

# Radio Polarimetry

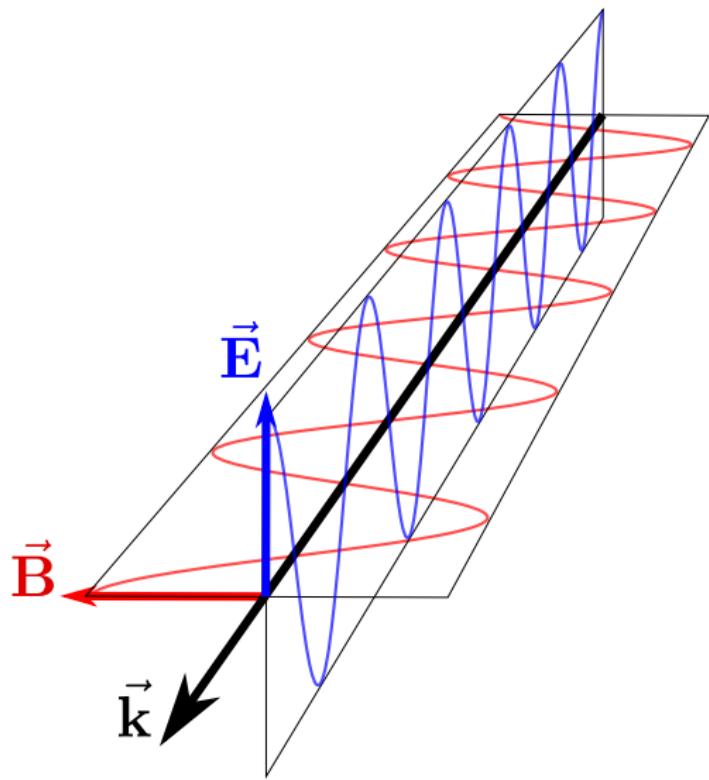
Michiel Brentjens

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ASTRON, Dwingeloo, The Netherlands

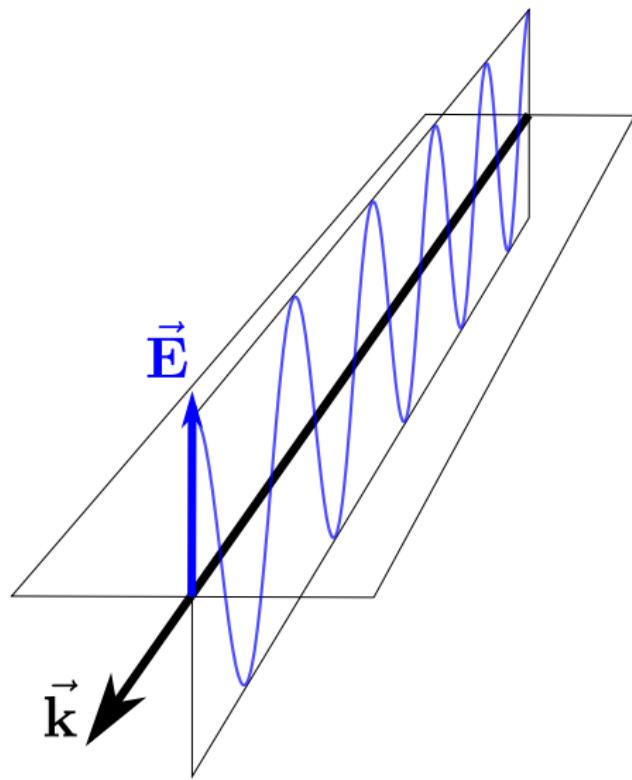
May 29, 2012

- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)

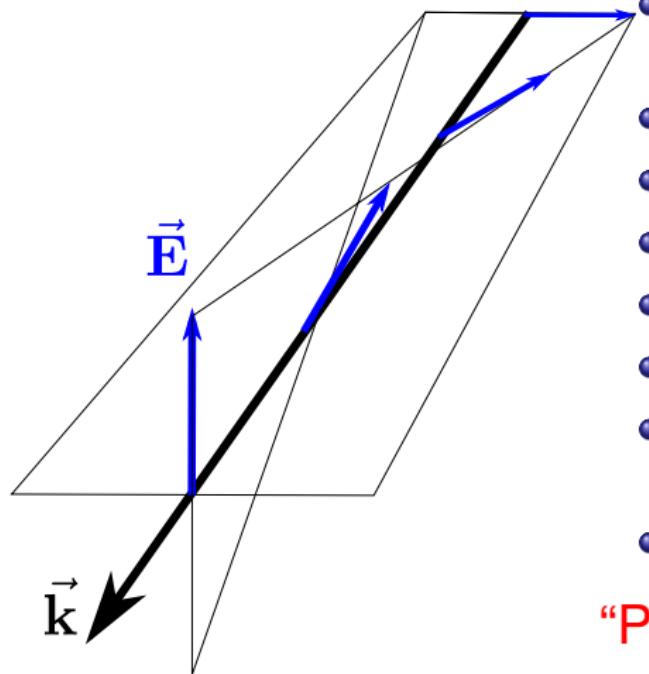
- 1 EM wave physics
- 2 Astrophysics
- 3 Polarized EM-waves
- 4 Interferometric polarimetry
- 5 Messy reality
- 6 Faraday rotation measure synthesis



- Vector phenomenon
- From Maxwell's equations:  
 $\hat{\mathbf{k}} = \hat{\mathbf{E}} \times \hat{\mathbf{B}}$
- We know  $\mathbf{k}$  (Rick Perley)
- Measure either  $\mathbf{E}$  or  $\mathbf{B}$
- $\mathbf{E}$  is easier (Bob Hayward)



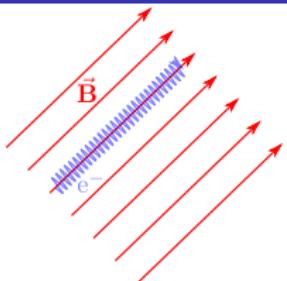
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- Measure either  $\mathbf{E}$  or  $\mathbf{B}$
- $\mathbf{E}$  is easier (Bob Hayward)
- But:
- $E_x$  and  $E_y$  not equal
- $\mathbf{E}$  may rotate as function of  $x$  and  $t$ .
- $\mathbf{E}$  traces ellipse

“Polarization”

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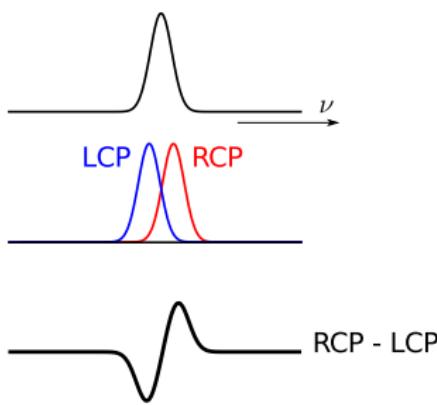
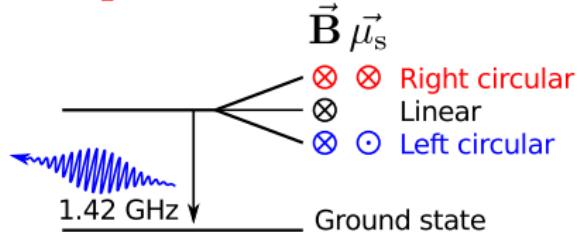


## Process

- Generates polarized emission
- Main emission mechanism at cm-m wavelength
- Up to 80% linearly polarized
- No circular
- $\langle \mathbf{E}_{\text{source}} \rangle \perp \mathbf{B}_{\text{source}}$

## Polarimetry provides

- $\mathbf{B}$ -field direction
- Turbulence
- Indirectly:  $\mathbf{B}$ -field strength

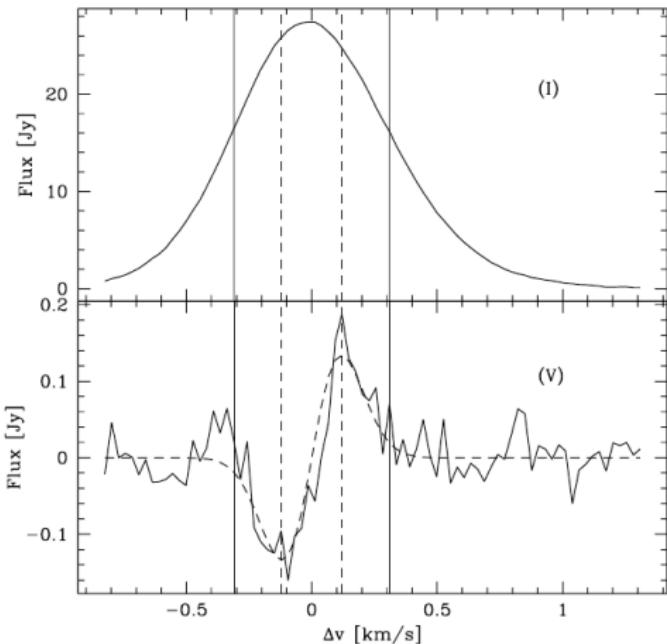
$H_I$  hyperfine transition

## Process

- Generates polarized emission
- Only in spectral lines
- If magnetic moment: e.g.  $H_I$ , OH, CN,  $H_2O$
- $\mathbf{B}$ -field splits RCP and LCP
- Separation:  $2.8 \text{ Hz mG}^{-1}$

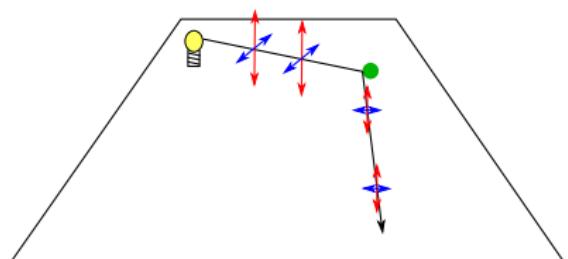
## Polarimetry provides

- $\mathbf{B}$ -field strength at source
- If detectable...



**Fig. 4.** Total power ( $I$ ) and circular polarization ( $V$ ) spectrum of the brightest  $\text{H}_2\text{O}$  maser feature around S Per. The dashed line is the fit of the synthetic  $V$ -spectrum to the observed spectrum. Also shown are the observed (dashed) and expected (solid) positions of the minimum and maximum of the  $V$ -spectrum.

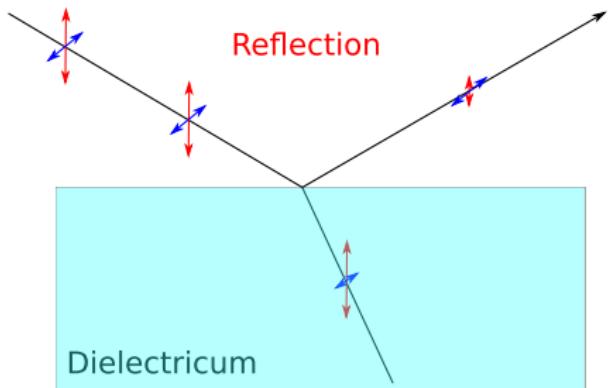
## Thomson scattering



## Process

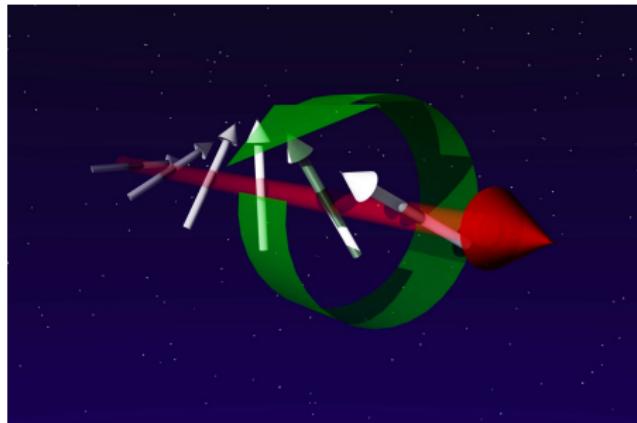
- Modifies polarization state
- Thomson scattering: no  $T$  dependence
- Planets / Moon: dielectric transition

## Reflection



## Polarimetry provides

- Electron densities in cool gas
- Dust properties
- Lunar dielectric constant

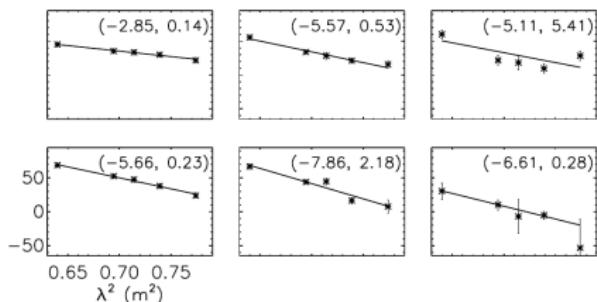


## Process

- Modifies polarization state
- Delay between LCP and RCP
- Rotates linear pol angle
- $\Delta\chi = \chi_0 + \phi\lambda^2$

$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot d\mathbf{l}$$

$\lambda^2$  law Haverkorn et al. (2001)

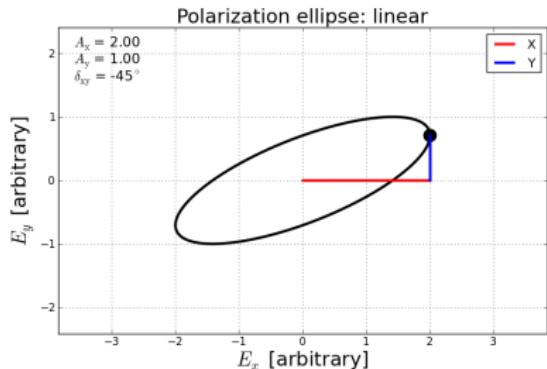


## Polarimetry provides

- Source plasma properties
- Intervening plasma properties
- Rare cases: 3D tomography

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## Geometry

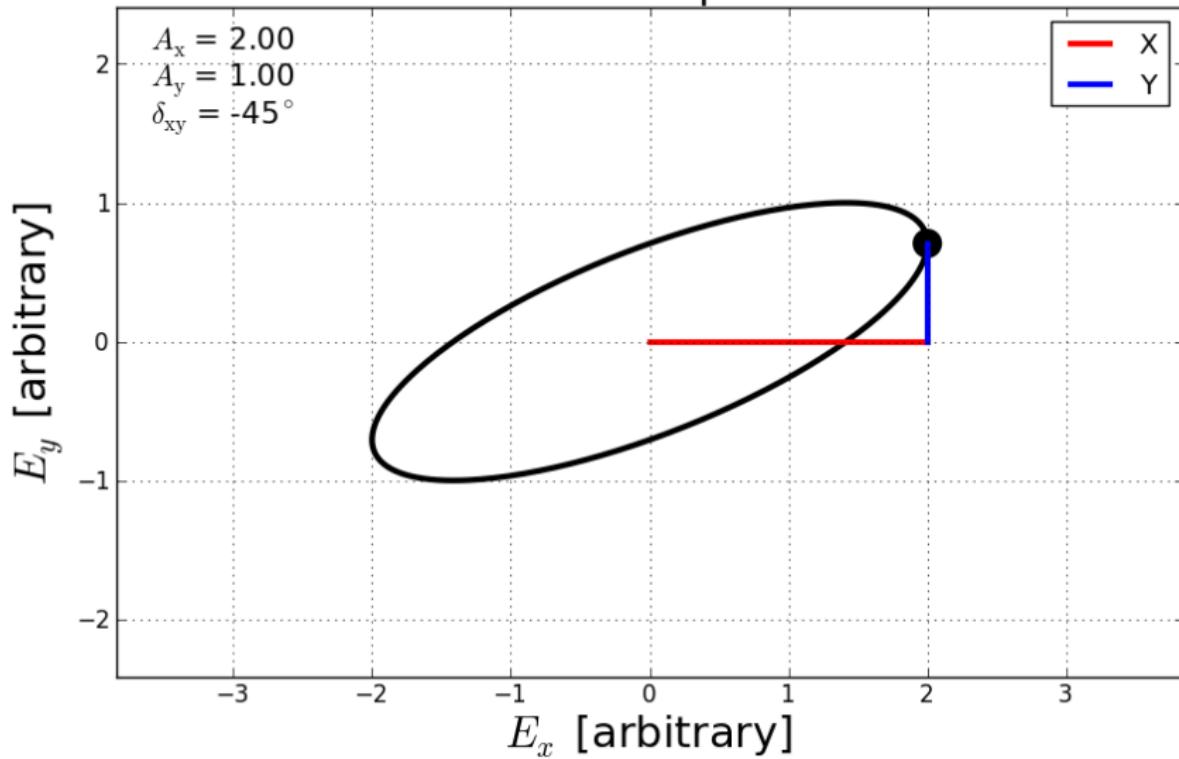


Viewing from antenna towards source, watching orientation and length of  $\mathbf{E}$  vector on a plane at a fixed location in space.

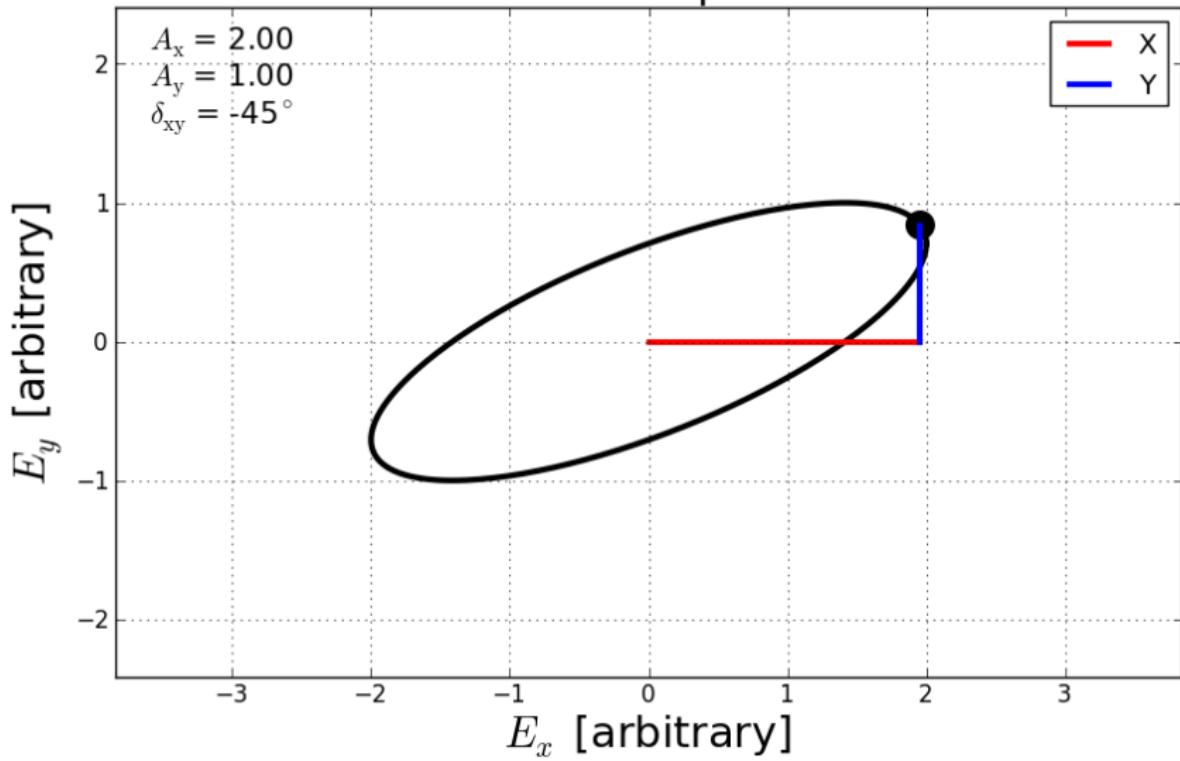
$$\begin{aligned}\mathbf{E} &= E_x \hat{\mathbf{e}}_x + E_y \hat{\mathbf{e}}_y \\ E_x &= A_x \cos(2\pi\nu t + \delta_x) \\ E_y &= A_y \cos(2\pi\nu t + \delta_y)\end{aligned}$$

- $A_x = x$ -amplitude
- $A_y = y$ -amplitude
- $\delta_{xy} = \delta_y - \delta_x$
- $\delta_{xy}$  = measure of ellipticity
- $\delta_{xy} > 0$ : CW rotation  $\Rightarrow$  LEP
- $\delta_{xy} = 0$ : linear polarization
- $\delta_{xy} < 0$ : CCW rotation  $\Rightarrow$  REP

