

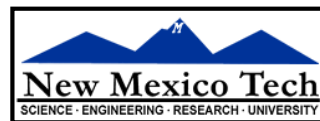
Calibration

George Moellenbrock, NRAO



Sixteenth Synthesis Imaging Workshop

16-23 May 2018



References

- ~Theory
 - Interferometry and Synthesis in Radio Astronomy (2nd or 3rd ed. Thompson, Moran, & Swenson)
 - Tools of Radio Astronomy (6th ed., Wilson, Rohlfs, & Huettemeister)
- “Practical Thinking”
 - ***These talks!***
 - Synthesis Imaging in Radio Astronomy II (Editors: Taylor, Carilli, & Perley)
- “Practical Doing”
 - VLA: <https://science.nrao.edu/facilities/vla/docs/manuals/obsguide>
 - VLBA: <https://science.nrao.edu/facilities/vlba/other/intro>
 - ALMA: <https://almascience.nrao.edu>
 - Tutorials!



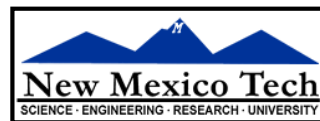
Calibration I

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Synopsis

- Calibration I
 - Why do we have to calibrate?
 - Review Idealistic formalism → Realistic practice
 - Fundamental Calibration Principles
 - Practical Calibration Considerations
 - Baseline-based vs. Antenna-based Calibration
 - Solving for calibration
 - An example Visibility dataset
 - Flagging
- Calibration II
 - Scalar Calibration Example
 - Generalizations & Specializations
 - Full Polarization
 - A Dictionary of Calibration Effects
 - Calibration Heuristics and ‘Bootstrapping’
 - New Calibration Challenges
 - Summary



Why Calibration?

- Synthesis radio telescopes, though exquisitely well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, gain stability, geometric model errors, etc.)
- Need to accommodate deliberate engineering (e.g., frequency down-conversion, analog/digital electronics, filter bandpass, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- Radio Frequency Interference (RFI)

Determining *instrumental and environmental properties* (calibration)
is a prerequisite to
determining *radio source properties*



From Idealistic to Realistic

- Formally, we wish to use our interferometer to obtain the visibility function:

$$V(u, v) = \int_{sky} I(l, m) e^{-i2\pi(ul+vm)} dl dm$$

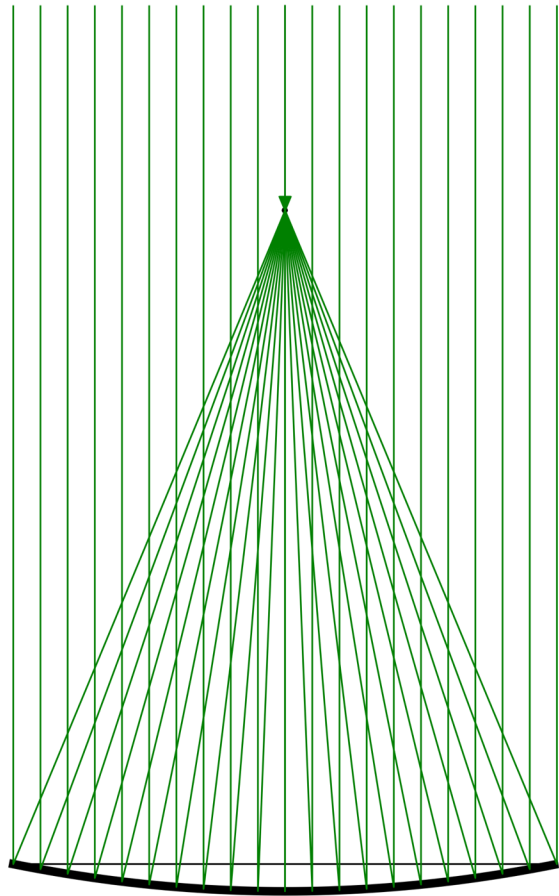
-a Fourier transform which we intend to invert to obtain an image of the sky:

$$I(l, m) = \int_{uv} V(u, v) e^{i2\pi(ul+vm)} du dv$$

- $V(u, v)$ describes the amplitude and phase of 2D sinusoids that add up to an image of the sky (a direction-dependent average)
 - Amplitude: “~how much & ~how concentrated?”
 - Phase: “~where?”
 - c.f. Young’s Double-Slit Interference Experiment (1804)
- To develop an intuitive feel for calibration, let’s review: What are the $V(u, v)$ and how do we measure them?

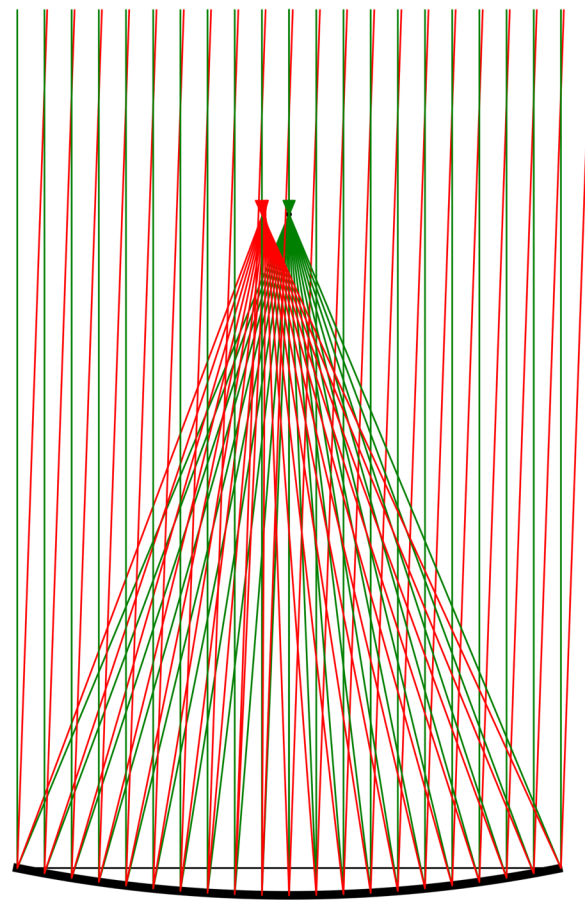
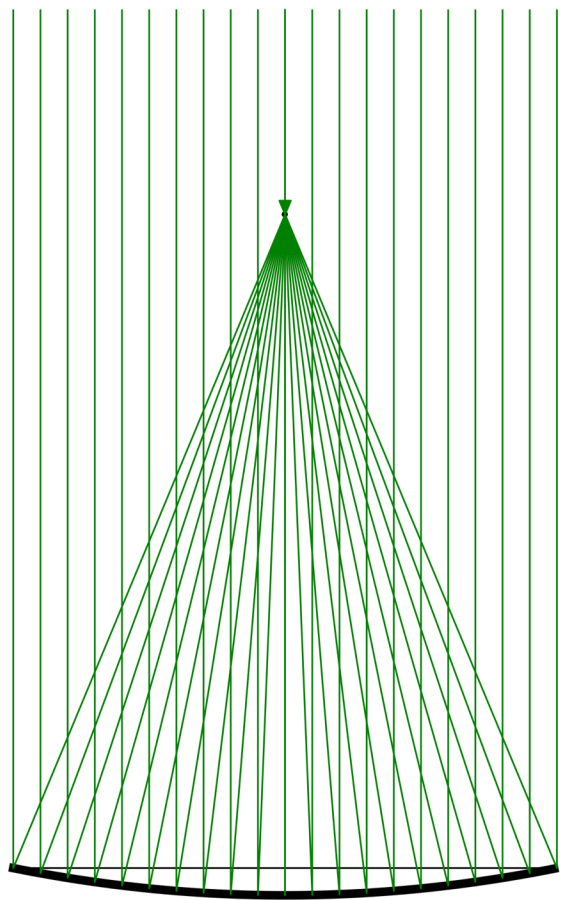


A Filled Aperture

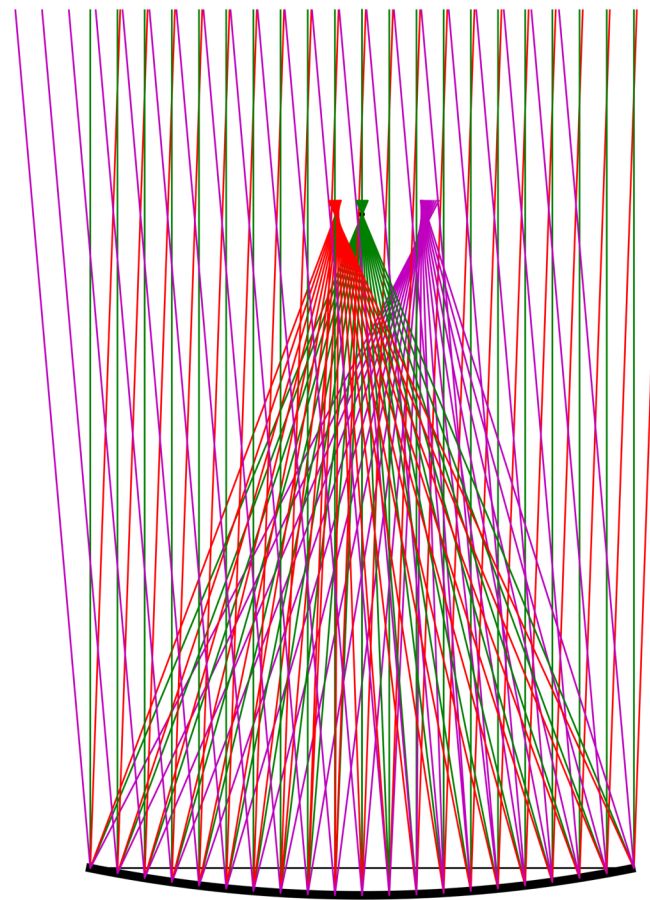
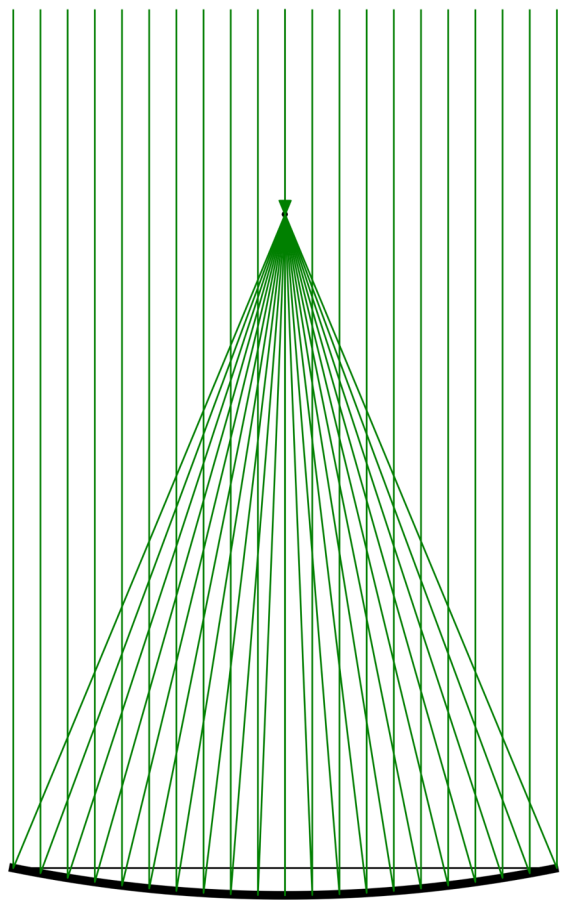


- EM wave focusing instruments
 - Your eye
 - A camera
 - A conventional telescope
- Properties
 - Gathering power (collecting area)
 - Resolution

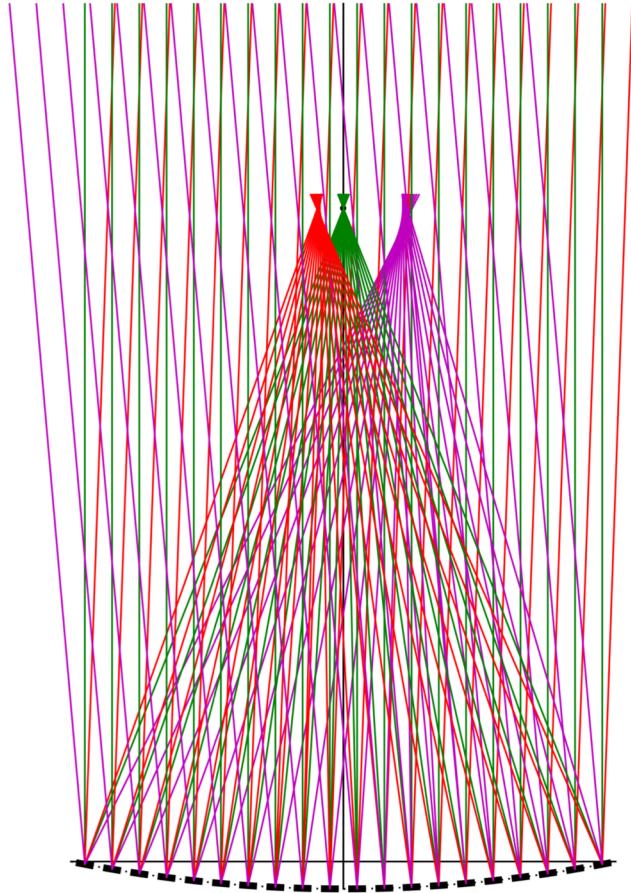
A Filled Aperture



A Filled Aperture

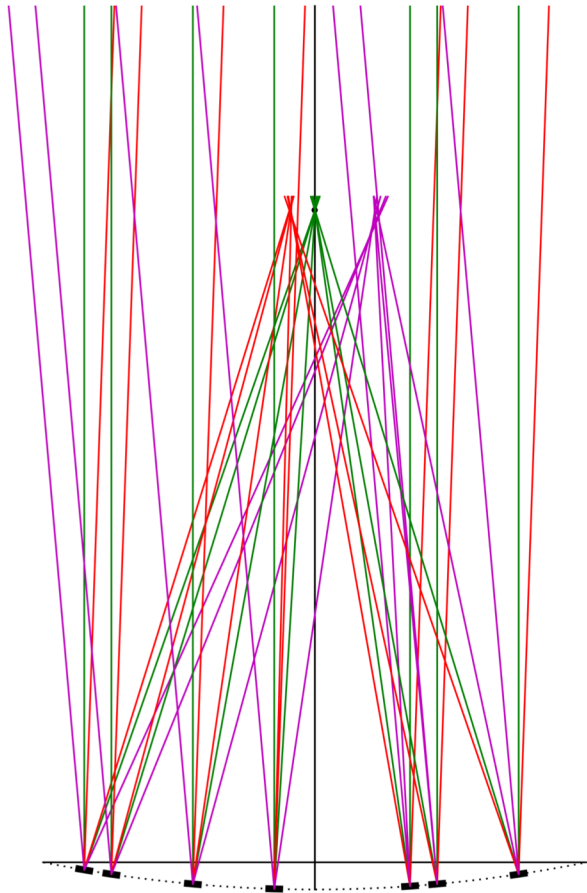


A Segmented (filled) Aperture



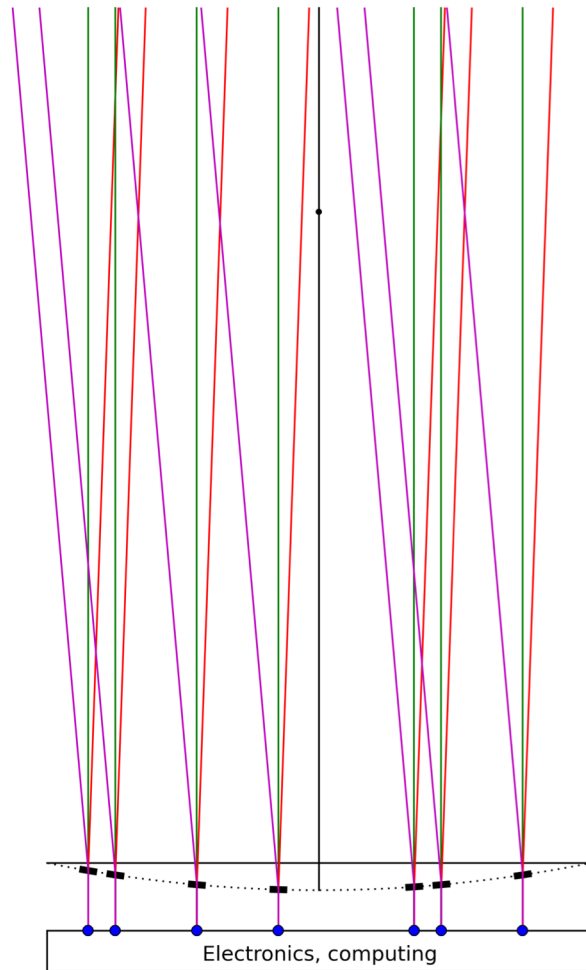
- Each segment ‘gathers’ EM field disturbances arriving from whole sky (*field superposition*)
- Parabolic figure redirects the net field and concentrates it in the focal plane
- *Diffraction* (EM waves!) dictates that each segment contributes complex (w/ phase) field to whole focal plane (*field superposition*)
- *Power is detected*: mean square of complex field sums per pixel: many *cross-products*...
 - Field disturbances from different directions (sources) are *independent*; no net contribution
 - Each surviving cross-product paints a sinusoid (a “*fringe*”) across focal plane, per source, per segment pair (*baseline*)
 - Per baseline source distribution sets the “*fringe visibility*” (*fringe superposition*)
- *Global Fringe Superposition* localizes direction-dependent source power at each pixel yielding a sensible image

An Unfilled Aperture



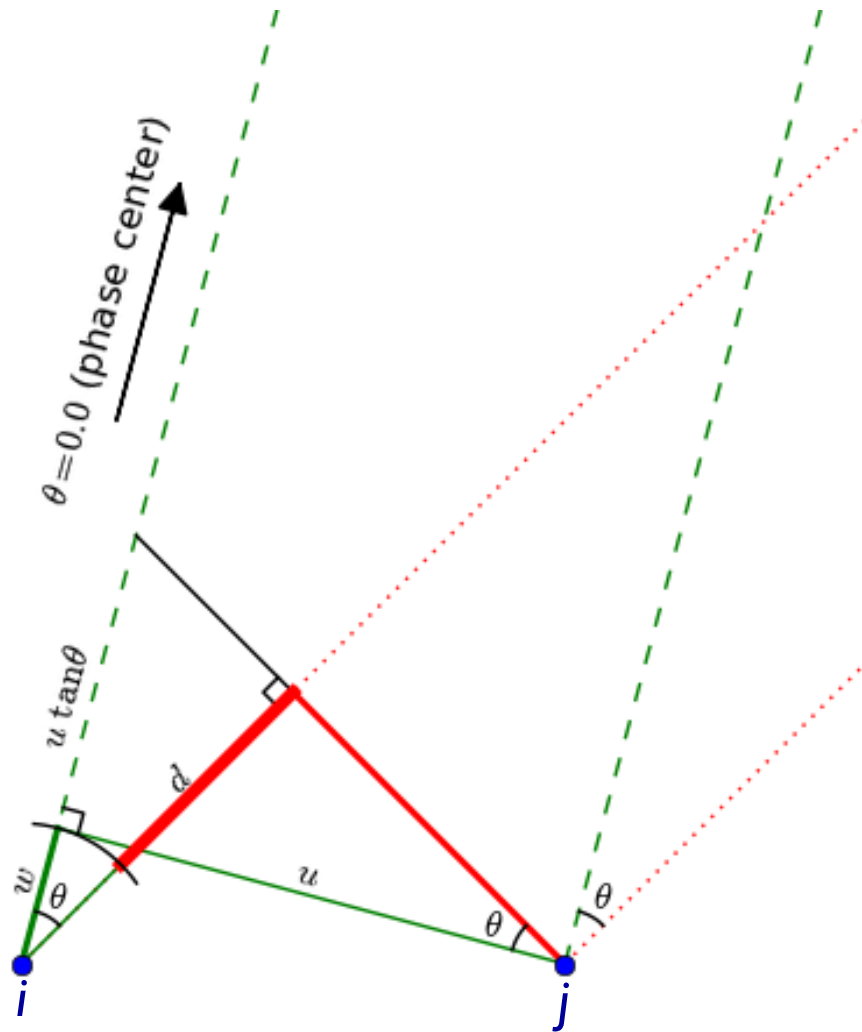
- Fewer segments, and pairs thereof
 - Less total collecting area
 - Uglier diffraction pattern
- Still, a sensible, if more modest image

An Unfilled Aperture – virtual focus



- Fewer segments, and pairs thereof
 - Less total collecting area
 - Uglier diffraction pattern
- Synthesis Interferometry:
 - Cross-products *explicitly* formed electronically
 - “Focus” is formed by computation, through correlation and imaging

The Geometry of Interferometry



- Consider direction-dependent arrival geometry for E-field disturbance reception at two points, i and j, relative to the phase center direction

$$d = (w_\lambda + u_\lambda \tan \theta) \cos \theta - w_\lambda$$

$$= u_\lambda \sin \theta + w_\lambda (\cos \theta - 1)$$

$$d(l) = u_\lambda l + w_\lambda (\sqrt{1 - l^2} - 1) \quad (1D)$$

$$(\sin \theta = l; \cos \theta = \sqrt{1 - l^2})$$

$$d(l, m) = u_\lambda l + v_\lambda m + w_\lambda (\sqrt{1 - l^2 - m^2} - 1) \quad (2D)$$

$$\approx u_\lambda l + v_\lambda m \quad (l, m \ll 1) \quad (\text{small angles})$$

Direction-dependent signals: $s_j = s_i e^{i2\pi d(l, m)}$

What are the $V(u,v)$ that we form?

- Correlate the net E-field disturbances, x_i & x_j arriving at spatially separate sensors, i & j
 - delay-aligned for the phase-center
 - s_i & s_j are the direction-dependent E-field disturbances
- Direction integral and product can be reversed, because the E-field disturbances from different directions don't correlate (**finite bandwidth**)
- s_i and s_j (for a specific direction) differ only by a phase factor given by the arrival geometry, d
- $\langle |s_i|^2 \rangle$ is proportional to the brightness distribution, $I(l,m)$

$$\begin{aligned}
 V_{ij}^{obs} &= \left\langle x_i \cdot x_j^* \right\rangle_{\Delta t} \\
 &= \left\langle \int_{sky} s_i dl_i dm_i \cdot \int_{sky} s_j^* dl_j dm_j \right\rangle_{\Delta t} \\
 &= \left\langle \int_{sky} s_i s_j^* dl dm \right\rangle_{\Delta t} \\
 &= \int_{sky} \left\langle |s_i|^2 \right\rangle e^{-i2\pi d(l,m)} dl dm \\
 &= \int_{sky} I(l,m) e^{-i2\pi d(l,m)} dl dm \\
 &= \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl dm
 \end{aligned}$$

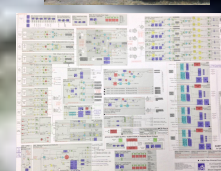
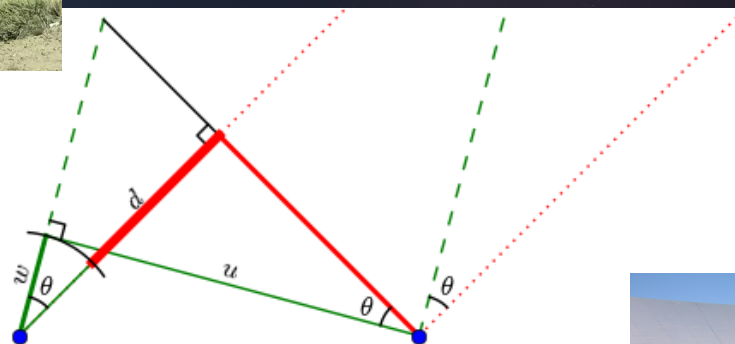


But in reality...

- Weather
- Realistic Antennas
- Electronics...
- Digital correlation
- Finite noise
- ...and the whole thing is moving!
- “Effective” geometry is not ideal



Drew Medlin



Realistic Visibility

- In practice, we obtain an imperfect visibility measurement per antenna pair:

$$\begin{aligned} V_{ij}^{obs}(u, v) &= \langle x_i(t) \cdot x_j^*(t) \rangle_{\Delta t} \\ &= J_{ij} V_{ij}^{true}(u, v) \end{aligned}$$

- x_i & x_j are mutually delay-compensated for the phase center
 - Averaging duration is set by the expected timescales for variation of the correlation result (\sim seconds)
- J_{ij} is a generalized *operator* characterizing the *net* effect of the observing process for antennas i and j on baseline ij , which we must *calibrate*
 - Includes any required scaling to physical units
- Sometimes J_{ij} corrupts the measurement irrevocably, resulting in data that must be *edited* or “*flagged*”



Realistic Visibility: Noise

- Normalized (fractional) visibility (Nyquist):

$$\sigma_{ij} = \frac{1}{\sqrt{2\Delta\nu\Delta t}}$$

- Absolute visibility:

$$\sigma_{ij} = \frac{\sqrt{T_i T_j}}{\sqrt{2\Delta\nu\Delta t}}$$

- $T_i T_j$ are the system temperatures (total sampled powers), in whatever units the corresponding visibility data are in (K or Jy)
- (The numerator, as measured by the correlator, is the factor by which visibilities are typically normalized, e.g. ALMA)

- Formal Visibility Weights:

$$w_{ij} = \frac{1}{\sigma_{ij}^2}$$

- The fundamental measure of statistical information content
- Uniform for normalized visibilities ($2\Delta\nu\Delta t$)



Practical Calibration Considerations

- Observatory housekeeping (optimizing default performance)
 - Nominal antenna positions, earth orientation and rate, clock(s), frequency reference
 - Antenna pointing/focus, voltage pattern, gain curve
 - Calibrator coordinates, flux densities, polarization properties
- Absolute *engineering* calibration (dBm, K, volts)?
 - Amplitude: episodic (ALMA) or continuous (EVLA/VLBA) T_{sys} or switched-power monitoring to enable calibration to nominal K (or J_{ν} , with antenna efficiency information)
 - Phase: Water Vapor Radiometry (ALMA), otherwise practically impossible (relative antenna phase)
 - Traditionally, we concentrate instead on ensuring *effective instrumental stability* on adequate timescales
- **Cross-calibration** a better practical choice
 - Observe strong astronomical sources near science target against which calibration (J_{ij}) can be solved, and interpolate solutions onto target observations
 - Choose appropriate calibrators; usually **point sources** because we can easily predict their visibilities (Amp \sim constant, phase \sim 0)
 - Choose appropriate timescales for calibration



“Absolute” Astronomical Calibrations

- Flux Density Calibration
 - Radio astronomy flux density scale set according to several “constant” radio sources, and planets/moons
 - Use resolved models where appropriate
- Astrometry
 - Most calibrators come from astrometric catalogs; sky coordinate accuracy of target images tied to that of the calibrators
 - Beware of resolved and evolving structures, and phase transfer biases due to troposphere (especially for VLBI)
- Polarization
 - Usual flux density calibrators also have significant stable linear polarization position angle for registration
 - Calibrator circular polarization usually assumed zero (?)
- Relative calibration solutions (and dynamic range) insensitive to errors in these “scaling” parameters



Baseline-based Cross-Calibration

$$V_{ij}^{obs} = J_{ij} V_{ij}^{mod}$$

- Simplest, most-obvious calibration approach: measure complex response of *each baseline* on a standard source, and scale science target visibilities accordingly
 - “Baseline-based” Calibration: $J_{ij} = \left\langle V_{ij}^{obs} / V_{ij}^{mod} \right\rangle_{\Delta t}$
- Only option for single baseline “arrays”
 - historical one-at-a-time visibilities
- Calibration precision same as calibrator visibility sensitivity (on timescale of calibration solution). Improves only with calibrator strength.
- Calibration accuracy sensitive to departures of calibrator from assumed structure
 - Un-modeled calibrator structure transferred (in inverse) to science target!



