Imaging and Deconvolution

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Atacama Large Millimeter/submillimeter Array
Expanded Very Large Array
Robert C. Byrd Green Bank Telescope
Very Large Baseline Array

NRAO

References

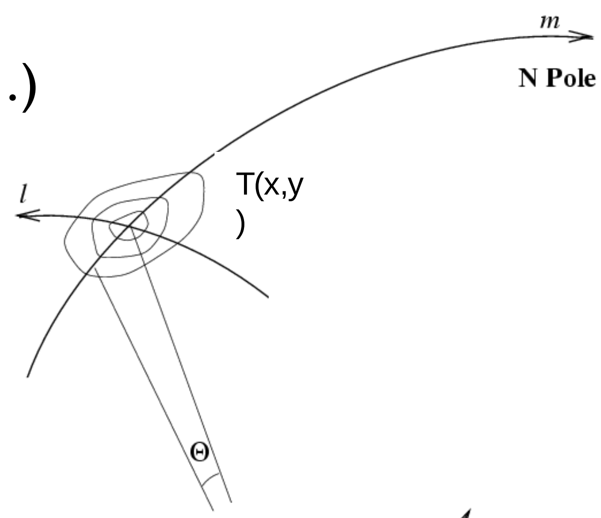
- Thompson, A.R., Moran, J.M. & Swensen, G.W. 2004, "Interferometry and Synthesis in Radio Astronomy" 2nd edition (Wiley-VCH)

- previous Synthesis Imaging workshop proceedings
  - Perley, R.A., Schwab, F.R., Bridle, A.H., eds. 1989, ASP Conf. Series 6, "Synthesis Imaging in Radio Astronomy" (San Francisco: ASP)
    - Ch. 6: Imaging (Sramek & Schwab), Ch. 8: Deconvolution (Cornwell)
  - http://www.aoc.nrao.edu/events/synthesis
    - Imaging and Deconvolution lectures by Cornwell 2002, Bhatnagar 2004, 2006
- IRAM Interferometry School proceedings
  - http://www.iram.fr/IRAM/FR/IS/IS2008/archive.html
    - Ch. 13: Imaging Principles (Guilloteau), Ch. 16: Imaging in Practice (Guilloteau)
    - Imaging and Deconvolution lectures by Pety 2004, 2006, 2008, 2010

- more interferometry school proceedings and pedagogical presentations are readily available: ALMA Cycle 1 primer, ATNF, CARMA, NAOJ, …

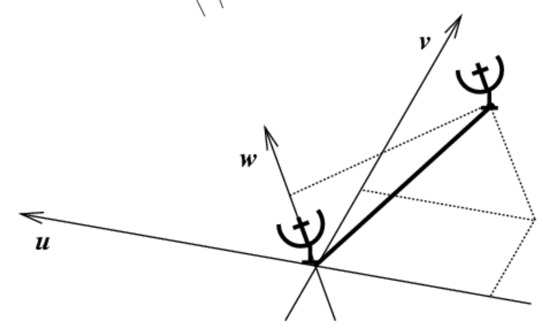
Visibility and Sky Brightness

- from the van Cittert-Zernike theorem (TMS Ch. 14)
  - the complex visibility V(u,v) is the 2-dimensional Fourier Transform of the sky brightness T(x,y) (incoherent source, small field of view, far field…)
  - u,v are E-W and N-S spatial frequencies
    units are wavelengths
  - x,y are E-W and N-S angles in the tangent plane
    units are radians



$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dx dy$$

$$T(x,y) = \int\int V(u,v) e^{-2\pi i(ux+vy)} du dv$$

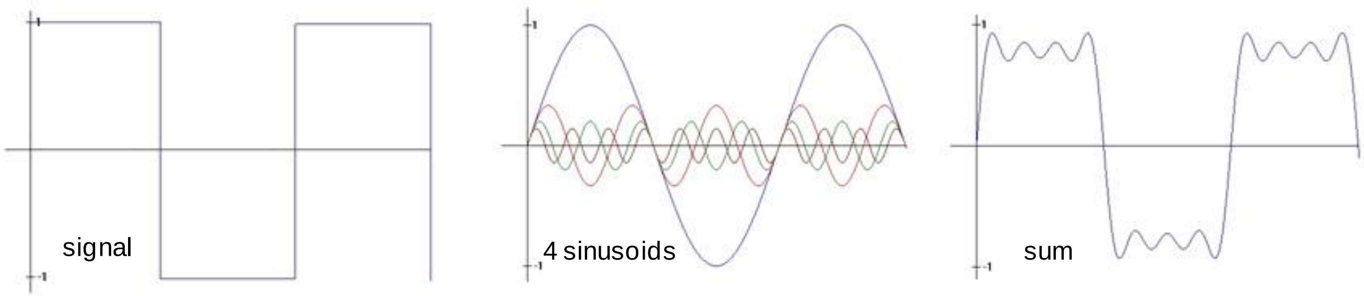


$$V(u,v) \rightleftharpoons T(x,y)$$

The Fourier Transform

- Fourier theory states that any well behaved signal (including images) can be expressed as the sum of sinusoids

Jean Baptiste Joseph Fourier 1768-1830



$$x(t) = \frac{4}{\pi}\left(\sin(2\pi f t) + \frac{1}{3}\sin(6\pi f t) + \frac{1}{5}\sin(10\pi f t) + \cdots\right)$$

- the Fourier transform is the mathematical tool that decomposes a signal into its sinusoidal components
- the Fourier transform of a signal contains *all* of the information of the original

The Fourier Domain

- acquire some comfort with the Fourier domain
  - in older texts, functions and their Fourier transforms occupy *upper* and *lower* domains, as if "functions circulated at ground level and their transforms in the underworld" (Bracewell 1965)

- a few properties of the Fourier transform $f(x) \rightleftharpoons F(s)$

adding $f(x) + g(x) = F(s) + G(s)$

scaling $f(\alpha x) = \alpha^{-1} F(s/\alpha)$

shifting $f(x - x_0) = F(s) e^{i 2\pi x_0 s}$

convolution/mulitplication $g(x) = f(x) \otimes h(x); \quad G(s) = F(s) H(s)$

Nyquist-Shannon sampling theorem $f(x) \subset \Theta$ completely determined if $F(s)$ sampled at intervals $\leq 1/\Theta$

Visibilities

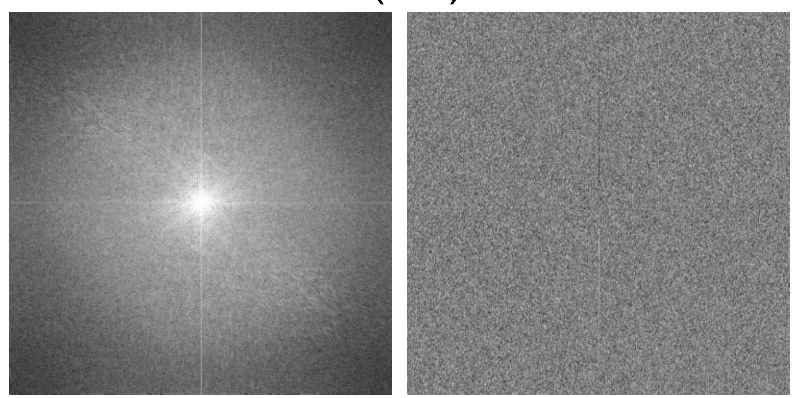
- each V(u,v) contains information on T(x,y) *everywhere*, not just at a given (x,y) coordinate or within a given subregion

- V(u,v) is a complex quantity
  - visibility expressed as (real, imaginary) or (amplitude, phase)

T(x,y)

$\rightleftharpoons$

V(u,v)



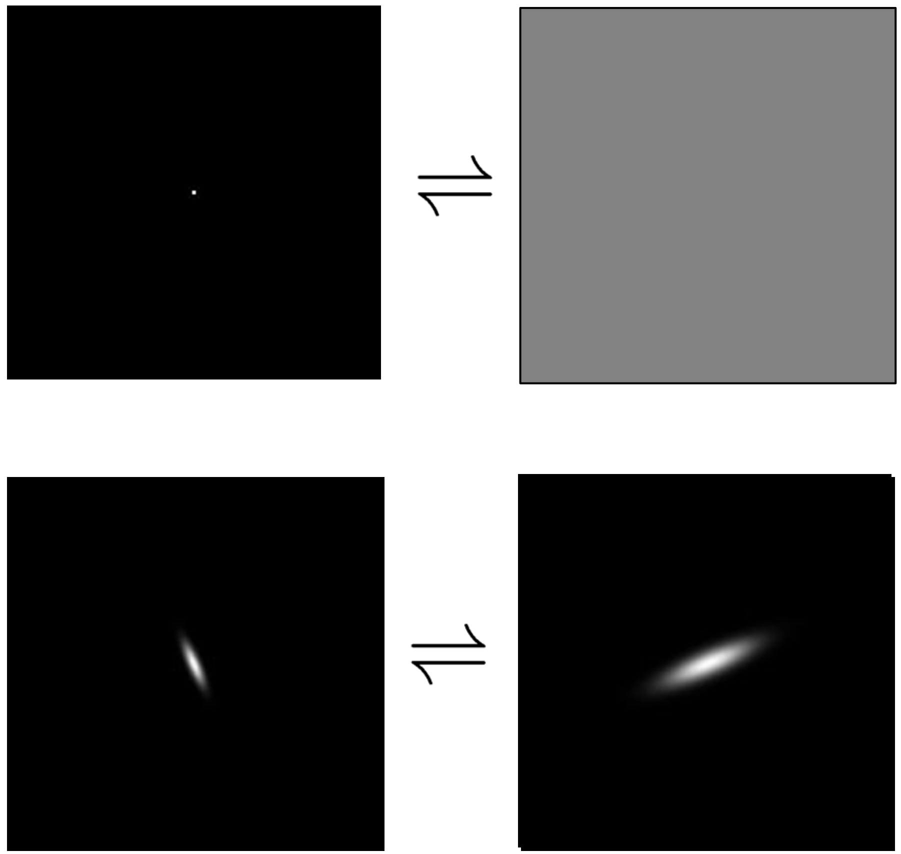
amplitude phase

Example 2D Fourier Transform Pairs

T(x,y) amp{V(u,v)}

δ function ⇌ constant

elliptical Gaussian ⇌ elliptical Gaussian

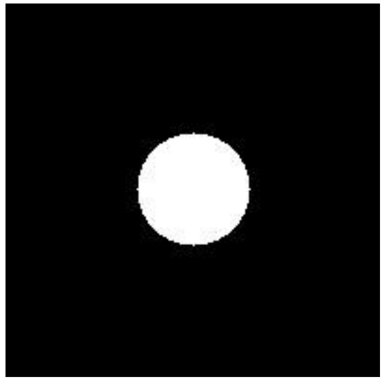


narrow features transform into wide features (and vice-versa)

Example 2D Fourier Transform Pairs

T(x,y) amp{V(u,v)}

disk



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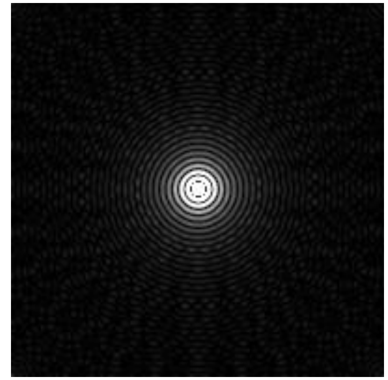
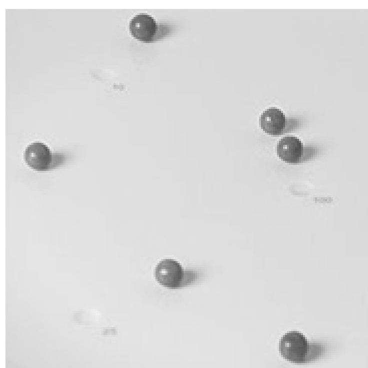


Bessel



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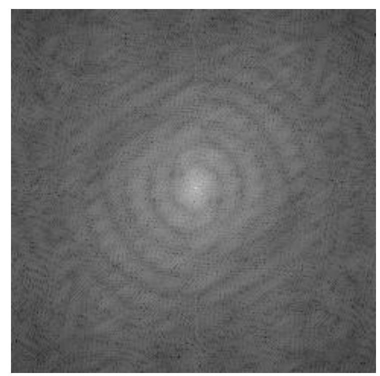
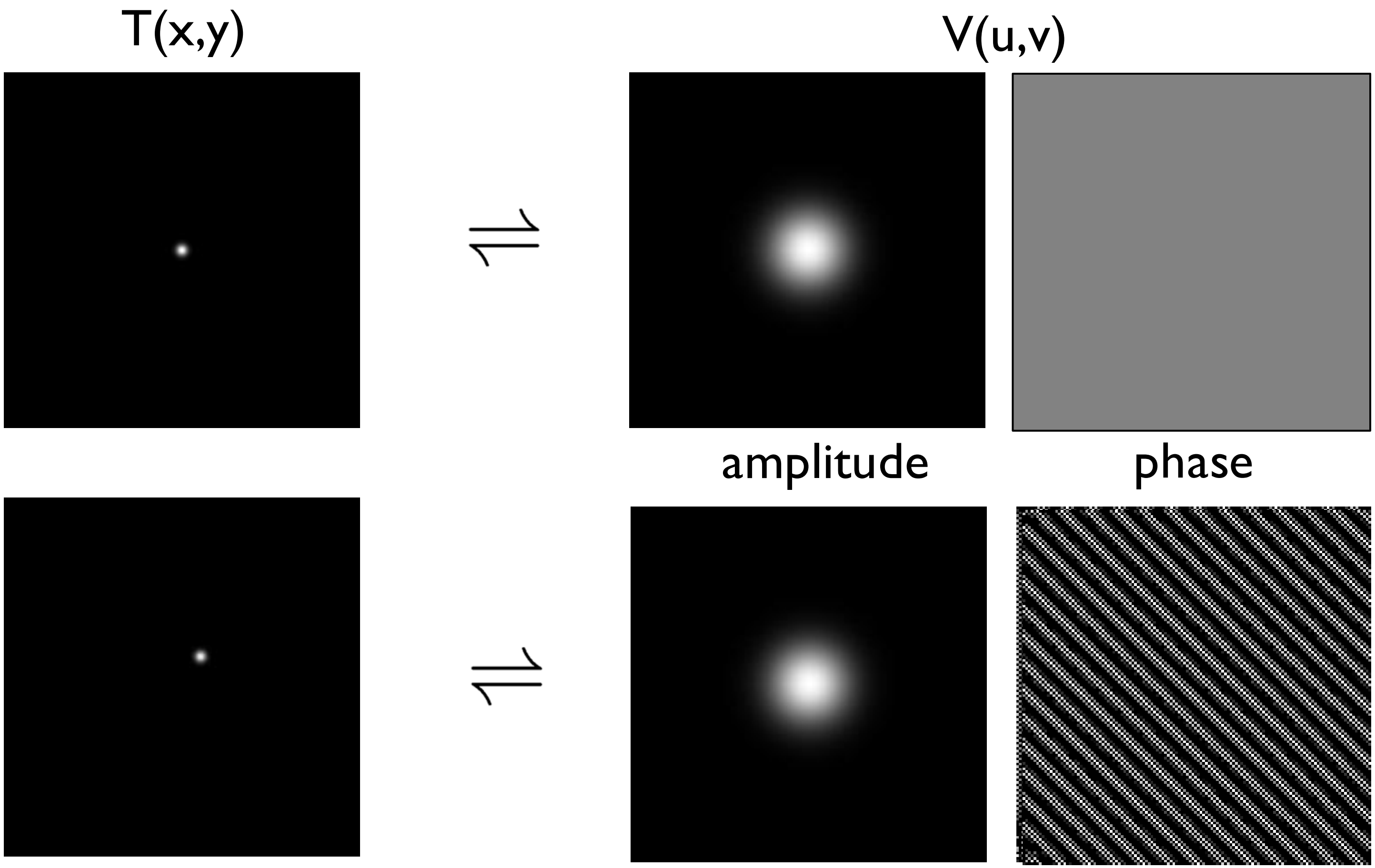


sharp edges result in many high spatial frequencies

Amplitude and Phase

- amplitude tells "how much" of a certain spatial frequency
- phase tells "where" this component is located



The Visibility Concept

$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dxdy$$

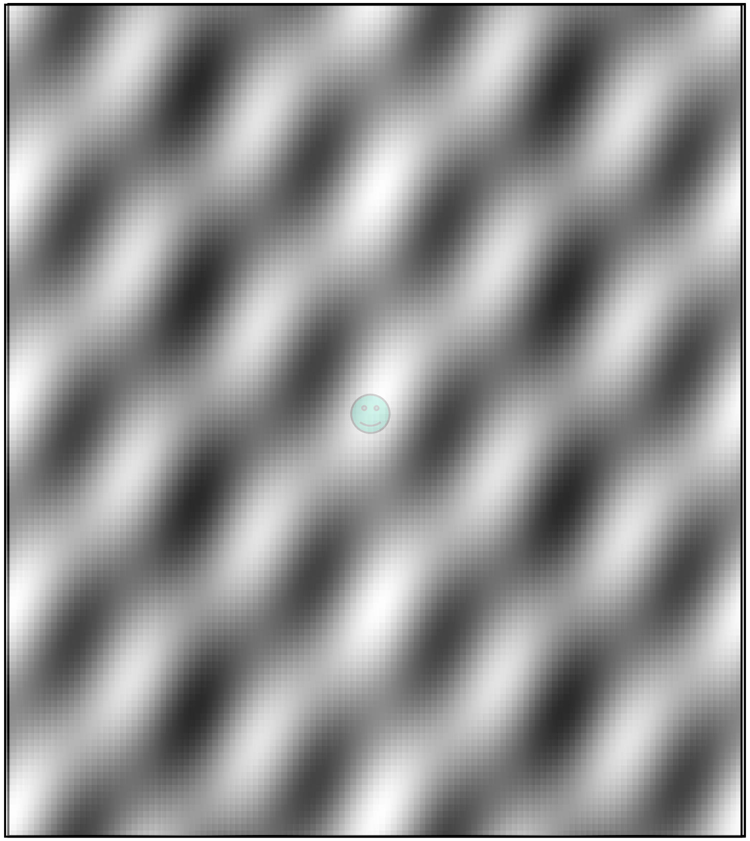
- visibility as a function of baseline coordinates (u,v) is the Fourier transform of the sky brightness distribution as a function of the sky coordinates (x,y)

- V(u=0,v=0) is the integral of T(x,y)dxdy = total flux

- since T(x,y) is real, V(u,v) is Hermitian: V(-u,-v) = V*(u,v)
  - get two visibilities for one measurement

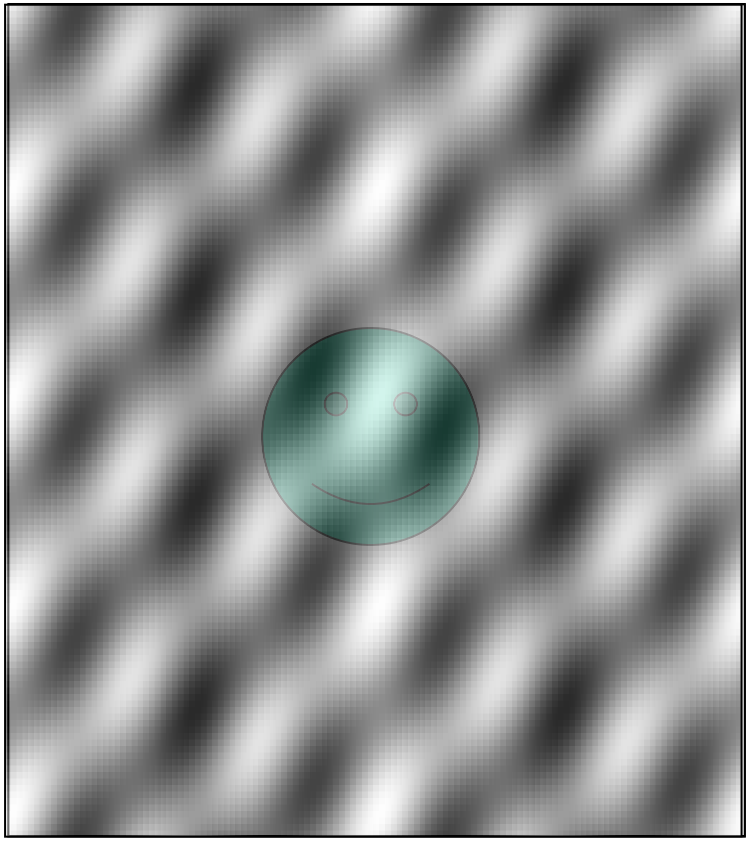
Visibility and Sky Brightness

$$V(u,v) = \int\int T(x,y) e^{2\pi i(ux+vy)} dx dy$$



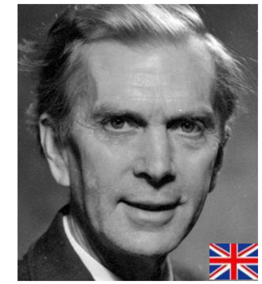
Visibility and Sky Brightness

$$V(u,v) = \int \int T(x,y) e^{2\pi i(ux+vy)} dx dy$$



Aperture Synthesis Basics

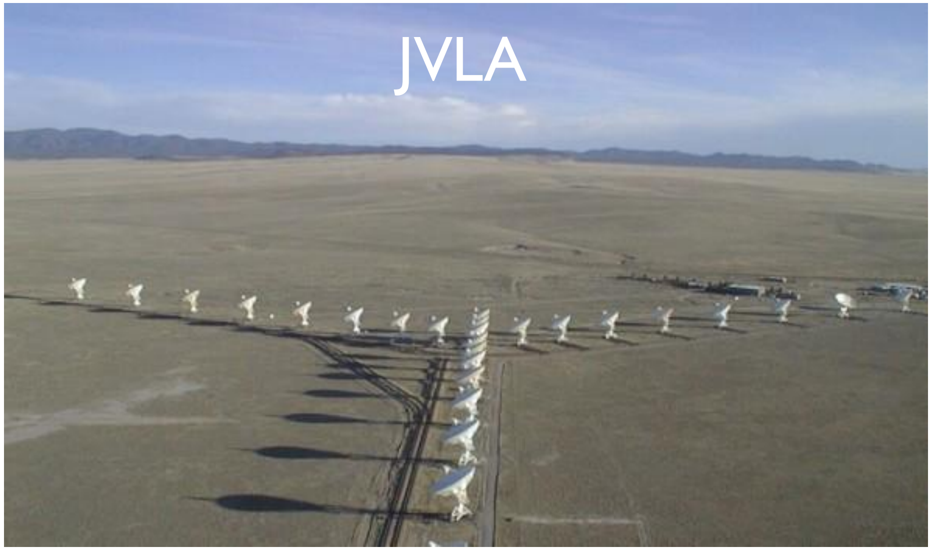
- idea: sample V(u,v) at *enough* baselines to synthesize a large aperture of size (u_{max}, v_{max})
  - one pair of telescopes = one baseline = one (u,v) sample at a time
  - N telescopes = N(N-1) (u,v) samples at a time
  - use Earth rotation to fill in (u,v) plane with time (Sir Martin Ryle 1974 Physics Nobel Prize)
  - reconfigure physical layout of N antennas for more
  - observe at multiple wavelengths simultaneously, if source spectrum amenable to simple characterization

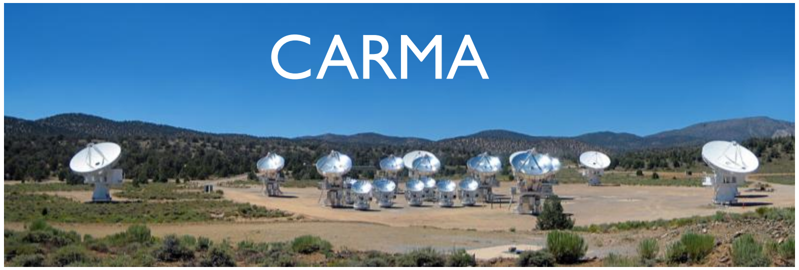


Sir Martin Ryle
1918-1984

- How many samples are enough?