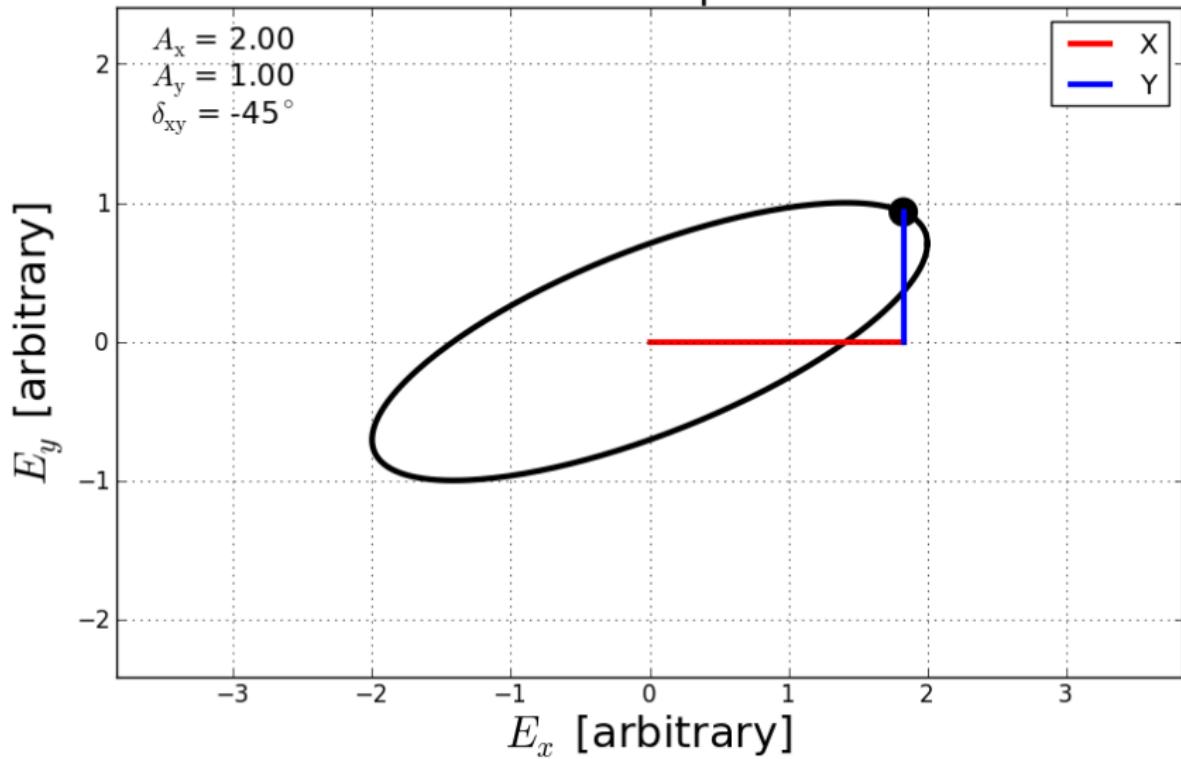
Polarization ellipse: linear



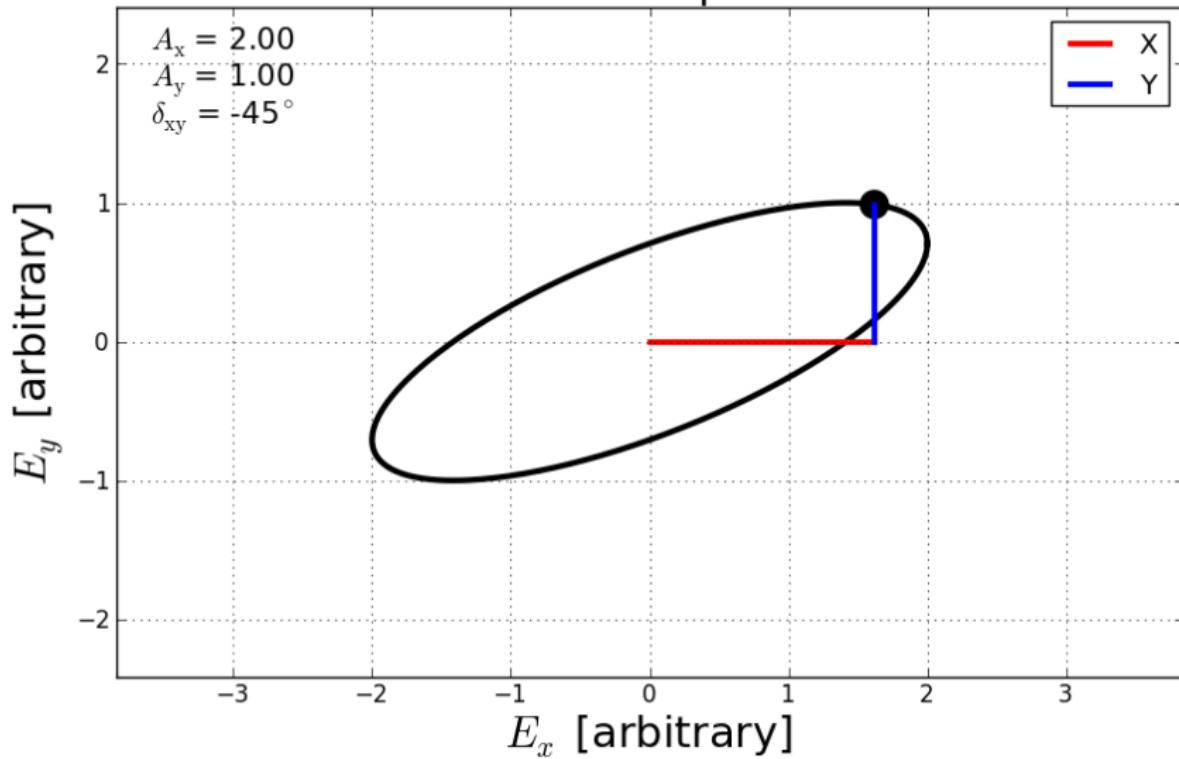
Polarization ellipse: linear



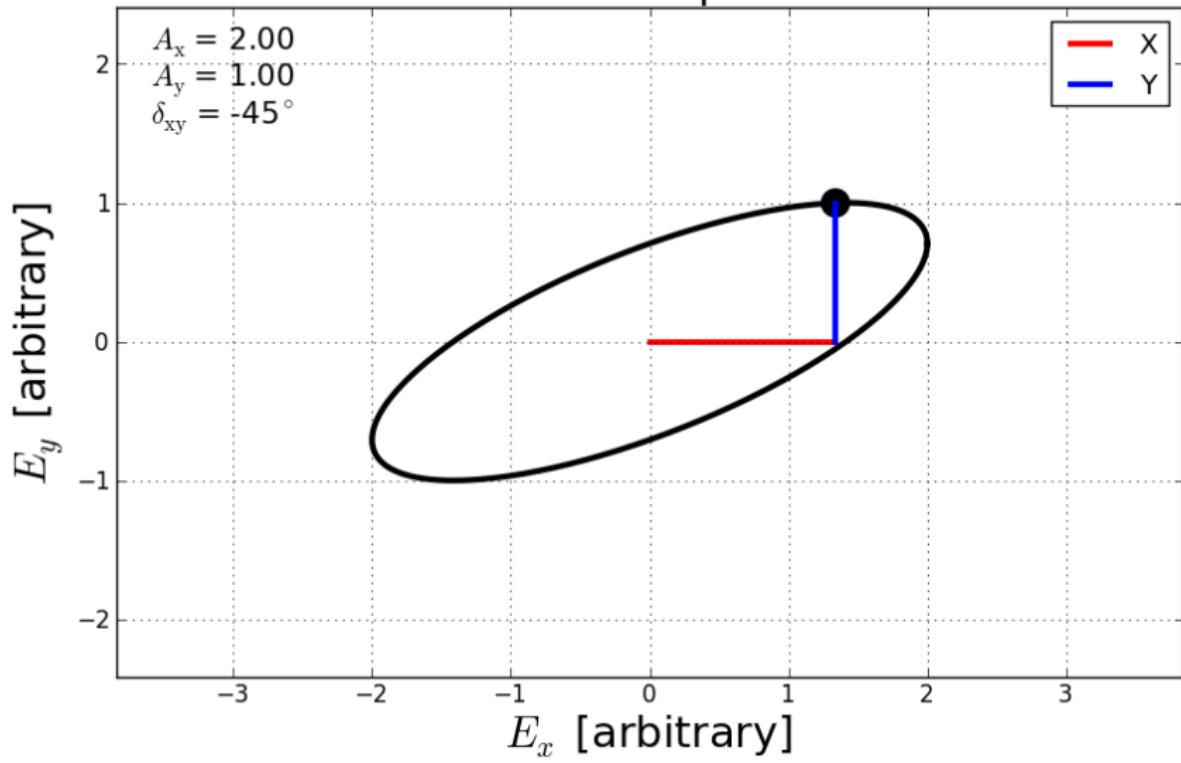
Polarization ellipse: linear



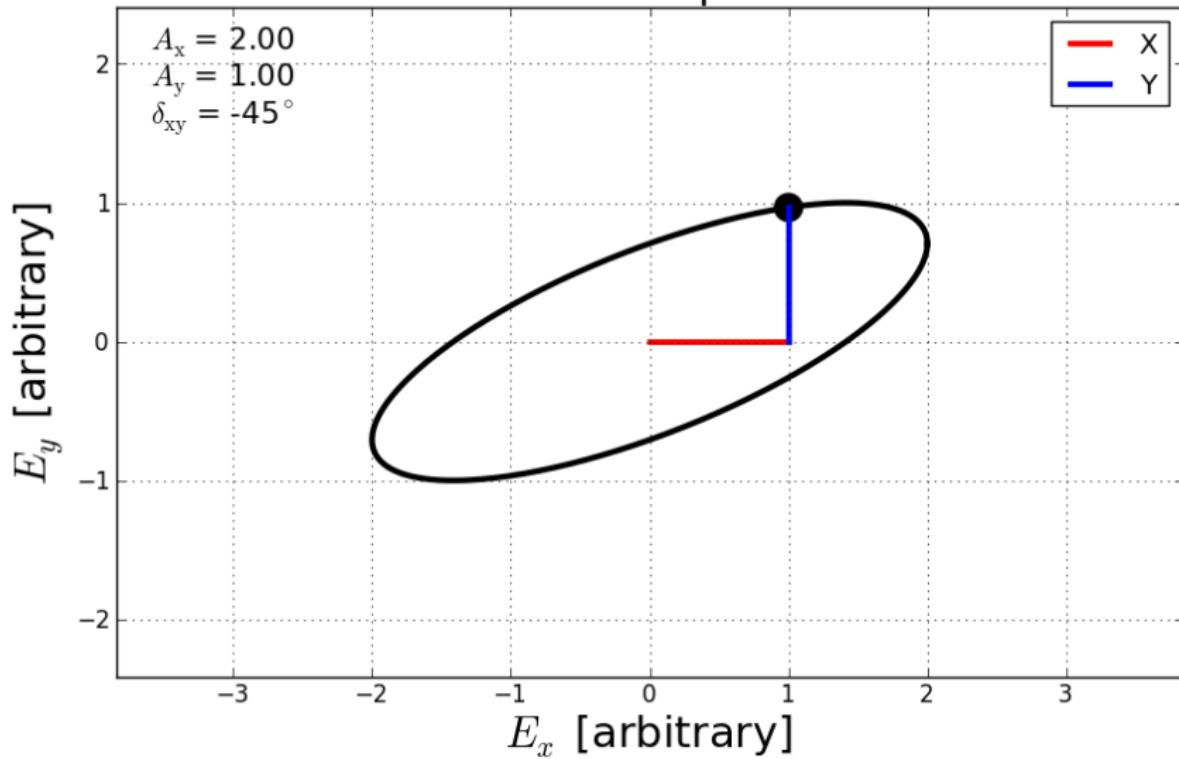
Polarization ellipse: linear



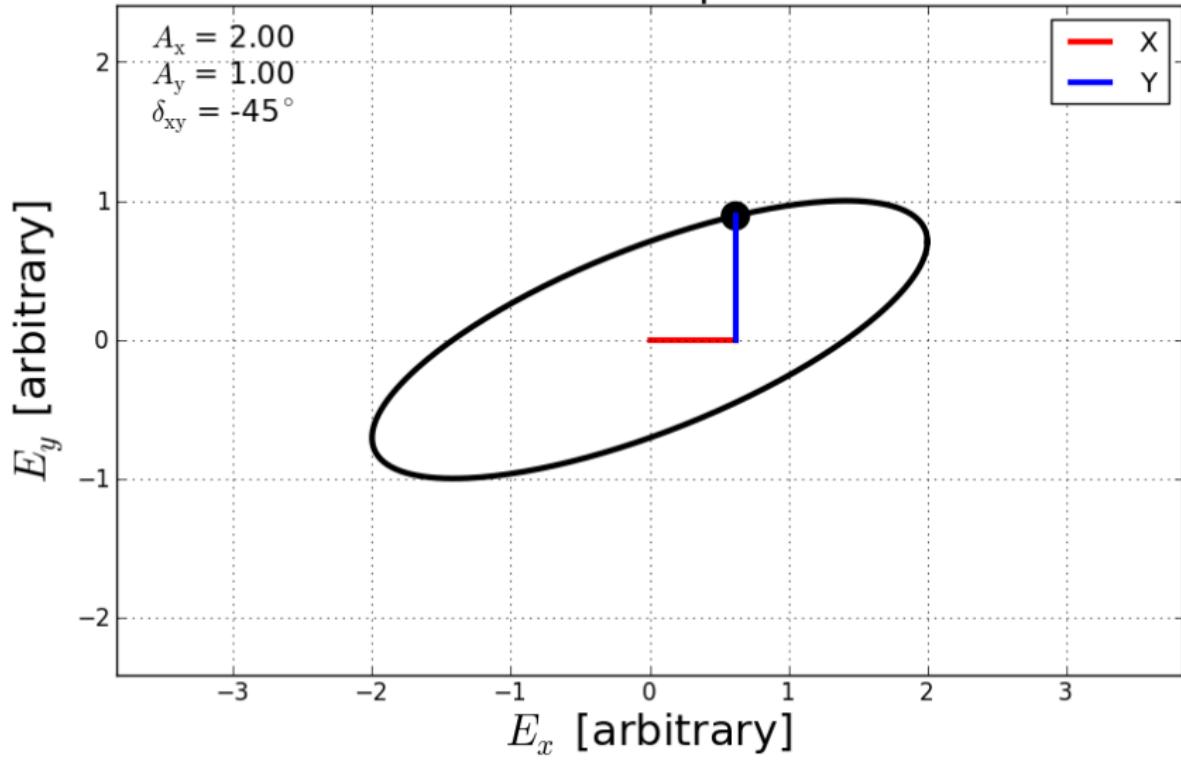
Polarization ellipse: linear



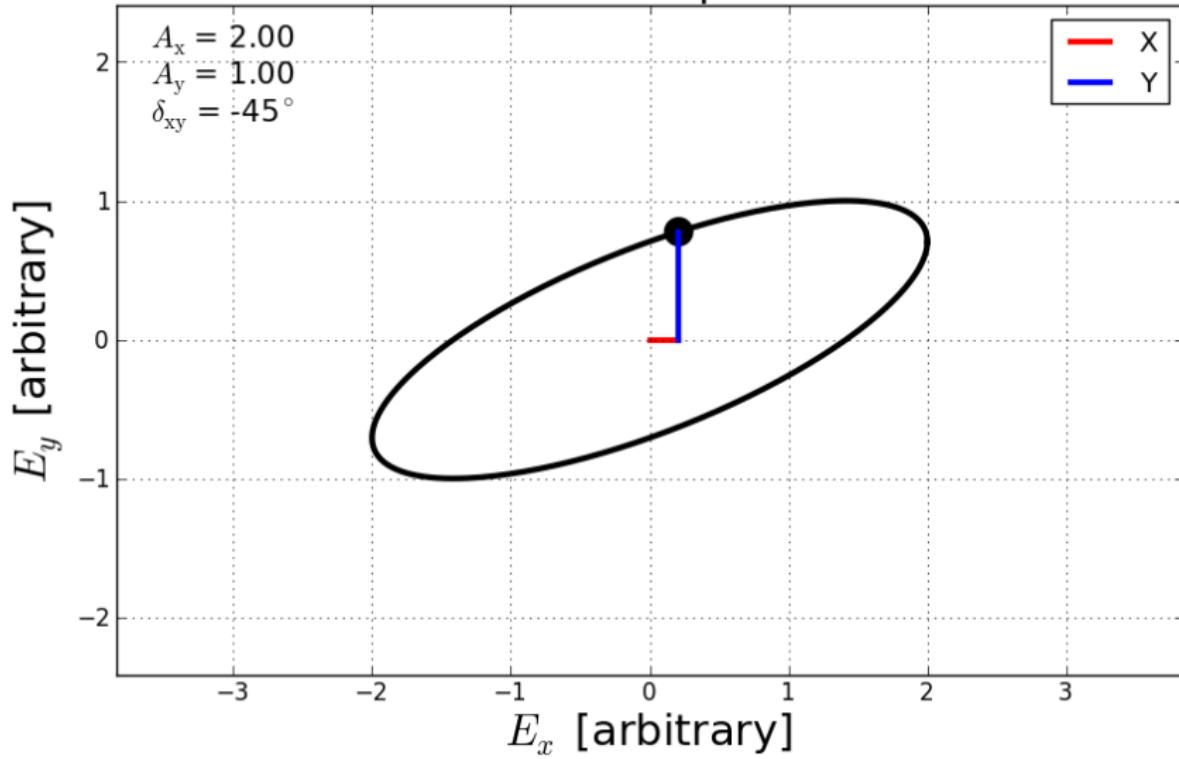
Polarization ellipse: linear



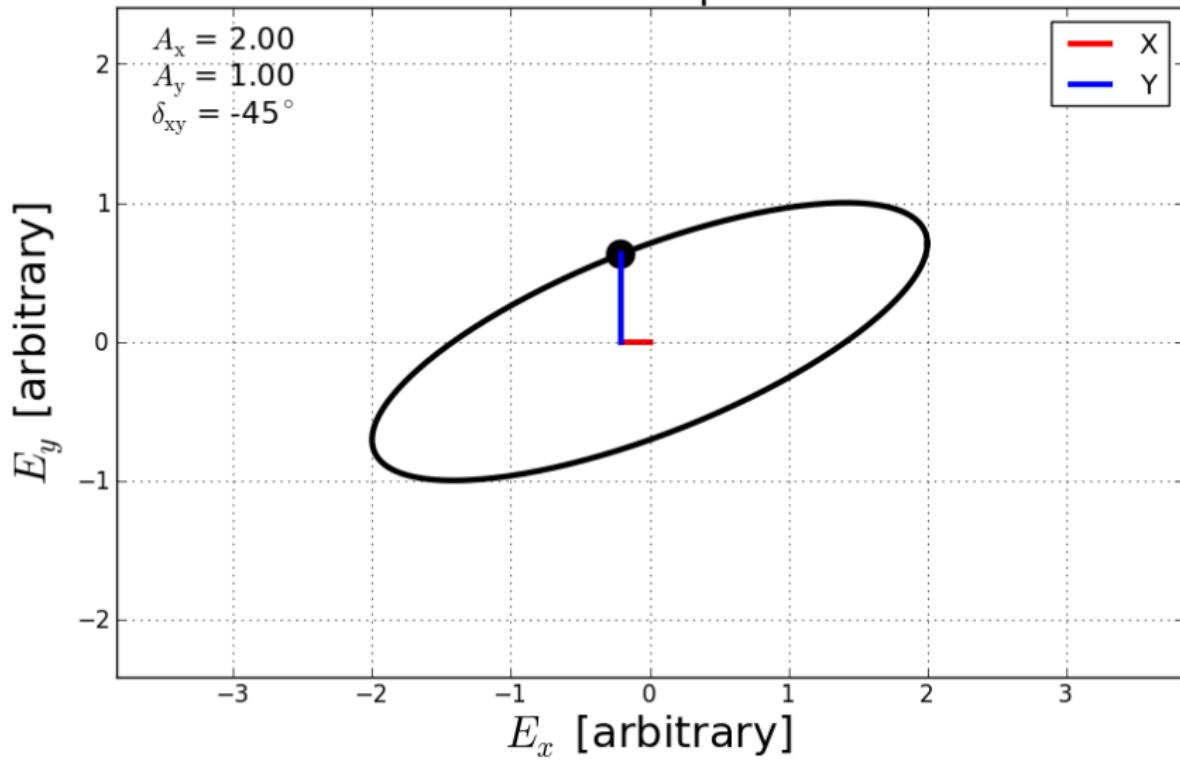
Polarization ellipse: linear



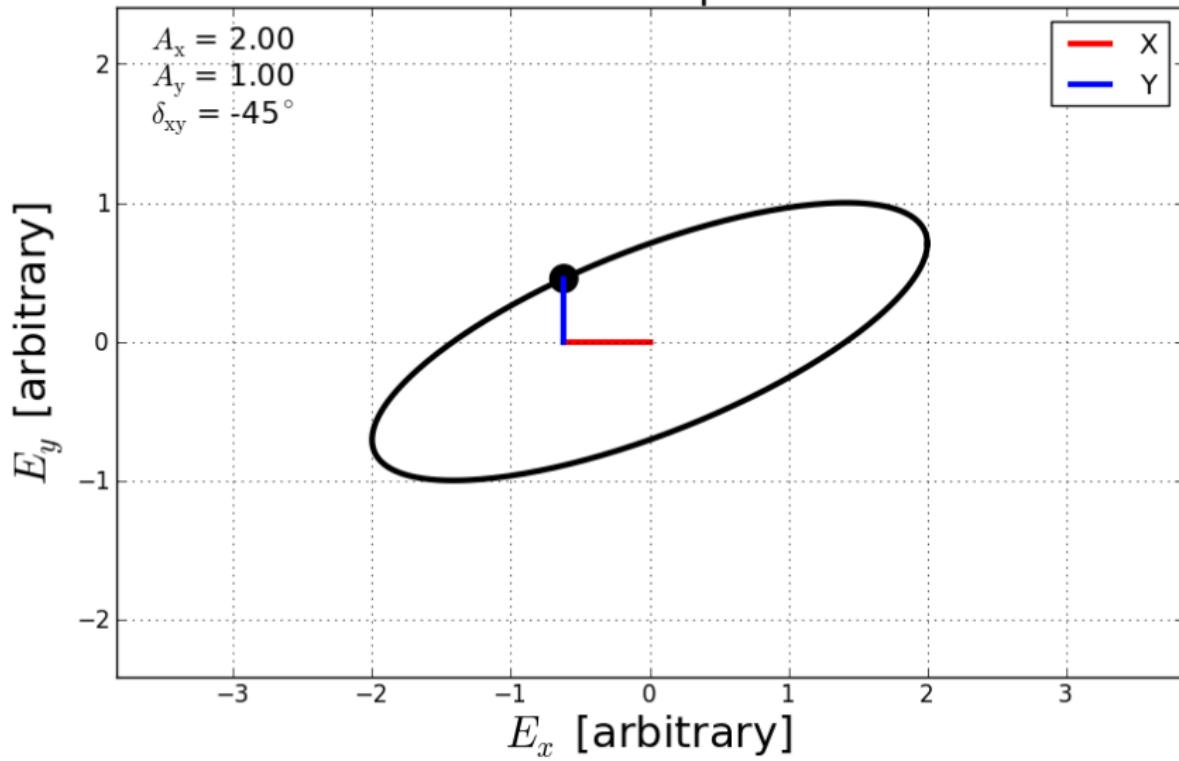
Polarization ellipse: linear



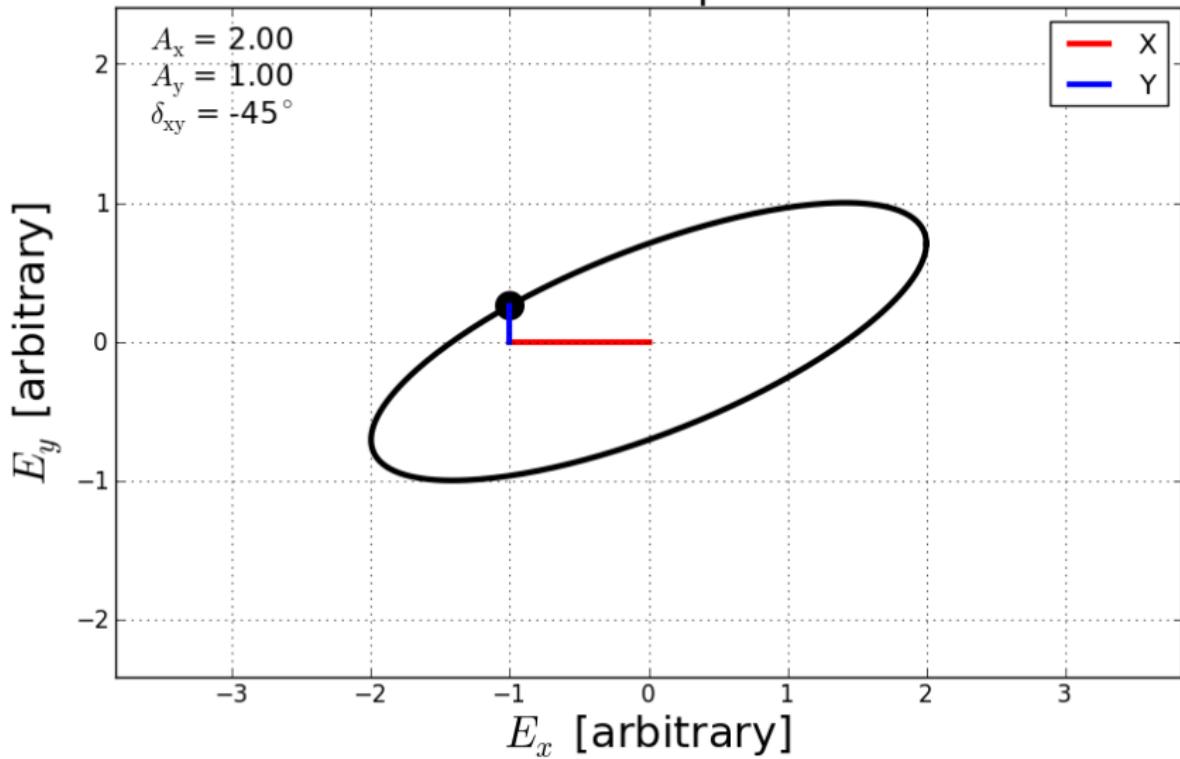
Polarization ellipse: linear



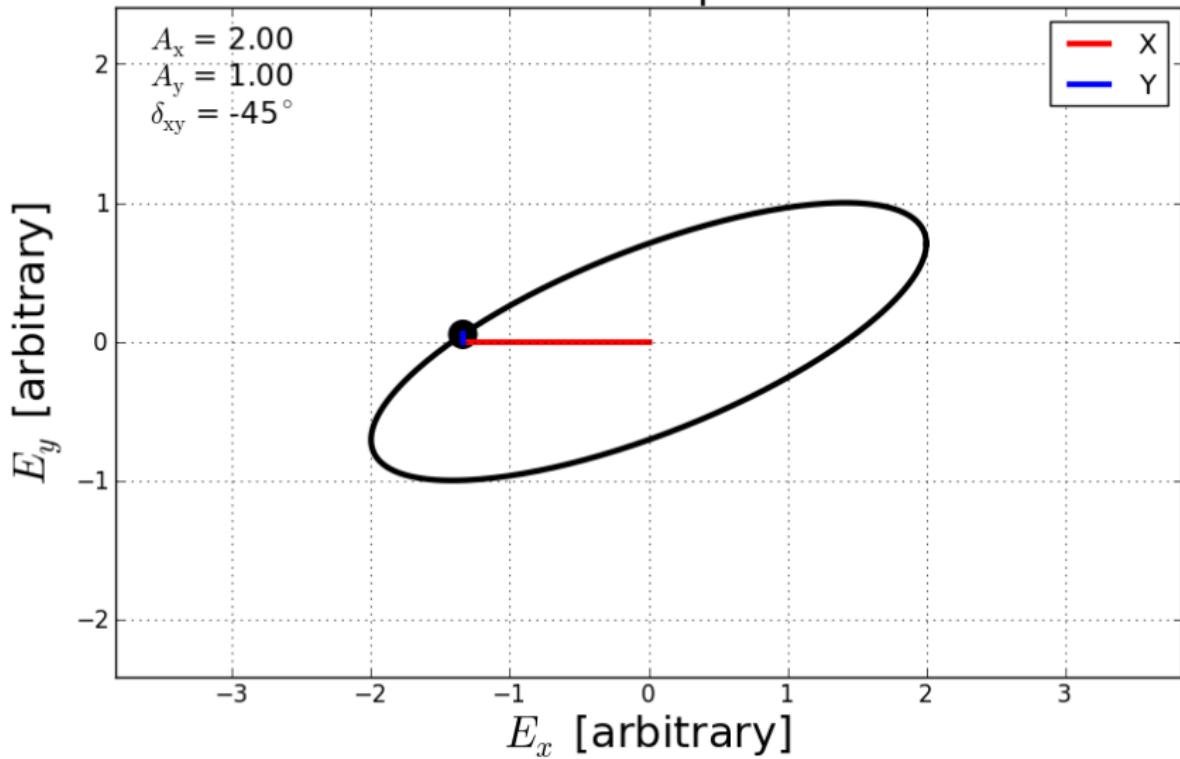
Polarization ellipse: linear



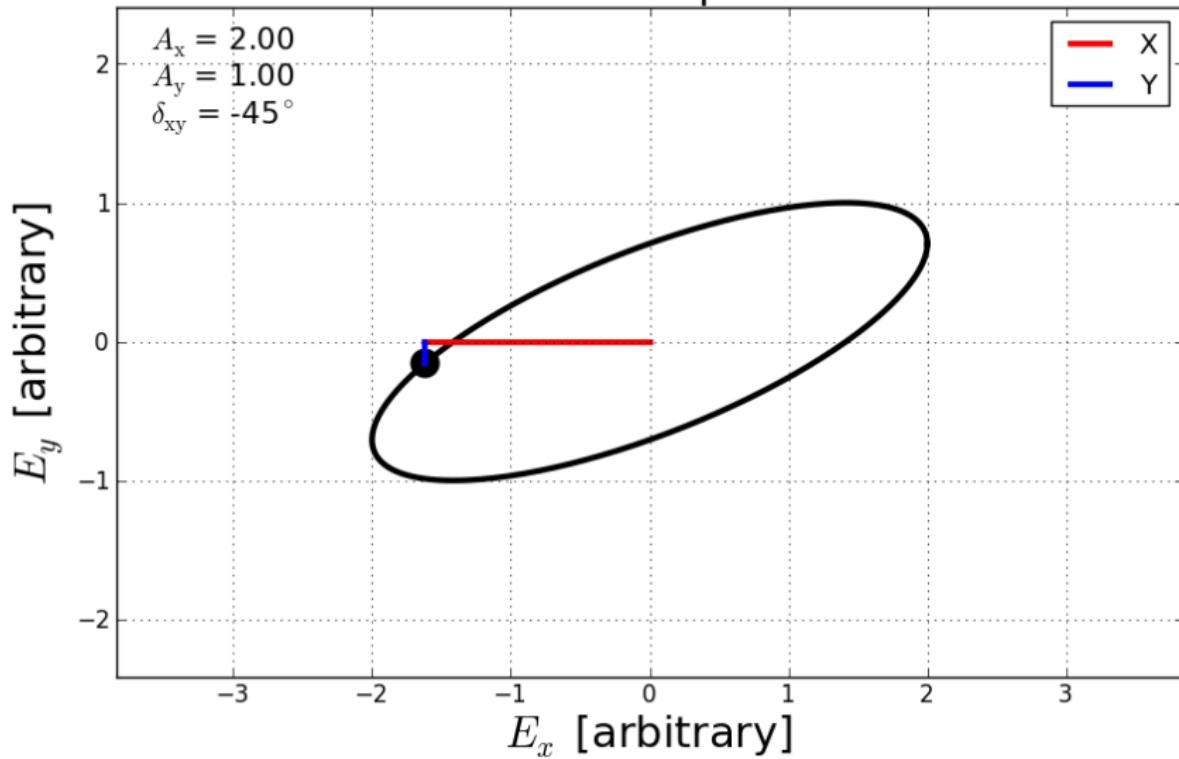
## Polarization ellipse: linear



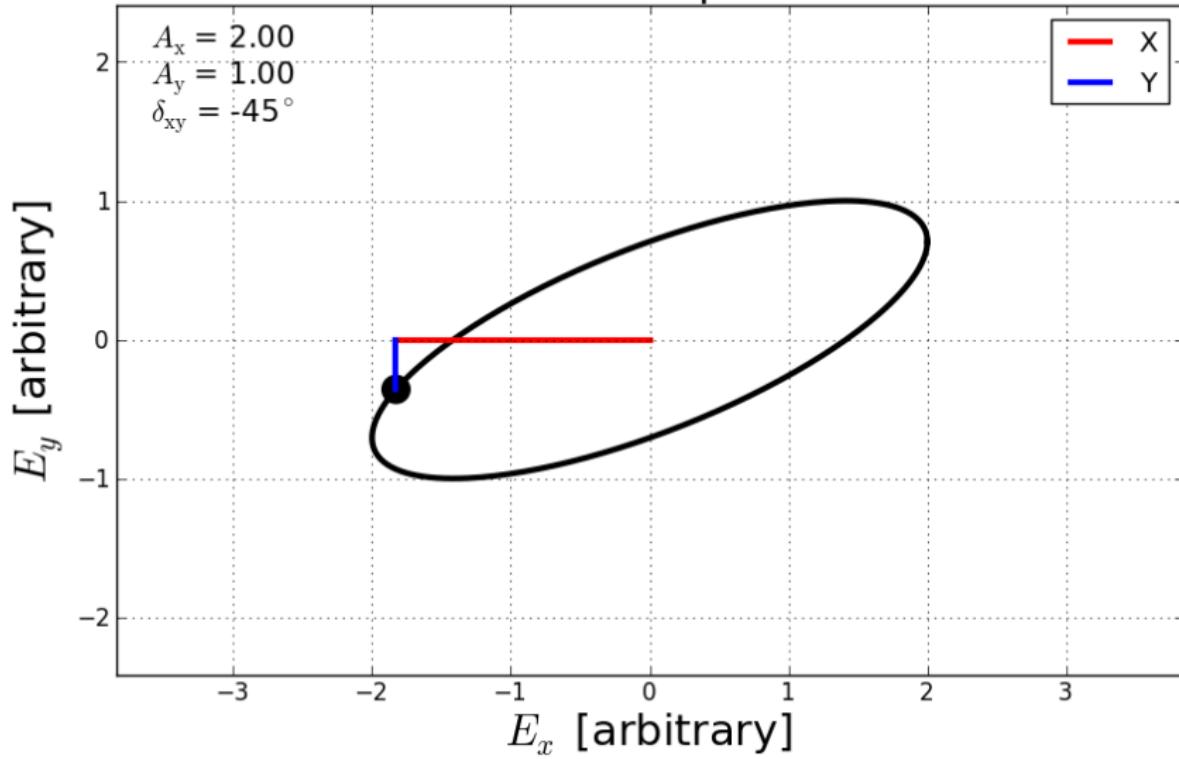
Polarization ellipse: linear



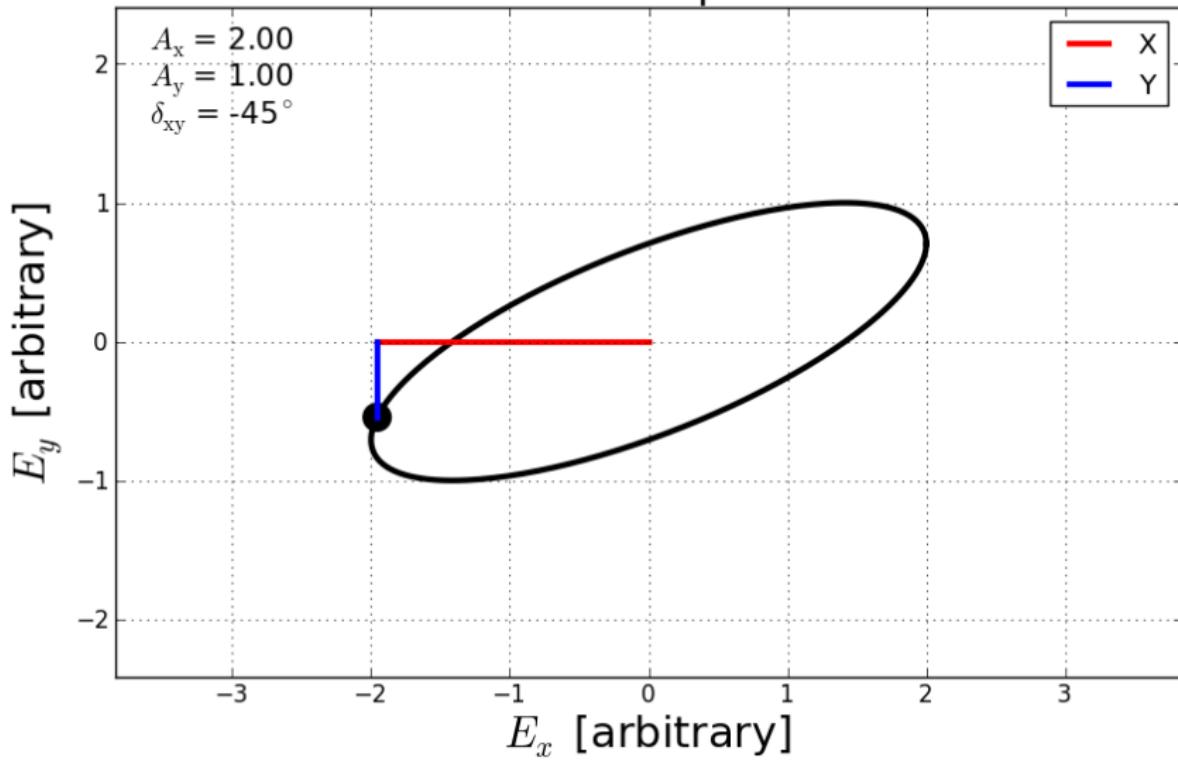
Polarization ellipse: linear



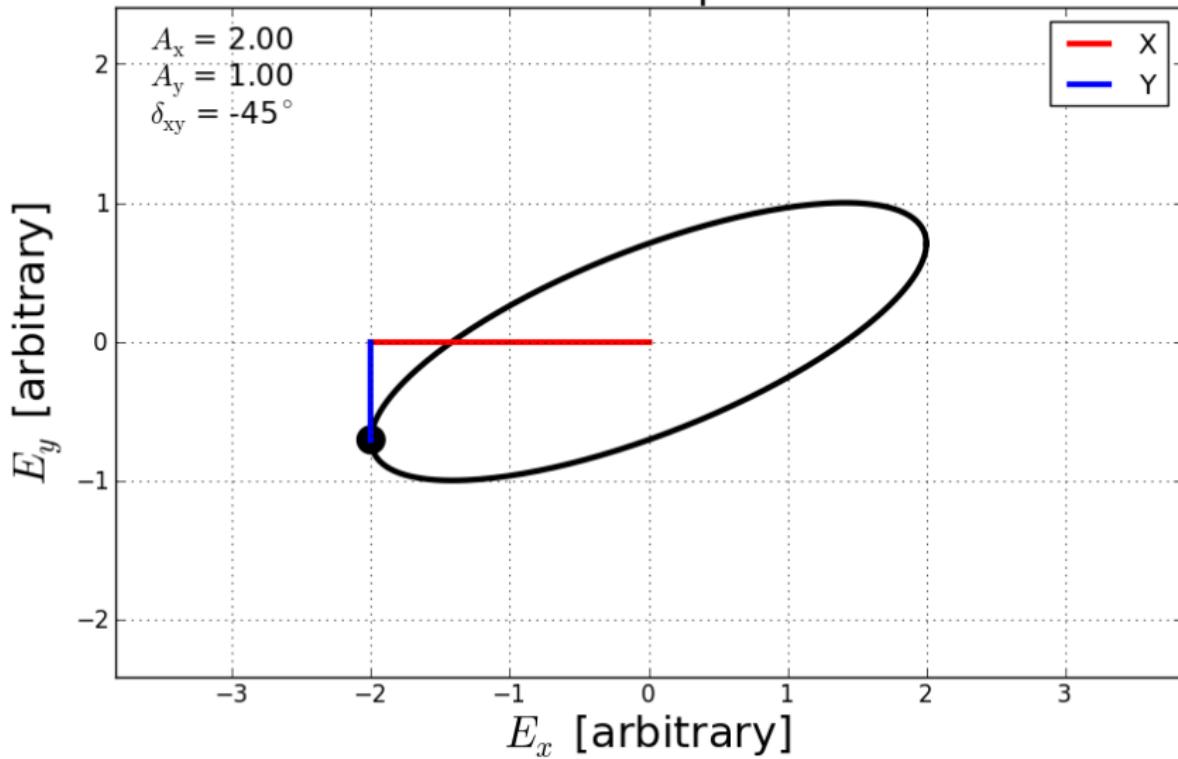
Polarization ellipse: linear



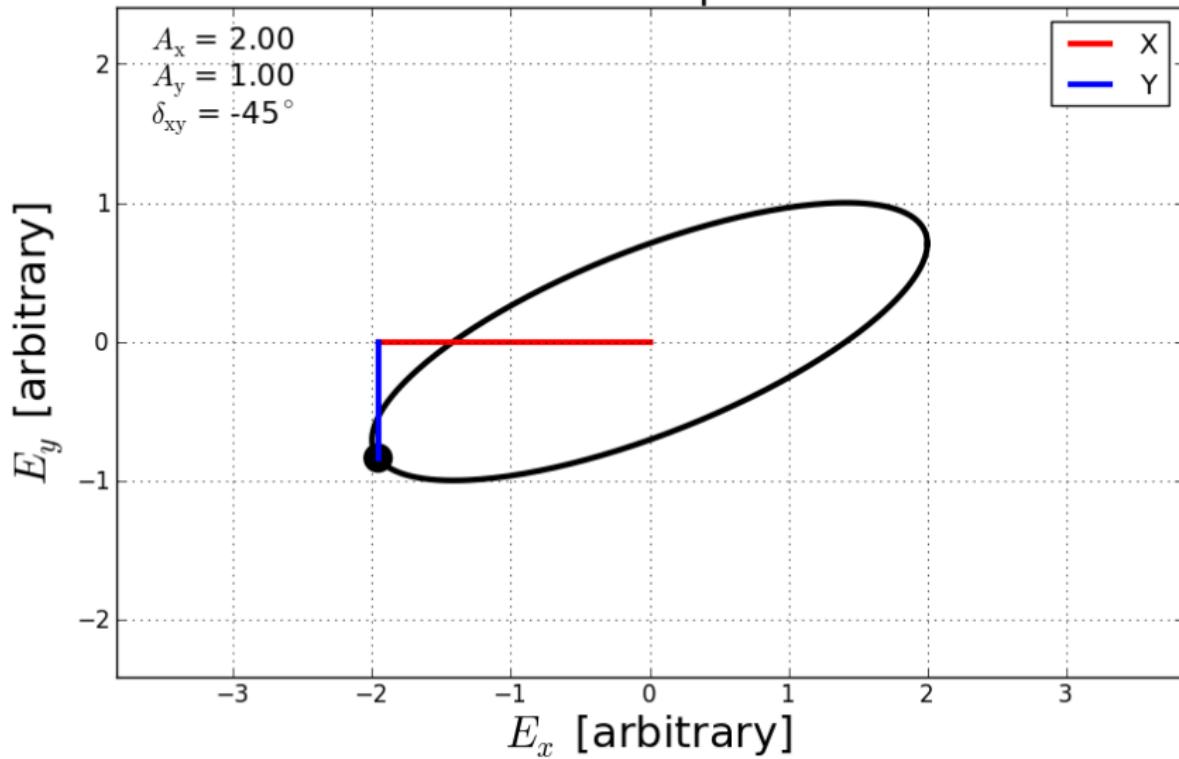
Polarization ellipse: linear



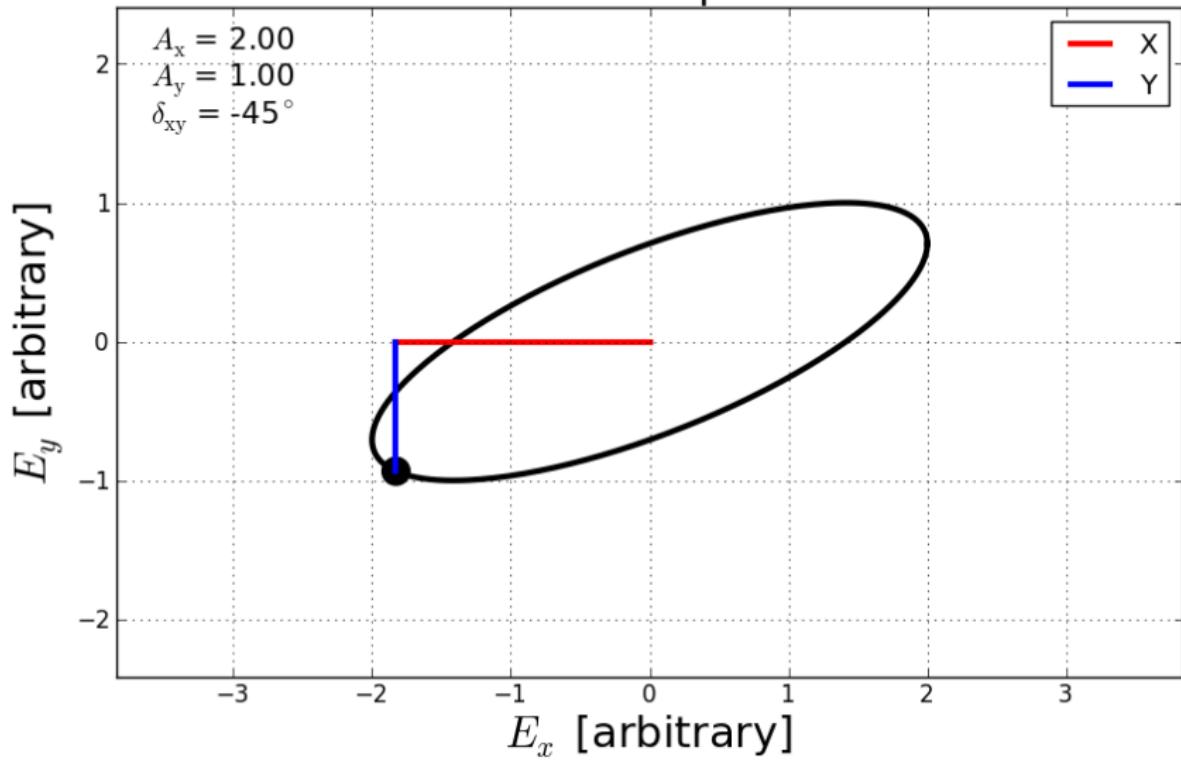
Polarization ellipse: linear



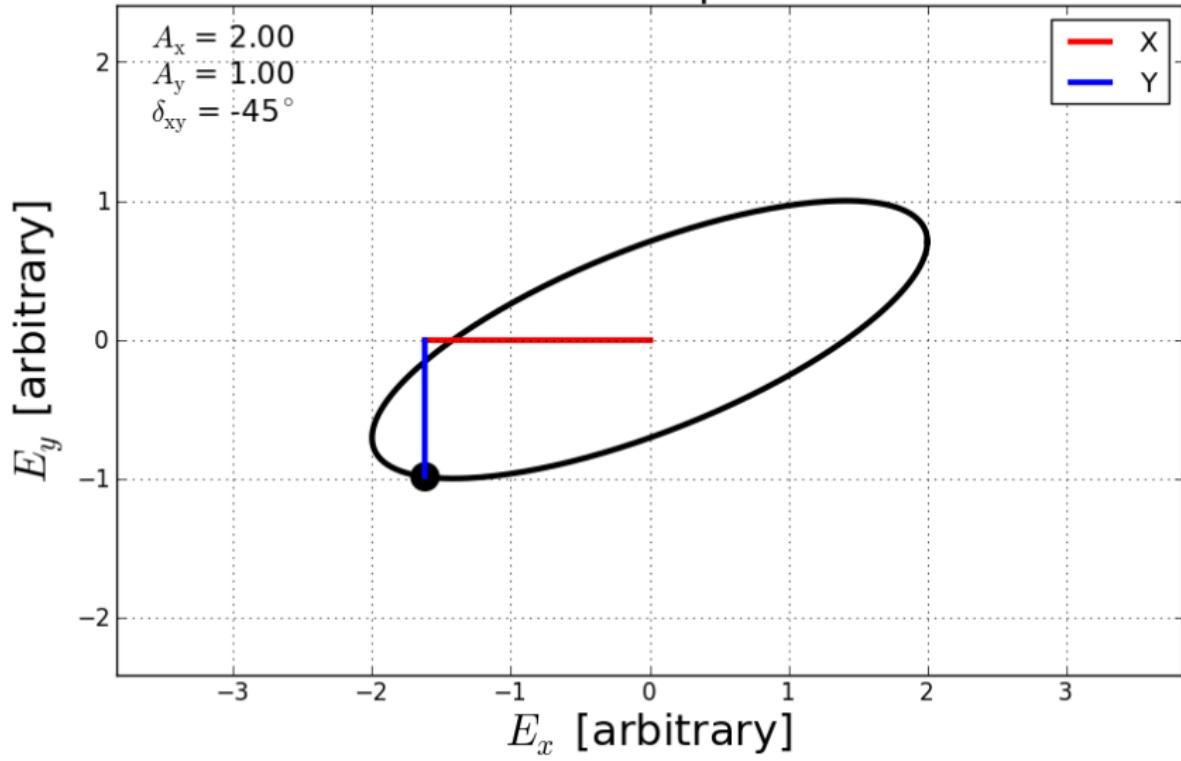
Polarization ellipse: linear



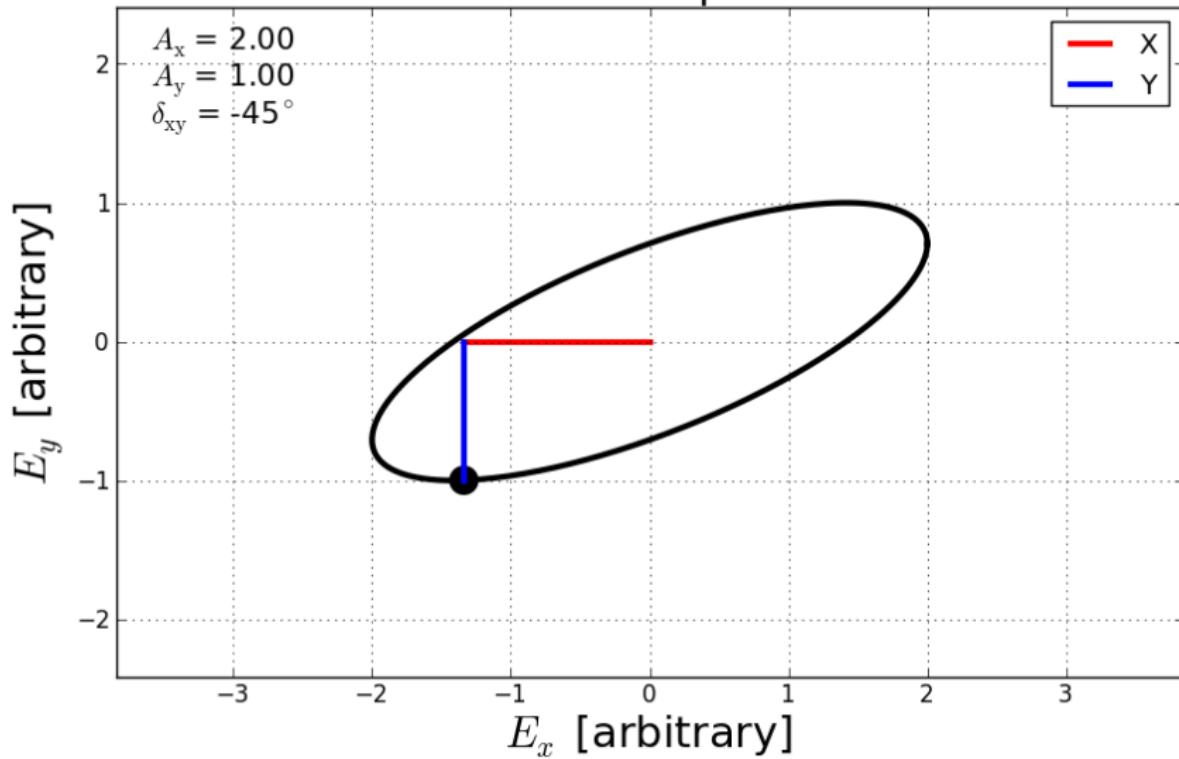
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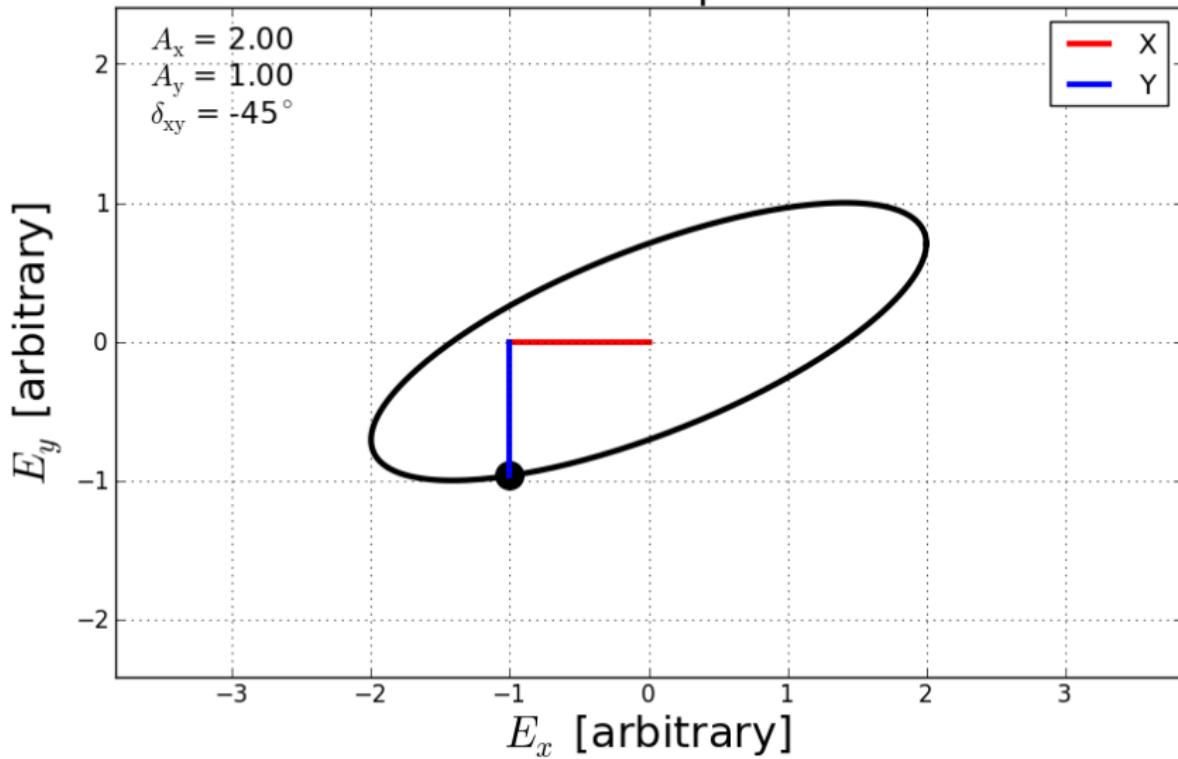
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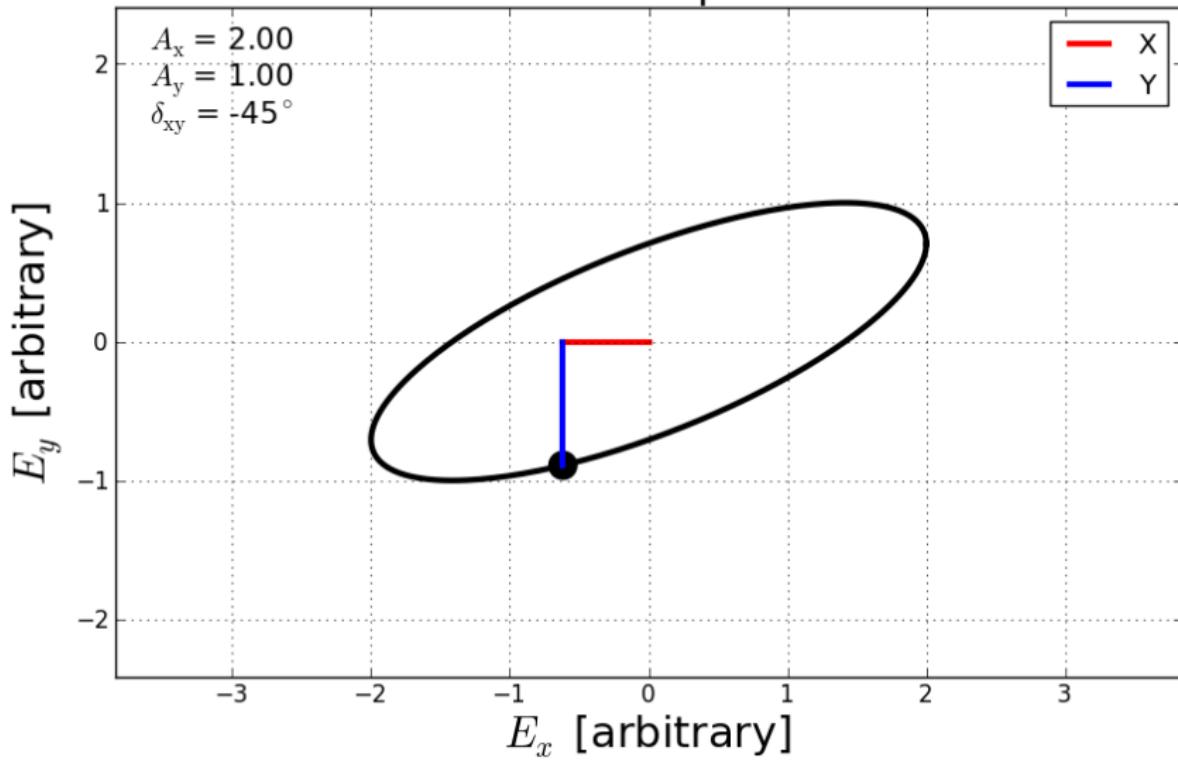
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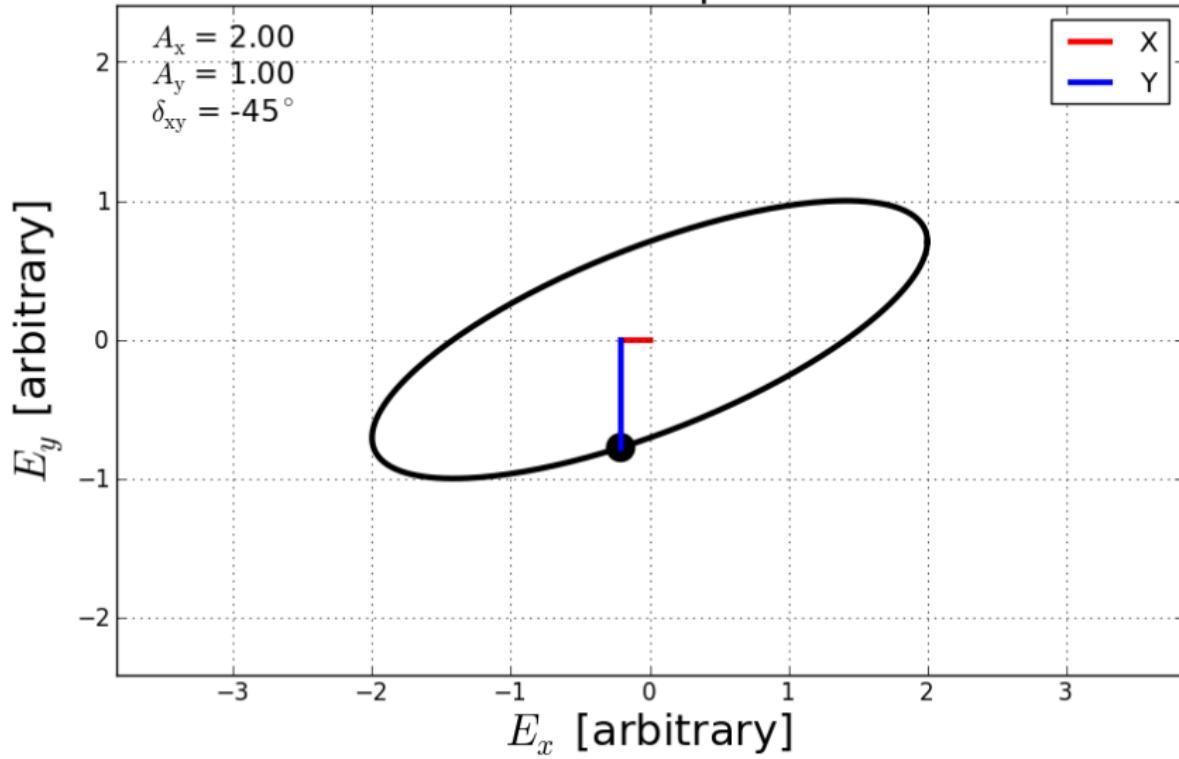
## Polarization ellipse: linear