Antenna-based Cross Calibration

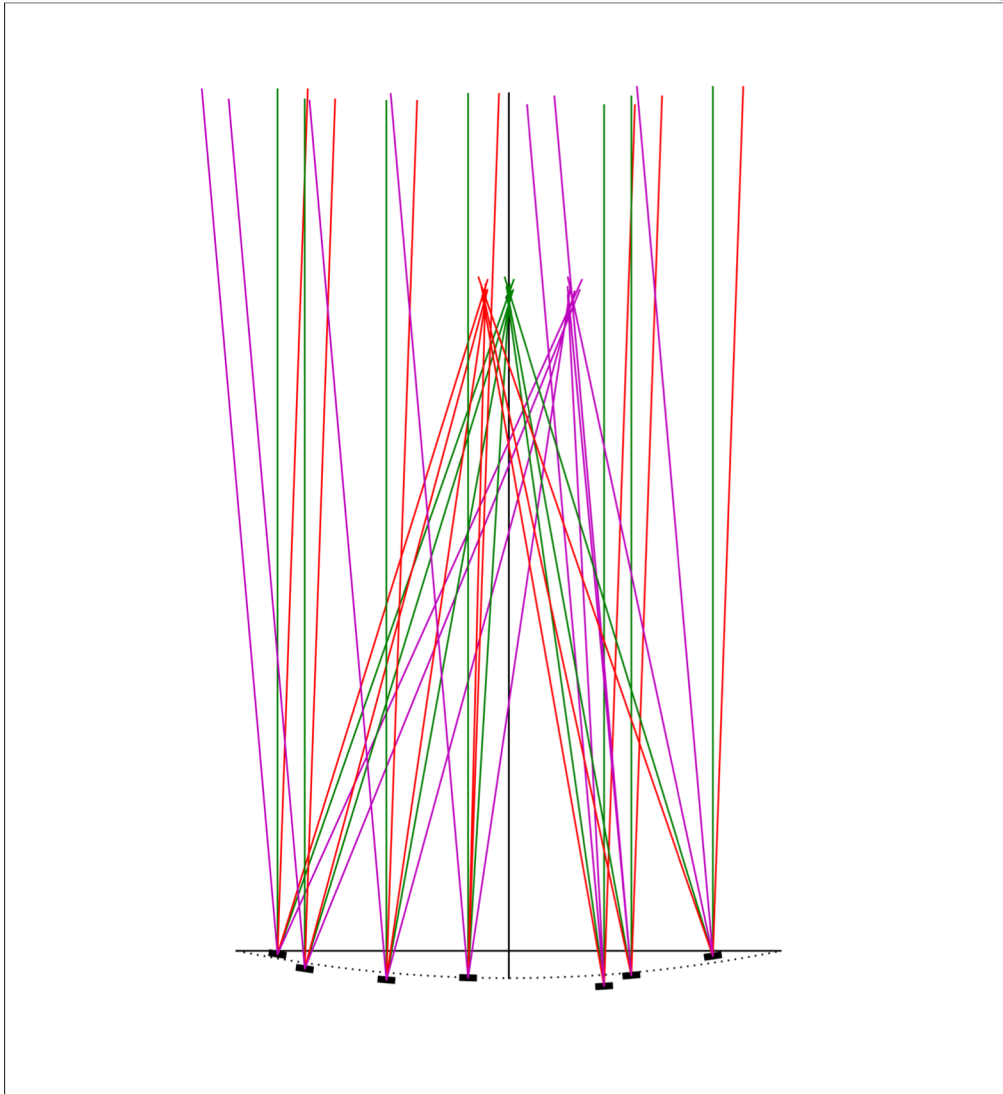
- Measured visibilities are formed from a product of *antenna-based* signals. Can we take advantage of this fact?

$$J_{ij} = J_i J_j^*$$

- *This is the fundamental insight that enabled the spectacular success of synthesis interferometers over the past 40 years.*
 - *Ryle (Nobel Prize in 1974 for developing aperture synthesis) very skeptical that atmospheric errors could be overcome...*

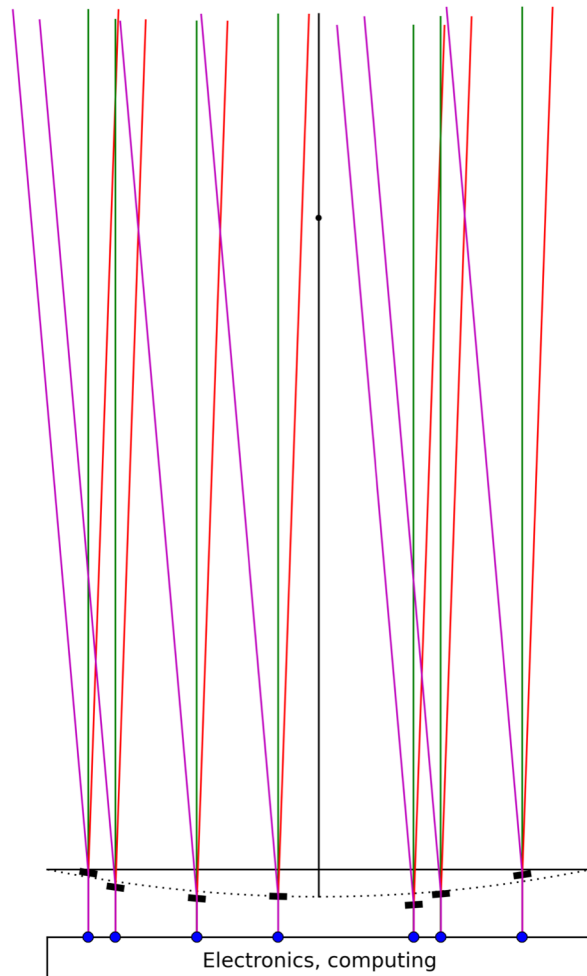


Distorted Unfilled Apertures



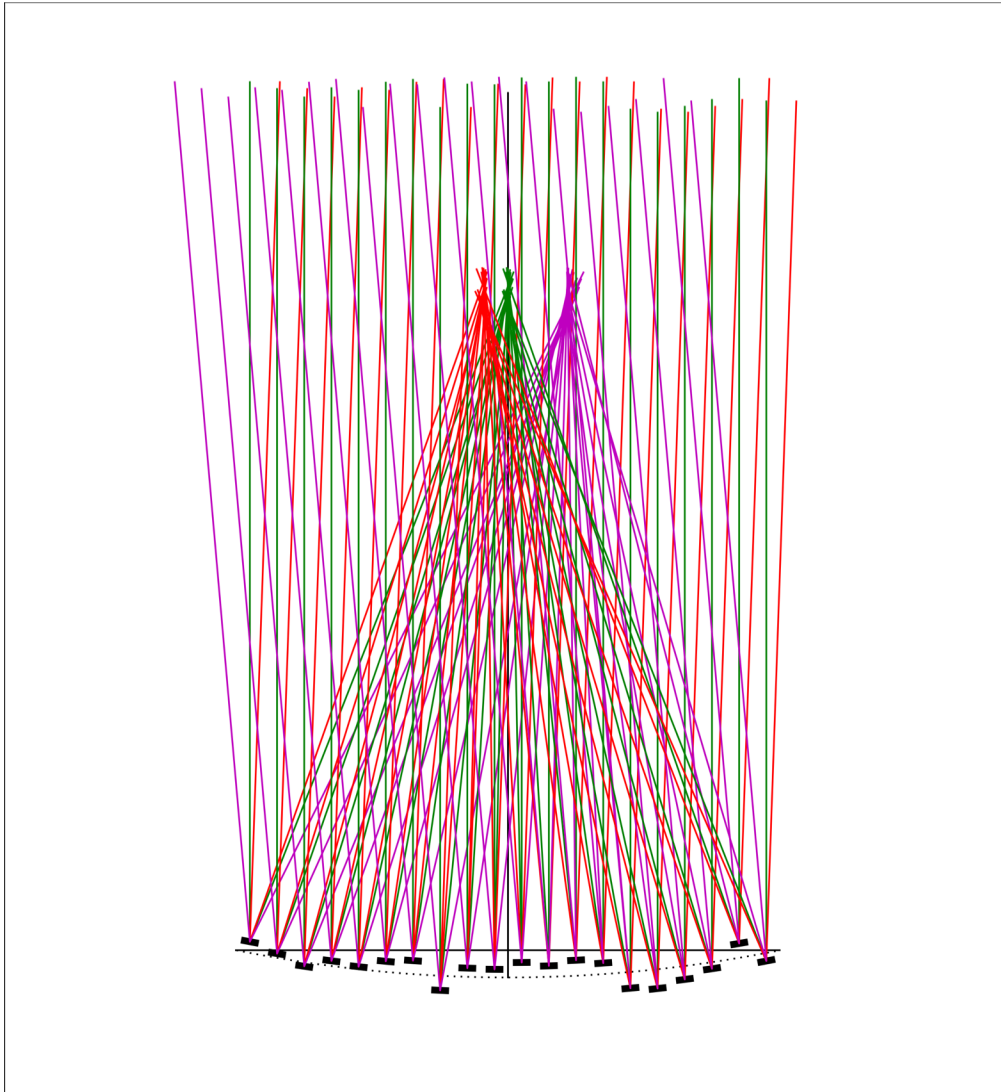
- Each unfilled aperture segment (antenna) has its own distinct properties that uniformly affect all correlations formed with other segments
 - E.g., unmodelled location and electronic path-length errors, atmosphere (delay errors)
 - Complex “Gain” (scale)

Distorted Unfilled Apertures



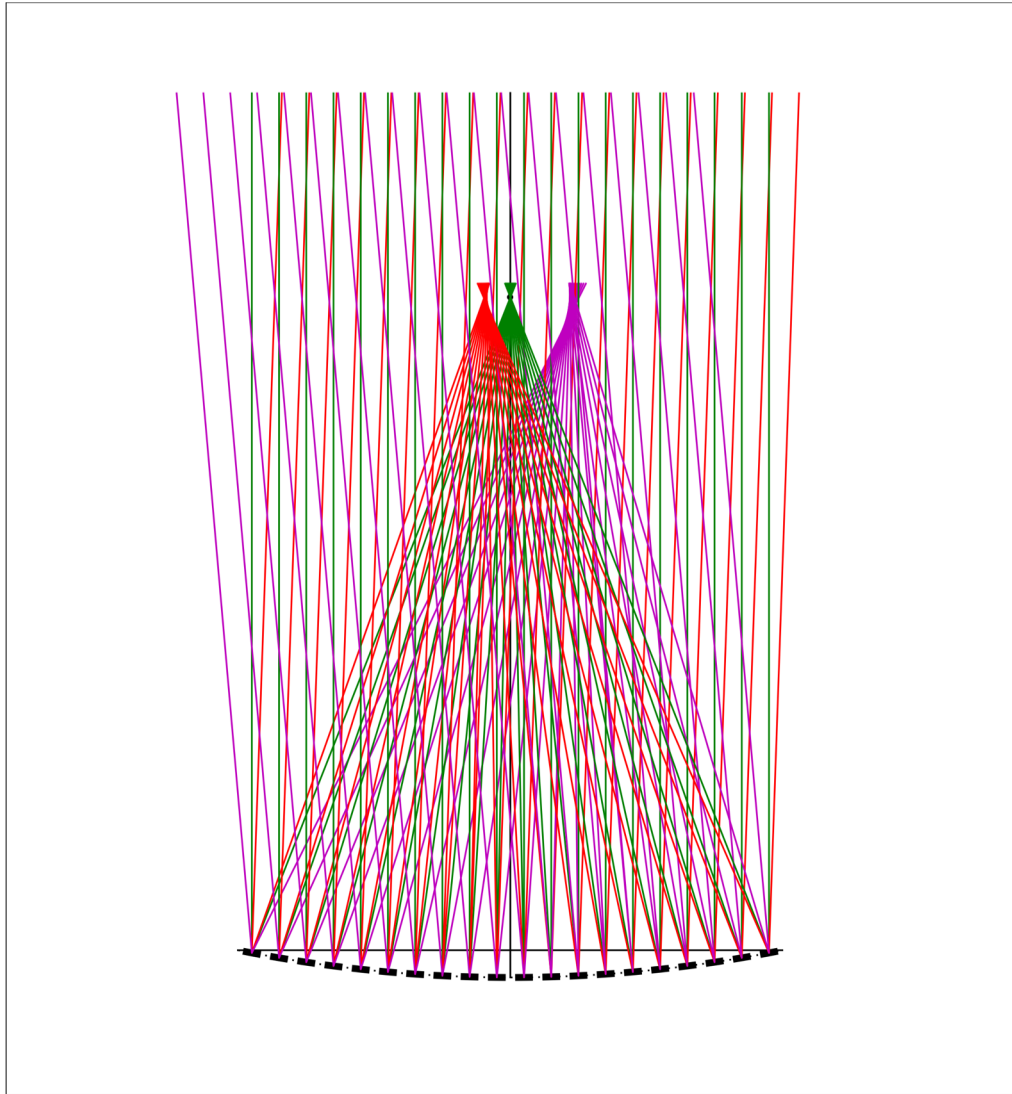
- Each unfilled aperture segment (antenna) has its own distinct properties that uniformly affect all correlations formed with other segments
 - E.g., unmodelled location and electronic path-length errors, atmosphere (delay errors)
 - Complex “Gain” (scale)
- Explicit formation and fine sampling of antenna-pair cross-products provides a post-observation—but “*pre-focus*”—opportunity to correct errors
 - I.e., to *calibrate*

Aside: Distorted Filled Apertures: AO



- Adaptive optics: Real-time, active correction of effective aperture geometry errors
 - Reflector figure (gravitational deflection, etc.)
 - Atmospheric propagation
- HST
 - Spherical aberration (constant)
 - Real-time “calibration” by introducing optical elements that correct wavefront *before* reaching the focus (otherwise, it was a *deconvolution* problem)
- Eyeglasses!
 - Calibration on an \sim annual timescale...

Aside: Distorted Filled Apertures: AO



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- Eyeglasses!
 - Calibration on an \sim annual timescale...

Antenna-based Cross Calibration

- The net time-dependent E-field signal sampled by antenna i , $x_i(t)$, is a combination of the desired signal, $s_i(t,l,m)$, corrupted by a factor $J_i(t,l,m)$ and integrated over the sky (l,m) , and diluted by noise, $n_i(t)$:

$$\begin{aligned}x_i(t) &= \int_{sky} J_i(t,l,m) s_i(t,l,m) dl dm + n_i(t) \\ &= s'_i(t) + n_i(t)\end{aligned}$$

- $x_i(t)$ is sampled (complex) voltage provided to the correlator input
- $J_i(t,l,m)$ is the product of a series of effects encountered by the incoming signal
- $J_i(t,l,m)$ is an *antenna-based* (one index) complex number
 - Amplitude: “gain” (also units)
 - Phase: geometry/directional distortion
- Usually, $|n_i|^2 \gg |s'_i|^2$ (i.e., noise power dominates)



Correlation of Realistic Signals

- The correlation of two realistic (aligned for a specific direction) signals from different antennas:
- Noise correlations have zero expectation—even if $|n_i|^2 \gg |s_i|^2$
 - the correlation process isolates desired signals amidst zero-mean noise
- Same analysis as before, except we carry $J_i J_j$ terms
 - J_i 's time- and frequency-dependence (and field-of-view) set the required timescale and frequency resolution

$$\begin{aligned}
 \langle x_i \cdot x_j^* \rangle_{\Delta t} &= \left\langle (s'_i + n_i) \cdot (s'_j + n_j)^* \right\rangle_{\Delta t} \\
 &= \langle s'_i \cdot s_j'^* \rangle_{\Delta t} + \langle s'_i \cdot n_j^* \rangle_{\Delta t} + \langle n_i \cdot s_j'^* \rangle_{\Delta t} + \langle n_i \cdot n_j^* \rangle_{\Delta t} \\
 &= \langle s'_i \cdot s_j'^* \rangle_{\Delta t}
 \end{aligned}$$

$$\begin{aligned}
 &= \left\langle \int_{sky} J_i s_i dl_i dm_i \cdot \int_{sky} J_j^* s_j^* dl_j dm_j \right\rangle_{\Delta t} \\
 &= \left\langle \int_{sky} J_i J_j^* s_i s_j^* dl dm \right\rangle_{\Delta t} \\
 &= \int_{sky} J_i J_j^* I(l, m) e^{-i2\pi(ul+vm)} dl dm
 \end{aligned}$$



The Scalar Measurement Equation

$$V_{ij}^{obs} = \int_{sky} J_i J_j^* I(l, m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dl dm$$

- First, isolate non-direction-dependent effects, and factor them from the integral:

$$= \left(J_i^{vis} J_j^{vis*} \right) \int_{sky} \left(J_i^{sky} J_j^{sky*} \right) I(l, m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dl dm$$

- Next, we recognize that over small fields of view, it is often possible to assume $J^{sky} = 1.0$, and we have a relationship between ideal and observed Visibilities:

$$= J_i J_j^* \int_{sky} I(l, m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dl dm$$

$$V_{ij}^{obs} = J_i J_j^* V_{ij}^{true}$$

- Standard calibration of most existing arrays reduces to solving this last equation for the J_i , assuming a visibility model V_{ij}^{mod} for a calibrator
- Visibilities corrupted by *difference of antenna-based phases*, and *product of antenna-based amplitudes*



Aside: Auto-correlations and Single Dishes

- The *auto-correlation* of a signal from a *single* antenna:

$$\begin{aligned}\langle x_i \cdot x_i^* \rangle_{\Delta t} &= \langle (s'_i + n_i) \cdot (s'_i + n_i)^* \rangle_{\Delta t} \\ &= \langle s'_i \cdot s'^*_i \rangle + \langle n_i \cdot n_i^* \rangle \\ &= \left\langle \int_{sky} |J_i|^2 |s_i|^2 dl dm \right\rangle_{\Delta t} + \langle |n_i|^2 \rangle \\ &= \int_{sky} |J_i|^2 I(l, m) dl dm + \langle |n_i|^2 \rangle\end{aligned}$$

- This is an integrated (sky) power measurement plus *non-zero-mean* noise, i.e., the T_{sys}
- Desired signal *not* simply isolated from noise
- Noise usually dominates the power
- Scalar calibration (c.f. single-baseline calibration)
- Single dish radio astronomy calibration strategies rely on switching (differencing) schemes to isolate desired signal from the noise



Solving for the J_i

- Observe point-like calibrator for which we know true visibilities, and...
- We can write:

$$V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$$

- ...and define chi-squared:

$$\chi^2 = \sum_{\substack{i,j \\ i \neq j}} |V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod}|^2 w_{ij} \quad \left(w_{ij} = \frac{1}{\sigma_{ij}^2} \right)$$

- ...and minimize chi-squared w.r.t. each $J_i^* \left(\frac{\partial \chi^2}{\partial J_i^*} = 0 \right)$, yielding:

$$J_i = \frac{\sum_{j \neq i} (V_{ij}^{obs} J_j V_{ij}^{mod*} w_{ij})}{\sum_{j \neq i} (|J_j|^2 |V_{ij}^{mod}|^2 w_{ij})}$$

$$= \frac{\sum_{j \neq i} \left(\frac{V_{ij}^{obs}}{J_j^* V_{ij}^{mod}} \right) w'_{ij}}{\sum_{j \neq i} w'_{ij}} \quad \left(w'_{ij} = |J_j|^2 |V_{ij}^{mod}|^2 w_{ij} \right)$$

- (Requires iteration to solve the ensemble)



Solving for J_i (cont)

- Formal errors:

$$\sigma_{J_i} = \sqrt{\frac{1}{\sum_{j \neq i} |V_{ij}^{mod}|^2 |J_j|^2 / \sigma_{ij, \Delta t}^2}}$$

- For a \sim uniform array (\sim same sensitivity on all baselines, \sim same calibration magnitude on all antennas) and point-like calibrator:

$$\sigma_{J_i} \approx \frac{\sigma_{ij, \Delta t}}{|V^{mod}| \sqrt{\langle |J_j|^2 \rangle (N_{ant} - 1)}}$$

- Calibration error decreases with increasing calibrator strength *and* square-root of N_{ant} (c.f. baseline-based calibration).
- Other properties of the antenna-based solution:
 - Minimal degrees of freedom (N_{ant} factors, $N_{ant}(N_{ant}-1)/2$ measurements)
 - Net calibration for a baseline involves a phase difference, so *absolute* directional information is lost ($N_{ant}-1$ phases)
 - Closure...



Antenna-based Calibration and Closure

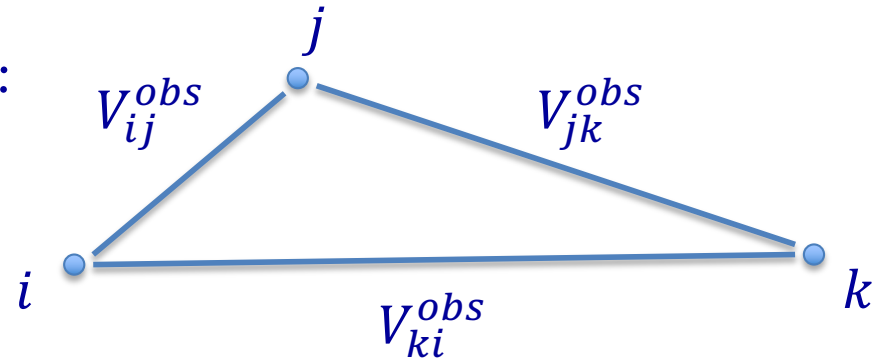
- Success of synthesis telescopes relies on antenna-based calibration
 - Fundamentally, any information that can be factored into antenna-based terms, could be antenna-based effects, and not true source visibility information
 - For $N_{ant} > 3$, non-trivial source visibility information cannot be *entirely* obliterated by any antenna-based calibration
- Observables *independent* of antenna-based errors: closure
 - Closure Phase (3 baselines)
 - Closure Amplitude (4 baselines)
- Baseline-based calibration formally violates closure!



Closure Phase

$$\begin{aligned}
 V_{ij}^{obs} &= A_{ij}^{obs} e^{i\phi_{ij}^{obs}} = G_i G_j^* V_{ij}^{true} \\
 &= g_i e^{i\theta_i} g_j e^{-i\theta_j} A_{ij} e^{i\phi_{ij}^{true}} = g_i g_j A_{ij}^{true} e^{i(\phi_{ij}^{true} + \theta_i - \theta_j)}
 \end{aligned}$$

- Form total phase around three baselines:



$$\begin{aligned}
 \phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} &= (\phi_{ij}^{true} + \theta_i - \theta_j) + (\phi_{jk}^{true} + \theta_j - \theta_k) + (\phi_{ki}^{true} + \theta_k - \theta_i) \\
 &= \phi_{ij}^{true} + \phi_{jk}^{true} + \phi_{ki}^{true}
 \end{aligned}$$

- Closure phase is independent of antenna-based phase errors
- $(N_{ant} - 1)(N_{ant} - 2)/2$ independent closure phases

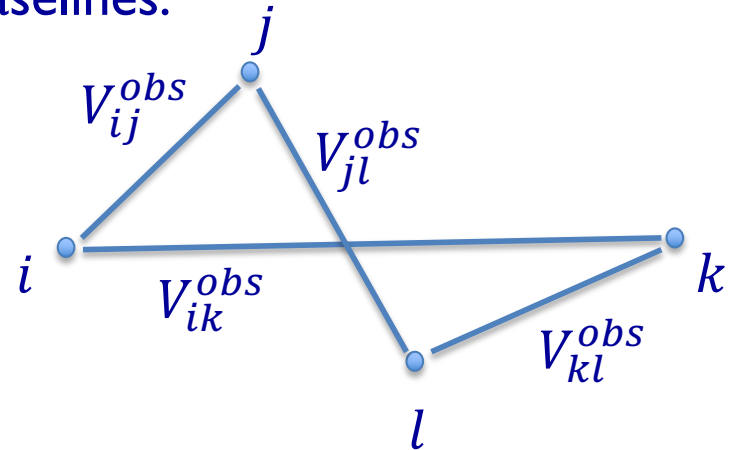
- Baseline-based calibration formally violates closure!

Closure Amplitude

$$\begin{aligned}
 V_{ij}^{obs} &= A_{ij}^{obs} e^{i\phi_{ij}^{obs}} = G_i G_j^* V_{ij}^{true} \\
 &= g_i e^{i\theta_i} g_j e^{-i\theta_j} A_{ij} e^{i\phi_{ij}^{true}} = g_i g_j A_{ij}^{true} e^{i(\phi_{ij}^{true} + \theta_i - \theta_j)}
 \end{aligned}$$

- Form ratios of amplitude products from four baselines:

$$\frac{A_{ij}^{obs} A_{kl}^{obs}}{A_{ik}^{obs} A_{jl}^{obs}} = \frac{g_i g_j A_{ij}^{true} g_k g_l A_{kl}^{true}}{g_i g_k A_{ik}^{true} g_j g_l A_{jl}^{true}} = \frac{A_{ij}^{true} A_{kl}^{true}}{A_{ik}^{true} A_{jl}^{true}}$$



- Closure amplitude is independent of antenna-based amplitude errors
- $N_{ant}(N_{ant} - 3)/2$ independent closure amplitudes

- Baseline-based calibration formally violates closure!

Reference Antenna

- Since the “antenna-based” phase solution is derived from antenna phase *differences*, we do not measure phase absolutely
 - *relative* astrometry (only as good as assumed calibrator astrometry)
- Phase solutions typically referred to a specific antenna, the refant, which is assumed to have constant phase (zero, in both polarizations)
 - refant typically near array center
 - The refant’s phase variation distributed to all other antennas’ solutions
 - Asserts unambiguous phase continuity, for adequate time sampling, thereby ensuring reliable interpolation of phase (c.f. arbitrary phase offsets between solutions)
 - Asserts stable cross-hand phase frame (which must be calibrated)
- Problems:
 - A single good refant not always available over whole observation (time, frequency), due to flagging, etc.
 - Effective cross-hand phase of refant (or over multiple refant changes) may not, in fact, be stable...