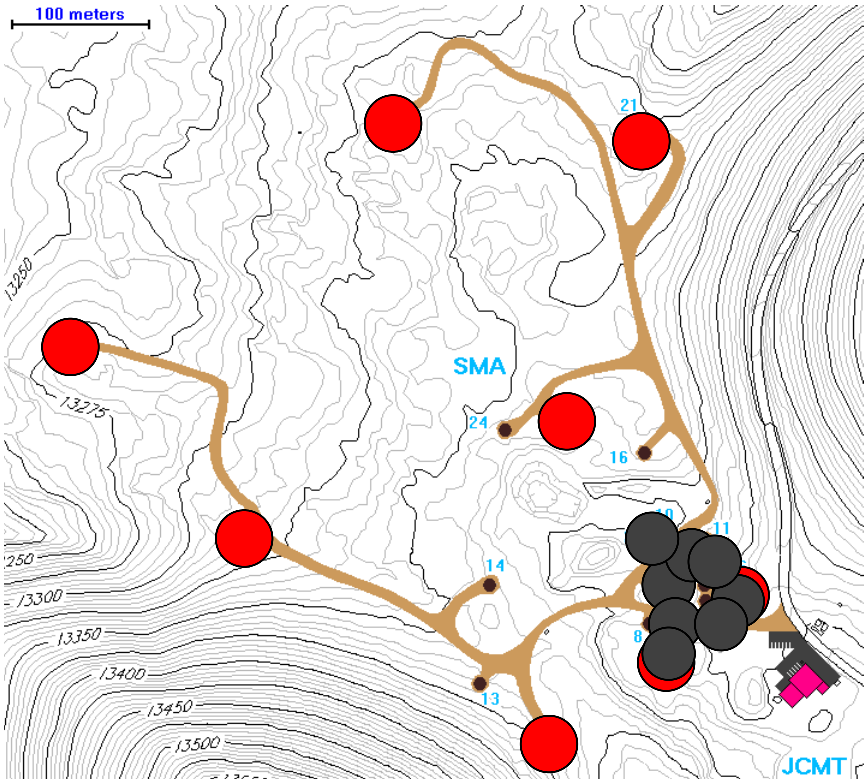
Examples of (Millimeter Wavelength) Aperture Synthesis Telescopes

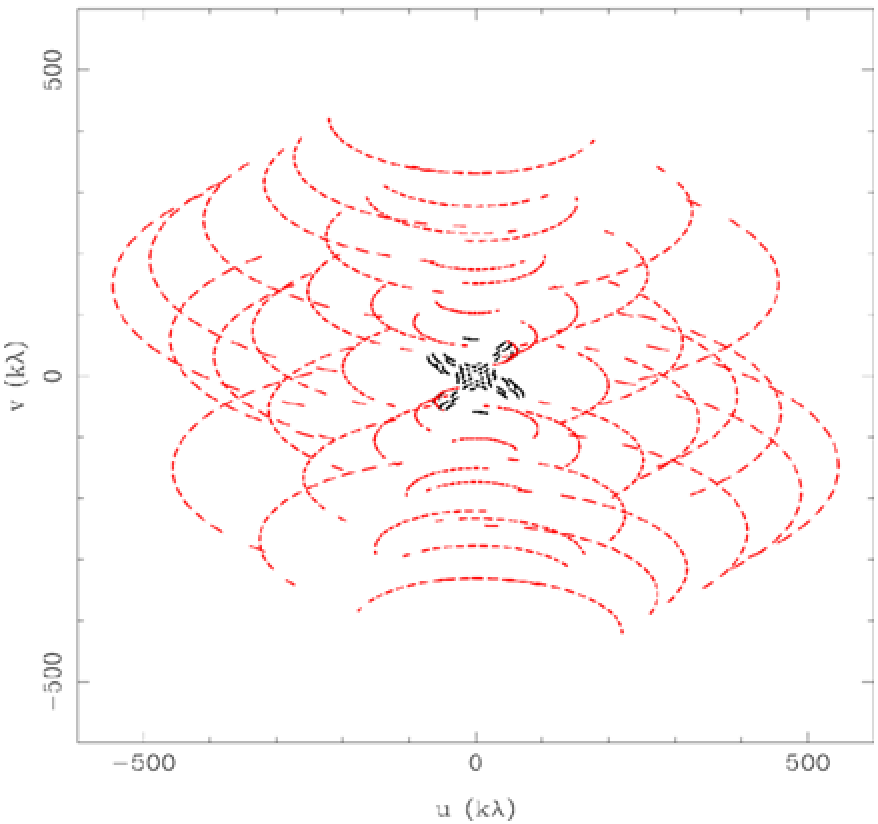




An Example of (u,v) plane Sampling

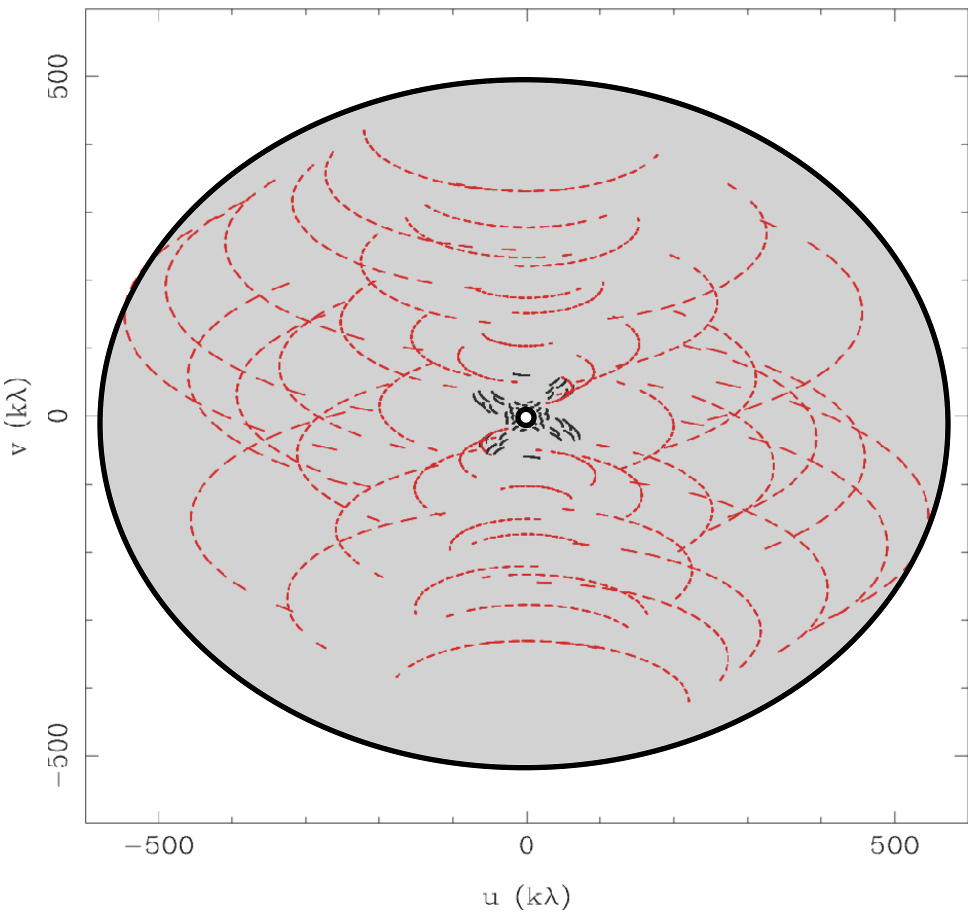
- 2 configurations of 8 SMA antennas, 345 GHz, Dec. -24 dec





Imaging: (u,v) plane Sampling

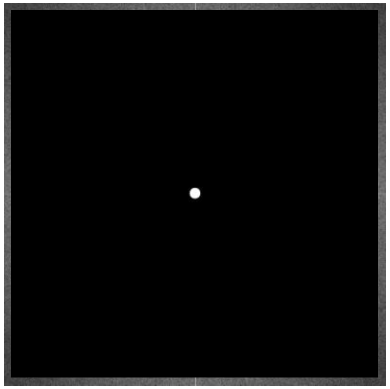
- in aperture synthesis, samples of V(u,v) are limited by the number of telescopes and the Earth-sky geometry



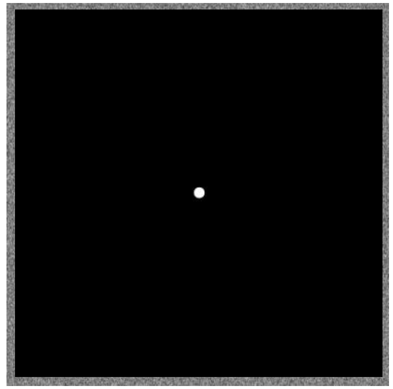
- outer boundary
  - no information on small scales
  - resolution limit
- inner hole
  - no information on large scales
  - extended structures invisible
- irregular coverage between inner and outer boundaries
  - sampling theorem violated
  - information missing

Inner and Outer (u,v) Boundaries

V(u,v)



amplitude

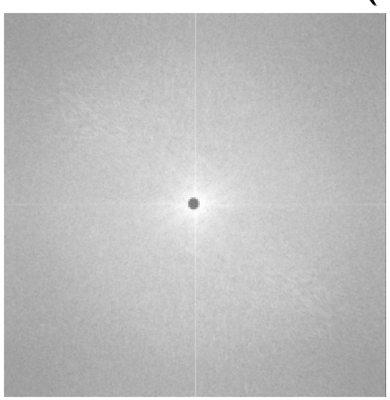


phase

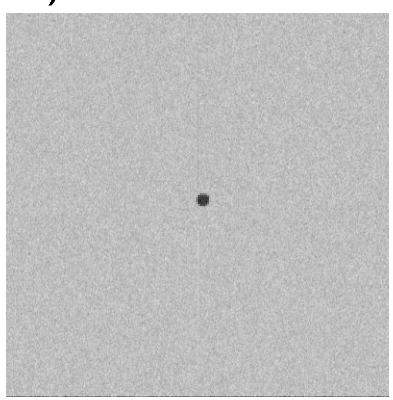
⇌

T(x,y)

V(u,v)



amplitude



phase

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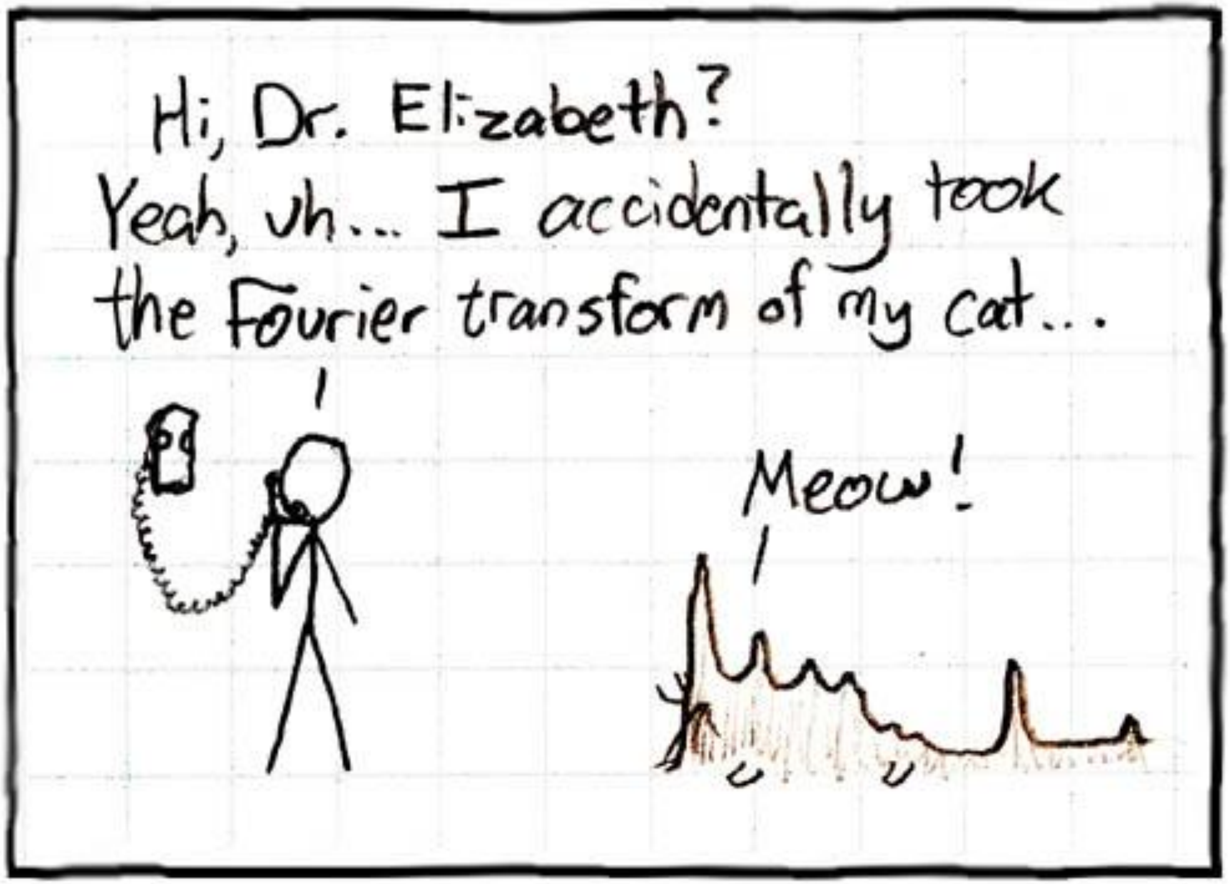
T(x,y)

xkcd.com/26/


Hi, Dr. Elizabeth?
Yeah, uh... I accidentally took the Fourier transform of my cat...
Meow!

Imaging: Formal Description

$$V(u,v) \rightleftharpoons T(x,y)$$

- sample Fourier domain at discrete points

  $B(u,v) = \sum_k (u_k, v_k)$

- the (inverse) Fourier transform is

  $T^D(x,y) = FT^{-1}\{B(u,v) \times V(u,v)\}$

- the convolution theorem tells us

  $T^D(x,y) = b(x,y) \otimes T(x,y)$

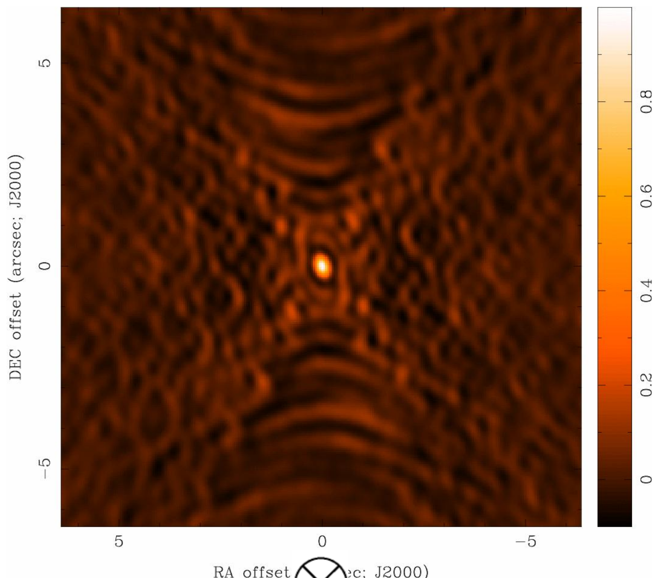
- where $b(x,y) = FT^{-1}\{B(u,v)\}$ (the point spread function)

the Fourier transform of the sampled visibilities yields the true sky brightness convolved with the point spread function

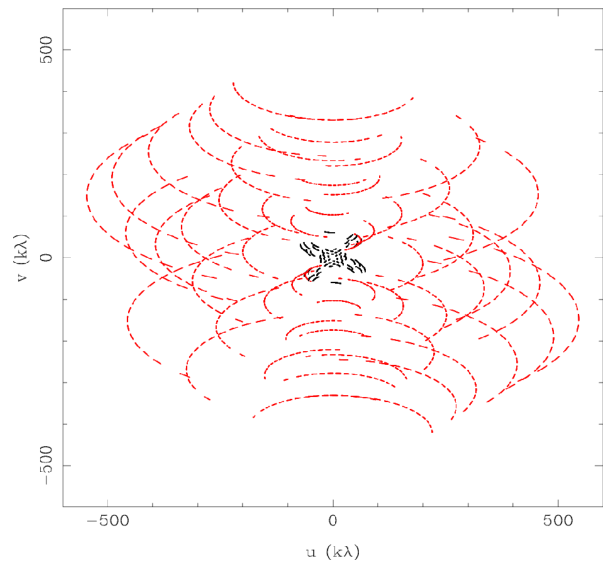
jargon: the "dirty image" is the true image convolved with the "dirty beam"

Dirty Beam and Dirty Image

$b(x,y)$
"dirty beam"



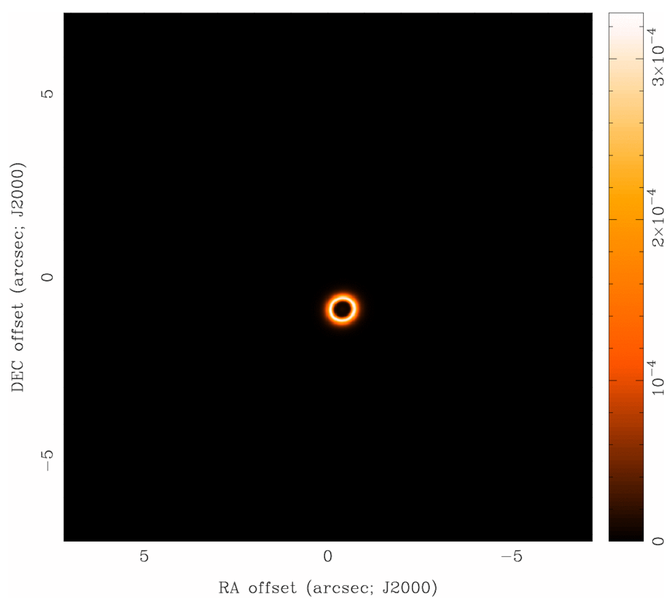
$\rightleftharpoons$



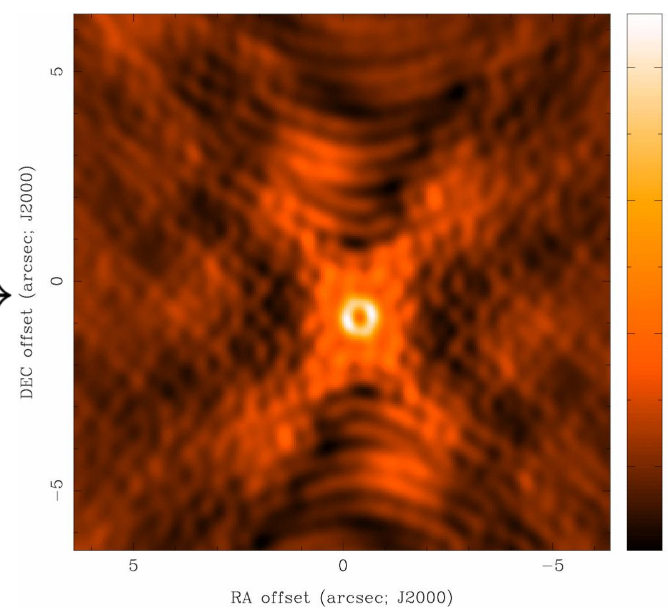
$B(u,v)$

$\otimes$

$T(x,y)$



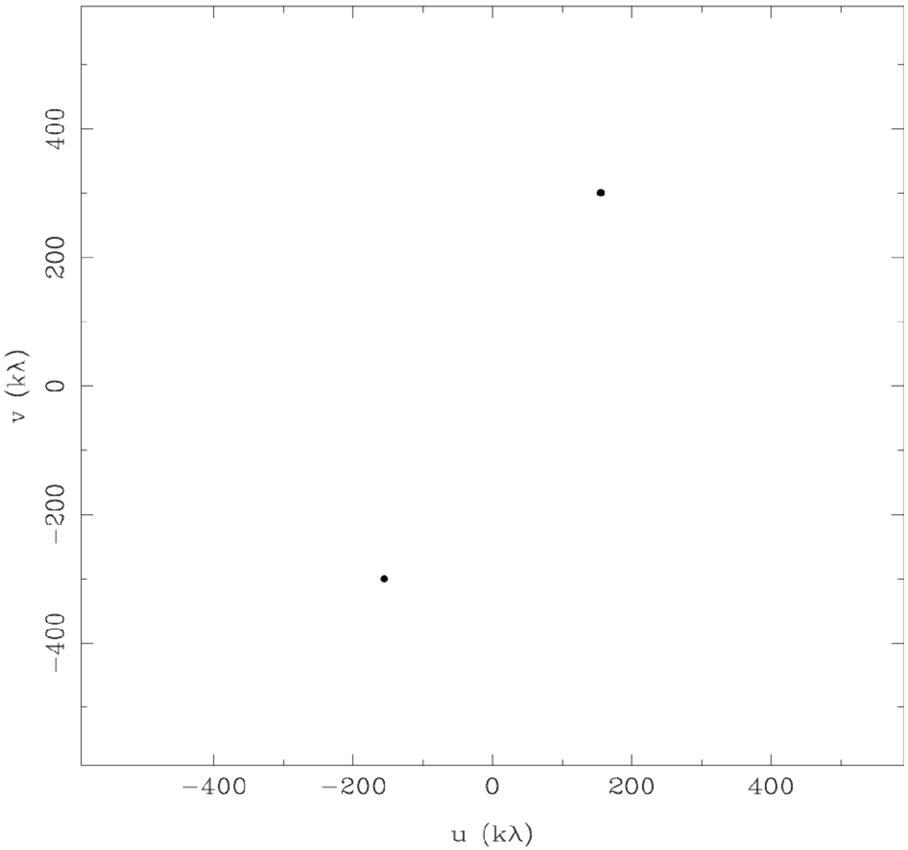
$\rightarrow$

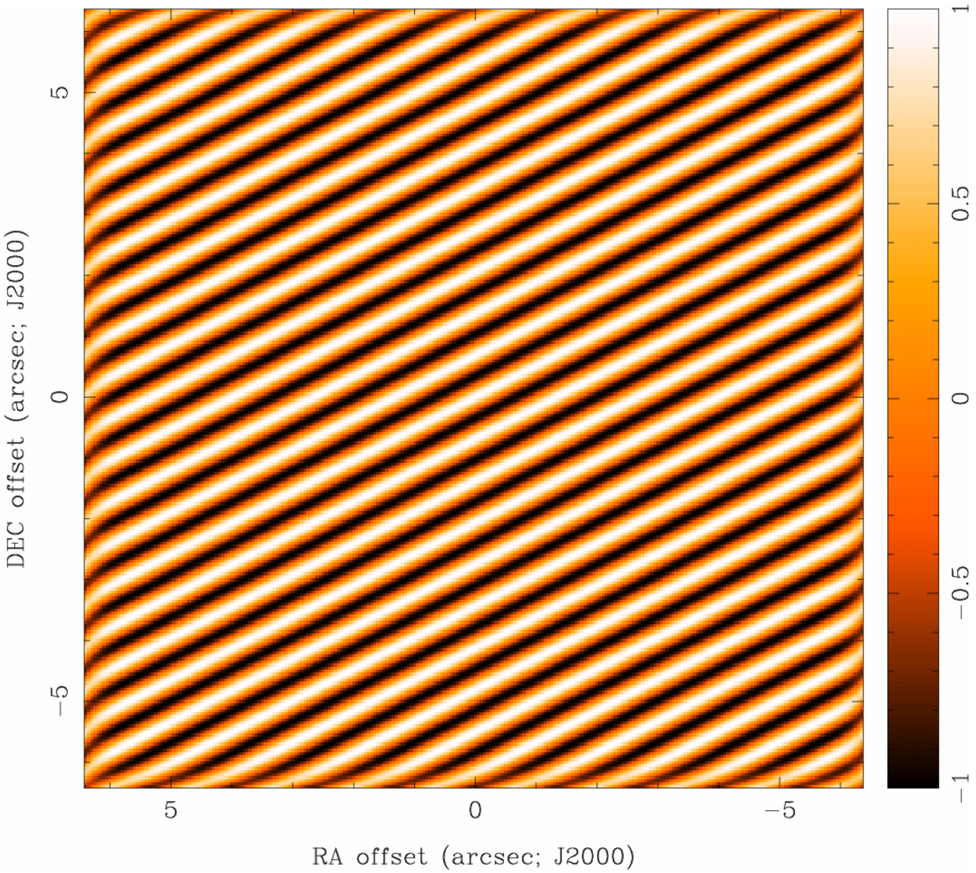


$T^D(x,y)$
"dirty image"

