Calibration

Joshua Marvil, NRAO

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National Radio Astronomy Observatory

References & Credits

- Interferometry and Synthesis in Radio Astronomy
  (3rd ed. , Thompson, Moran, & Swenson)

- Synthesis Imaging in Radio Astronomy II
  (Editors: Taylor, Carilli, & Perley)

- Tutorials, telescope observing guides, software documentation

- Previous workshops (especially G. Moellenbrock's and G. Heald's previous talks on which parts of this talk are based)

Idealized Visibilities

We wish to use our interferometer to obtain the visibility function:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl dm$$

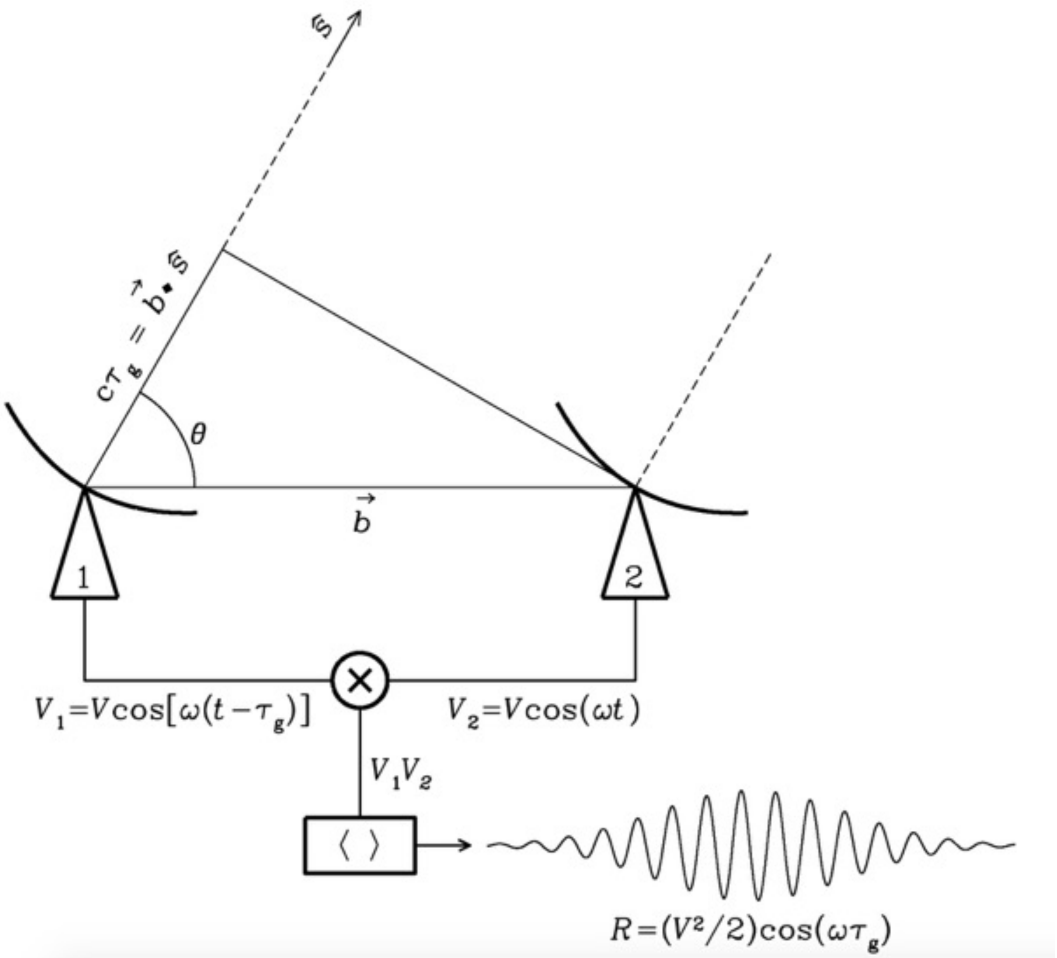
which we intend to invert to obtain an image of the sky:

$$I(l,m) = \int_{uv} V(u,v) e^{i2\pi(ul+vm)} du dv$$

V(u,v) describes the amplitude and phase of 2D sinusoids that add up to an image of the sky:

- Amplitude: "how bright and how compact is the source?"
- Phase: "where is the source?"

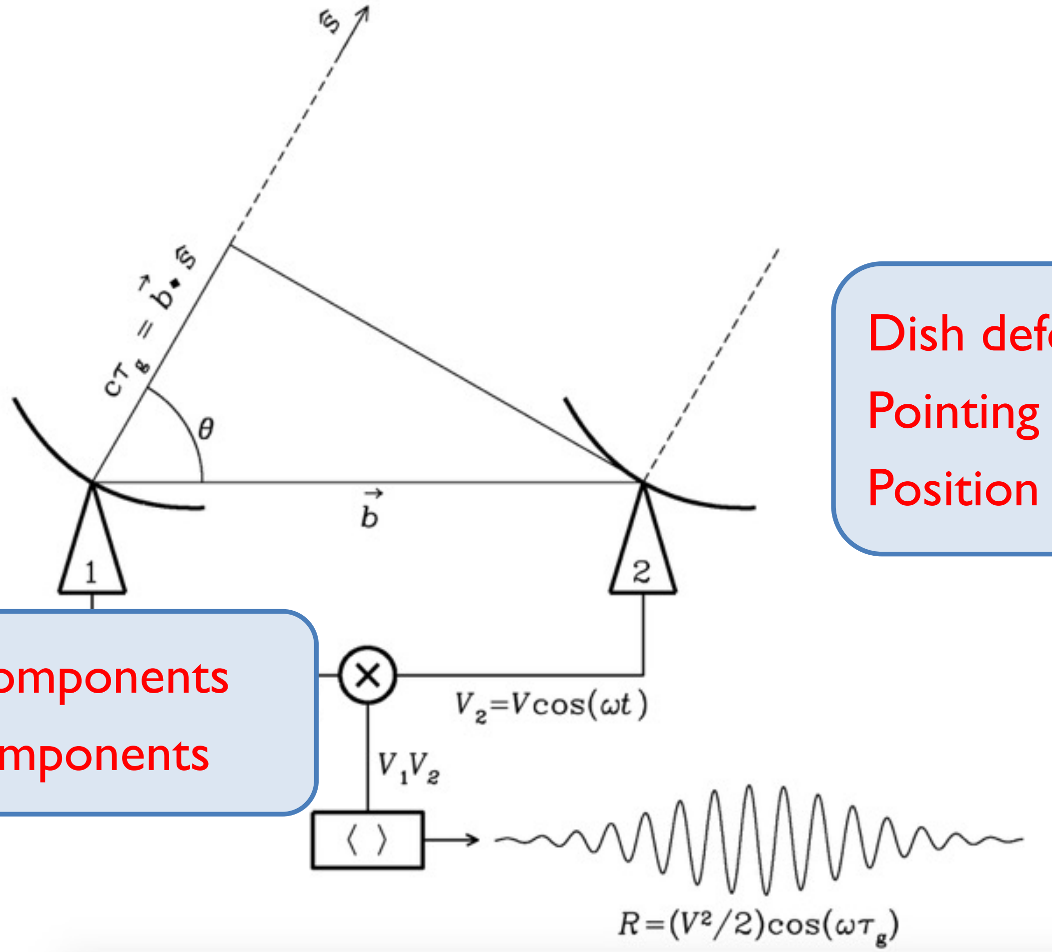
Ideal Case



$\hat{s}$
$c\tau_g = \vec{b} \bullet \hat{s}$
θ
$\vec{b}$
1
2
$V_1 = V\cos[\omega(t - \tau_g)]$
$V_2 = V\cos(\omega t)$
$V_1 V_2$
$\langle\ \rangle$
$R = (V^2/2)\cos(\omega\tau_g)$