Corrected Visibility

- Visibility...

$$V_{ij}^{obs} = J_i J_j^* V_{ij}^{true} \quad \rightarrow \quad V_{ij}^{cor} = J_i^{-1} J_j^{*-1} V_{ij}^{obs}$$

- ...and weights!
 - calibrate the sigmas!

$$w_{ij}^{cor} = w_{ij}^{obs} |J_i|^2 |J_j|^2 = \frac{|J_i|^2 |J_j|^2}{\sigma_{ij}^2}$$

- Statistical information content becomes baseline-dependent
- Imaging will be a non-trivially-weighted direction-dependent average of the visibilities...



What Is Delivered by a Synthesis Array?

- An *enormous* list of complex visibilities! (*Enormous!*)
 - At each timestamp (~1-10s intervals): $N(N-1)/2$ baselines
 - EVLA: 351 baselines
 - VLBA: 45 baselines
 - ALMA: 1225+ baselines
 - For each baseline: up to 64 Spectral Windows (“spws”, “subbands” or “IFs”)
 - For each spectral window: tens to thousands of channels ($\Delta\nu < 10$ MHz)
 - For each channel: 1, 2, or 4 complex correlations (polarizations)
 - EVLA or VLBA: RR or LL or (RR,LL), or (RR,RL,LR,LL)
 - ALMA: XX or YY or (XX,YY) or (XX,XY,YX,YY)
 - With each correlation, a weight value and a flag (T/F)
 - Meta-info: Coordinates, antenna, field, frequency label info
- $N_{\text{total}} = N_t \times N_{\text{bl}} \times N_{\text{spw}} \times N_{\text{chan}} \times N_{\text{corr}}$ visibilities
 - $\sim \text{few } 10^6 \times N_{\text{spw}} \times N_{\text{chan}} \times N_{\text{corr}}$ vis/hour \rightarrow 10s to 100s of GB per observation

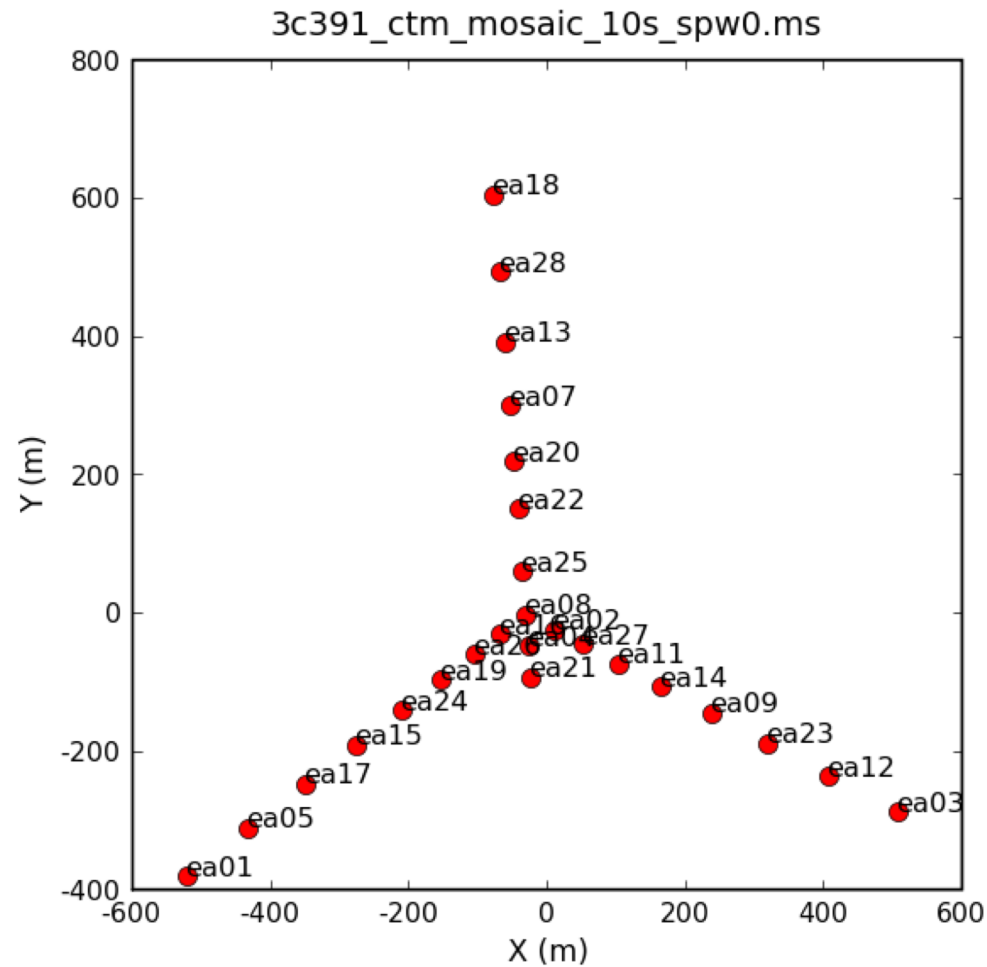


A Typical Dataset

- Array:
 - EVLA D-configuration (Apr 2010)
- Sources:
 - Science Target: 3C391, a SNR (7 mosaic pointings)
 - Near-target calibrator: J1822-0938 (~11 deg from target)
 - Flux Density calibrator: 3C286
 - Instrumental Polarization Calibrator: 3c84
- Signals:
 - RR,RL,LR,LL correlations
 - One spectral window centered at 4600 MHz, 128 MHz bandwidth, 64 channels

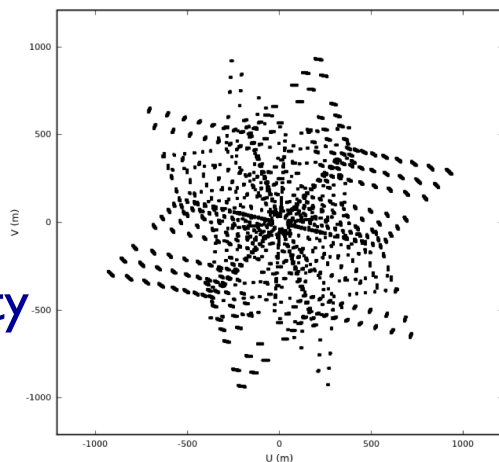


The Array

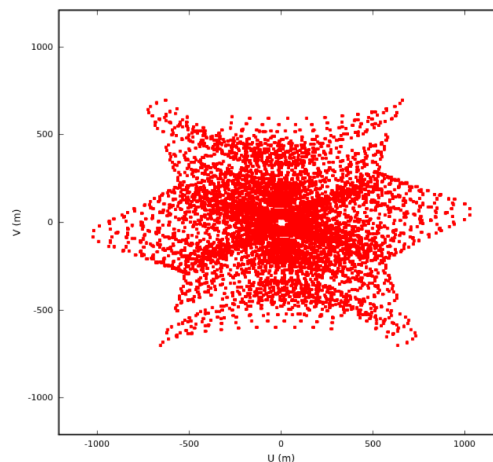


UV-coverages

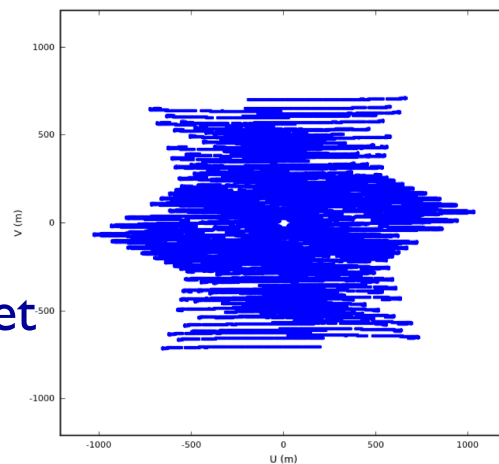
3C286
Flux Density



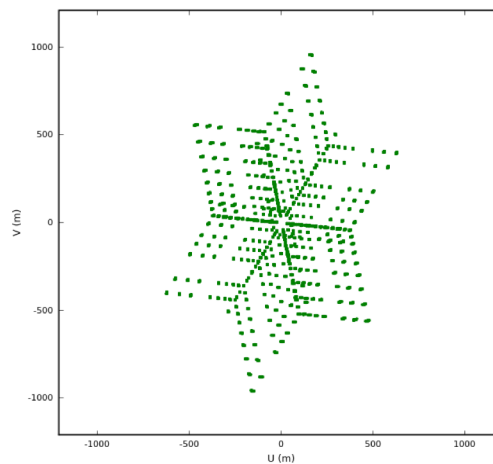
J1822-0938
Gain Calibrator



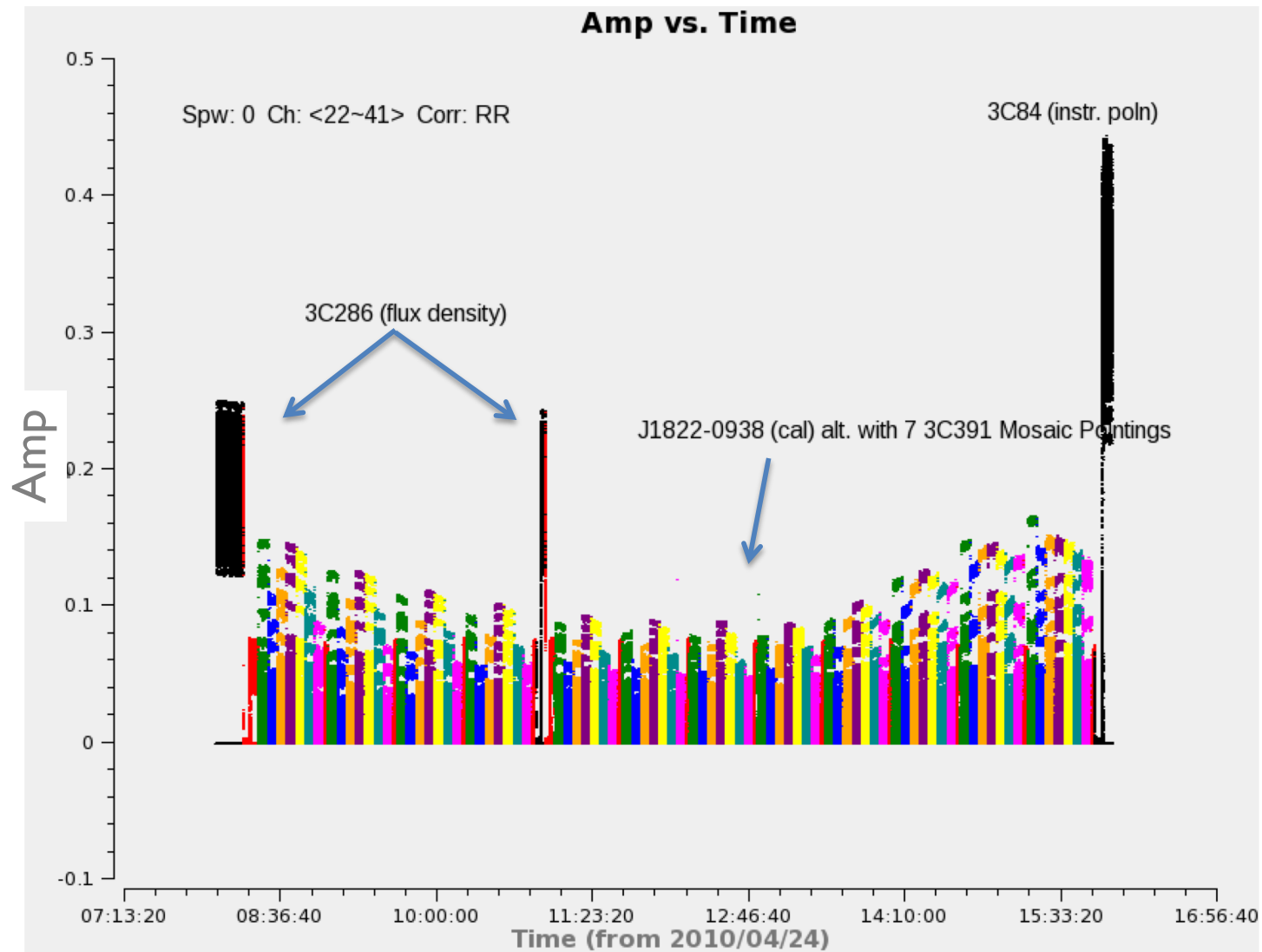
3C391
Science Target



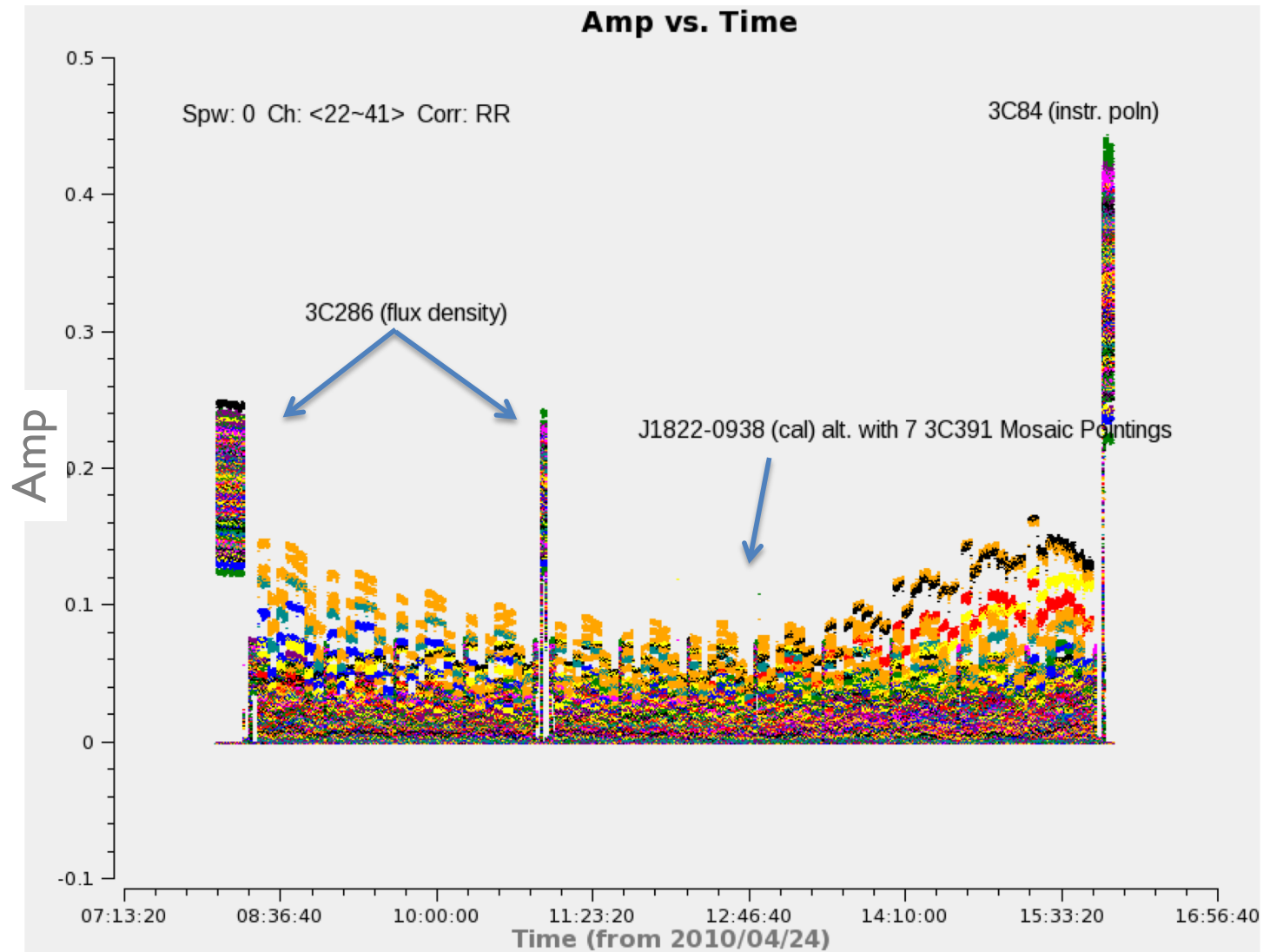
3C84
Instr. Poln Calibrator



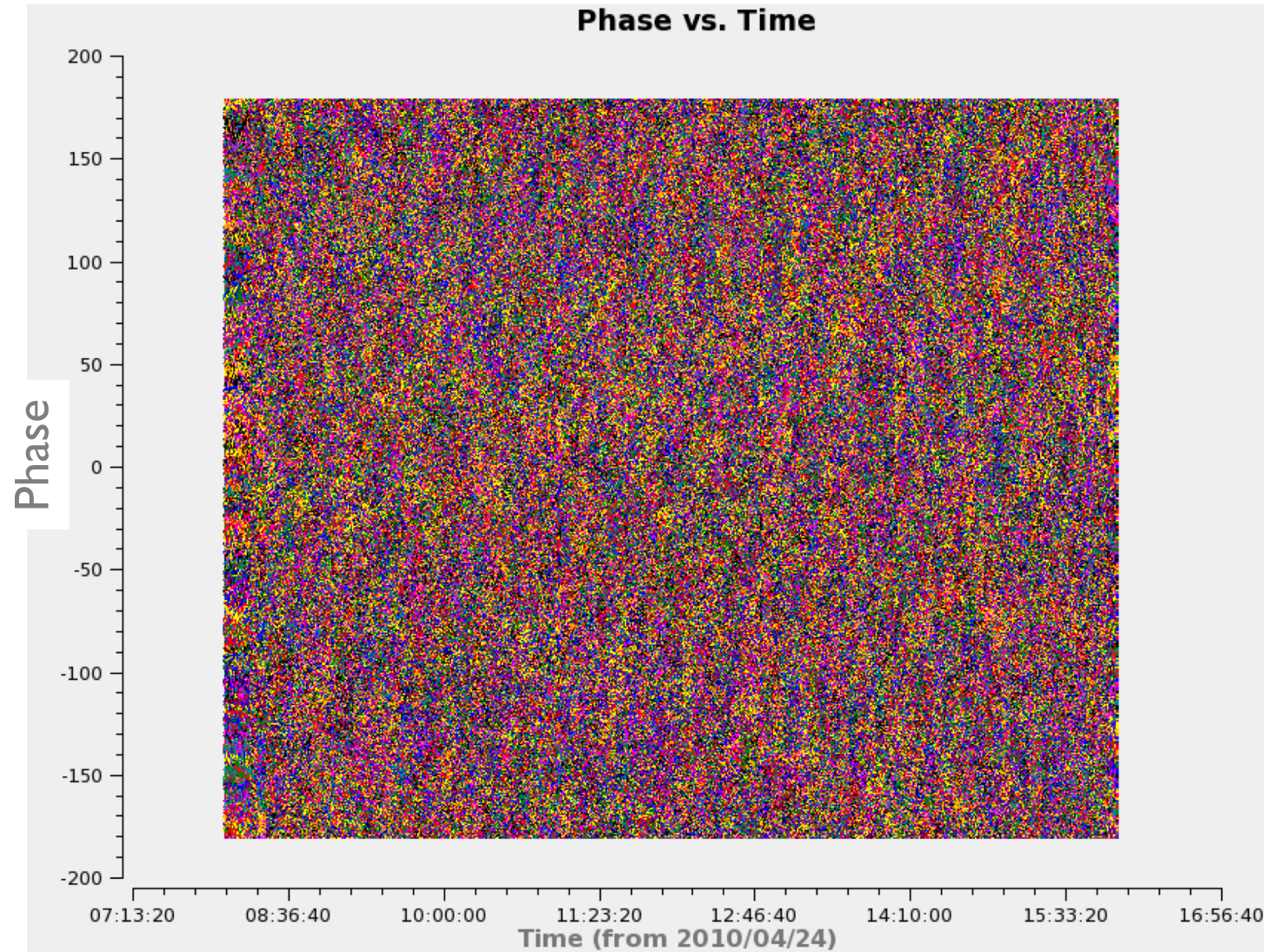
The Visibility Data (source colors)



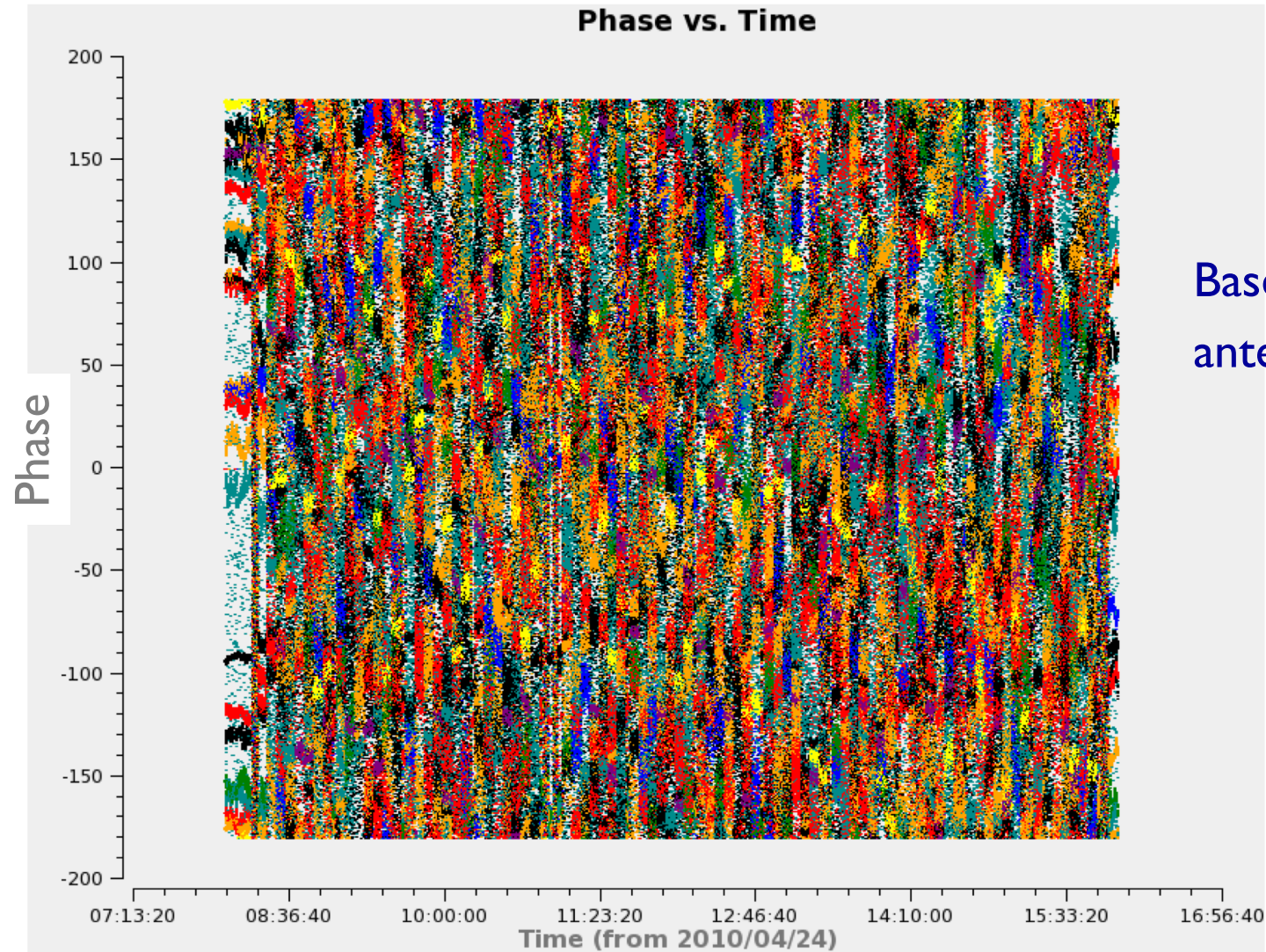
The Visibility Data (baseline colors)



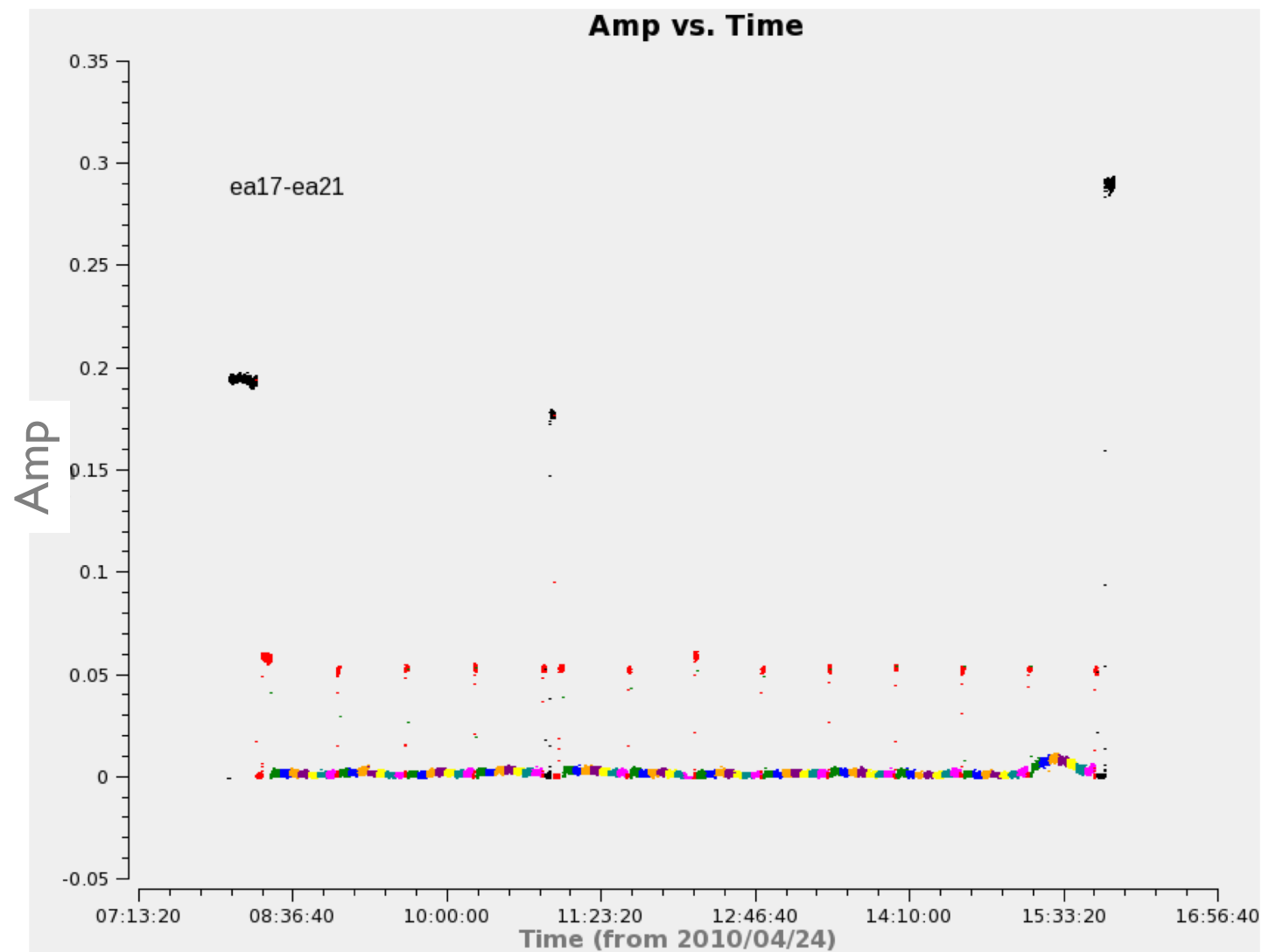
The Visibility Data (baseline colors)