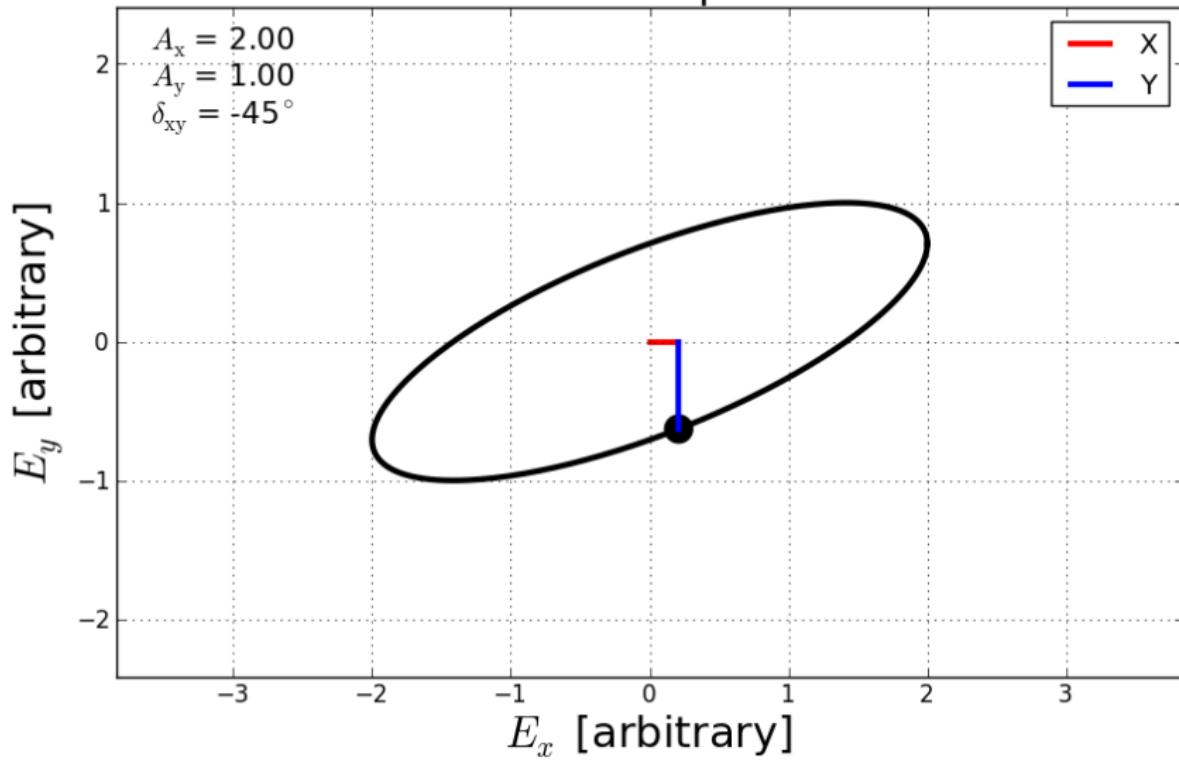
## Polarization ellipse: linear



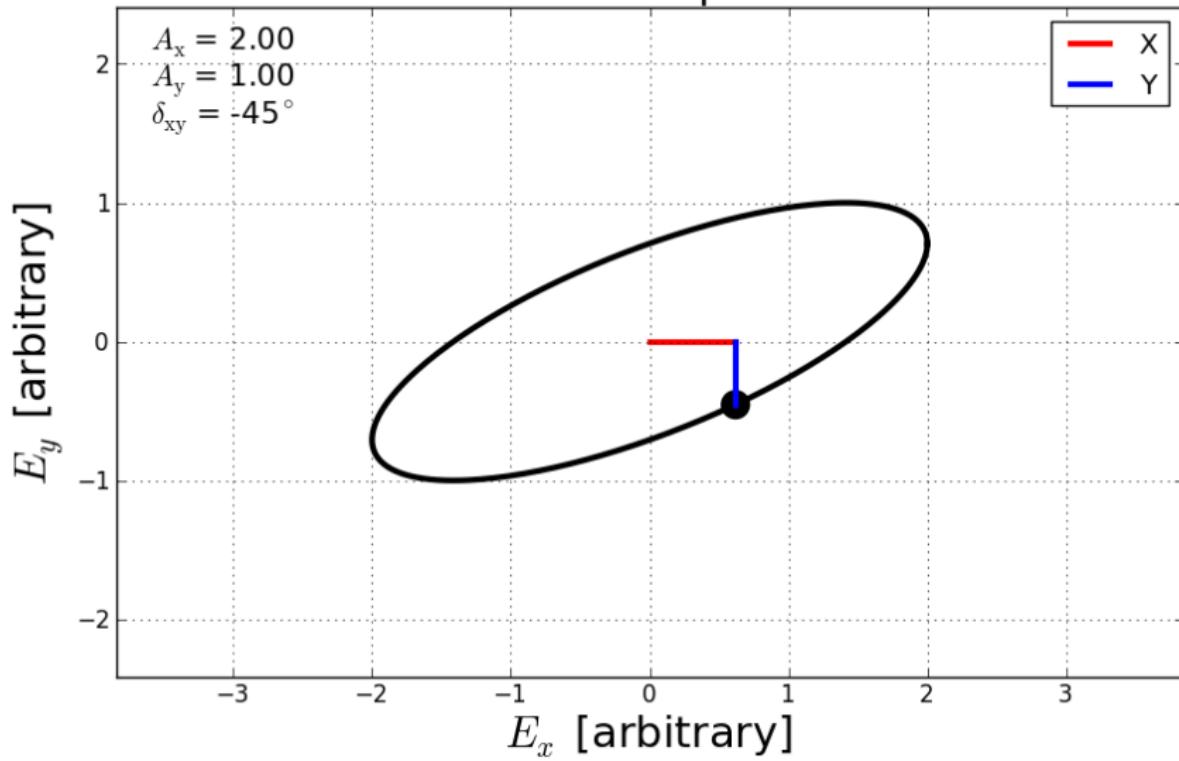
Polarization ellipse: linear



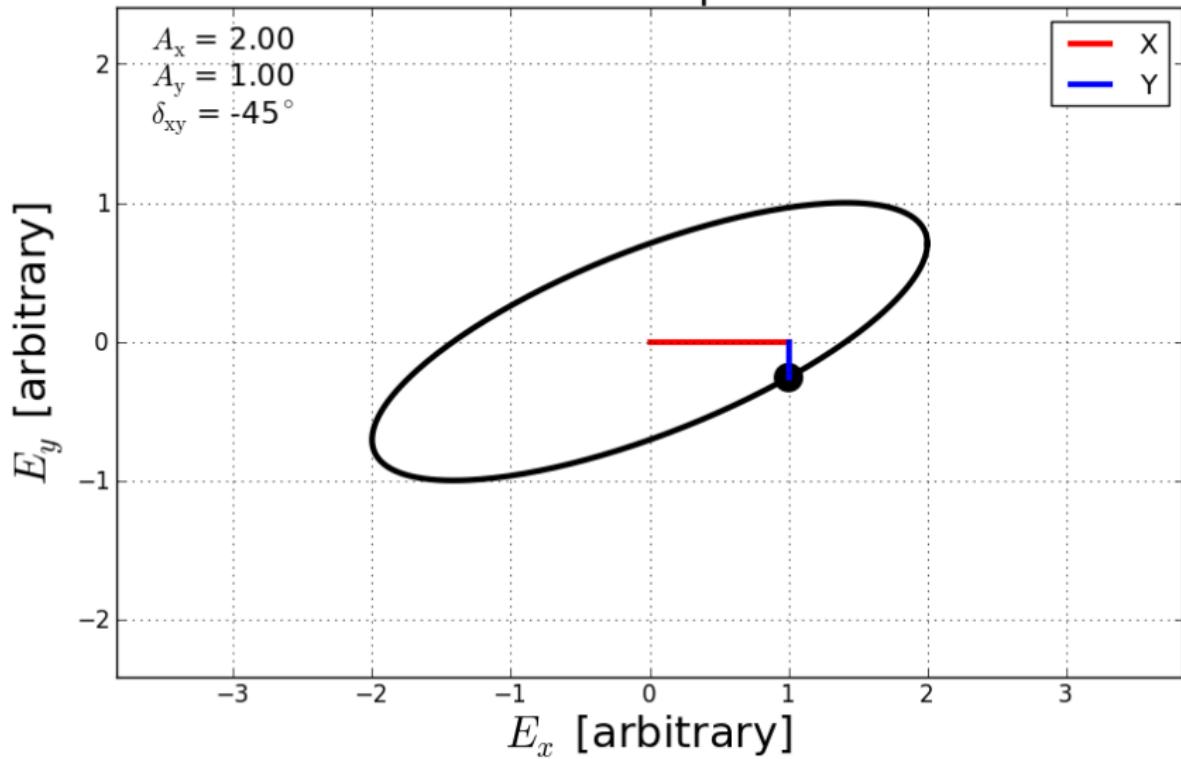
Polarization ellipse: linear



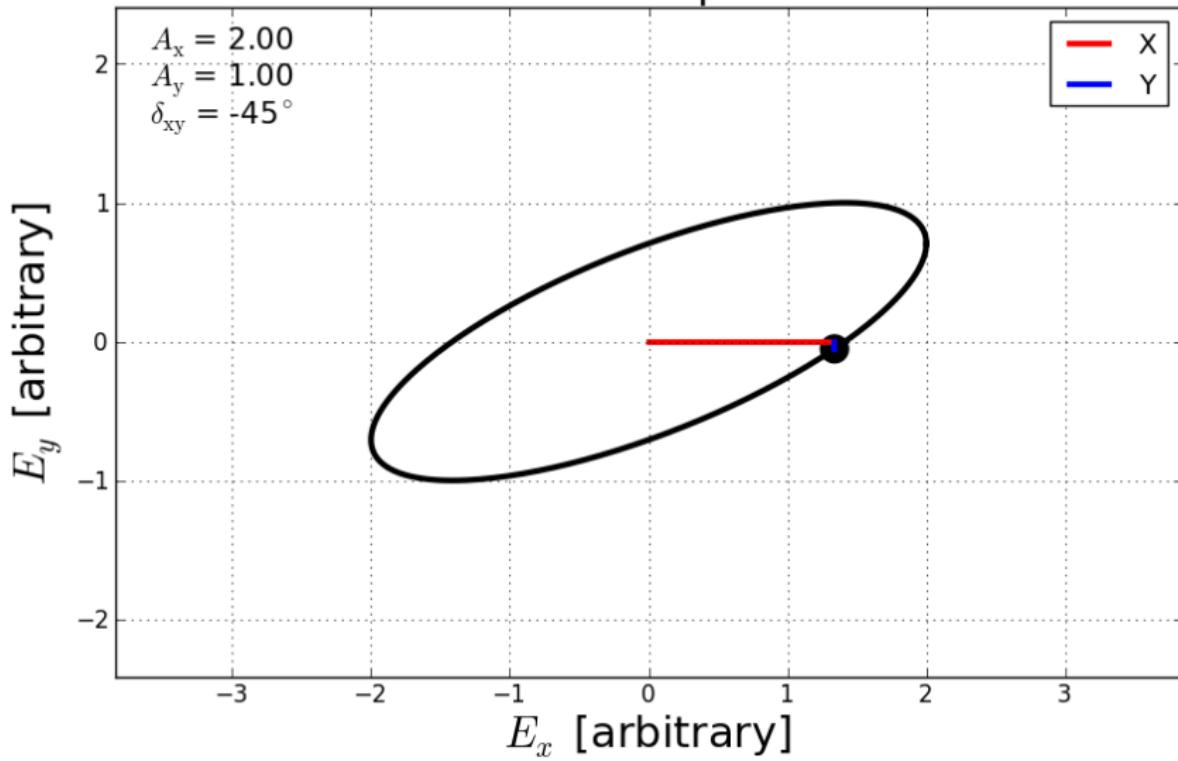
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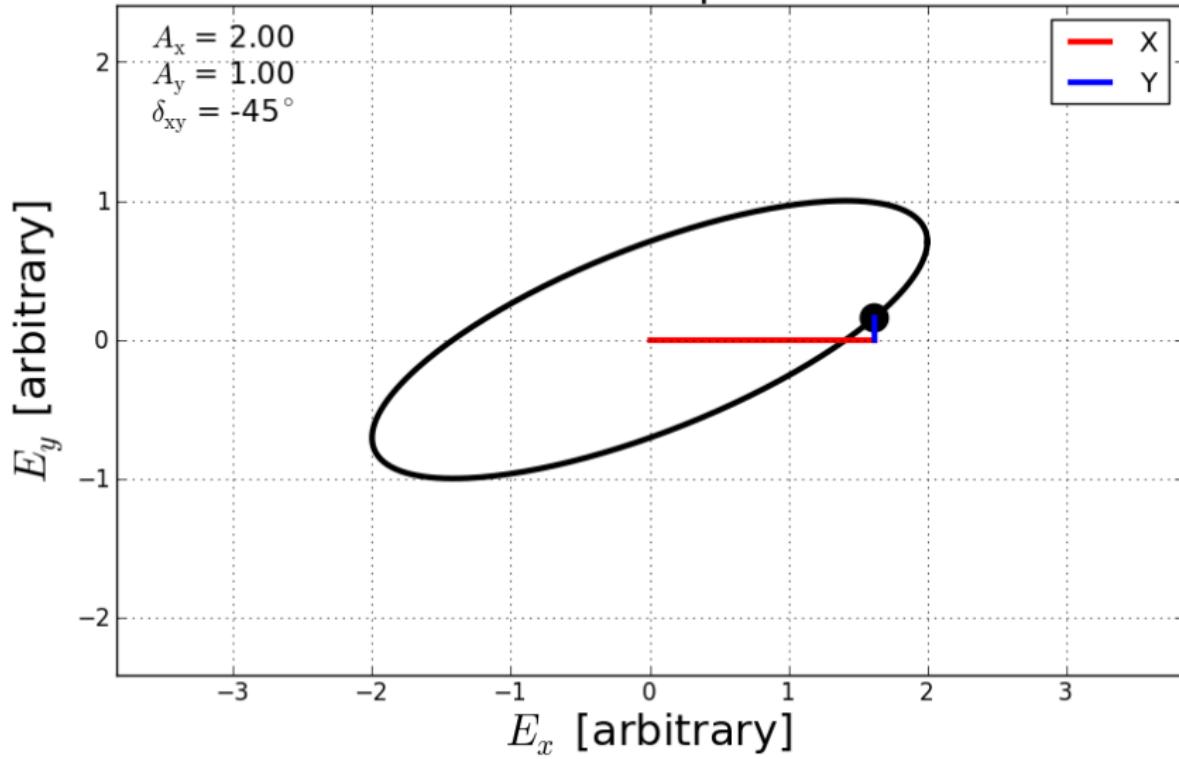
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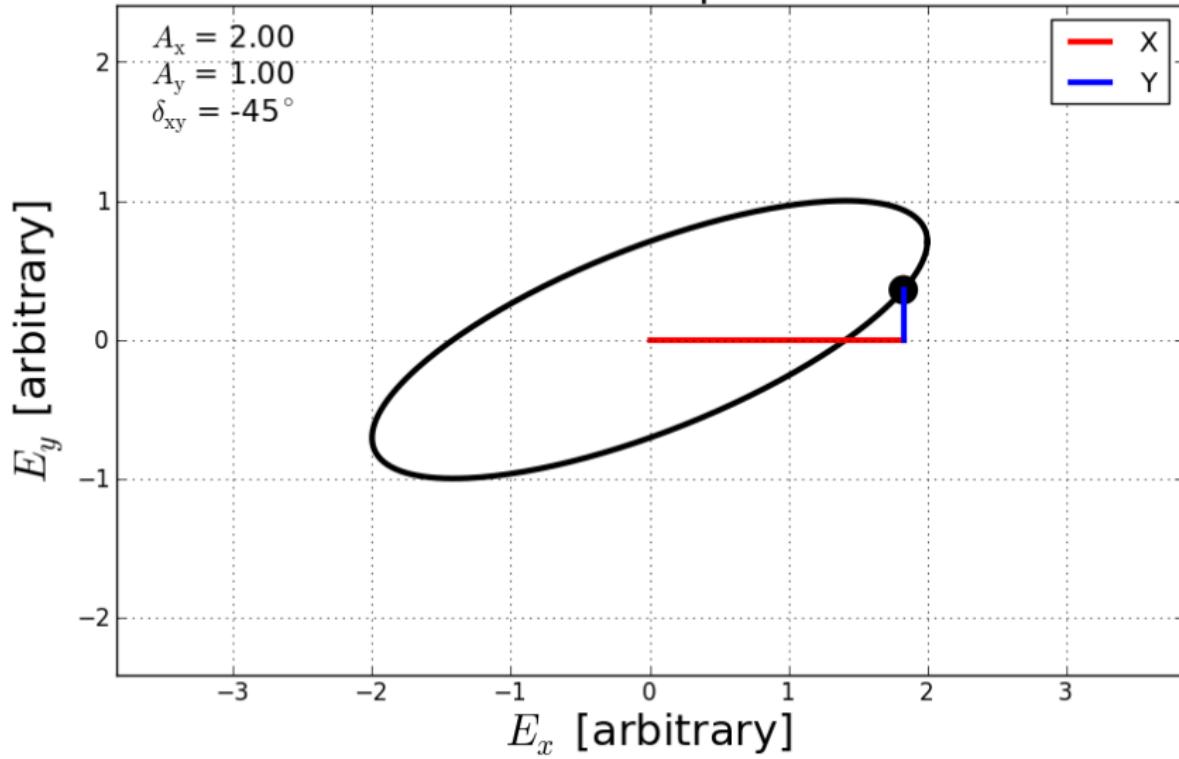
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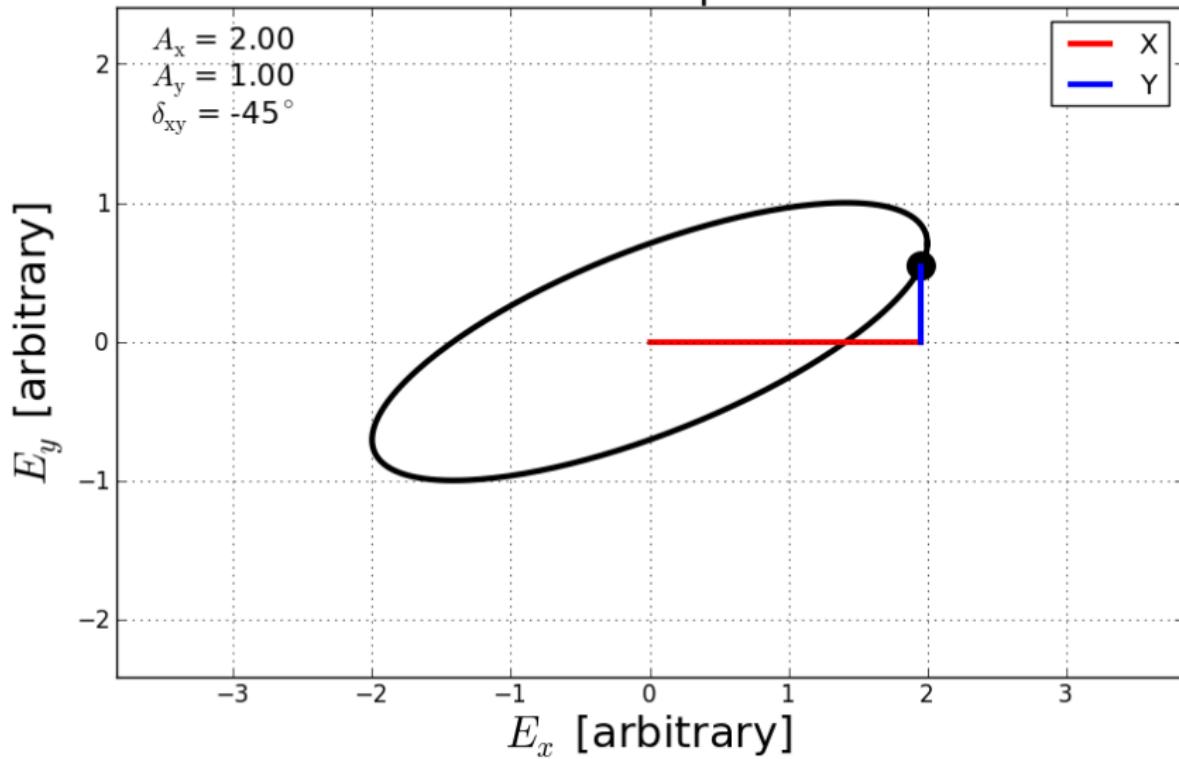
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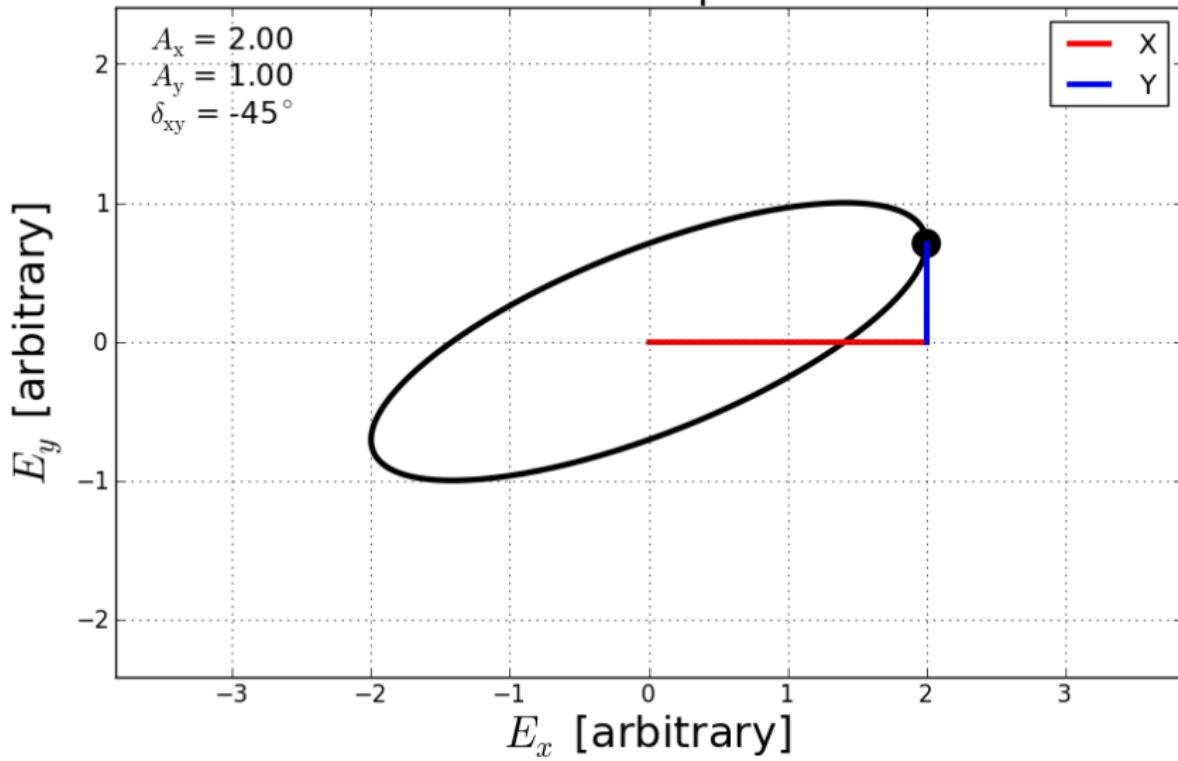
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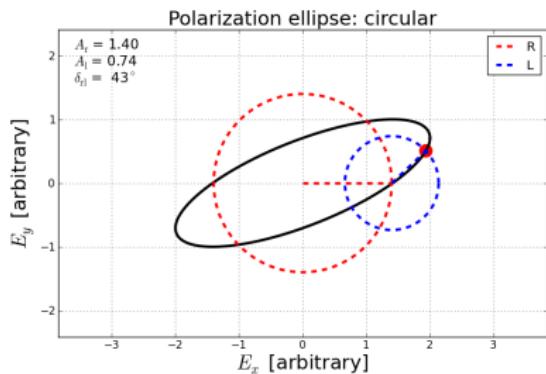
Polarization ellipse: linear



Polarization ellipse: linear



## Geometry

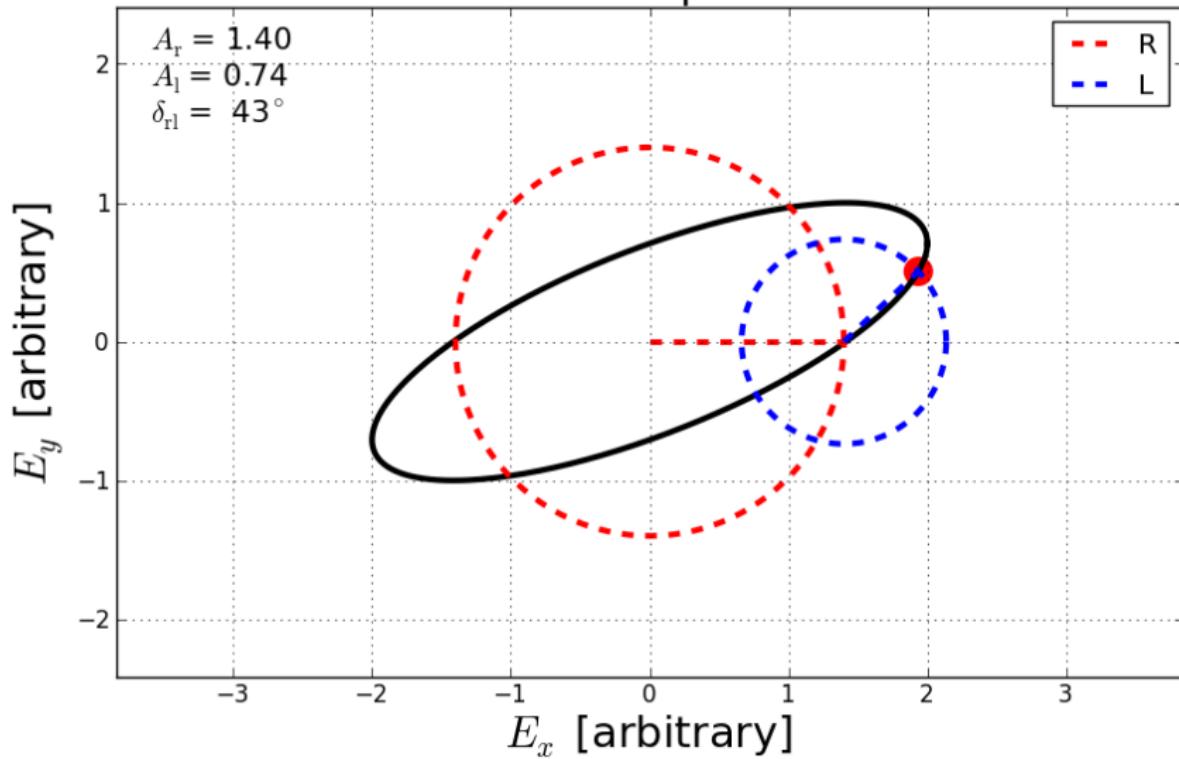


Viewing from antenna towards source, watching orientation and length of  $\mathbf{E}$  vector on a plane at a fixed location in space.

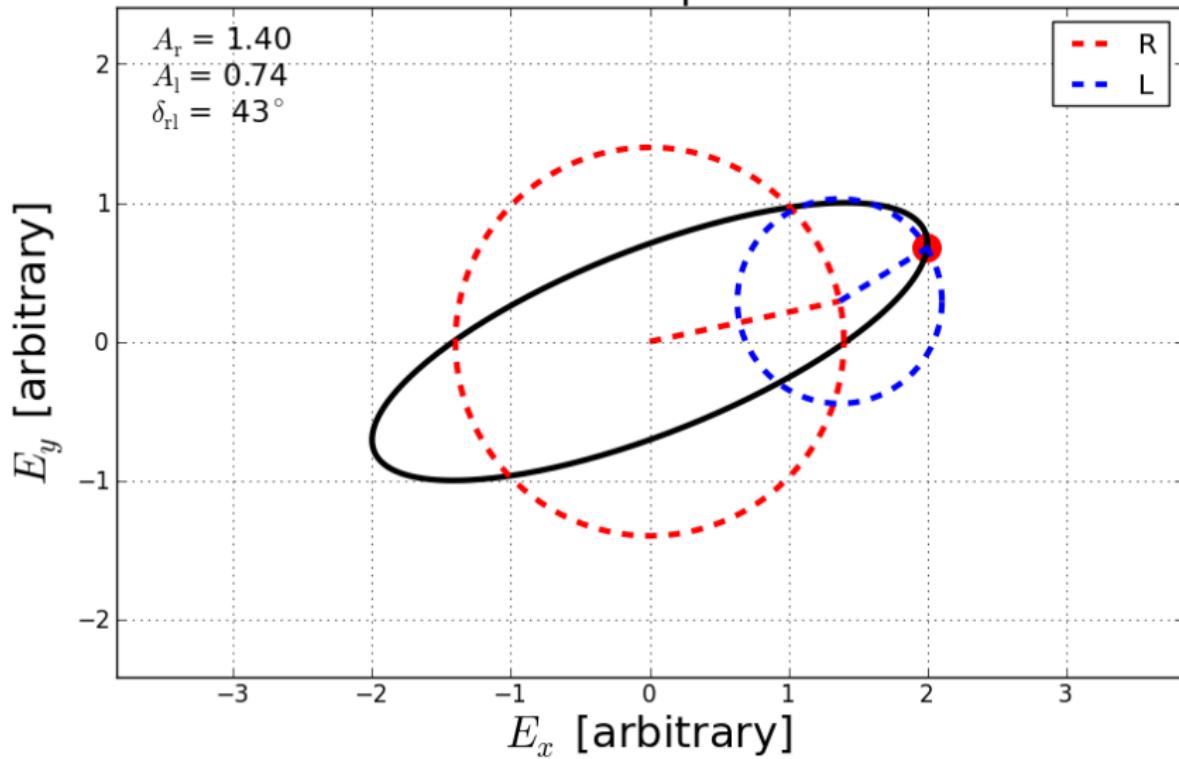
$$\begin{aligned}\mathbf{E} &= A_r \hat{\mathbf{e}}_r + A_l \hat{\mathbf{e}}_l \\ \hat{\mathbf{e}}_r &= \begin{pmatrix} \cos(2\pi\nu t + \delta_r) \\ \sin(2\pi\nu t + \delta_r) \end{pmatrix} \\ \hat{\mathbf{e}}_l &= \begin{pmatrix} \cos(2\pi\nu t + \delta_l) \\ -\sin(2\pi\nu t + \delta_l) \end{pmatrix}\end{aligned}$$

- $A_r + A_l$  = semi-major axis
- $\|A_r - A_l\|$  = semi-minor axis
- $\delta_{rl} = \delta_r - \delta_l$
- $\delta_{rl}$  = orientation of major axis
- $\delta_{rl} > 0$ : MA rotated CCW
- $\delta_{rl} = 0$ : MA along  $x$ -axis
- $\delta_{rl} < 0$ : MA rotated CW

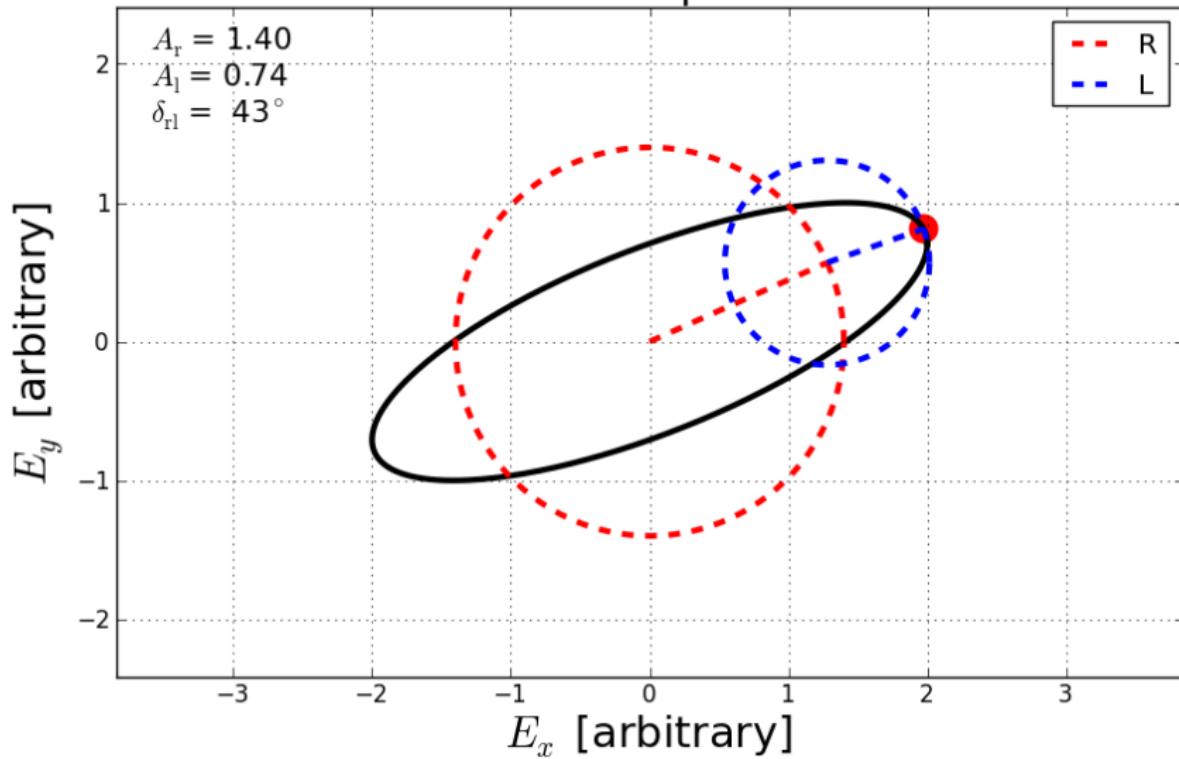
Polarization ellipse: circular



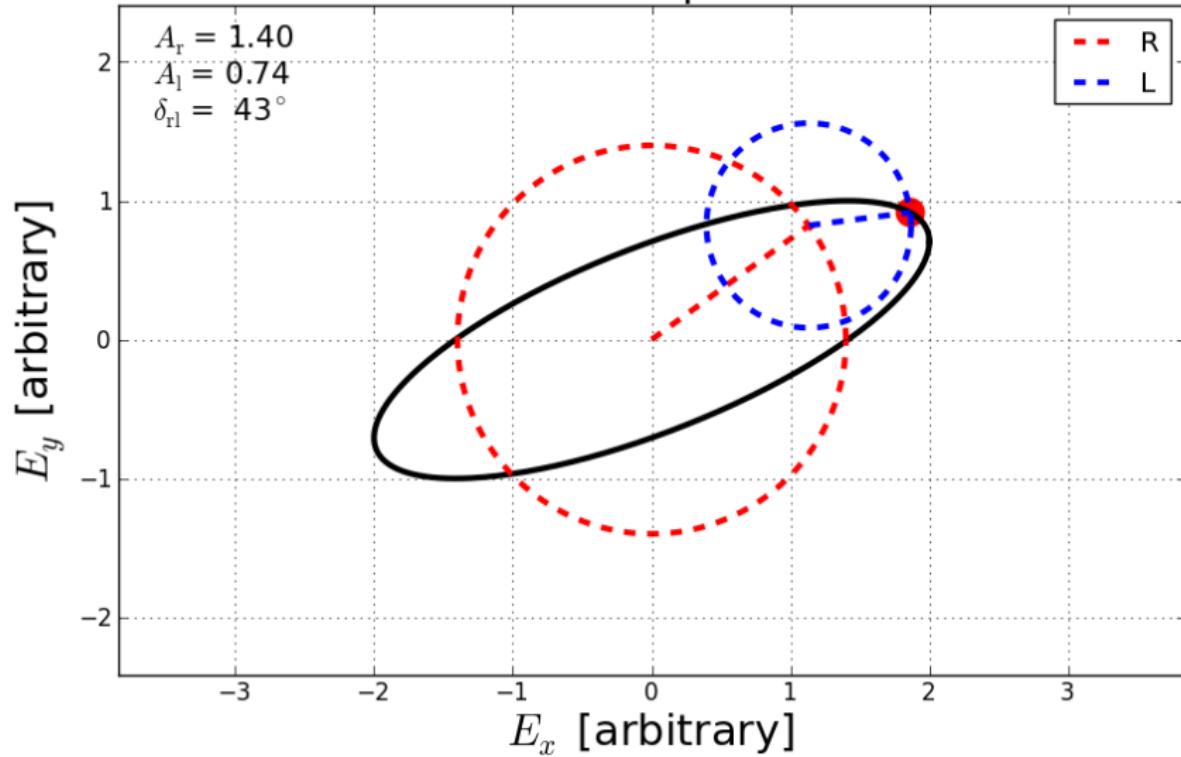
Polarization ellipse: circular



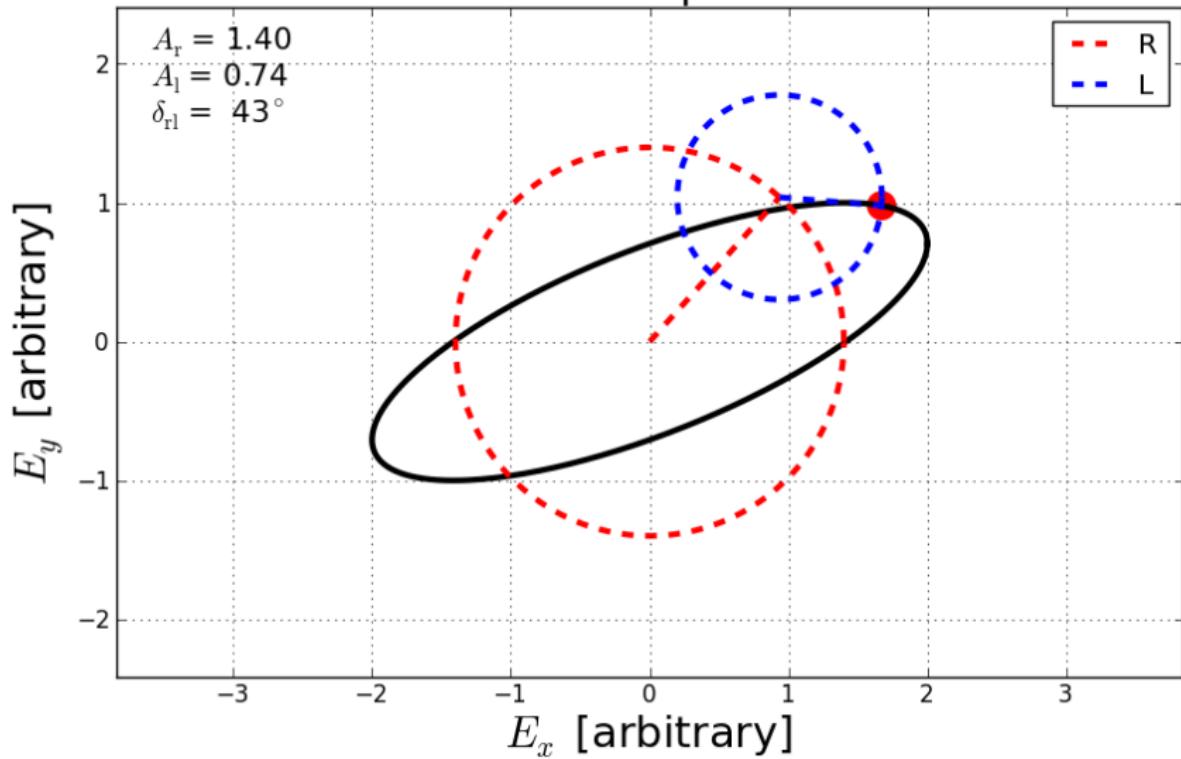
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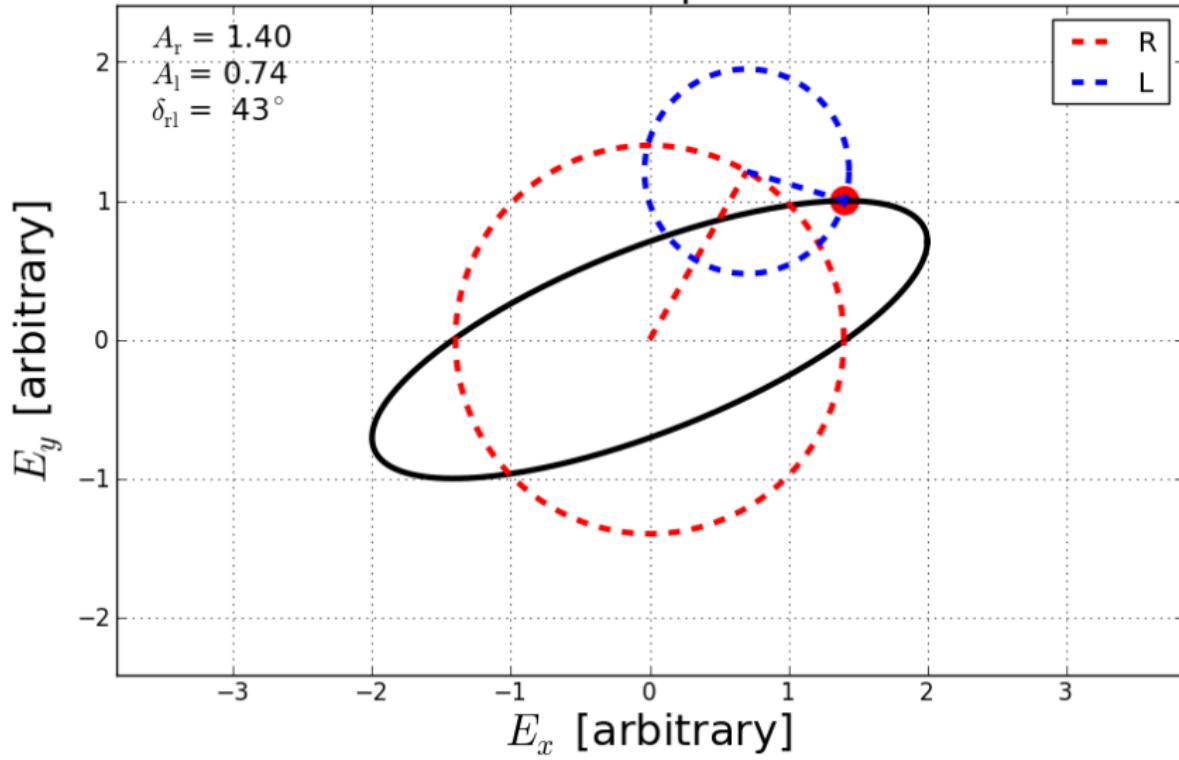
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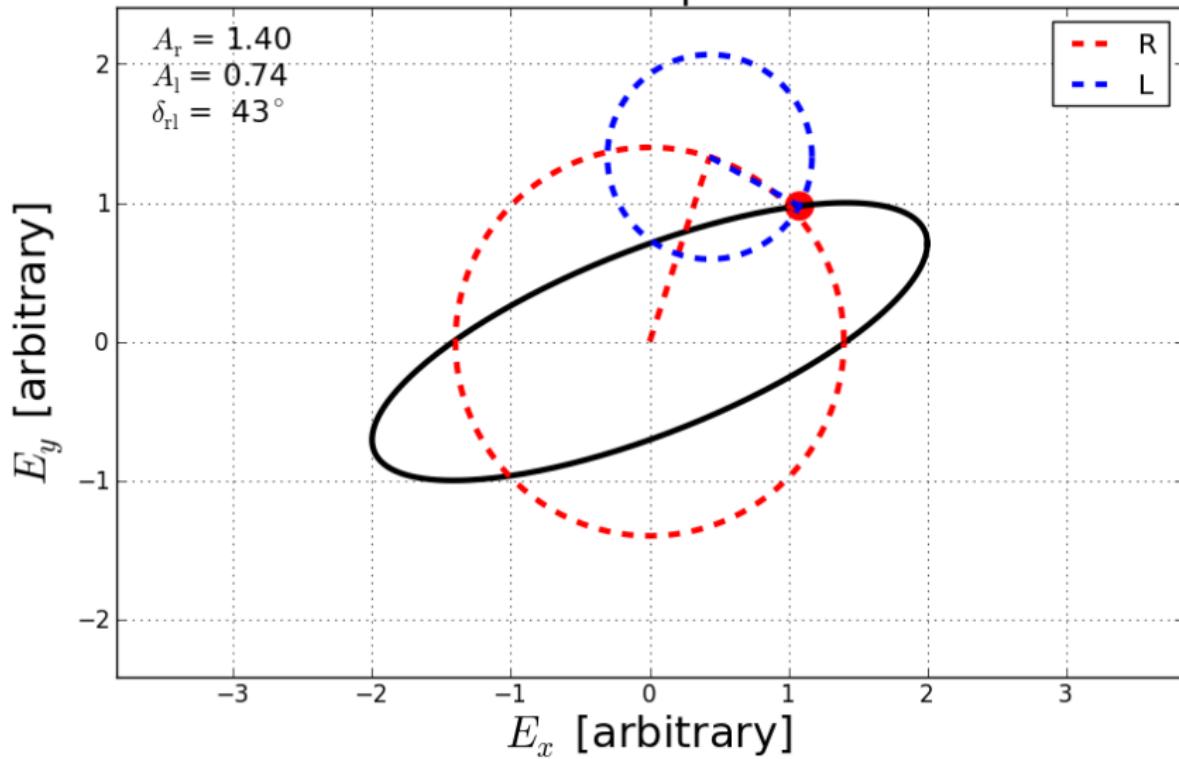
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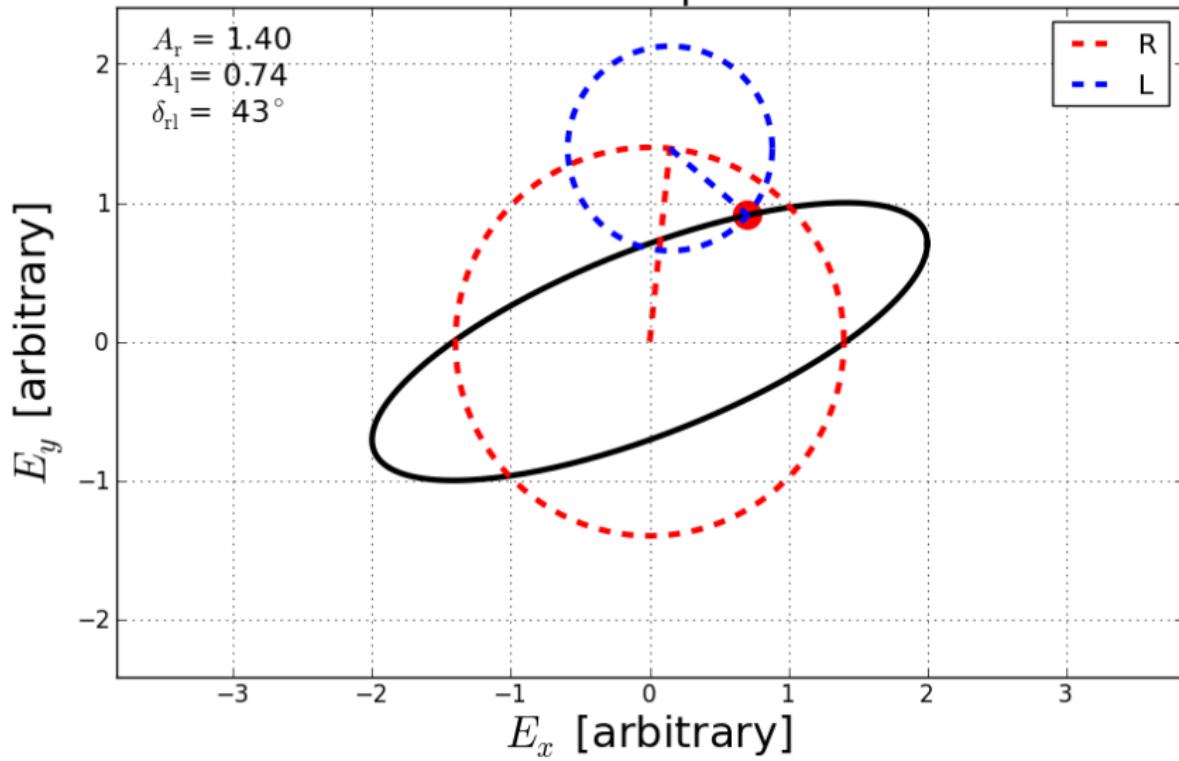
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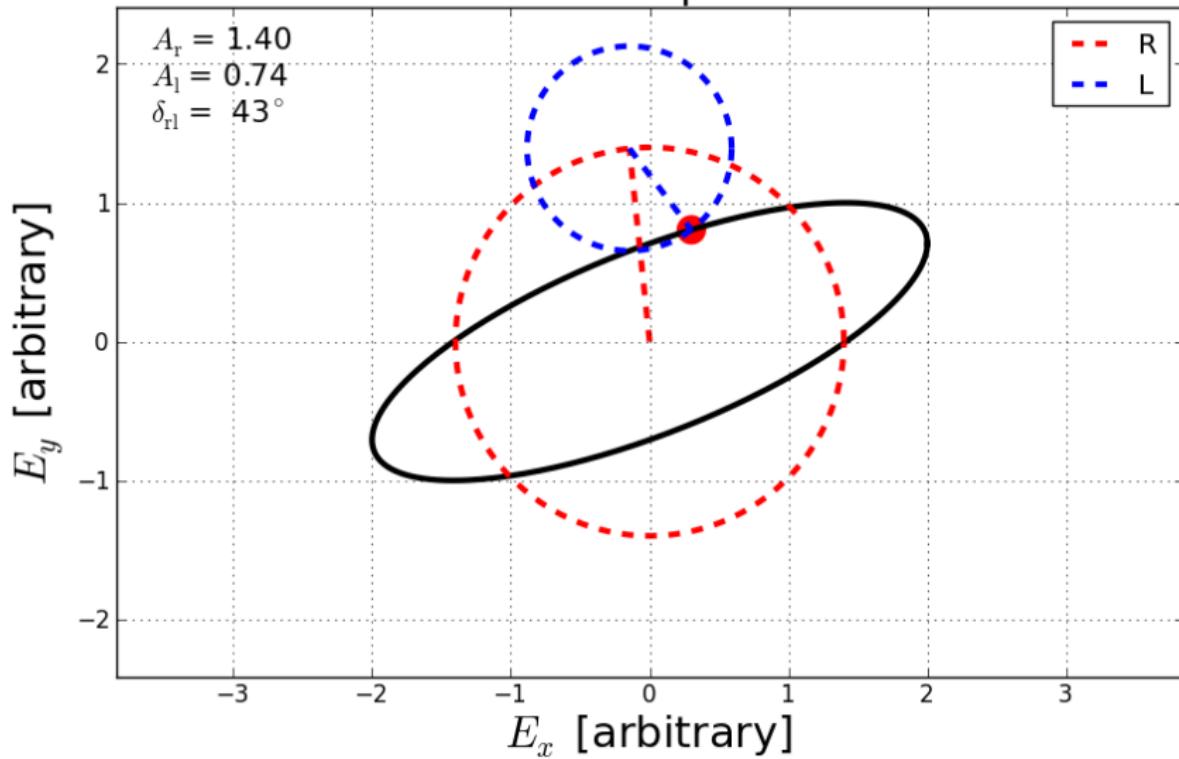
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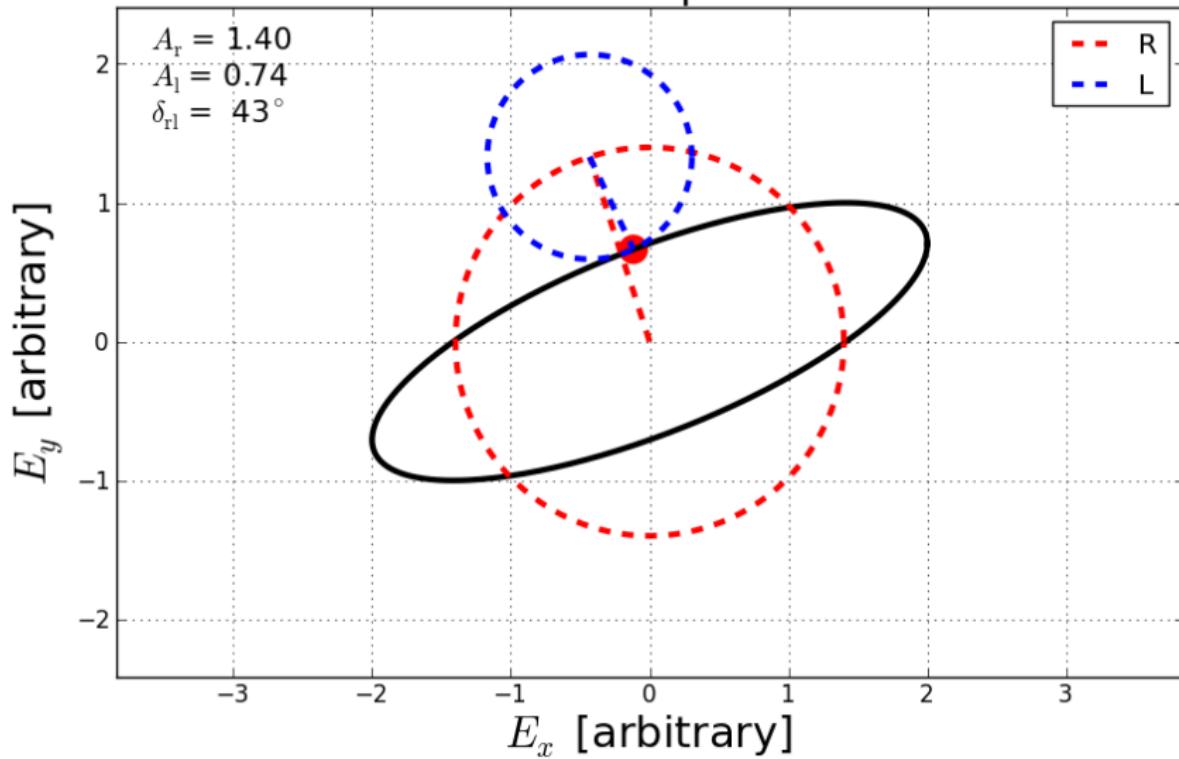
Polarization ellipse: circular