Real Case

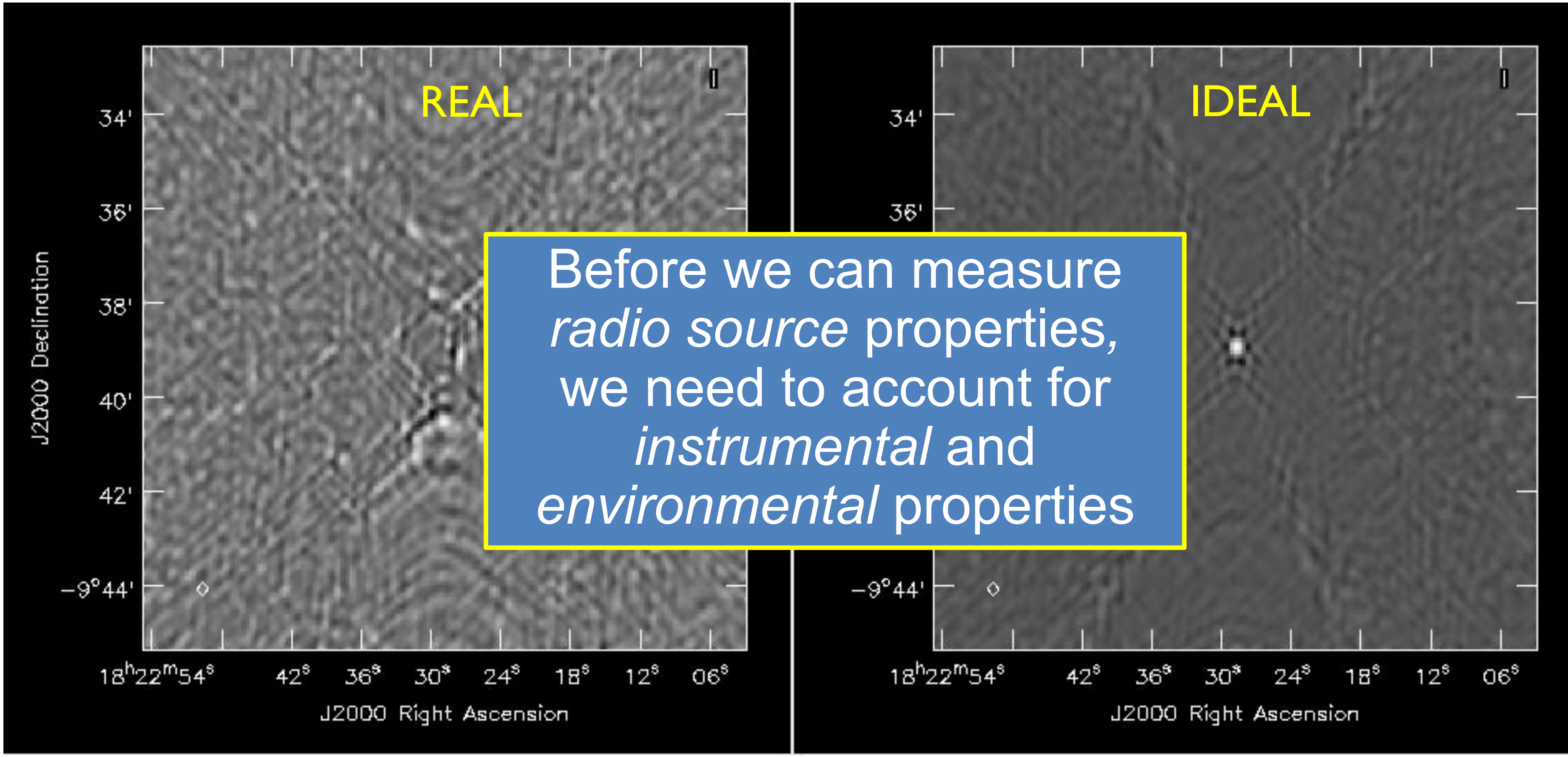
Ionosphere
Troposphere



Dish deformation
Pointing errors
Position errors

Waveguide components
Electronic components

Images of Real vs. Ideal Visibilities



Before we can measure *radio source* properties, we need to account for *instrumental* and *environmental* properties

Realistic Visibilities

The real signal sampled by antenna *i*, $x_i(t)$, is a combination of the desired signal, $s_i(t,l,m)$, corrupted by a factor $J_i(t,l,m)$ and with added noise, $n_i(t)$:

$$x_i(t) = \int_{sky} \underline{J_i(t,l,m)} s_i(t,l,m) dl dm + n_i(t)$$

So we have an imperfect visibility measurement per antenna pair:

$$V_{ij}^{obs}(u,v) = \langle x_i(t) \cdot x_j^*(t) \rangle_{\Delta t}$$

$$= \underline{J_{ij}} V_{ij}^{true}(u,v) \qquad \text{(for } J_i, J_j \text{ constant in } l,m\text{)}$$

The ***Jones matrix*** $J_{ij} = J_i J_j^*$ is a generalized *operator* characterizing the *net* effect of the observing process for antennas *i* and *j* on baseline *ij*

Solving for Calibration

Observe a celestial calibration source for which we have a model,

$$V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$$

define chi-squared:

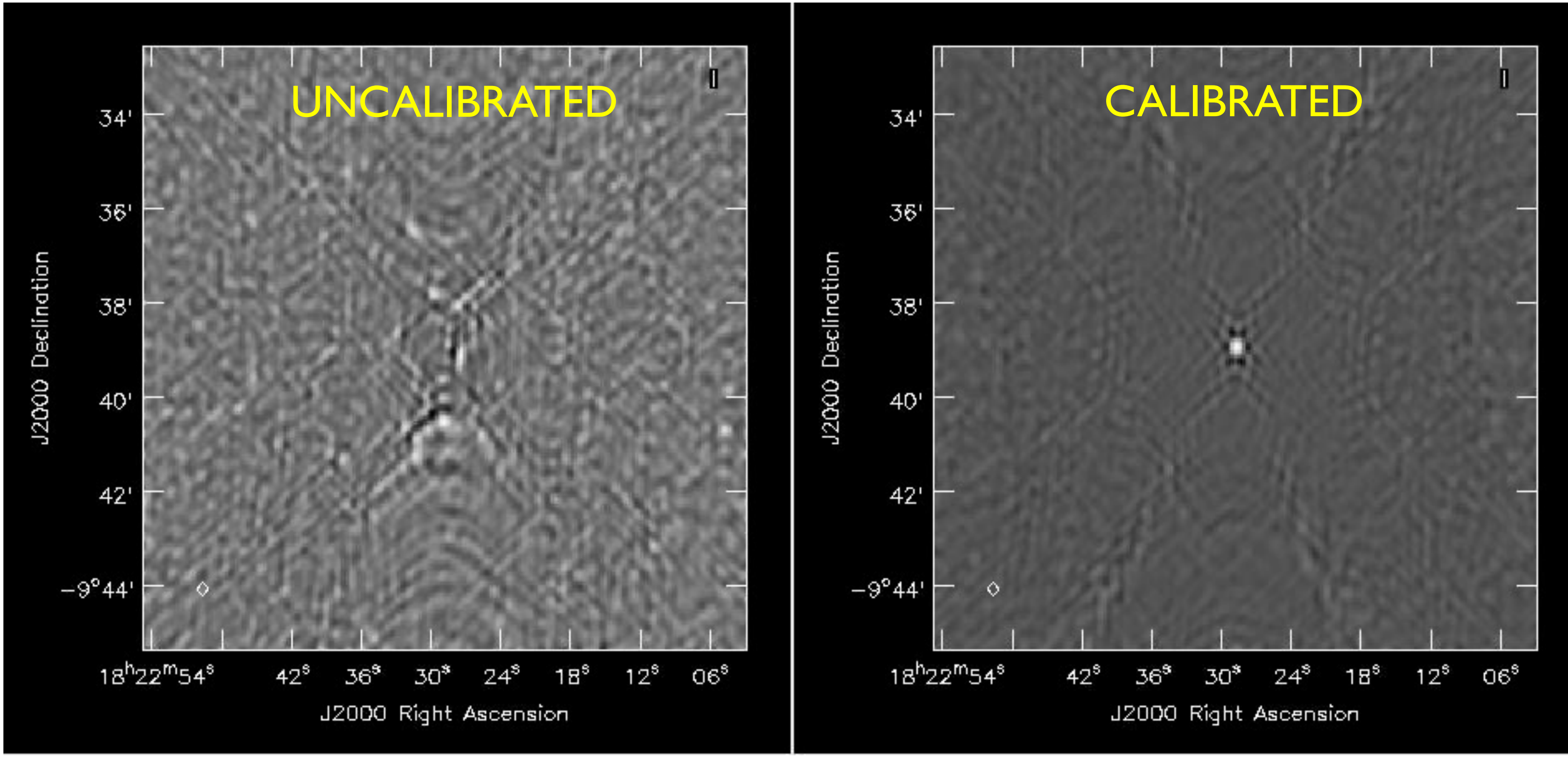
$$\chi^2 = \sum_{\substack{i,j \\ i \neq j}} \left| V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} \right|^2 w_{ij} \qquad \left(w_{ij} = \frac{1}{\sigma_{ij}^2} \right)$$

and minimize chi-squared w.r.t. each J_i^* $\left(\frac{\partial \chi^2}{\partial J_i^*} = 0 \right)$