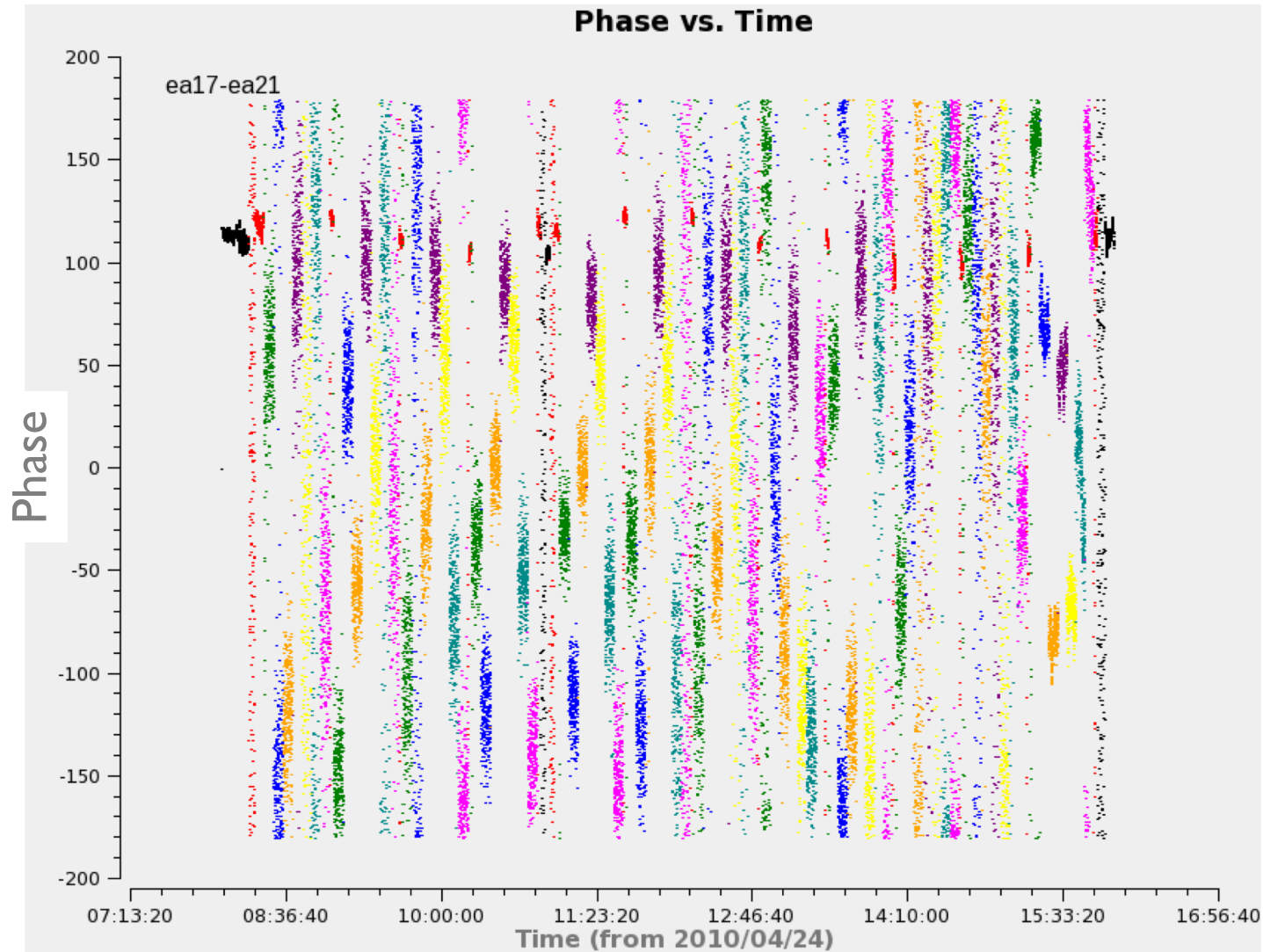
The Visibility Data (baseline colors)



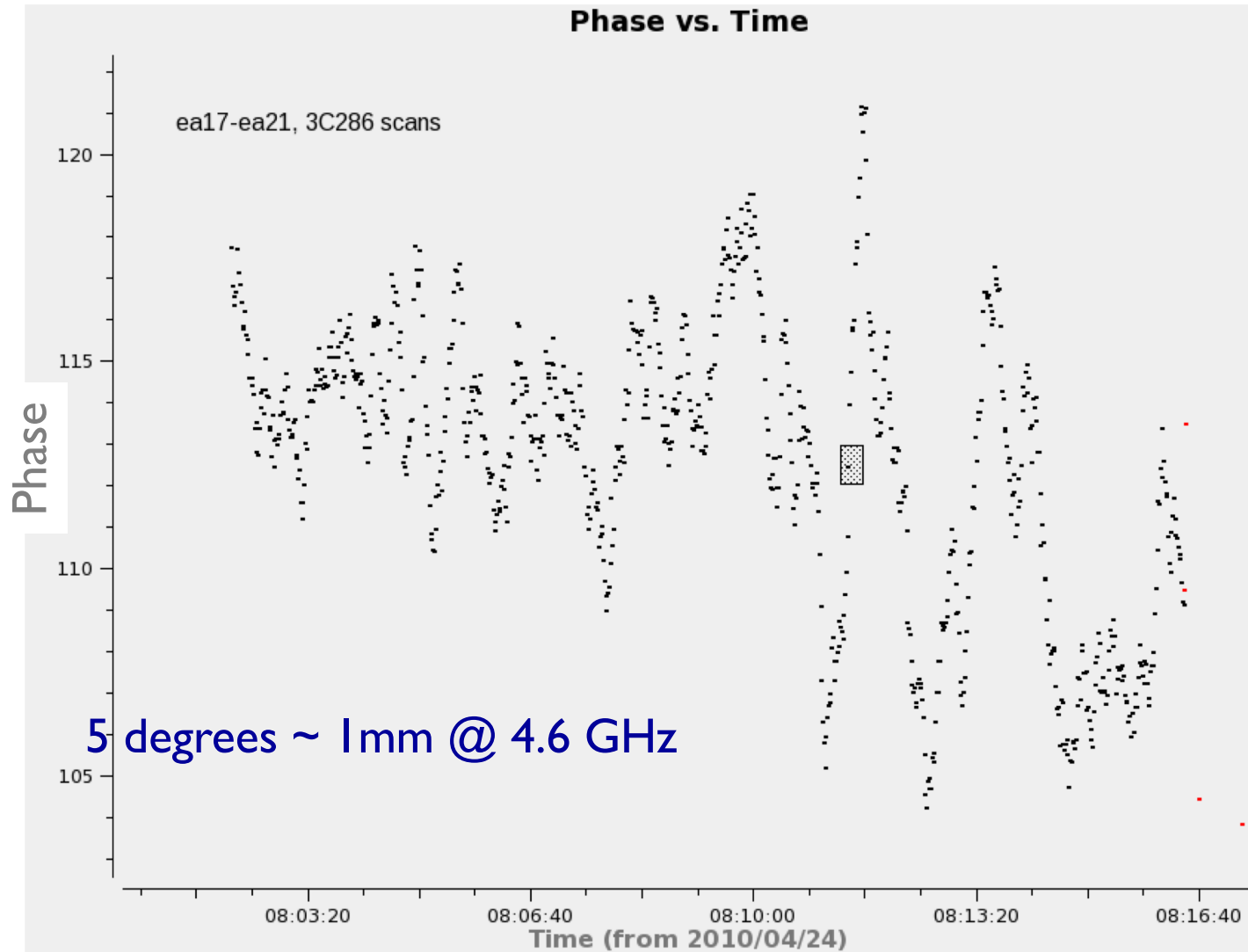
A Single Baseline – Amp (source colors)



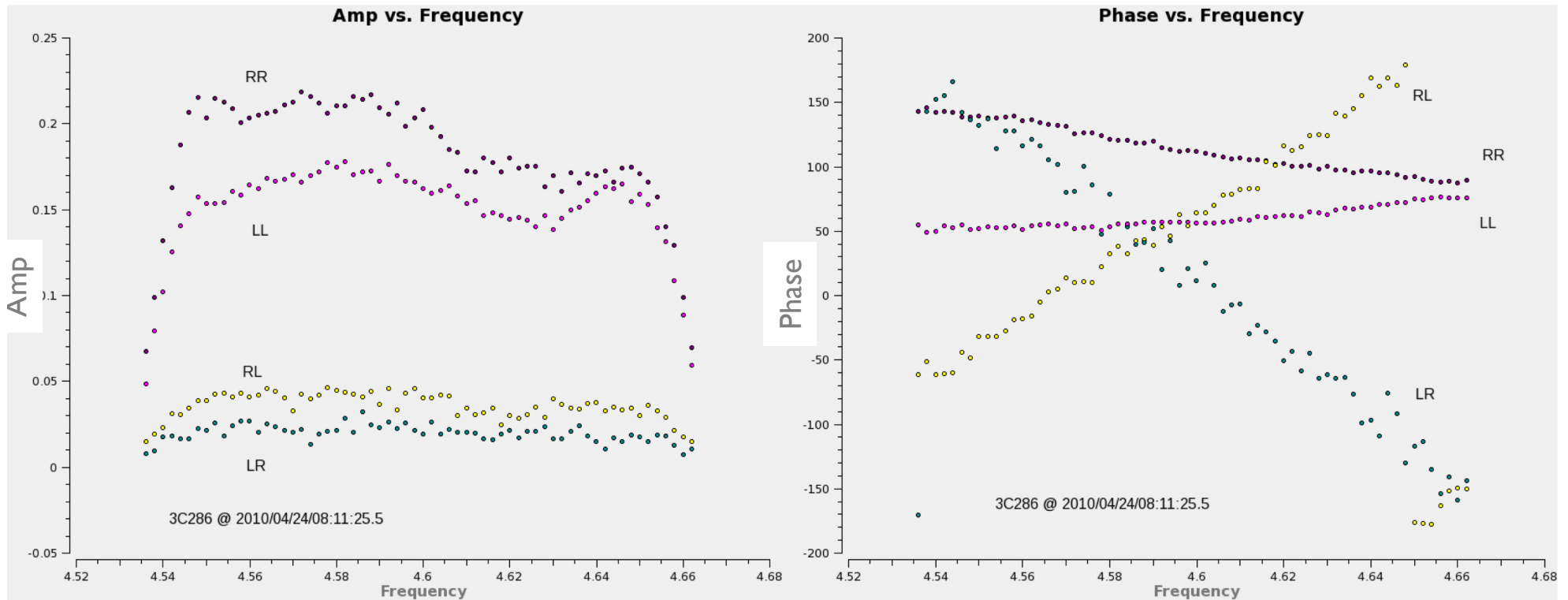
A Single Baseline – Phase (source colors)



A Single Baseline – 2 scans on 3C286



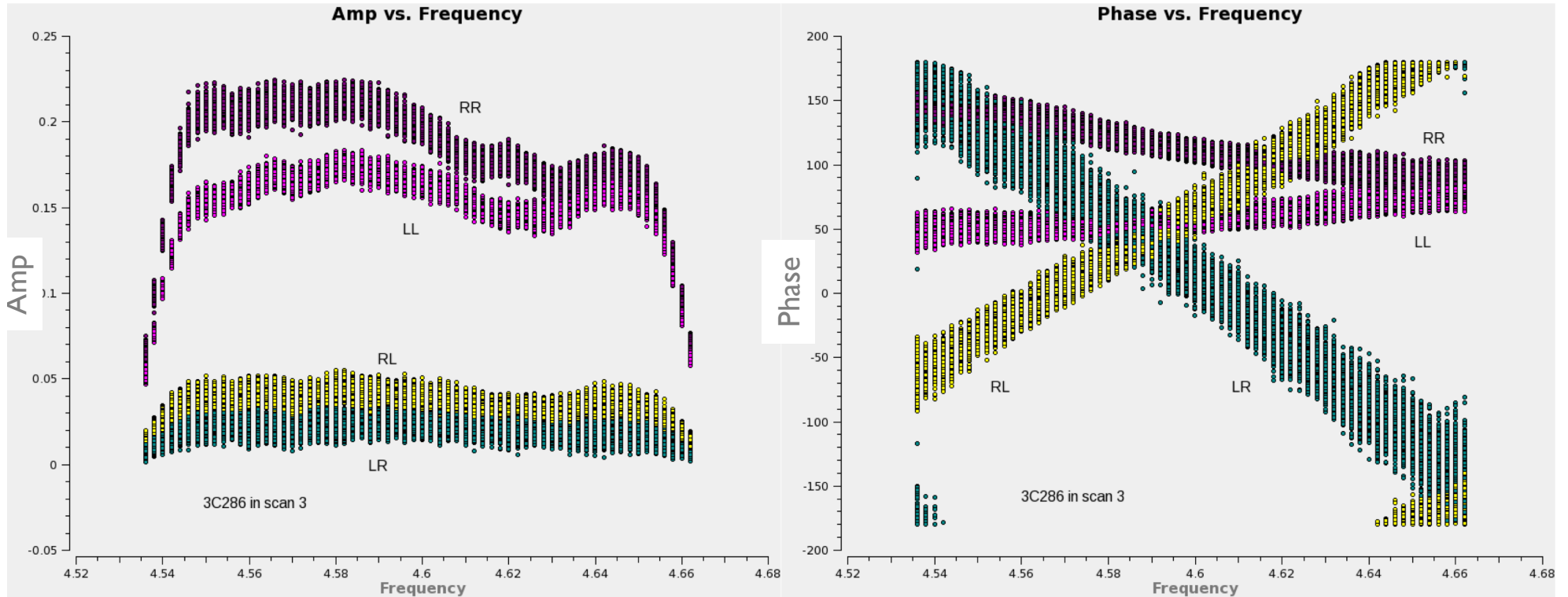
Single Baseline, Single Integration Visibility Spectra (4 correlations)



baseline ea17-ea21



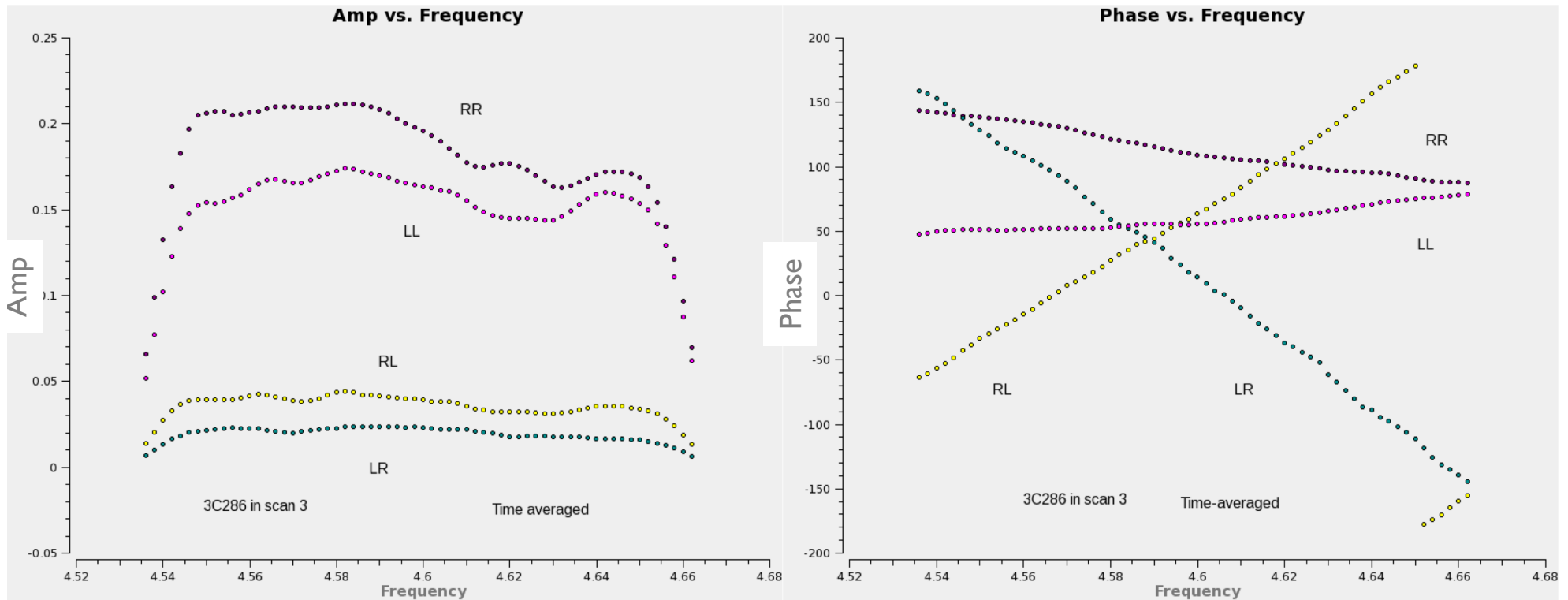
Single Baseline, Single Scan Visibility Spectra (4 correlations)



baseline ea17-ea21



Single Baseline, Single Scan (time-averaged) Visibility Spectra (4 correlations)



baseline ea17-ea21

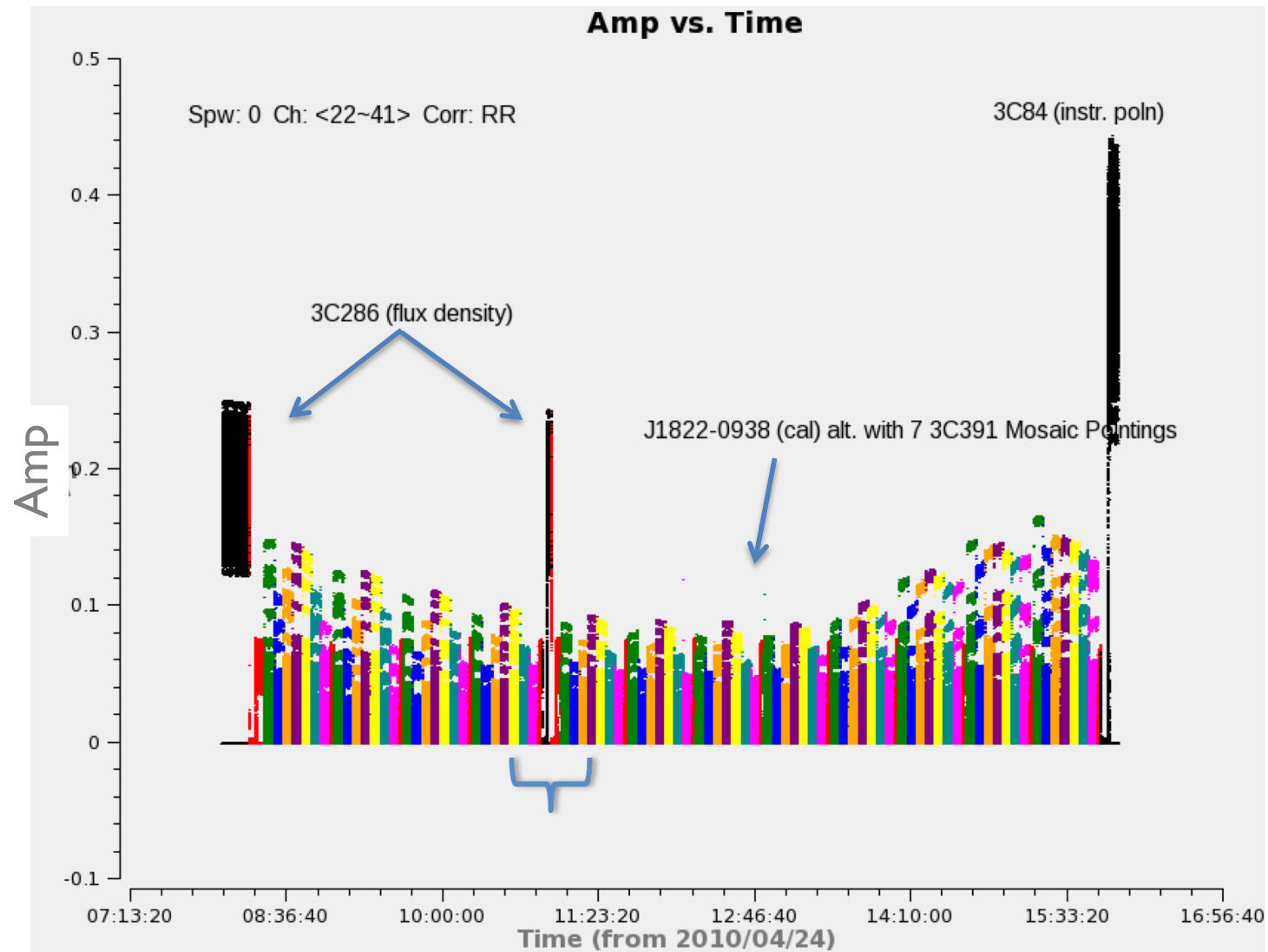


Data Examination and Editing

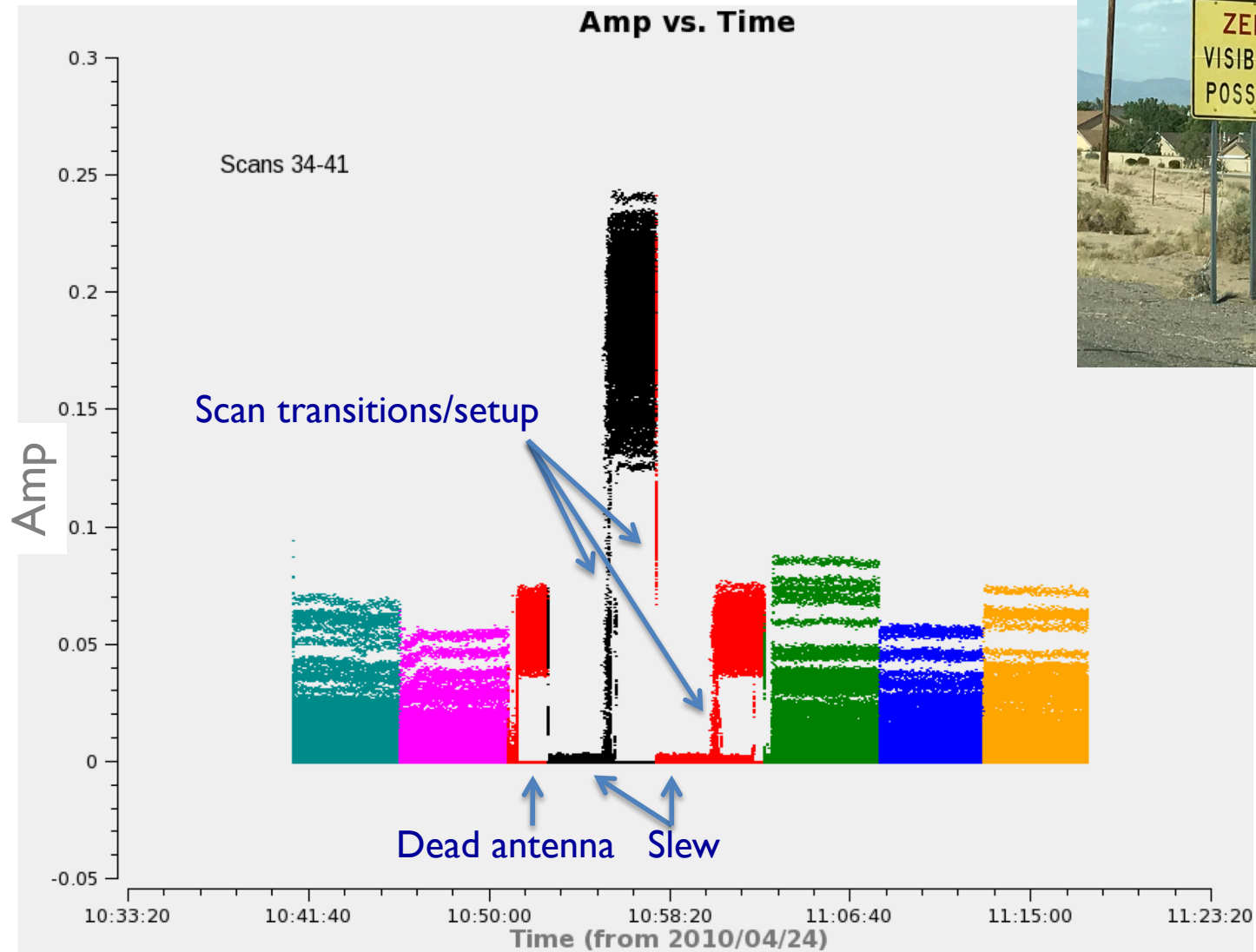
- After observation, initial data examination and editing very important
 - Will observations meet goals for calibration and science requirements?
- What to edit (much of this is now automated):
 - Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)
 - Any persistently 'dead' antennas (check operator's logs)
 - Periods of especially poor weather? (check operator's log)
 - Any antennas shadowing others? Edit such data.
 - Amplitude and phase should be continuously varying—edit outliers
 - Radio Frequency Interference (RFI)?
- Caution:
 - Be careful editing noise-dominated data.
 - Be conservative: those antennas/timeranges which are obviously bad on calibrators are probably (less obviously) bad on weak target sources—edit them
 - Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have significantly different amplitude response which may not be fatal—it may only need to be calibrated
 - Choose (phase) reference antenna wisely (ever-present, stable response)
- Increasing data volumes increasingly demand automated editing algorithms...