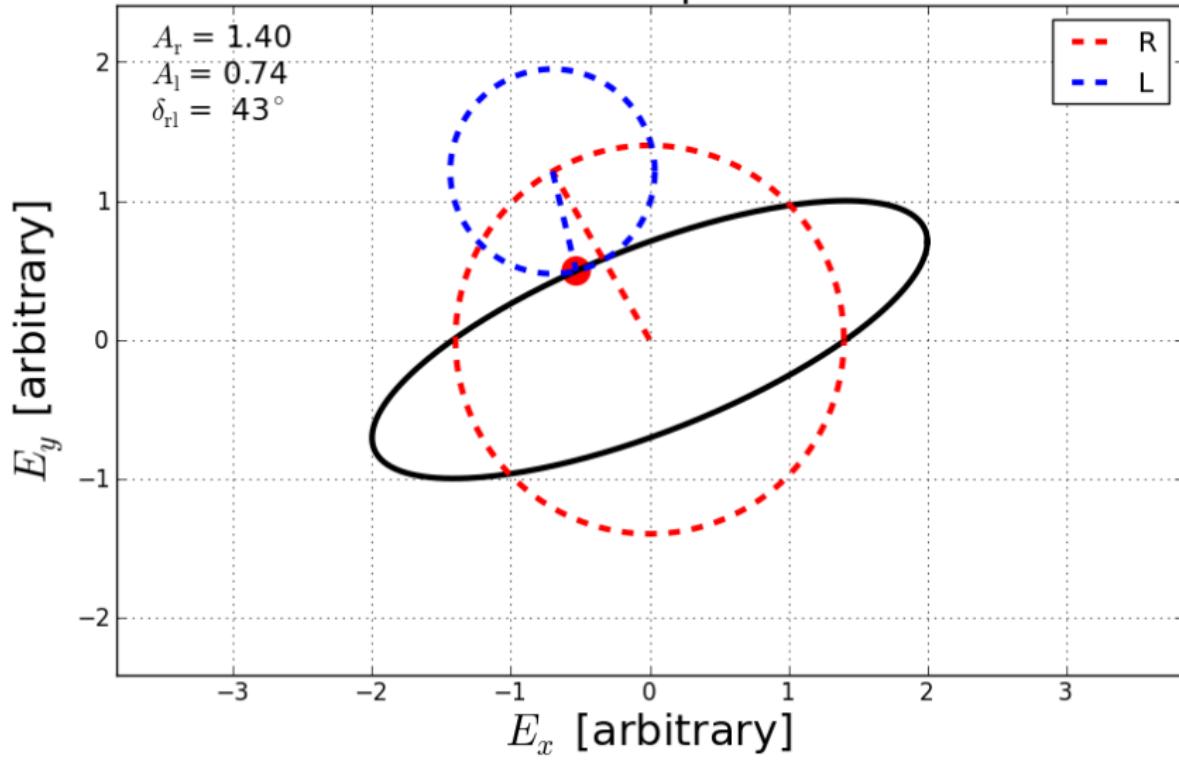
Polarization ellipse: circular



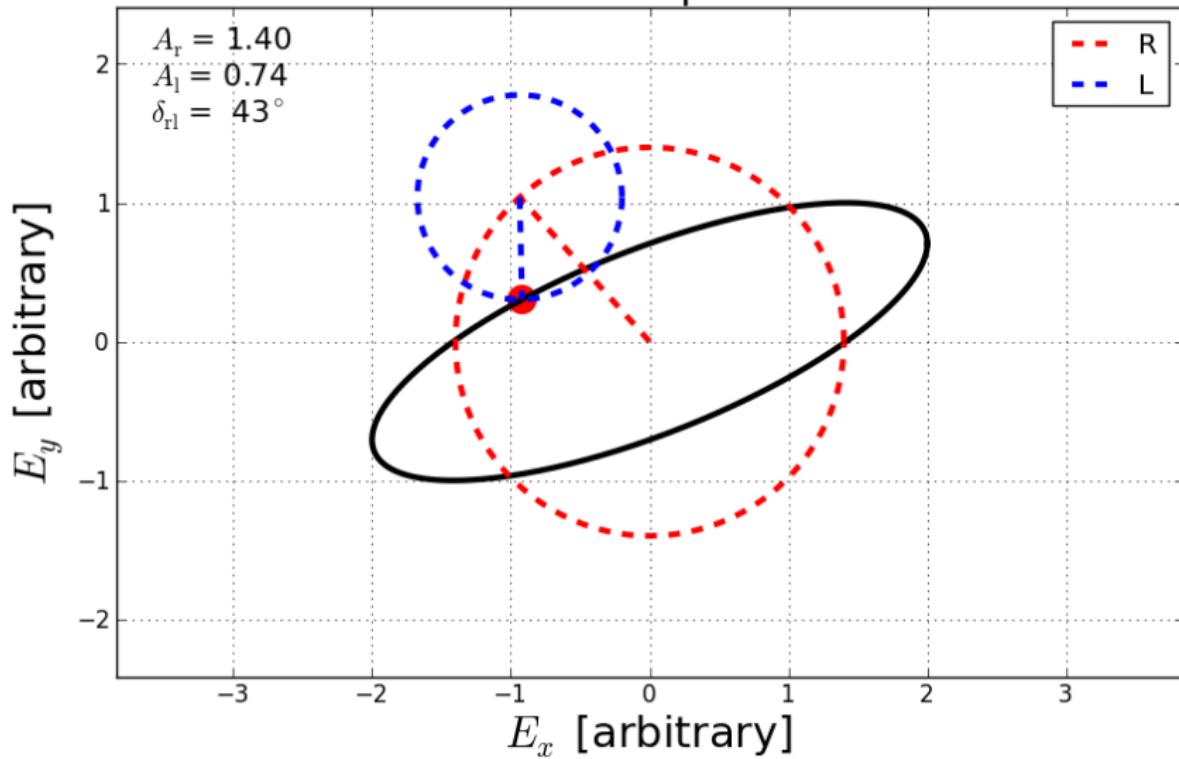
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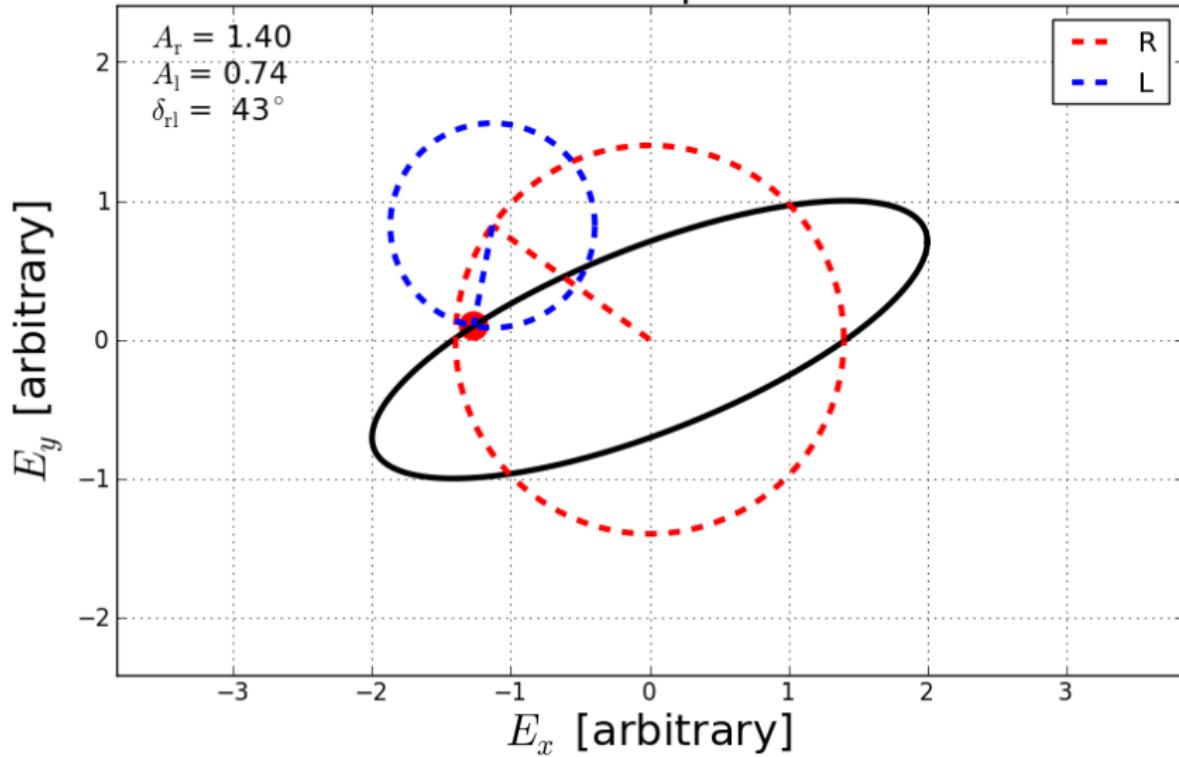
Polarization ellipse: circular



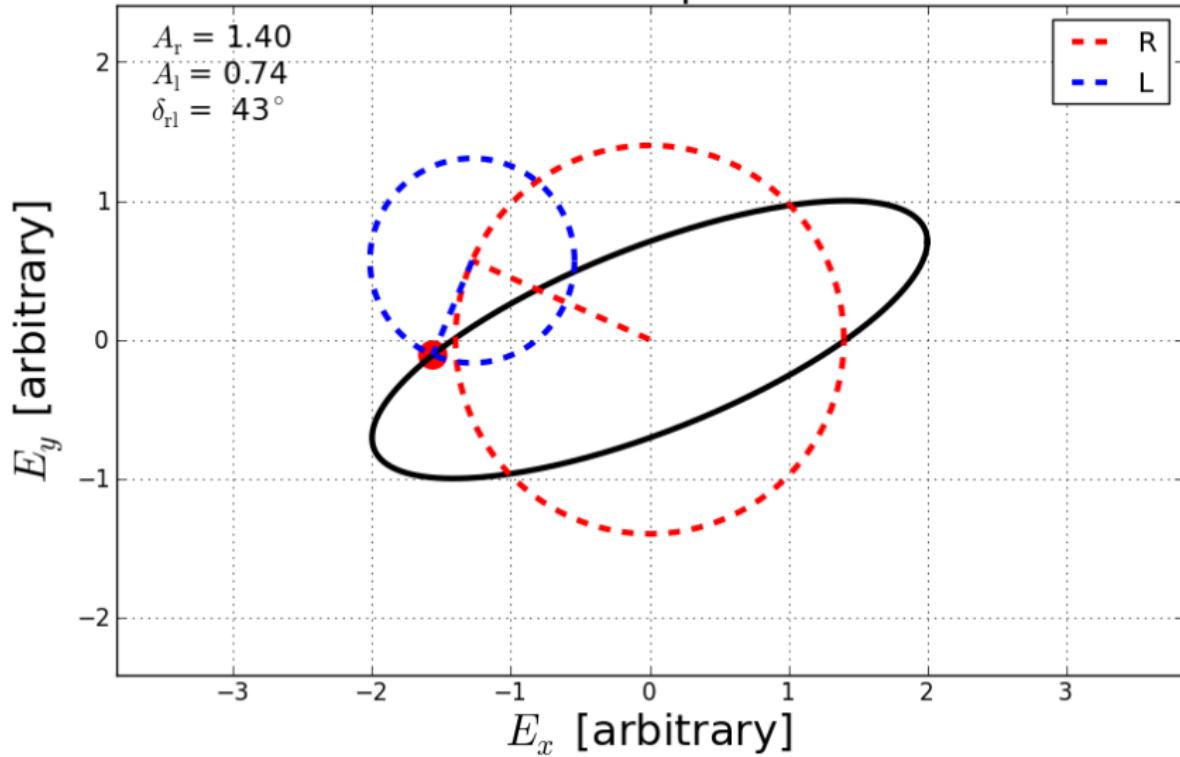
Polarization ellipse: circular



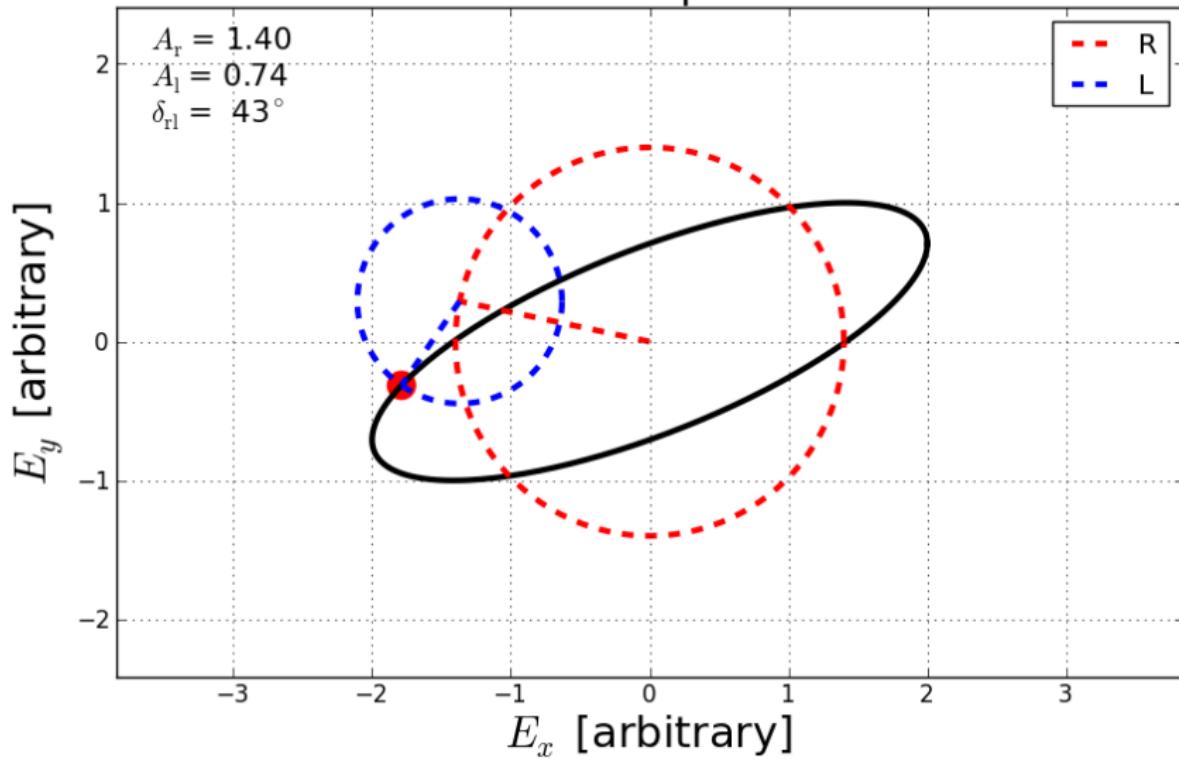
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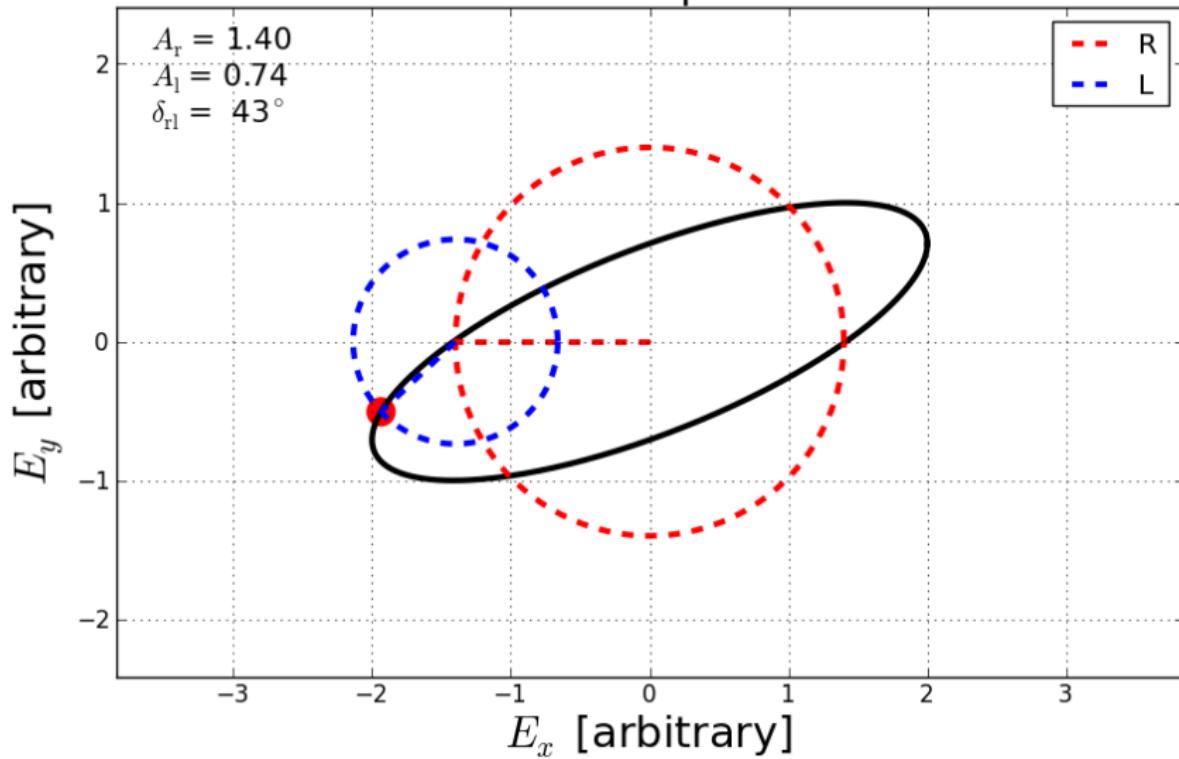
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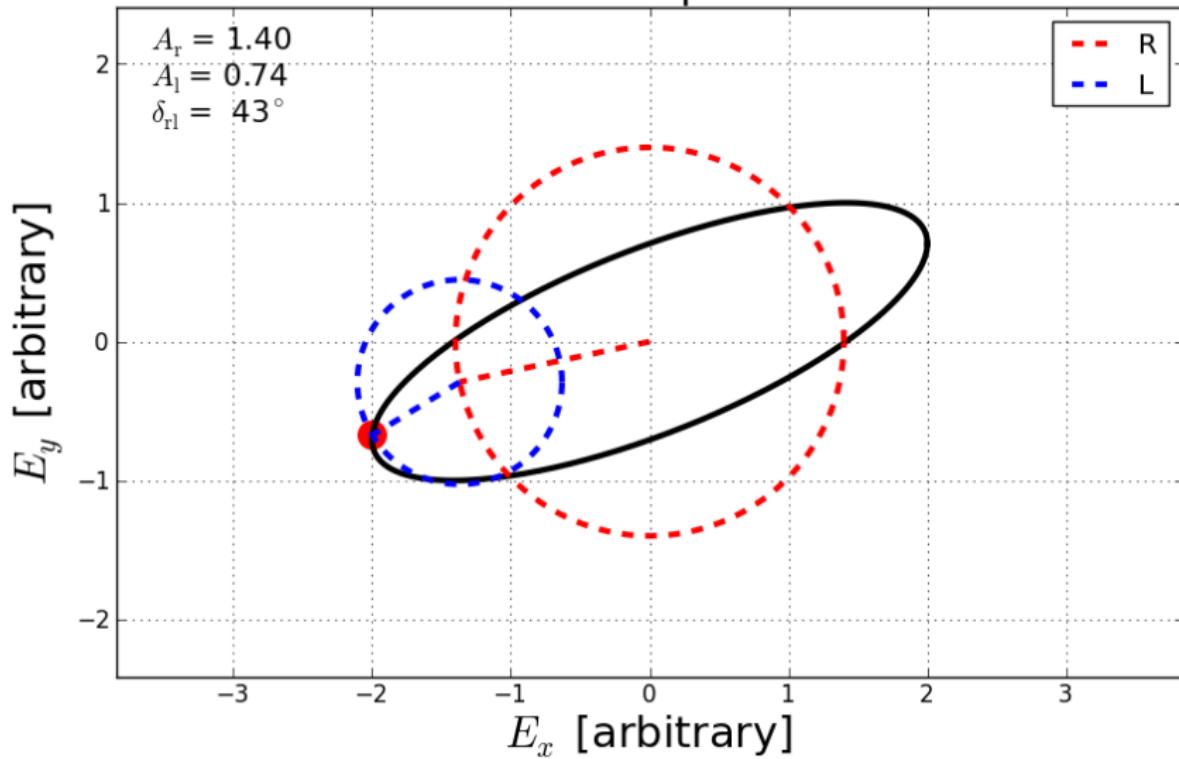
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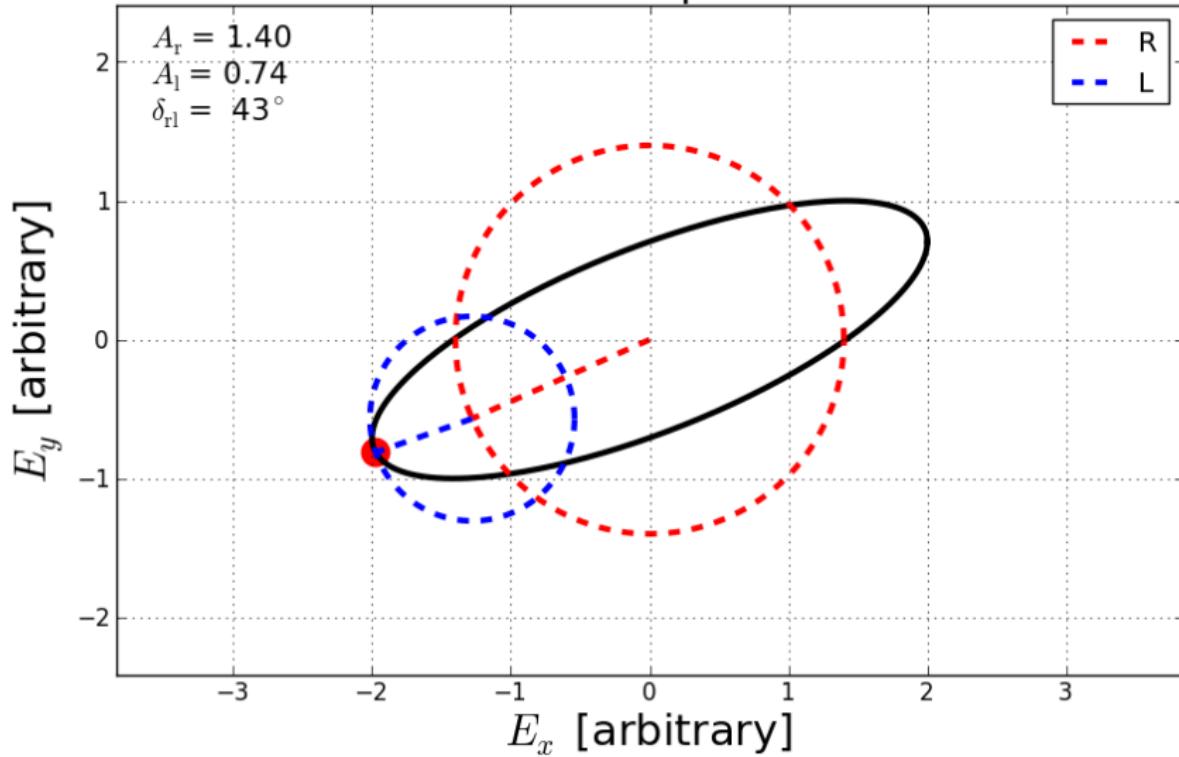
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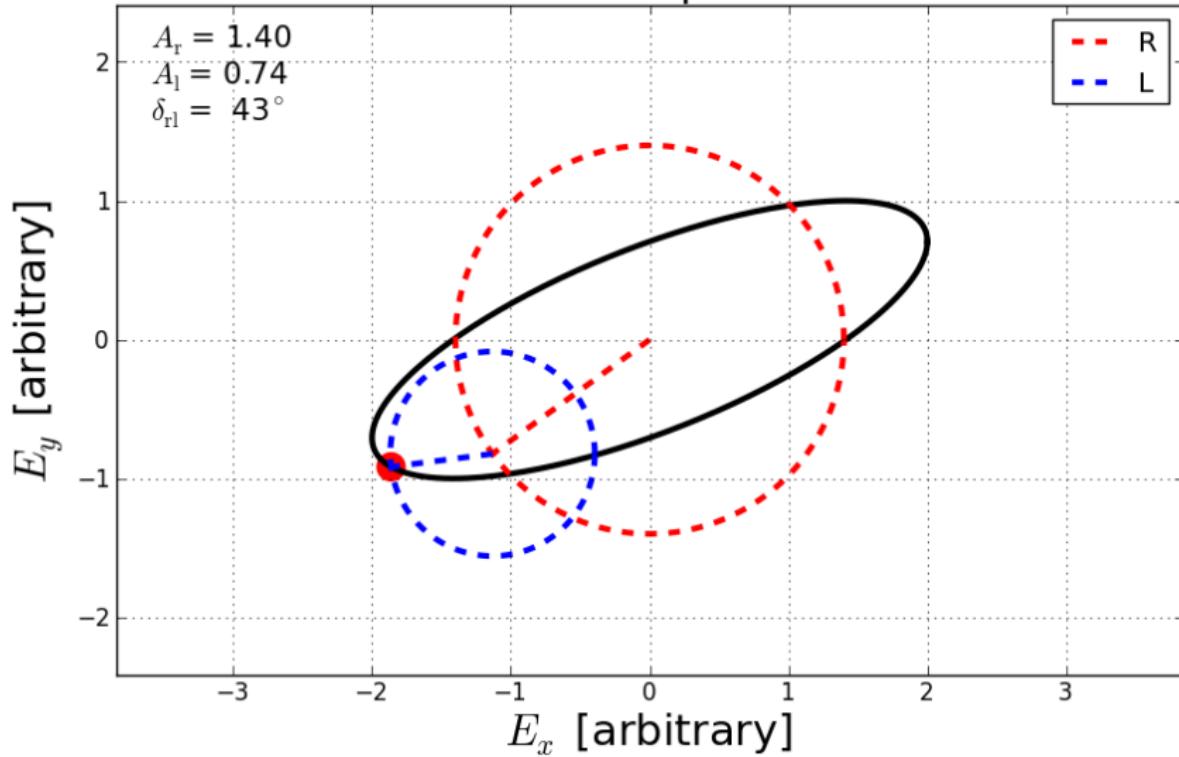
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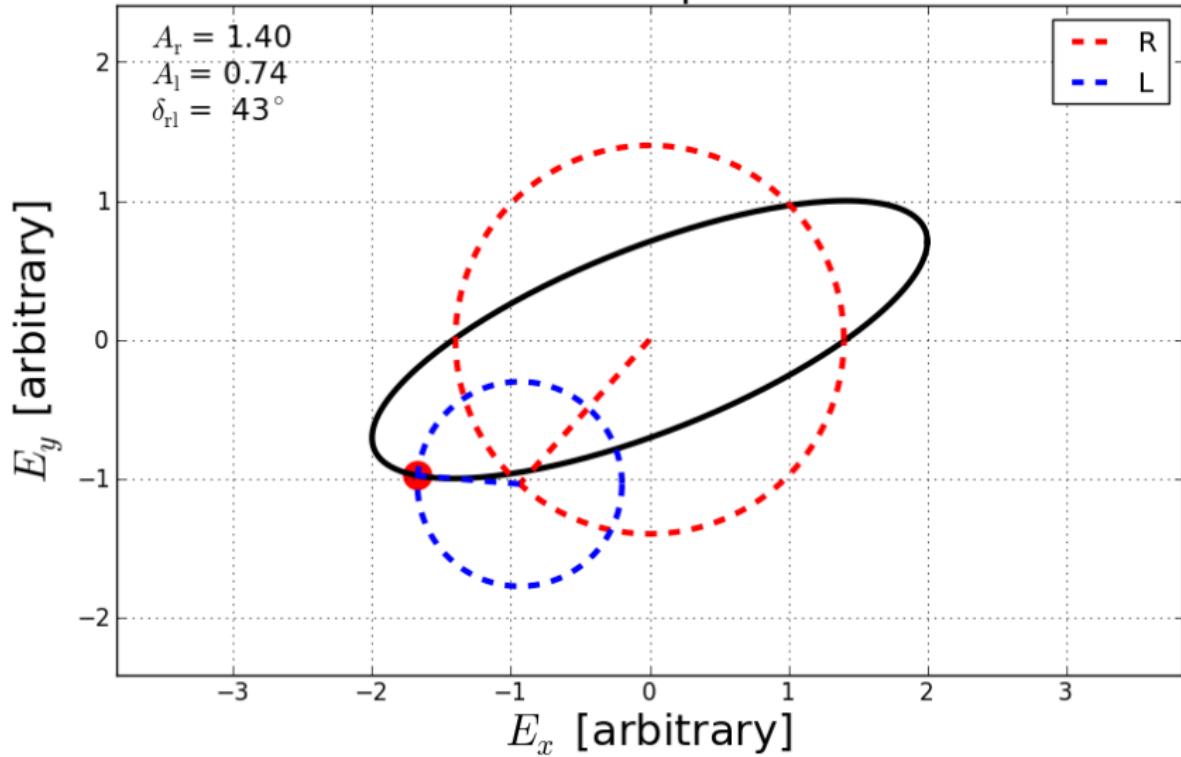
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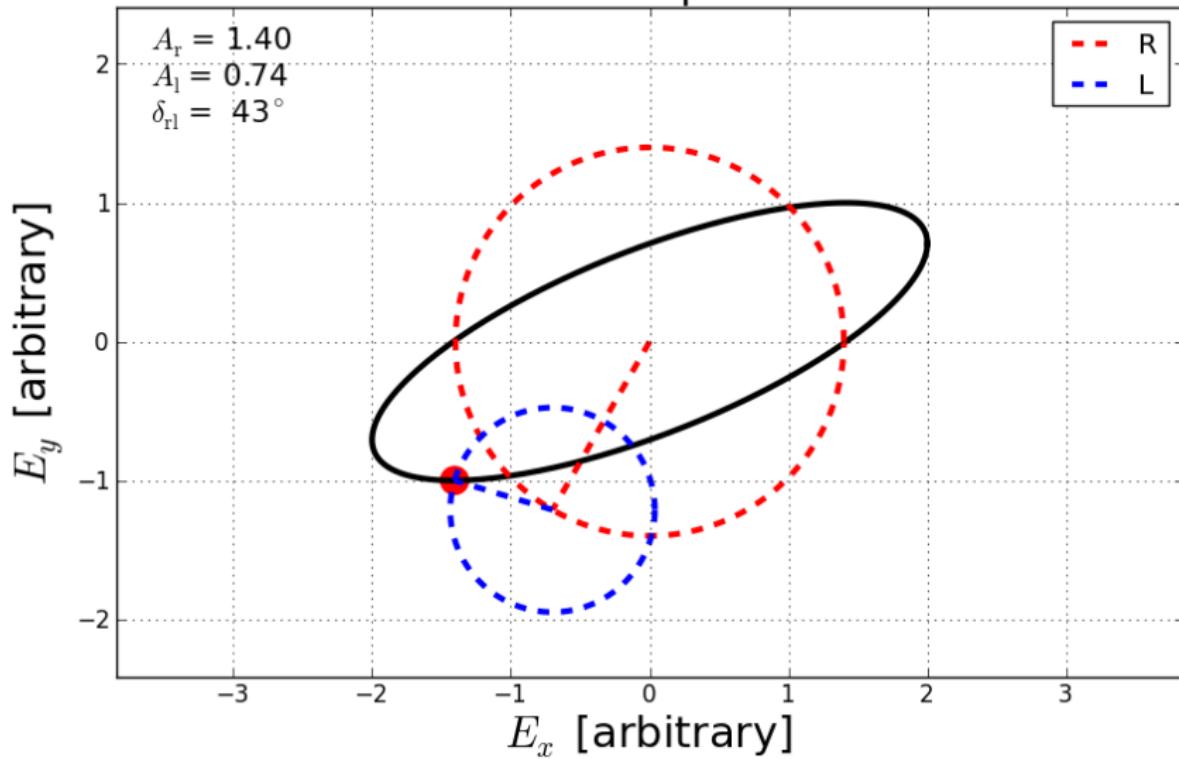
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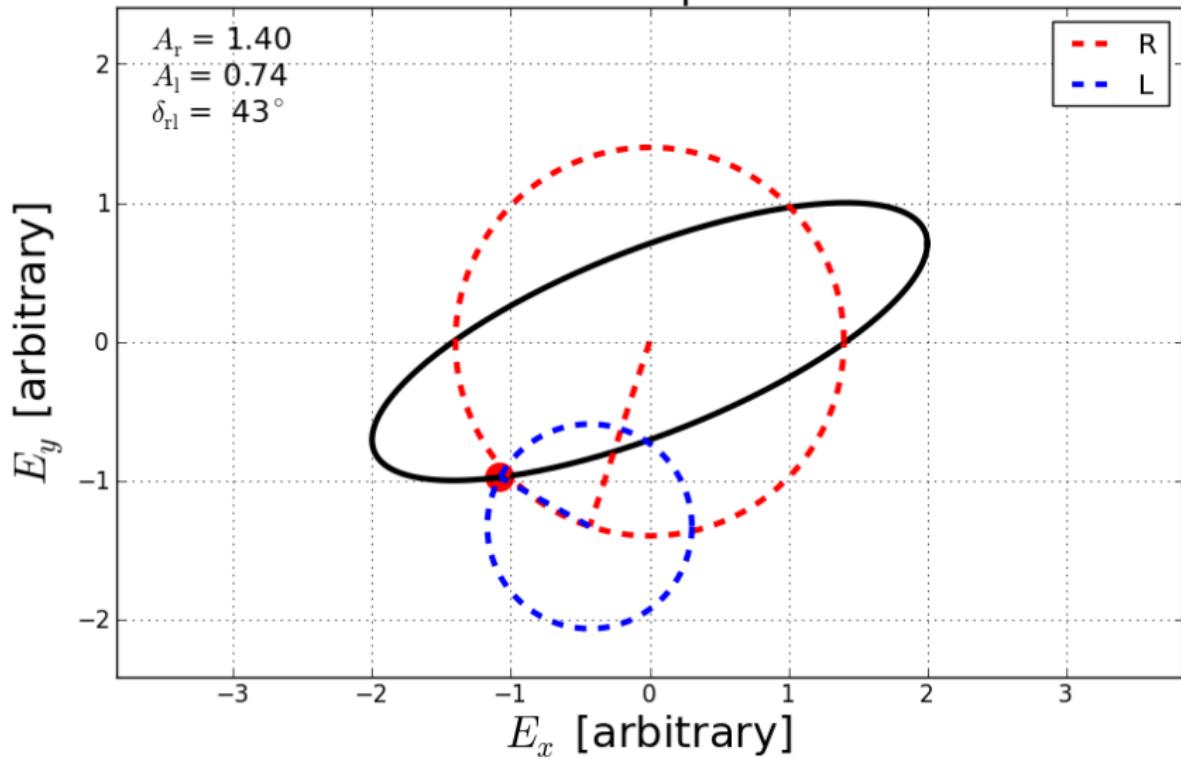
Polarization ellipse: circular