Then apply ***J*** to each visibility:

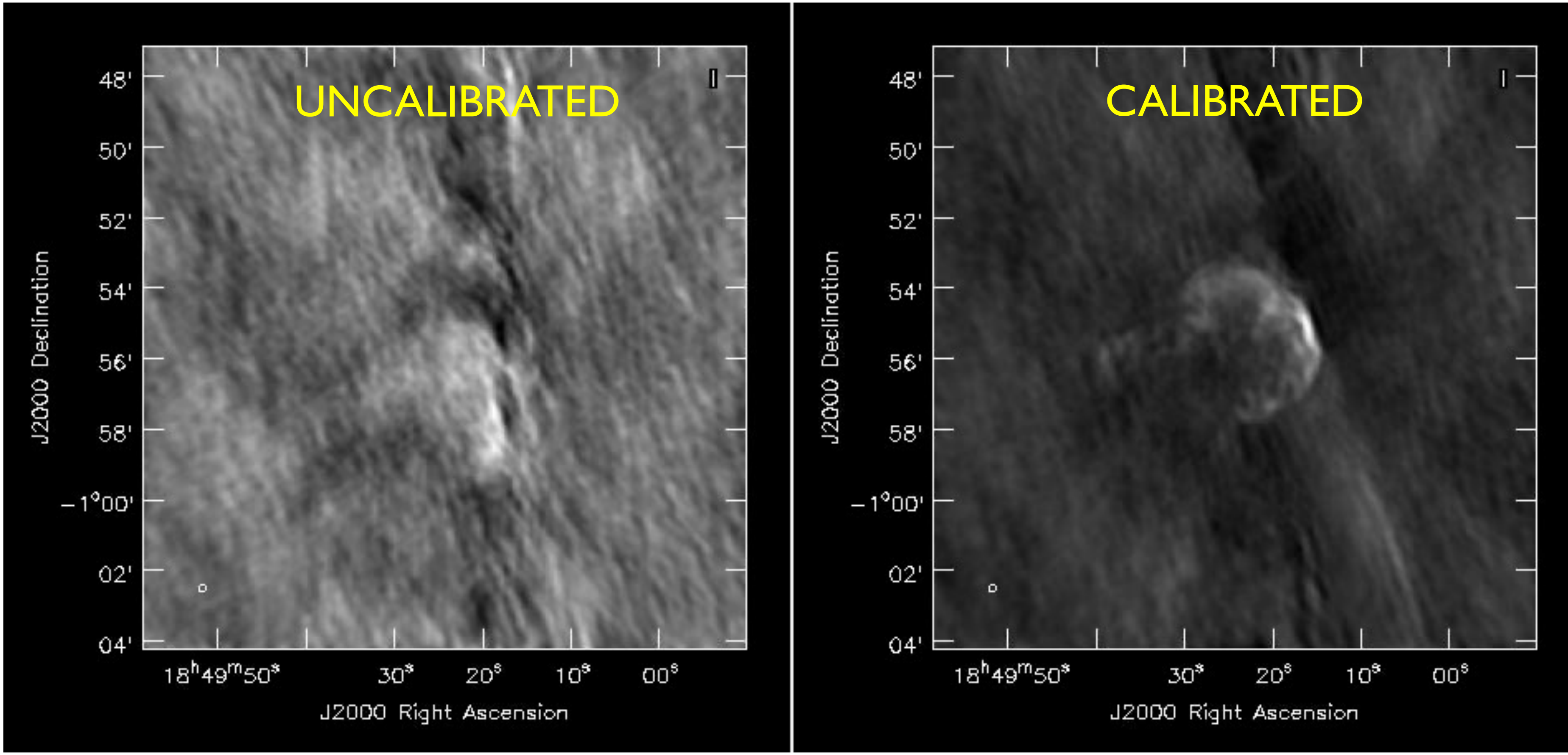
$$V_{ij}^{obs} = J_i J_j^* V_{ij}^{true} \quad \rightarrow \quad V_{ij}^{cor} = J_i^{-1} J_j^{*-1} V_{ij}^{obs}$$

Calibration's Effect on Imaging



(Calibration applied to calibrator field)

Calibration's Effect on Imaging



(Calibration applied to target field)

Solving for Calibration - Practical

- There typically is not one single calibrator which we can use to solve for all of the corrupting effects
- We don't want to solve for all effects at once
  - High SNR solutions often require averaging
  - Averaging uncalibrated phases creates decorrelation
  - Some calibration terms should be determined in a similar direction as your science target(s)
- More accurate to decompose the net corruption into separate components that can be addressed individually

Calibration Components

In principle, J_i contains many components:

F = ionospheric effects

T = tropospheric effects

P = parallactic angle

X = linear polarization position angle

E = antenna voltage pattern, gaincurve

D = polarization leakage

G = electronic gain

B = bandpass response

K = geometry

$$\overleftrightarrow{J}_i = \overleftrightarrow{K}_i \overleftrightarrow{B}_i \overleftrightarrow{G}_i \overleftrightarrow{D}_i \overleftrightarrow{E}_i \overleftrightarrow{X}_i \overleftrightarrow{P}_i \overleftrightarrow{T}_i \overleftrightarrow{F}_i$$

- J_i is a function of time, as are most of its components

Ionospheric Effects, *F*

$$\overleftrightarrow{F}^{RL} = e^{i\Delta\phi}\begin{pmatrix} e^{-i\varepsilon} & 0 \\ 0 & e^{i\varepsilon} \end{pmatrix}; \ \overleftrightarrow{F}^{XY} = e^{i\Delta\phi}\begin{pmatrix} \cos\varepsilon & \sin\varepsilon \\ -\sin\varepsilon & \cos\varepsilon \end{pmatrix}$$

The ionosphere introduces a dispersive path-length offset:

$$\Delta\phi \propto \frac{\int n_e \, dl}{\nu}$$

And also Faraday rotation:

$$\varepsilon \propto \frac{\int B_{\parallel} n_e \, dl}{\nu^2}$$

More important at lower frequencies ($\lesssim$5 GHz)

Tropospheric Effects, *T*

$$\overset{\leftrightarrow}{T} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Polarization-independent effects due to the lower atmosphere

- Most important at $\nu \gtrsim 15$ GHz where path length errors are a larger fraction of the wavelength, and water vapor and oxygen absorb/emit
- Zenith-angle-dependent opacity (higher air mass near horizon)
- One of the most highly time-variable calibration terms

Parallactic Angle, *P*

$$\overleftrightarrow{P}^{RL} = \begin{pmatrix} e^{-i\chi} & 0 \\ 0 & e^{i\chi} \end{pmatrix}; \ \overleftrightarrow{P}^{XY} = \begin{pmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{pmatrix}$$

Accounts for changing orientation of sky in telescope's frame

- Constant for equatorial telescopes
- Varies for alt-az-mounted telescopes

$$\chi(t) = \arctan\left(\frac{\cos l \sin h(t)}{\sin l \cos\delta - \cos l \sin\delta \cos h(t)}\right)$$

$$l = \text{latitude}, h(t) = \text{hour angle}, \delta = \text{declination}$$

- Analytically known based on geometry
- Rotates the position angle of linearly polarized radiation

Linear Polarization Position Angle, X

$$\overleftrightarrow{X}^{RL} = \begin{pmatrix} e^{-i\Delta\chi} & 0 \\ 0 & e^{i\Delta\chi} \end{pmatrix}; \quad \overleftrightarrow{X}^{XY} = \begin{pmatrix} \cos\Delta\chi & \sin\Delta\chi \\ -\sin\Delta\chi & \cos\Delta\chi \end{pmatrix}$$

- Configuration of optics and electronics (and use of a refant) causes a net linear polarization position angle offset
- Can be treated as an offset to the parallactic angle, P
- For circular feeds, this is a phase difference between the R and L polarizations, which is frequency-dependent (a R-L phase bandpass)
- For linear feeds, this is the orientation of the dipoles (in the frame of the telescope) projected onto sky coordinates