Dirty Beam Shape and N Antennas

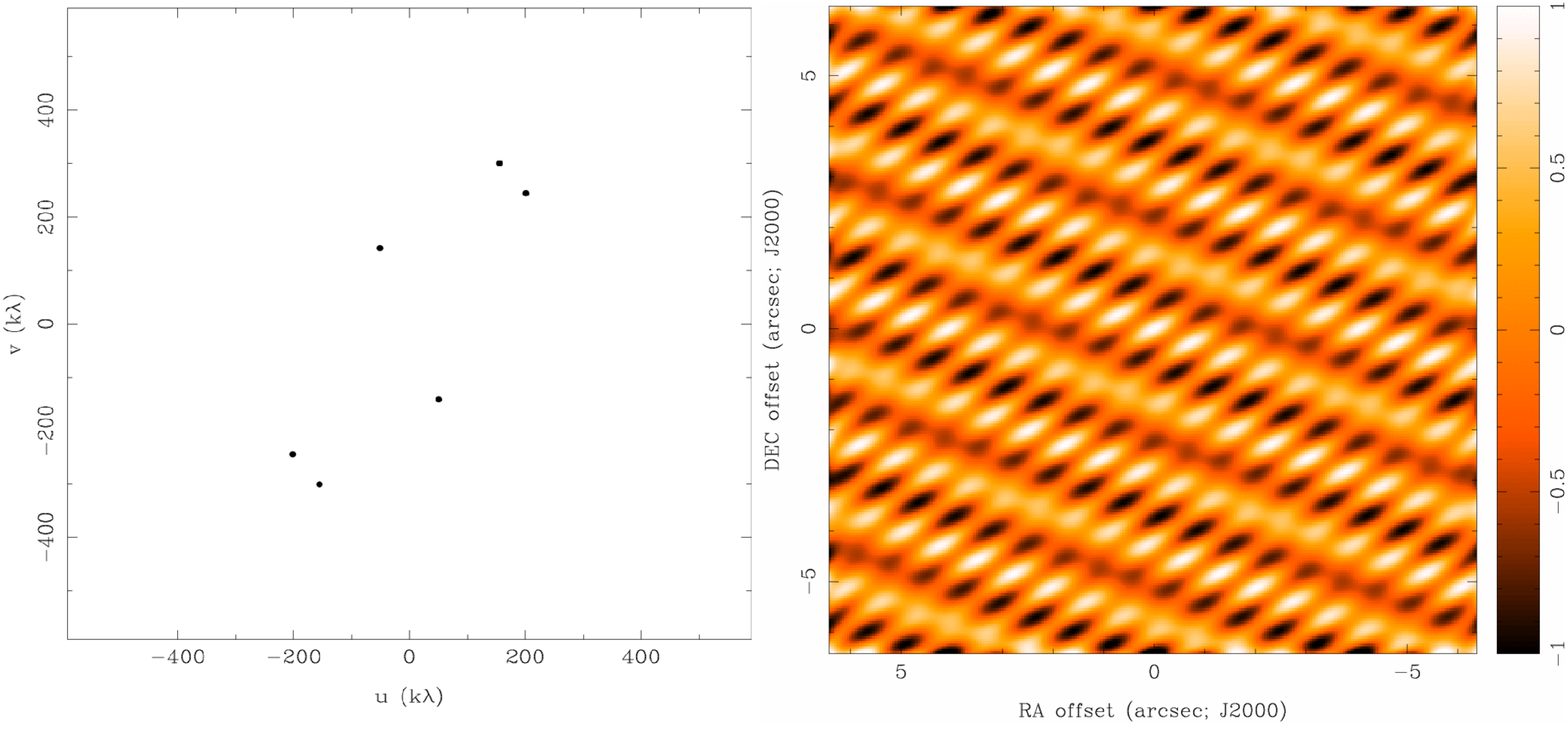
2 Antennas





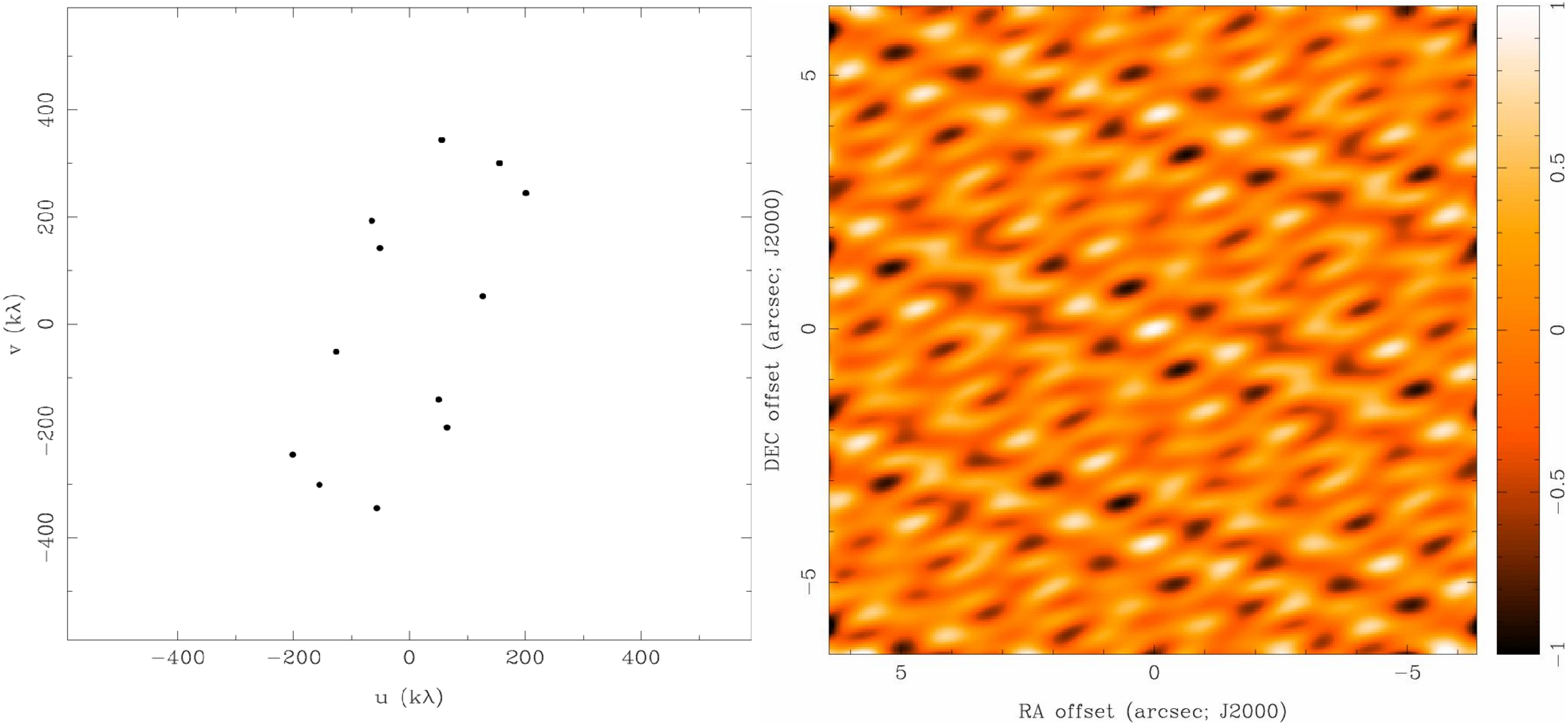
Dirty Beam Shape and N Antennas

3 Antennas



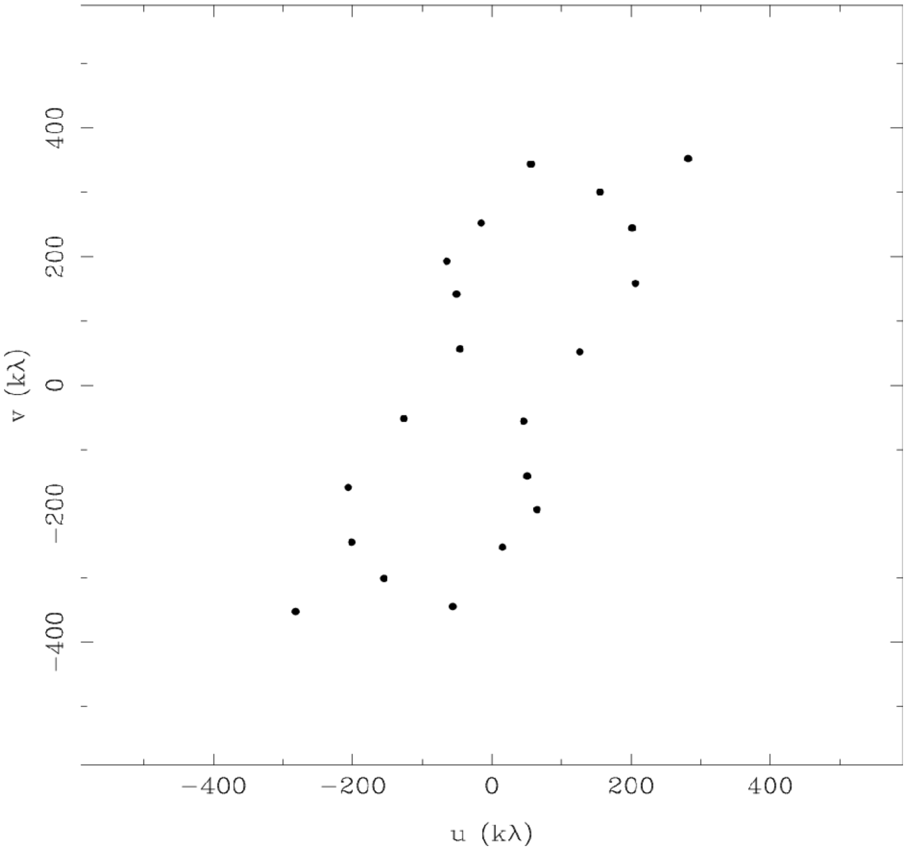
Dirty Beam Shape and N Antennas

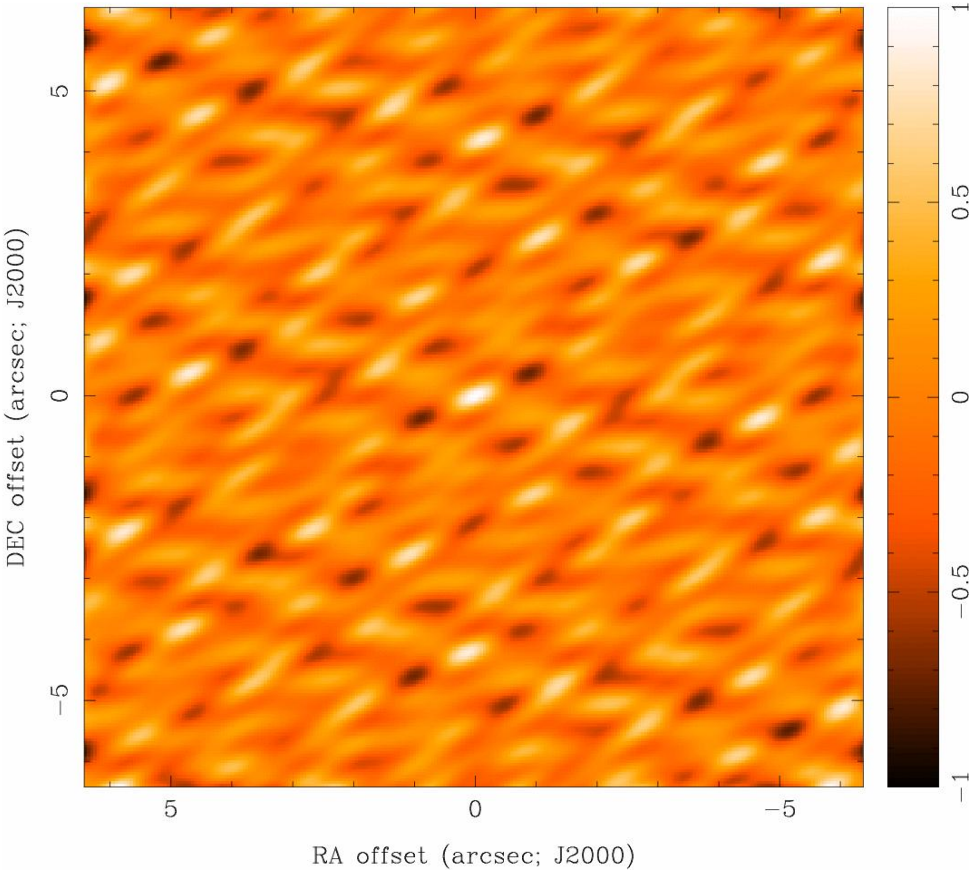
4 Antennas



Dirty Beam Shape and N Antennas

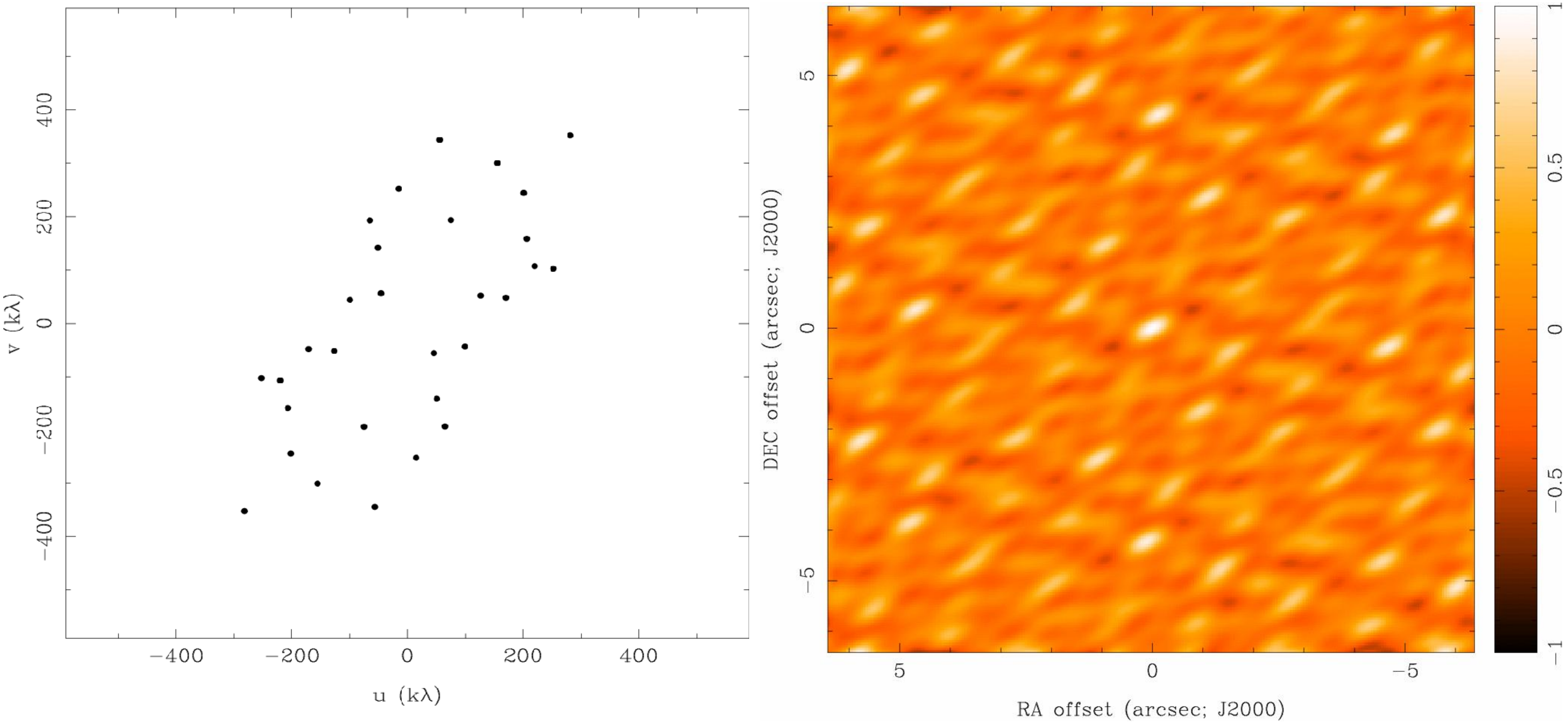
5 Antennas





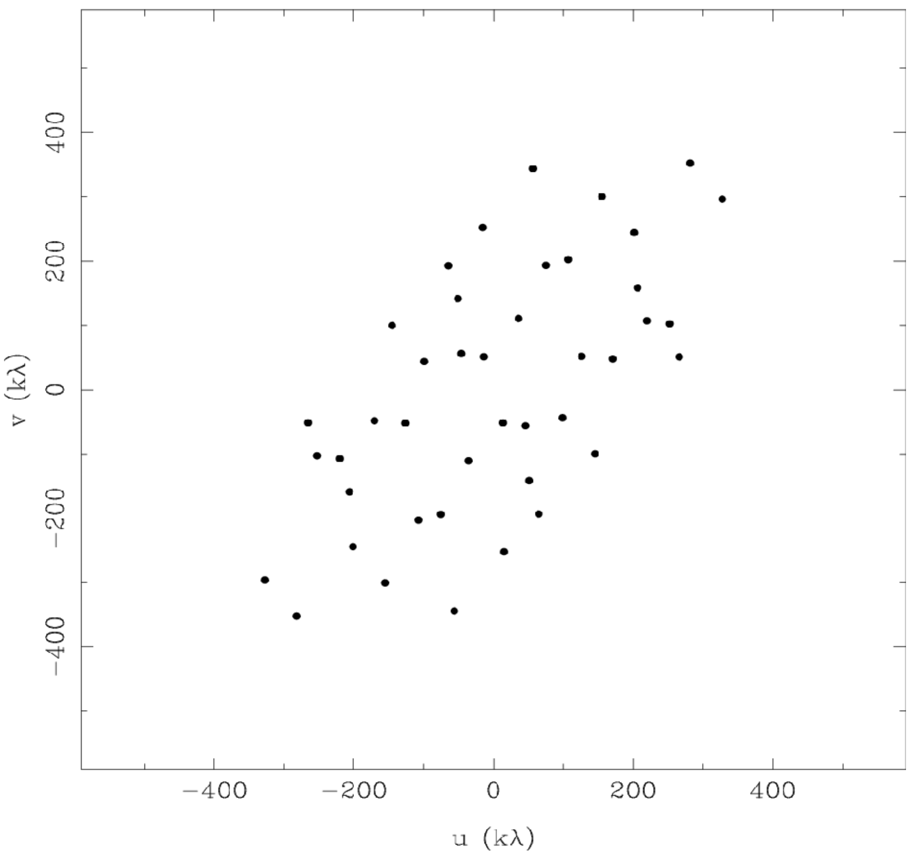
Dirty Beam Shape and N Antennas

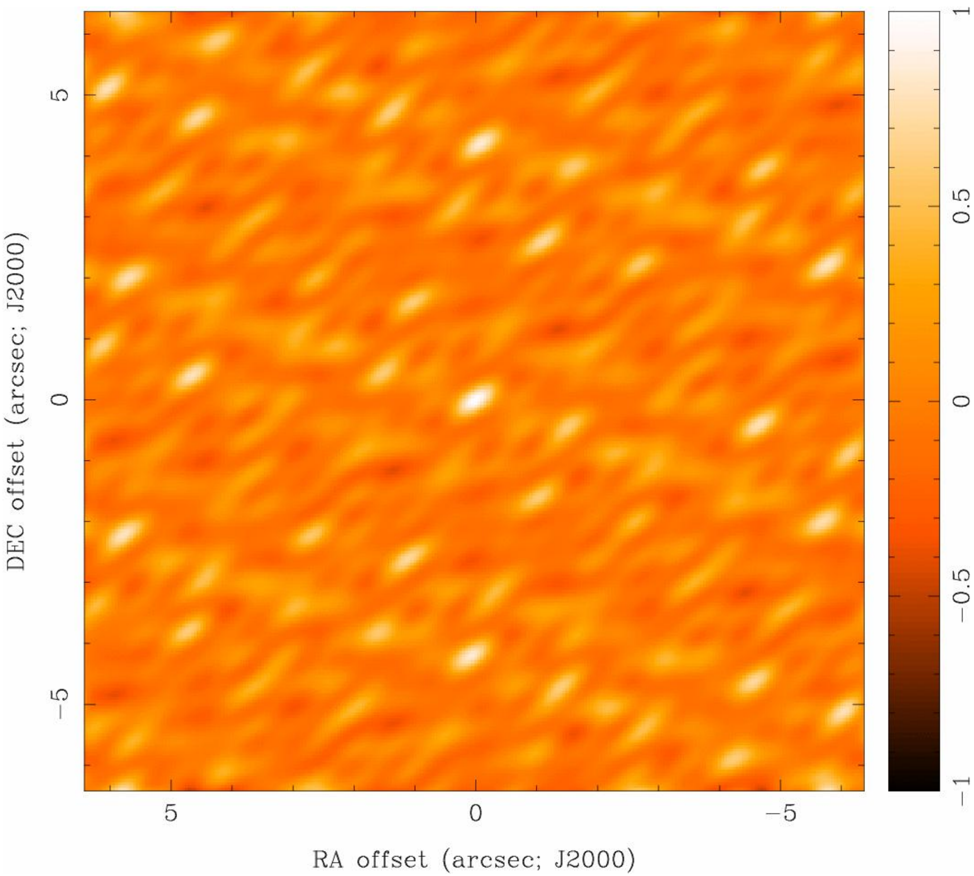
6 Antennas



Dirty Beam Shape and N Antennas

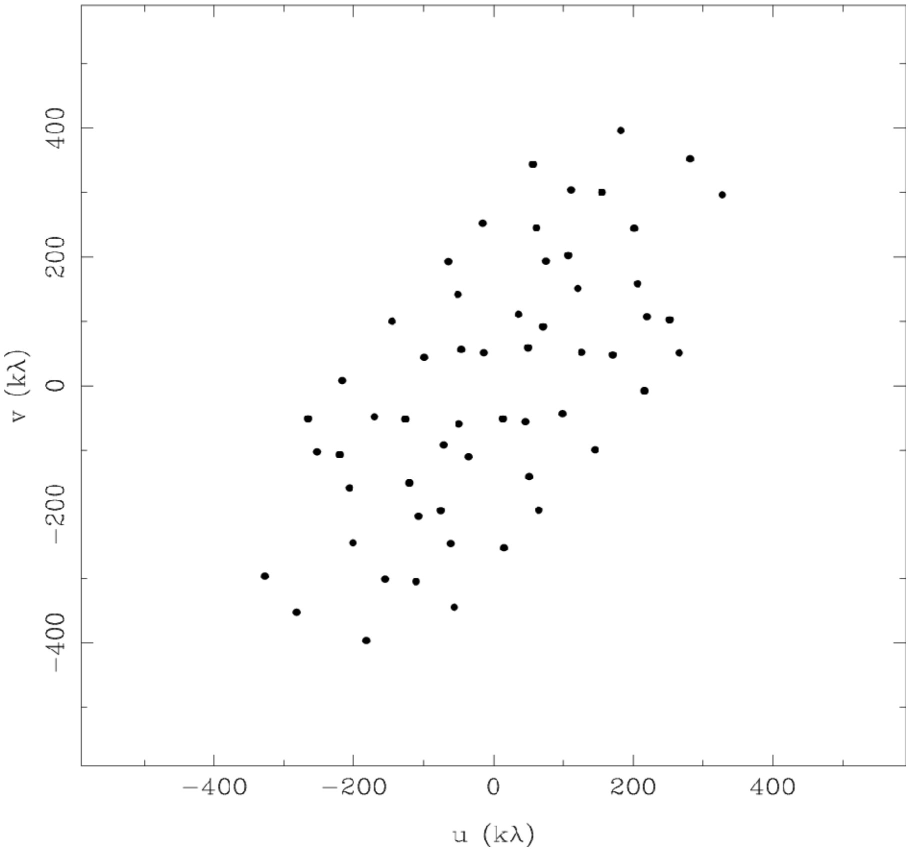
7 Antennas

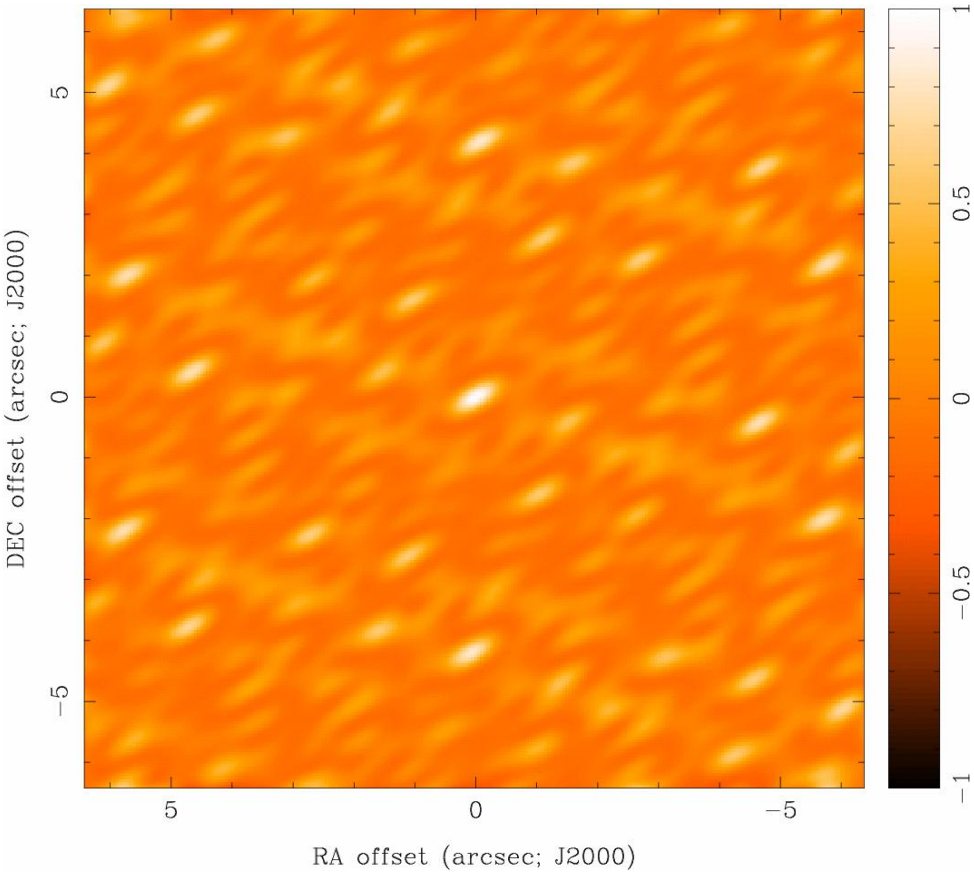




Dirty Beam Shape and N Antennas

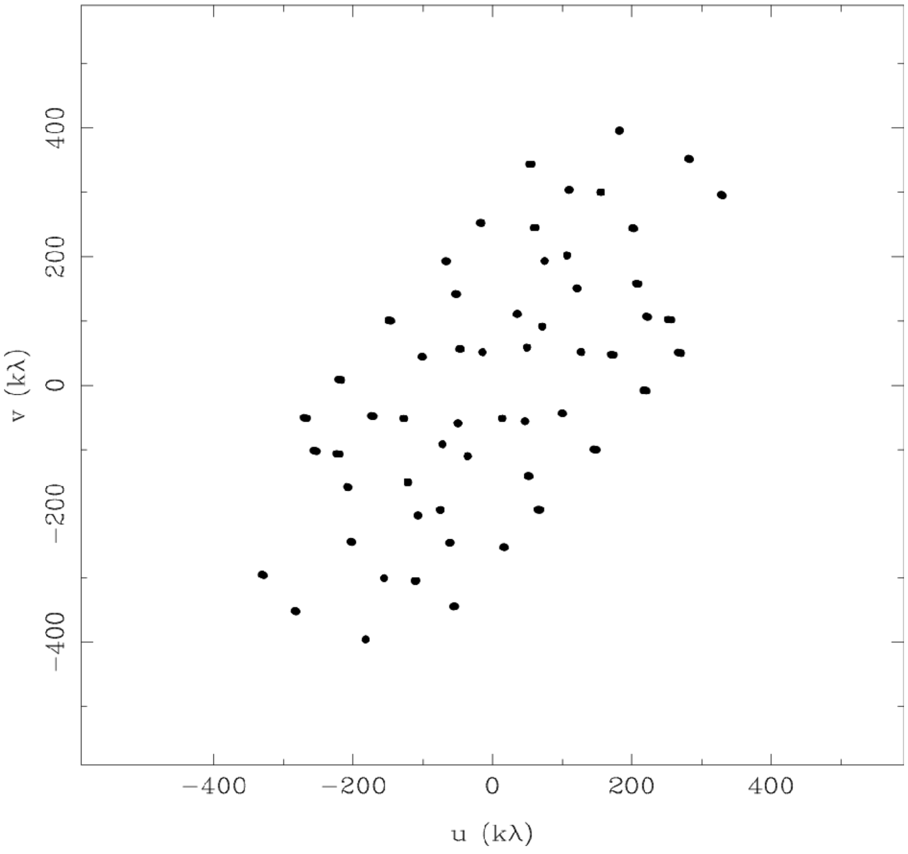
8 Antennas

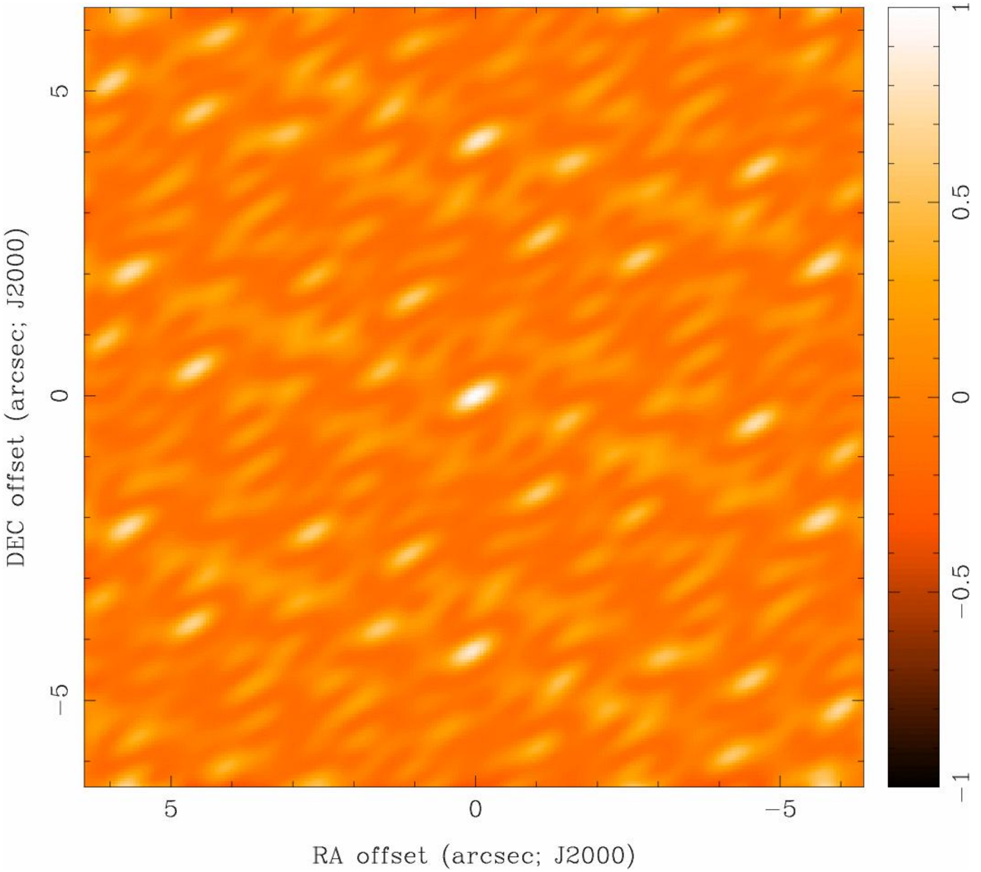




Dirty Beam Shape and N Antennas

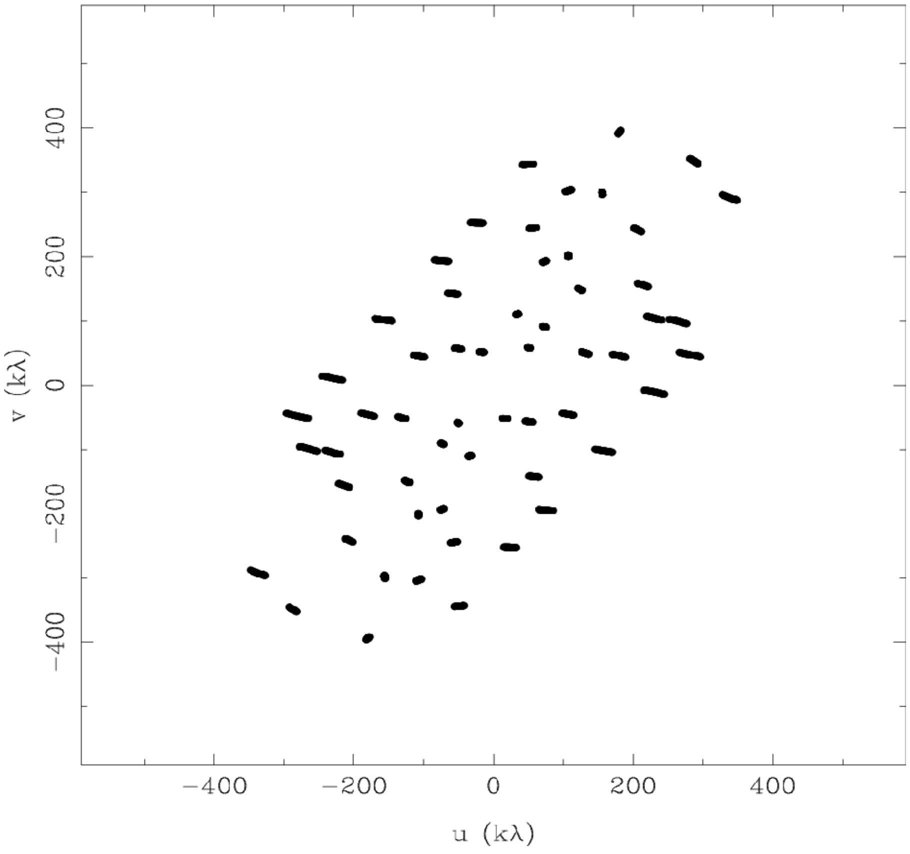
8 Antennas x 6 samples

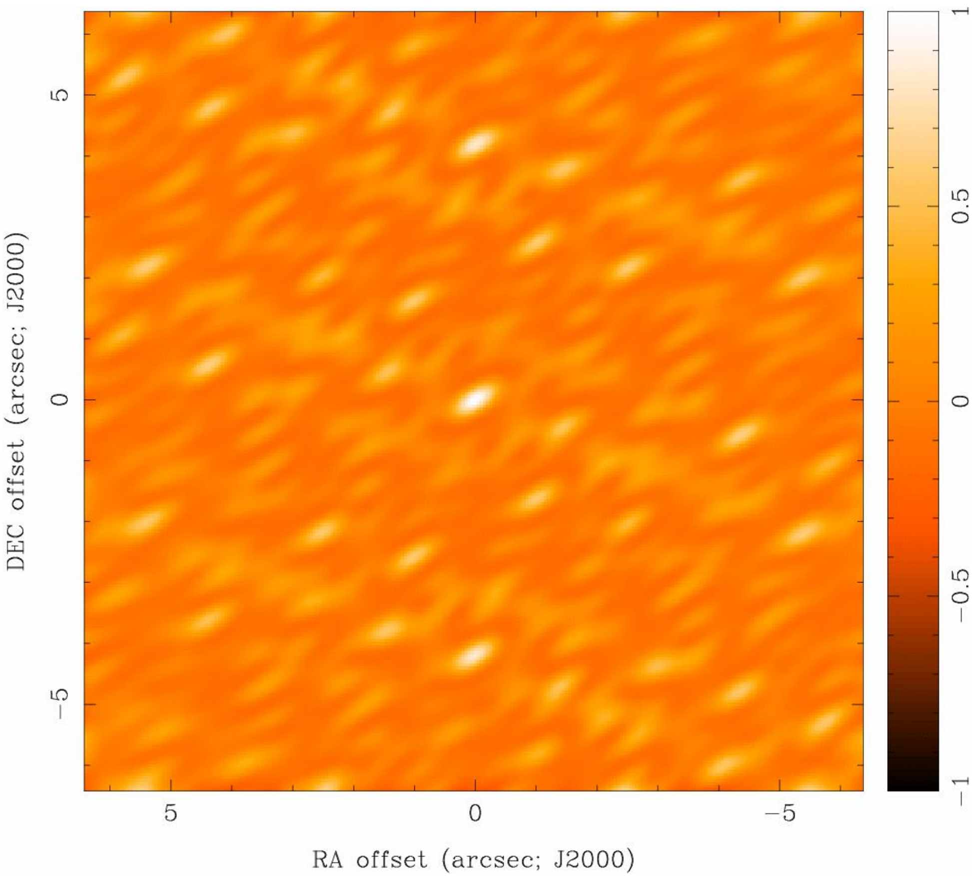




Dirty Beam Shape and N Antennas

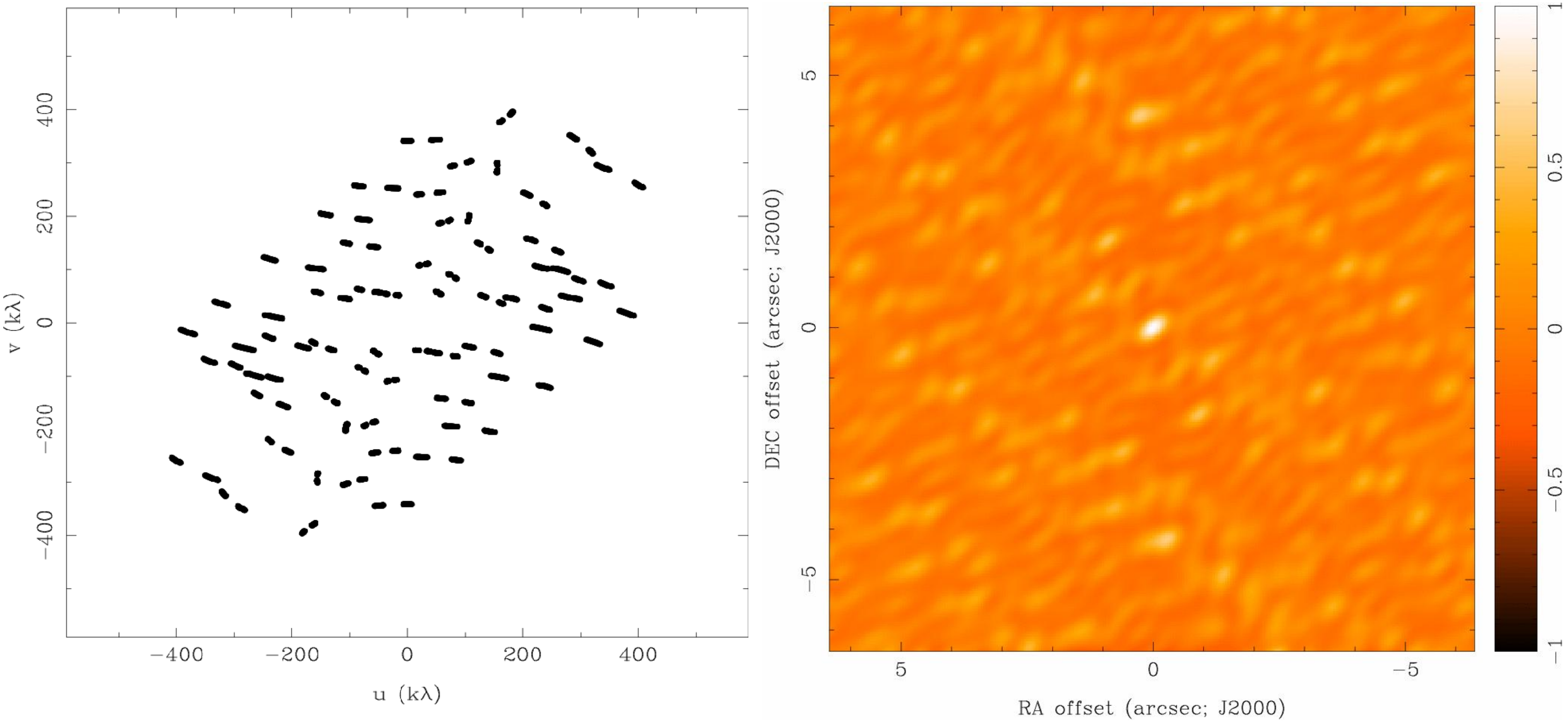
8 Antennas x 30 samples





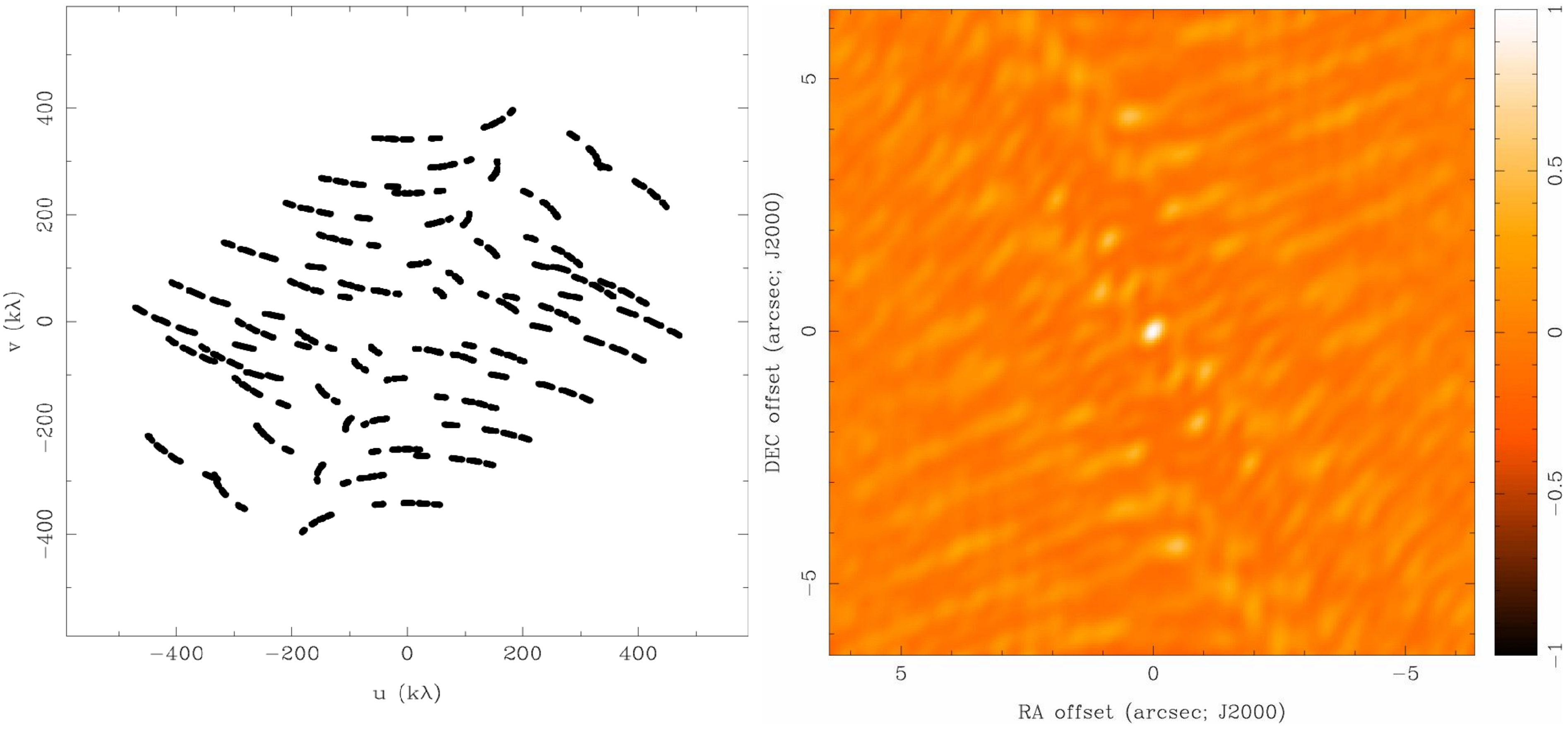
Dirty Beam Shape and N Antennas

8 Antennas x 60 samples



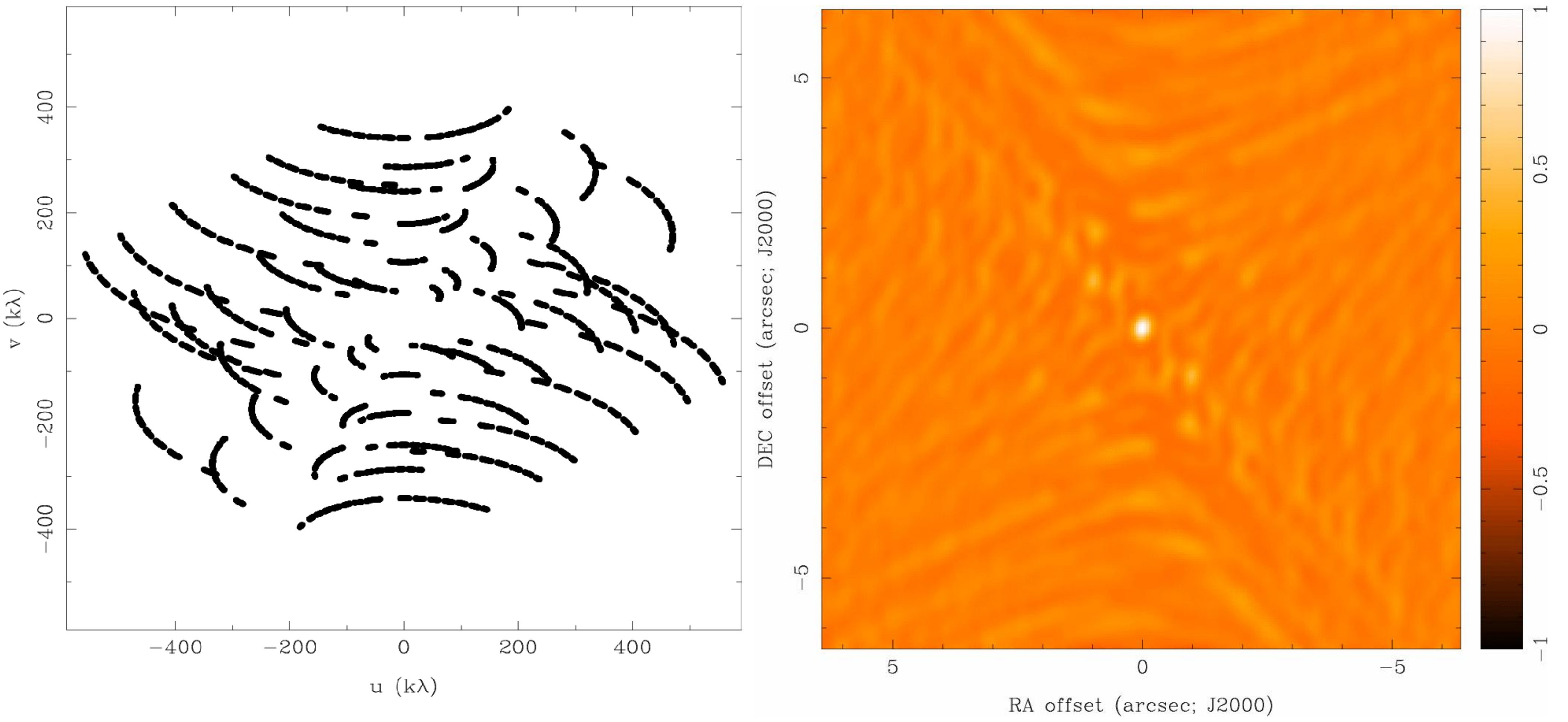
Dirty Beam Shape and N Antennas

8 Antennas x 120 samples



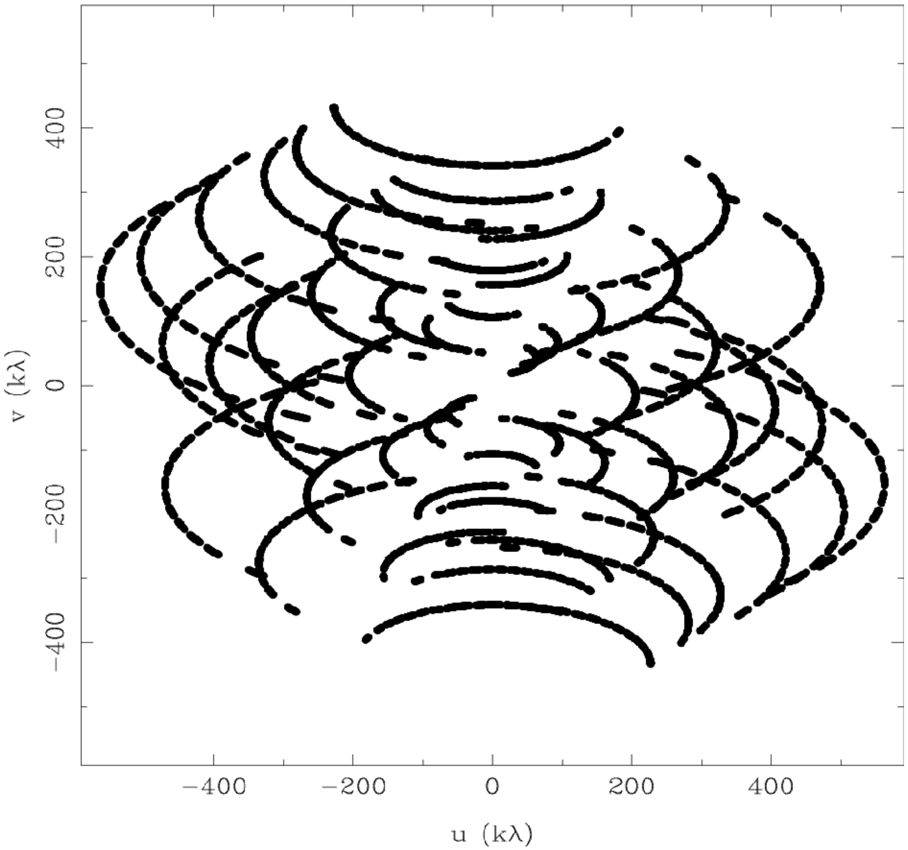
Dirty Beam Shape and N Antennas

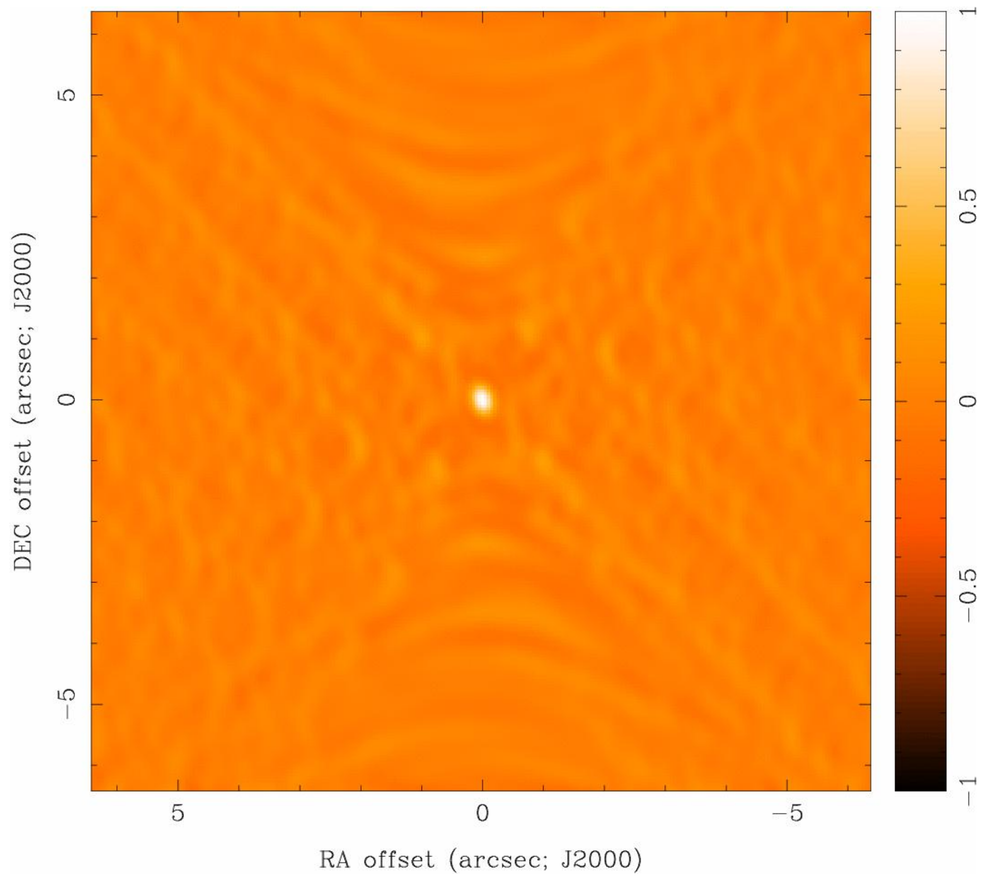
8 Antennas x 240 samples



Dirty Beam Shape and N Antennas

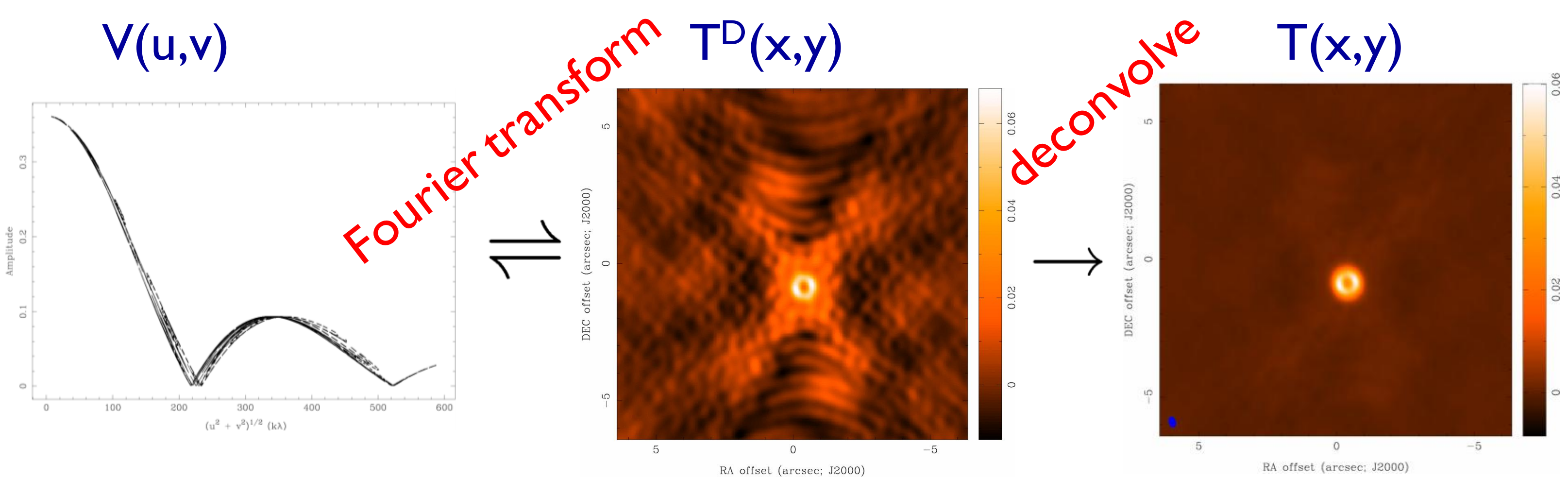
8 Antennas x 480 samples





Calibrated Visibilities- What Next?

- analyze V(u,v) samples directly by model fitting
  - best for "simple" structures, e.g. point sources, disks
- recover an image from the observed incomplete and noisy samples of its Fourier transform to analyze
  - Fourier transform V(u,v) samples to get $T^D(x,y)$
  - but difficult to do science on this dirty image
  - deconvolve b(x,y) from $T^D(x,y)$ to determine (a model of) T(x,y)



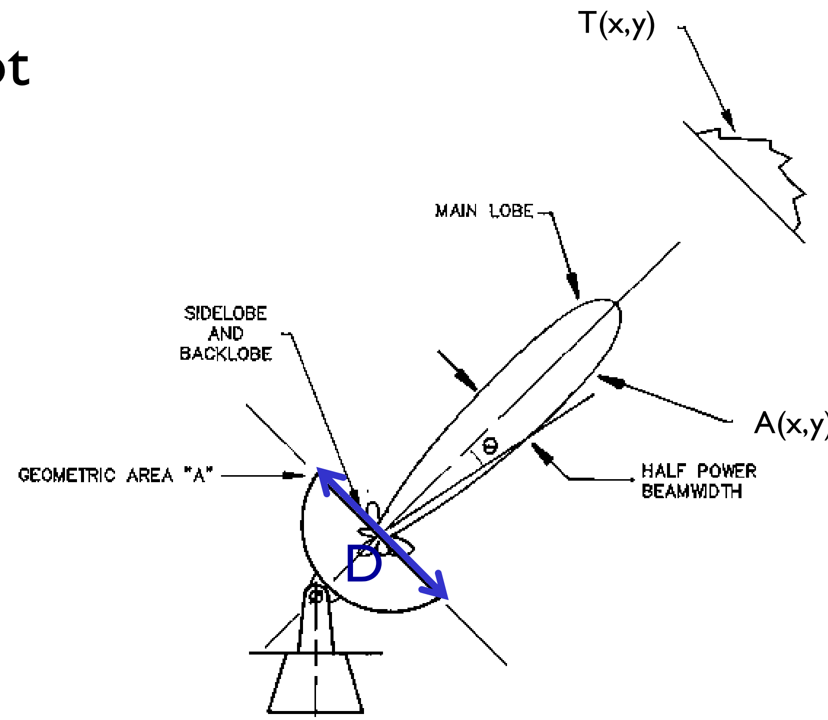
Some Details of the Dirty Image

- "Fourier transform"
  - Fast Fourier Transform (FFT) algorithm much faster than simple Fourier summation, $O(N\log N)$ for $2^N \times 2^N$ image
