Outline

Quick overview of imaging concepts
  van-Cittert Zernike Theorem
  Imaging as a minimization problem

Applying imaging fundamentals to real data.
  Mapping data to images
  Choosing an imaging algorithm
  Deconvolution and algorithmic consequences.
  Computational requirements
van Cittert - Zernike Theorem

\[ V(u, v) \Rightarrow I^{sky}(l, m) \]
Useful Fourier Pairs

Aperture

UV Coverage

Primary Beam

Synthesized Beam - PSF
Imaging - Iterative Minimization

**Major Cycle (Imaging)**

1. **DATA MODEL**
2. **RESIDUAL**
3. **GRIDDING**
   - Use Flags and Weights
4. **iFFT**
5. **DE-GRIDDING**

**Minor Cycle (Deconvolution)**

- **RESIDUAL IMAGE**
- **MODEL IMAGE**
- **Image Definition**
- **FFT**

**Use Flags and Weights**
Mapping Data to Images - I

Cube Imaging

+ Clean M channels out of N data channels
+ Natively parallel way of CLEANing WB data
+ Can derive spectra for bright sources.

- Limited sensitivity per channel.
- Varying resolution across channels

MFS – Continuum Imaging

- Single sensitive continuum image
- Resolution given by the highest frequencies
- Better UV coverage and PSF behaviour
- Trouble dealing with steep spectrum sources
- Does not model the psf frequency behaviour
Mapping Data to Images - II

Continuum

Multi Term Multi Frequency Synthesis

- Sensitive continuum image
- PSF and sky frequency behaviour modelled
- Derives in band spectral index.
  - Point sources SNR >50
  - Extended sources SNR >100

Polarimetric Imaging

- Maps feed correlations to Stokes
- Provides the ability to measure linear polarization and even perform RM synthesis

More info in Frank’s Talk
Stitched Mosaics – Linear Mosaics

- Equivalent to optical mosaic construction.
- Separate pointings stitched together to provide a large coverage in the sky.
- Largest angular scale is still given by the largest angular scale sampled by the interferometer.
- Improved sensitivity in overlapping regions.

Joint Mosaic

- Imaged on a single UV grid.
- Convolution function + phase. [See Urvashi’s talk for theory]
- Increased sensitivity in overlap regions.
- Important to perform PB correction.
- No region is free from chromatic PB effects.
Imaging - First Image (Dirty Image)

If you only knew about vCZ theorem and had radio data what would you do?

Historically the first interferometers did this by hand. VLA was the interferometer to introduce deconvolution to the mainstream. Hogbom 1974, Schwab & Cotton 1975 are the first papers to introduce CLEAN.
Dataset Supernova Remnant G55.7+3.4

- It is a dataset available as a part of the VLA Imaging CASAguide - http://casa.nrao.edu/Data/EVLA/SNRC55/SNR_G55_10s.calib.tar.gz
- Dataset contains calibrated scans of the SNR at L-band in D-configuration.
- Pick a cell size to allow for 3 to 5 pixels across the PSF.
- Make an image of the entire FoV.
- L-Band FWHM of the primary beam is 30arcmin at 1.5GHz.

UV Coverage
Four colors one for each spectral window (IF/subband)
First Widefield Image

- Fourier Transform of the gridded visibilities. Gridding kernel used is a prolate spheroidal function.
- The imprint of the sampling function (psf) is clearly present.
- Source well outside PB mainlobe.
- Note bright sources far from the object of interest.
- Field has point sources and extended emission.

RMS: 21 microJy
First Widefield CLEAN Image

- Deconvolution clearly improved the image.
- Every CLEAN produces a model, residual, psf and pb images.
- The distortions around the bright sources in the edges are showing widefield effects.
- What do we call widefield?
  - Corrections for non co-planar baseline effects
  - Corrections for the rotational asymmetry of the PB
  - Corrections for the frequency or polarization dependent effects
  - Noise limited imaging at low frequencies

RMS : 14 microJy
First Widefield CLEAN Image

- Model image shows that you need a lot of point sources to model extended emission.
- Need a better set of basis to model the emission from the sky.
- The residuals are not truly noise-like so we have not hit thermal noise yet.
- The negatives around the source arise from missing zero spacings. [remember Brian Mason’s lecture.]

RMS : 14 microJy
Imaging - Iterative Minimization

DATA → MODEL → RESIDUAL

GRIDDING
Use Flags and Weights

DE-GRIDDING

W-Projection
Facetting

Major Cycle (Imaging)

RESIDUAL IMAGE

MODEL IMAGE

iFFT

FFT
Mapping Data to Images - IV
Widefields

Outlier Fields

- Image additional regions outside your image FoV with separate phase centers.
- Widefields distortions are reduced around each new phase center.
- Also distorts the geometry of the sky.

Facetting

- Treat each facet of the image as isoplantic
- Grid the facets separately but combine to produce a single image.
- Number of facets is given by

\[ N_f = \frac{2B_{\text{max}} \lambda_{\text{max}}}{f^2 D^2} \]
Outlier Fields

RMS : 20 microJy
Facetting vs W-Projection

9x9 facets. 13 microJy
Support kernel 9x9

128 w planes. 12.7 microJy
Max Support kernel 56x56
Widefield Correction

The source distortion from the W phase term is removed. Both facetting and W-projection produce the same result. W-Projection uses a fresnel kernel spanning 56x56 uv pixels. Facetting used the standard prolate spheroidal kernel in its 81 facets to grid them independently.
Wideband Widefield Imaging

Fractional bandwidth \( \sim 40\% \)

Have to take into account frequency dependence of sky and psf.