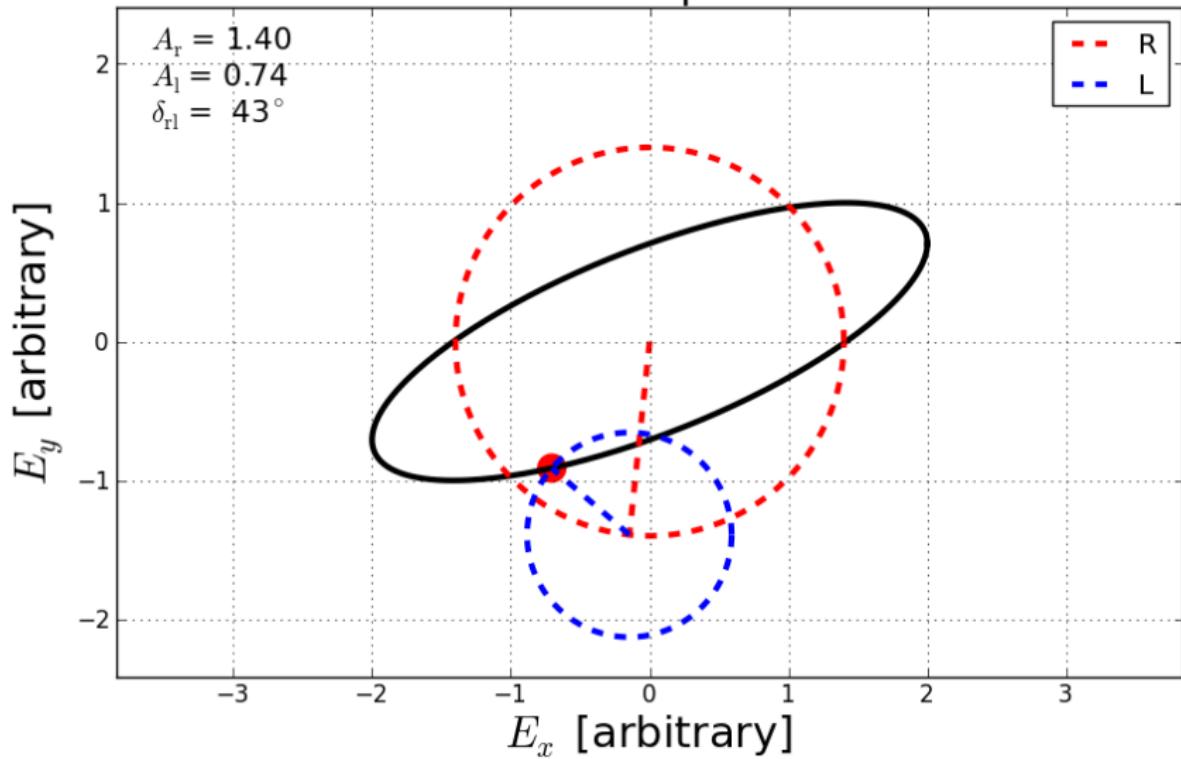
Polarization ellipse: circular



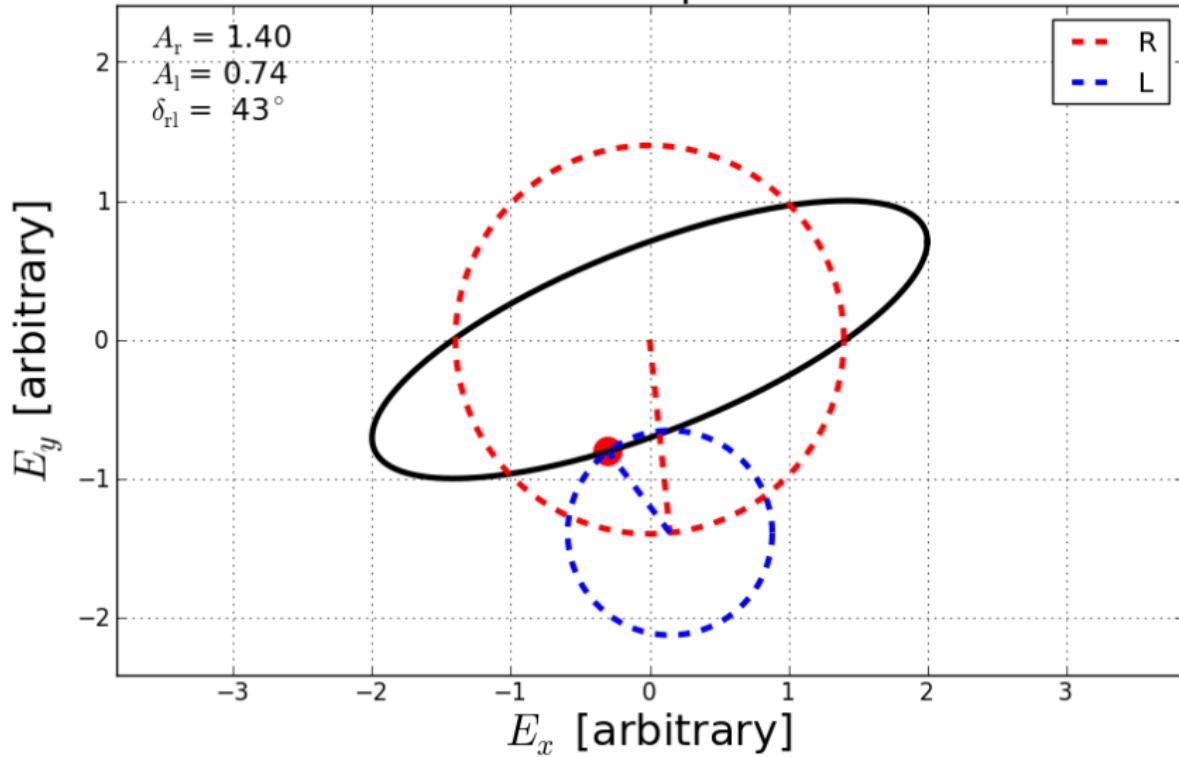
Polarization ellipse: circular



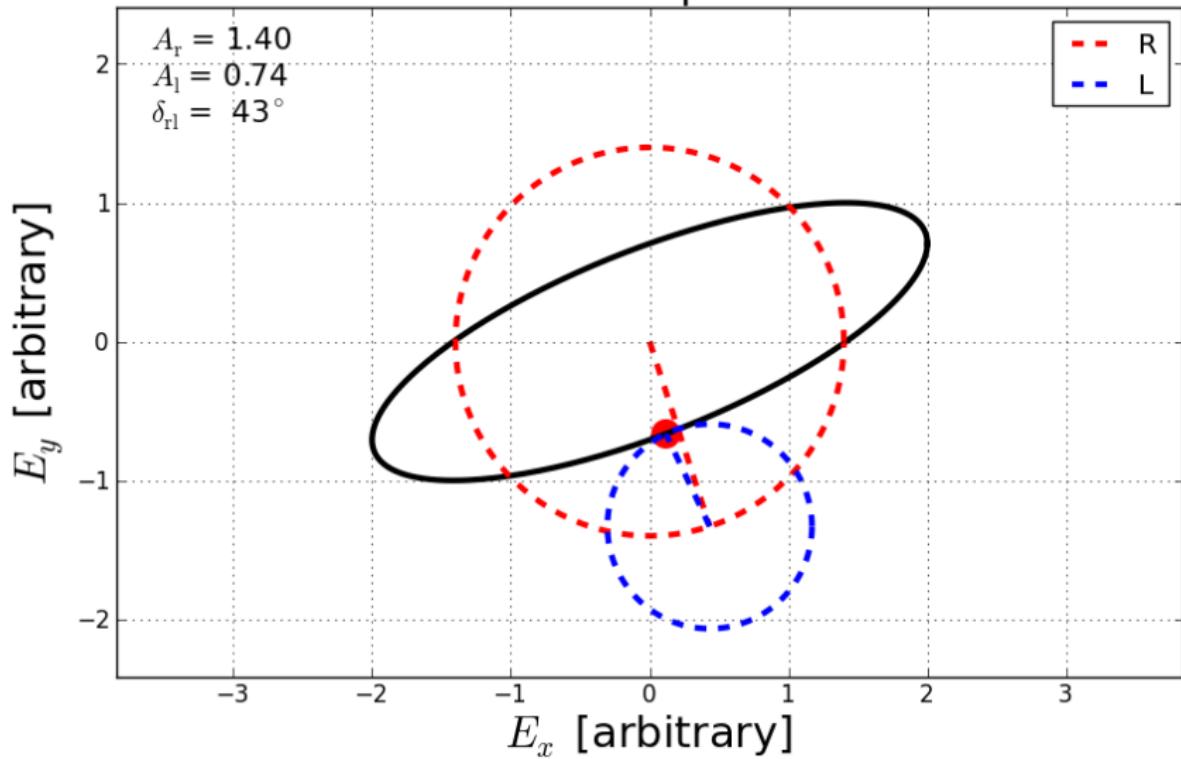
Polarization ellipse: circular



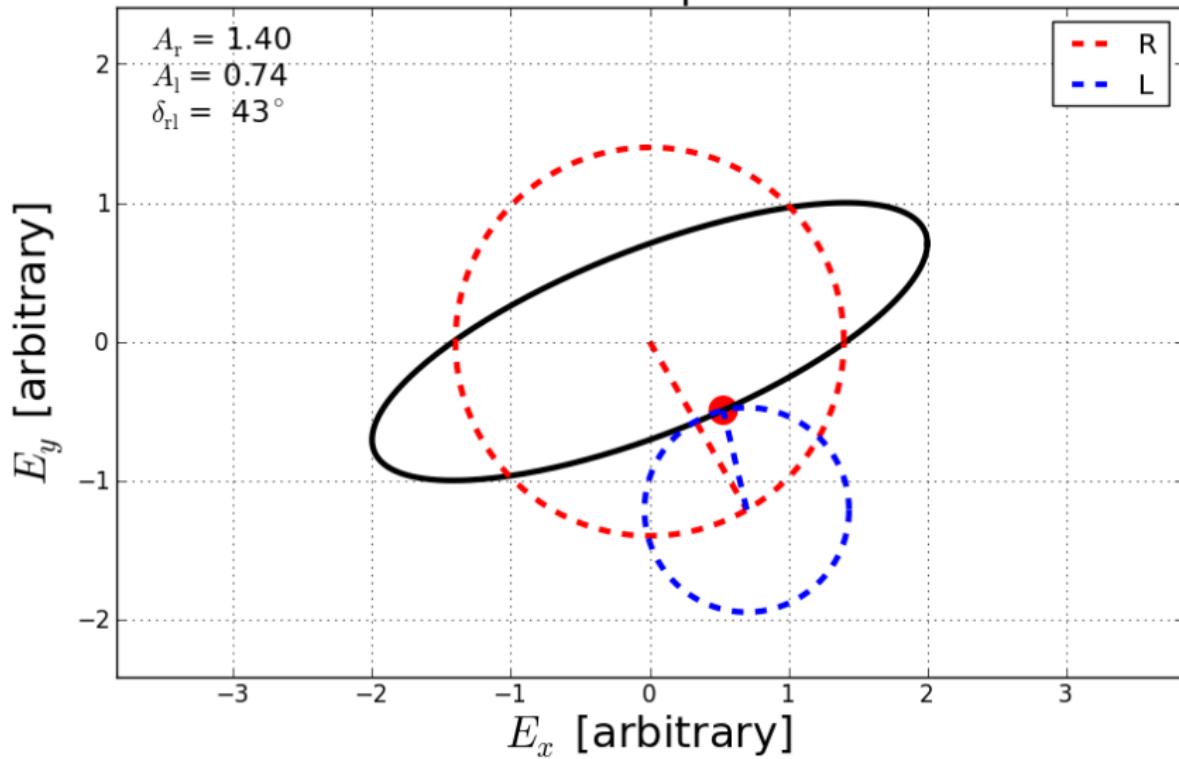
Polarization ellipse: circular



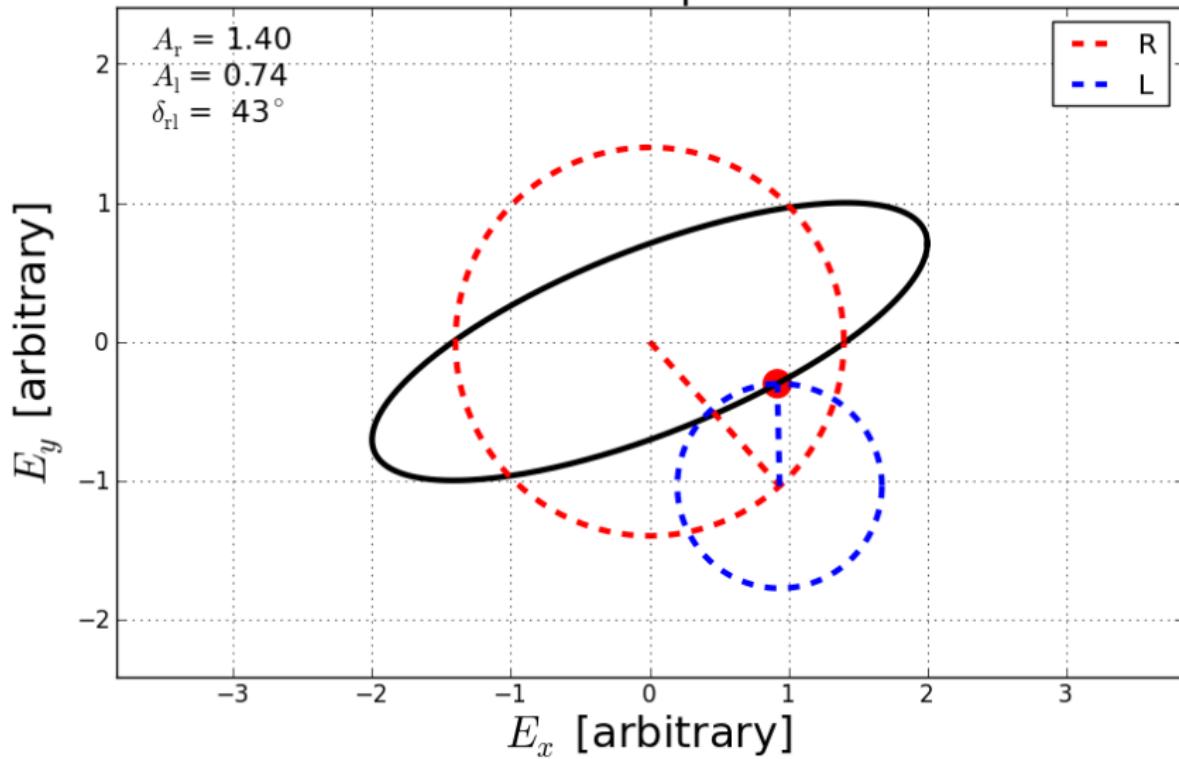
Polarization ellipse: circular



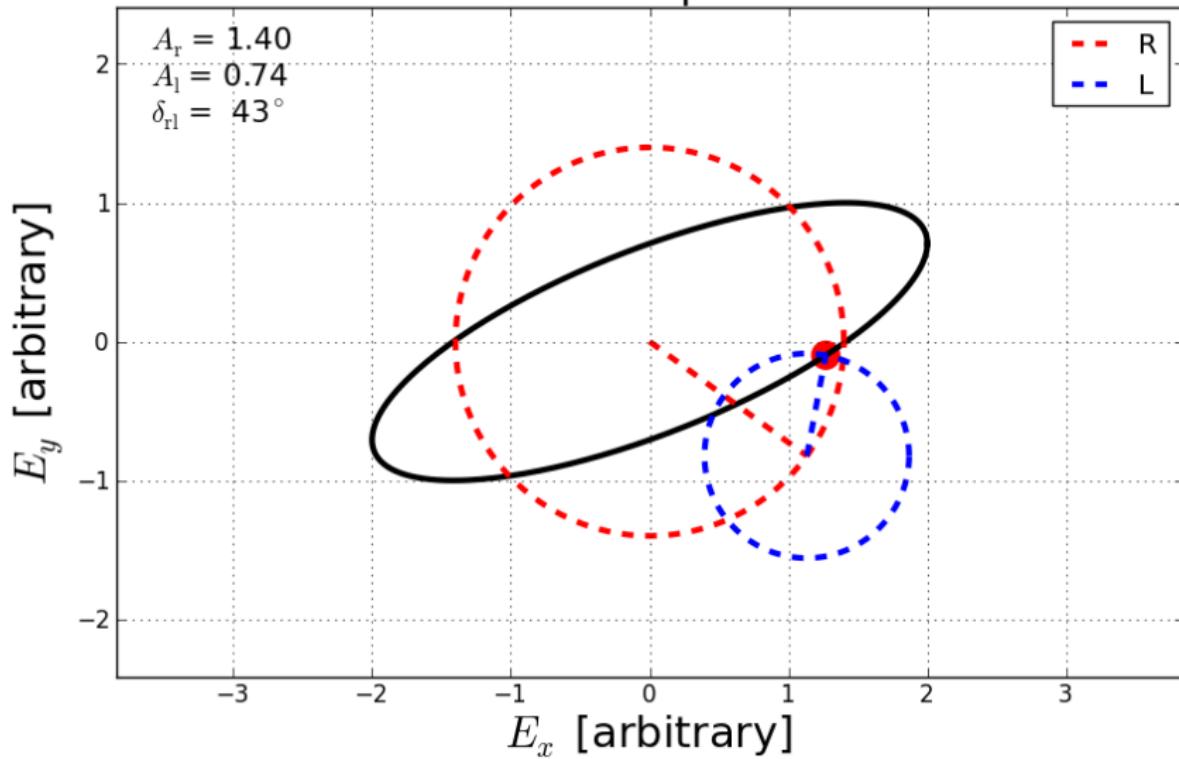
Polarization ellipse: circular



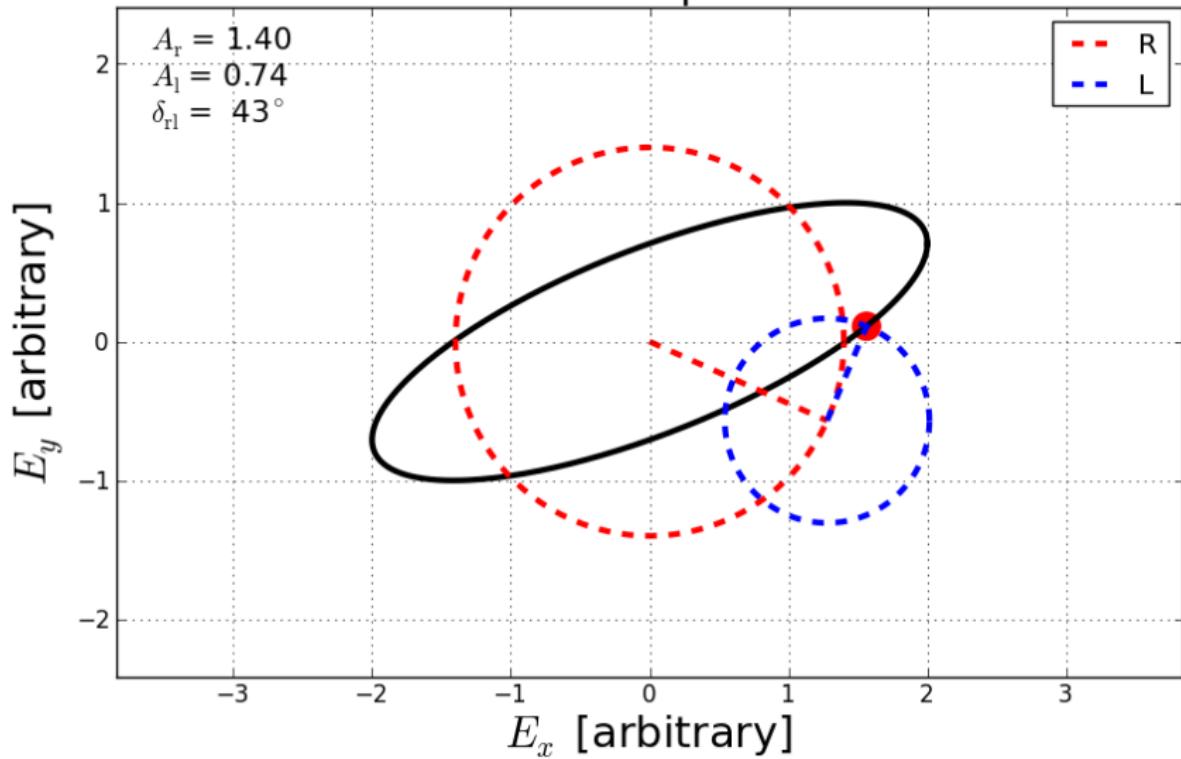
Polarization ellipse: circular



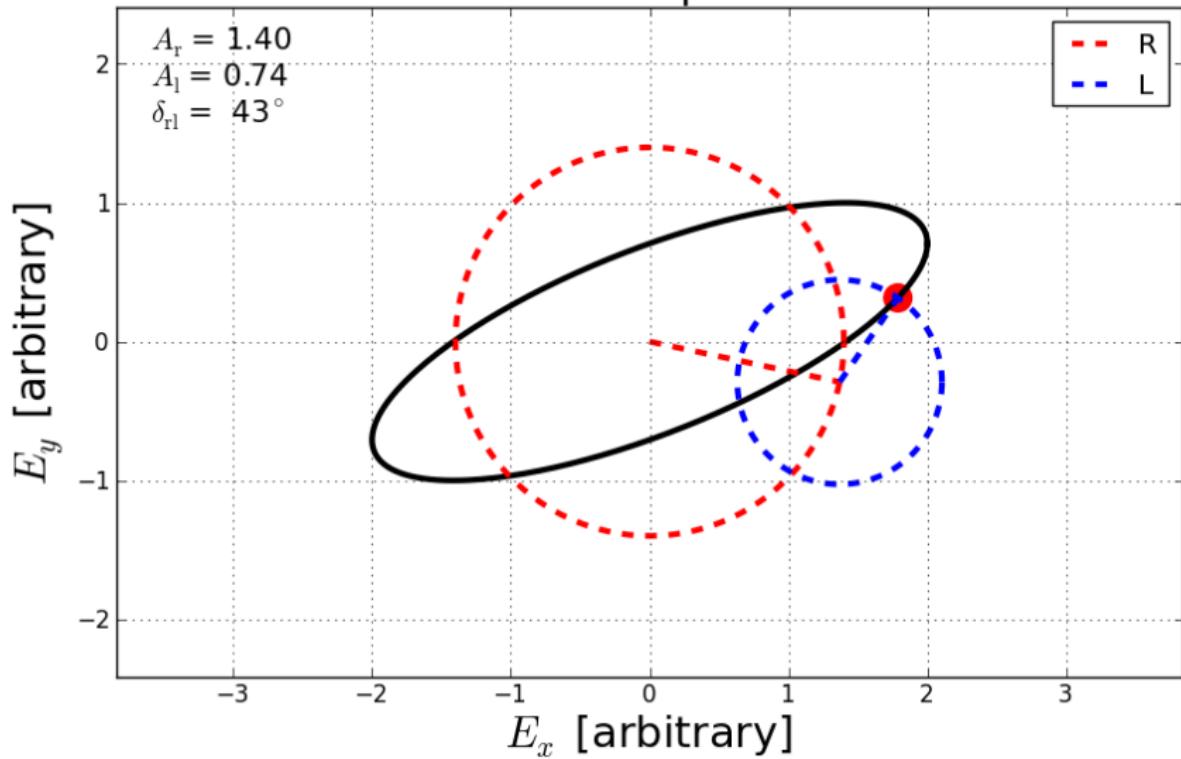
## Polarization ellipse: circular



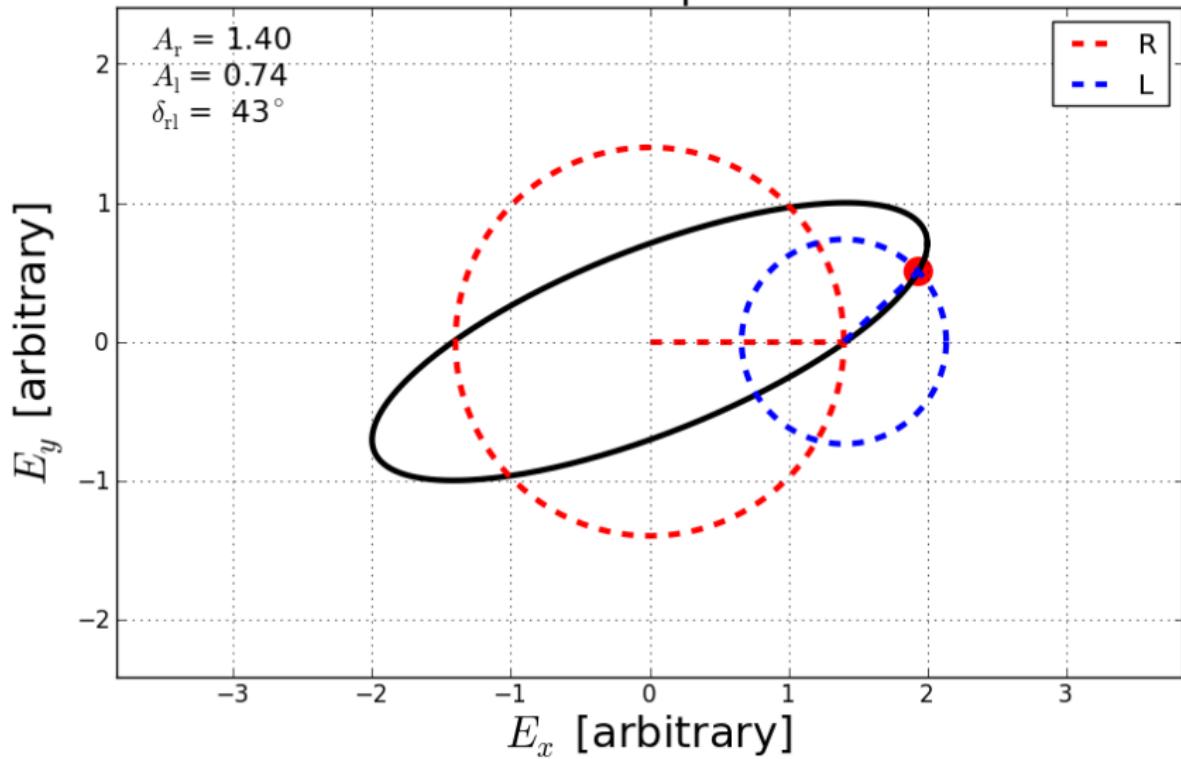
Polarization ellipse: circular



Polarization ellipse: circular



Polarization ellipse: circular

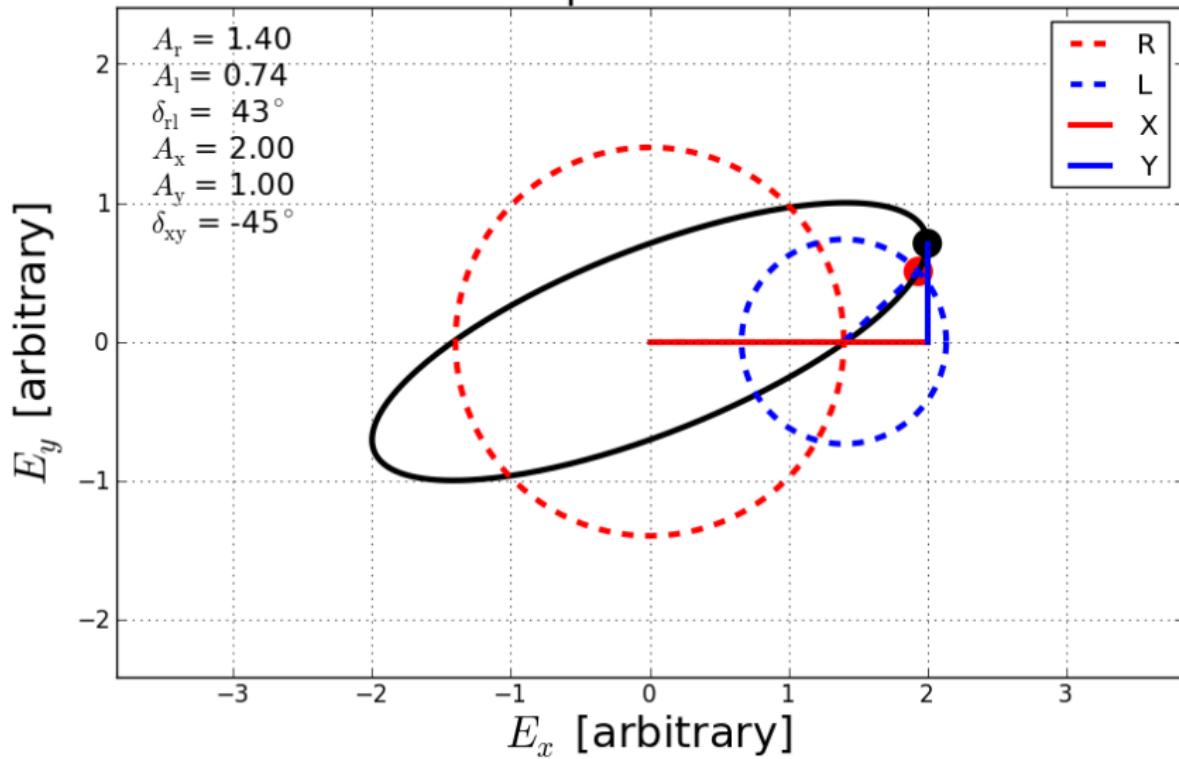


$$A_r = \frac{1}{2} \sqrt{A_x^2 + A_y^2 - 2A_x A_y \sin \delta_{xy}}$$

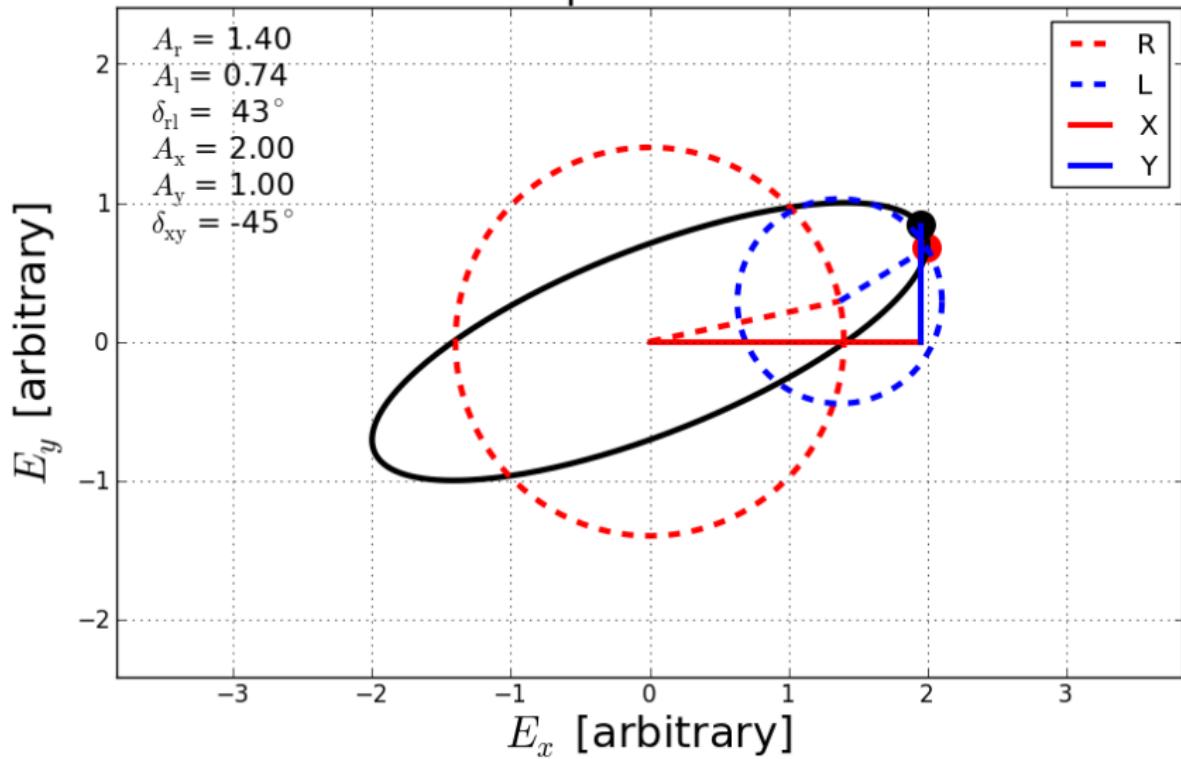
$$A_l = \frac{1}{2} \sqrt{A_x^2 + A_y^2 + 2A_x A_y \sin \delta_{xy}}$$

$$\tan \delta_{rl} = \frac{2A_x A_y \cos \delta_{xy}}{A_x^2 - A_y^2}$$

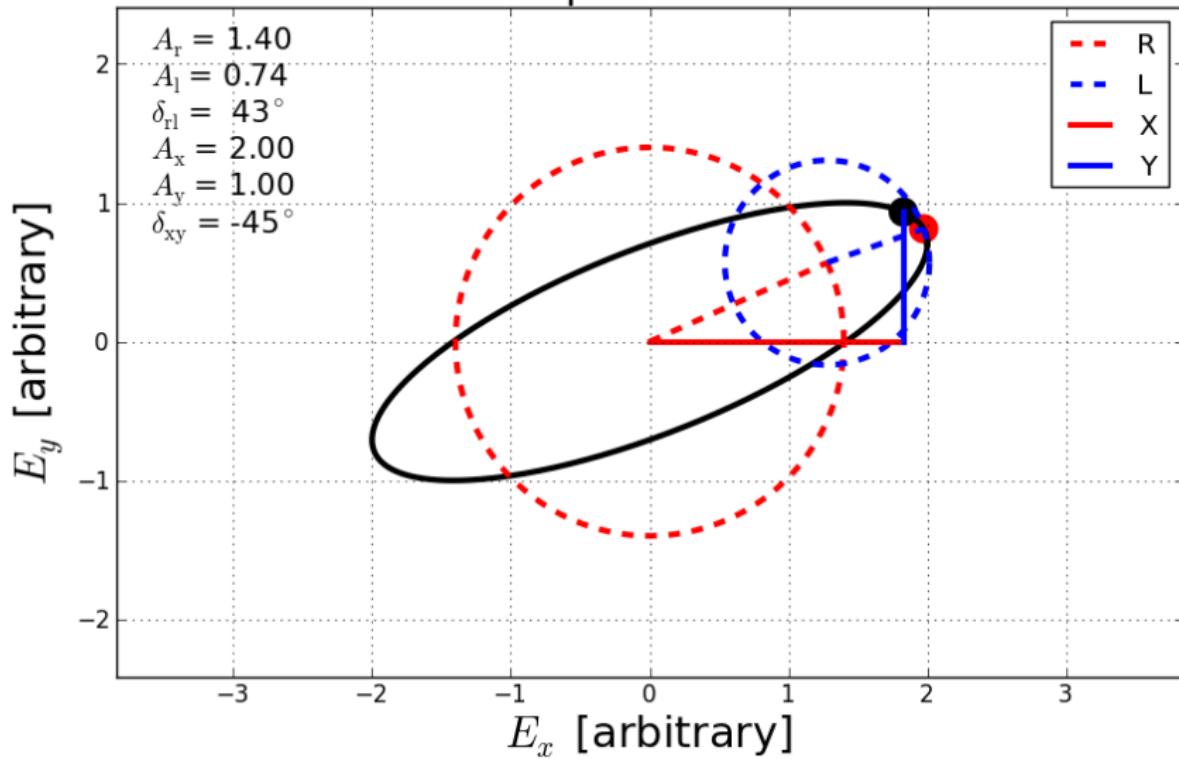
## Polarization ellipse: circular and linear



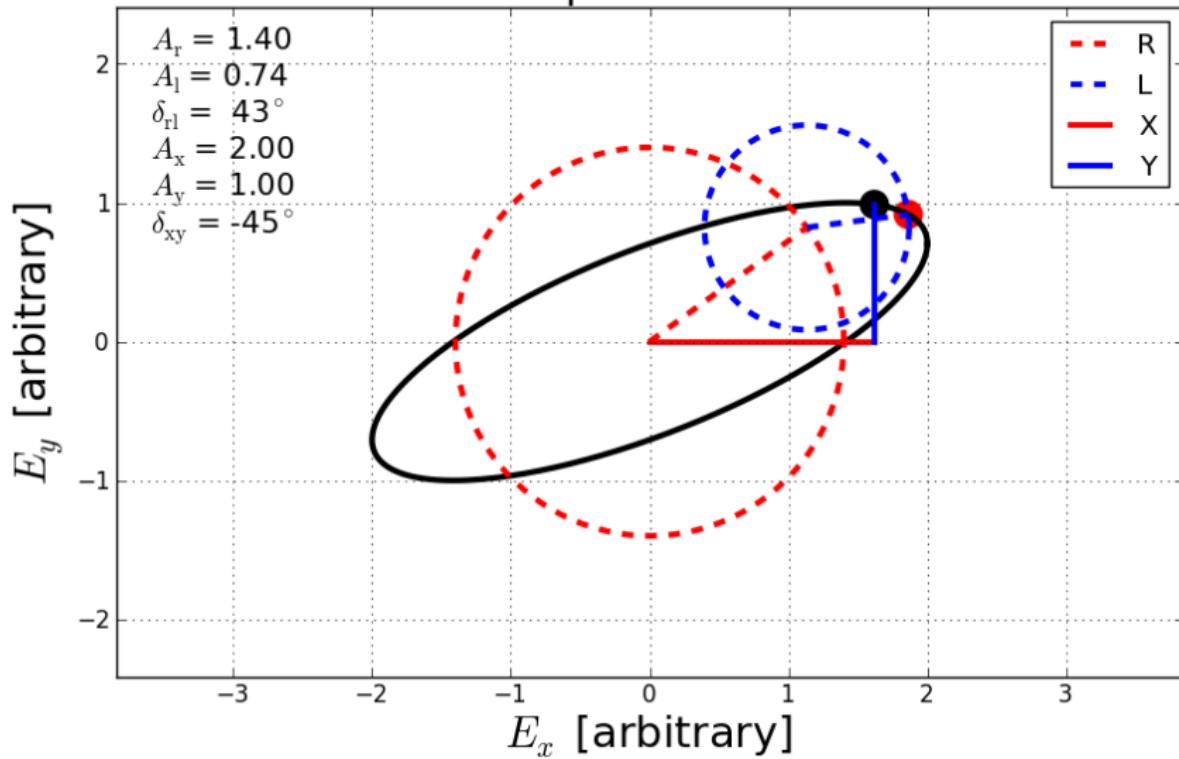
## Polarization ellipse: circular and linear



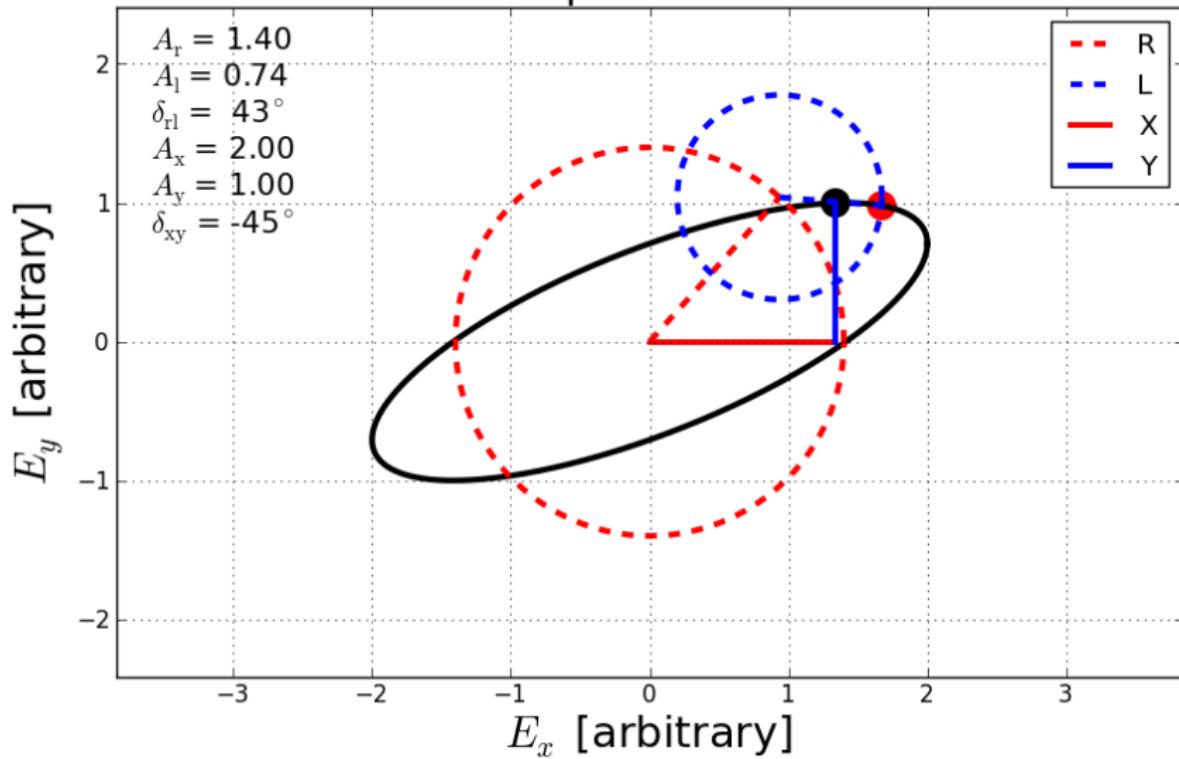
## Polarization ellipse: circular and linear



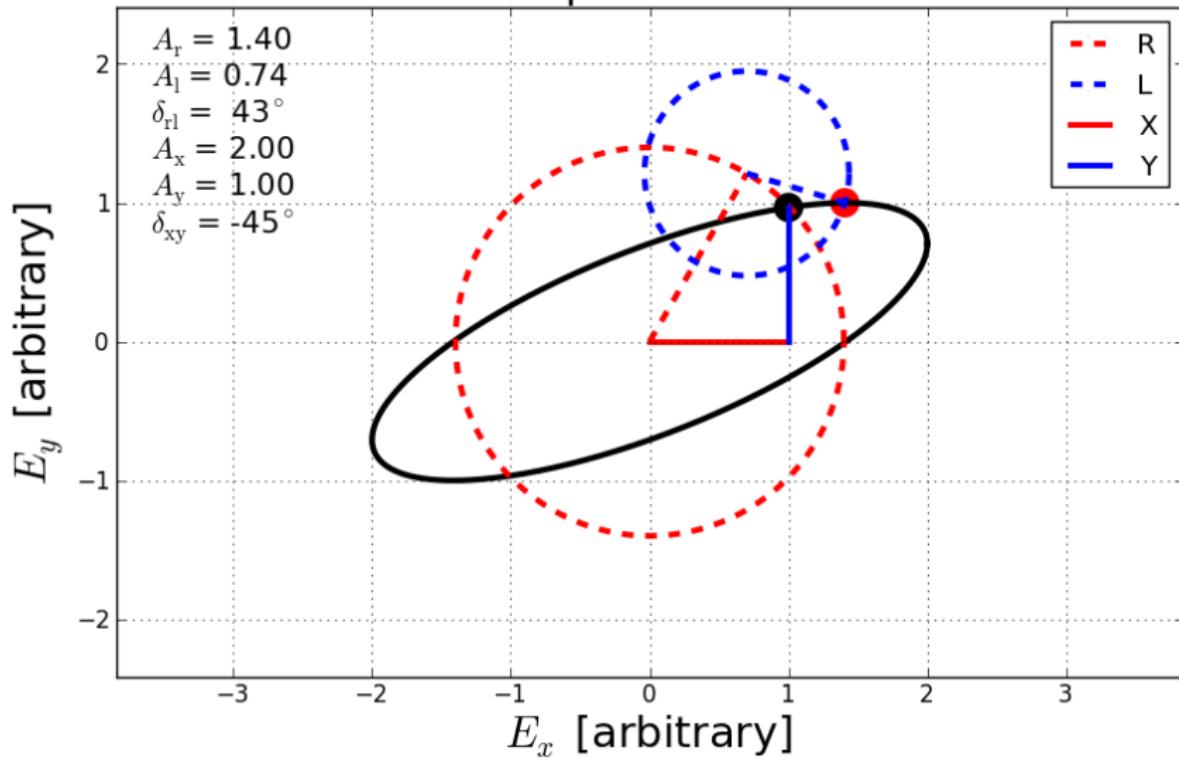
## Polarization ellipse: circular and linear



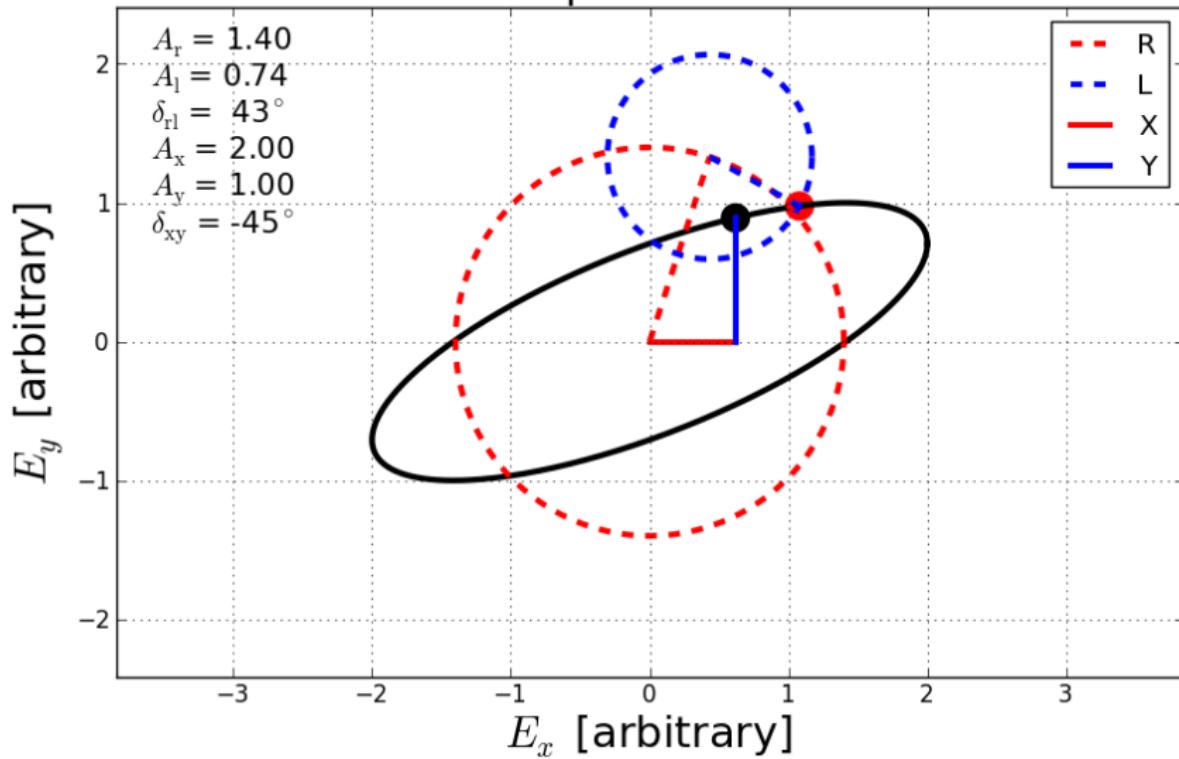
## Polarization ellipse: circular and linear



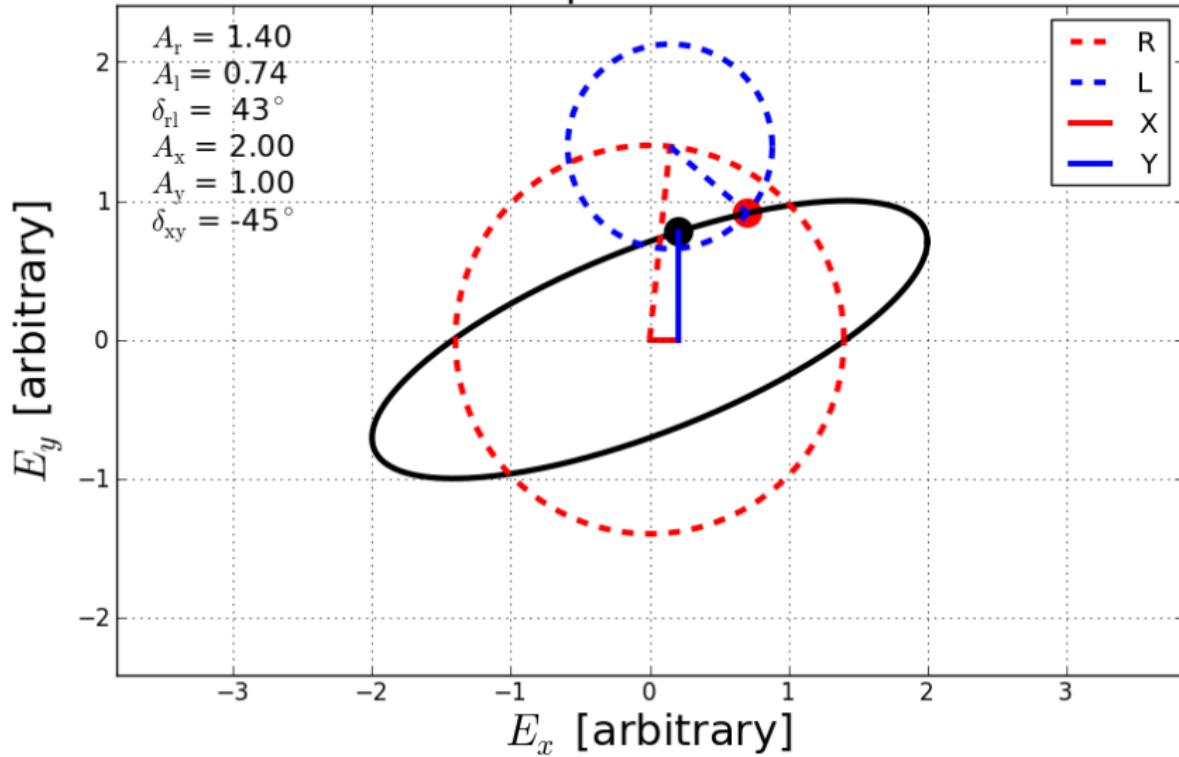
## Polarization ellipse: circular and linear



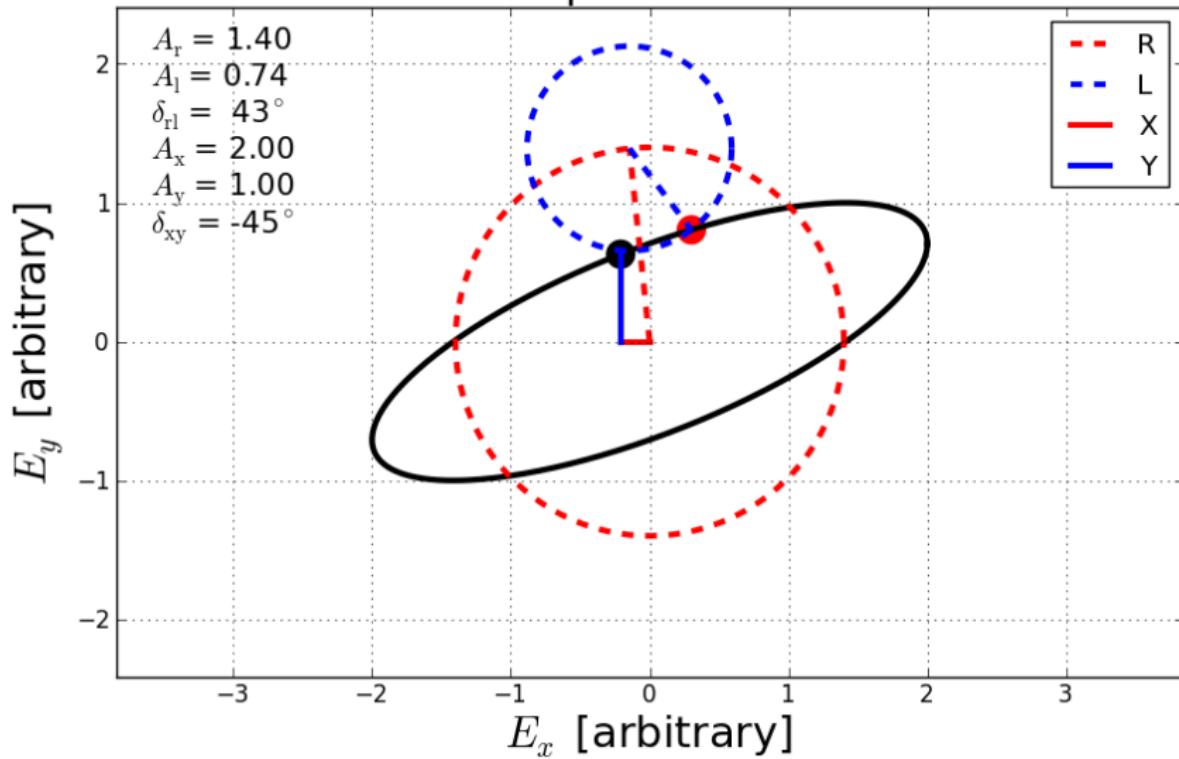
## Polarization ellipse: circular and linear



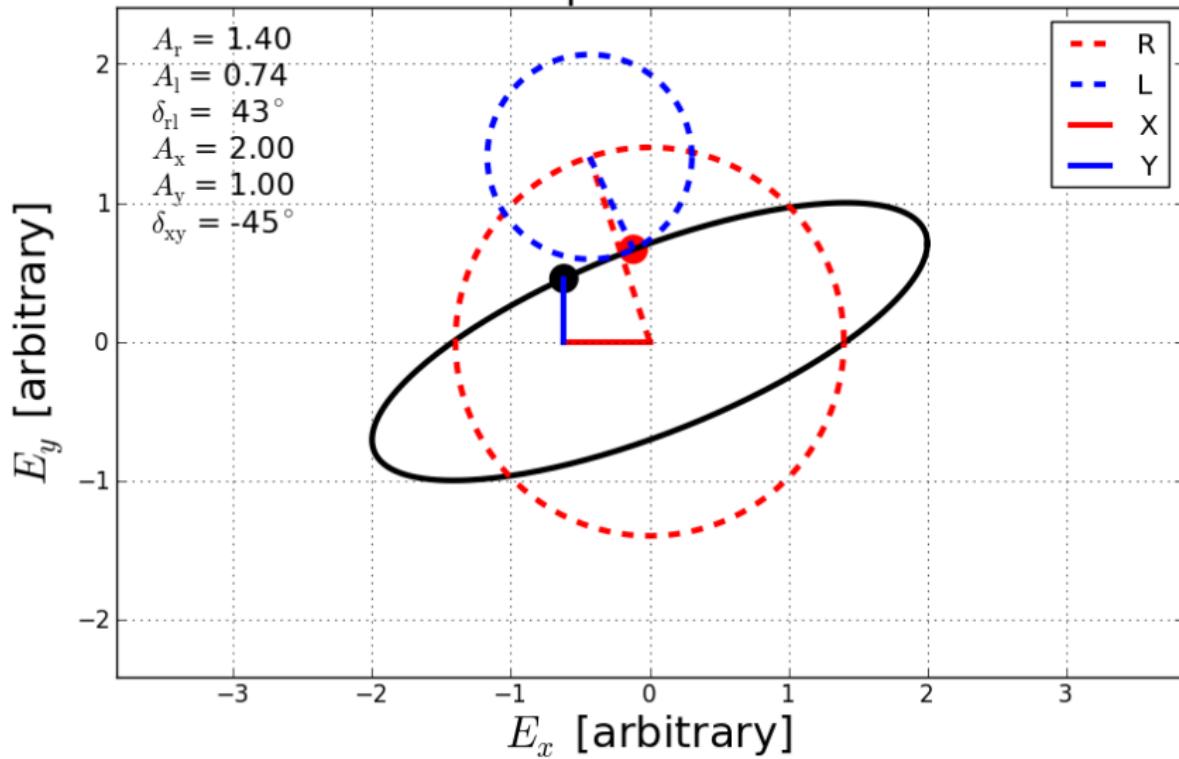
## Polarization ellipse: circular and linear



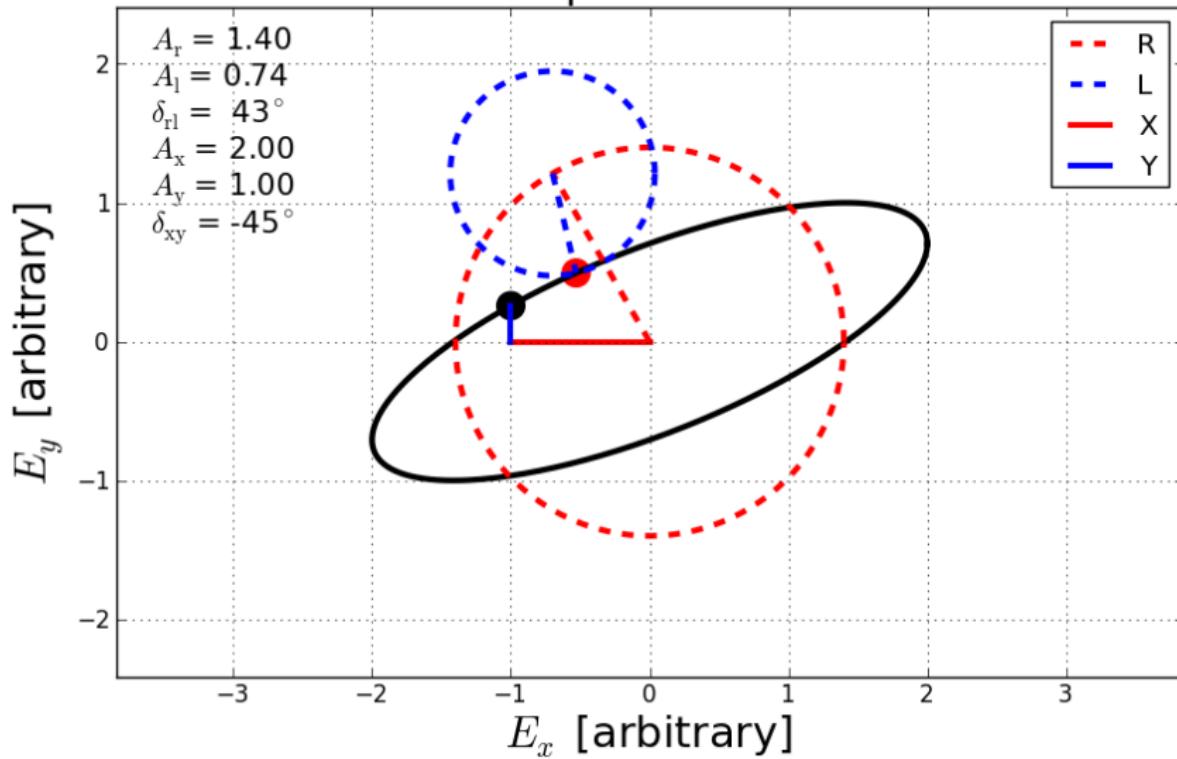
## Polarization ellipse: circular and linear



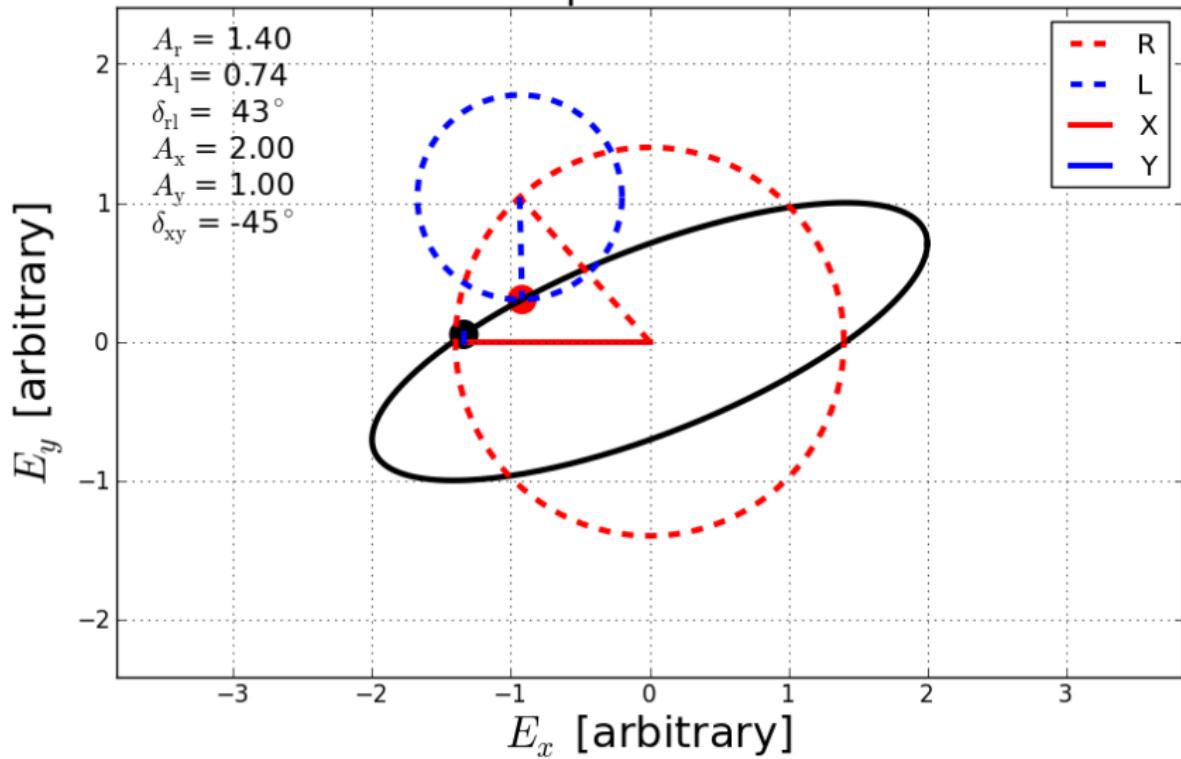
## Polarization ellipse: circular and linear