  - FFT requires data on a regularly spaced grid
  - aperture synthesis observations do not provide samples of V(u,v) on a regularly spaced grid, so…
- "gridding" is used to resample V(u,v) for FFT
  - customary to use a convolution method
    - visibilities are noisy samples of a smooth function
    - nearby visibilities are not independent
  - use special ("Spheroidal") functions with nice properties
    - fall off quickly in (u,v) plane: not too much smoothing
    - fall off quickly in image plane: avoid aliasing

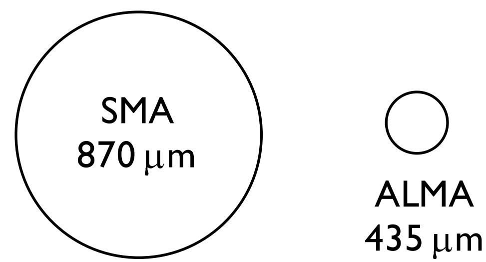
$$V^G(u,v) = V(u,v)B(u,v) \otimes G(u,v) \rightleftharpoons T^D(x,y)g(x,y)$$

Telescope Primary Beam

- telescope response A(x,y) is not uniform across the entire sky
  - main lobe fwhm ~ 1.2λ/D, "primary beam"
  - limits field of view
  - region beyond primary beam sometimes important (sidelobes, error beam)



- telescope beam modifies the sky brightness distribution
  - T(x,y) → T(x,y)A(x,y)
  - can correct with division by A(x,y) in the image plane
  - large sources require multiple telescope pointings = mosaicking



Pixel Size and Image Size

- pixel size
  - satisfy sampling theorem for longest baselines

$$\Delta x < \frac{1}{2u_{max}} \quad \Delta y < \frac{1}{2v_{max}}$$

  - in practice, 3 to 5 pixels across main lobe of dirty beam to aid deconvolution
  - e.g., SMA 870 μm, 500 m baselines → 600 kλ → pixels < 0.1 arcsec

- image size
  - natural choice: span the full extent of the primary beam A(x,y)
  - e.g., SMA 870 μm, 6 m telescope → 2x 35 arcsec
  - if there are bright sources in the sidelobes of A(x,y), then the FFT will alias them into the image → make a larger image (or equivalent)

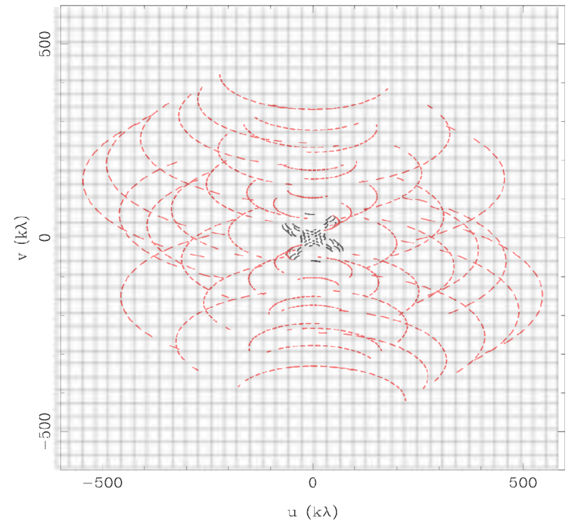
Dirty Beam Shape and Weighting

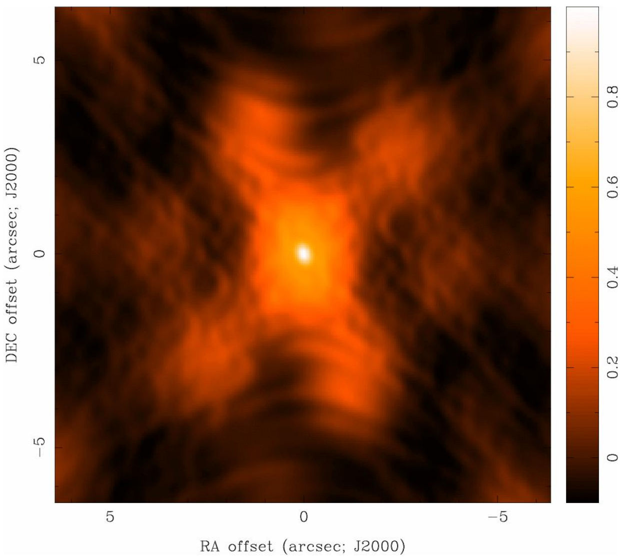
- introduce weighting function W(u,v)

$$b(x, y) = FT^{-1}\{W(u, v)B(u, v)\}$$

  - W(u,v) modifies sidelobes of dirty beam (W(u,v) also gridded for FFT)

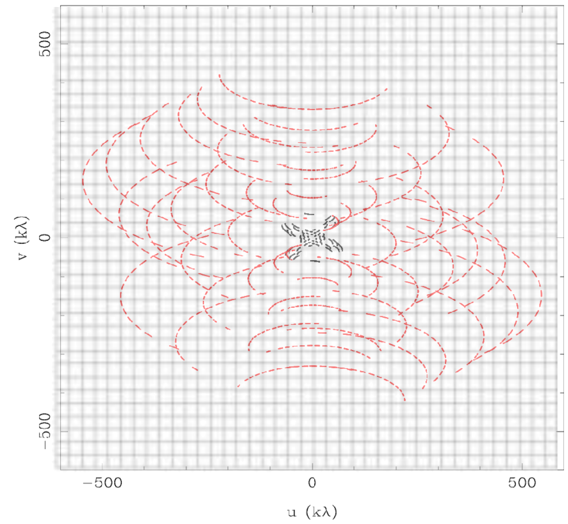
- "natural" weighting