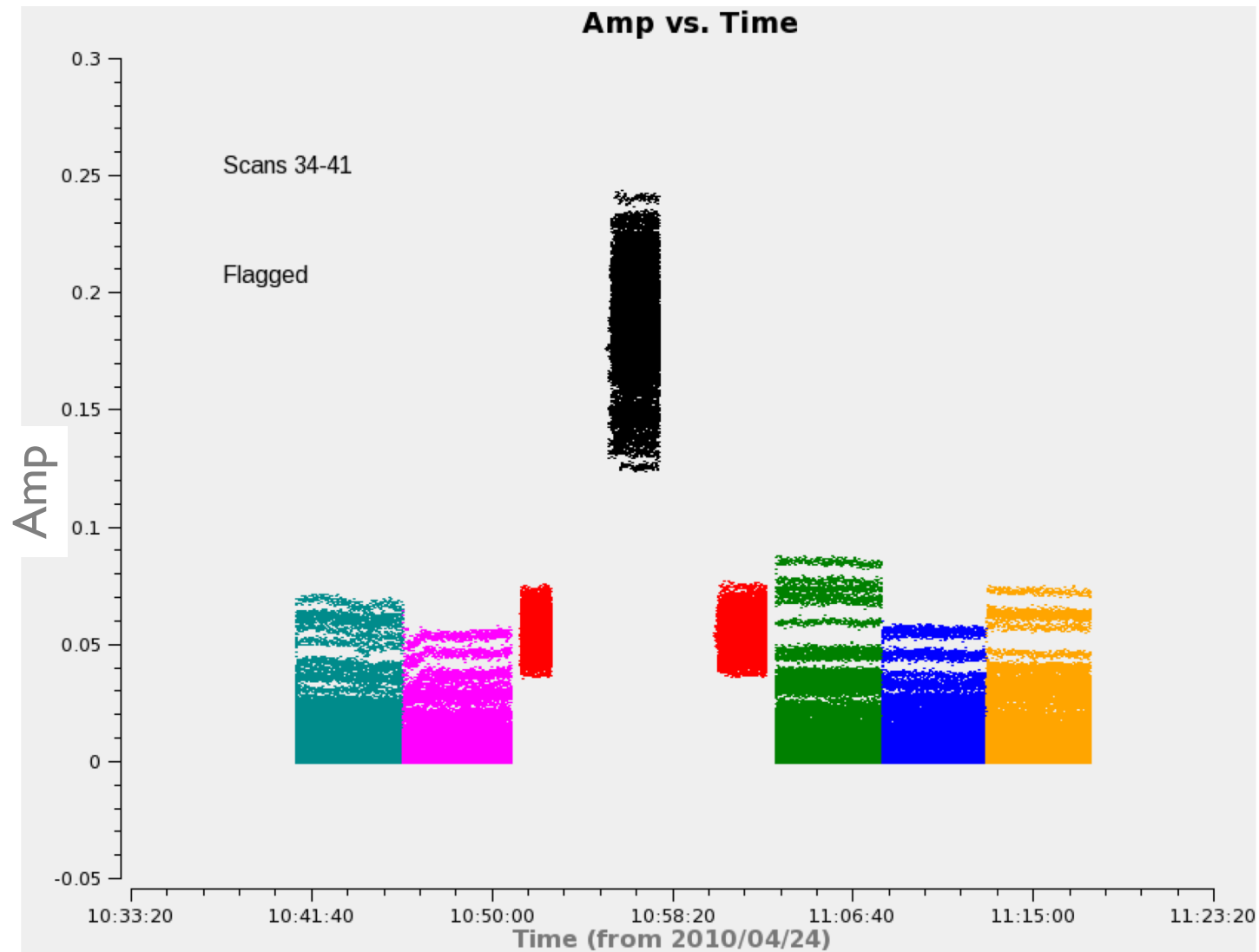
Editing Example



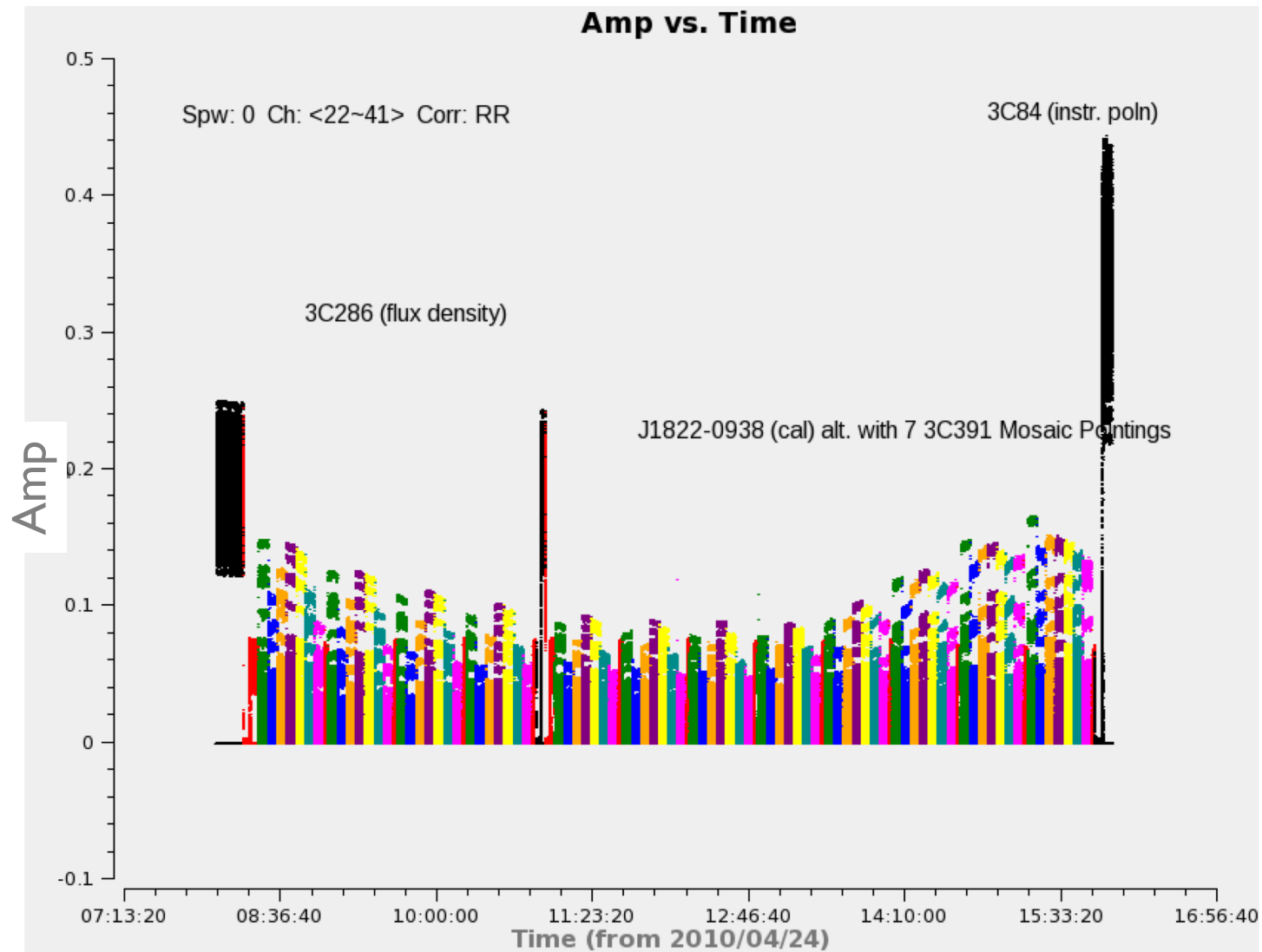
Editing Example



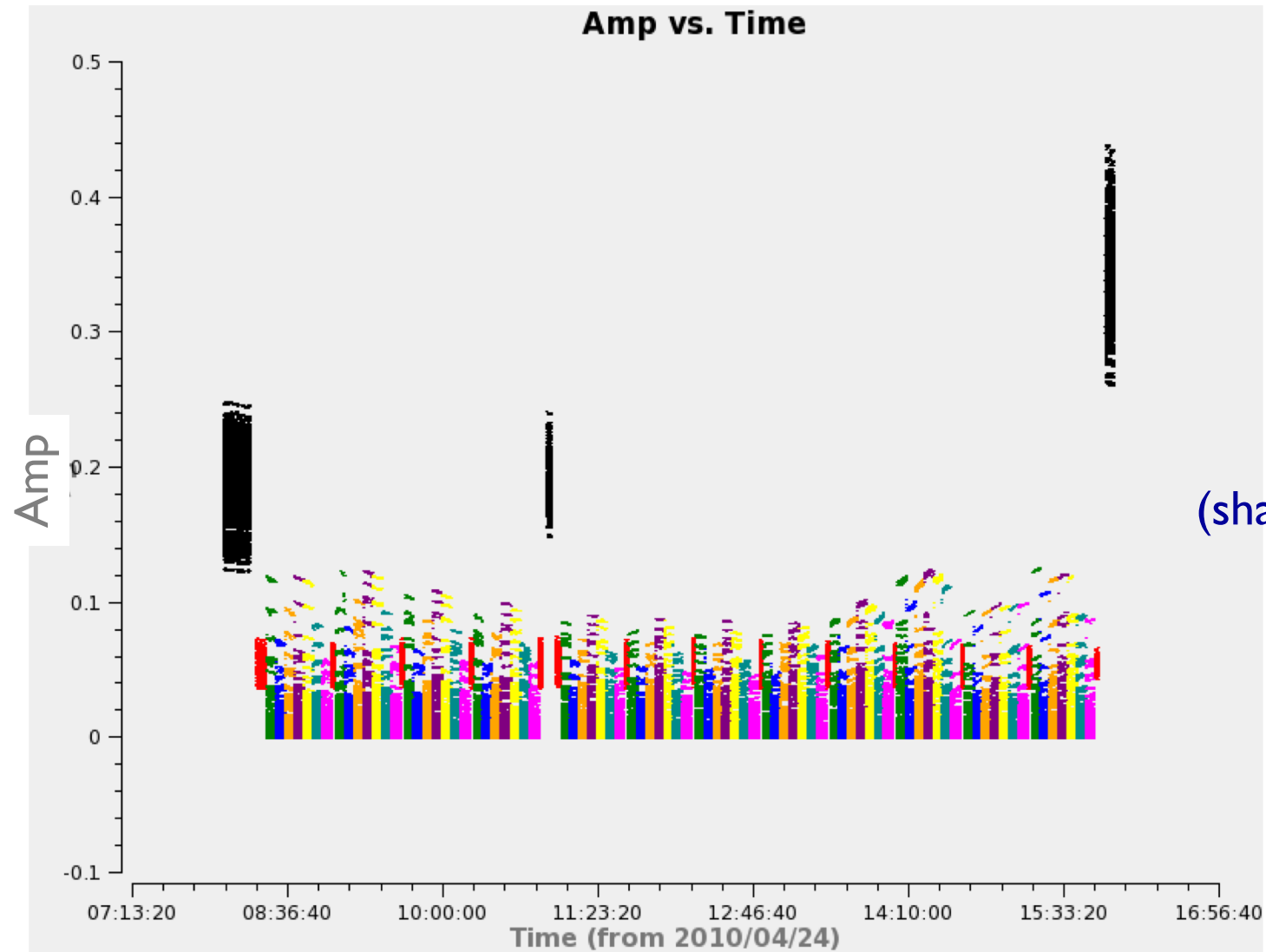
Editing Example



Editing Example (before)



Editing Example (after)



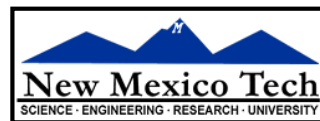
Calibration II

George Moellenbrock, NRAO



Sixteenth Synthesis Imaging Workshop

16-23 May 2018



Synopsis

- Calibration I
 - Why do we have to calibrate?
 - Review Idealistic formalism → Realistic practice
 - Fundamental Calibration Principles
 - Practical Calibration Considerations
 - Baseline-based vs. Antenna-based Calibration
 - Solving for calibration
 - An example Visibility dataset
 - Flagging
- Calibration II
 - Scalar Calibration Example
 - Generalizations & Specializations
 - Full Polarization
 - A Dictionary of Calibration Effects
 - Calibration Heuristics and ‘Bootstrapping’
 - New Calibration Challenges
 - Summary



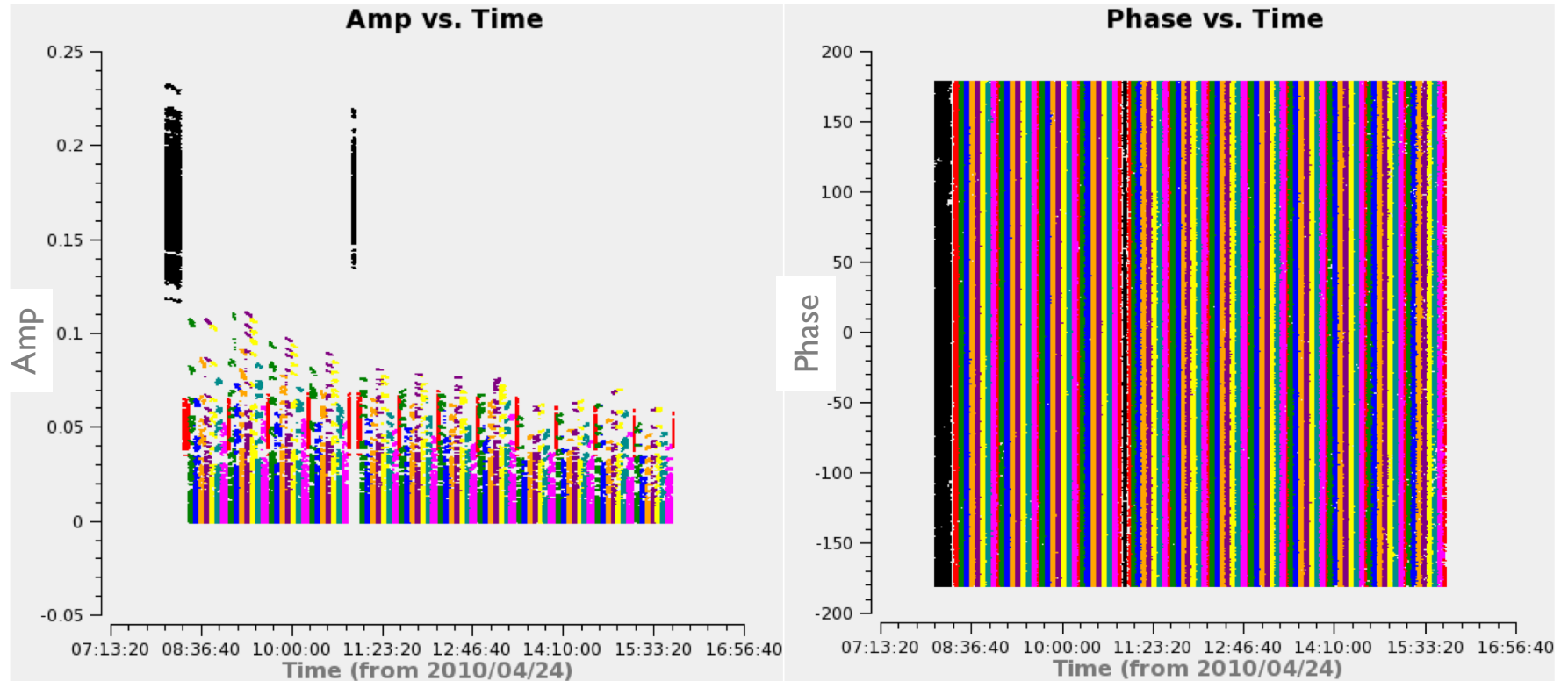
Simple Scalar Calibration Example

- Array:
 - EVLA D-configuration (Apr 2010)
- Sources:
 - Science Target: 3C391, a SNR (7 mosaic pointings)
 - Near-target calibrator: J1822-0938 (~11 deg from target; unknown flux density, assumed 1 Jy)
 - Flux Density calibrator: 3C286 (7.747 Jy, essentially unresolved)
- Signals (simplified for this example):
 - RR correlation only for this illustration (total intensity only)
 - One spectral window centered at 4600 MHz, 128 MHz bandwidth
 - 64 observed spectral channels averaged with normalized bandpass calibration applied (this illustration considers only the time-dependent 'gain' calibration)
 - (extracted from a continuum polarimetry mosaic observation)