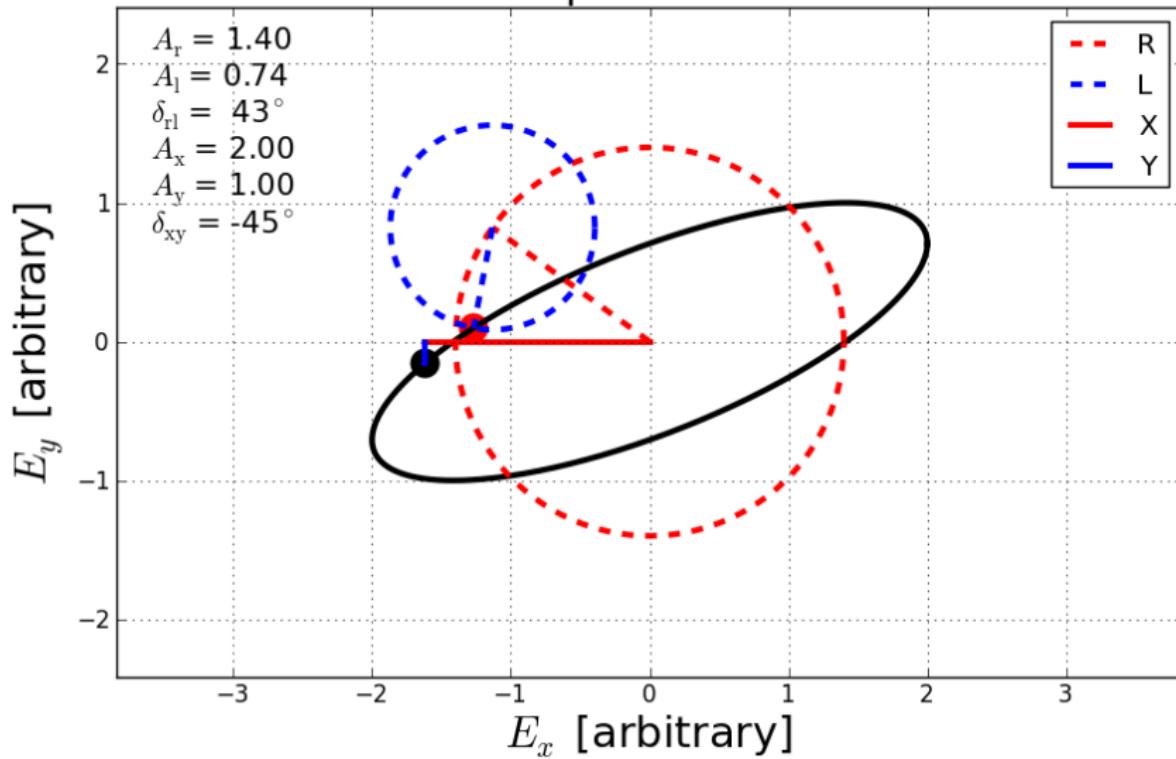
## Polarization ellipse: circular and linear



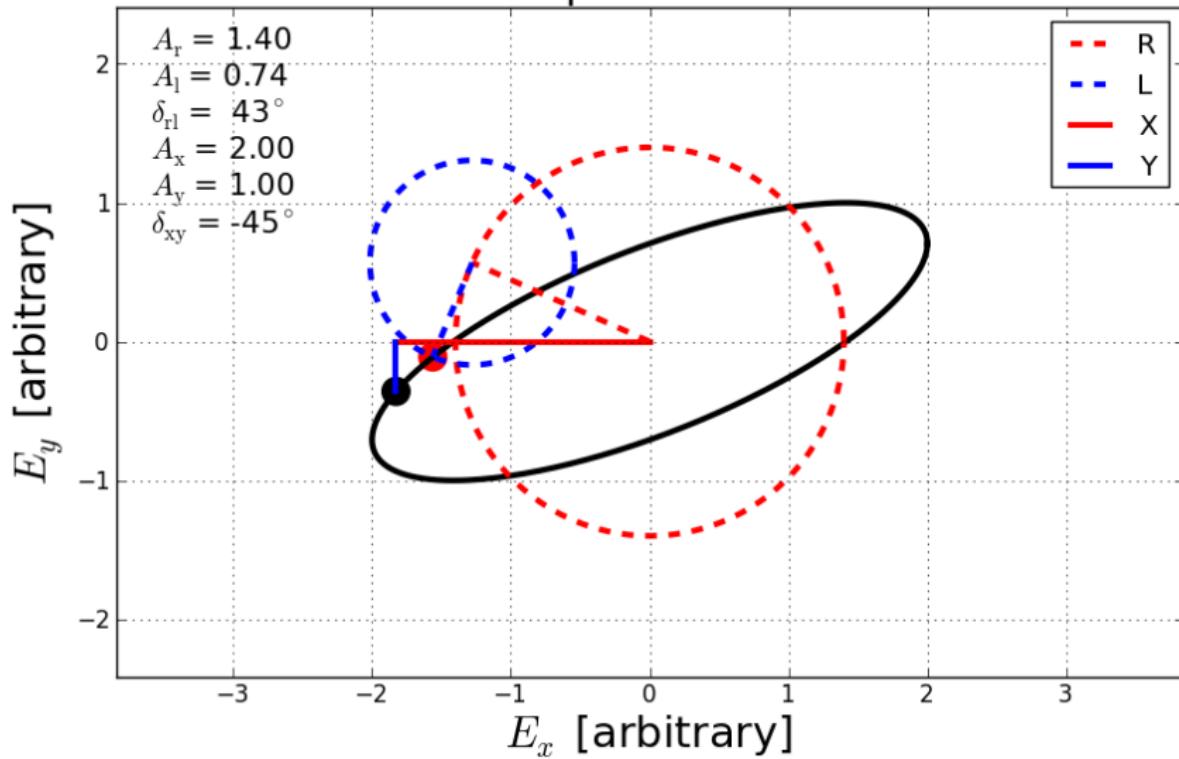
## Polarization ellipse: circular and linear



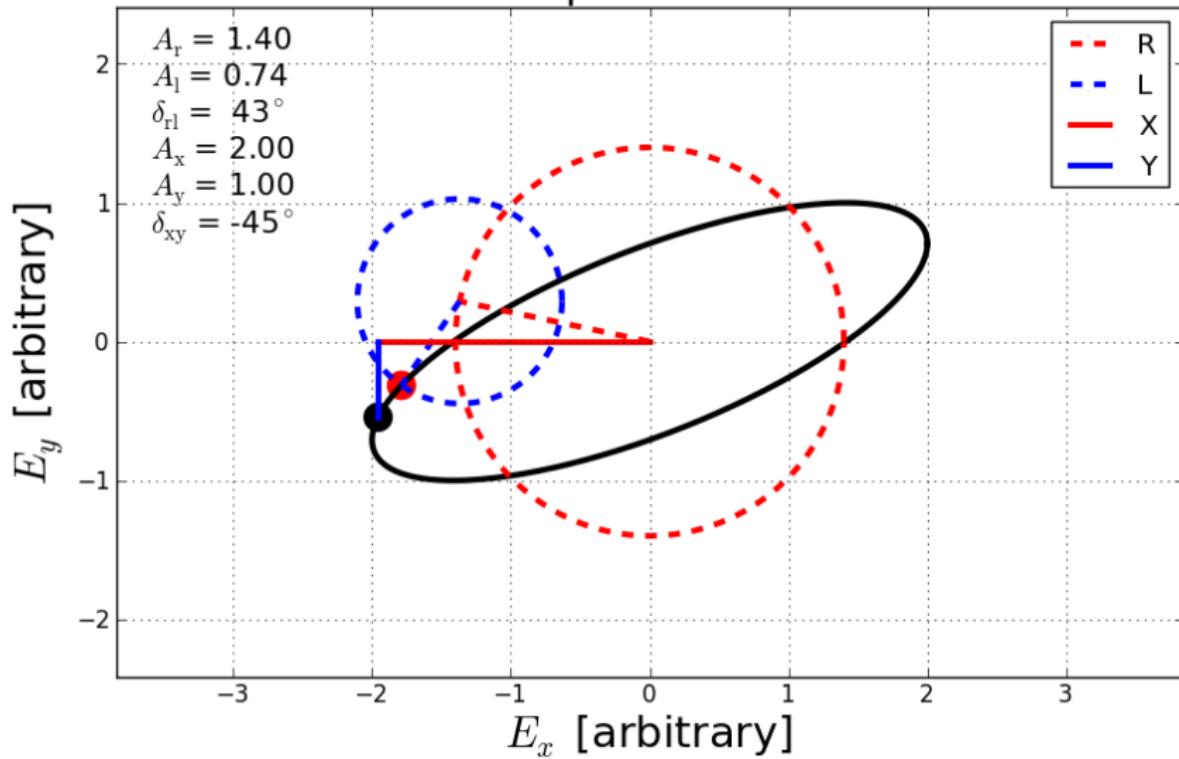
## Polarization ellipse: circular and linear



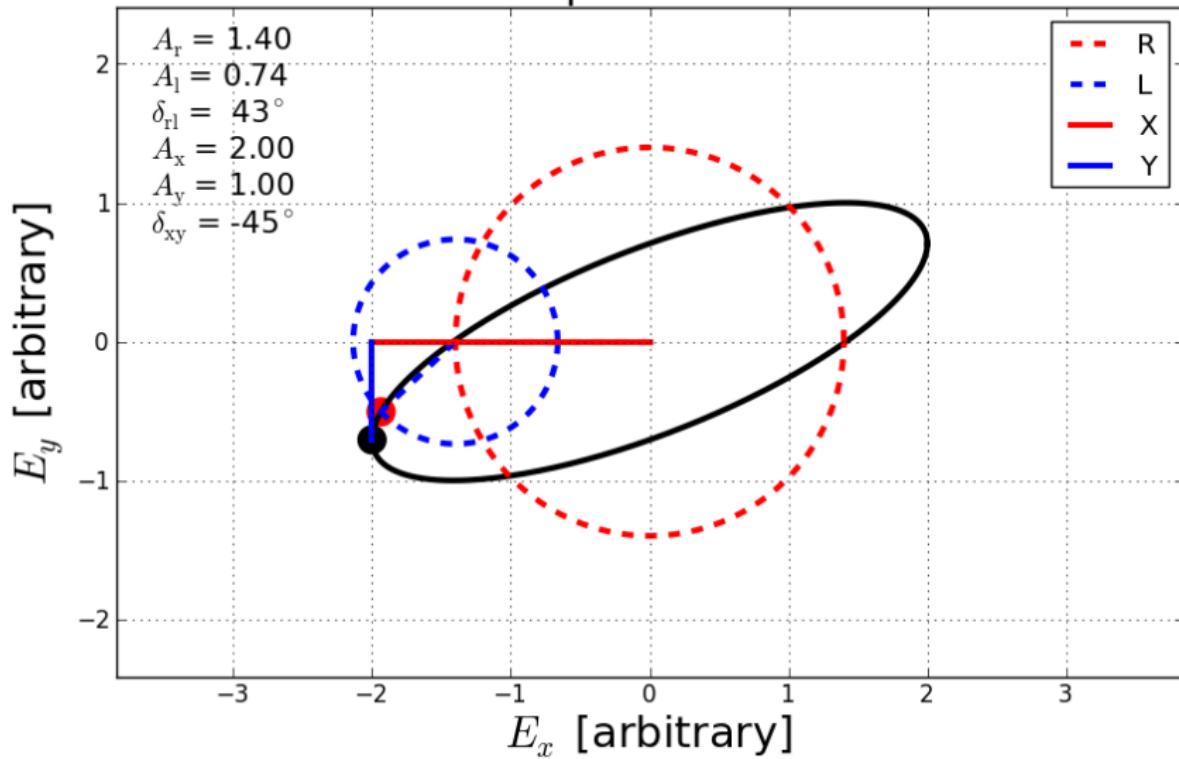
## Polarization ellipse: circular and linear



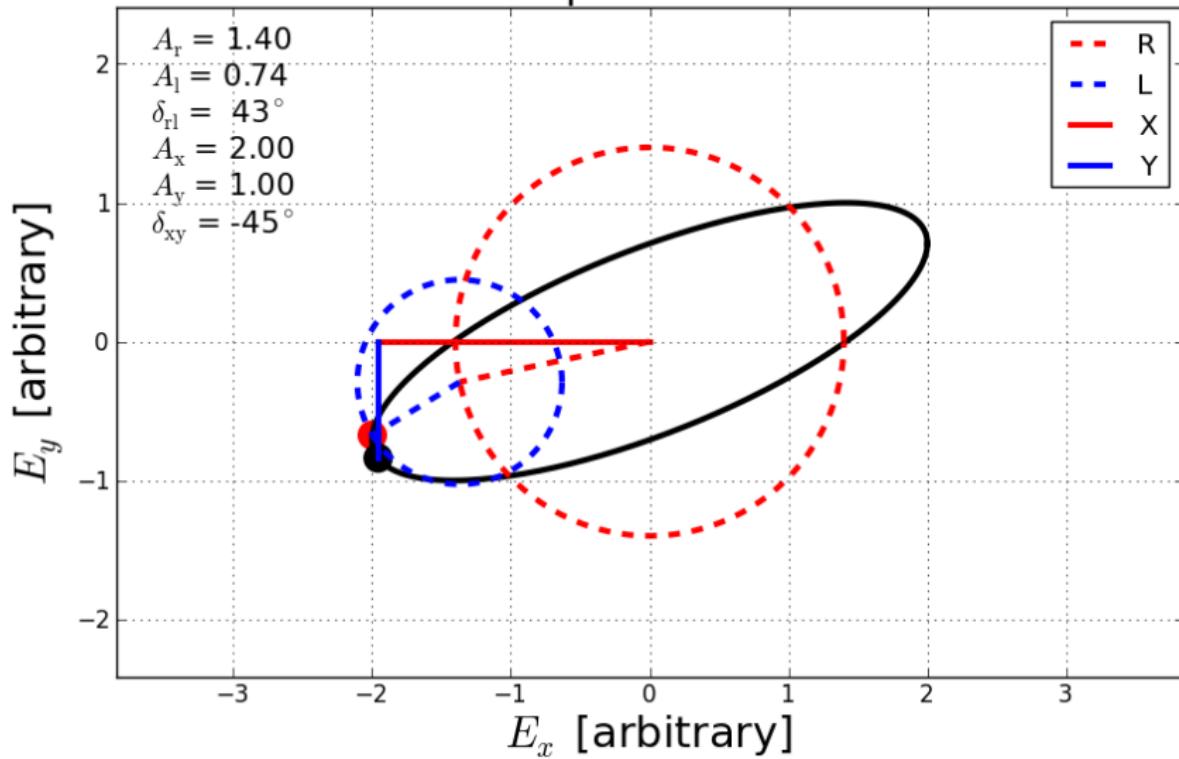
## Polarization ellipse: circular and linear



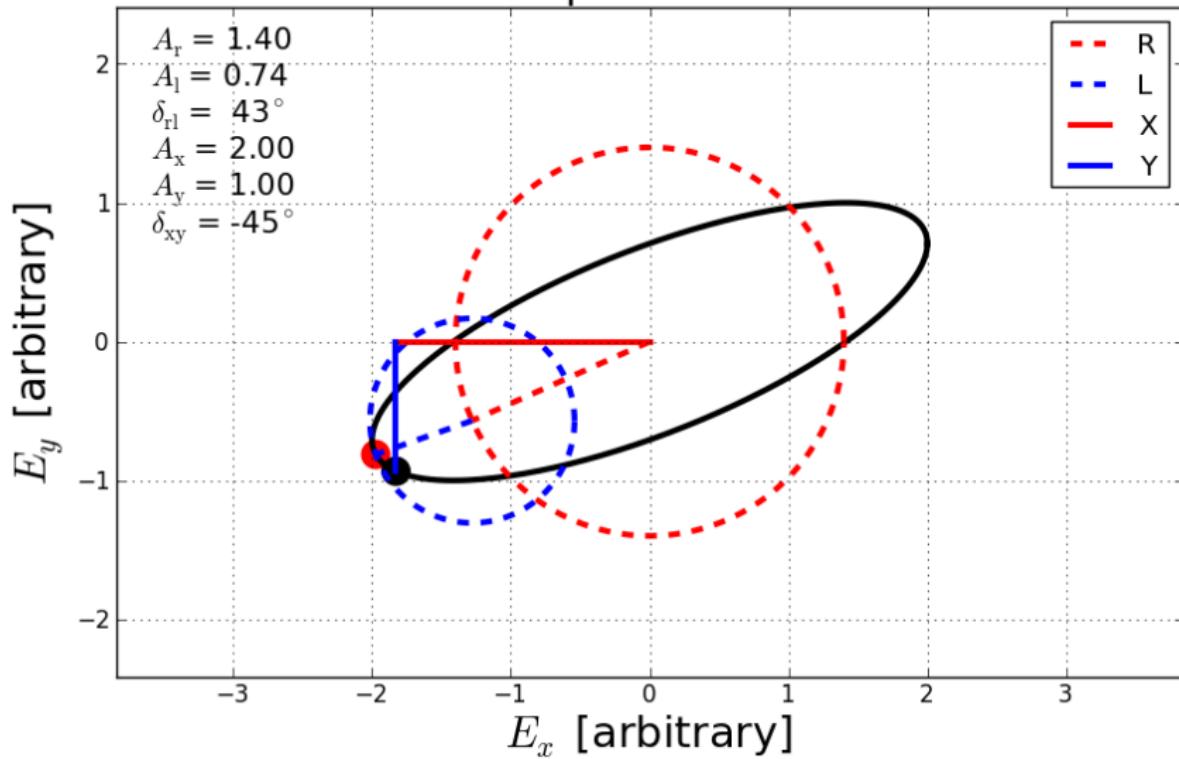
## Polarization ellipse: circular and linear



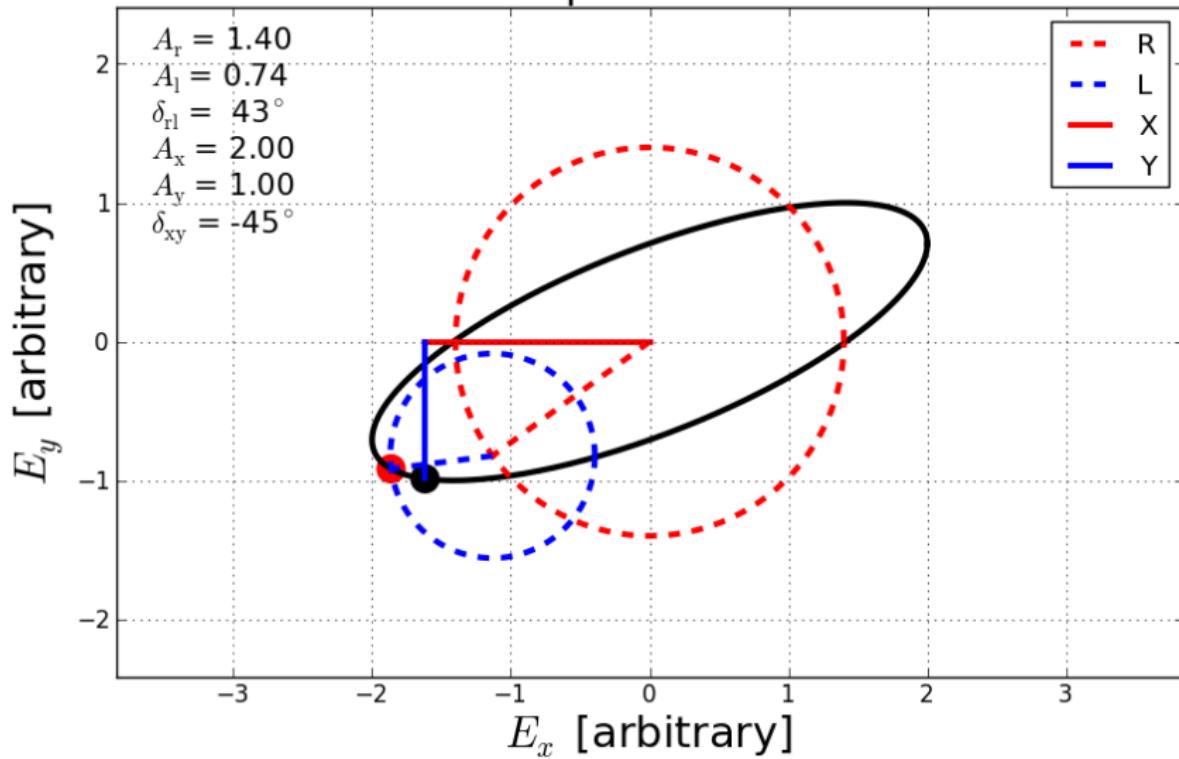
## Polarization ellipse: circular and linear



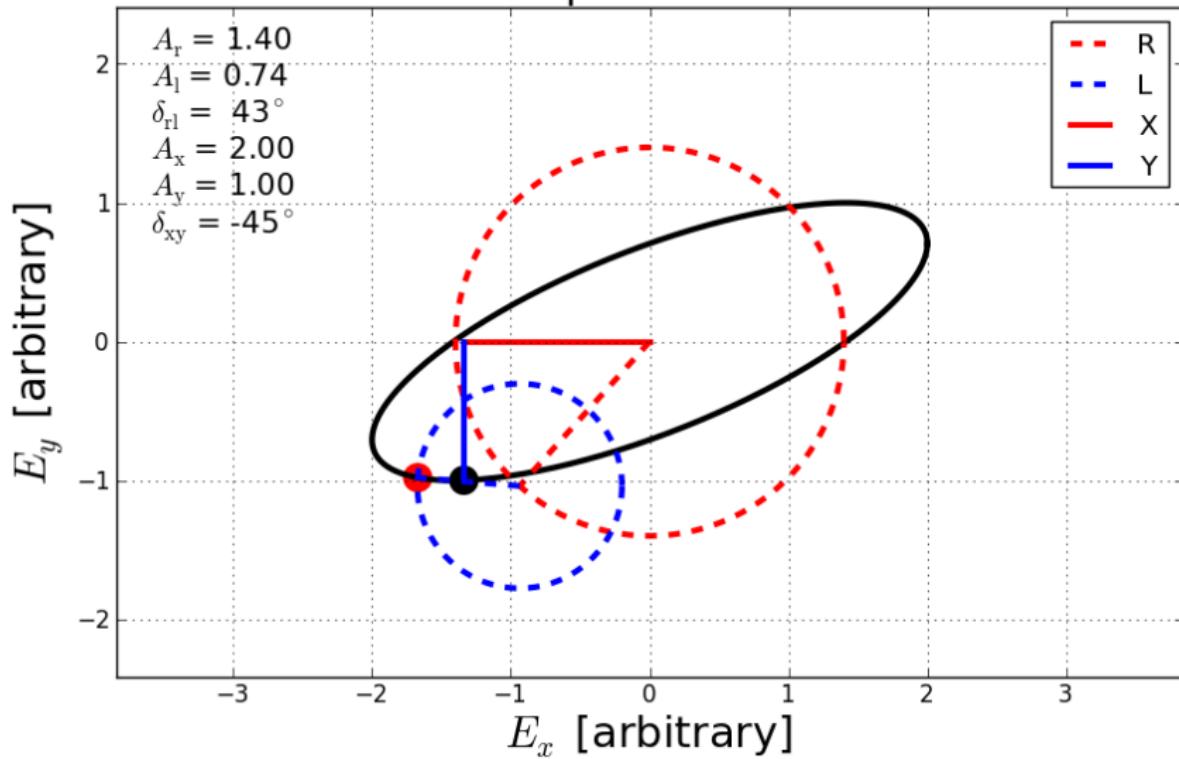
## Polarization ellipse: circular and linear



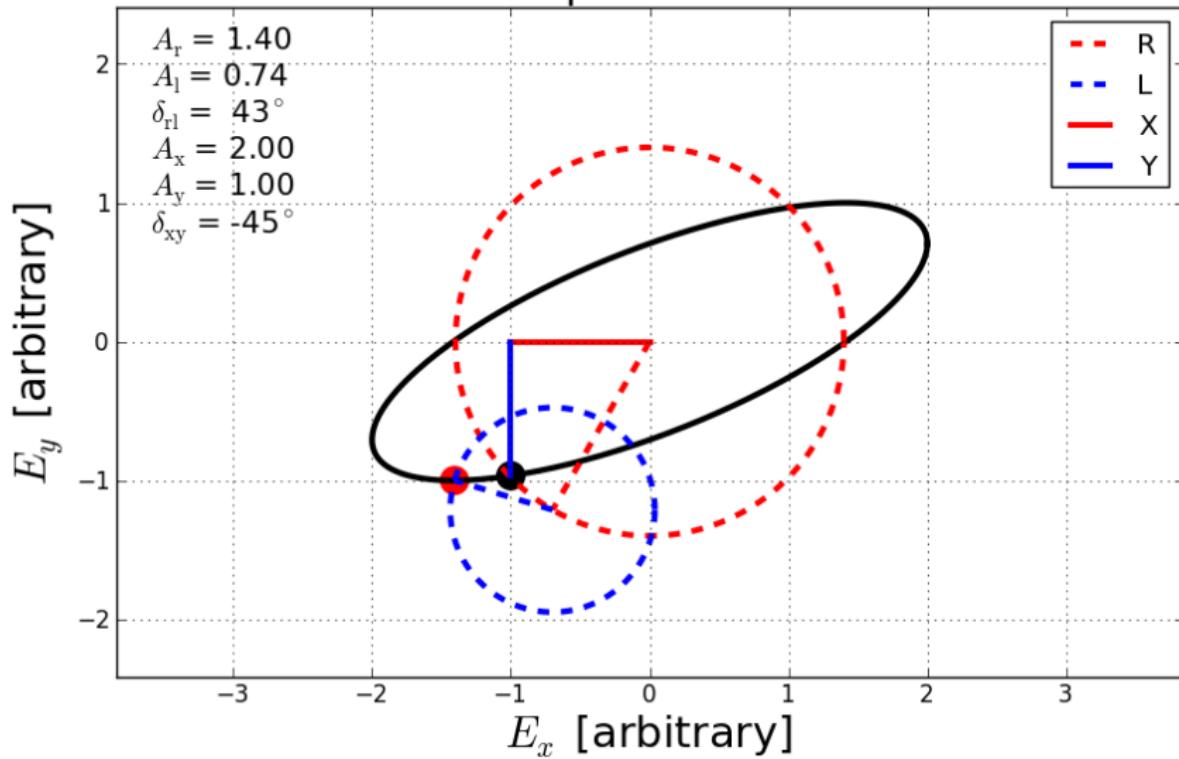
## Polarization ellipse: circular and linear



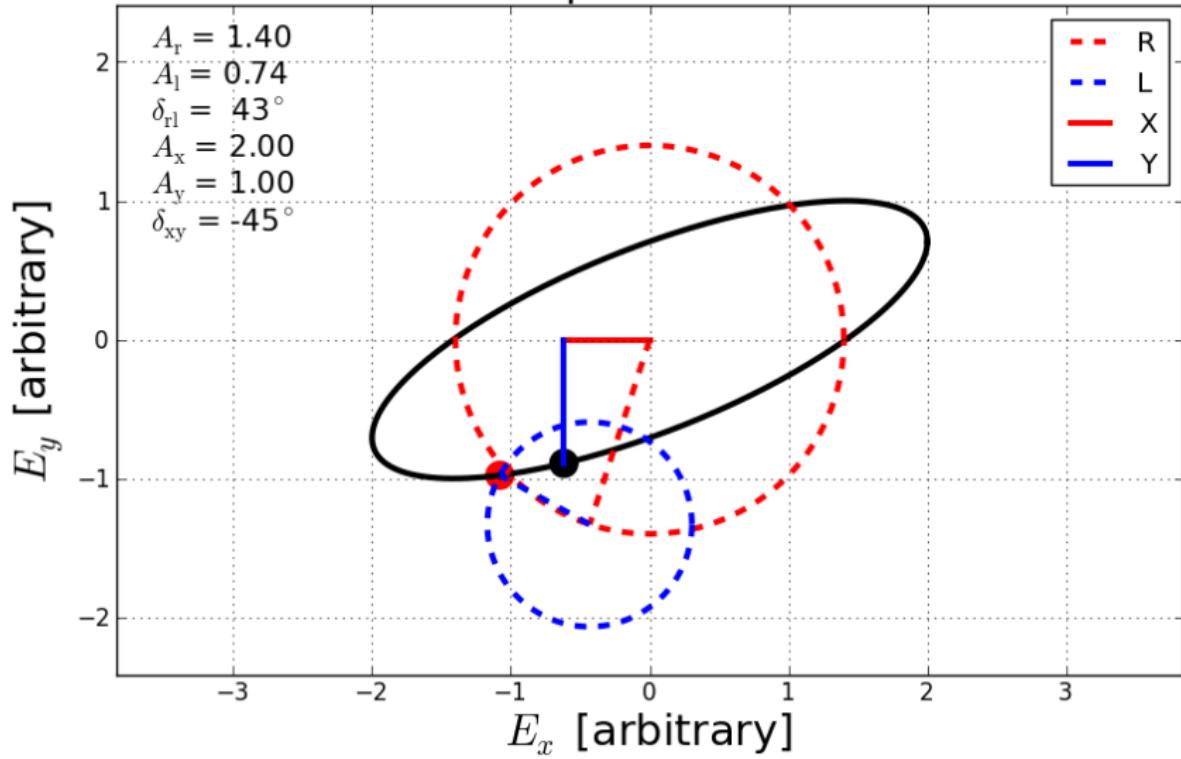
## Polarization ellipse: circular and linear



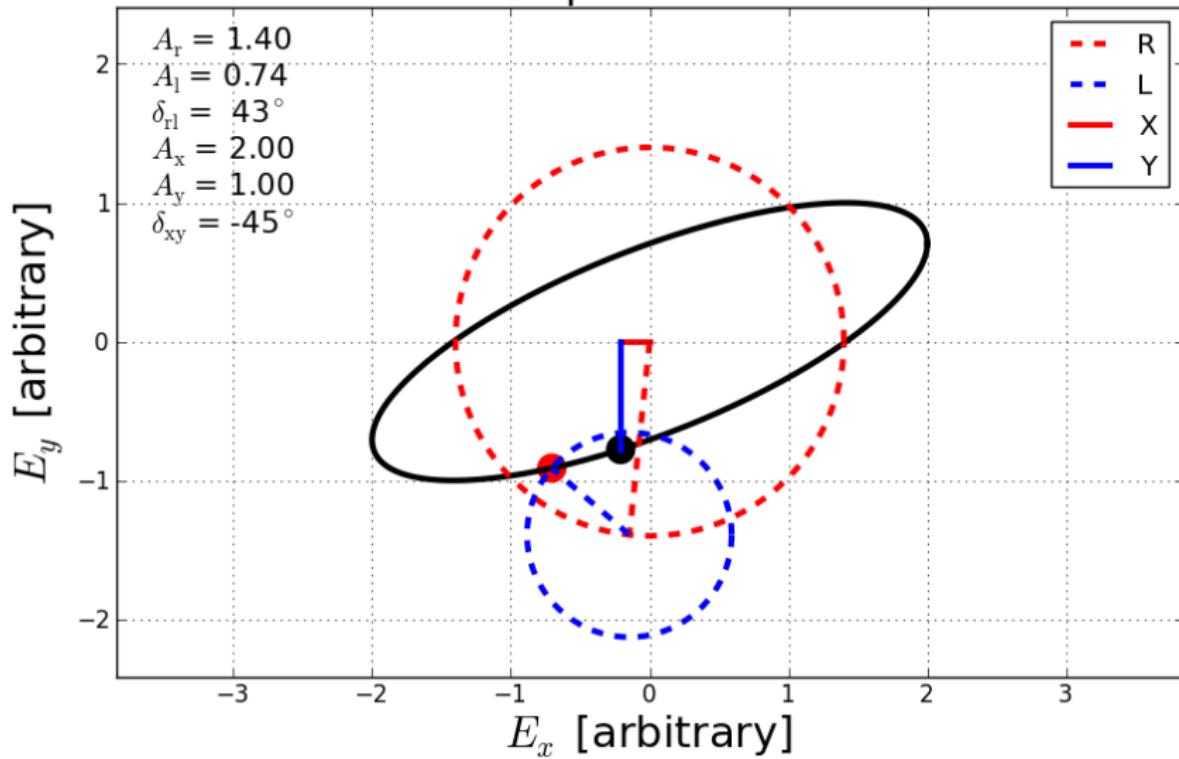
## Polarization ellipse: circular and linear



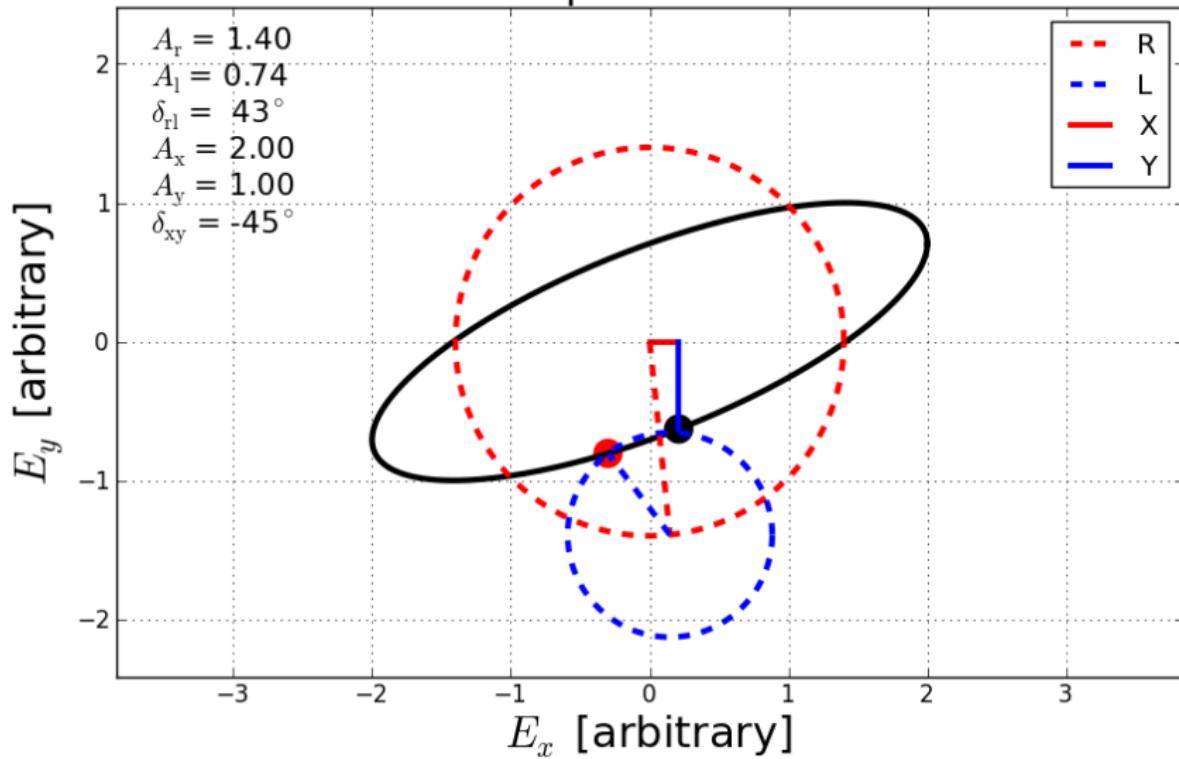
## Polarization ellipse: circular and linear



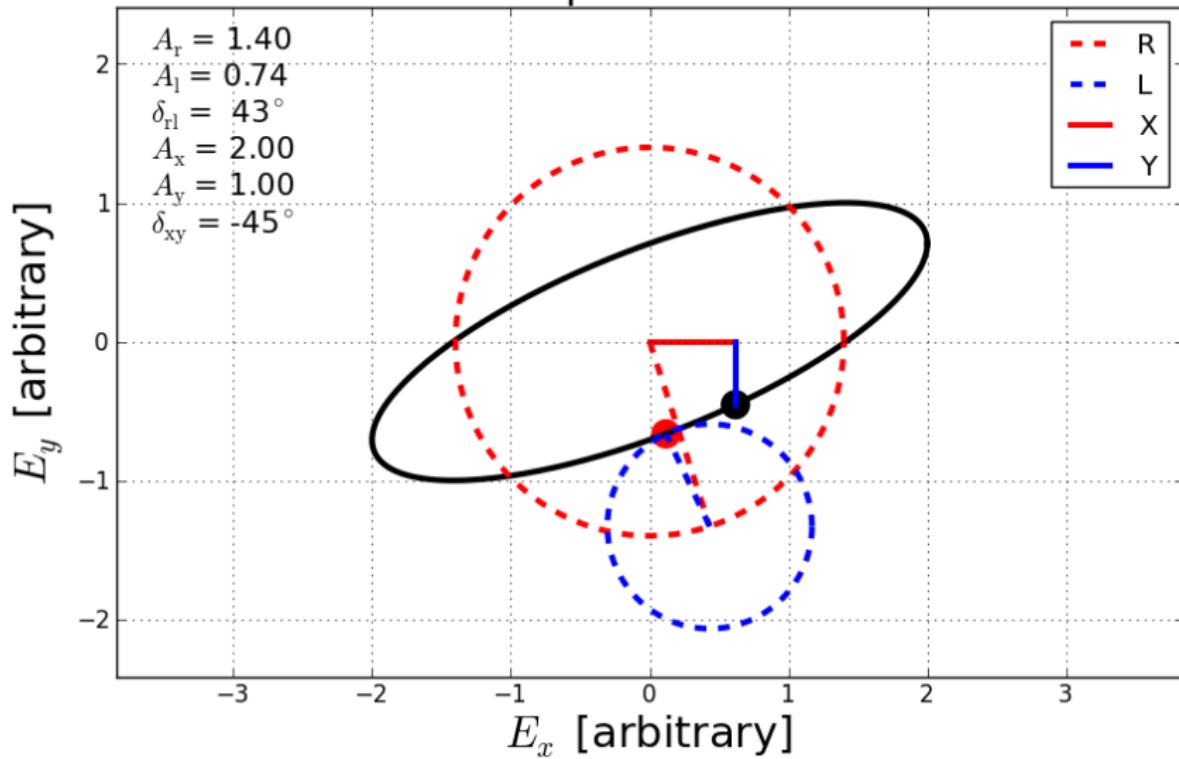
## Polarization ellipse: circular and linear



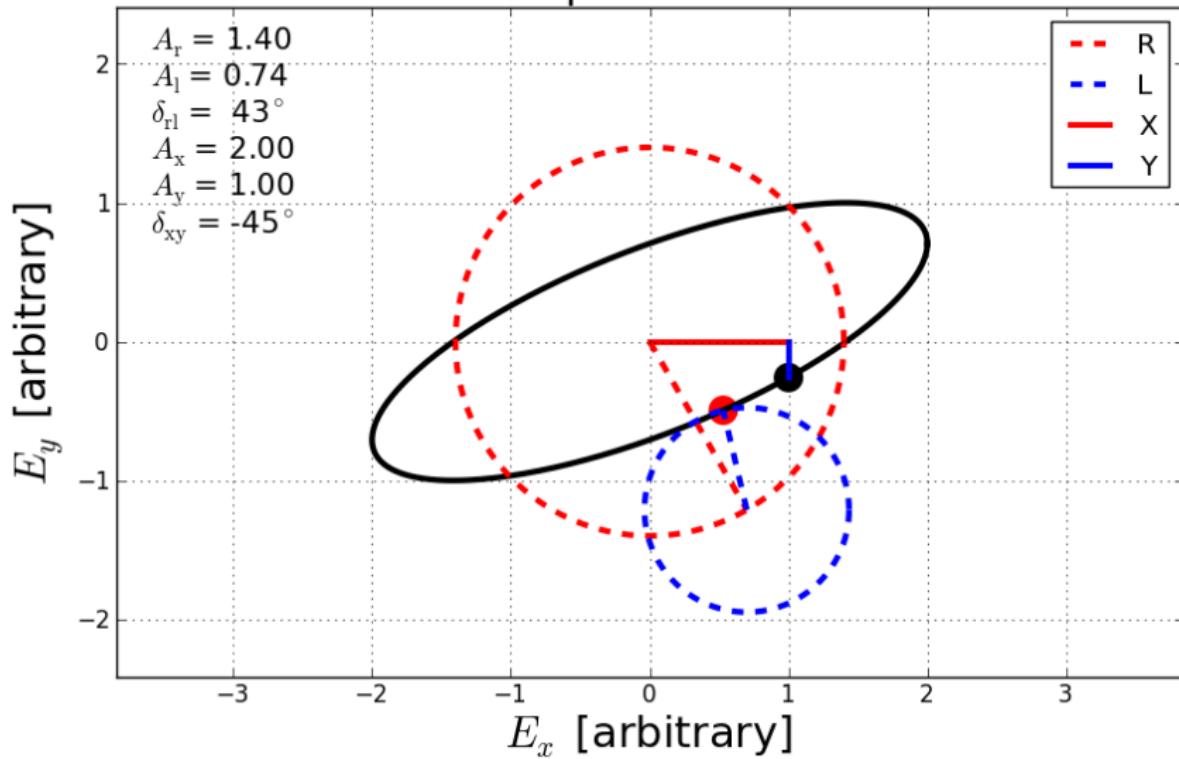
## Polarization ellipse: circular and linear



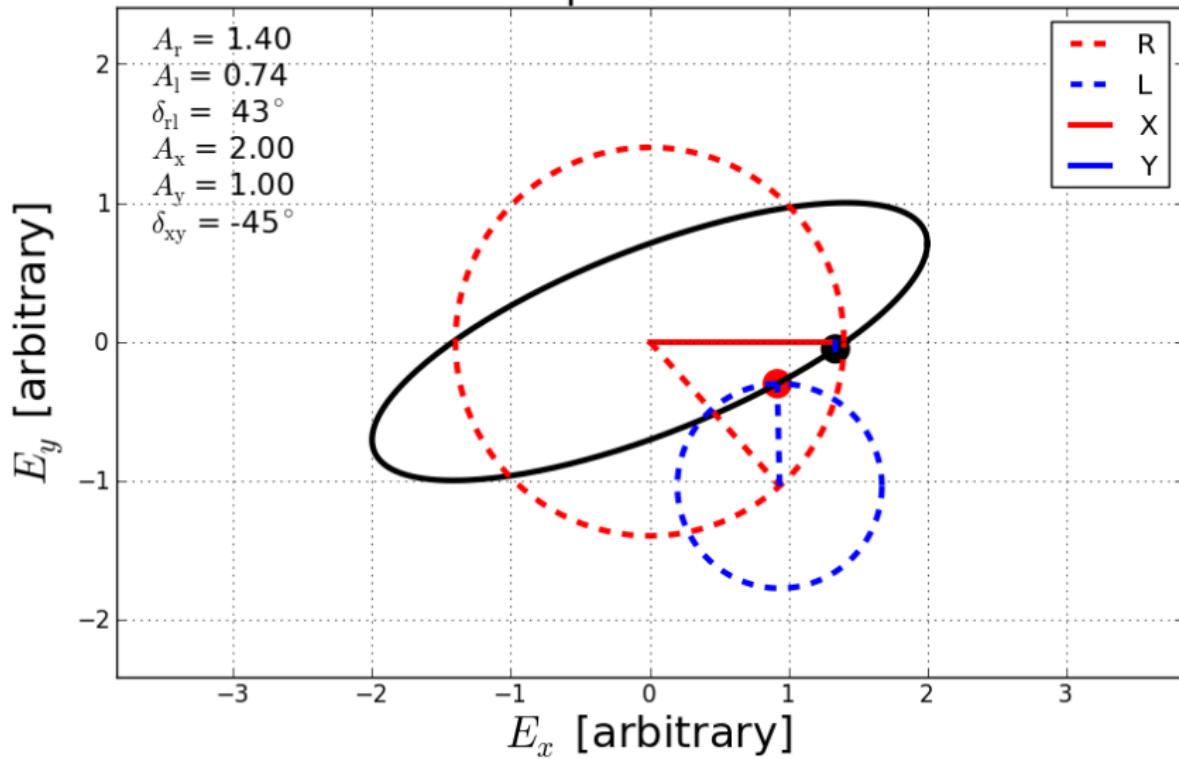
## Polarization ellipse: circular and linear