  - W(u,v) = $1/\sigma^2$ in (u,v) cells, where σ^2 is the noise variance of the data, and W(u,v) = 0 everywhere else
  - maximizes the point source sensitivity (lowest rms in image)
  - generally gives more weight to short baselines (low spatial frequencies), so angular resolution is degraded

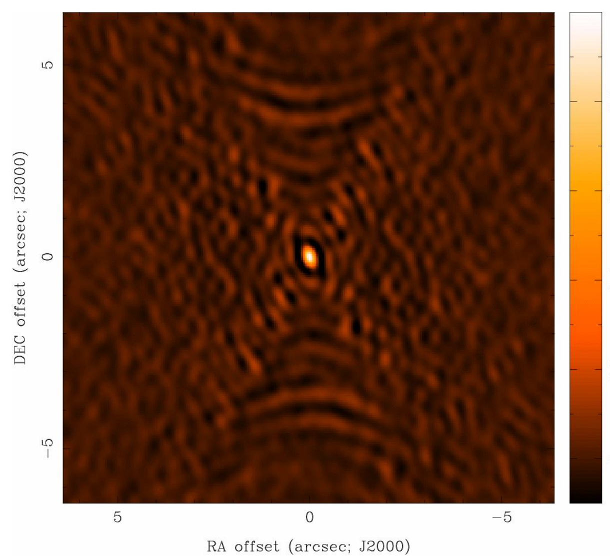




Dirty Beam Shape and Weighting

- "uniform" weighting
  - W(u.v) is inversely proportional to local density of (u,v) points, so sum of weights in a (u,v) cell is a constant (zero for the empty cells)
  - fills (u,v) plane more uniformly, so dirty beam sidelobes are lower
  - gives more weight to long baselines (high spatial frequencies), so angular resolution is enhanced
  - downweights data, so degrades point source sensitivity
  - can be trouble with sparse sampling: cells with few data points have same weight as cells with many data points





Dirty Beam Shape and Weighting

- "robust" (Briggs) weighting
  - variant of "uniform" that avoids giving too much weight to (u,v) cells with low natural weight
  - software implementations differ
  - example: $$W(u,v) = \frac{1}{\sqrt{1+S_N^2/S_{thresh}^2}}$$

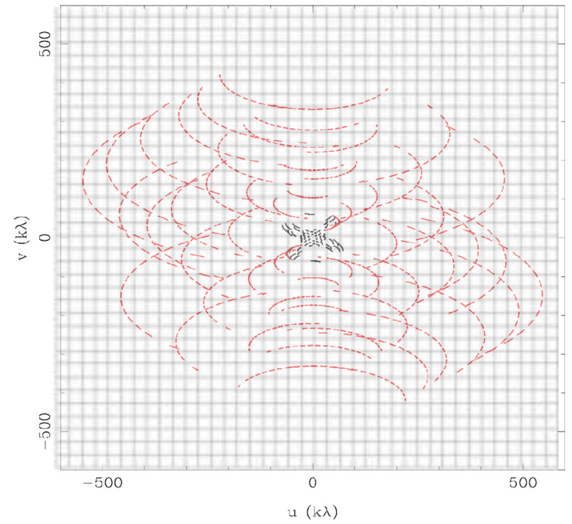
    S_N is natural weight of cell

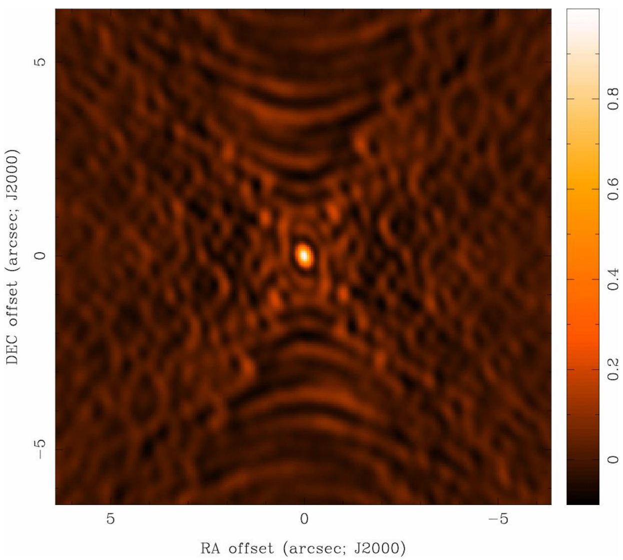
    S_{thresh} is a threshold

    high threshold $\rightarrow$ natural weighting

    low threshold $\rightarrow$ uniform weighting

  - an adjustable parameter that allows for continuous variation between the maximum point source sensitivity and the highest angular resolution



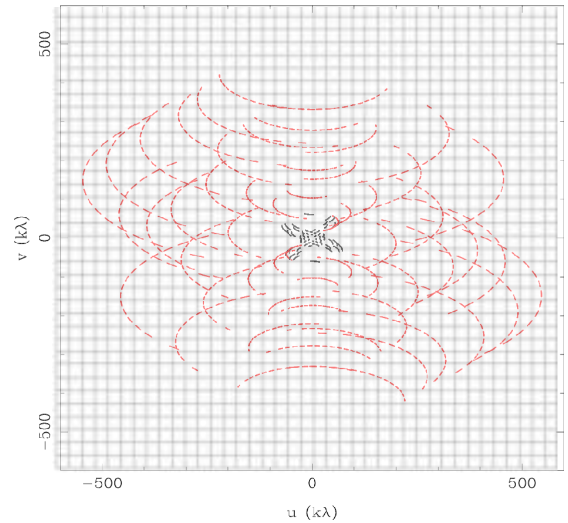


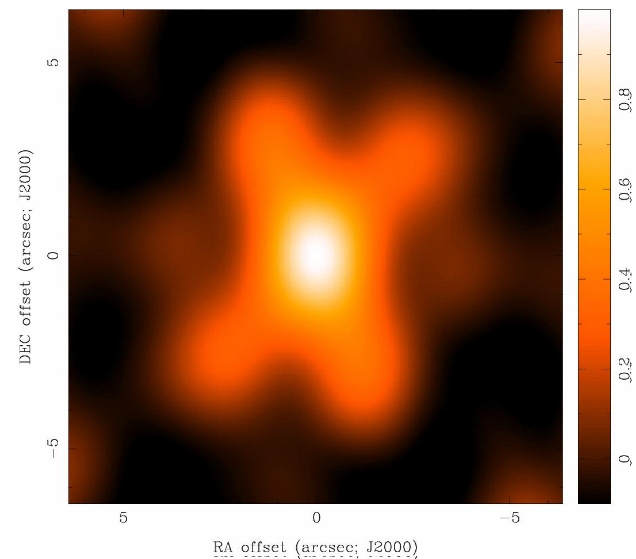
Dirty Beam Shape and Weighting

- "tapering"
  - apodize (u,v) sampling by a Guassian

$$W(u,v) = exp\left\{-\frac{(u^2+v^2)}{t^2}\right\}$$

    t = adjustable tapering parameter (usually in λ units)
  - like smoothing in the image plane (convolution by a Gaussian)
  - gives more weight to short baselines, degrades angular resolution
  - degrades point source sensitivity but can improve sensitivity to extended structure sampled by short baselines
  - limits to usefulness