Antenna Voltage Pattern, *E*

$$\overleftrightarrow{E}^{pq} = \begin{pmatrix} E^p(l,m) & 0 \\ 0 & E^q(l,m) \end{pmatrix}$$

- Antennas of all designs have direction-dependent gain (primary beam)
- Antenna forward gain may change with elevation: 'gain curve'
- We typically only include direction-independent effects in ***E***, and defer (*l*,*m*) effects to be handled during imaging

Polarization Leakage, *D*

$$\overleftrightarrow{D} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

Orthogonal polarizations are not perfectly isolated

- Potential origins include alignment errors, EM induction, polarizer
- Well-designed systems have d ~ a few percent or less on-axis

D does not include off-axis leakage, a separate issue for polarimetry

“Electronic” Gain, *G*

$$\overleftrightarrow{G}^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

A catch-all for most time-dependent amplitude and phase effects introduced by antenna electronics and other generic effects

- Includes scaling from engineering units to radio astronomy units (Jy)
- Includes any internal system monitoring such as a noise source
- Often includes residual tropospheric and ionospheric effects

Bandpass Response, *B*

$$\overleftrightarrow{B}^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

Like G but as a function of frequency

- Filters used to select frequency passband not square
- Optical and electronic reflections introduce ripples
- Often assumed time-independent, but not necessarily so
- Typically (but not necessarily) normalized

Geometry, *K*

$$\overleftrightarrow{K}^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

- *K* is a clock- & geometry-parameterized version of *G*
- Typical contributions to ***K*** are antenna positions and delay refinements
- Must have correct geometry for Fourier Transform relation to work

Decoupling Calibration Components

$$\overleftrightarrow{J}_i = \overleftrightarrow{K}_i \overleftrightarrow{B}_i \overleftrightarrow{G}_i \overleftrightarrow{D}_i \overleftrightarrow{E}_i \overleftrightarrow{X}_i \overleftrightarrow{P}_i \overleftrightarrow{T}_i \overleftrightarrow{F}_i$$

- Make an informed decision about which terms to ignore
- Use external (*a priori*) information: ***F***, ***T***, ***P***, ***E*** and parts of ***G***, ***K***
- Parameterize terms appropriately (e.g. zenith angle vs. time)
- Other effects need to be solved for by observing calibrator sources

Observing Calibrator Sources

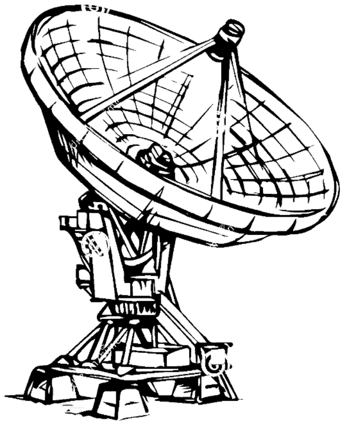
Flux Density: 'standard candle' with known structure and spectral energy distribution. Typically observe once per observation.

Delay, Bandpass: very bright, preferably unresolved. Typically observe once per observation.

Gain: bright, preferably unresolved, accurate position. Observe before and after the target source more frequently than the coherence time.

Pol Angle: polarized source with full-Stokes model

Pol Leakage: unpolarized, preferably unresolved or multiple scans over a range of parallactic angles



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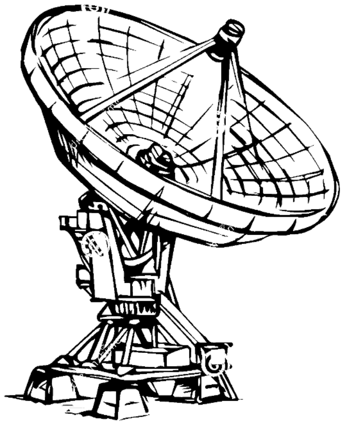
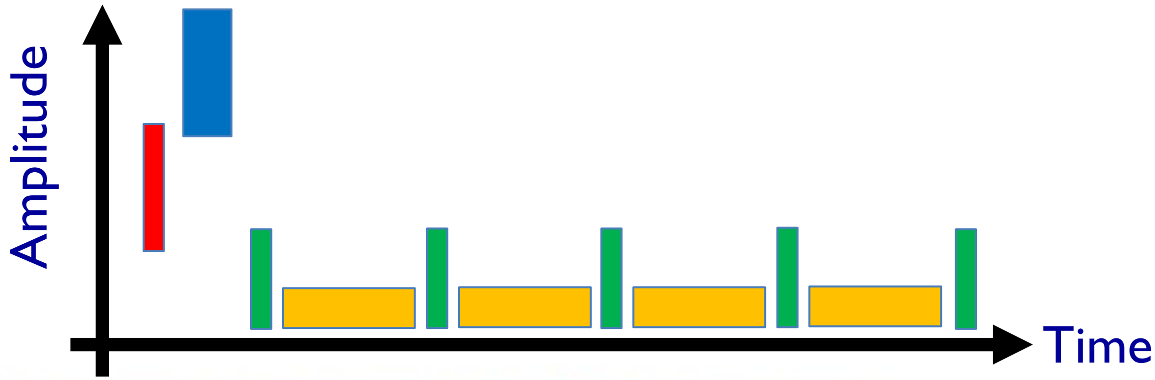




Flux-density Calibration

- Bootstrap the flux density scale by enforcing gain amplitude consistency over all calibrators:

$$\left\langle \frac{|G_i|}{|G_i(fd\ cal)|} \right\rangle_{time,antennas} = 1.0$$

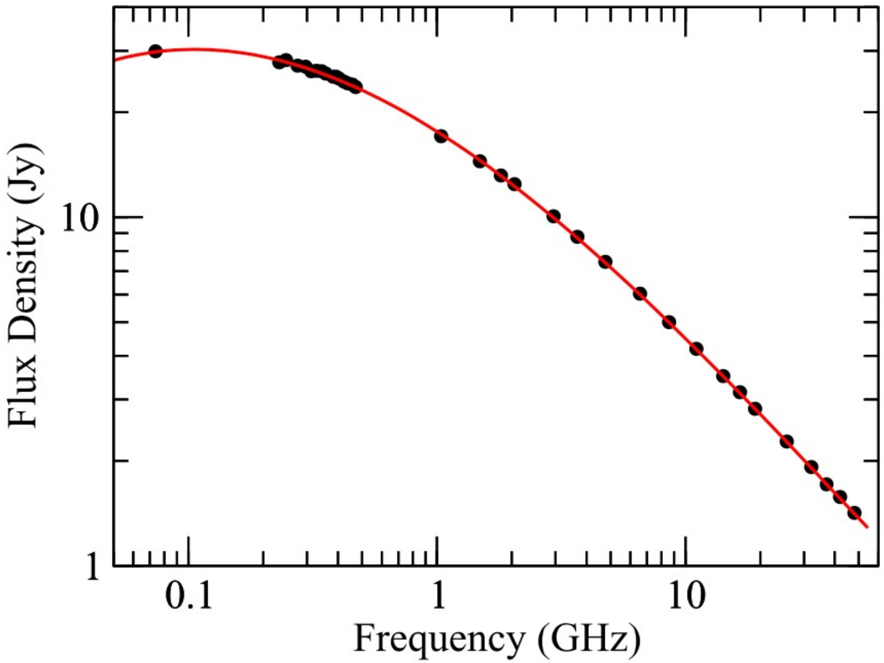
- Important to have previously removed other known gain effects, e.g. elevation-dependence

- I.e., assume the system is linear and time-independent:

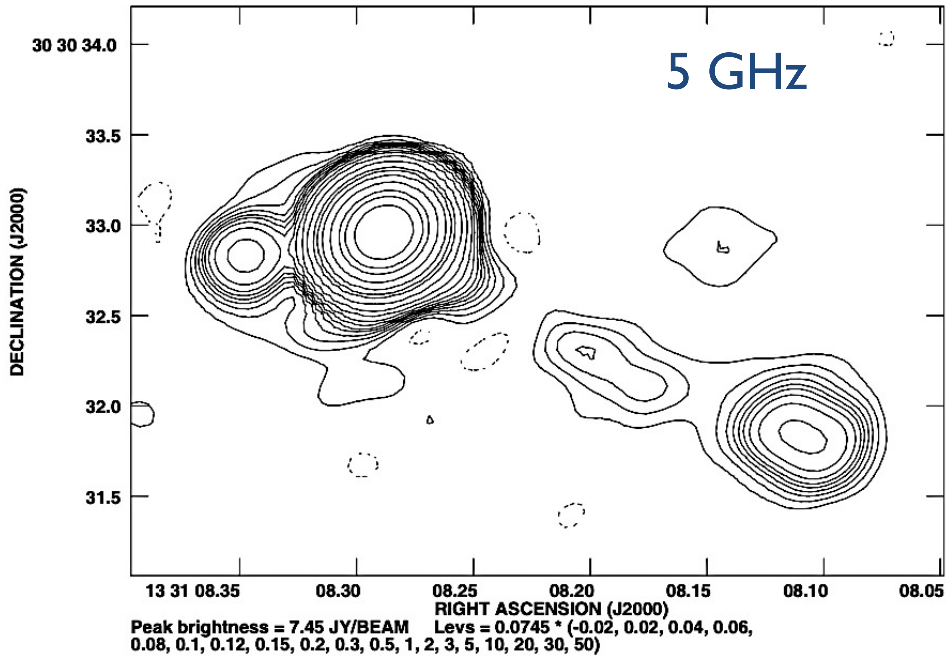
$$\frac{\left|V_{u,v\rightarrow 0}(fd\ cal)\right|}{F(fd\ cal)} = \frac{\left|V_{u,v\rightarrow 0}(other\ cal)\right|}{F(other\ cal)}$$

Flux-density Calibration

- Use observatory-provided flux calibrator models, e.g. 3C 286



(Perley & Butler 2017)



(Perley & Butler 2013)

Reference Antenna

- Phase solutions are typically referred to a specific antenna, the ***refant***, which is assumed to have constant (zero) phase
  - The ***refant's*** phase variations are distributed to the solutions of all other antennas
  - Asserts phase continuity and a cross-hand phase frame
  - Assists in the inspection and interpolation of solutions
- An ideal ***refant*** should be:
  - Available over the entire observation (time, frequency)
  - Stable and generally 'well behaved'

Examination and Editing

- Calibration is very susceptible to 'bad data' and radio interference
  - Initial data examination and 'flagging' are very important
  - J_i will try to make the bad data match the model!

$$V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$$

- Evaluate calibration tables– are solutions continuous and sensible?
- Iterate– provisional calibration can make bad data easier to see

Advanced Techniques

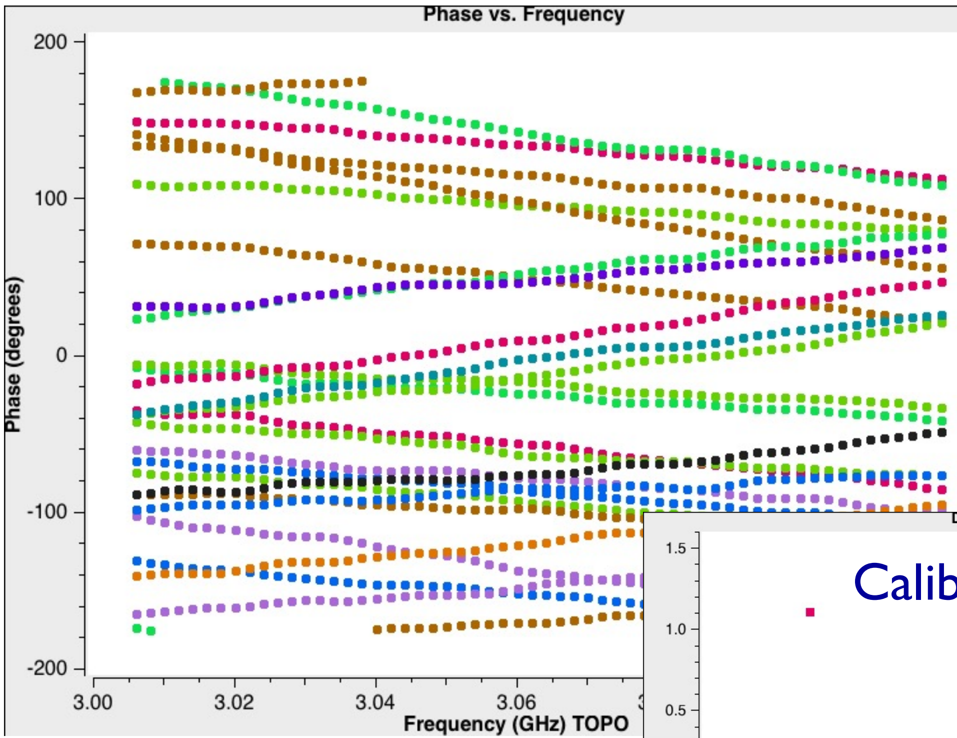
- Make use of ancillary hardware, when available:
  - Ionospheric instrumentation to measure TEC
  - Water vapor radiometer to infer tropospheric phase
  - Tipping scans to fit atmospheric opacity
  - Pulse cal to track radiometer gain, X-Y phase
  - Paddle scans to calibrate flux density
- Self-calibration: create and use source model to refine $\boldsymbol{J}_i$
- Baseline-based calibration– solve for each $\boldsymbol{J}_{ij}$ instead of $\boldsymbol{J}_i$
- Direction-dependent (wide-field) calibration

A Practical Demonstration

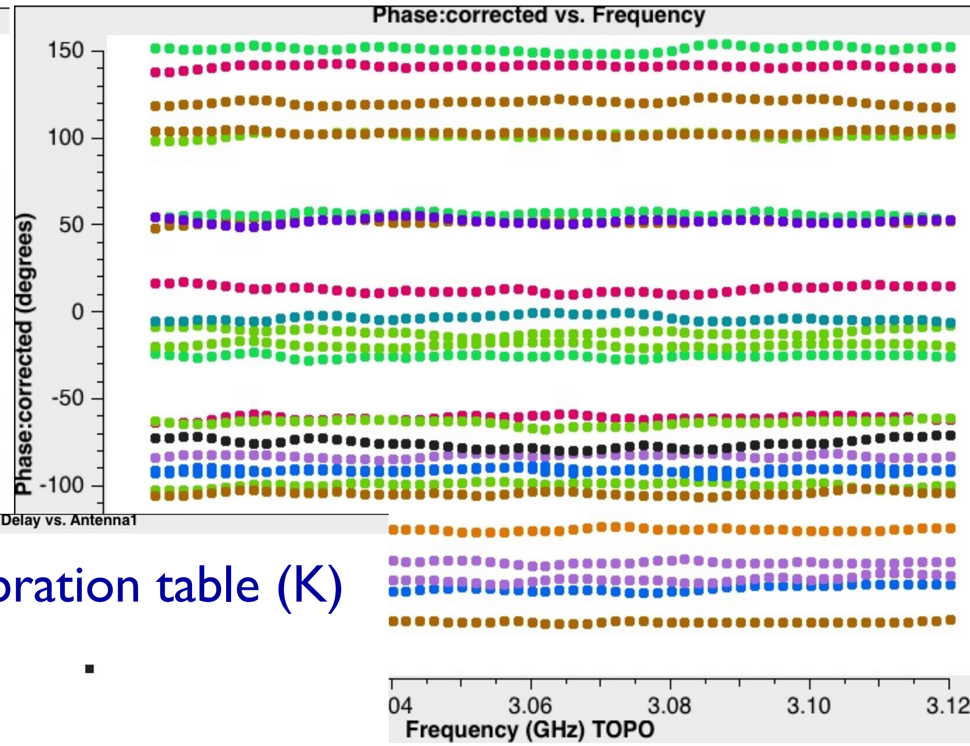
- Example calibrator scan from VLA S-band (~3 GHz)
  - 27 antennas
  - 351 baselines
  - 64 frequency channels
- Assume point source of 1 Jy
- Solve for delay, bandpass and complex gain (amplitude and phase)
- Correct the visibilities by applying the calibration solutions

