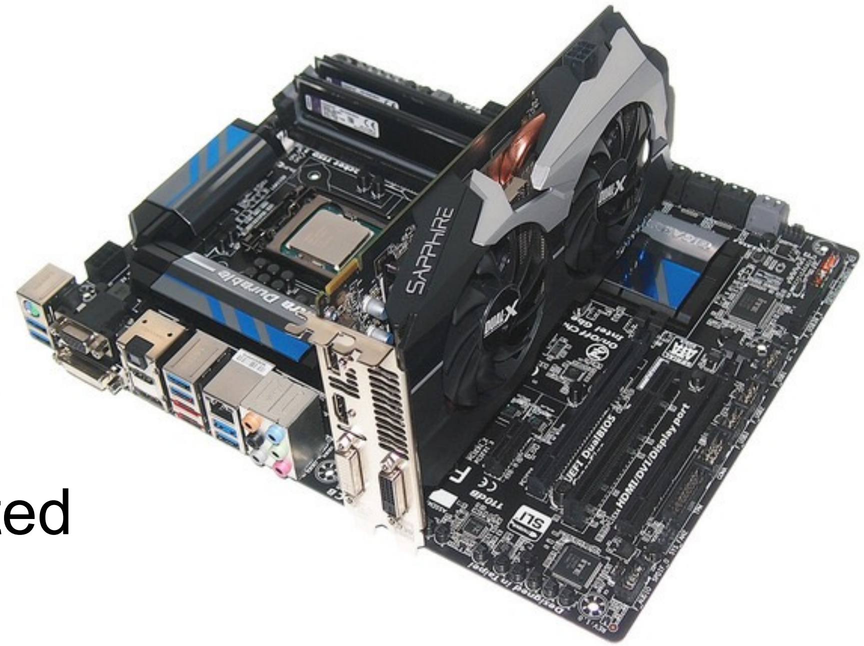
The European VLBI Network, ~30 stations



The Long Baseline Array, Australia, ~6 stations

Correlators on GPUs



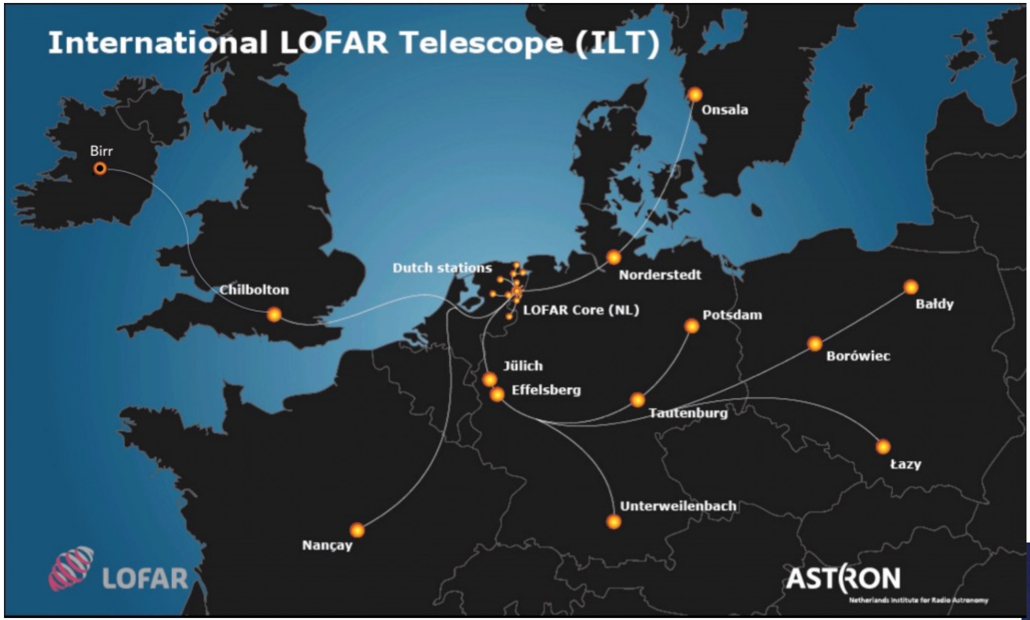


Like CPUs, GPUs are mounted on a standard motherboard

Correlators on GPUs

- Advantages:
  - More powerful and more efficient than CPUs
  - Also good at floating point
- Disadvantages:
  - Writing code is more difficult (GPUs are more specialized, less flexible: need to carefully manage data transfers)
  - Fewer expert GPU programmers available
  - Transfer-ability of code across hardware generations harder (capabilities change faster, need new code to use)