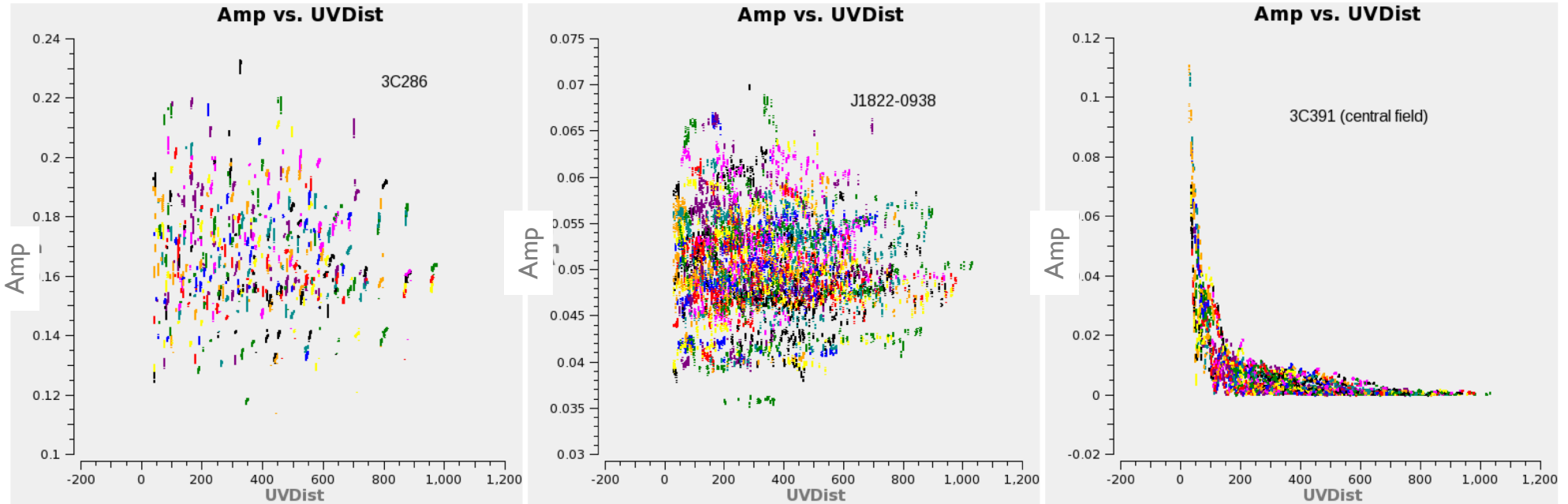
Views of the Uncalibrated Data

Field colors



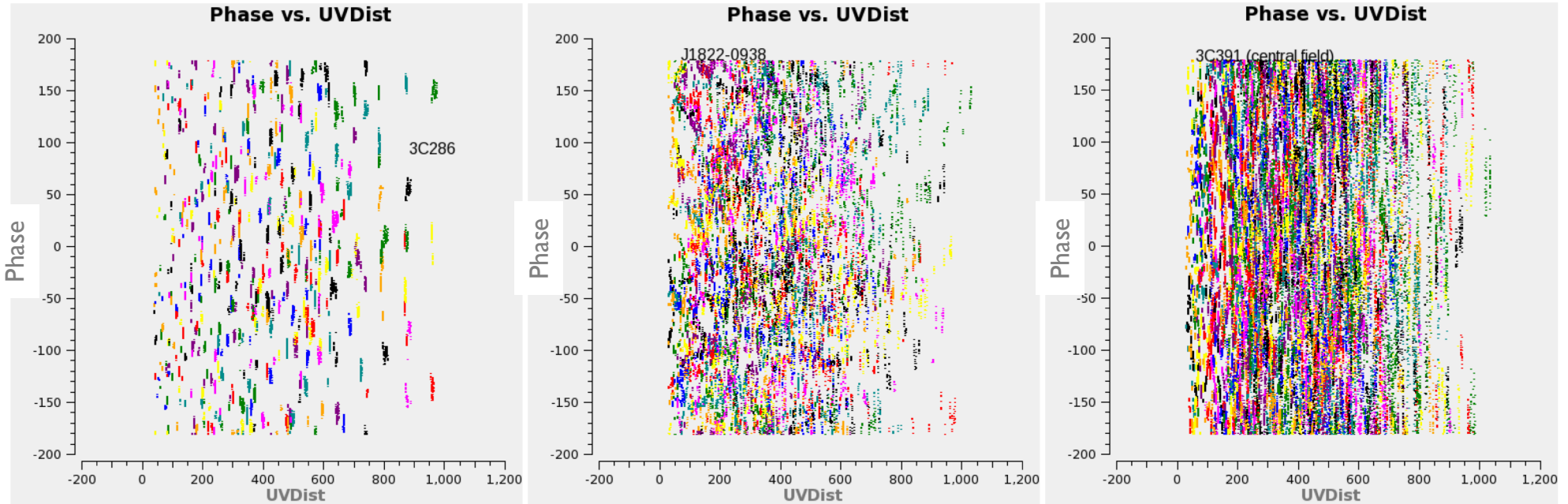
Views of the Uncalibrated Data

Baseline colors

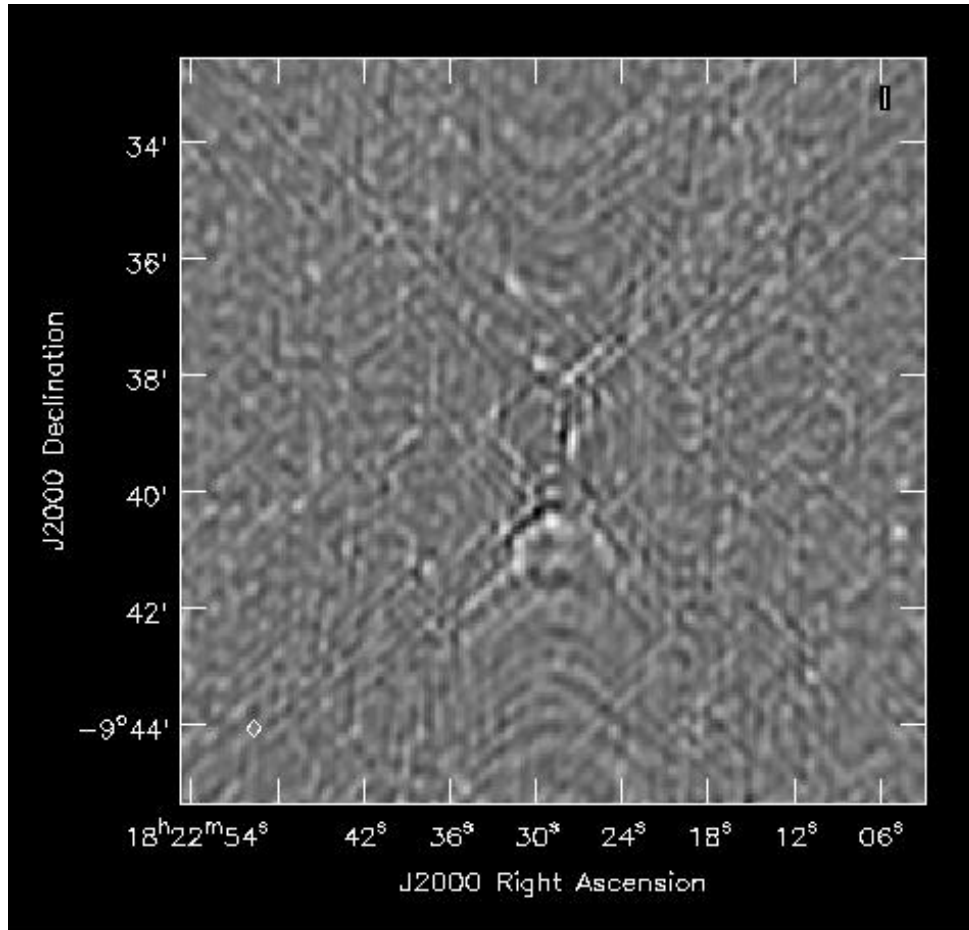


Views of the Uncalibrated Data

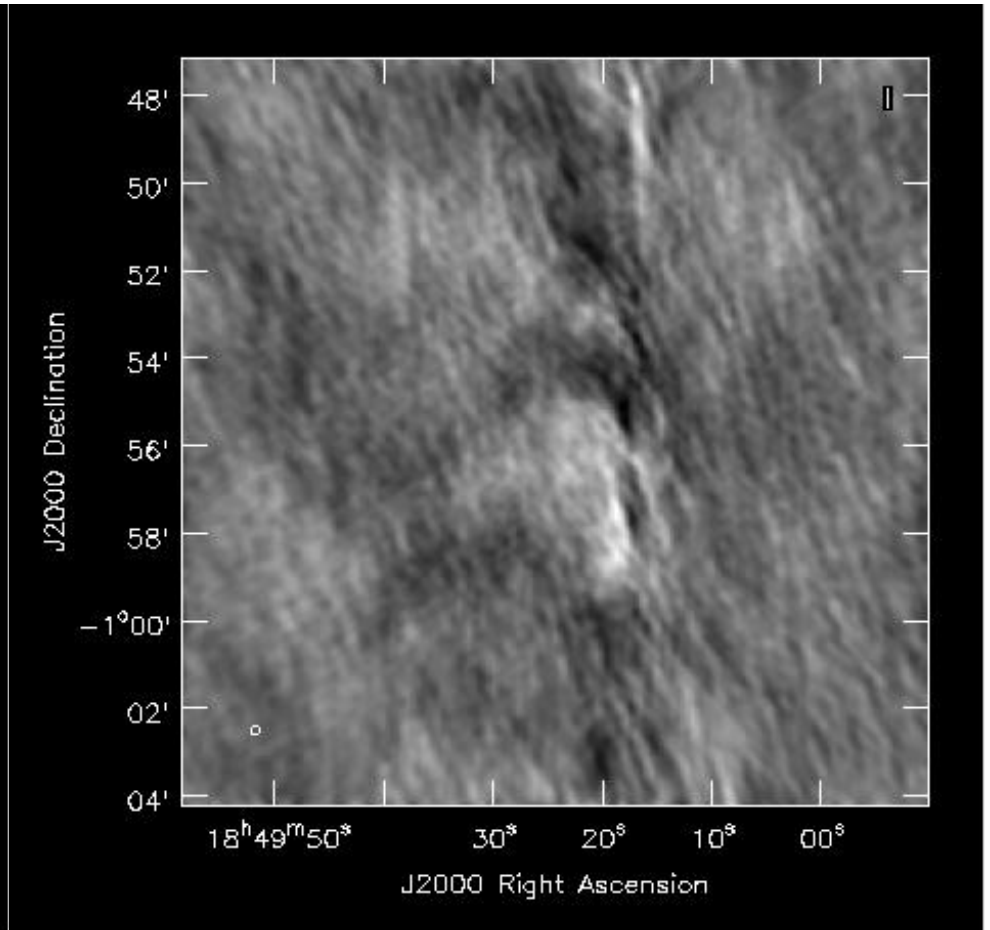
Baseline colors



Uncalibrated Images



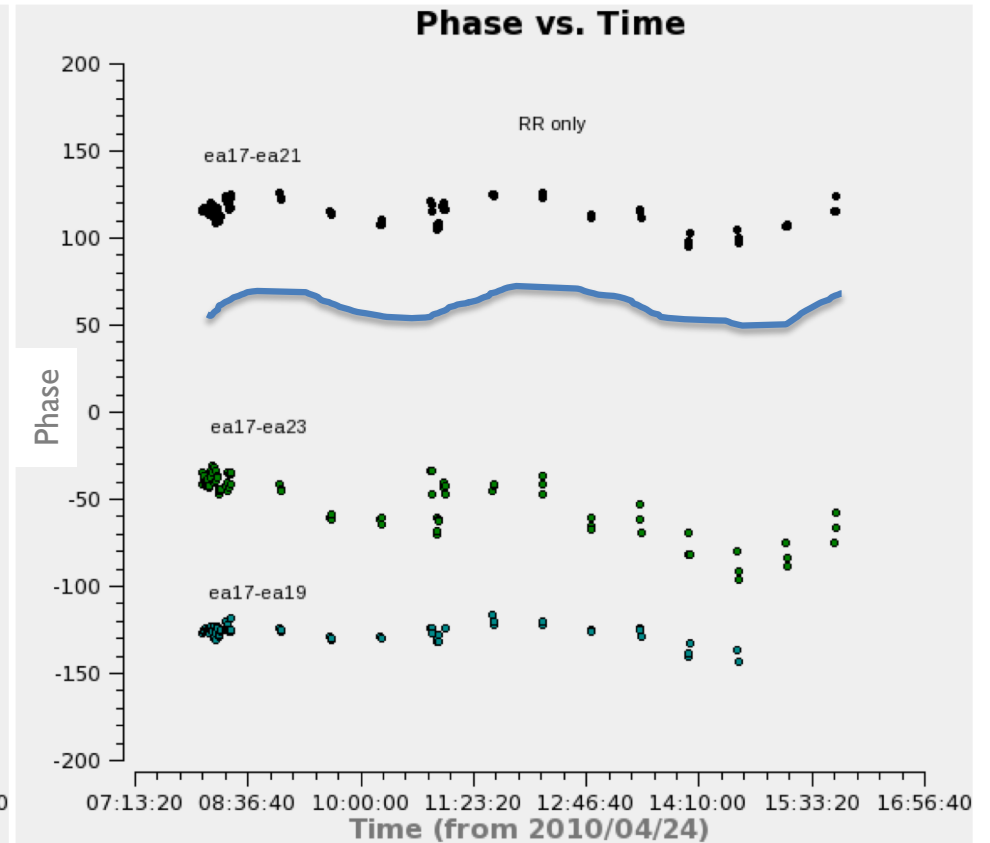
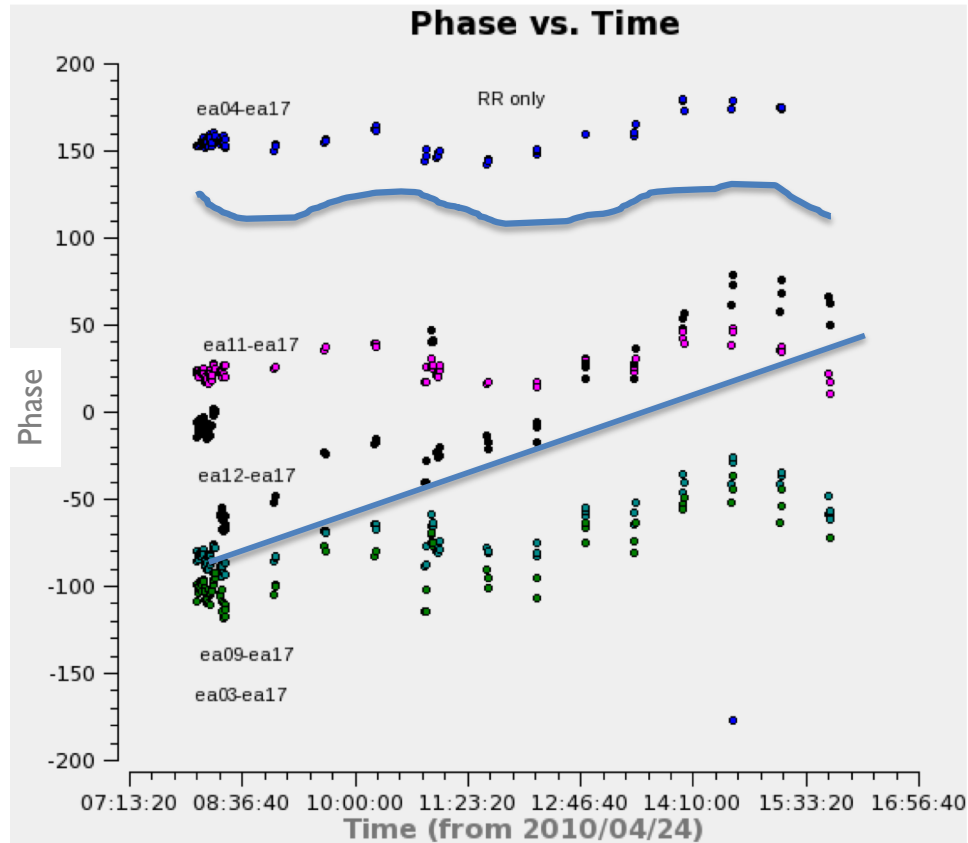
J1822-0938
(calibrator)



3C391
(science)



Rationale for Antenna-based Calibration



The Calibration Process

- Solve (LS) for antenna-based gain factors for each scan on all calibrators ($V^{mod}=S$ for f.d. calibrator; $V^{mod}=1.0$ for others) :

$$V_{ij}^{obs} = G_i G_j^* V_{ij}^{mod}$$

- Bootstrap flux density scale by enforcing gain amplitude consistency over all calibrators:

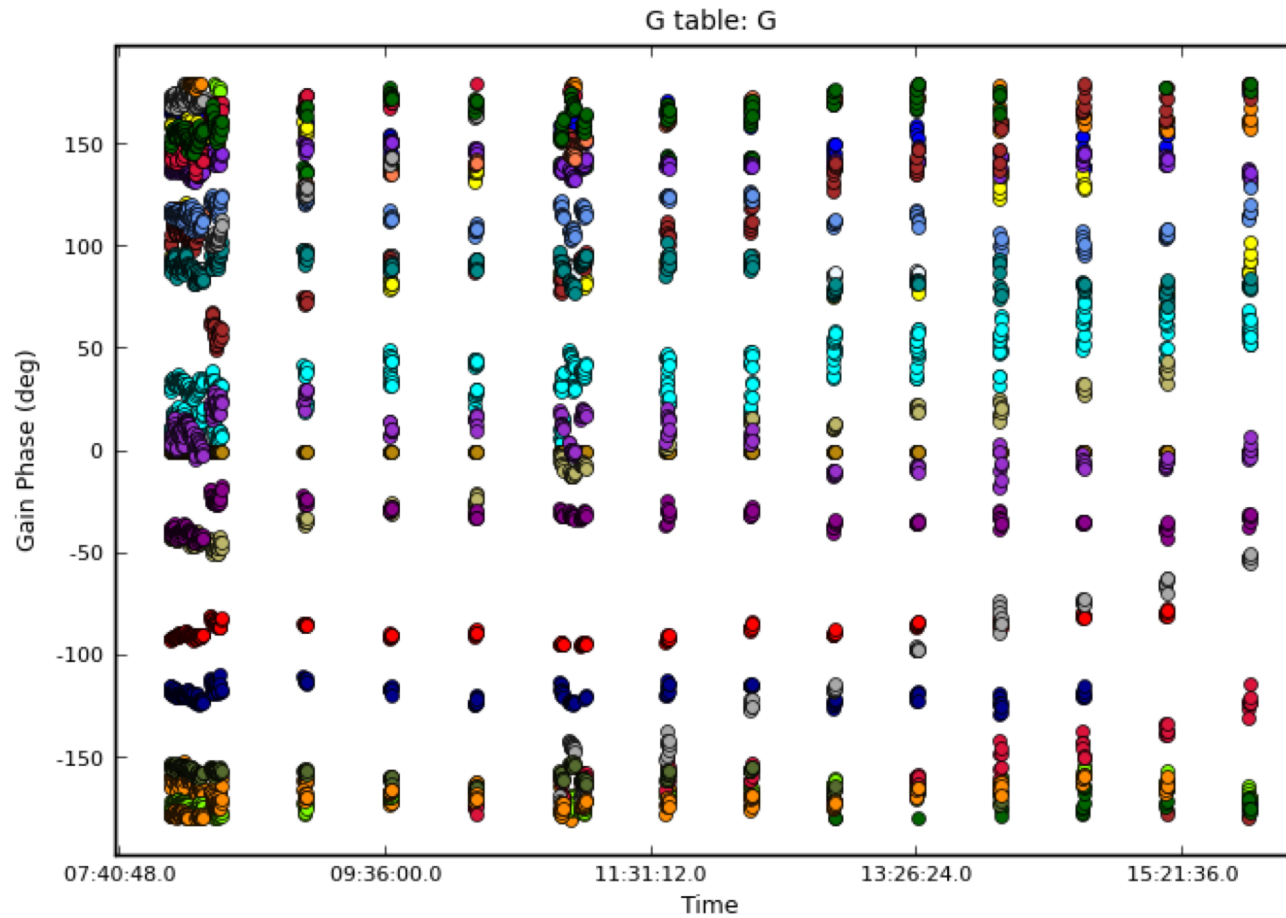
$$\left\langle \frac{|G_i|}{|G_i(fd\ cal)|} \right\rangle_{time, antennas} = 1.0$$

- Correct data (interpolate, as needed):

$$V_{ij}^{cor} = G_i^{-1} G_j^{*-1} V_{ij}^{obs}$$

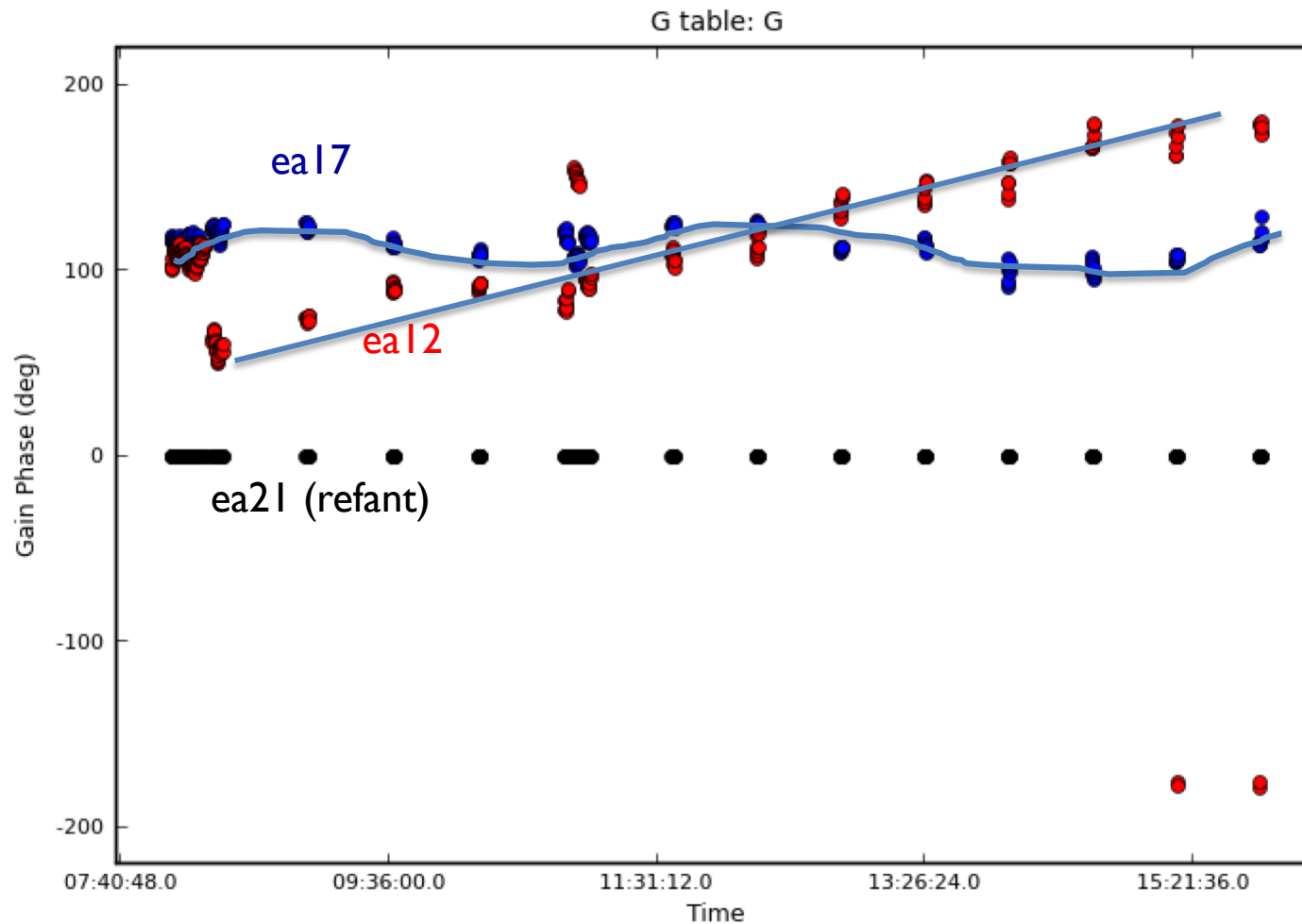


The Antenna-based Calibration Solution

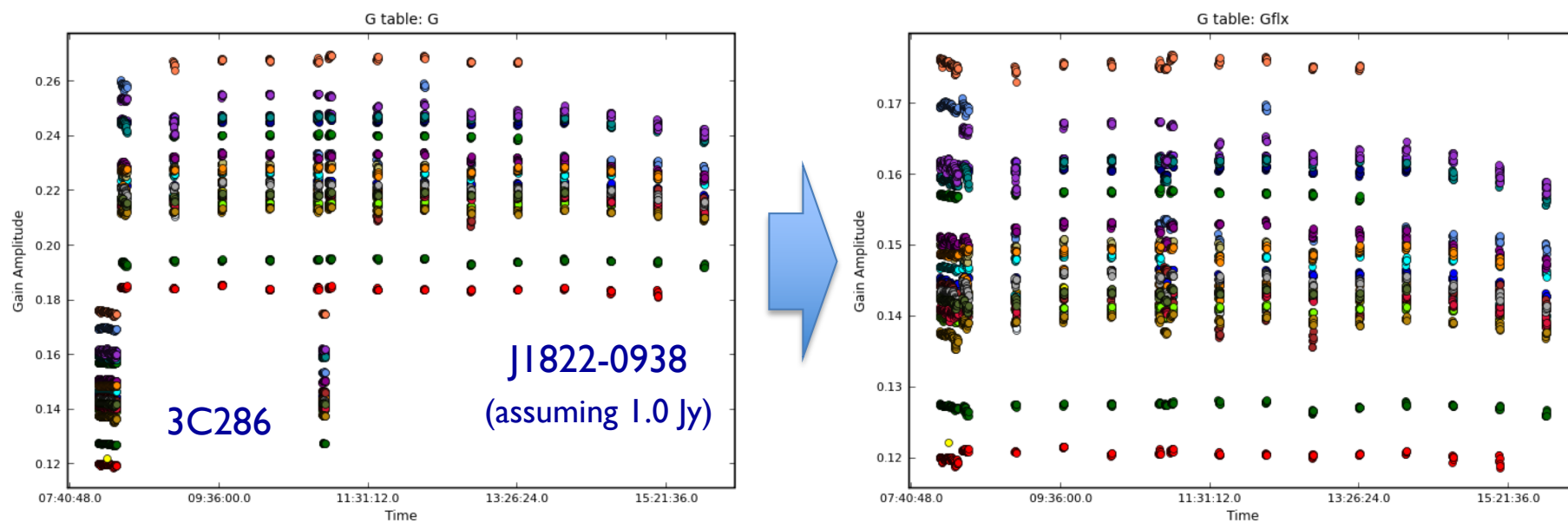


- Reference antenna: ea21 (phase = 0)

The Antenna-based Calibration Solution

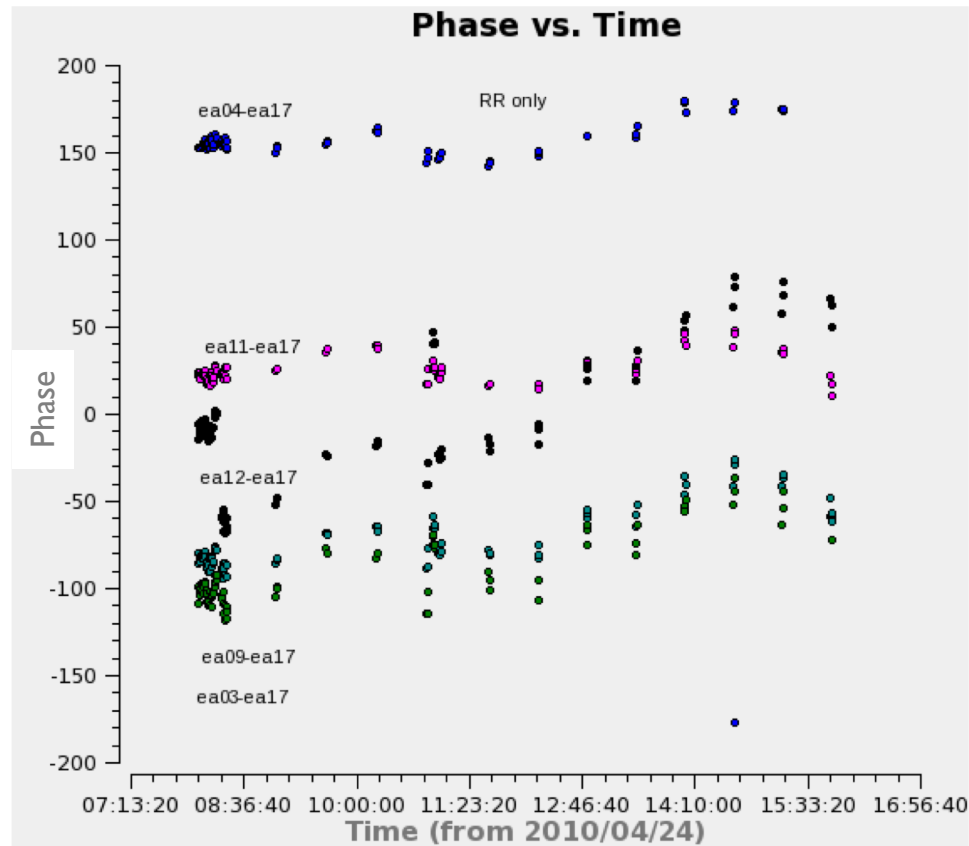


Flux Density Bootstrapping

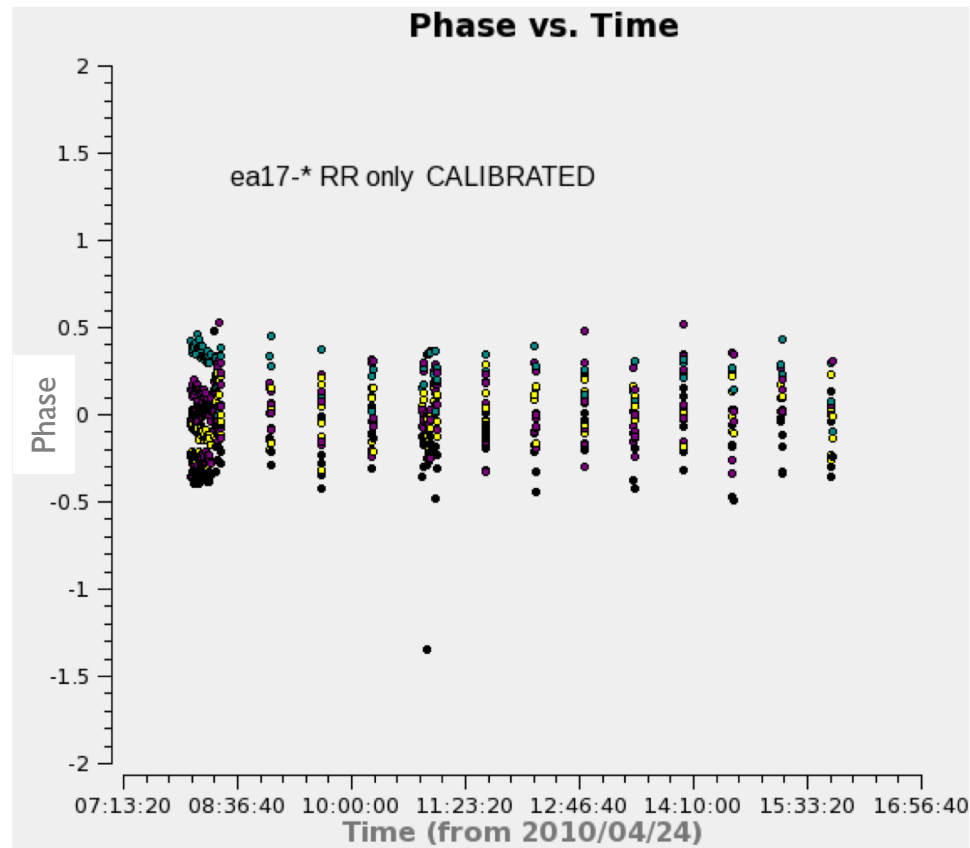


- 3C286's gains have correct scale ($\sim\sqrt{\text{Jy}^{-1}}$)
- Thus, J1822-0938 is 2.32 Jy (not 1.0 Jy, as assumed)

Effect of Antenna-based Calibration: Phase (before)

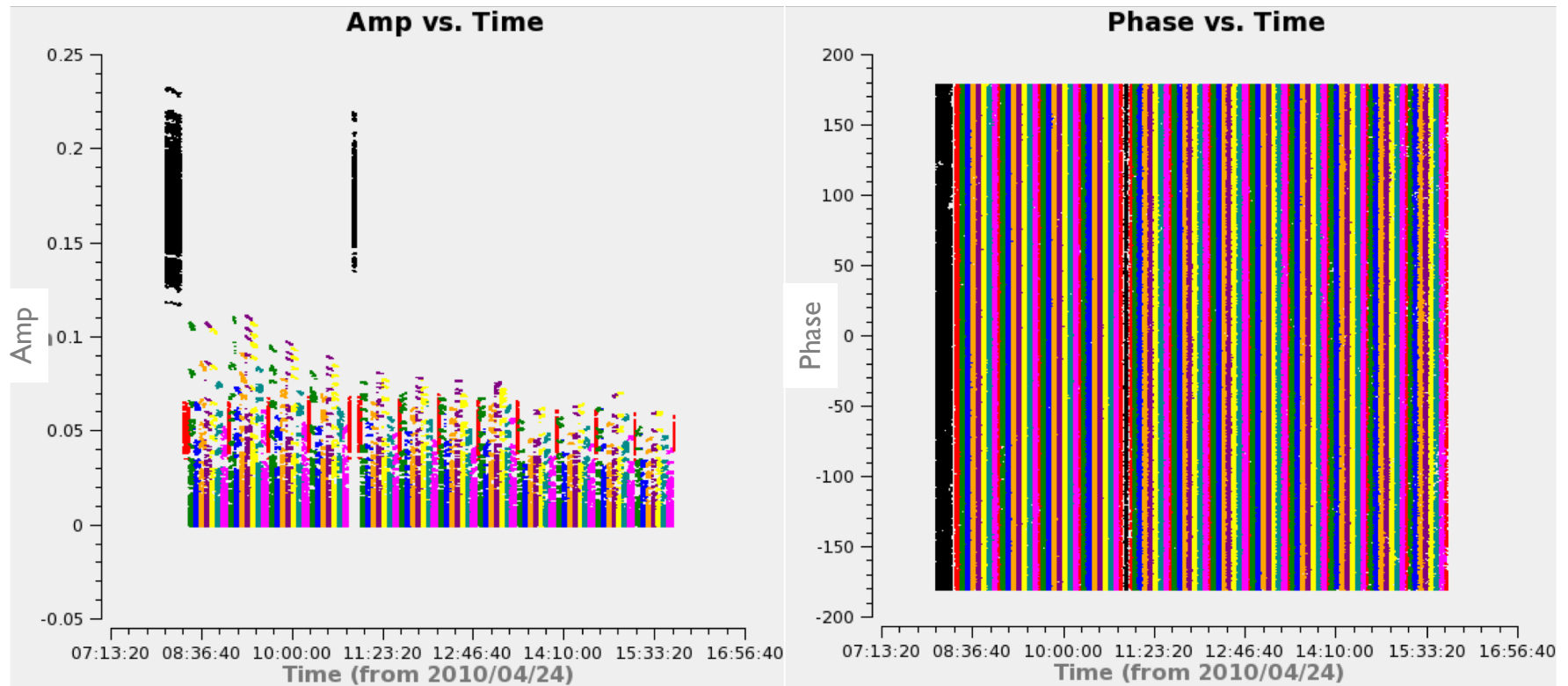


Effect of Antenna-based Calibration Phase (after)