A Practical Demonstration

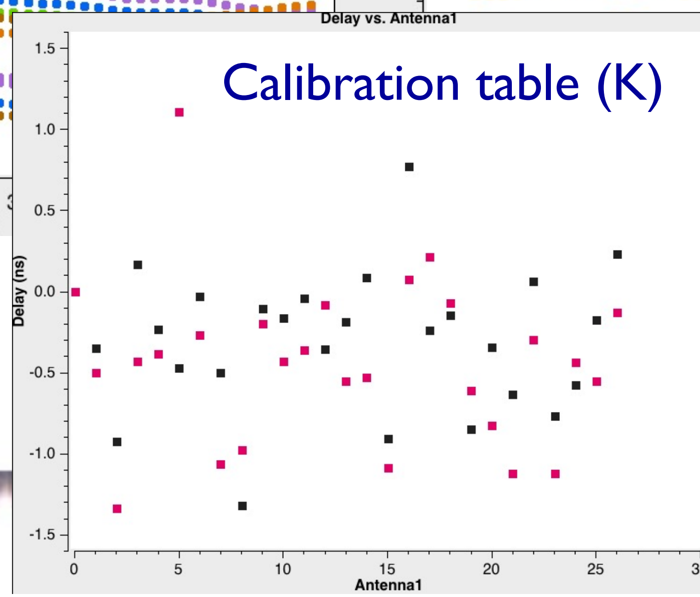
Uncalibrated



Calibrated

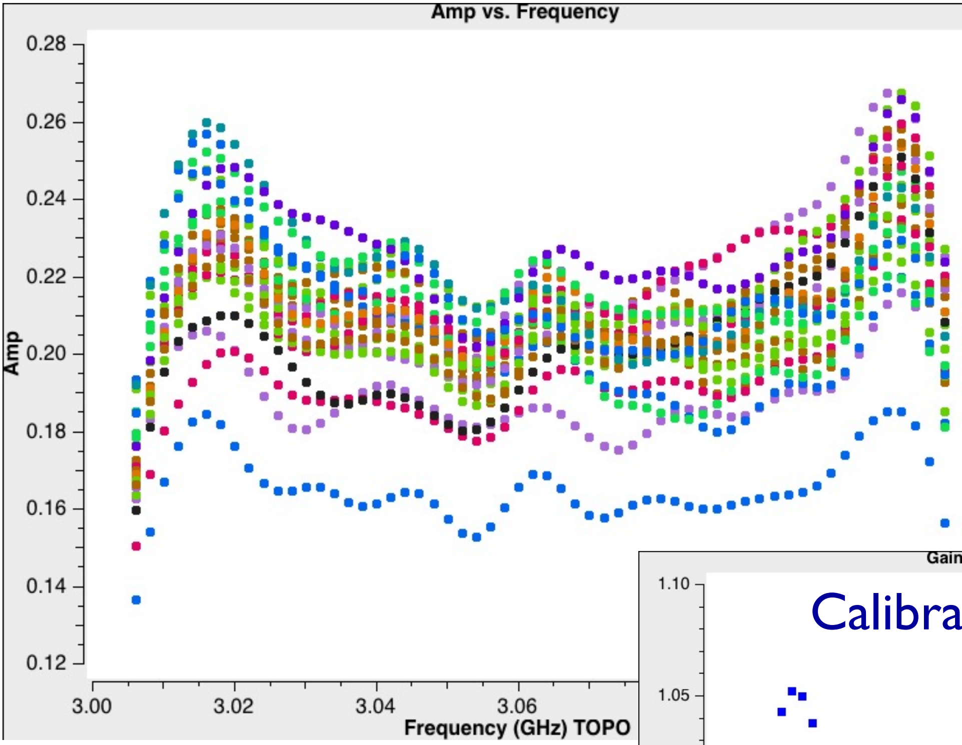


Calibration table (K)

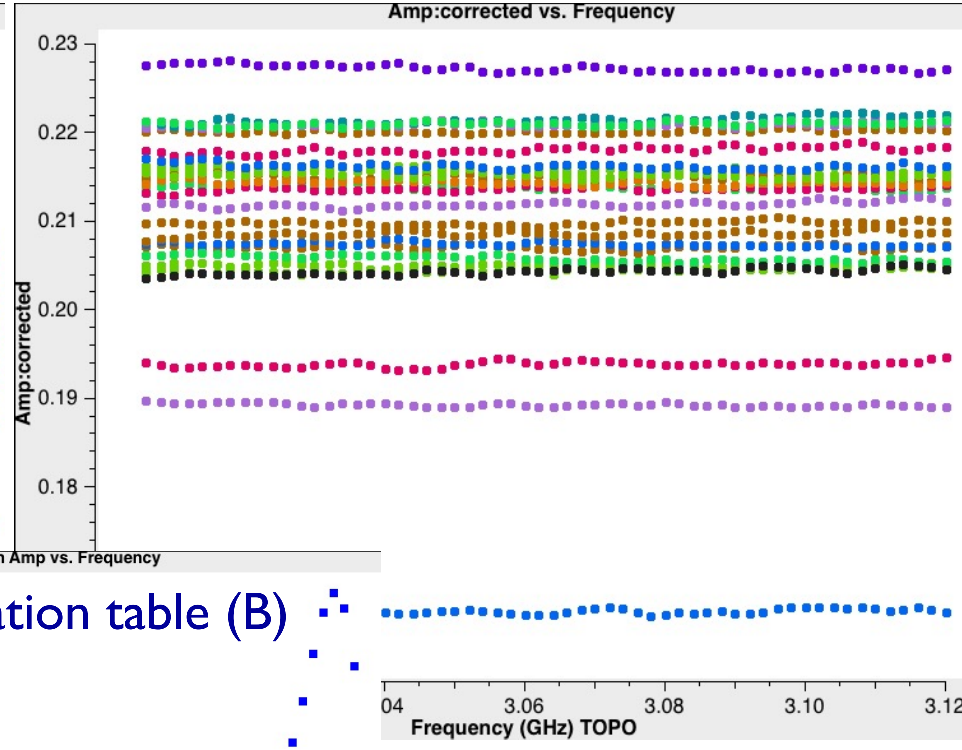


A Practical Demonstration

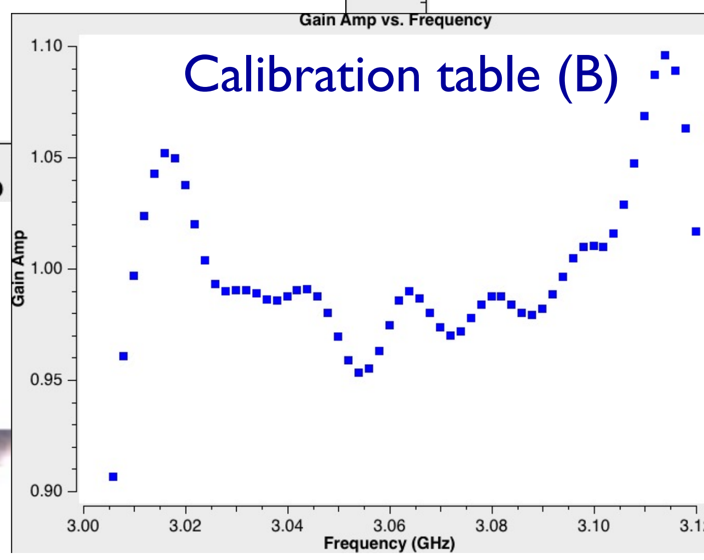
Uncalibrated



Calibrated

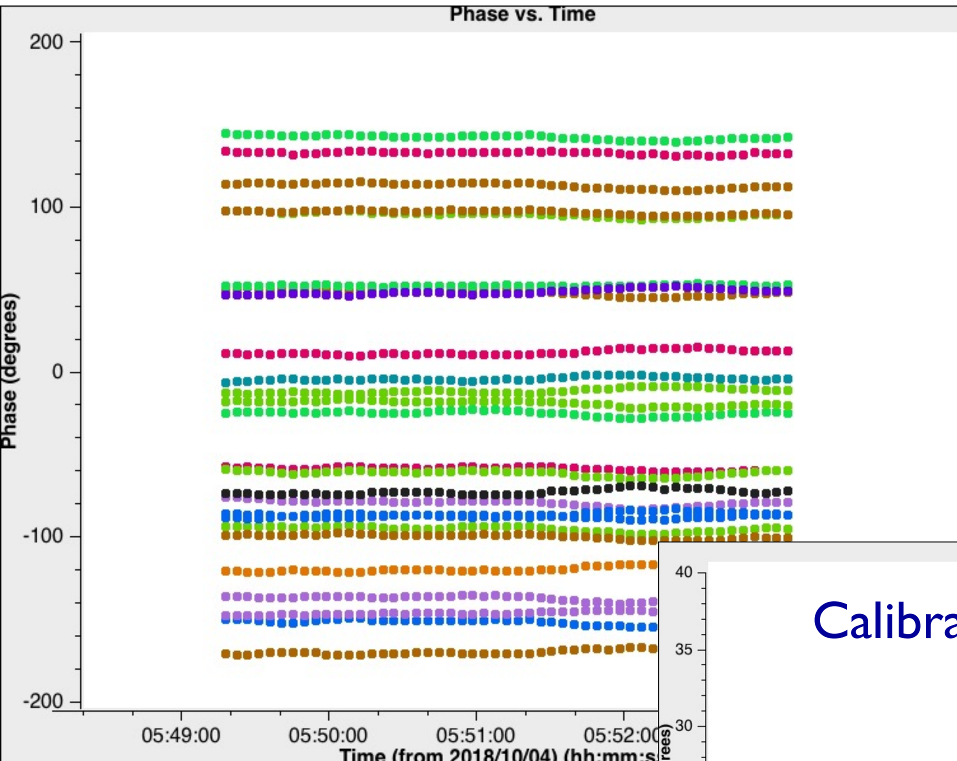


Calibration table (B)