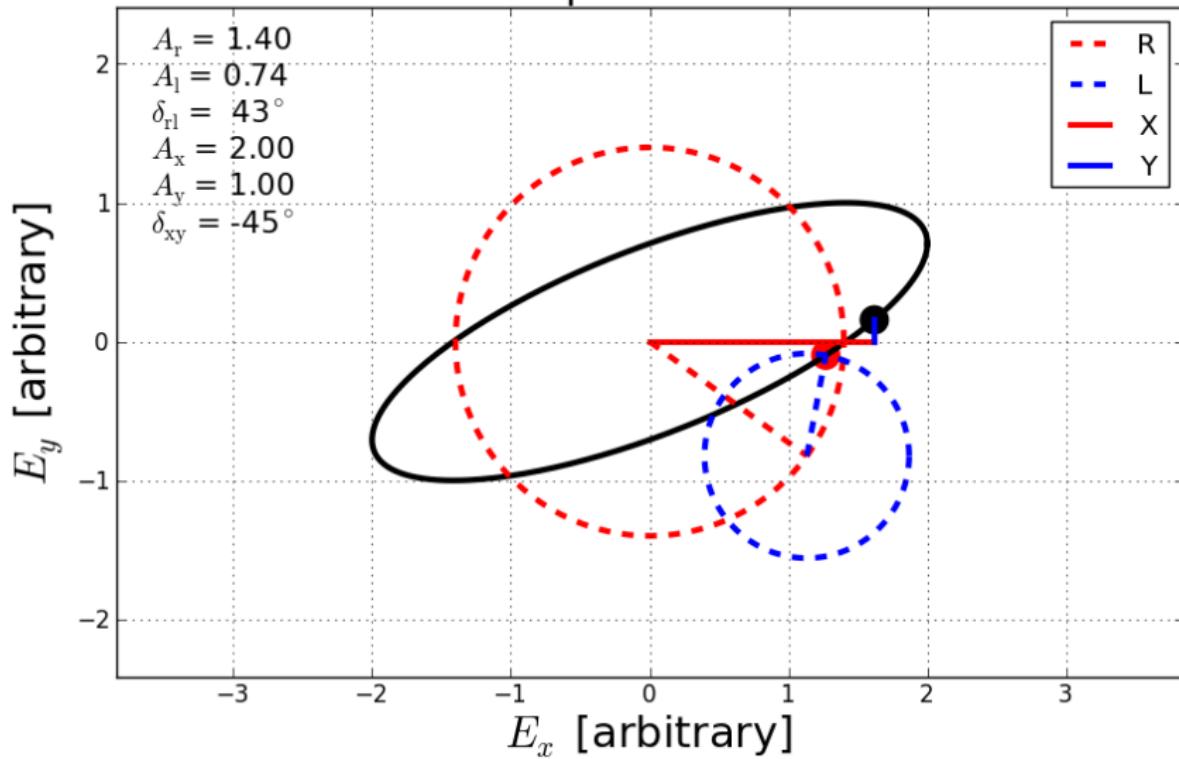
## Polarization ellipse: circular and linear



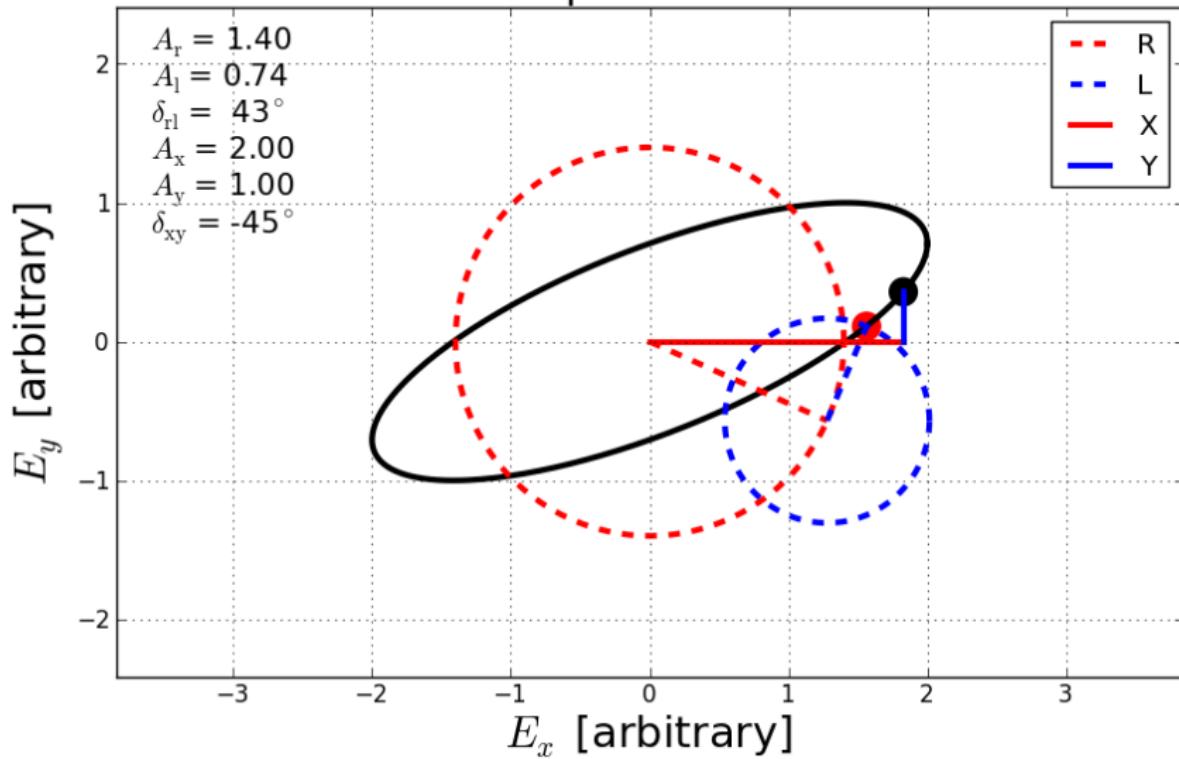
## Polarization ellipse: circular and linear



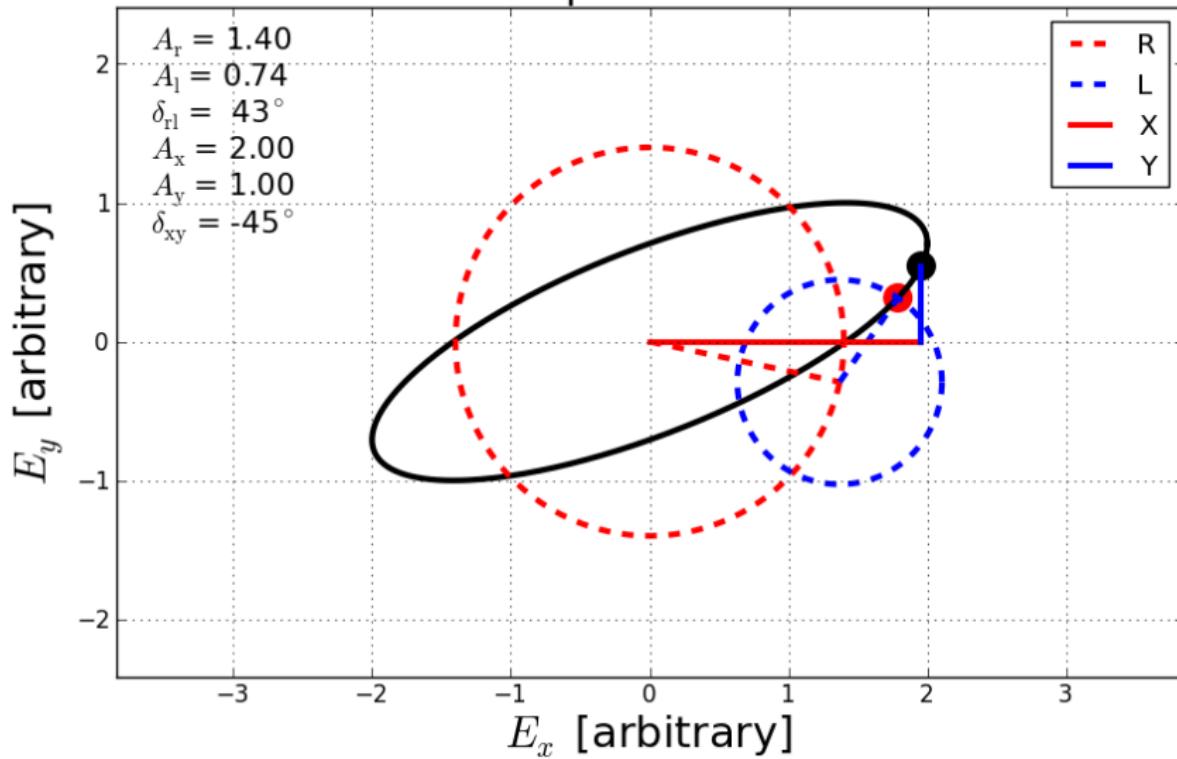
## Polarization ellipse: circular and linear



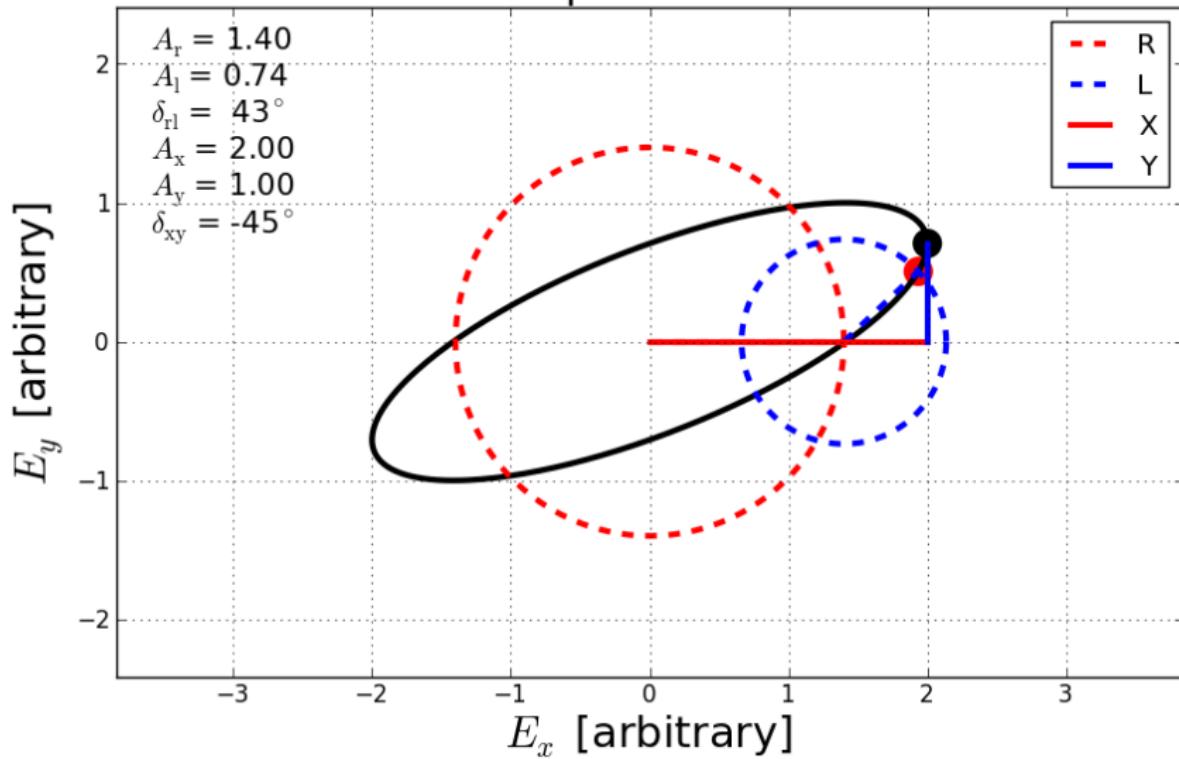
## Polarization ellipse: circular and linear



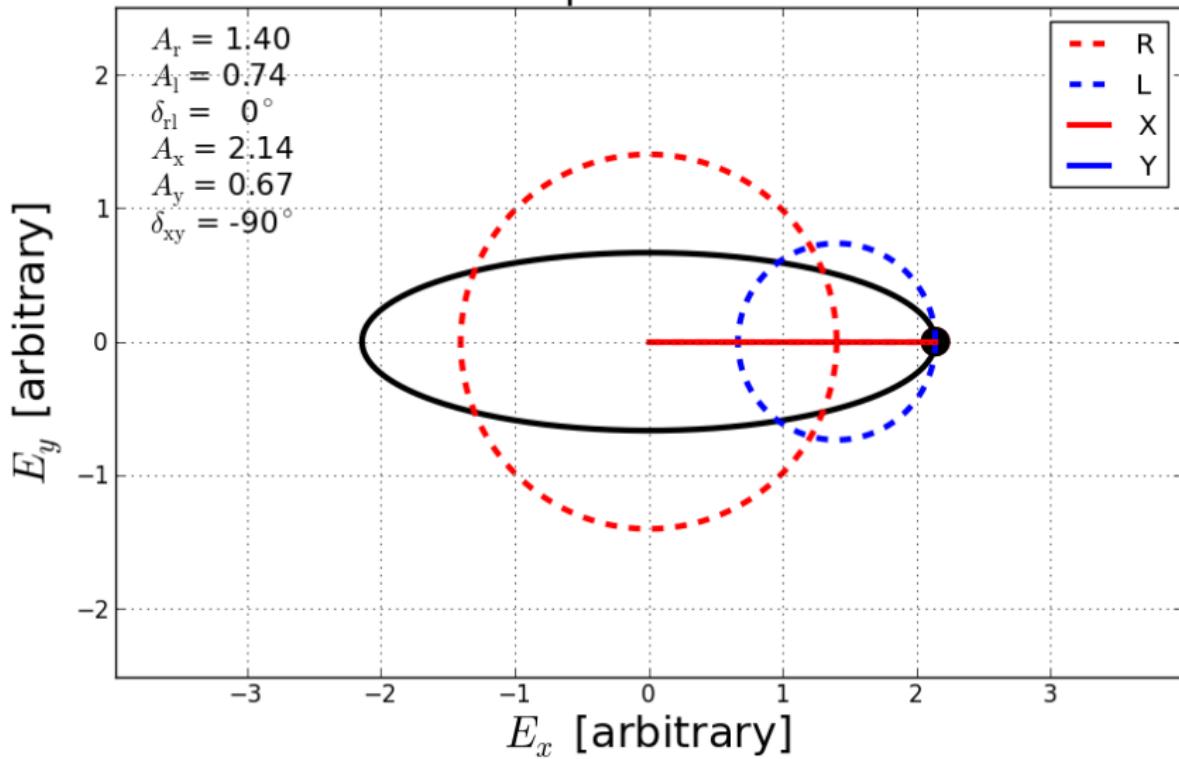
## Polarization ellipse: circular and linear



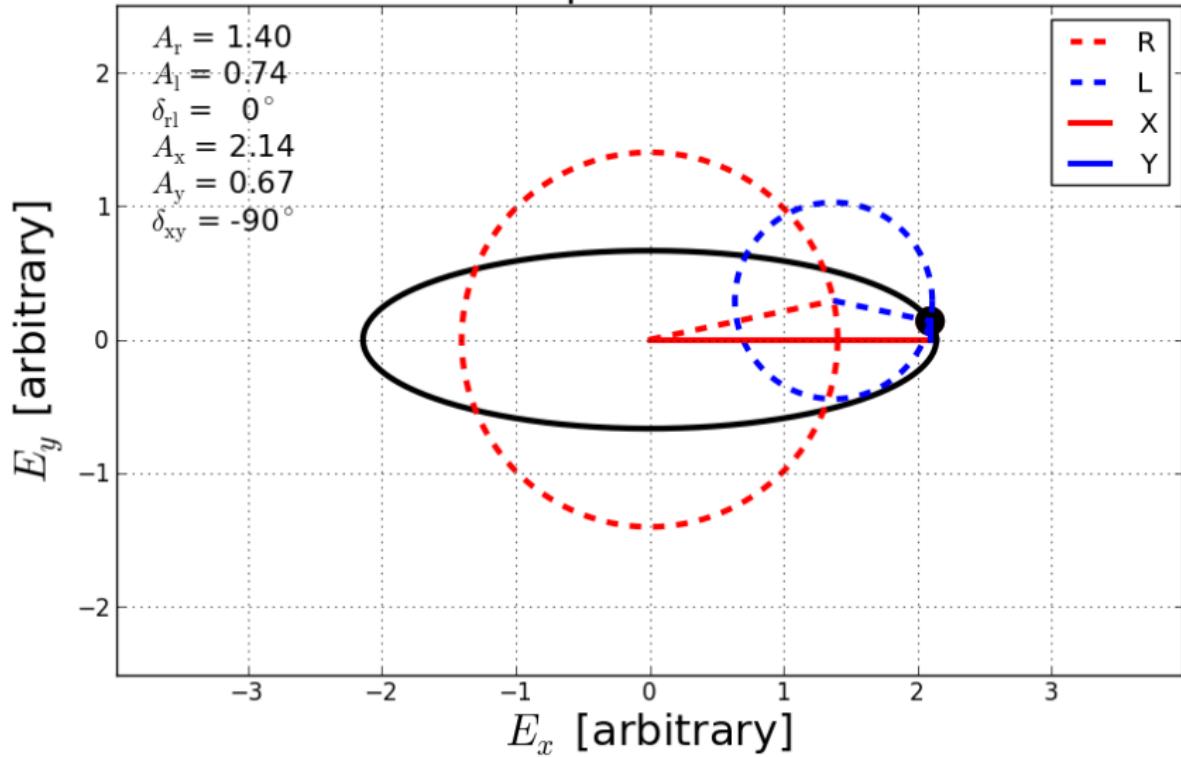
## Polarization ellipse: circular and linear



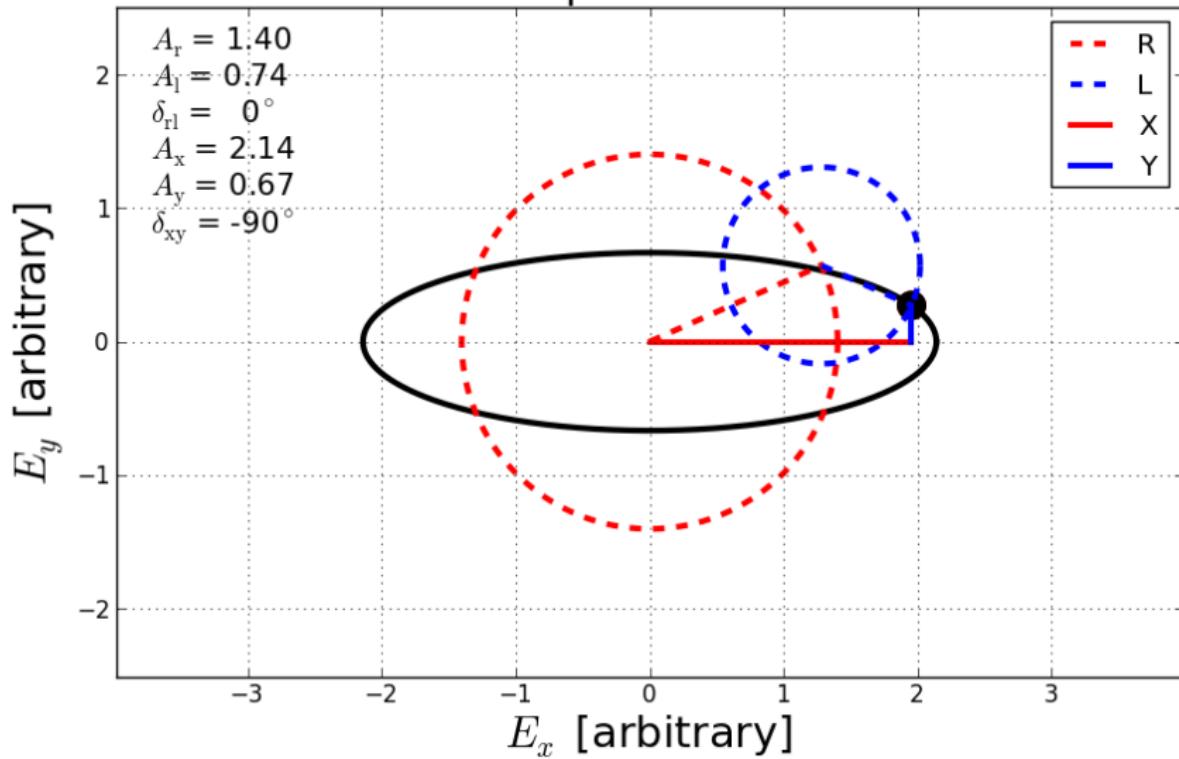
## Polarization ellipse: circular and linear



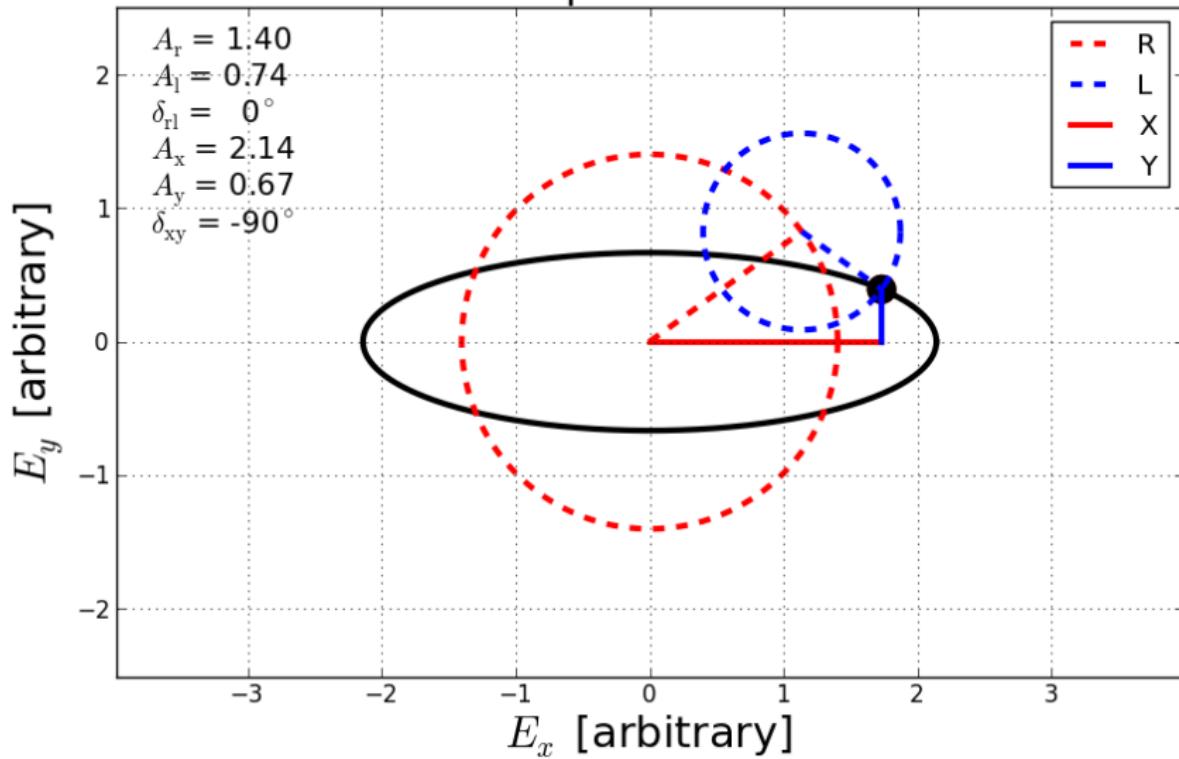
## Polarization ellipse: circular and linear



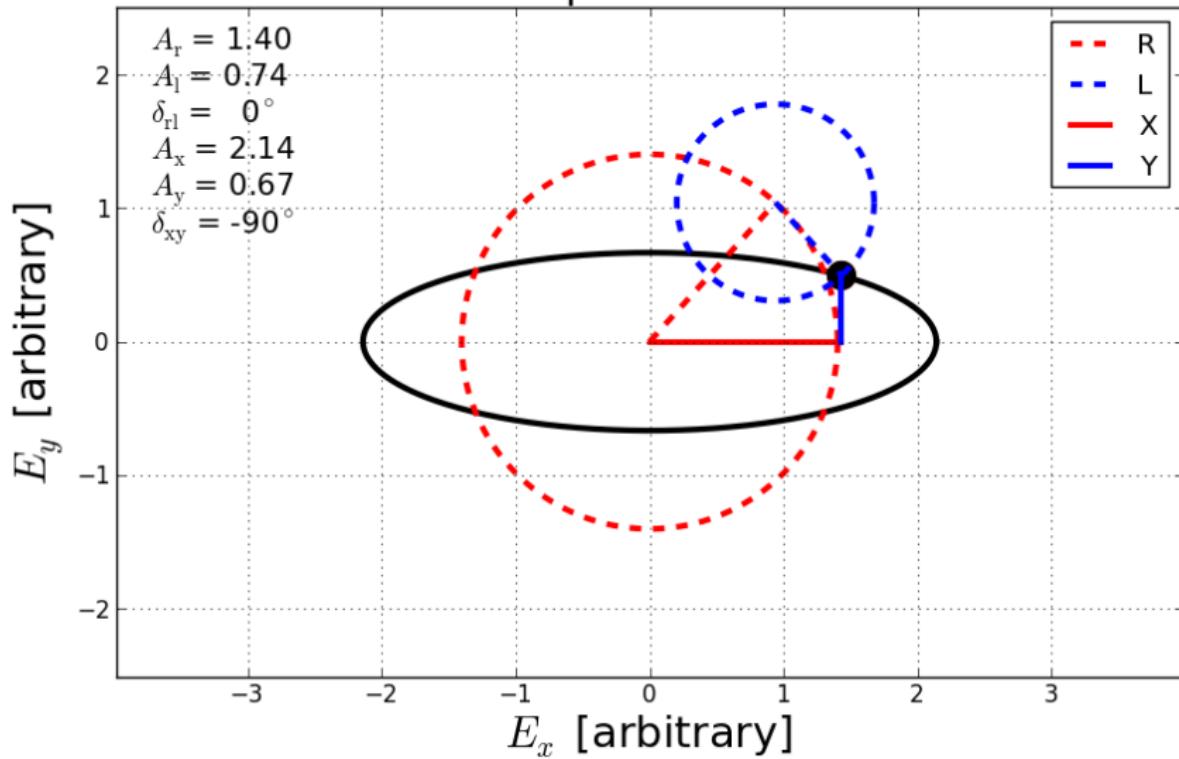
## Polarization ellipse: circular and linear



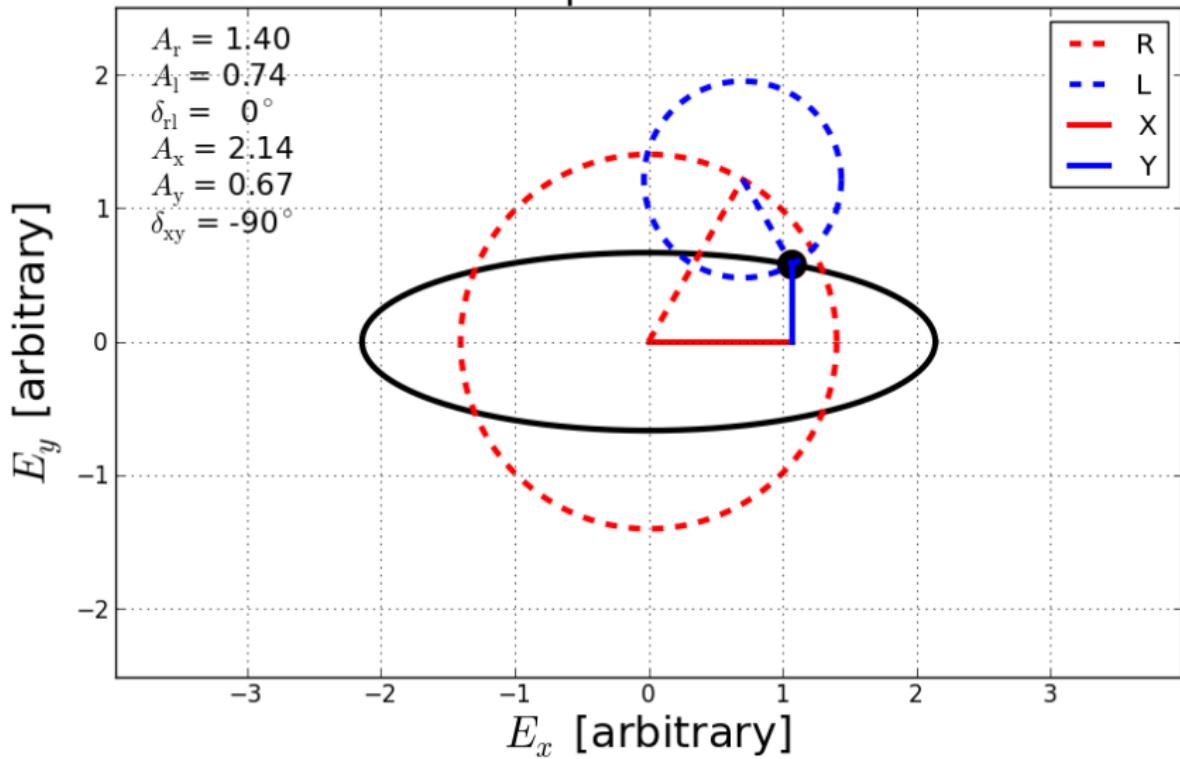
## Polarization ellipse: circular and linear



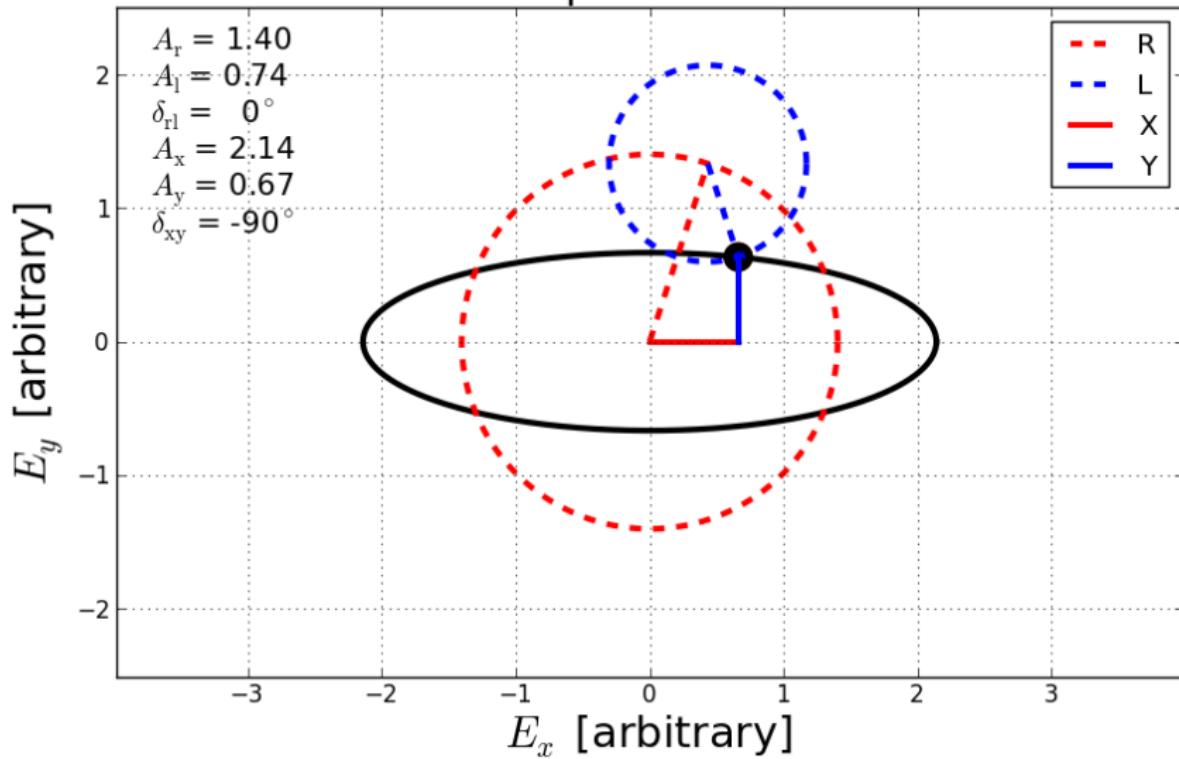
## Polarization ellipse: circular and linear



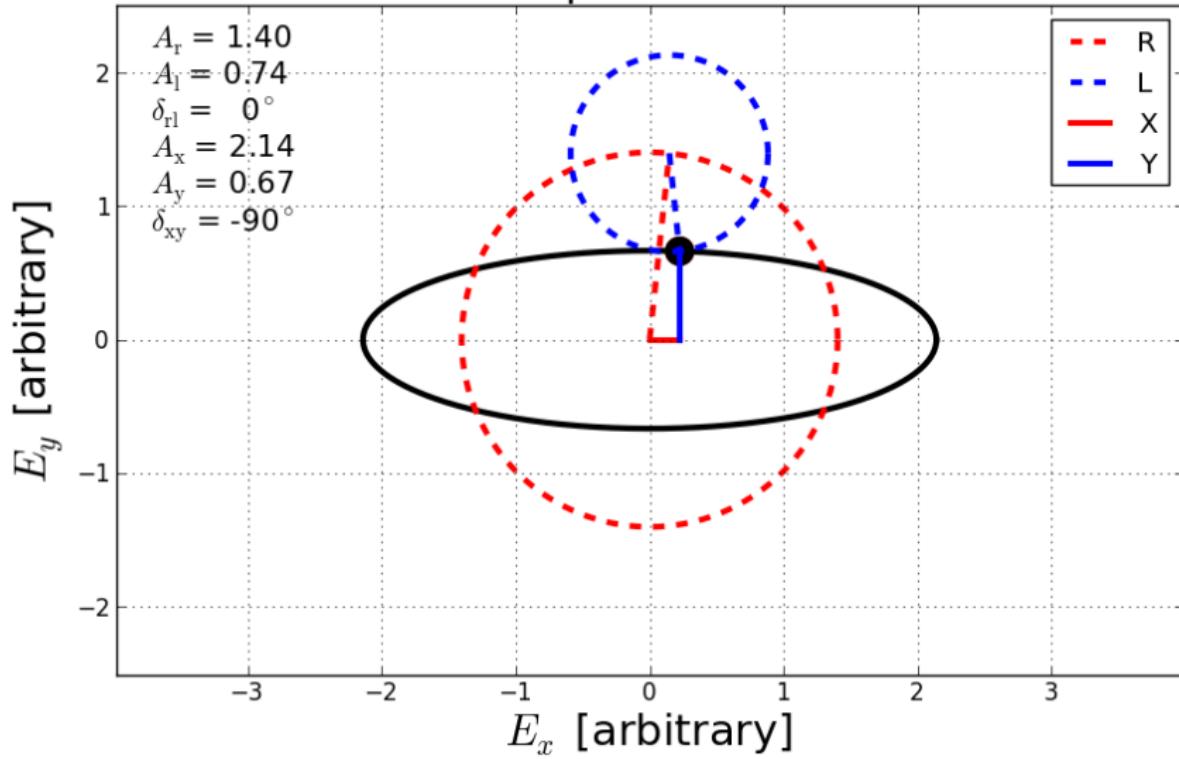
## Polarization ellipse: circular and linear



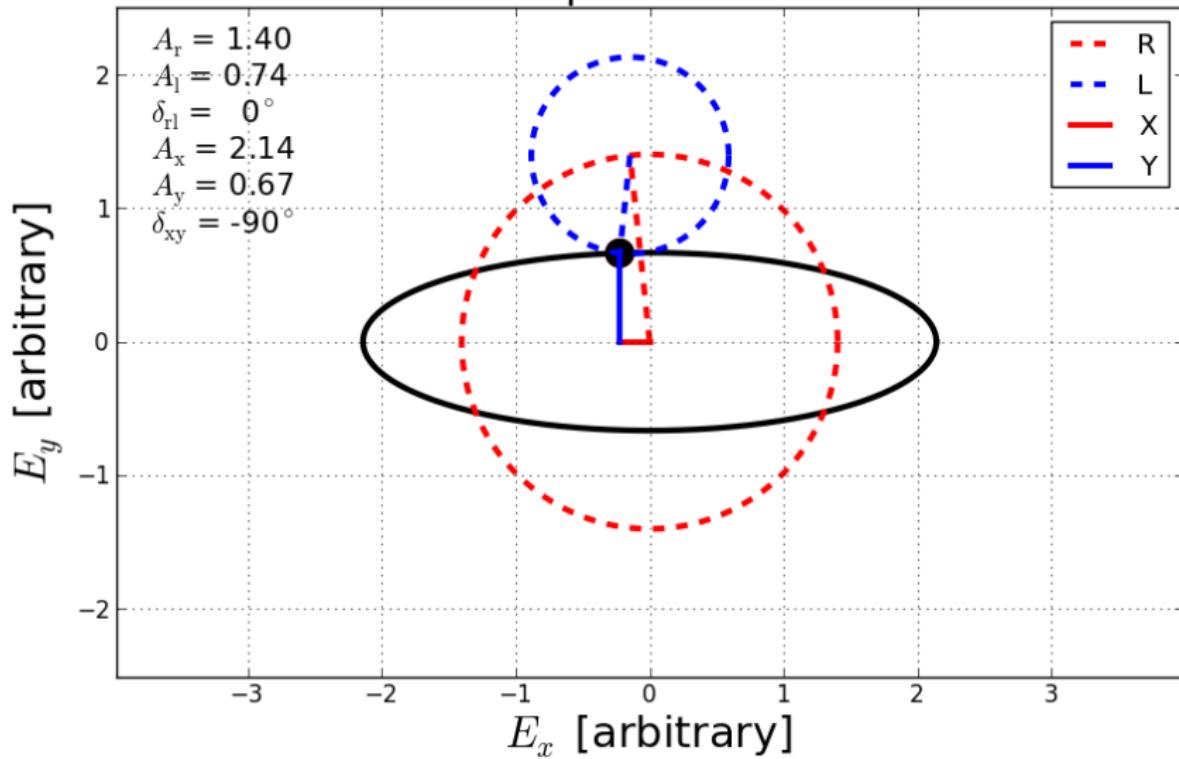
## Polarization ellipse: circular and linear



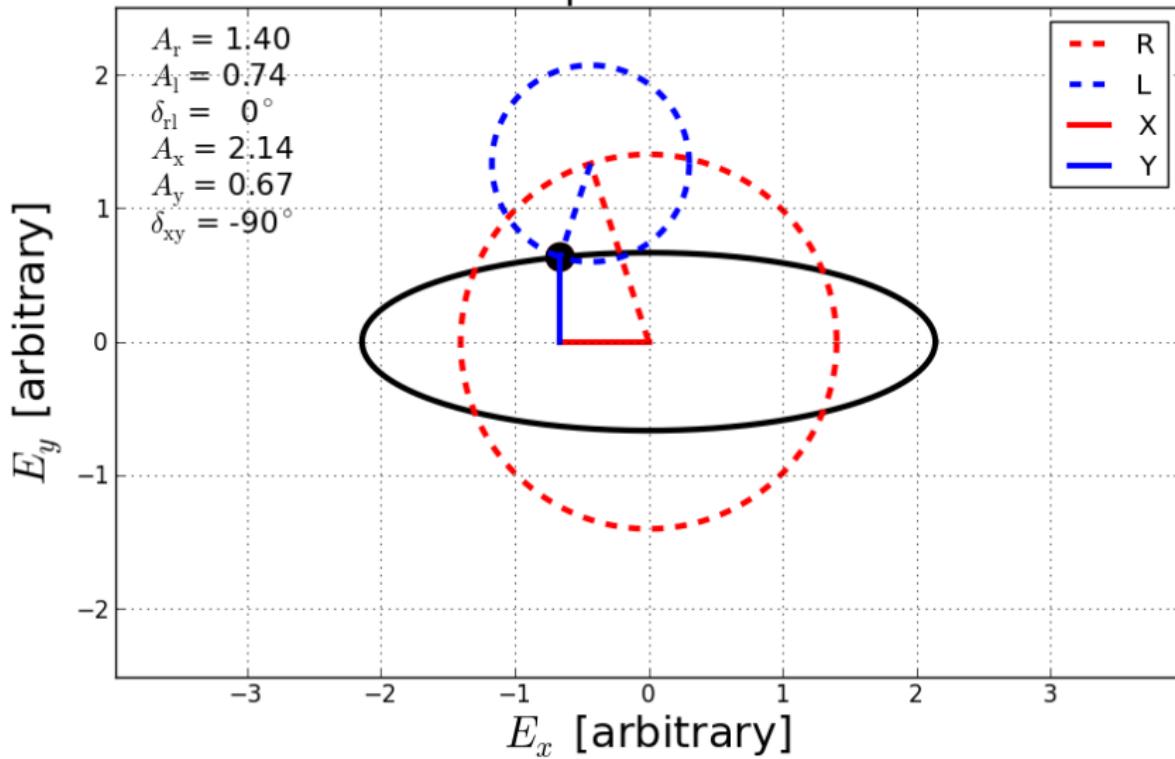
## Polarization ellipse: circular and linear



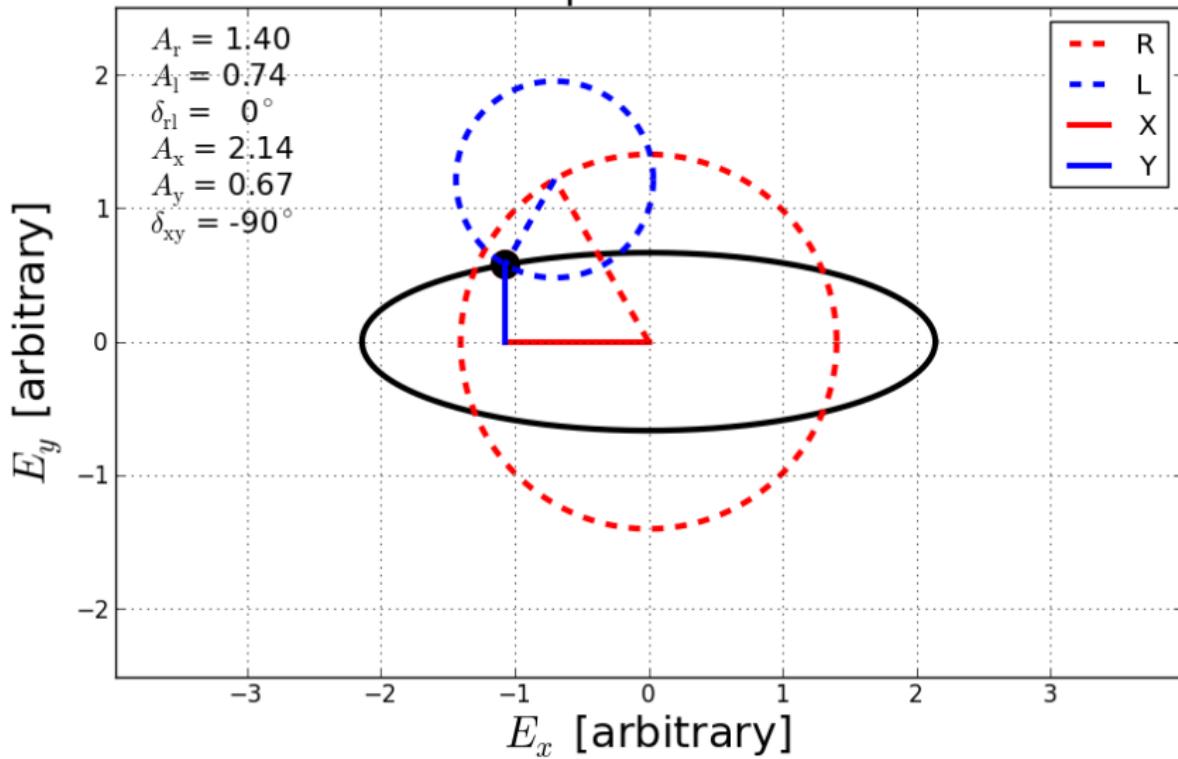
## Polarization ellipse: circular and linear



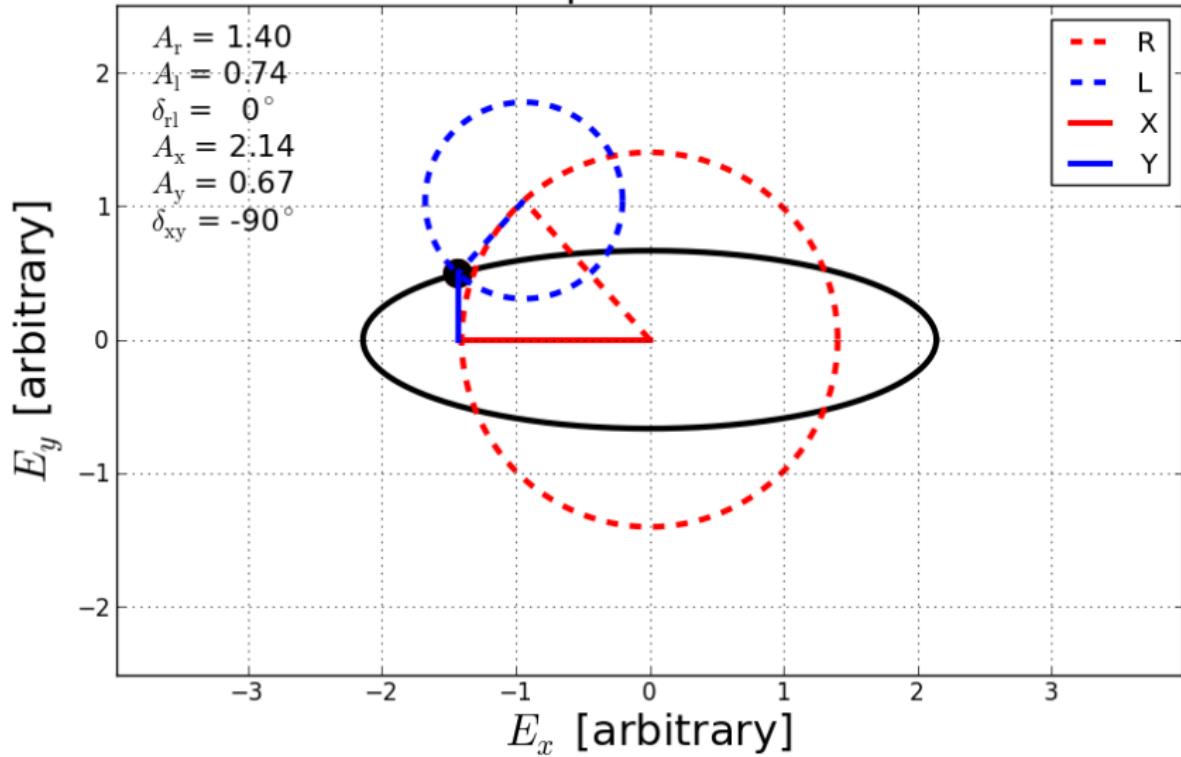
## Polarization ellipse: circular and linear