Multi Term Multi Frequency Synthesis. Expands the PSF into \( n \) Taylor terms in frequency. The algorithm derives an in-band spectral index for the sources.

In order to model the extended emission better, we use the multiscale algorithm. It uses circular Gaussian of different sizes which are more representative of the model.
Multiscale vs Hogbom

Scales = [6, 10, 20, 60]
Primary beams are the forward gain pattern of the antenna. They rotate in the sky for an Alt-Az mounted Telescope. The PB are also highly chromatic varying by the same factor as your fractional bandwidth in size. Typically only a concern when performing widefield imaging. Corrections are only as good as the models we have for them.
Gridding kernels and FoV - A-Projection

Imaging FoV out to 20% point

Support size 3x3

Imaging FoV out to 1\textsuperscript{st} side-lobe

Support size 9x9
Computational Cost - Imaging

Gridding Cost
- Directly proportional to number of visibilities
- Scales as the square of the support size of gridding kernel or convolution function (CF)
  - Support size Prolate spheroidal – 3x3
  - Support size W-Projection – 10 – few hundreds
  - Support size A-Projection – 9 -20

Computational Cost of Gridding kernel can be high too.

Currently in A-Projection we compute and cache them so they can be reused.

Memory requirement for these projection algorithms scales as number of copies of images kept in memory for A-Projection 5*Nx*Ny*float.

Total compute is gridding for the number of images specified + Number of CF

Deconvolution adds its own compute and memory requirements which nominally scale with number of nterms used and the number of scales deployed in multiscale.
WB A-Projection + MTMFS

RMS : 10 microJy

Image courtesy – U.Rau
WB A-Projection + MTMFS – Before & After

Alpha = -3.9

Alpha = -2.7

Alpha = -2.3

Alpha = -0.8

Images courtesy – U.Rau
Intensity-weighted Spectral Index

300GB calibrated dataset, 106 pointings over 1.5x2 deg, imaged with MS-MT-MFS (NT=2) + WB-A-Projection.

=> Mosaic primary beam spectral index of ~ -1.5 has been removed prior to the wideband sky model fitting.
Widefield Wideband Imaging - Summary

**Basic MFS imaging**
(no WB, WF corrections)

**MT-MFS wideband imaging**
(No WF corrections, PB freq dependence part of sky model)

**MT-MFS wideband imaging + A-Proj**
(PB^2 freq dependence part of sky model)

**MT-MFS wideband imaging + WB-A-Proj**
(PB freq dependence removed during gridding)
Always know your source before imaging.

Only choose the level of algorithmic complexity required to achieve your science goal.

Every algorithmic choice comes with an associated computational cost.

What qualifies as widefield imaging
  - Corrections for non coplanar arrays
  - Imaging a large FoV single pointing or in mosaic.
  - Polarization corrections arising from the beam

For widefield wideband imaging for DR >10000 you need direction dependent corrections for Primary Beams, Pointing Accuracy, Source squint etc.

Trust in band spectral index fits for point source of SNR>50 and extended source of SNR>100, when using MTMFS.
The measured power at the correlator is a cross correlation of the voltages received by two antennas. Note that the antennas can be of different types i,j. The basis a,b are the sky stokes basis. The matrix $s$ performs the transfer of basis from stokes to an orthogonal feed basis p,q. vCZ theorem relates the sky brightness distribution to the measured interferometric voltages.
Measurement Equation - II

\[ \vec{I}_{ab} = (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}^*_b \vec{J}^*_j) \]

\[ \vec{V}^{obs}_{pq} = \mathcal{F}_{pq} S^{ab} (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}^*_b \vec{J}^*_j) \]

\[ \vec{V}^{obs}_{pq} = p_q S^{ab} (\vec{A}_i \otimes \vec{A}^*_j) \star \vec{V}_{ab} \]

Our goal then is to be able to reconstruct true sky brightness or the true sky coherence function.
A-Projection

If the antenna A term were (approximately hermitian) we could consider an inversion operation of the form

$$(\vec{A}_i \otimes \vec{A}_j)^M \star \vec{V}_{obs}^a = |A_{ij}|^2 \star \vec{V}_{ab}$$

This term that is applied on the left is the kernel for A-projection at the time of gridding. The term in the modulus is the square of the antenna PB (forward gain) in the image plane so we divide it out after we take the FT to go from data to the image.

$$\mathcal{F}^\dagger (\vec{A}_i \otimes \vec{A}_j)^M \star \vec{V}_{obs}^a = PB \cdot I_{ab} \quad \mathcal{F}^\dagger (\vec{A}_{ij}) = \vec{M}_{ij}$$
Widefield Polarimetry

\[ \vec{I}_{ab} = (\vec{J}_i \vec{\epsilon}_a \otimes \vec{\epsilon}_b^* \vec{J}_j^*) \]

The antenna jones can be measured through holography.

\[ \vec{J}_i = \begin{array}{c}
\end{array} \]

S-band ~ 3GHz, In feed basis
I.e. pq