Correlators on GPUs



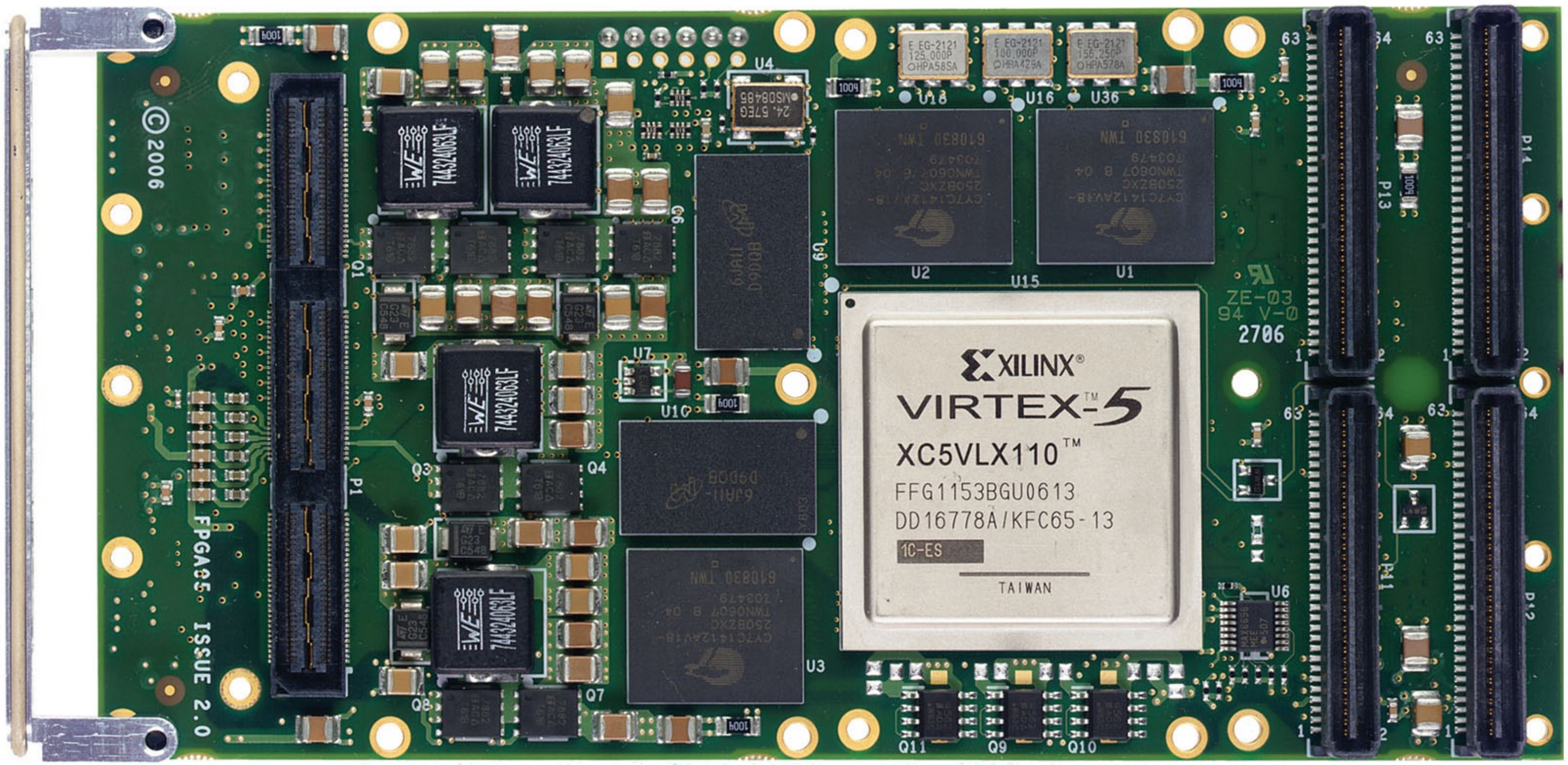
The Low Frequency Array (LOFAR), 76 stations