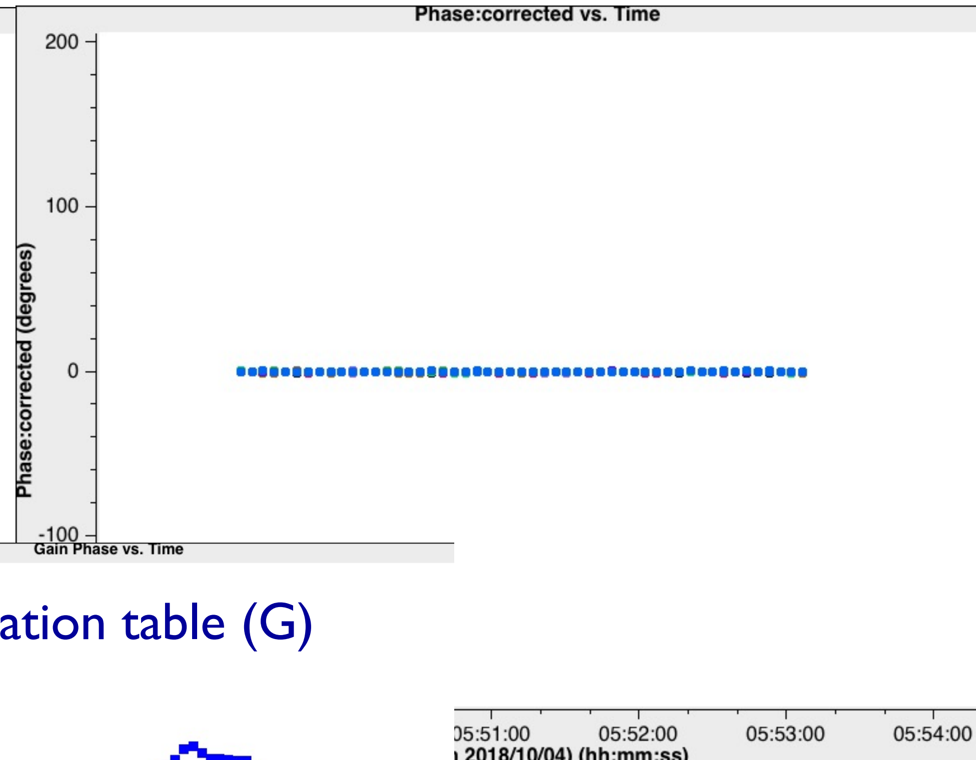


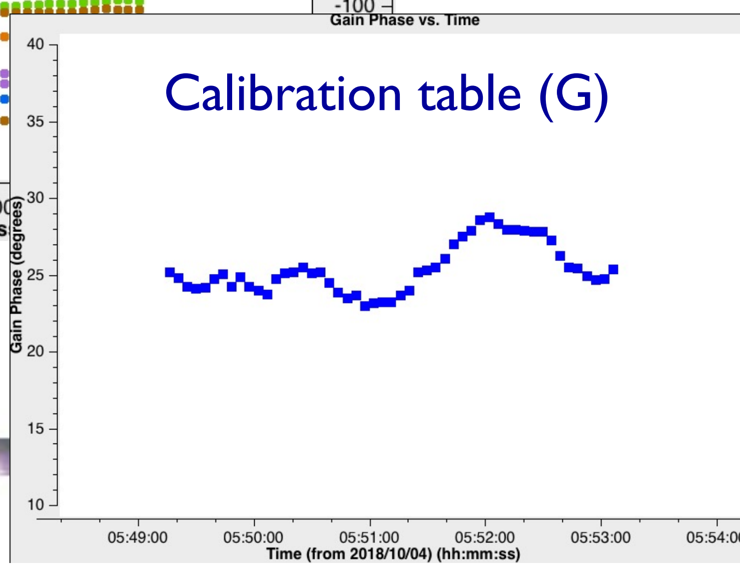
A Practical Demonstration

Uncalibrated



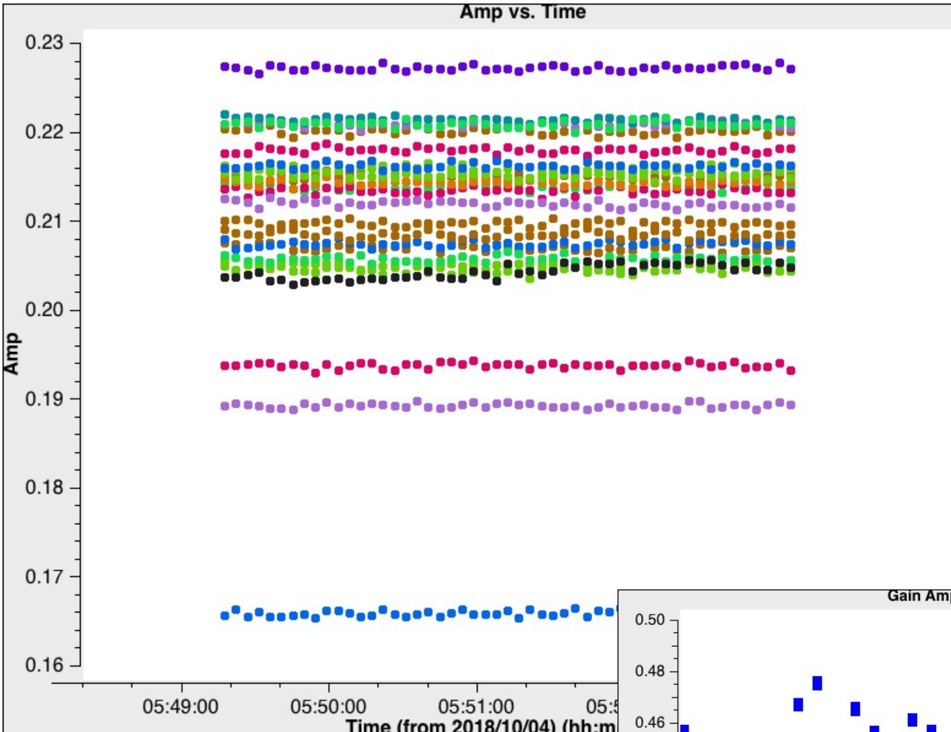
Calibrated



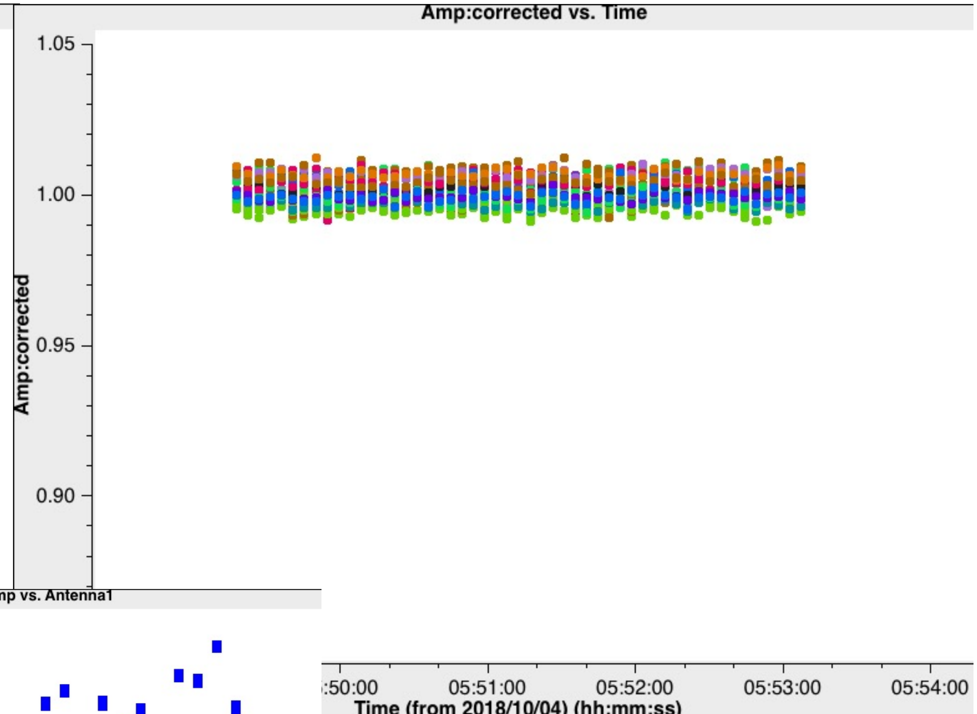


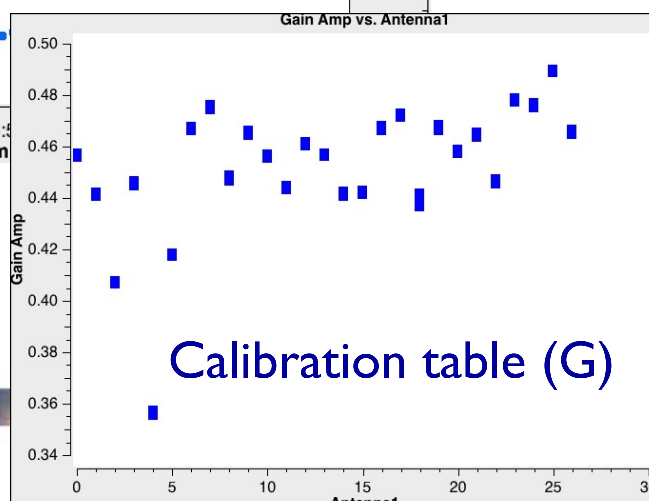
A Practical Demonstration

Uncalibrated



Calibrated



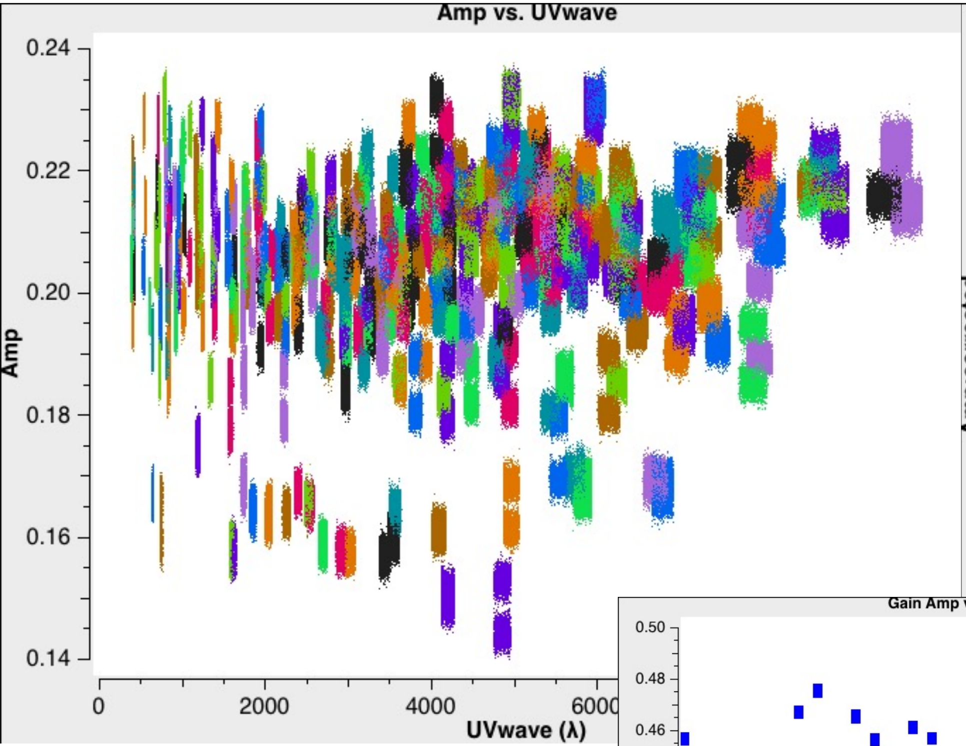


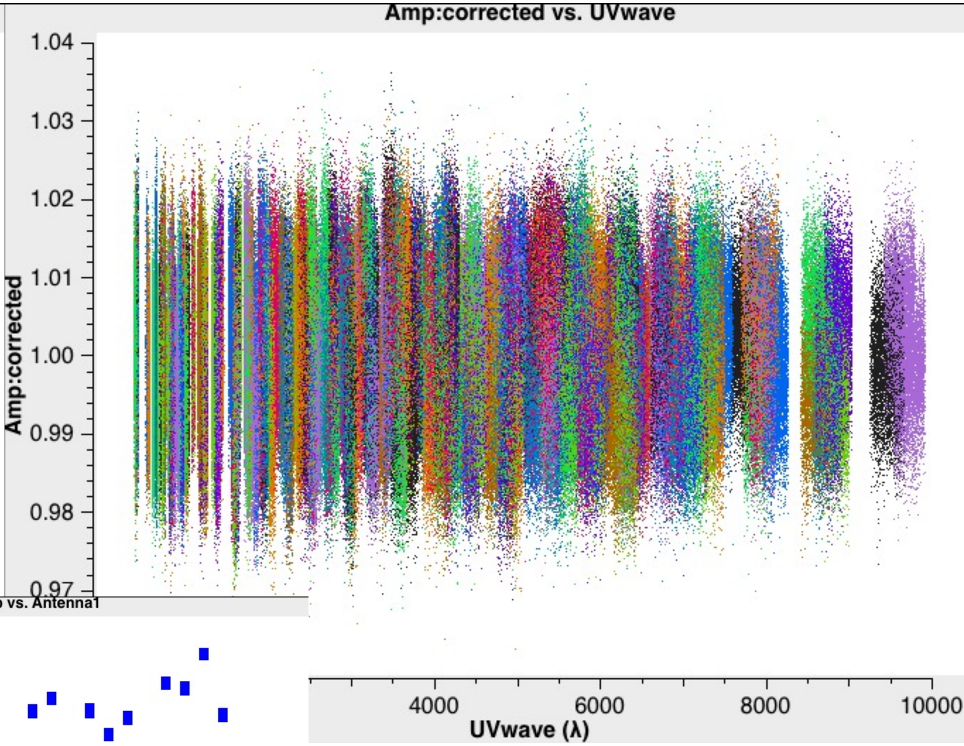
Calibration table (G)