Effect of Antenna-based Calibration

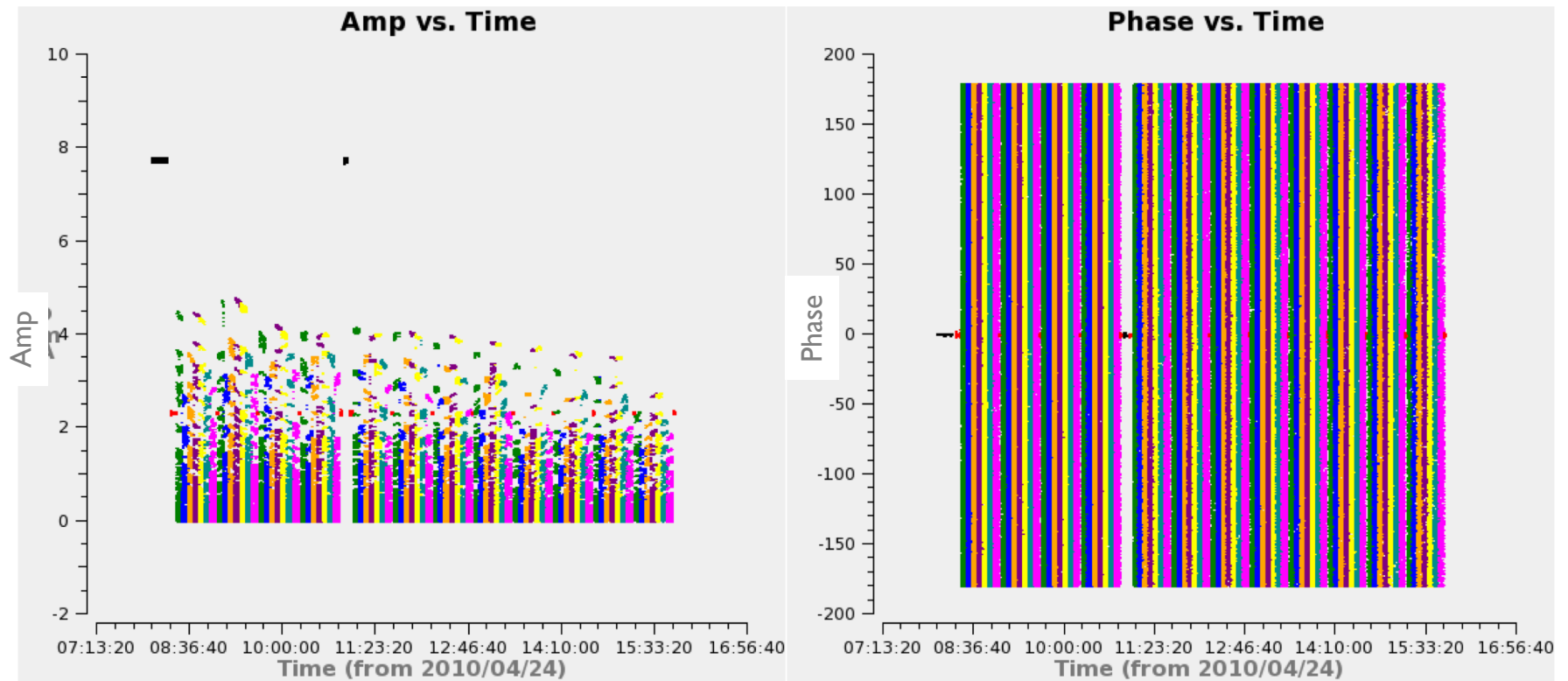
Field colors



UNCALIBRATED

Effect of Antenna-based Calibration

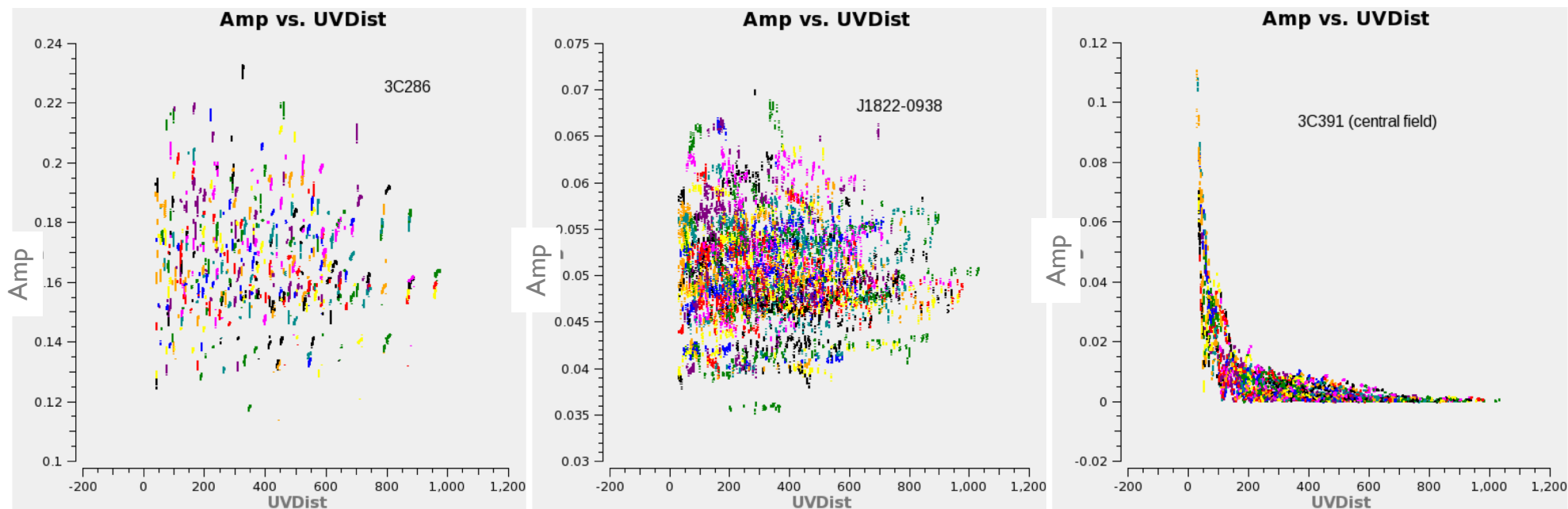
Field colors



CALIBRATED

Effect of Antenna-based Calibration

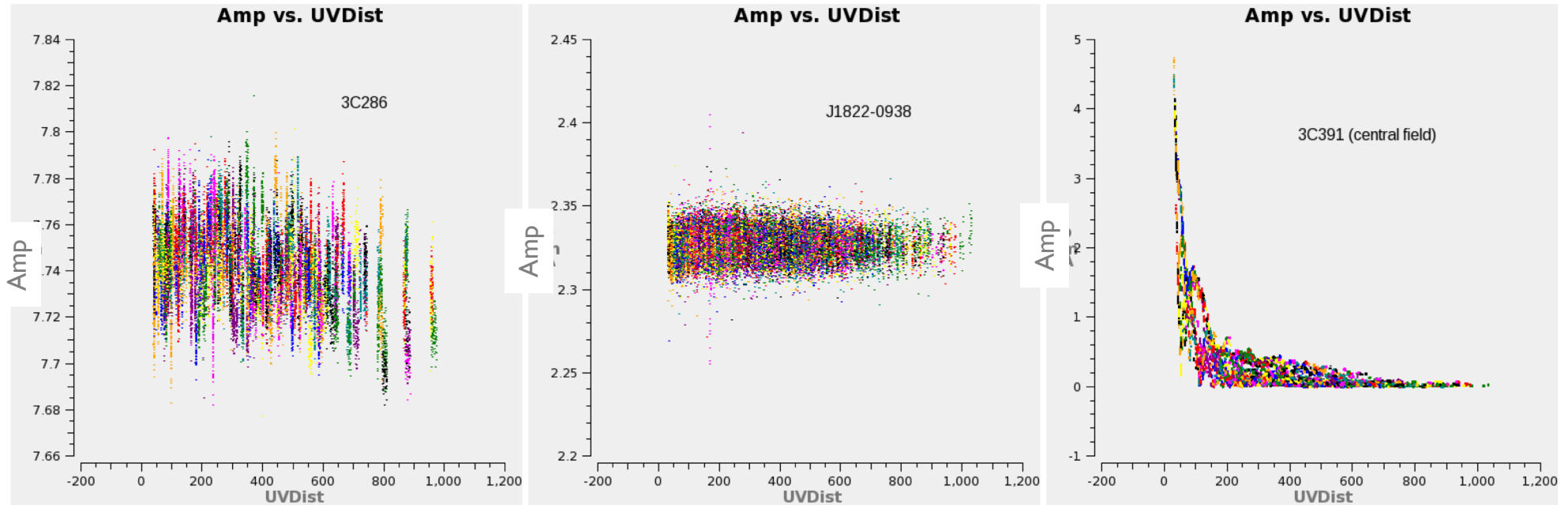
Baseline colors



UNCALIBRATED

Effect of Antenna-based Calibration

Baseline colors

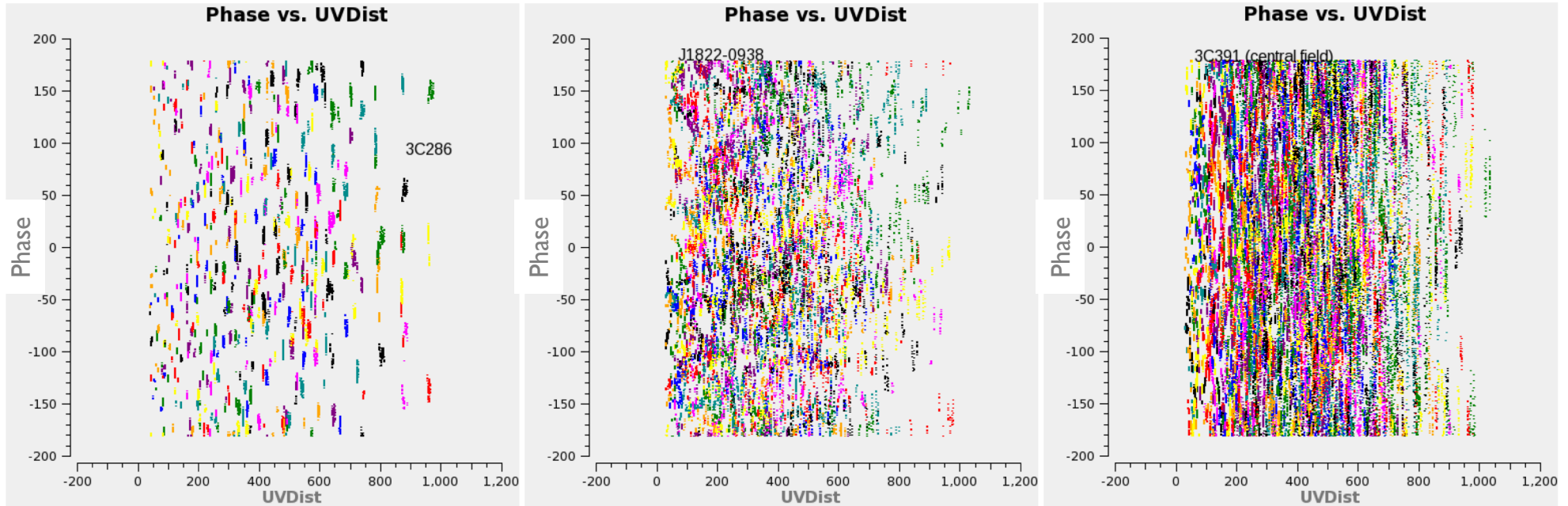


CALIBRATED



Effect of Antenna-based Calibration

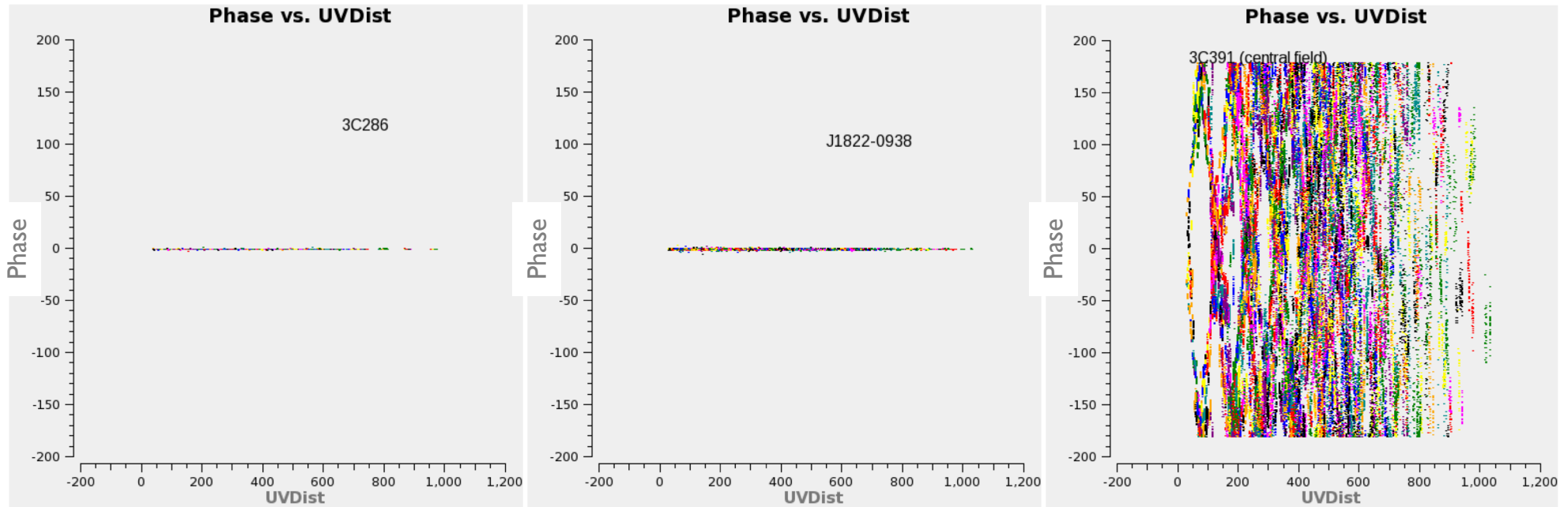
Baseline colors



UNCALIBRATED

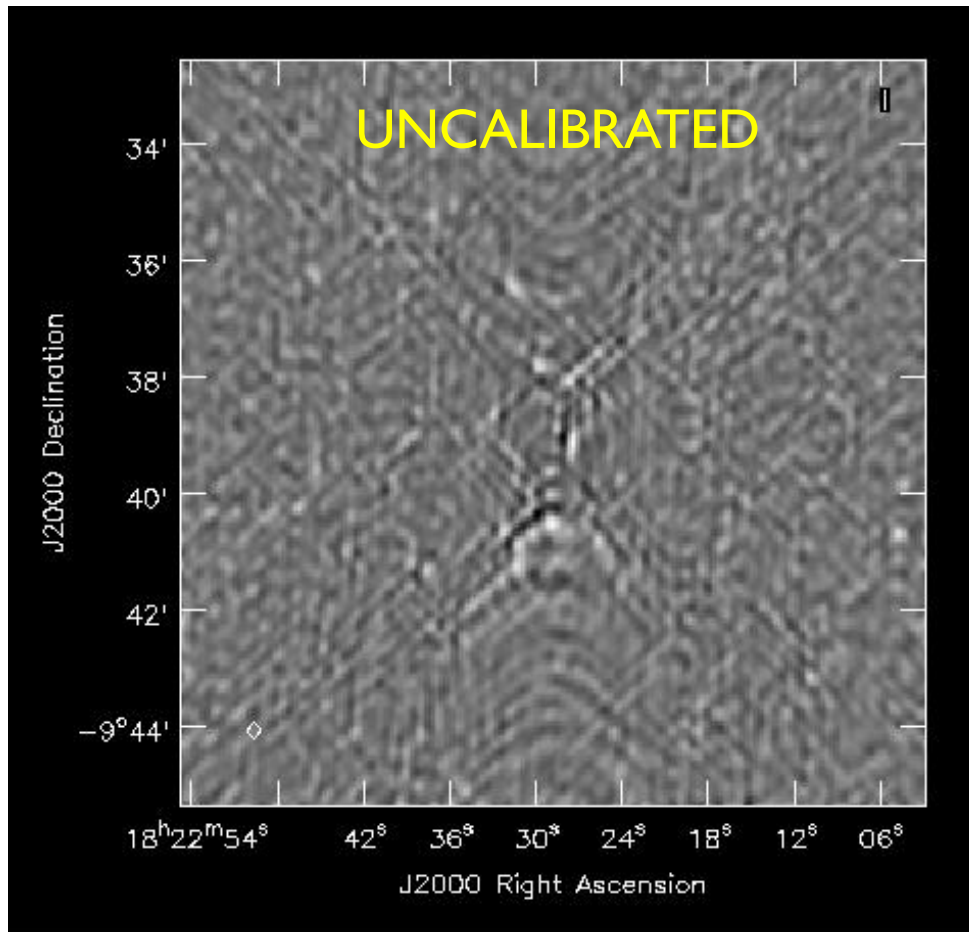
Effect of Antenna-based Calibration

Baseline colors

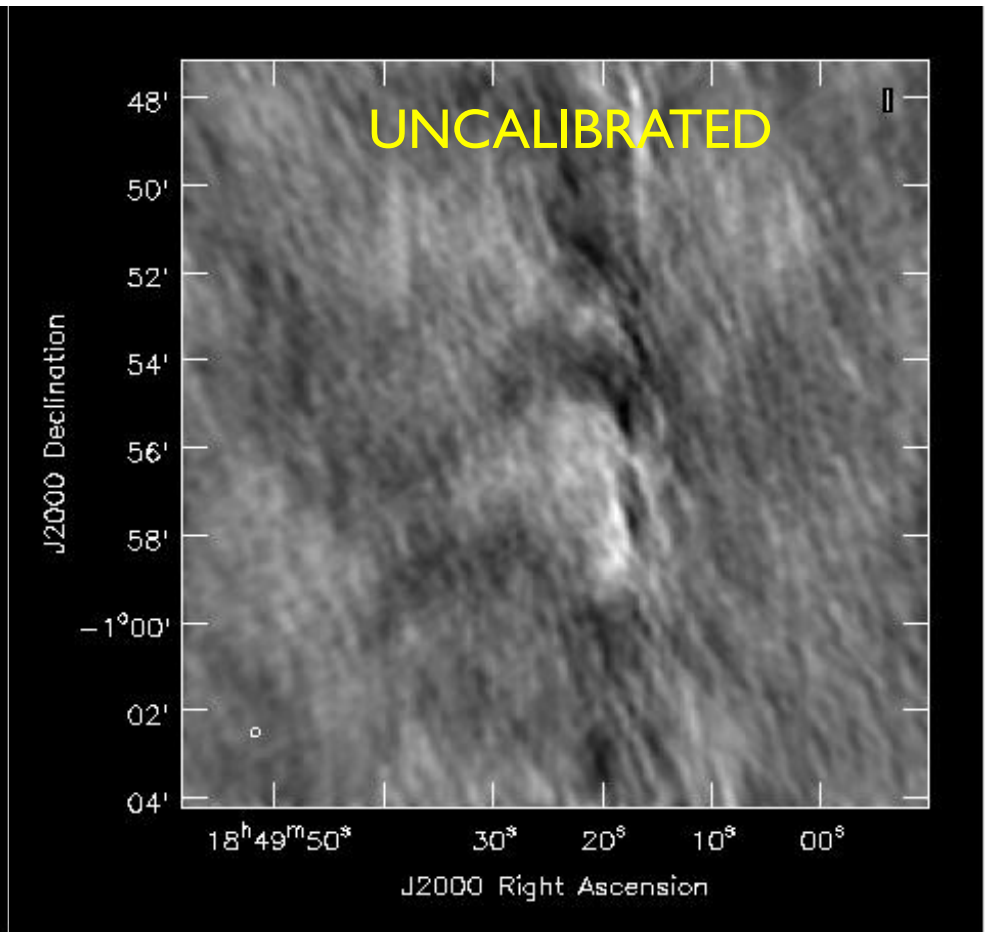


CALIBRATED

Calibration Effect on Imaging

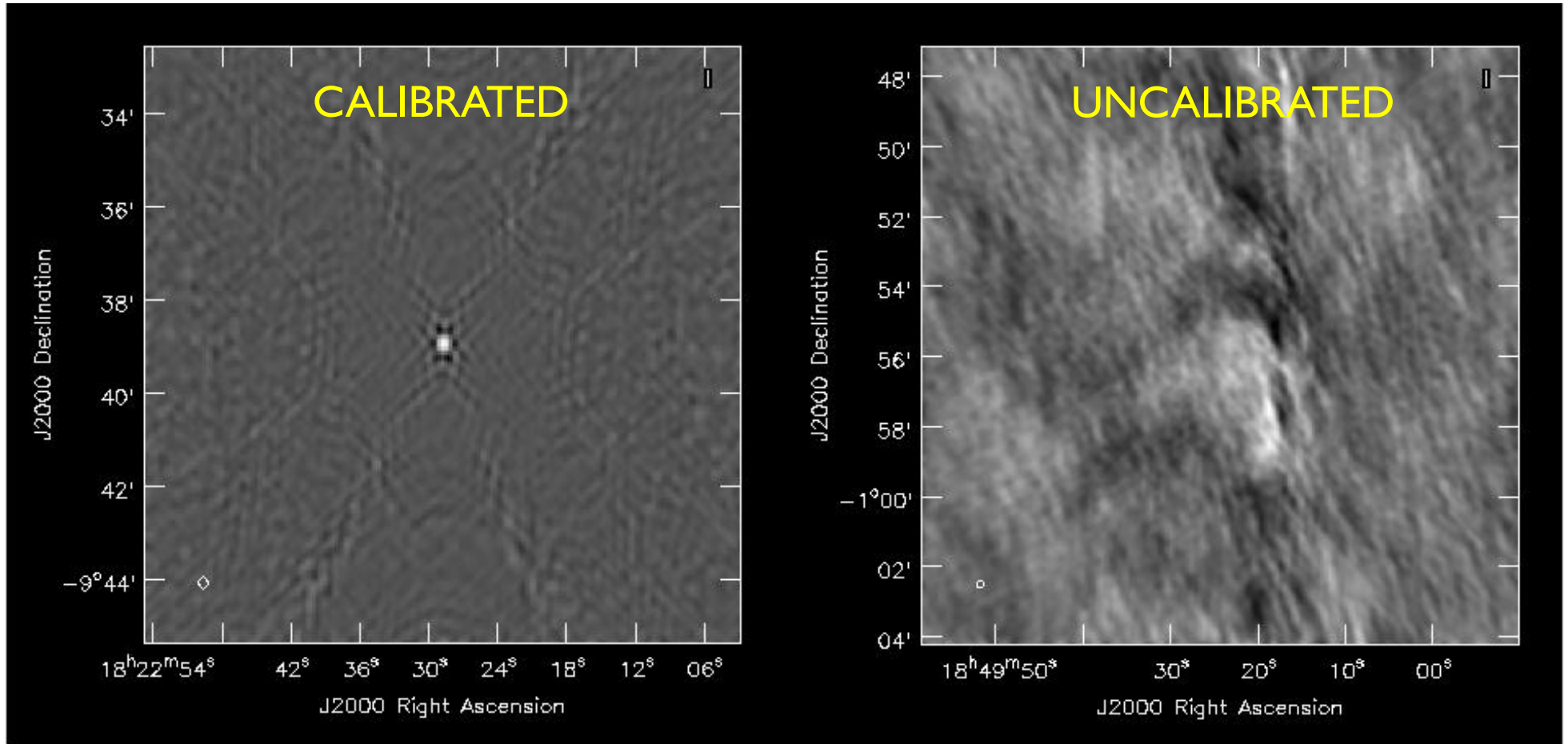


J1822-0938
(calibrator)



3C391
(science)

Calibration Effect on Imaging

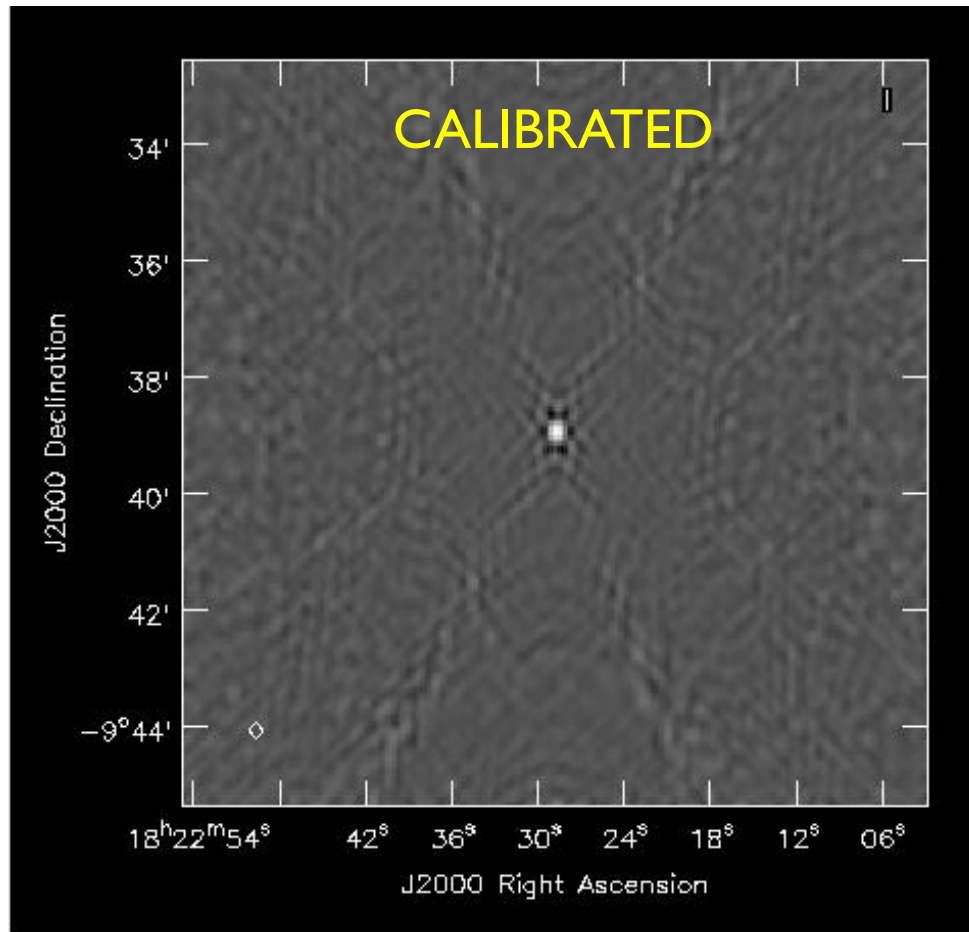


J1822-0938
(calibrator)

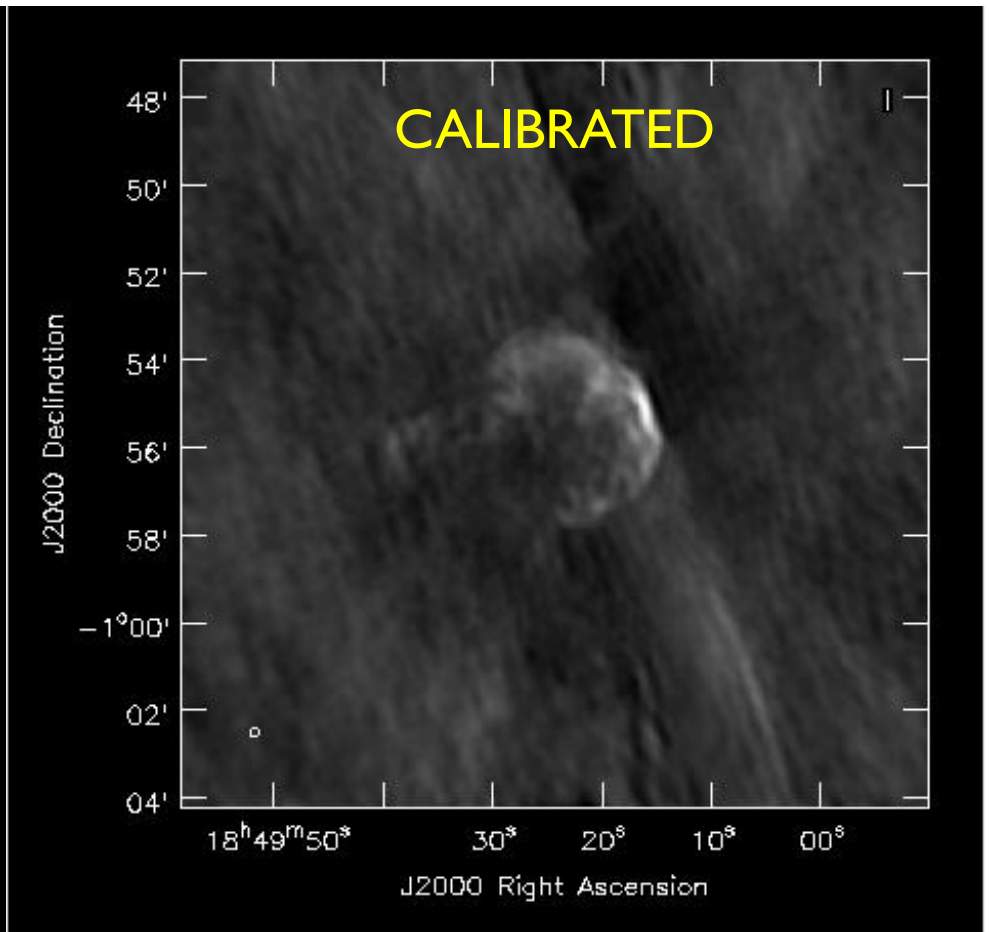
3C391
(science)