GMRT, India, 30 stations

Now underway: adding GPU acceleration to “general purpose” software correlators

SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlators on FPGAs

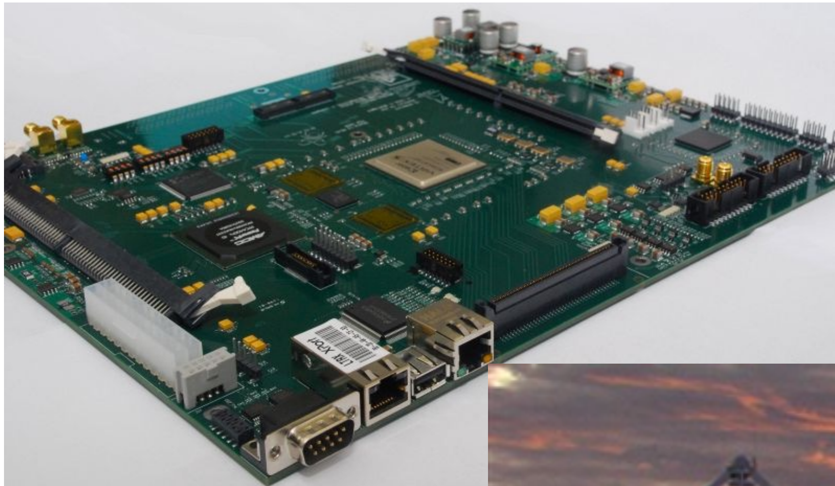


Correlators on FPGAs

- Advantages:
  - More efficient than CPUs or GPUs, particularly for integer multiplication – big power savings
- Disadvantages:
  - Programming is harder again (especially debugging), yet fewer experts
  - Transfer-ability across hardware generations even more limited
  - Synchronous (clocked) system, less robust to perturbations c.f. CPUs/GPUs

SWINBURNE UNIVERSITY OF TECHNOLOGY

Correlators on FPGAs

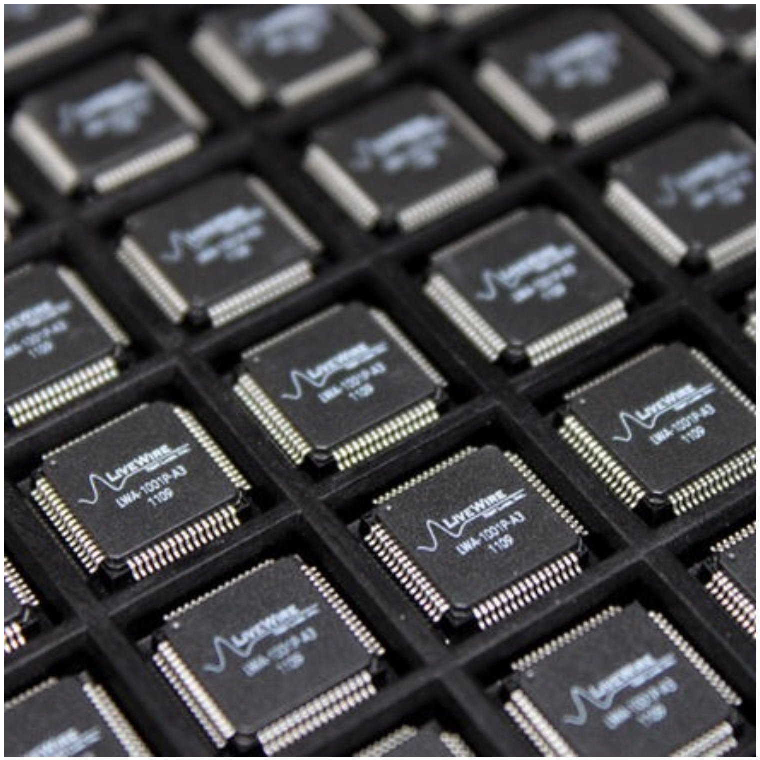


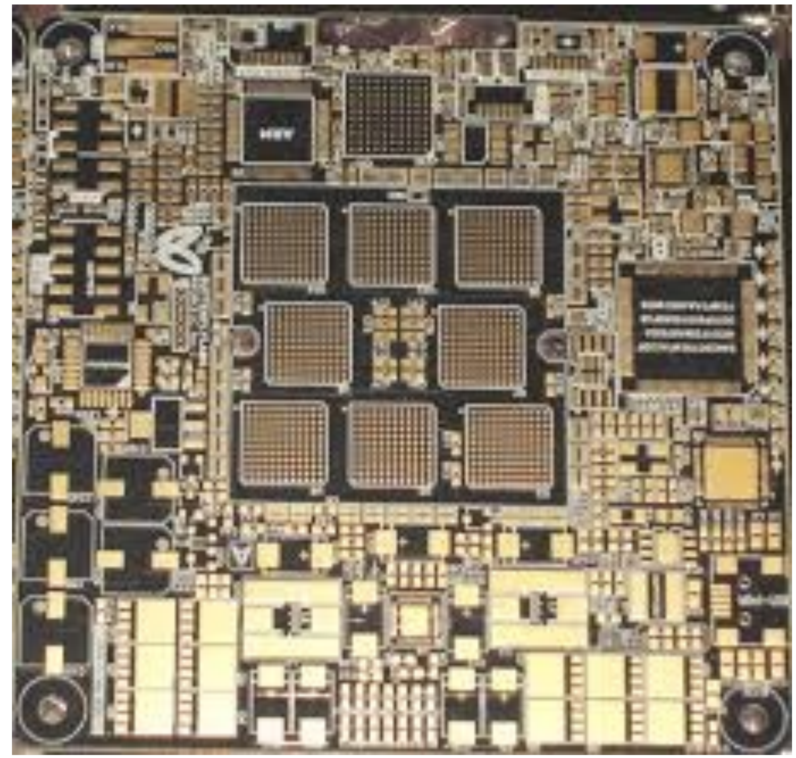
“Roach” reconfigurable FPGA board used for correlation