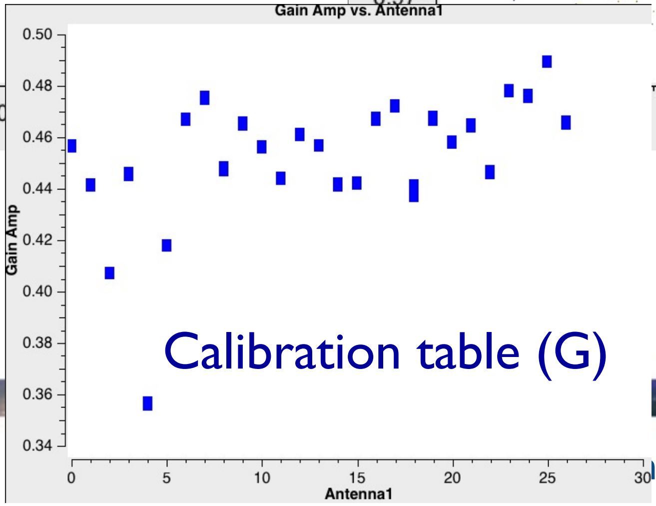
A Practical Demonstration

Uncalibrated

Calibrated







Calibration table (G)

Summary

- Calibration deals primarily with correcting the (baseline-based) visibilities for antenna-based effects
- Corrected visibilities are a prerequisite for imaging and analysis
- Accurate calibration relies on separating the corrupting effects
- Have a calibration strategy, not a recipe!