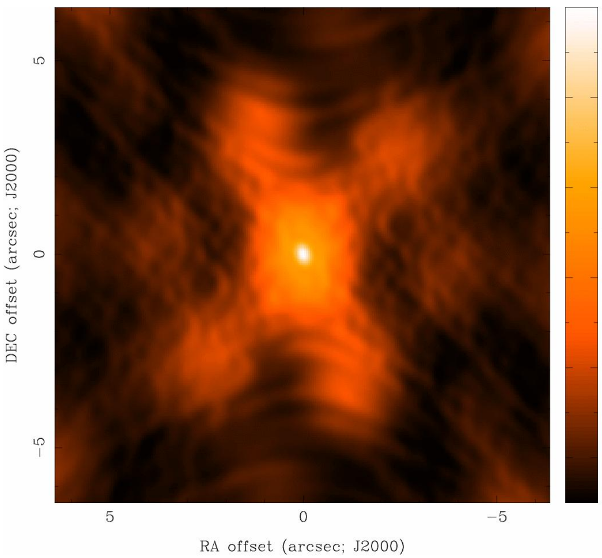


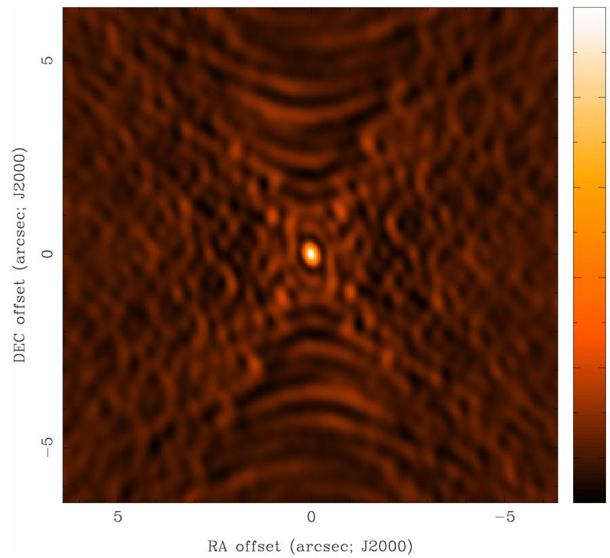


Weighting and Tapering: Noise

natural
0.77x0.62

σ=1.0



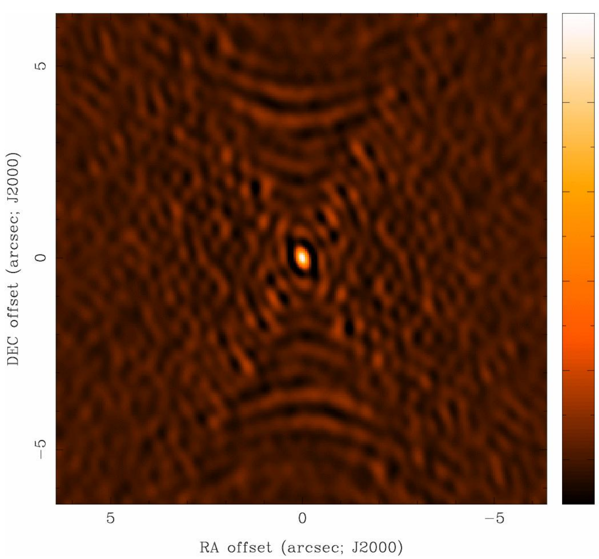


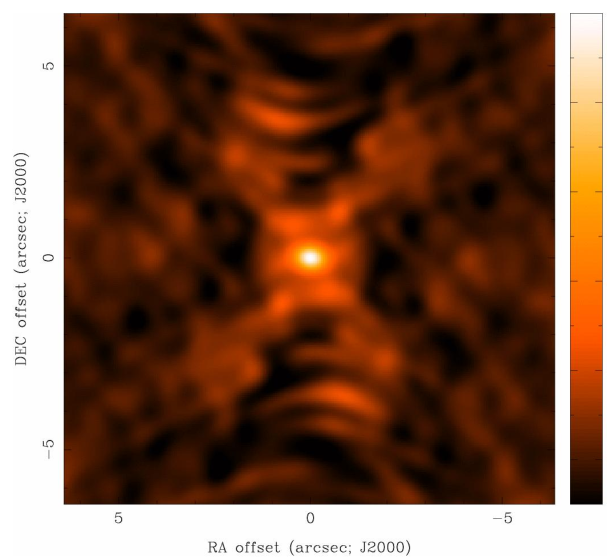
robust=0
0.41x0.36

σ=1.6

uniform
0.39x0.31

σ=3.7





robust=0
+ taper
0.77x0.62

σ=1.7

Weighting and Tapering: Summary

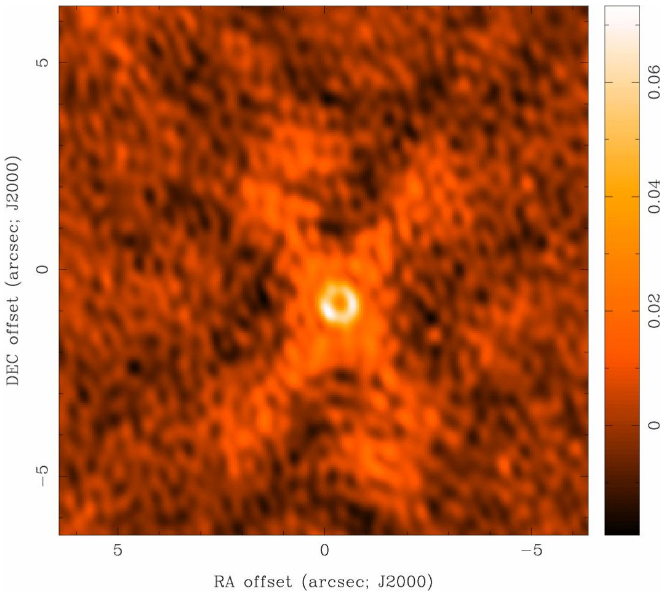
- imaging parameters provide a lot of freedom
- appropriate choice depends on science goals

	Robust/Uniform	Natural	Taper
Resolution	higher	medium	lower
Sidelobes	lower	higher	depends
Point Source Sensitivity	lower	maximum	lower
Extended Source Sensitivity	lower	medium	higher

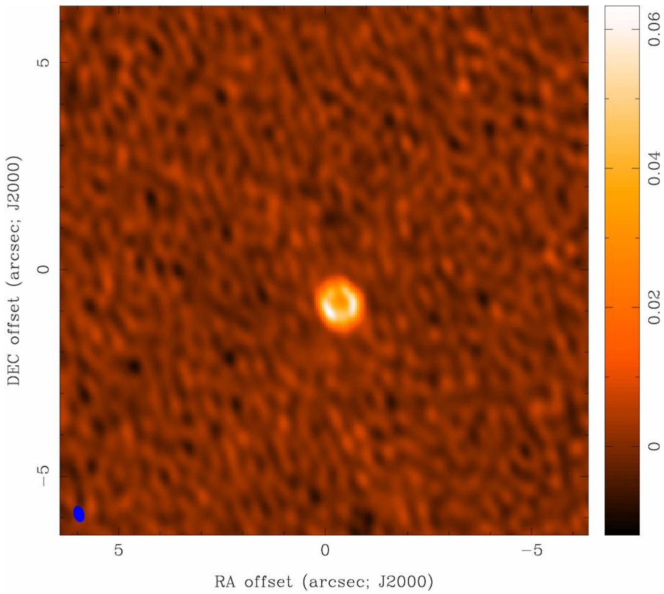
Deconvolution: Beyond the Dirty Image

- calibration and Fourier transform go from the V(u,v) samples to the best possible dirty image, $T^D(x,y)$
- in general, science requires to deconvolve b(x,y) from $T^D(x,y)$ to recover (a model of) T(x,y) for analysis
- information is missing, so be careful (there's noise, too)

dirty image



"CLEAN" image



Deconvolution Philosophy

- to keep you awake at night
  - ∃ an infinite number of T(x,y) compatible with sampled V(u,v), i.e. "invisible" distributions R(x,y) where $b(x,y) \otimes R(x,y) = 0$
    - no data beyond u_{max},v_{max} → unresolved structure
    - no data within u_{min},v_{min} → limit on largest size scale
    - holes in between → sidelobes
  - noise → undetected/corrupted structure in T(x,y)
  - no unique prescription for extracting optimum estimate of T(x,y)

- deconvolution
  - uses non-linear techniques effectively to interpolate/extrapolate samples of V(u,v) into unsampled regions of the (u,v) plane
  - aims to find a sensible model of T(x,y) compatible with data
  - requires *a priori* assumptions about T(x,y) to pick plausible "invisible" distributions to fill unmeasured parts of the Fourier plane

Deconvolution Algorithms

- Clean: dominant deconvolution algorithm in radio astronomy
  - *a priori* assumption: T(x,y) is a collection of point sources
  - fit and subtract the synthesized beam iteratively
  - original version by Högbom (1974) purely image based
  - variants developed for higher computational efficiency, model visibility subtraction, to deal with extended structure, … (Clark, Cotton-Schwab, Steer-Dewdney-Ito, etc.)

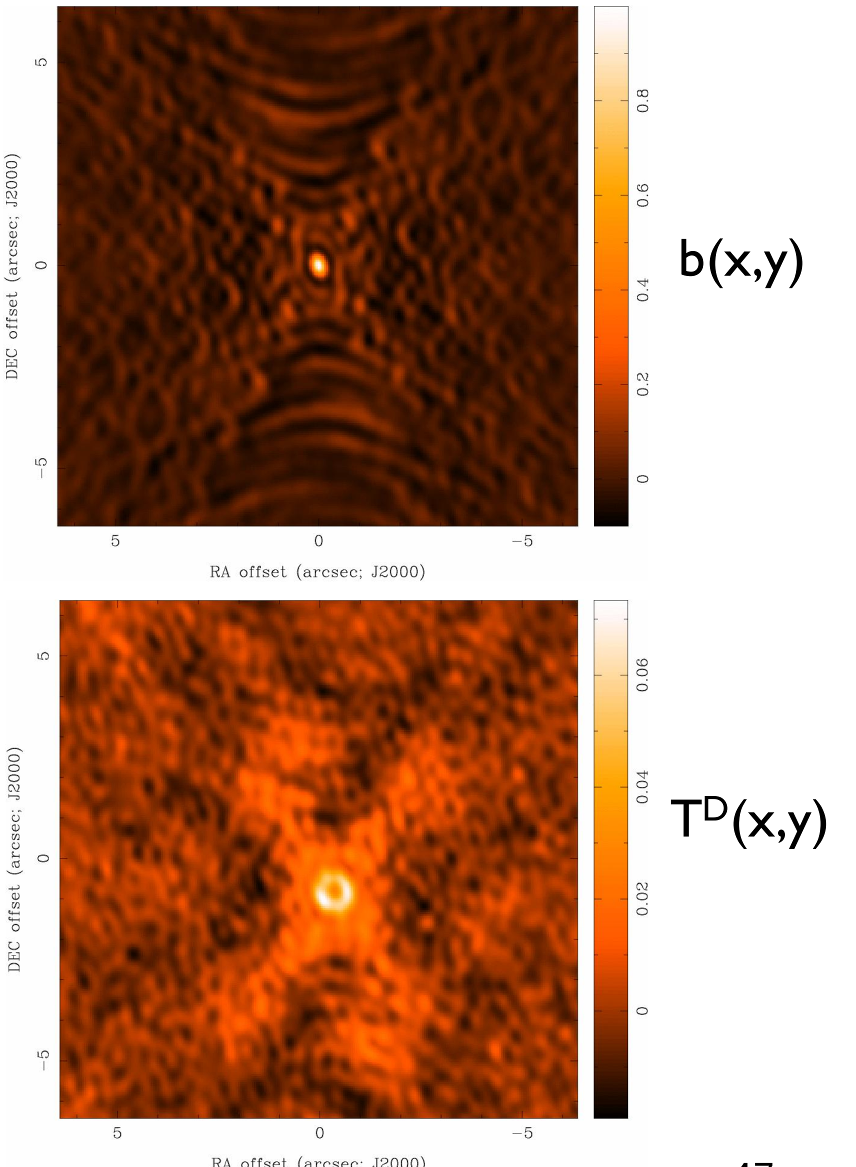
- Maximum Entropy: used in some situations
  - *a priori* assumption: T(x,y) is smooth and positive
  - define "smoothness" via a mathematical expression for entropy, e.g. Gull and Skilling 1983, find smoothest image consistent with data
  - vast literature about the deep meaning of entropy as information content

- an active research area, e.g. compressive sensing methods

Basic Clean Algorithm

1. Initialize
   - a *residual* map to the dirty map
   - a *Clean Component* list to empty

2. identify highest peak in the *residual* map as a point source
3. subtract a fraction of this peak from the *residual* map using a scaled (loop gain g) dirty beam b(x,y)
4. add this point source location and amplitude to *Clean Component* list
5. goto step 2 (an iteration) unless stopping criterion reached



Basic Clean Algorithm (cont)

- stopping criteria
  - *residual map* max < multiple of rms (when noise limited)
  - *residual map* max < fraction of dirty map max (dynamic range limited)
  - max number of *Clean Components* reached (no justification)
- loop gain
  - good results for g ~ 0.1 to 0.3
  - lower values can work better for smoother emission, g ~ 0.05
- easy to include *a priori* information about where in image to search for *Clean Components* (using "boxes" or "windows")
  - very useful but potentially dangerous
- Schwarz (1978): in the absence of noise, Clean algorithm is equivalent to a least squares fit of sinusoids to visibilities

Basic Clean Algorithm (cont)

- last step: make the "restored" image
  - take *residual map*, which consists of noise and weak source structure below the Clean cutoff limit
  - add point source *Clean components* convolved with an elliptical Gaussian fit to the main lobe of the dirty beam ("Clean beam") to avoid super-resolution of point source component model
  - resulting image is an estimate of the true sky brightness
  - units are (mostly) Jy per Clean beam area = intensity, or brightness temperature
  - there is information from baselines that sample beyond the Clean beam FWHM, so modest super-resolution may be OK
  - the restored image does not actually fit the observed visibilities

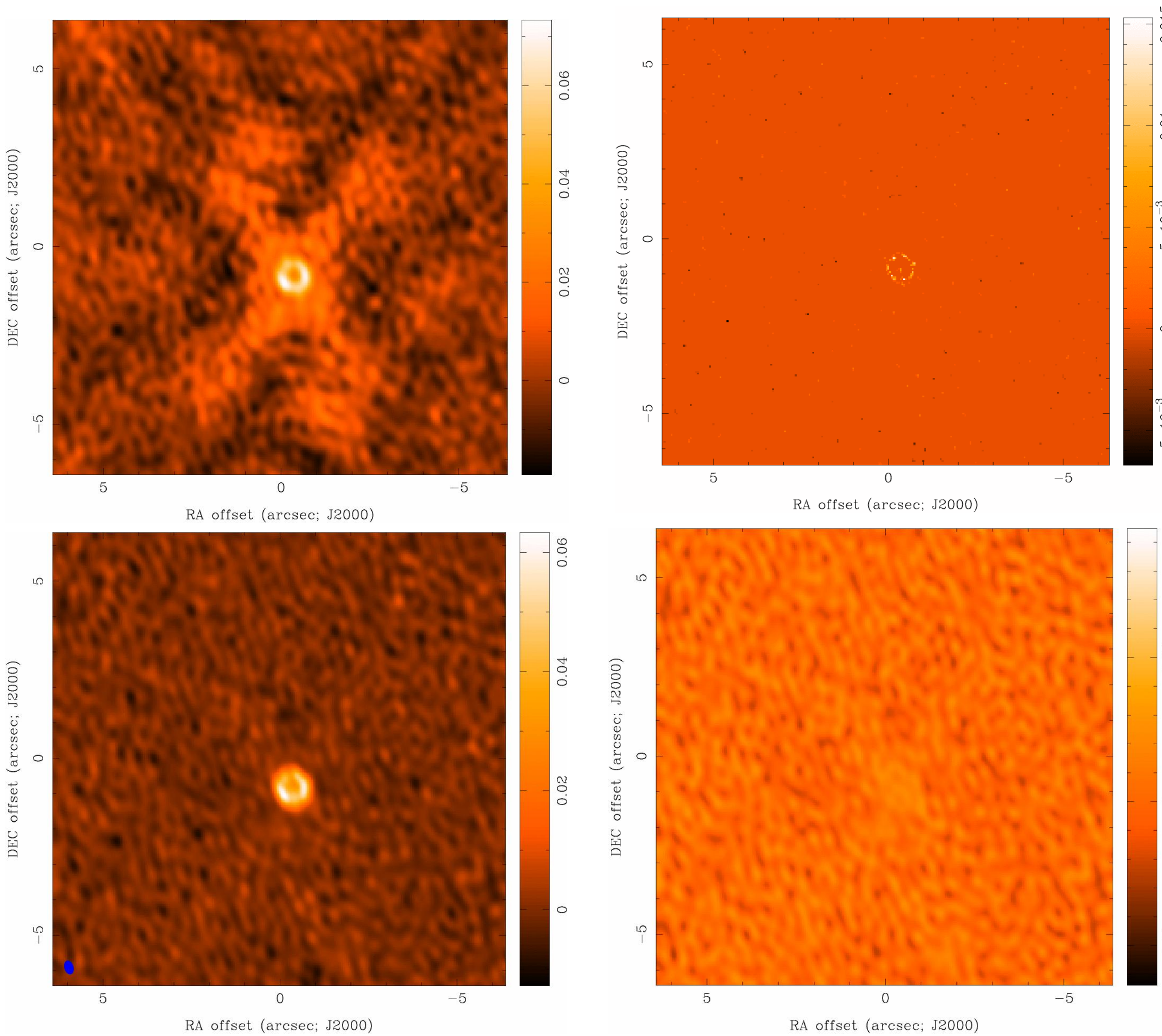
Clean Example

$T^D(x,y)$

CC model

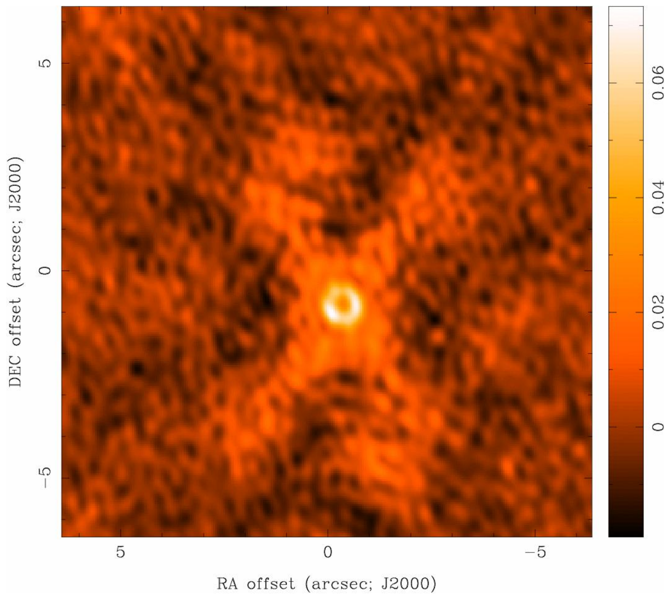
restored image

residual map

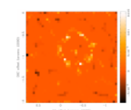


Clean with a “box”

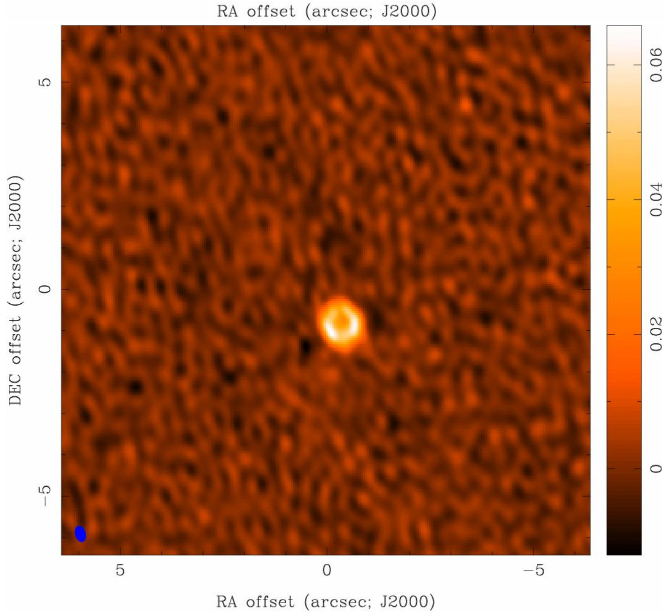
$T^D(x,y)$

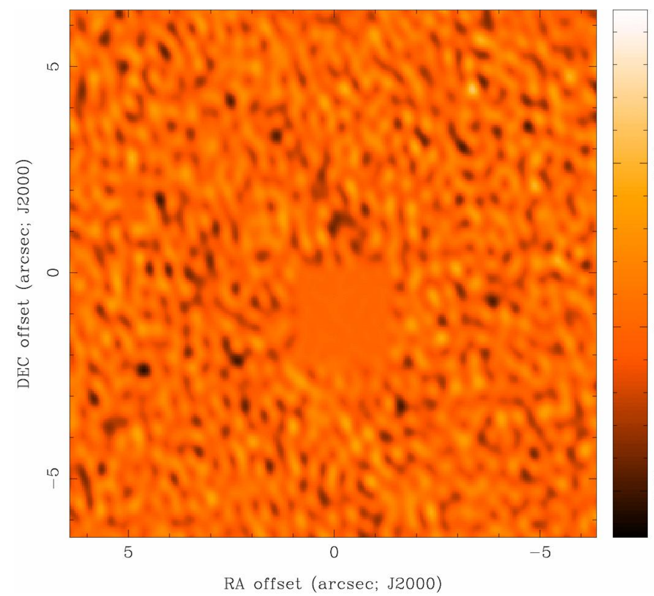


CC model



restored image

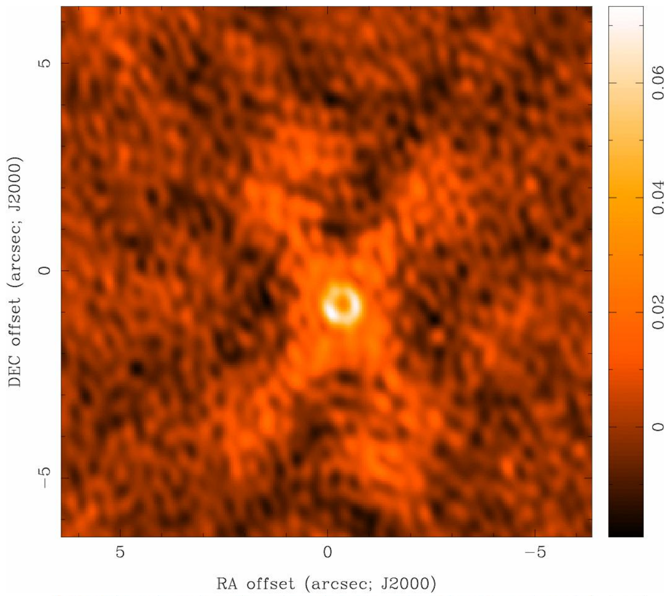




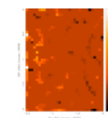
residual map

Clean with poor choice of “box”