MeerKAT, 64 dishes

SWINBURNE UNIVERSITY OF TECHNOLOGY

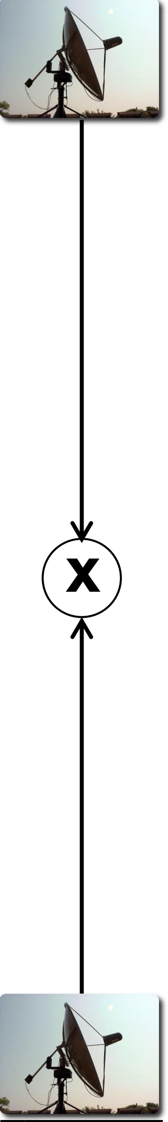
Correlators on ASICs





As with FPGAs, ASICs are mounted on boards

SWINBURNE UNIVERSITY OF TECHNOLOGY



Correlators on ASICs

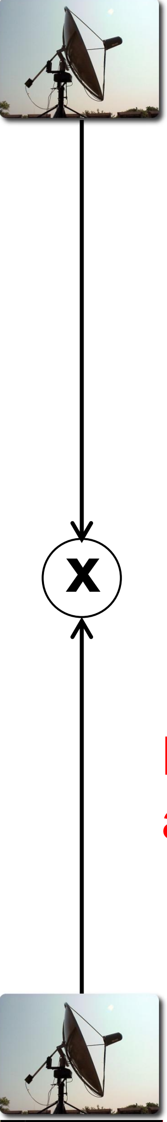
- Advantages:
  - Highest possible efficiency, low per-unit cost
- Disadvantages:
  - Highest development cost (time and manufacturing setup)
  - Specialized knowledge required
  - Can't be changed / very difficult to upgrade during lifetime