Calibration Effect on Imaging



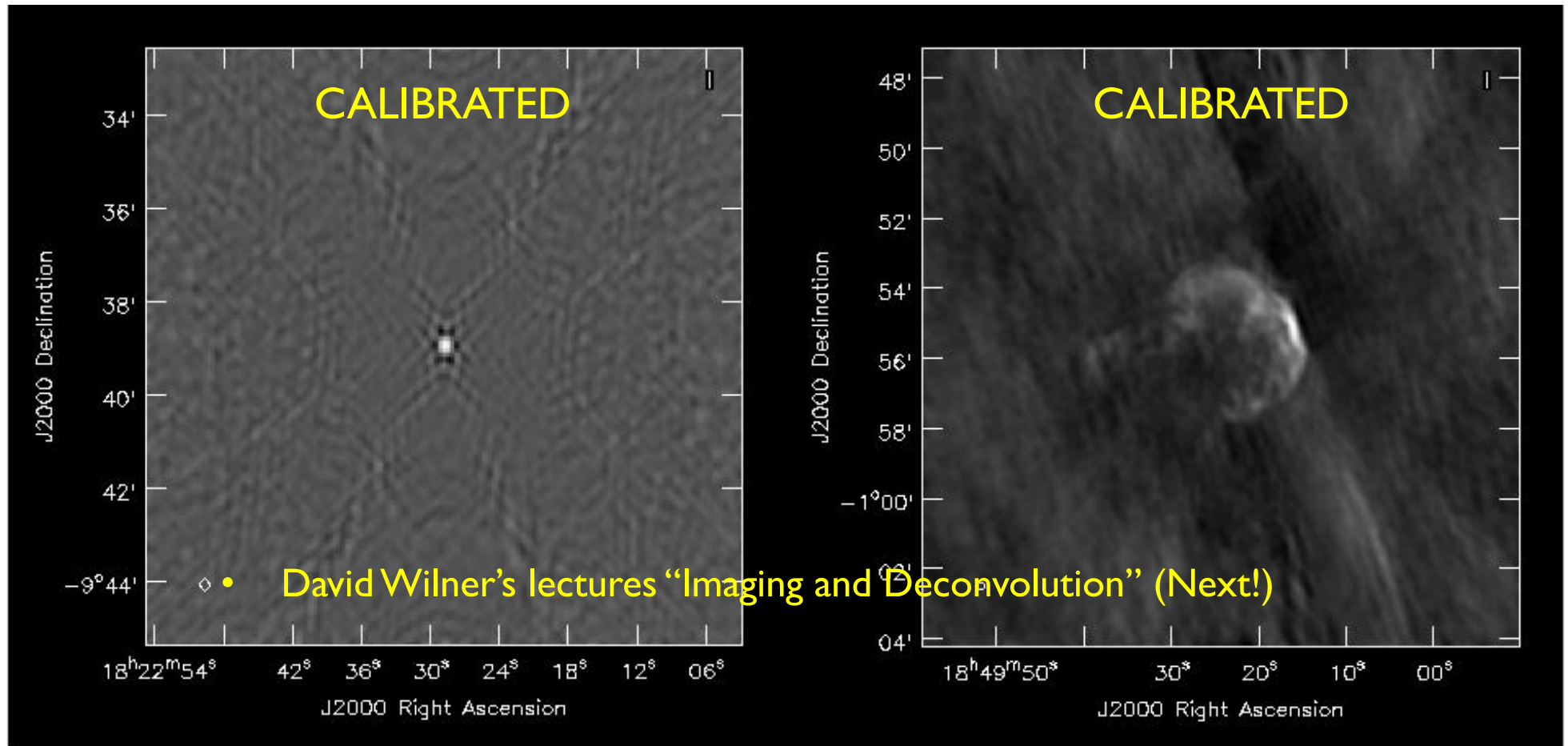
J1822-0938
(calibrator)



3C391
(science)



Calibration Effect on Imaging



J1822-0938
(calibrator)

3C391
(science)



Evaluating Calibration Performance

- Are solutions ~continuous?
 - Noise-like solutions are just that—noise (beware: calibration of pure noise generates a spurious point source)
 - Discontinuities may indicate instrumental glitches (interpolate with care)
 - Any additional editing required?
 - Provisional calibration can make bad data easier to see
 - Evidence of unsampled variation?
 - Flag uncalibrateable data
 - (Consider faster cadence next time!)
- Are calibrator data fully described by antenna-based effects?
 - Phase and amplitude *closure errors* are the baseline-based residuals
 - Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration
 - Crystal Brogan's lectures: "Advanced Calibration" (this afternoon)
- Greg Taylor's lecture: "Error Recognition" (Tuesday)



Summary of Scalar Example

- Dominant calibration effects are ***antenna-based***
 - Minimizes degrees of freedom
 - More precise
 - Preserves closure
 - Permits higher dynamic range *safely!*
- Point-like calibrators effective
- Flux density bootstrapping
- Deconvolution necessary (“Imaging”)



Generalizations and Specializations

- Full-polarization Matrix Formalism
- Calibration Effects Factorization
- Calibration Heuristics and ‘Bootstrapping’



Full-Polarization Formalism (Matrices!)

- Need dual-polarization basis (p,q) to fully sample the incoming EM wave front, where $p,q = R,L$ (circular basis) or $p,q = X,Y$ (linear basis):

$$\vec{I}_{circ} = \vec{S}_{circ} \vec{I}_{Stokes}$$

$$\begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

$$\vec{I}_{lin} = \vec{S}_{lin} \vec{I}_{Stokes}$$

$$\begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

- Stokes Parameters:

I = Total Intensity; Q,U = Linear Polarization; V = Circular Polarization

- Devices can be built to sample these circular (R,L) or linear (X,Y) basis states in the signal domain (Stokes Vector is defined in “power” domain)
- Some components of J_i involve mixing of basis states, so dual-polarization matrix description desirable or even required for proper calibration



Full-Polarization Formalism: Signal Domain

- Substitute:

$$s_i \rightarrow \vec{s}_i = \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i, \quad J_i \rightarrow \vec{J}_i = \begin{pmatrix} J^{p \rightarrow p} & J^{q \rightarrow p} \\ J^{p \rightarrow q} & J^{q \rightarrow q} \end{pmatrix}_i$$

- The *Jones matrix* thus corrupts the vector wavefront signal as follows:

$$\begin{aligned} \vec{s}'_i &= \vec{J}_i \vec{s}_i \\ \begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i &= \begin{pmatrix} J^{p \rightarrow p} & J^{q \rightarrow p} \\ J^{p \rightarrow q} & J^{q \rightarrow q} \end{pmatrix}_i \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i \\ &= \begin{pmatrix} J^{p \rightarrow p} s^p + J^{q \rightarrow p} s^q \\ J^{p \rightarrow q} s^p + J^{q \rightarrow q} s^q \end{pmatrix}_i \end{aligned}$$

Full-Polarization Formalism: Correlation - I

- Four correlations are possible from two polarizations. The *coherency matrix* represents correlation in the matrix formalism:

$$\vec{V}_{ij}^{true} = \langle \vec{s}_i \cdot \vec{s}_j^{*+} \rangle = \left\langle \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i \cdot \begin{pmatrix} s^{p*} & s^{q*} \end{pmatrix}_j \right\rangle = \begin{pmatrix} \langle s_i^p \cdot s_j^{p*} \rangle & \langle s_i^p \cdot s_j^{q*} \rangle \\ \langle s_i^q \cdot s_j^{p*} \rangle & \langle s_i^q \cdot s_j^{q*} \rangle \end{pmatrix}$$

- Observed visibilities:

$$\vec{V}_{ij}^{obs} = \langle \vec{s}'_i \cdot \vec{s}'_j^{*+} \rangle = \left\langle \left(\vec{J}_i \vec{s}_i \right) \cdot \left(\vec{J}_j^* \vec{s}_j^* \right)^+ \right\rangle = \vec{J}_i \langle \vec{s}_i \cdot \vec{s}_j^{*+} \rangle \vec{J}_j^{*+} = \vec{J}_i \vec{V}_{ij}^{true} \vec{J}_j^{*+}$$

Full-Polarization Formalism: Correlation - II

- And finally, **for fun**, expand the correlation of corrupted signals:

$$\vec{V}_{ij}^{obs} = \vec{J}_i \langle \vec{s}_i \cdot \vec{s}_j^{*+} \rangle \vec{J}_j^{*+}$$

$$= \begin{pmatrix} J_i^{p \rightarrow p} J_j^{*p \rightarrow p} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p \rightarrow p} J_j^{*q \rightarrow p} \langle s_i^p \cdot s_j^{*q} \rangle + & J_i^{p \rightarrow p} J_j^{*p \rightarrow q} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p \rightarrow p} J_j^{*q \rightarrow q} \langle s_i^p \cdot s_j^{*q} \rangle + \\ J_i^{q \rightarrow p} J_j^{*p \rightarrow p} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q \rightarrow p} J_j^{*q \rightarrow p} \langle s_i^q \cdot s_j^{*q} \rangle & J_i^{q \rightarrow p} J_j^{*p \rightarrow q} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q \rightarrow p} J_j^{*q \rightarrow q} \langle s_i^q \cdot s_j^{*q} \rangle \\ J_i^{p \rightarrow q} J_j^{*p \rightarrow p} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p \rightarrow q} J_j^{*q \rightarrow p} \langle s_i^p \cdot s_j^{*q} \rangle + & J_i^{p \rightarrow q} J_j^{*p \rightarrow q} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p \rightarrow q} J_j^{*q \rightarrow q} \langle s_i^p \cdot s_j^{*q} \rangle + \\ J_i^{q \rightarrow q} J_j^{*p \rightarrow p} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q \rightarrow q} J_j^{*q \rightarrow p} \langle s_i^q \cdot s_j^{*q} \rangle & J_i^{q \rightarrow q} J_j^{*p \rightarrow q} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q \rightarrow q} J_j^{*q \rightarrow q} \langle s_i^q \cdot s_j^{*q} \rangle \end{pmatrix}$$

- UGLY, but we rarely, if ever, need to worry about algebraic detail at this level---just let this occur “inside” the matrix formalism, and work (think) with the matrix short-hand notation
- Synthesis instrument design driven by minimizing off-diagonal terms in J_i



The Matrix Measurement Equation

- We can now write down the Measurement Equation in matrix notation:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\vec{J}_i \vec{I}_c(l, m) \vec{J}_j^{*+} \right) e^{-i2\pi(u_{ij}l + v_{ij}m)} dl dm$$

- $I_c(l, m)$ is the 2x2 matrix of Stokes parameter combinations corresponding to the coherency matrix of correlations (basis-dependent)

- Circular basis: $I_c = \begin{pmatrix} RR & RL \\ LR & LL \end{pmatrix} = \begin{pmatrix} I + V & Q + iU \\ Q - iU & I - V \end{pmatrix}$
- Linear basis: $I_c = \begin{pmatrix} XX & XY \\ YX & YY \end{pmatrix} = \begin{pmatrix} I + Q & U + iV \\ U - iV & I - Q \end{pmatrix}$

- ...and consider how the J_i are products of many effects...



A Dictionary of Calibration Components

- J_i contains many components, in principle:

- F = ionospheric effects
- T = tropospheric effects
- P = parallactic angle
- X = linear polarization position angle
- E = antenna voltage pattern, gaincurve
- D = polarization leakage
- G = electronic gain
- B = bandpass response
- K = geometry
- M, A = baseline-based corrections

$$\vec{J}_i = \vec{K}_i \vec{B}_i \vec{G}_i \vec{D}_i \vec{E}_i \vec{X}_i \vec{P}_i \vec{T}_i \vec{F}_i$$

- Order of terms ~follows signal path (right to left)
- Each term has matrix form of J_i with terms embodying its particular algebra (on- vs. off-diagonal terms, etc.)
- Direction-dependent terms must stay inside FT integral
- ‘Full’ calibration is traditionally a bootstrapping process wherein relevant terms (usually a minority of above list) are considered in decreasing order of dominance, relying on approximate separability



Ionospheric Effects, F

$$\vec{F}^{RL} = e^{i\Delta\phi} \begin{pmatrix} e^{-i\varepsilon} & 0 \\ 0 & e^{i\varepsilon} \end{pmatrix}; \quad \vec{F}^{XY} = e^{i\Delta\phi} \begin{pmatrix} \cos \varepsilon & \sin \varepsilon \\ -\sin \varepsilon & \cos \varepsilon \end{pmatrix}$$

- The ionosphere introduces a dispersive path-length offset: $\Delta\phi \propto \frac{\int n_e dl}{\nu}$
 - More important at lower frequencies (<5 GHz)
 - Varies more at solar maximum and at sunrise/sunset, when ionosphere is most active and variable
 - Direction-dependent within wide field-of-view
- The ionosphere is *birefringent*: Faraday rotation: $\varepsilon \propto \frac{\int B_{\parallel} n_e dl}{\nu^2}$
 - as high as 20 rad/m² during periods of high solar activity will rotate linear polarization position angle by $\varepsilon = 50$ degrees at 1.4 GHz
 - Varies over the array, and with time as line-of-sight magnetic field and electron density vary, violating the usual assumption of stability in position angle calibration
- Frank Schinzel’s lecture: “Polarization” (Friday)
- Tracy Clark’s lecture: “Low Frequency Interferometry” (Friday)



Tropospheric Effects, T

$$\vec{T} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- The troposphere causes polarization-independent amplitude and phase effects due to emission/opacity and refraction, respectively
 - Up to 2.3m excess path length at zenith compared to vacuum
 - Higher noise contribution, less signal transmission: Lower SNR
 - Most important at $\nu > 15$ GHz where water vapor and oxygen absorb/emit, and where path length errors are a larger fraction (or multiple!) of the wavelength
 - Zenith-angle-dependent (more troposphere path nearer horizon)
 - Clouds, weather = variability in phase and opacity; may vary across array
 - Water vapor radiometry (estimate phase from power measurements)
 - Phase transfer from low to high frequencies (delay calibration)
- ALMA!
 - Crystal Brogan’s lectures: “Advanced Calibration” (today)



Parallactic Angle, P

$$\vec{P}^{RL} = \begin{pmatrix} e^{-i\chi} & 0 \\ 0 & e^{i\chi} \end{pmatrix}; \quad \vec{P}^{XY} = \begin{pmatrix} \cos \chi & \sin \chi \\ -\sin \chi & \cos \chi \end{pmatrix}$$

- Changing orientation of sky in telescope's field of view
 - Constant for equatorial telescopes
 - Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos l \sin h(t)}{\sin l \cos \delta - \cos l \sin \delta \cos h(t)}\right)$$

l = latitude, $h(t)$ = hour angle, δ = declination

- Rotates the position angle of linearly polarized radiation
 - Analytically known, and its variation provides leverage for determining polarization-dependent effects
-
- Frank Schinzel's lecture: "Polarization" (Friday)

Linear Polarization Position Angle, χ

$$\vec{X}^{RL} = \begin{pmatrix} e^{-i\Delta\chi} & 0 \\ 0 & e^{i\Delta\chi} \end{pmatrix}; \quad \vec{X}^{XY} = \begin{pmatrix} \cos \Delta\chi & \sin \Delta\chi \\ -\sin \Delta\chi & \cos \Delta\chi \end{pmatrix}$$

- Configuration of optics and electronics (and refant) causes a net linear polarization position angle offset
- Can be treated as an offset to the parallactic angle, P
- Calibrated by registration with a strongly polarized source with known polarization position angle (e.g., flux density calibrators)
- For circular feeds, this is a phase difference between the R and L polarizations, which is frequency-dependent (a R-L phase bandpass)
- For linear feeds, this is the orientation of the dipoles (in the frame of the telescope) projected onto sky coordinates
- Frank Schinzel's lecture: "Polarization" (Friday)

Antenna Voltage Pattern, E

$$\vec{E}^{pq} = \begin{pmatrix} E^p(l, m) & 0 \\ 0 & E^q(l, m) \end{pmatrix}$$

- Antennas of all designs have direction-dependent gain within field-of-view
 - Important when region of interest on sky comparable to or larger than λ/D radians
 - Important at lower frequencies where radio source surface density is greater and wide-field imaging techniques required
 - Beam squint: E^R and E^L offset, yielding spurious Stokes V polarization
 - Sky rotates within field-of-view for alt-az antennas, so off-axis sources move through the pattern
 - Direction dependence of polarization leakage (D) may be included in E (off-diagonal terms then non-zero)
- Shape and efficiency of the voltage pattern may change with zenith angle: ‘gain curve’
 - Brian Mason’s lecture: “Mosaicking” (Friday)
 - Urvashi Rao Venkata’s lecture: “Widefield Imaging” (Monday)



Polarization Leakage, D

$$\vec{D} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

- Antenna & polarizer are not ideal, so orthogonal polarizations not perfectly isolated
 - Well-designed feeds have $d \sim$ a few percent or less
 - A geometric property of the optics design, so frequency-dependent
 - For R,L systems, total-intensity imaging affected as $\sim dQ, dU$, so only important at high dynamic range (Q,U,d each \sim few %, typically)
 - For R,L systems, linear polarization imaging affected as $\sim dI$, so almost always important
 - For small arrays (no *differential* parallactic angle coverage), only relative D solution is possible from standard linearized solution, so parallel-hands cannot be corrected absolutely (closure errors)
- Best calibrator: Strong, point-like, observed over large range of parallactic angle (to separate source polarization from D)
- Frank Schinzel's lecture: "Polarization" (Friday)

“Electronic” Gain, G

$$\vec{G}^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

- Catch-all for most amplitude and phase effects introduced by antenna electronics and other generic effects
 - Most commonly treated calibration component
 - Dominates other effects for most standard observations
 - Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
 - Includes any internal system monitoring, like EVLA switched power calibration
 - Often also includes tropospheric and (on-axis) ionospheric effects which are typically difficult to separate uniquely from the electronic response
 - Excludes frequency dependent effects (see B)
- Best calibrator: strong, point-like, near science target; observed often enough to track expected variations
 - Also observe a flux density standard



Bandpass Response, B

$$\vec{B}^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

- G-like component describing frequency-dependence of antenna electronics, etc.
 - Filters used to select frequency passband not square
 - Optical and electronic reflections introduce ripples across band
 - Often assumed time-independent, but not necessarily so
 - Typically (but not necessarily) normalized
 - ALMA Tsys is a “bandpass” (freq-dependent calibration to K)
- Best calibrator: strong, point-like; observed long enough to get sufficient *per-channel* SNR, and often enough to track variations
- Ylva Pihlstrom’s lecture: “Spectral Line Data Analysis” (Friday)

Geometry, K

$$\vec{K}^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

- Must get geometry right for Synthesis Fourier Transform relation to work in real time
 - Antenna positions (geodesy)
 - Source directions (time-dependent in topocenter!) (astrometry)
 - Clocks
 - Electronic path-lengths introduce delays (polarization, spw differences)
 - Longer baselines generally have larger relative geometry errors, especially if clocks are independent (VLBI)
 - Importance scales with frequency *and* bandwidth
- K is a clock- & geometry-parameterized version of G
 - All-sky observations used to isolate geometry parameters

– Adam Deller’s lecture: “Very Long Baseline Interferometry” (Friday)



Non-closing Effects: M, A

- Baseline-based errors which do not factor into antenna-based components
 - Digital correlators designed to limit such effects to well-understood and **uniform** (not dependent on baseline) scaling laws (absorbed in *f.d.* calibration)
 - Simple noise (additive)
 - Averaging in time and frequency over variation in antenna-based effects and visibilities (practical instruments are finite!)
 - Instrumental polarization effects in parallel hands (not properly factored)
 - Correlated “noise” (e.g., RFI)
- Difficult to distinguish from source structure (visibility) effects
 - Geodesy and astrometry observers consider determination of radio source structure—a baseline-based effect—as a required *calibration* if antenna positions are to be determined accurately
- Separate factors for each element of the coherency matrix; M multiplies, A adds

Solving the Measurement Equation

- Formally, solving for any antenna-based visibility calibration component is always the same general non-linear fitting problem:

$$\vec{V}_{ij}^{\text{corrected}\cdot\text{obs}} = \vec{J}_i \vec{V}_{ij}^{\text{corrupted}\cdot\text{mod}} \vec{J}_j^{*+}$$

- Generalization of scalar non-linear LS approach
 - Observed and Model visibilities are corrected/corrupted by available prior calibration solutions/information downstream and upstream of the solved-for component, respectively
 - Resulting solution used as prior in subsequent solves, as necessary
 - Each solution is relative to priors and assumed source model
 - Iterate sequences, as needed → generalized self-calibration
- Viability and accuracy of the overall calibration depends on isolation of different effects using *proper calibration observations*, and *appropriate solving strategies (heuristics)*



Measurement Equation Heuristics

- When considering which effects are relevant to a particular observation, and how to sequence calibration determination, it is convenient to express the Measurement Equation in a “Heuristic Operator” notation:

$$V^{obs} = M B G D E X P T F V^{true} + A$$

- Rigorous notation, antenna-basedness, etc., suppressed
- **Usually, only a subset of terms are considered, though highest-dynamic range observations may require more**
- An expression of a “Calibration Model”
 - Order is important (handled in software)
 - Solve for terms in decreasing order of dominance, iterate to isolate
 - NB: Non-trivial direction-dependent solutions involve convolutional treatment of the visibilities, and is coupled to the imaging and deconvolution process---see advanced imaging lectures...)



Decoupling Calibration Effects

- All calibration terms are a function of prior information!
- Multiplicative gain (G) term will soak up many different effects; known priors should be compensated for *explicitly*, especially when direction-dependent differences (e.g., between calibrator and target) will limit the accuracy of calibration transfer:
 - Zenith angle-dependent atmospheric opacity, phase (T,F)
 - Zenith angle-dependent gain curve (E)
 - Antenna position errors (K)
- Early calibration solves (e.g., G) are always subject to more subtle, uncorrected effects
 - Instrumental polarization (D), which introduces gain calibration errors and causes apparent closure errors in *parallel-hand* correlations
 - When possible, iterate and alternate solves to decouple effects...



Calibration Heuristics – Spectral Line

Total Intensity Spectral Line (K=antenna positions, B=bandpass, G=gain, E=gaincurve, T=opacity):

$$V^{obs} = K \textcolor{red}{B} \textcolor{red}{G} E T V^{true}$$

1. Preliminary Gain solve on B-calibrator:

$$(K' V^{obs}) = \underline{\textcolor{red}{G}}_B (E T V^{mod})$$

2. Bandpass Solve (using G_B) on B-calibrator (then discard G_B):

$$(K' V^{obs}) = \underline{\textcolor{red}{B}} (G_B E T V^{mod})$$

3. Time-dependent Gain solve (using inverse of B) on all calibrators:

$$(B' K' V^{obs}) = \underline{\textcolor{red}{G}} (E T V^{mod})$$

4. Flux Density scaling:

$$G \rightarrow \textcolor{red}{G}_f \quad (\text{enforce gain consistency})$$

5. Correct with inverted (primes) solutions:

$$V^{cor} = T' E' G_f' B' K' V^{obs}$$

6. Image!



Calibration Heuristics – Polarimetry

Polarimetry (B=bandpass, G=gain, D=instr. poln, X=pos. ang., P=parallactic ang.):

$$V^{obs} = B G D X P V^{true}$$

1. Preliminary Gain solve on B-calibrator:

$$V^{obs} = \underline{G}_B V^{mod}$$

2. Bandpass (B) Solve (using G_B) on B-calibrator (then discard G_B):

$$V^{obs} = \underline{B} (G_B V^{mod})$$

3. Gain (G) solve (using parallactic angle P , inverse of B) on calibrators:

$$(B' V^{obs}) = \underline{G} (P V^{mod})$$

4. Instrumental Polarization (D) solve (using P , inverse of G, B) on instrumental polarization calibrator:

$$(G' B' V^{obs}) = \underline{D} (P V^{mod})$$



Calibration Heuristics – Polarimetry

5. Polarization position angle solve (using P , inverse of D, G, B) on position angle calibrator:

$$(D' G' B' V^{obs}) = \underline{X} (P V^{mod})$$

6. Flux Density scaling:

$$G \rightarrow G_f \quad (\text{enforce gain consistency})$$

7. Correct with inverted solutions:

$$V^{cor} = P' X' D' G_f' B' V^{obs}$$

8. Image!

- To use external priors, e.g., T (opacity), K (ant. position errors), E (gaincurve), revise step 3 above as:

$$3. \quad (B' K' V^{obs}) = G (E P T V^{mod})$$

- and carry T, K , and E forward along with G to subsequent steps



Modern Calibration Challenges

- Gain calibration optimizations
 - ‘Delay-aware’ gain (self-) calibration: Troposphere and Ionosphere introduce time-variable phase effects easily parameterized as functions of frequency
 - Inter-band gain transfer (high-frequency ALMA)
 - Water Vapor Radiometry
- Polarization calibration optimizations
 - Frequency-dependent Instrumental Polarization $\sqrt{}$
 - High Dynamic Range (I, Q, U, & V)
 - More robust gain refant algorithms
 - Routine Full Polarization Treatments
- Voltage pattern for wide fields of view, mosaicking
 - Frequency-dependent voltage pattern
 - Wide-field accuracy (sidelobes, rotation)
 - Instrumental polarization (incl. frequency-dependence)
- RFI mitigation
- Pipelines/Science Ready Data Products (SRDP)
 - Generalized Heuristics vs. observational flexibility...
 - Modern instruments’ sensitivity to more subtle effects...
- Increasing sensitivity: Can implied dynamic range be reached by our calibration and imaging techniques?



Summary

- Determining calibration is as important as determining source structure—can't have one without the other
- Data examination and editing an important part of calibration
- Calibration dominated by antenna-based effects
 - permits efficient, accurate and scientifically defensible separation of calibration from astronomical information (satisfies closure)
- Full calibration formalism algebra-rich, but is *modular*
- Calibration an iterative process, improving various components in turn, as needed
- Point sources are the best calibrators
- Observe calibrators according requirements of calibration components