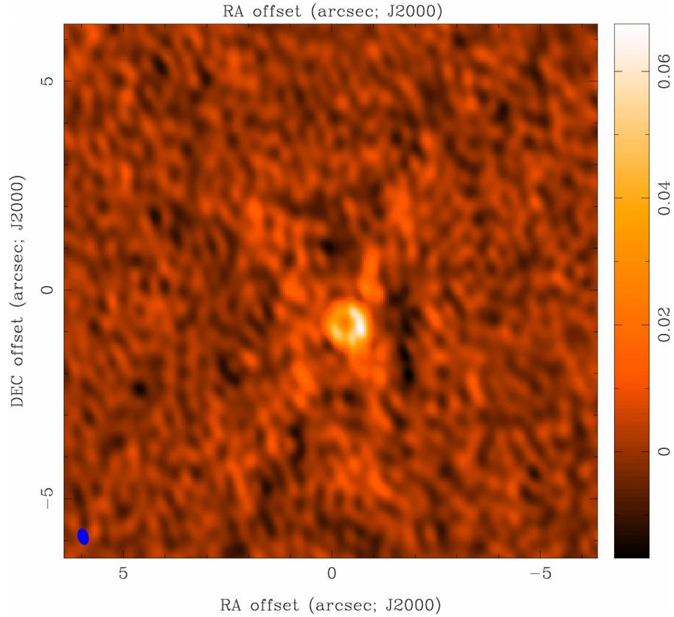
$T^D(x,y)$

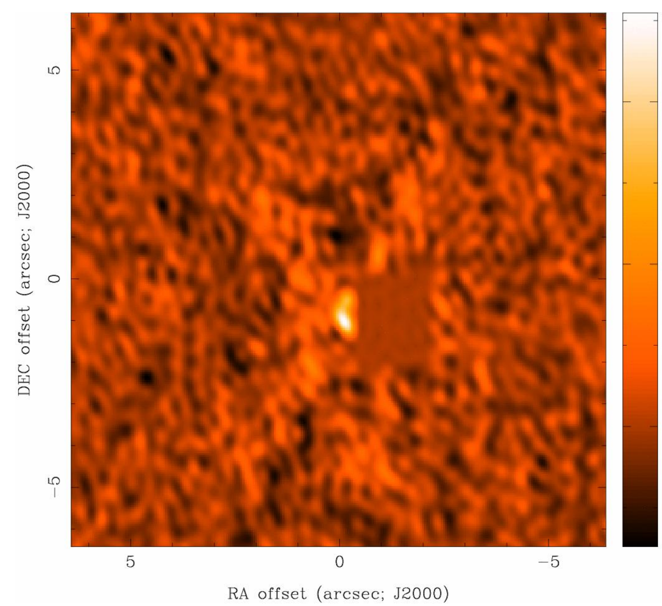


CC model



restored image





residual map

Maximum Entropy Algorithm

- Maximize a measure of smoothness (the entropy)

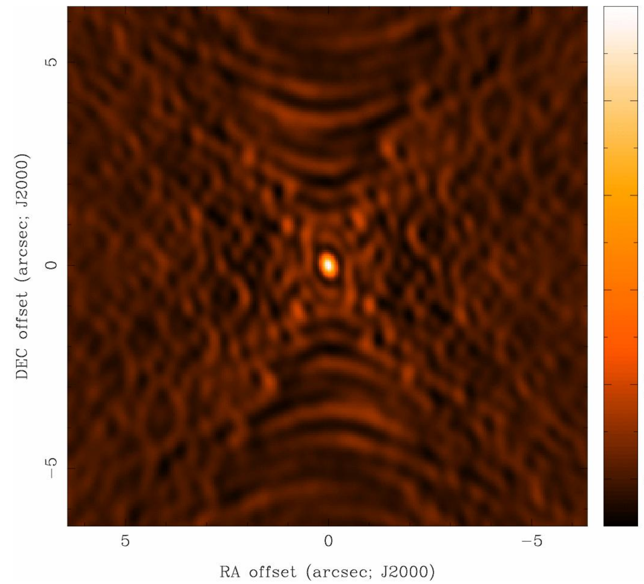
$$H = -\sum_k T_k \log\left(\frac{T_k}{M_k}\right)$$

subject to the constraints

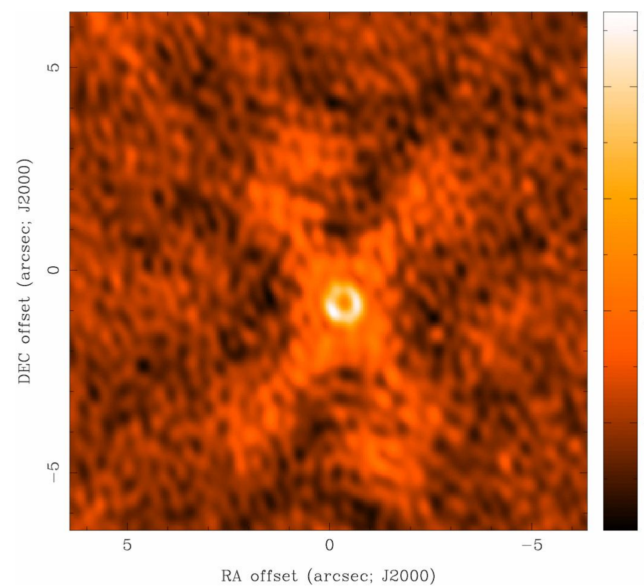
$$\chi^2 = \sum_k \frac{|V(u_k,v_k) - \mathrm{FT}\{T\}|^2}{\sigma_k^2}$$

$$F = \sum_k T_k$$

- M is the “default image”
- fast (NlogN) non-linear optimization solver due to Cornwell and Evans (1983)
- optional: convolve model with elliptical Gaussian fit to beam and add residual map to make image



$b(x,y)$



$T^D(x,y)$

Maximum Entropy Algorithm (cont)

- easy to include *a priori* information with default image
  - flat default best only if nothing known
- straightforward to generalize χ^2 to combine observations from different telescopes and obtain an optimal image
- many measures of "entropy" available
  - replace log with cosh $\rightarrow$ "emptiness" (does not enforce positivity)
- works well for smooth, extended emission
- super-resolution regulated by signal-to-noise

- less robust and harder to drive than Clean
- can have trouble with point source sidelobes (could remove those first with Clean)