SWINBURNE UNIVERSITY OF TECHNOLOGY

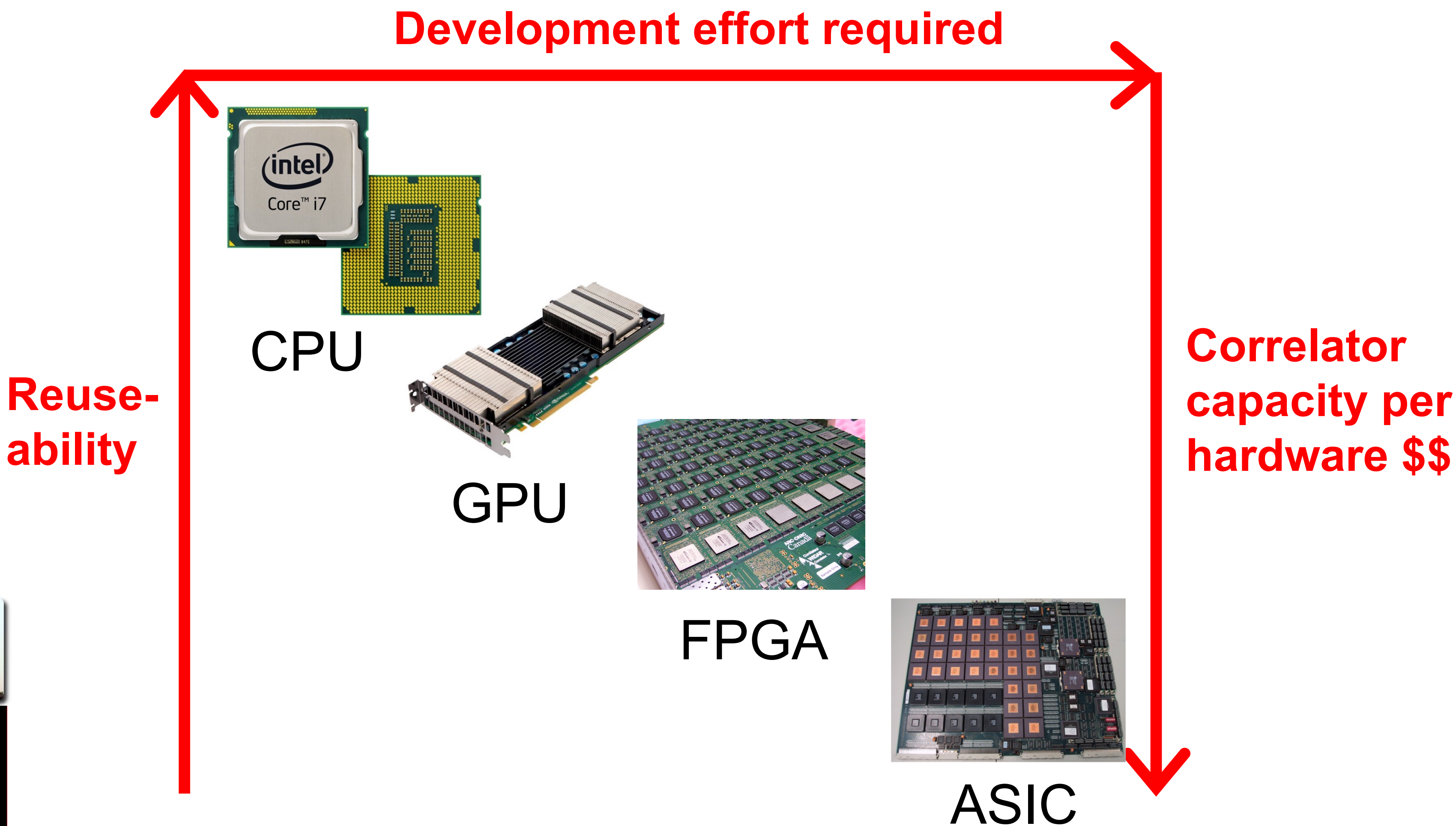
Correlators on ASICs

The Atacama Large Millimetre Array, Chile

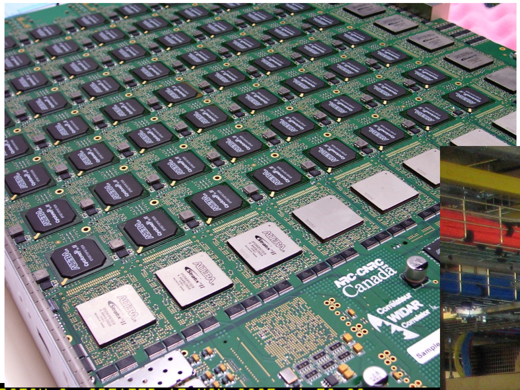
The Very Large Array, New Mexico

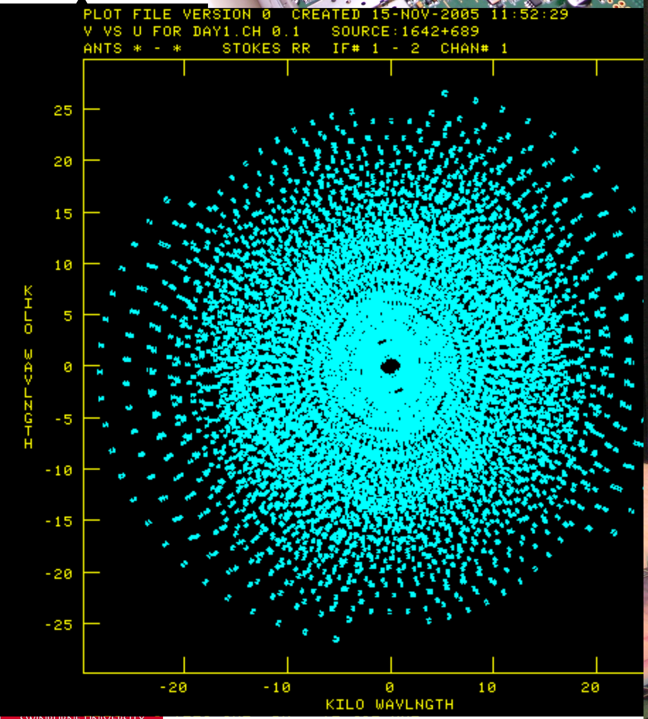


Correlator platform overview



The end




PLOT FILE VERSION 0 CREATED 15-NOV-2005 11:52:29
V VS U FOR DAY1.CH 0.1 SOURCE:1642+689
ANTS * - * STOKES RR IF# 1 - 2 CHAN# 1
KILO WAVLNGTH
KILO WAVLNGTH
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-10
0
10
20