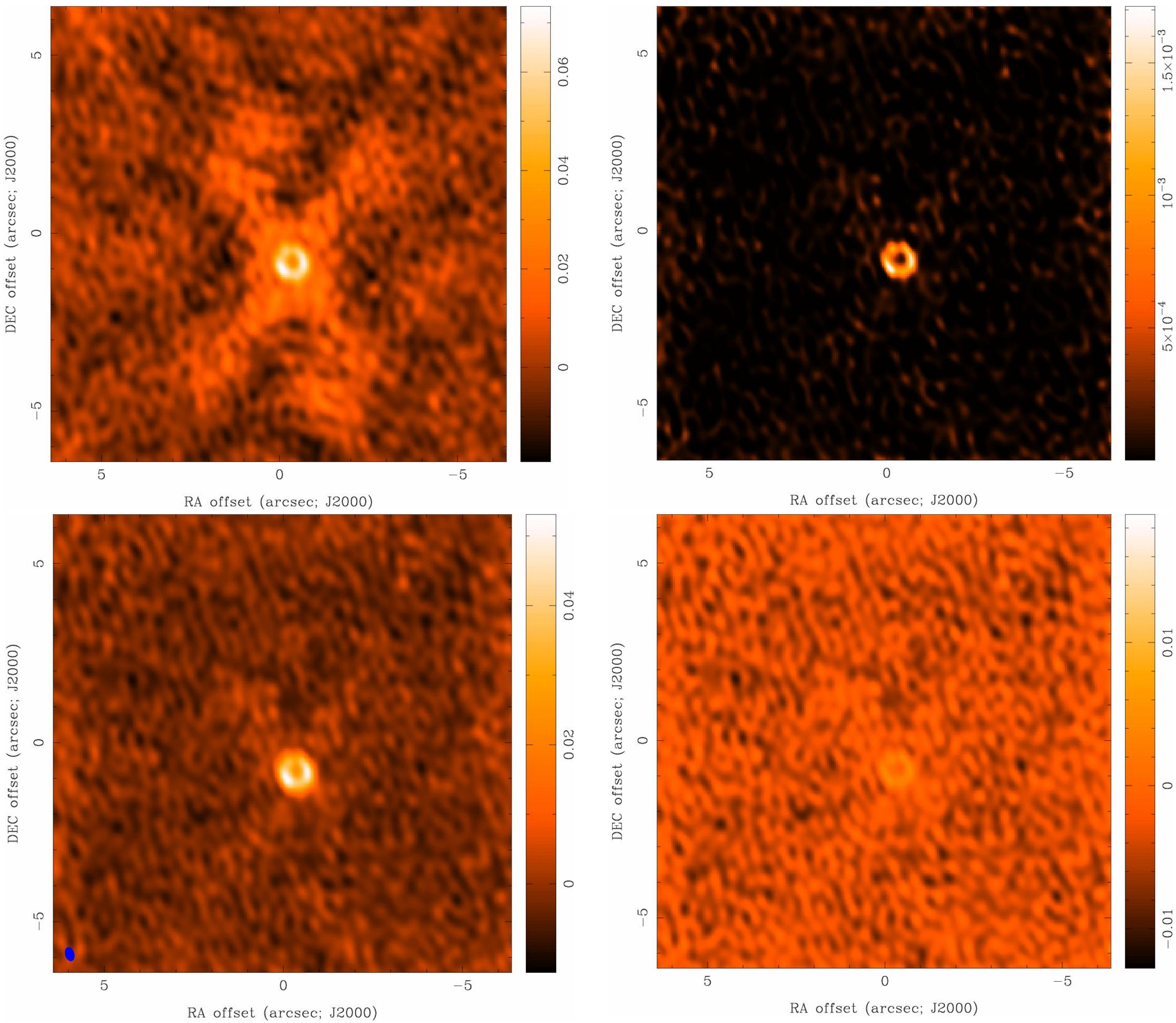
Maximum Entropy Example

$T^D(x,y)$

maxen model

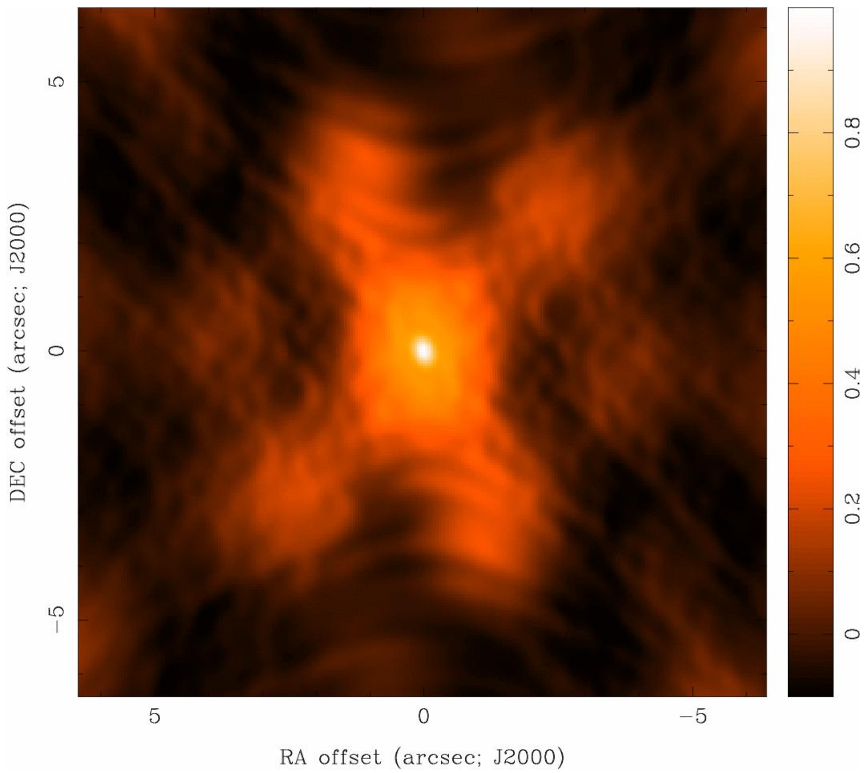
restored image

residual map

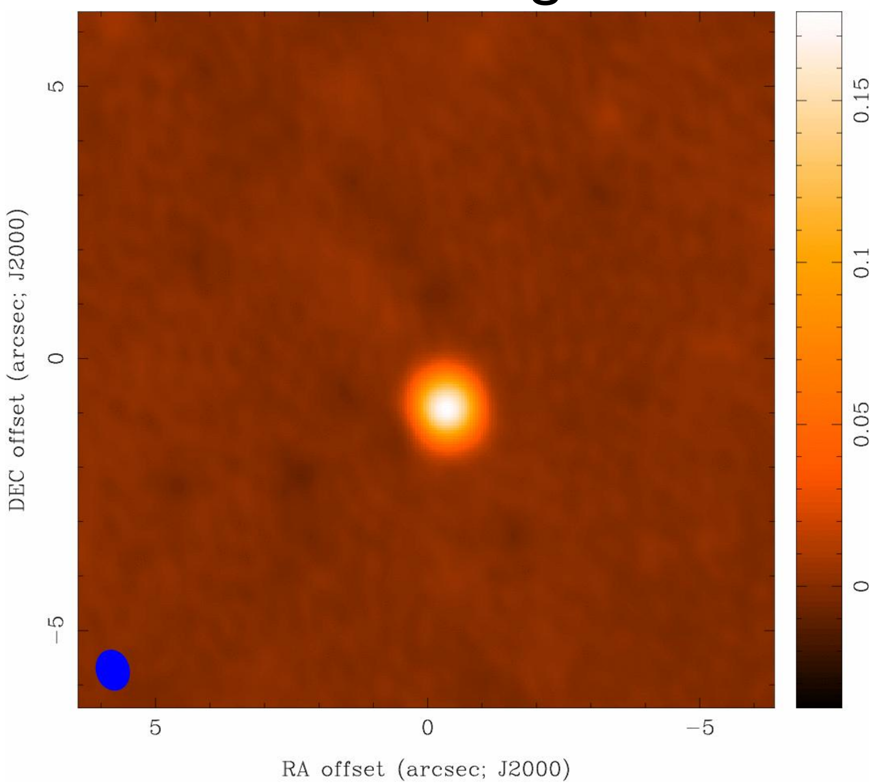


Summary of Imaging Results

Natural Weight Beam

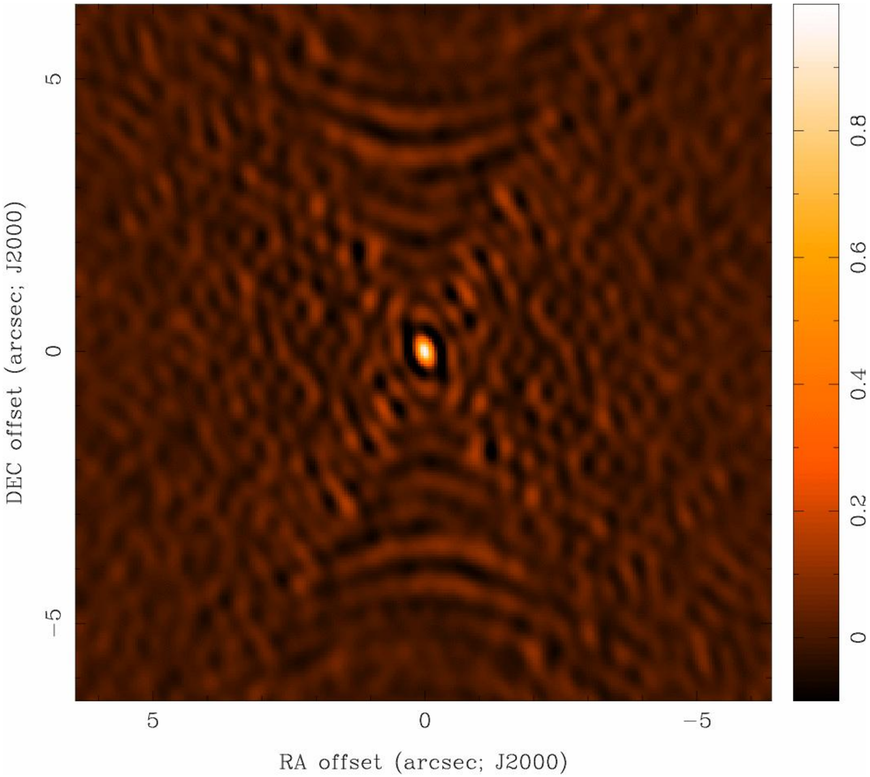


Clean image

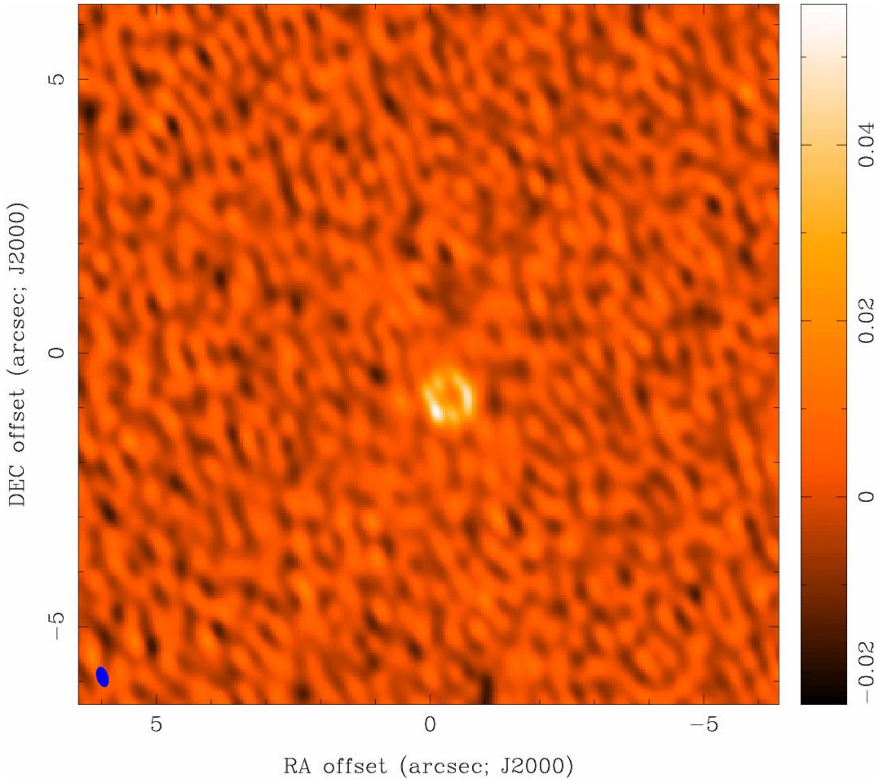


Summary of Imaging Results

Uniform Weight Beam

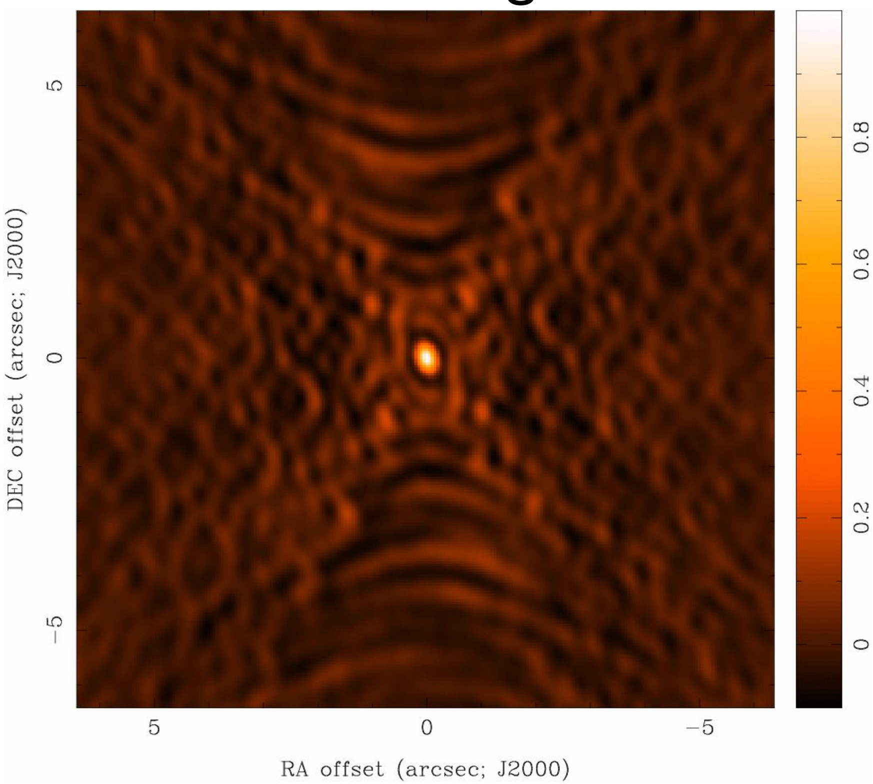


Clean image

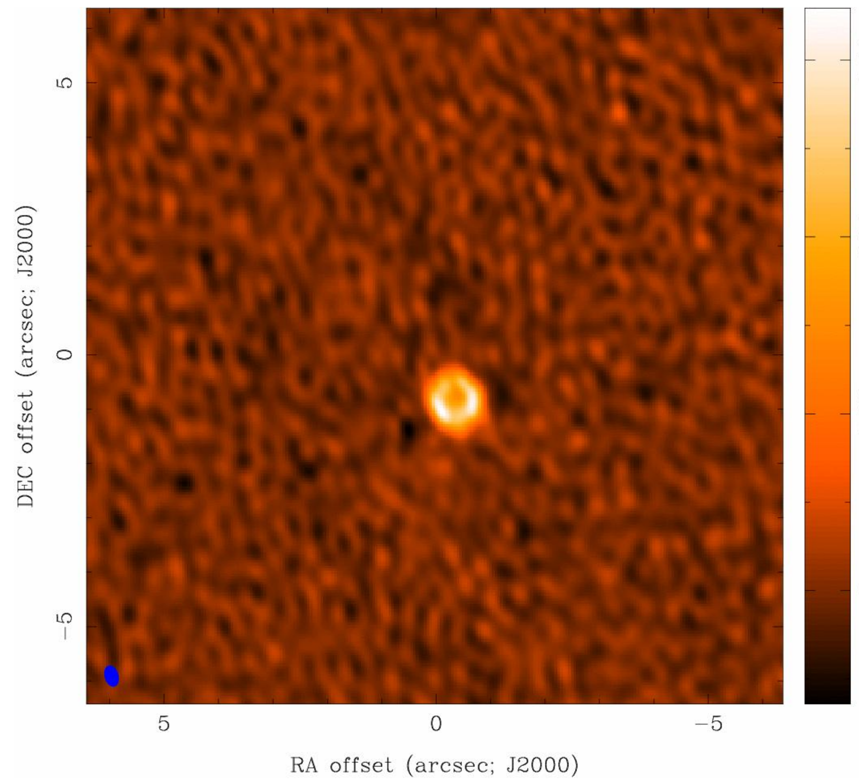


Summary of Imaging Results

Robust=0 Weight Beam

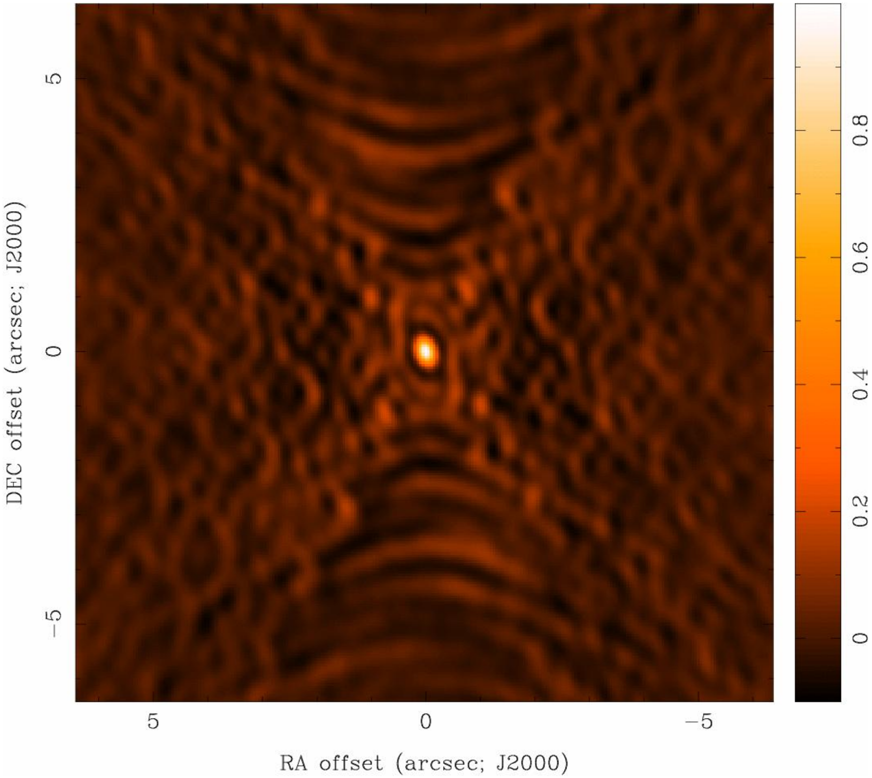


Clean image

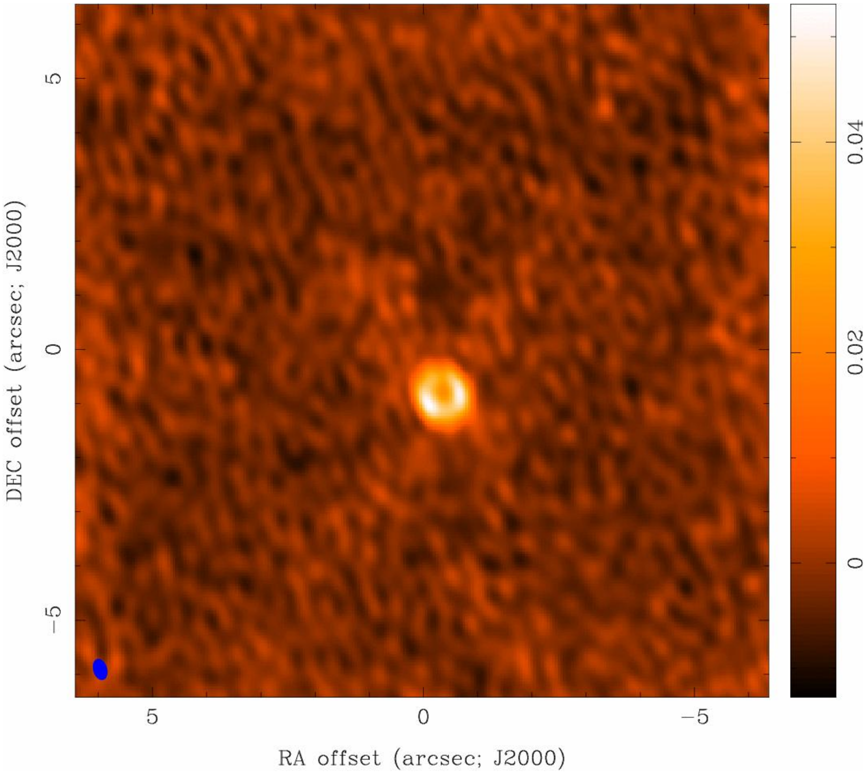


Summary of Imaging Results

Robust=0 Weight Beam

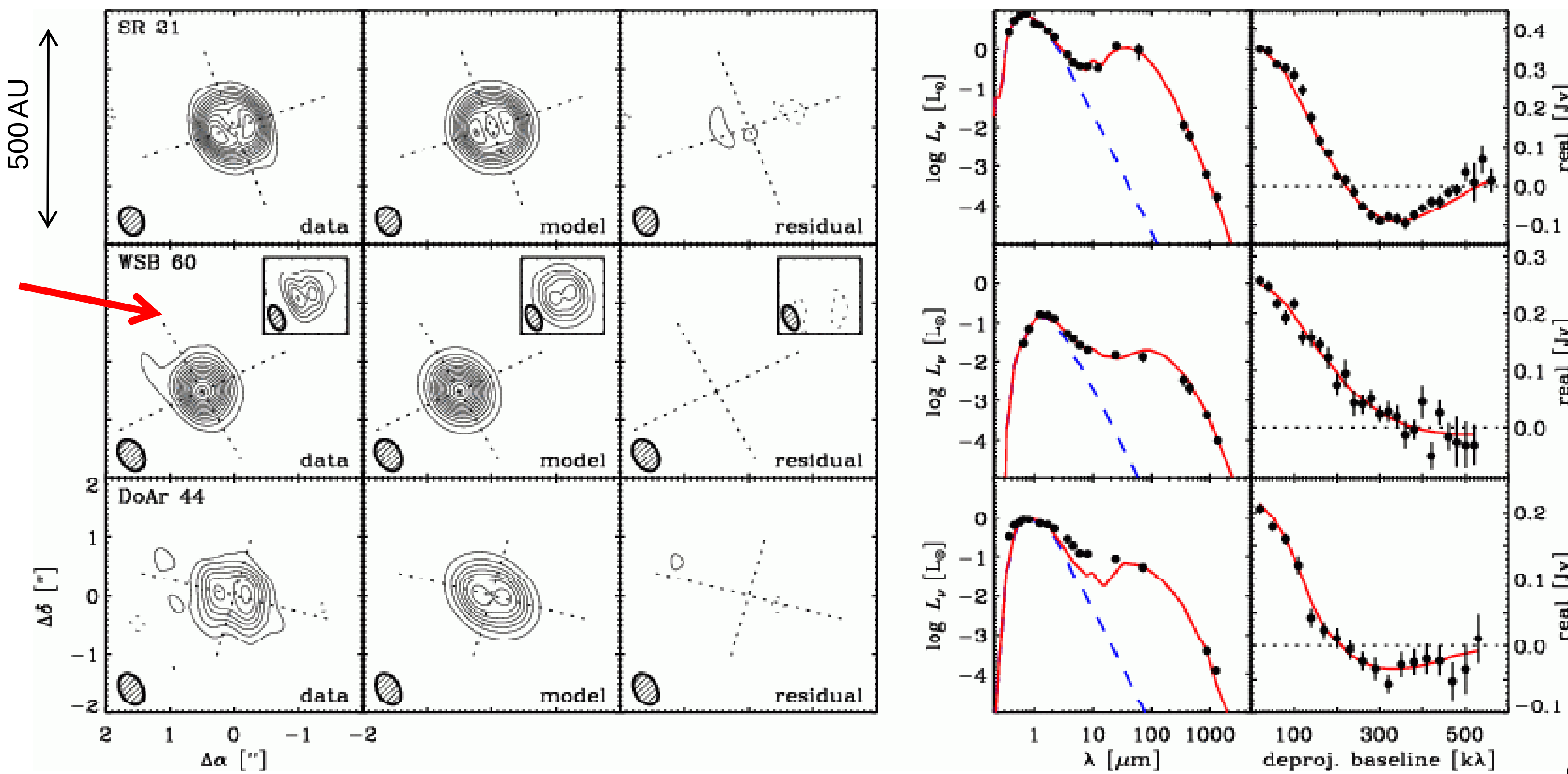


Maximum Entropy image



Tune Resolution/Sensitivity to suit Science

- e.g. SMA 870 mm images of protoplanetary disks with resolved inner holes (Andrews, Wilner et al. 2009, ApJ, 700, 1502)



Noise in Images

- photometry of extended sources requires caution
  - Clean does not conserve flux (extrapolates)
  - extended structure can be missed, attenuated, distorted

- be very careful with low signal-to-noise images
  - if source position known, 3σ is OK for point source detection
  - if position unknown, then 5σ required (and flux is biased up)
  - if < 6σ, then cannot measure the source size
    (require ~3σ difference between "long" and "short" baselines)
  - spectral line emission may have unknown position, velocity, width

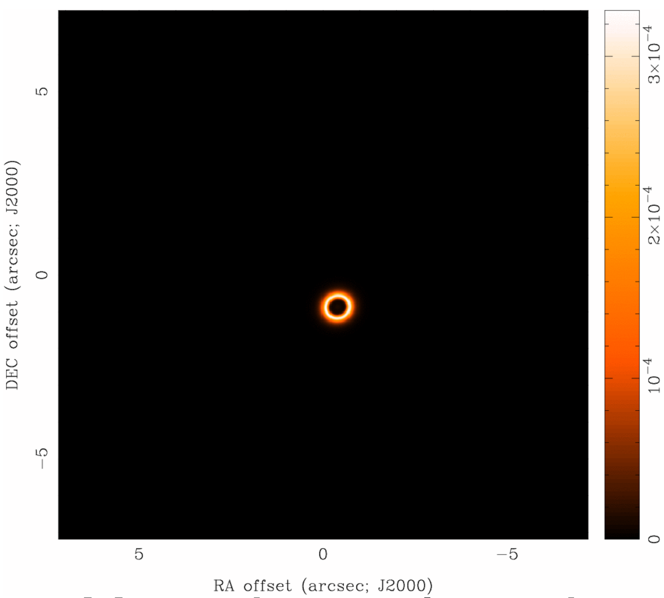
Scale Sensitive Deconvolution Algorithms

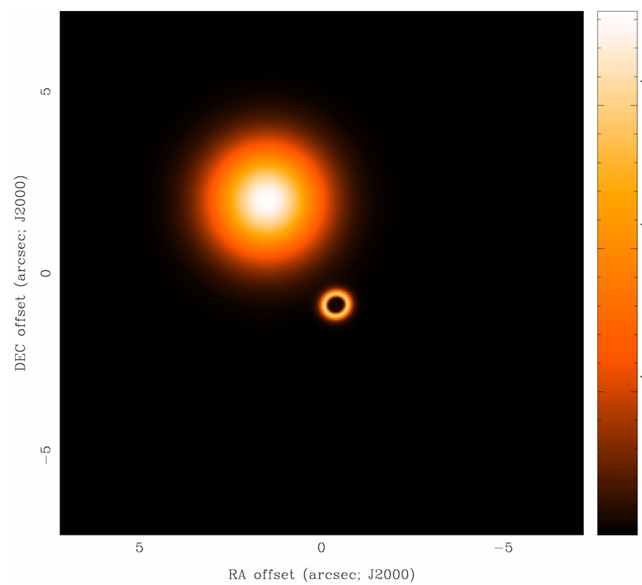
- basic Clean and Maximum Entropy are scale-free and treat each pixel as an independent degree of freedom
  - they have no concept of source size
- adjacent pixels in an image are not independent
  - resolution limit
  - intrinsic source size, e.g. a Gaussian source covering 1000 pixels can be characterized by only 5 parameters, not 1000
- scale sensitive algorithms try to employ fewer degrees of freedom to model plausible sky brightness distributions
  - MS-Clean (Multi-Scale Clean)
  - Adaptive Scale Pixel (Asp) Clean

"Invisible" Large Scale Structure

- missing short spacings (= large scale emission) can be problematic
  - to estimate? simulate observations, or check simple expressions for a Gaussian and a disk (appendix of Wilner & Welch 1994, ApJ, 427, 898)

- do the visibilities in our example discriminate between these two models of the sky brightness distribution T(x,y)?

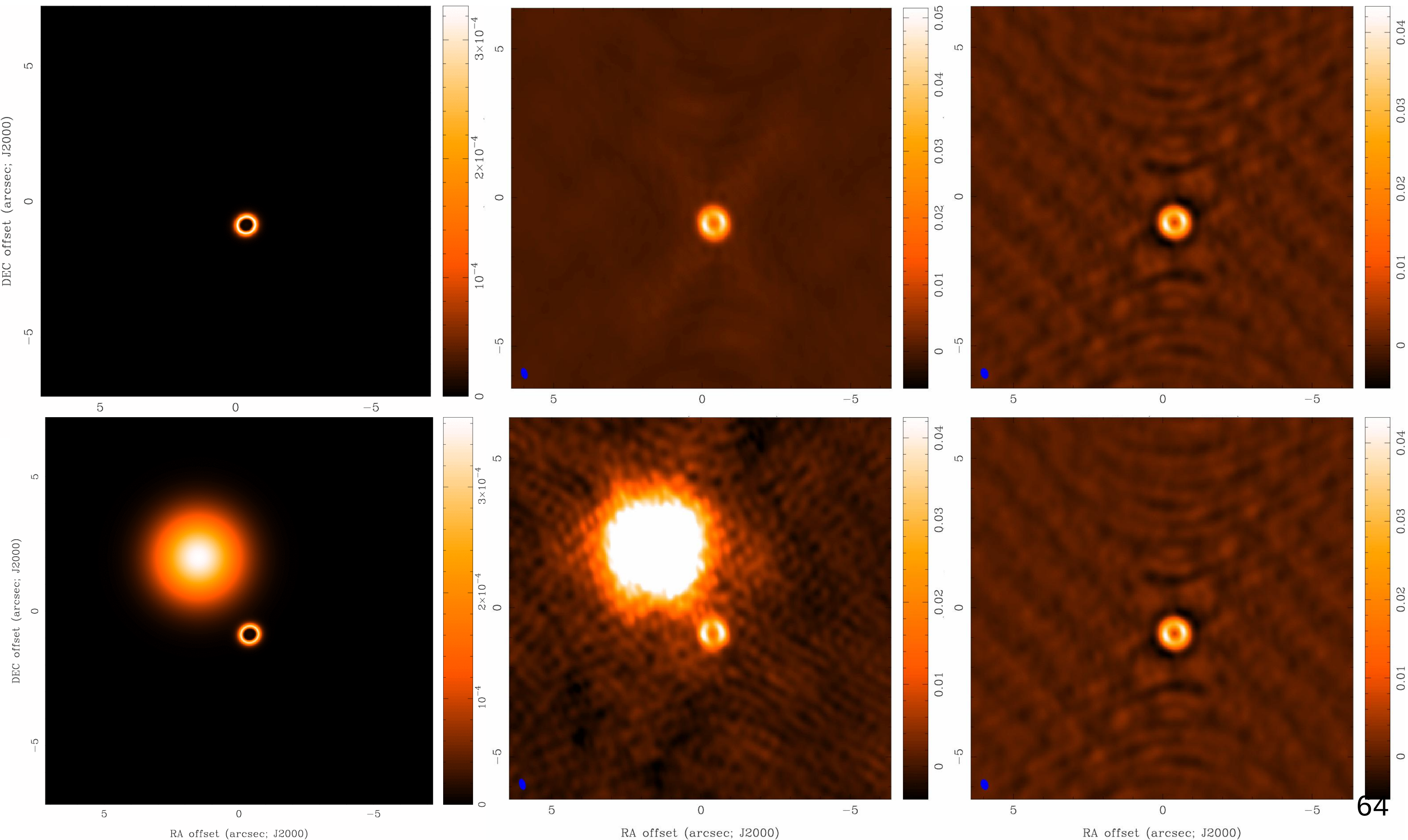




Yes… but only on baselines shorter than ~100 kλ.

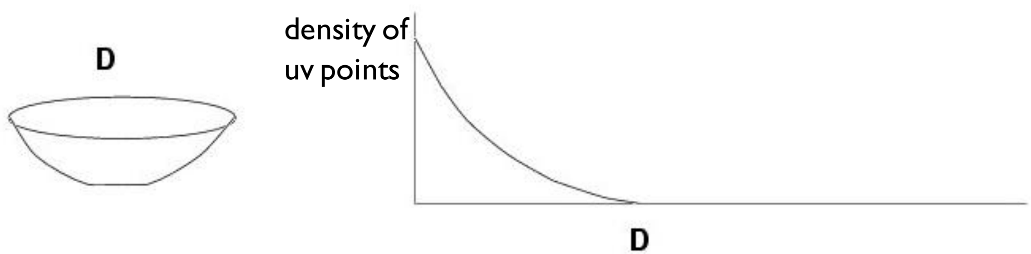
Missing Short Spacings: Demonstration

T(x,y) | Clean image | >100 kλ Clean image



Techniques to Obtain Short Spacings (1)

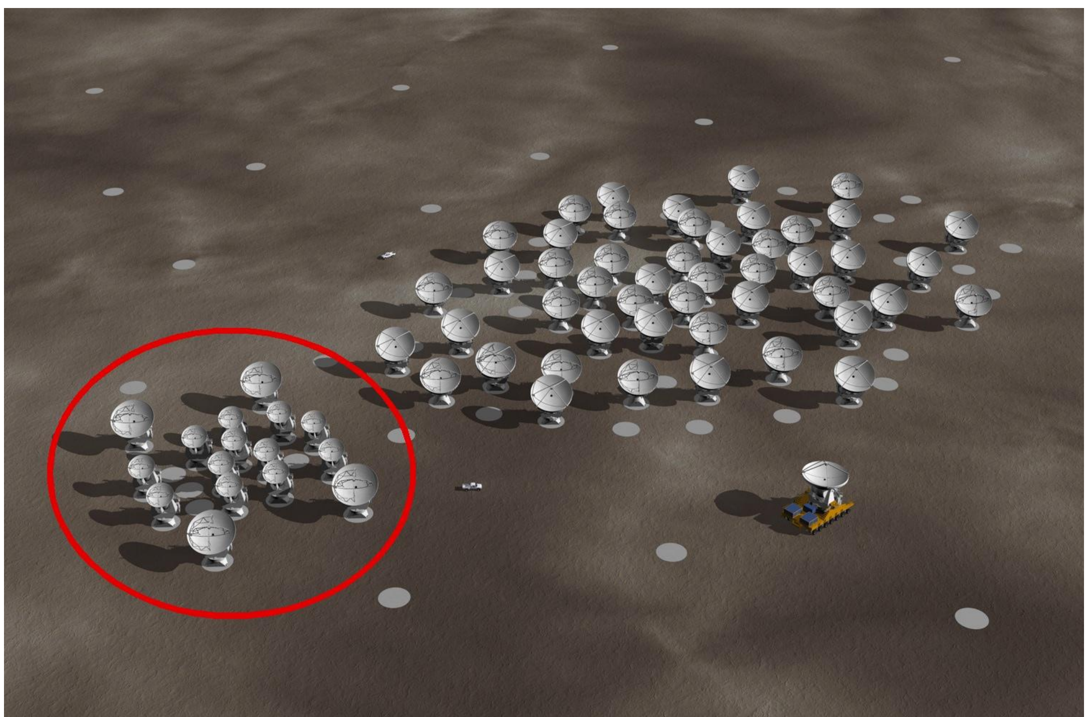
- a large single dish telescope
  - examples: JVLA & GBT, IRAM PdbI & 30 m telescope,
    SMA & JCMT
  - scan single dish across the sky to make an image
  - all Fourier components from 0 to D sampled, where D is the telescope diameter (weighting depends on illumination)



  - Fourier transform single dish map = $T(x,y) \otimes A(x,y)$,
    then divide by $a(x,y) = FT\{A(x,y)\}$ to estimate $V(u,v)$
  - choose D large enough to overlap interferometer samples of $V(u,v)$ and avoid using data where $a(x,y)$ becomes small

Techniques to Obtain Short Spacings (II)

- a separate array of smaller telescopes
  - example: ALMA main array & ACA
  - use smaller telescopes to observe short baselines not accessible to larger telescopes
  - use the larger telescopes as single dishes to make images with Fourier components not accessible to smaller telescopes



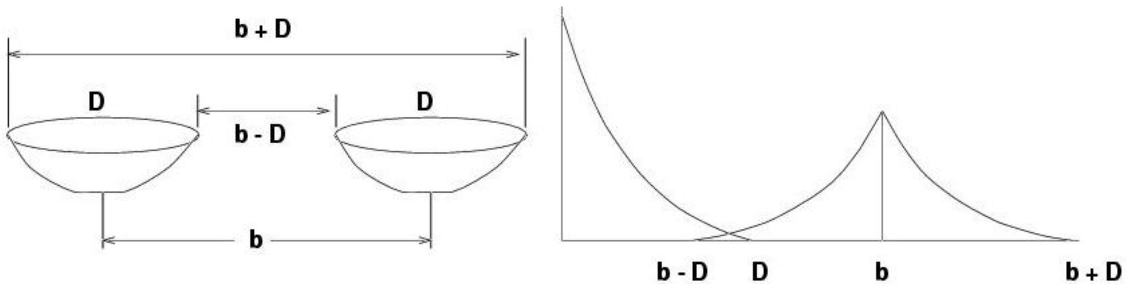
ALMA with ACA

50 x 12 m: 12 m to 14+ km

+12 x 7 m: fills 7 m to 12 m
+ 4 x 12 m: fills 0 m to 7 m

Techniques to Obtain Short Spacings (III)

- mosaic with a homogeneous array
  - recover a range of spatial frequencies around the nominal baseline b using knowledge of A(x,y) (Ekers and Rots 1979), and get shortest baselines from single dish maps



  - V(u,v) is linear combination of baselines from b-D to b+D
  - depends on pointing direction (x_o,y_o) as well as (u,v)

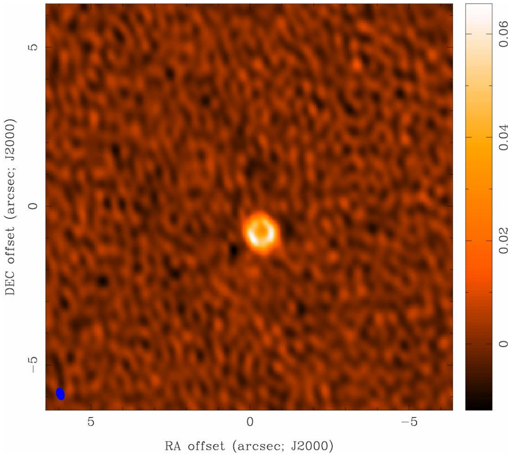
$$V(u, v; x_o, y_o) = \int\int T(x, y) A(x - x_o, y - y_o) e^{2\pi i(ux+vy)} dx dy$$

  - Fourier transform with respect to pointing direction (x_o,y_o)

$$V(u - u_o, v - v_o) = \frac{\int\int V(u,v;x_o,y_o) e^{2\pi i(u_o x_o + v_o y_o)} dx_o dy_o}{a(u_o,v_o)}$$

Measures of Image Quality

- "dynamic range"
  - ratio of peak brightness to rms noise in a region void of emission (common in radio astronomy)
  - an easy to calculate lower limit to the error in brightness in a non-empty region



- "fidelity"
  - difference between any produced image and the correct image
  - convenient measure of how accurately it is possible to make an image that reproduces the brightness distribution on the sky
  - need a priori knowledge of the correct image to calculate
  - fidelity image = input model / difference

    $= \text{model} \otimes \text{beam} \;/\; \text{abs}(\,\text{model} \otimes \text{beam} - \text{reconstruction}\,)$

    = inverse of the relative error
  - in practice, lowest values of difference need to be truncated

Self Calibration

- *a priori* calibration is not perfect
  - interpolated from different time, different sky direction from source
- basic idea of self calibration is to correct for antenna based phase and amplitude errors *together with imaging*
- works because
  - at each time, measure N complex gains and N(N-1)/2 visibilities
  - source structure can be represented by small number of parameters
  - highly overconstrained problem if N large and source simple
- in practice: an iterative, non-linear relaxation process
  - assume initial model → solve for time dependent gains → form new sky model from corrected data using e.g. Clean → solve for new gains…
  - requires sufficient signal-to-noise at each solution interval
- loses absolute phase and therefore position information
- dangerous with small N, complex source, low signal-to-noise

Concluding Remarks

- interferometry samples visibilities that are related to a sky brightness image by the Fourier transform
- deconvolution attempts to correct for incomplete sampling

- remember… there are usually an infinite number of images compatible with the sampled visibilities
- missing (or corrupted) visibilities affect the entire image
- astronomers must use judgement in the process of imaging and devonvolution
- it's fun and worth the trouble $\rightarrow$ high angular resolution!

- many, many issues not covered: see the References and upcoming talks at this workshop

End