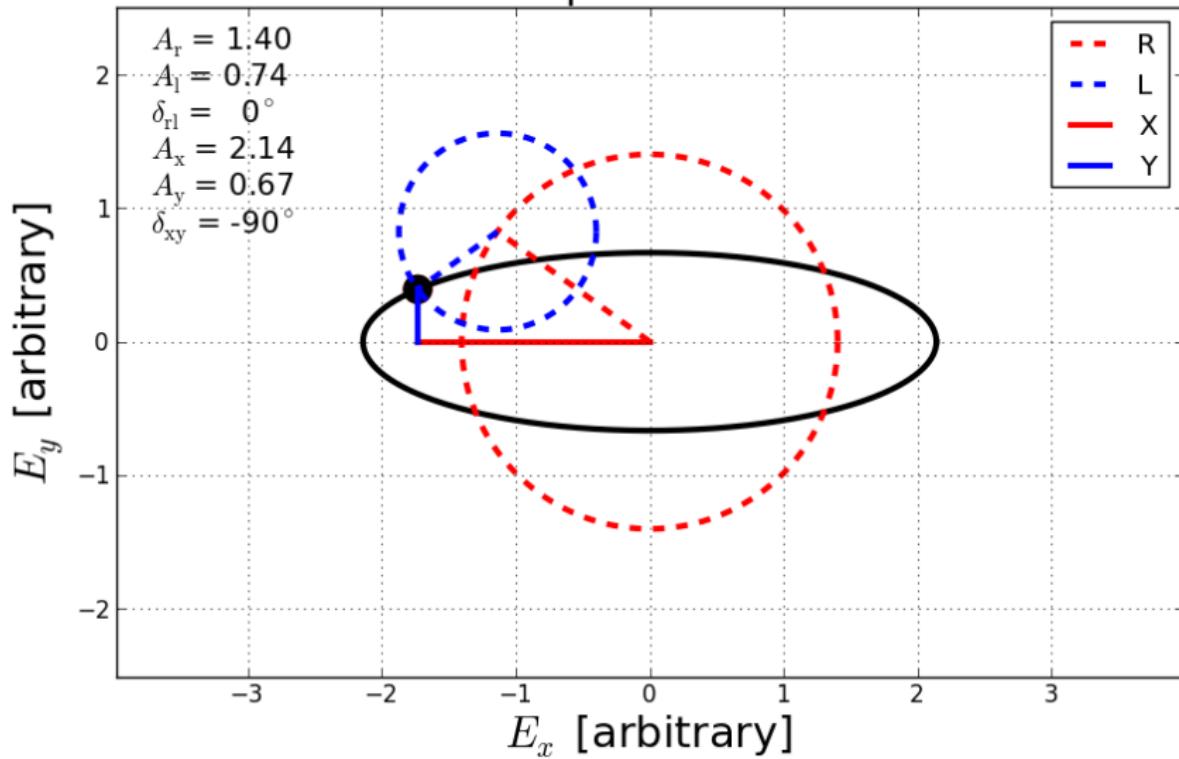
## Polarization ellipse: circular and linear



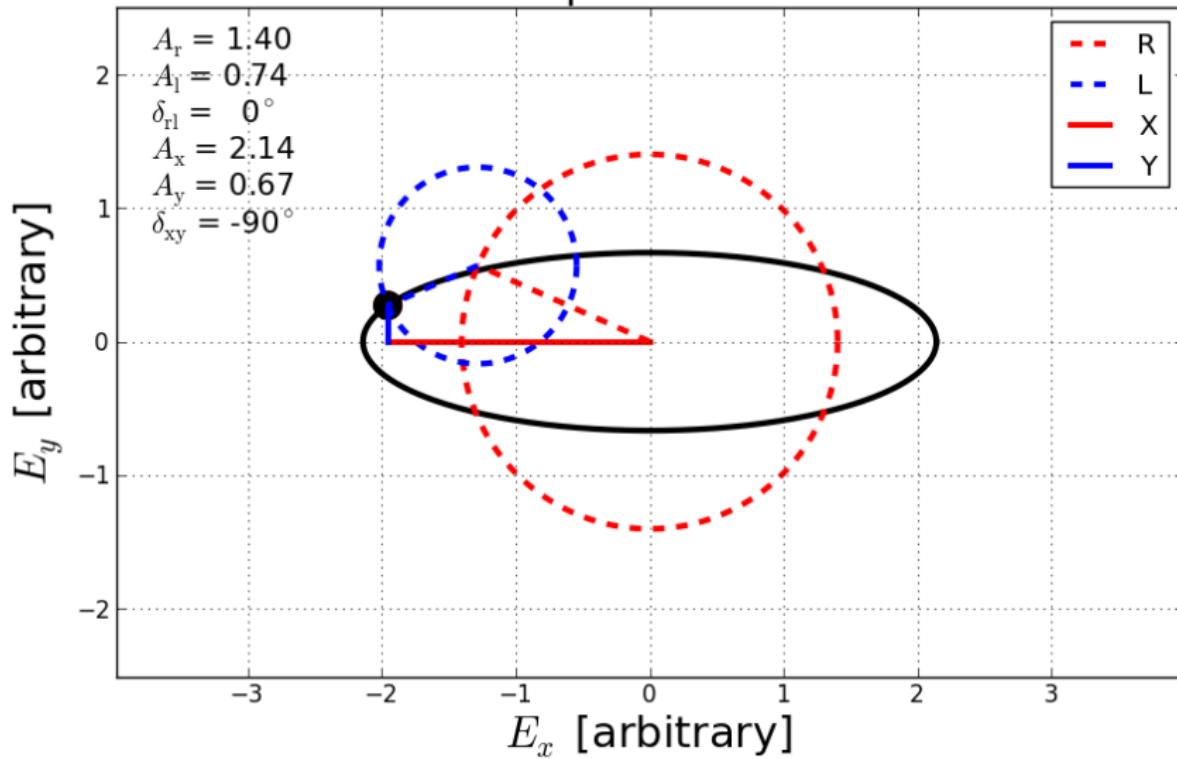
## Polarization ellipse: circular and linear



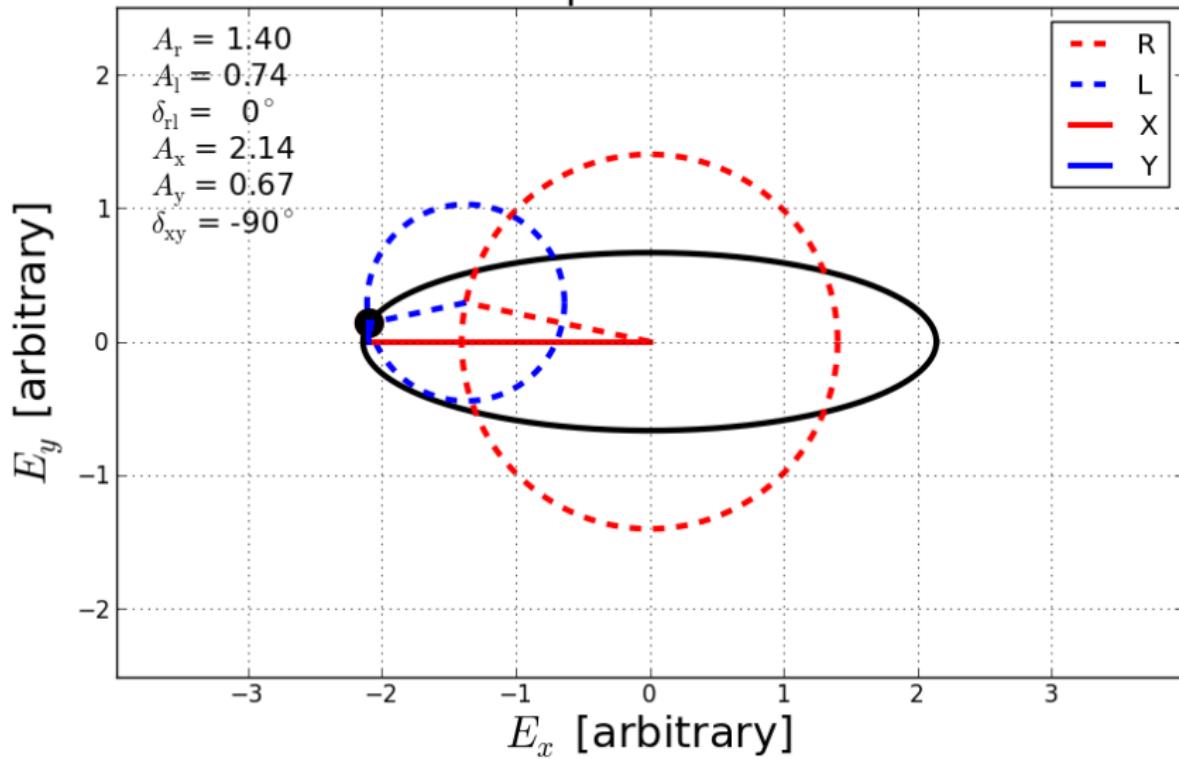
## Polarization ellipse: circular and linear



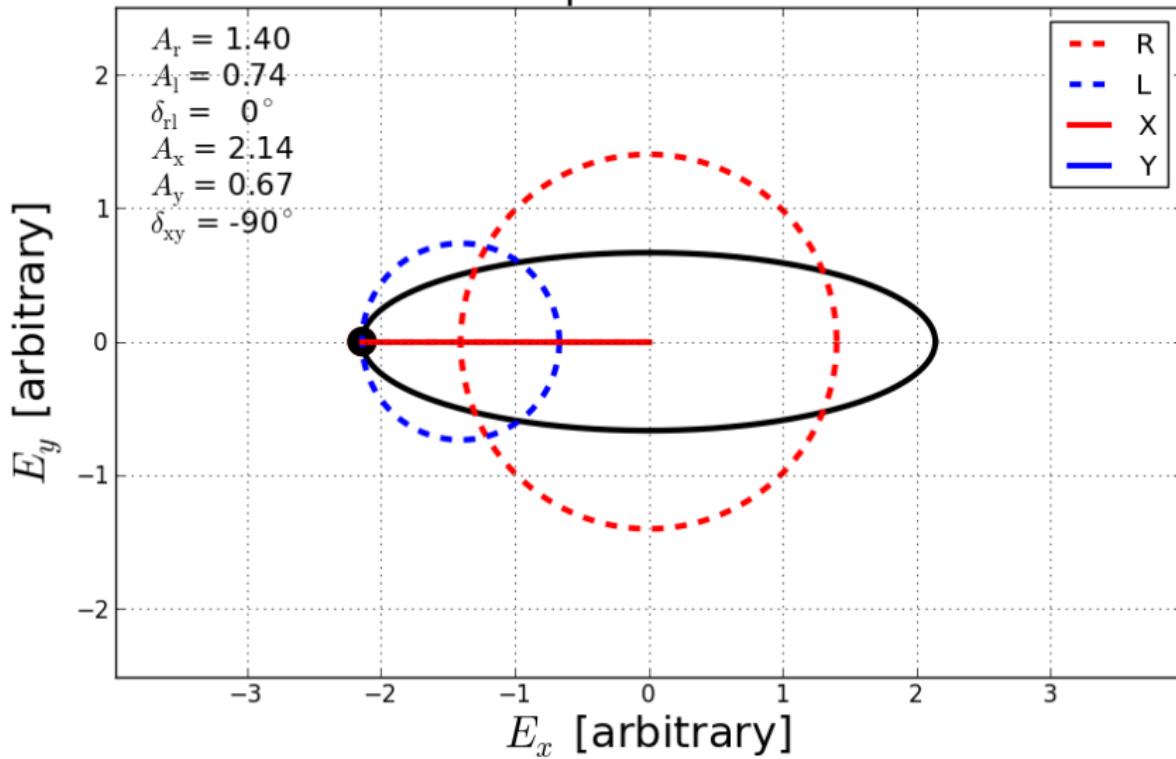
## Polarization ellipse: circular and linear



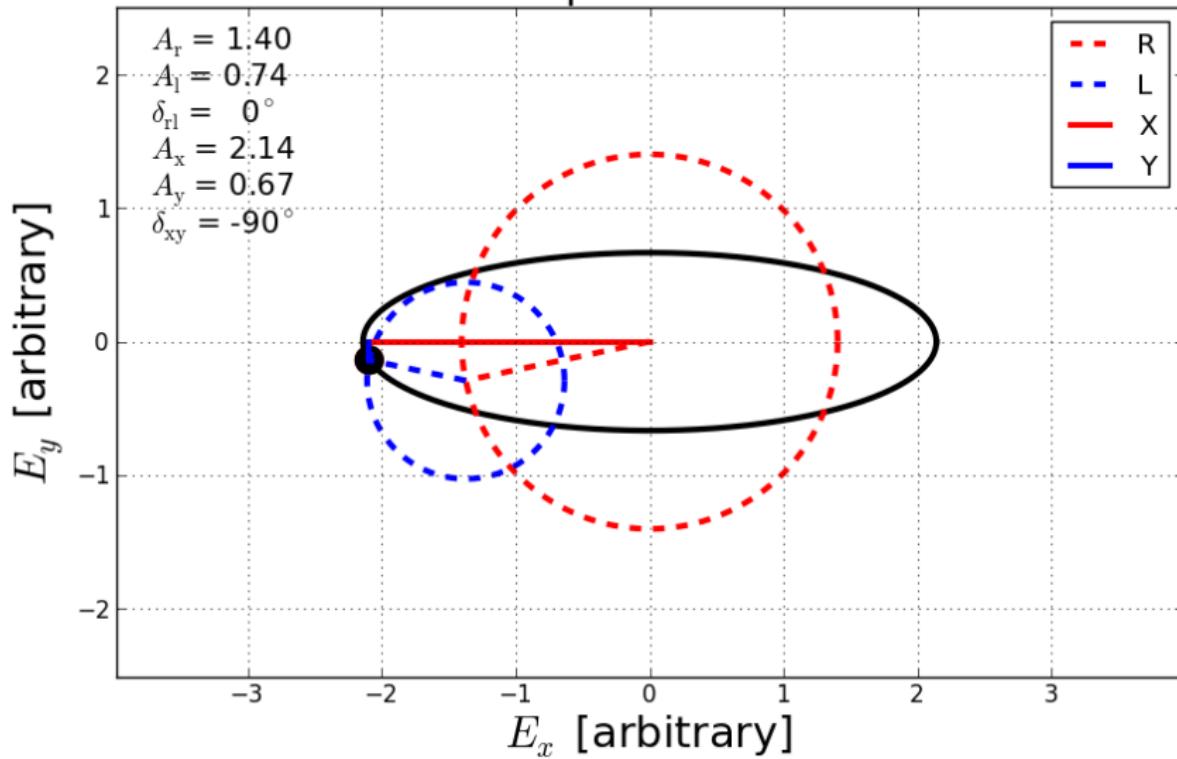
## Polarization ellipse: circular and linear



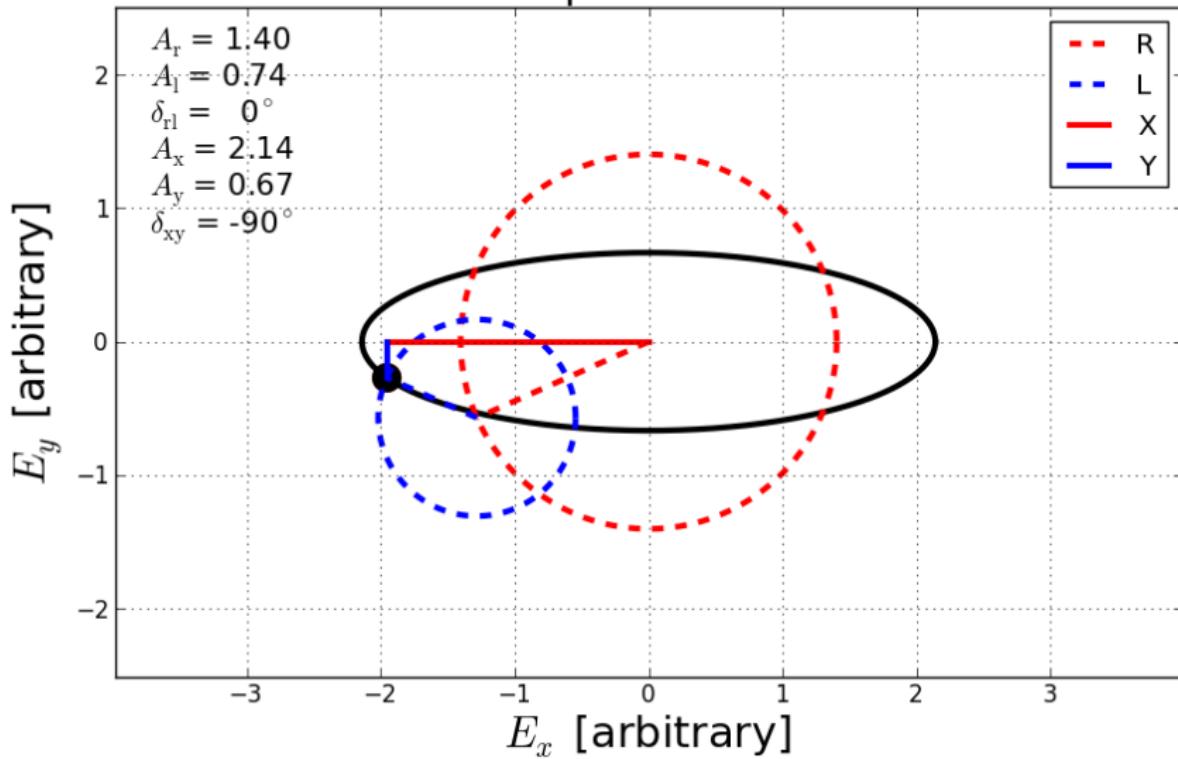
## Polarization ellipse: circular and linear



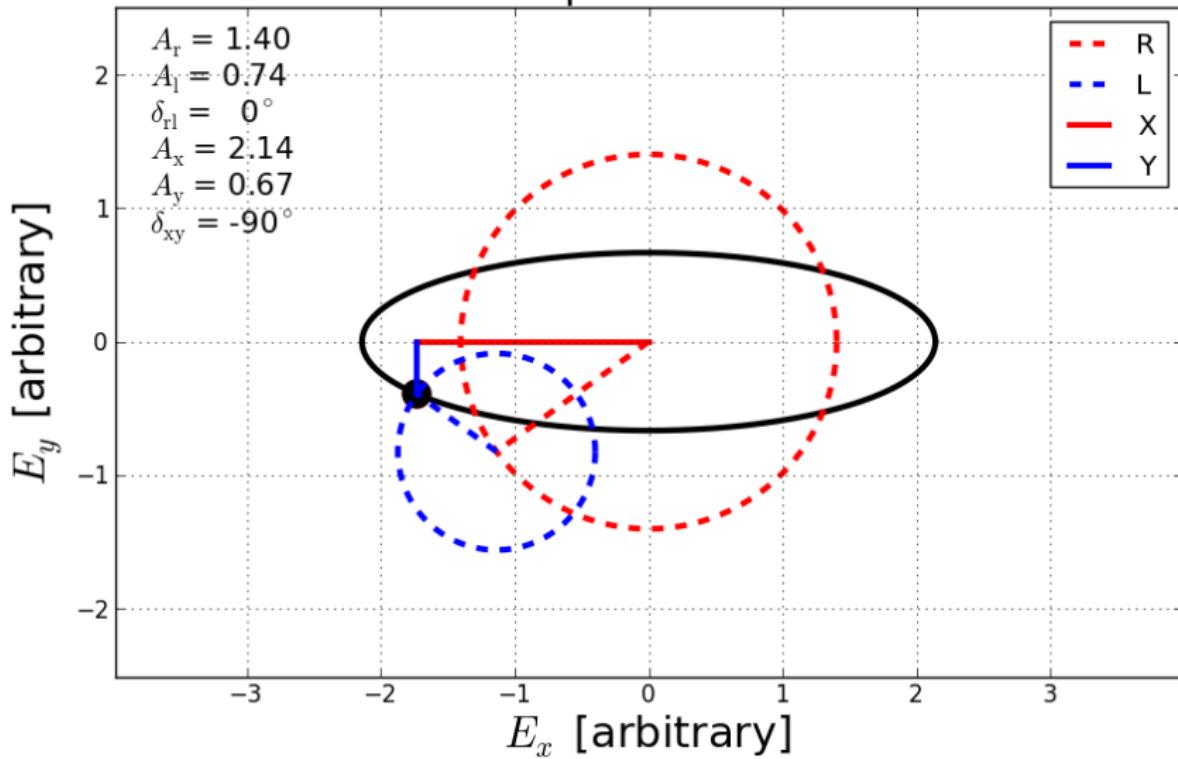
## Polarization ellipse: circular and linear



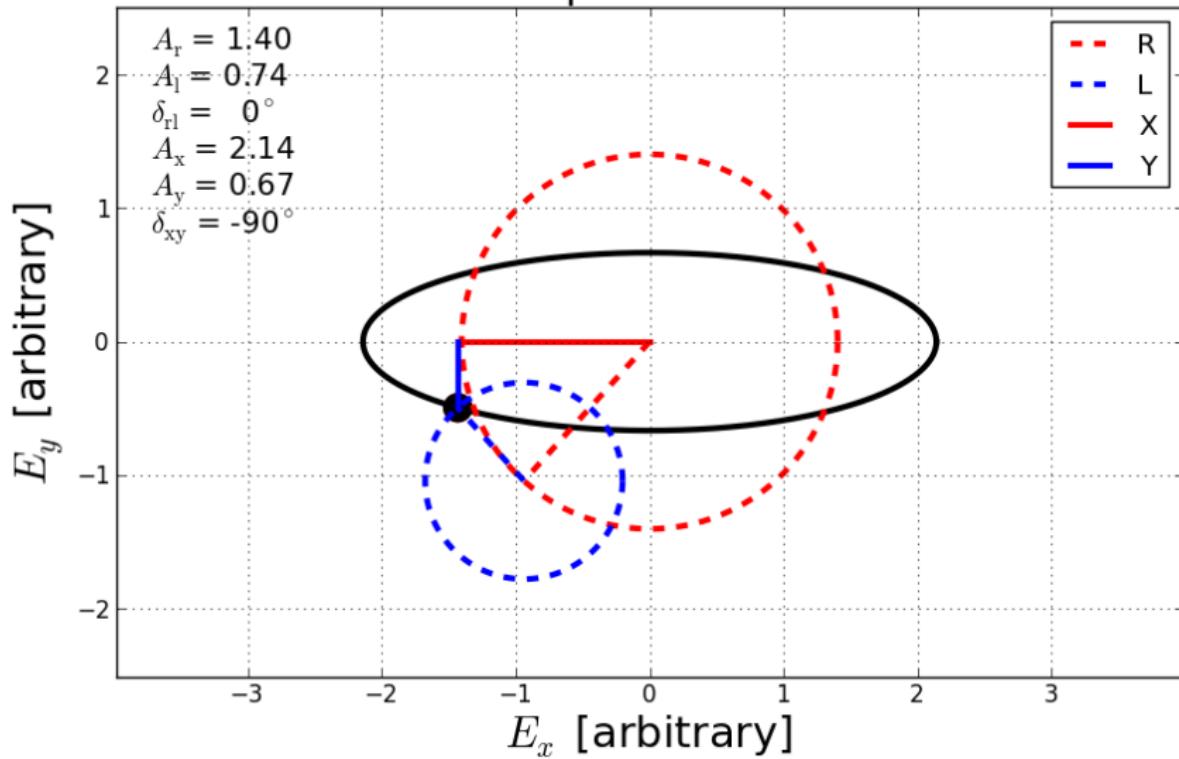
## Polarization ellipse: circular and linear



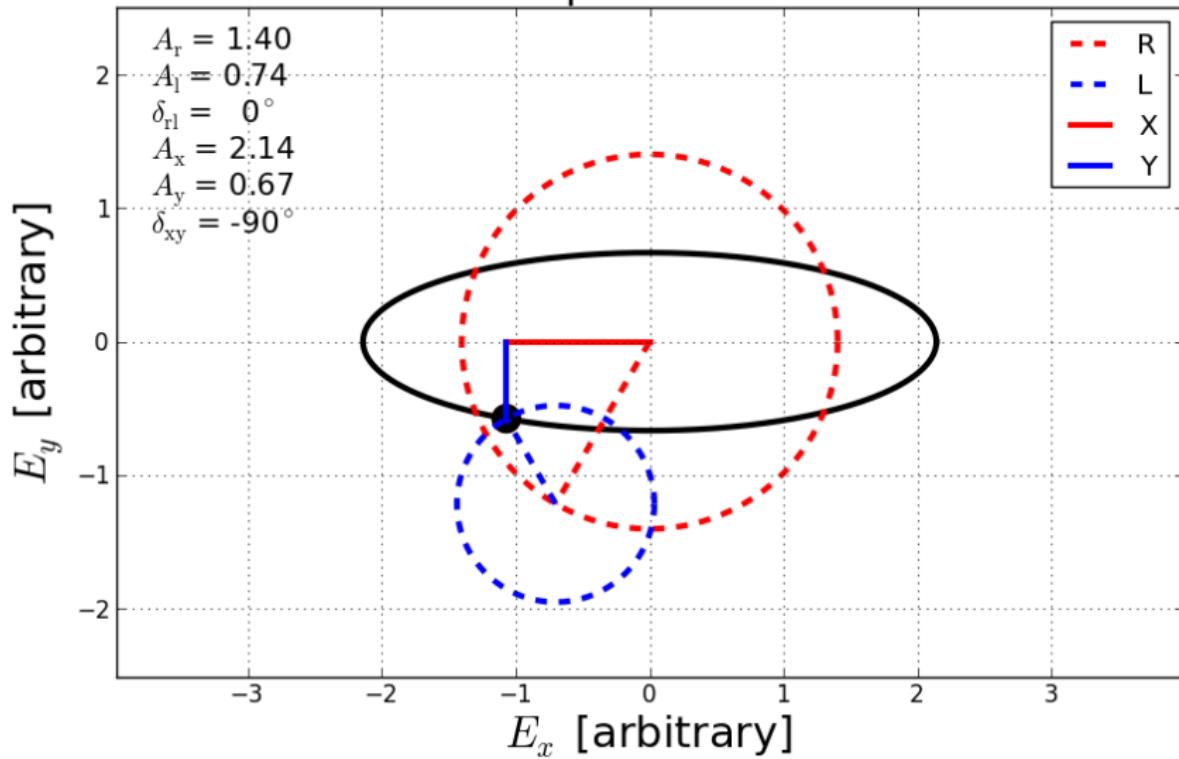
## Polarization ellipse: circular and linear



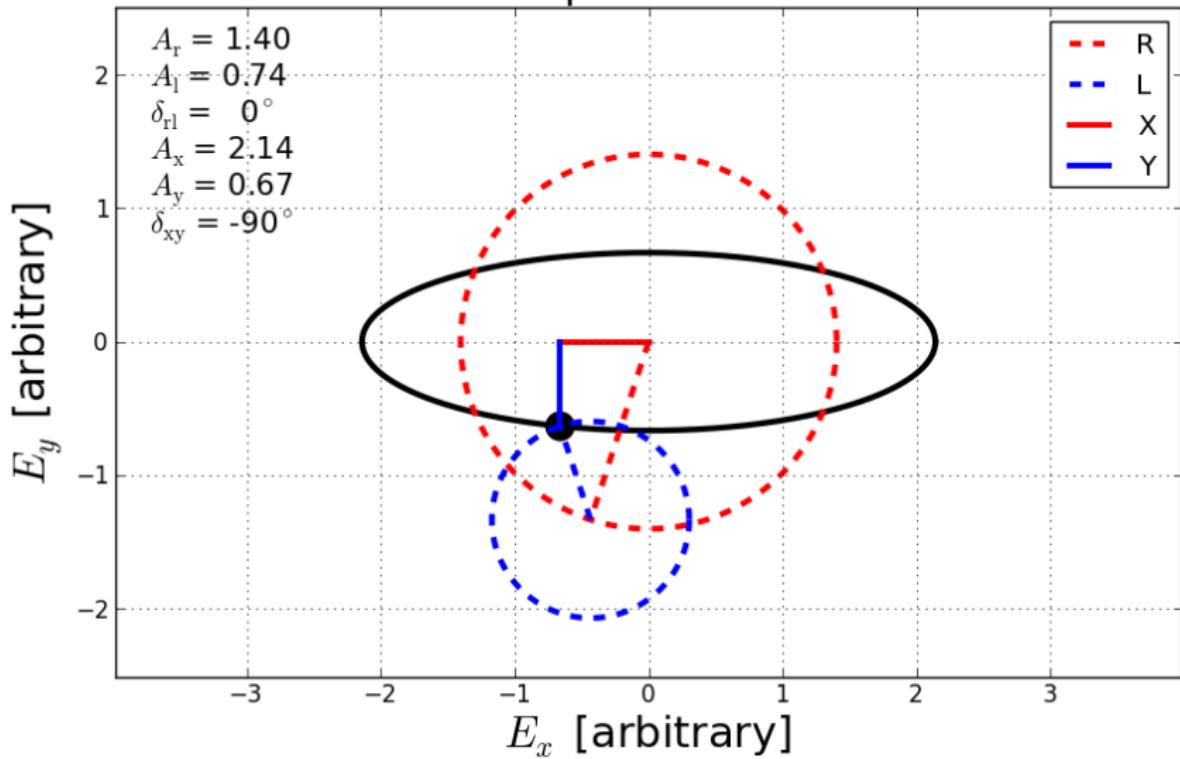
## Polarization ellipse: circular and linear



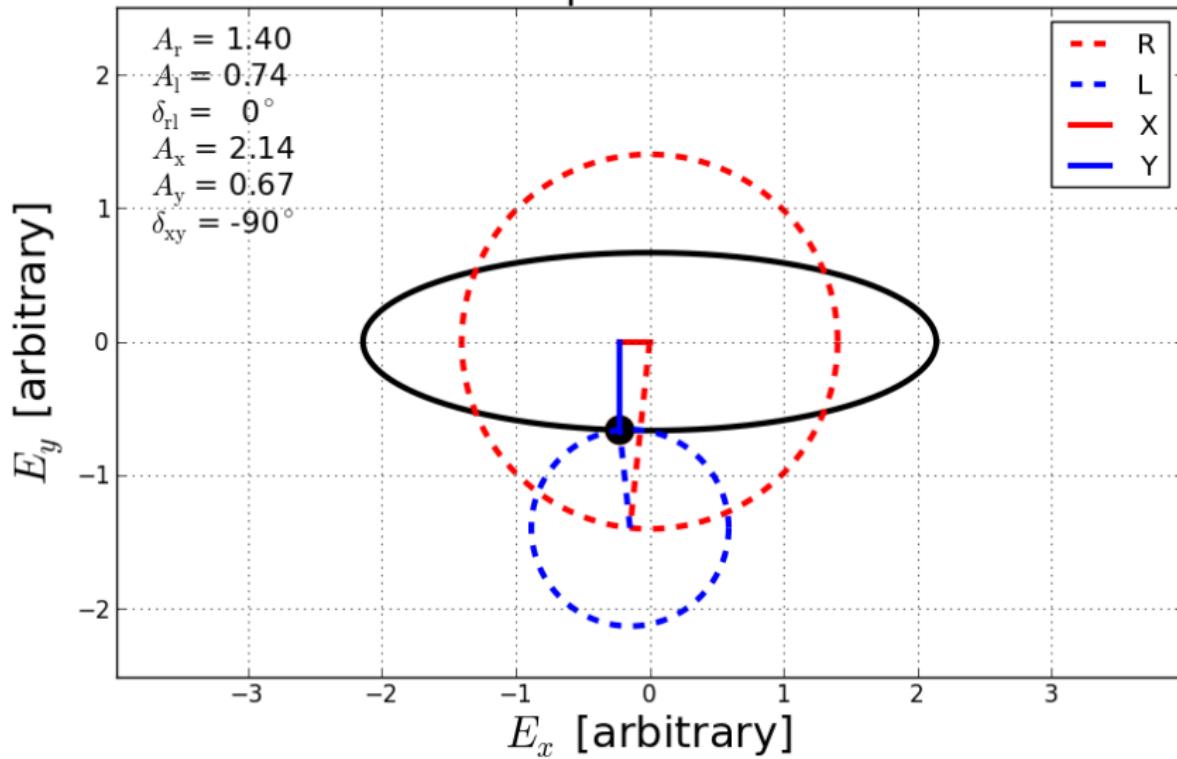
## Polarization ellipse: circular and linear



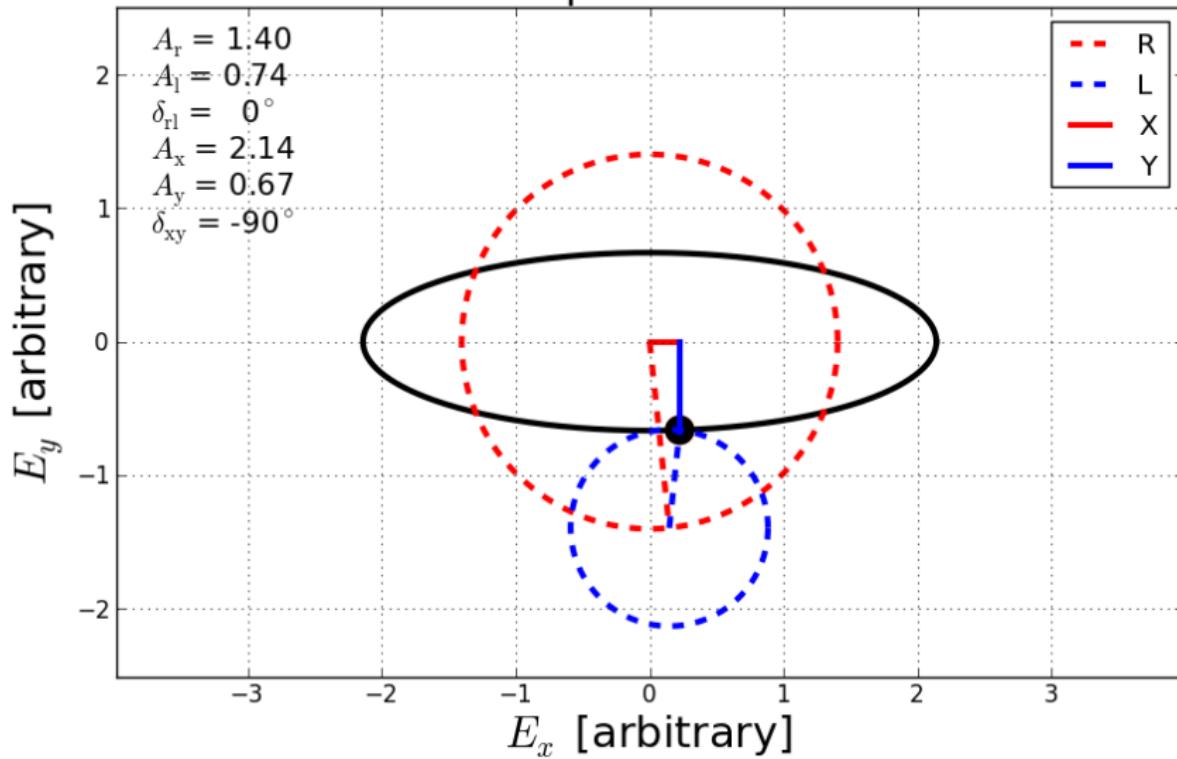
## Polarization ellipse: circular and linear



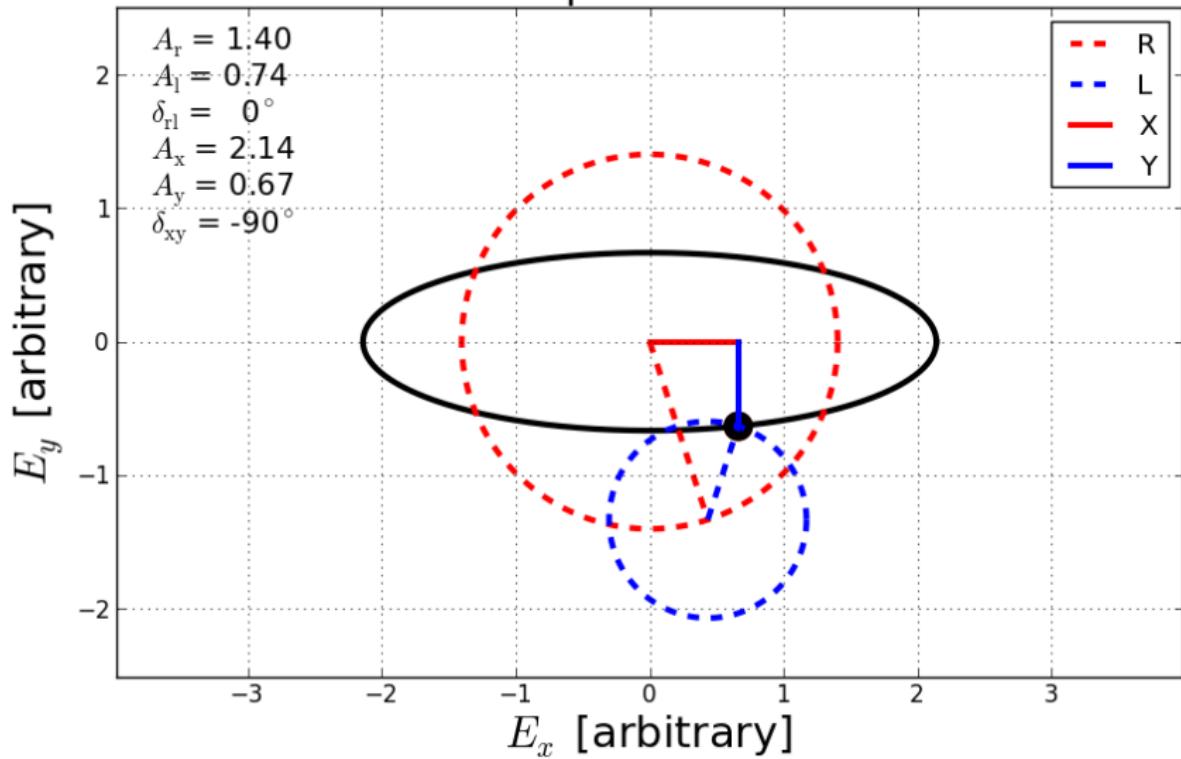
## Polarization ellipse: circular and linear



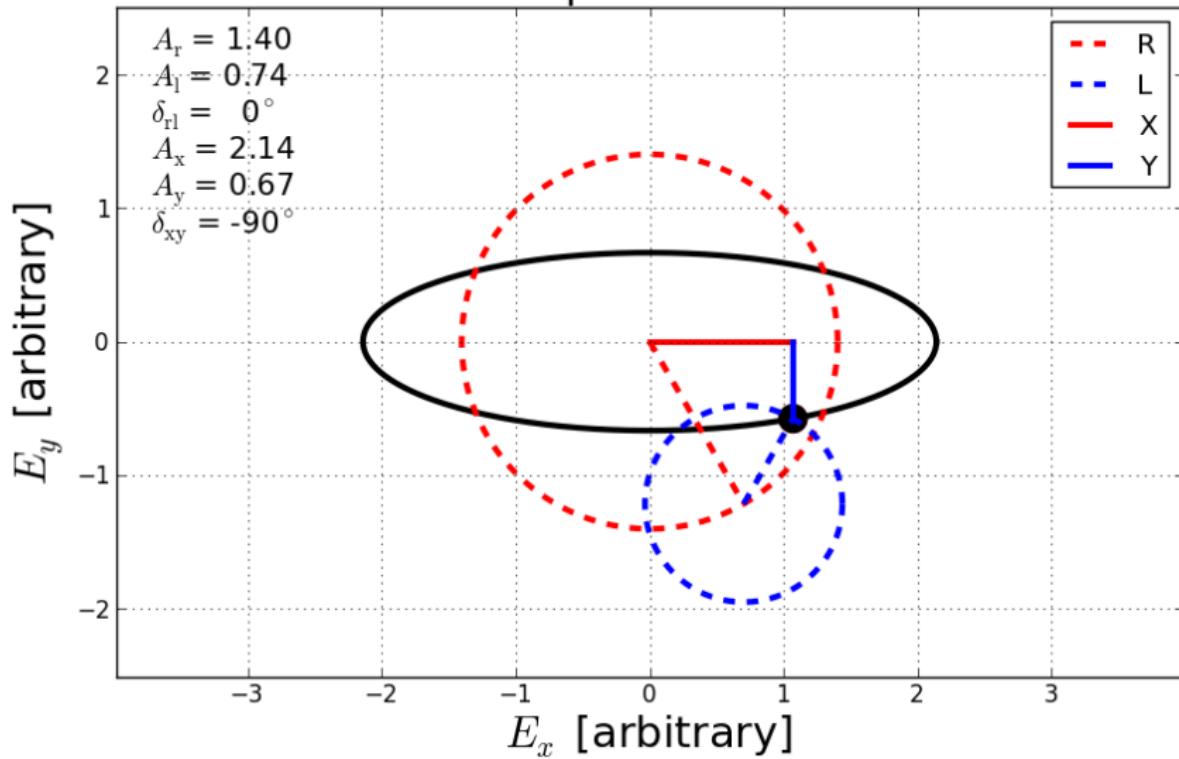
## Polarization ellipse: circular and linear



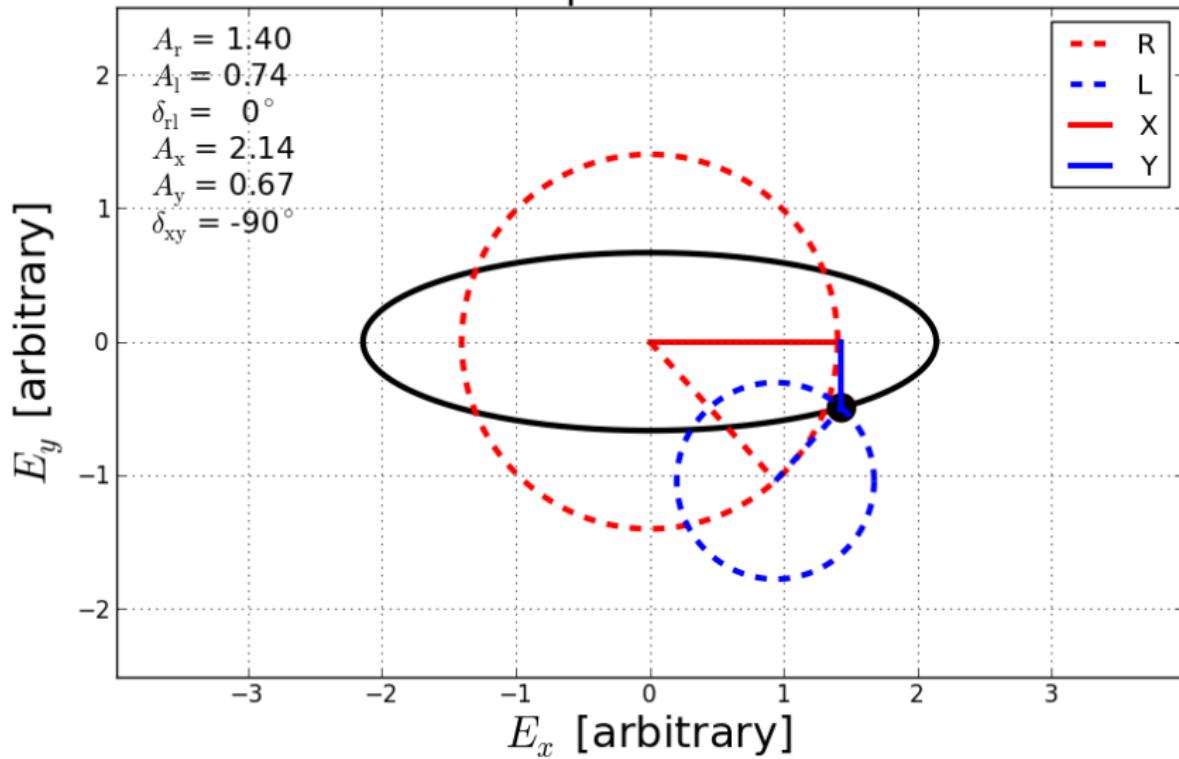
## Polarization ellipse: circular and linear



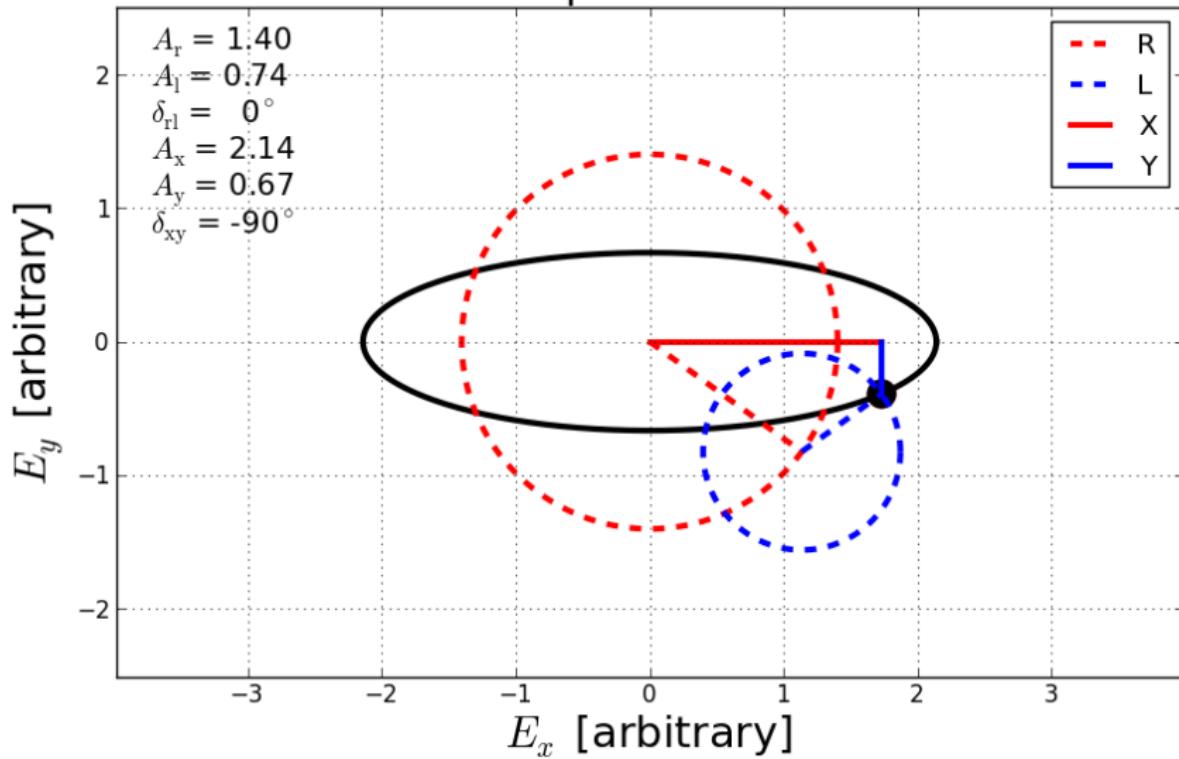
## Polarization ellipse: circular and linear



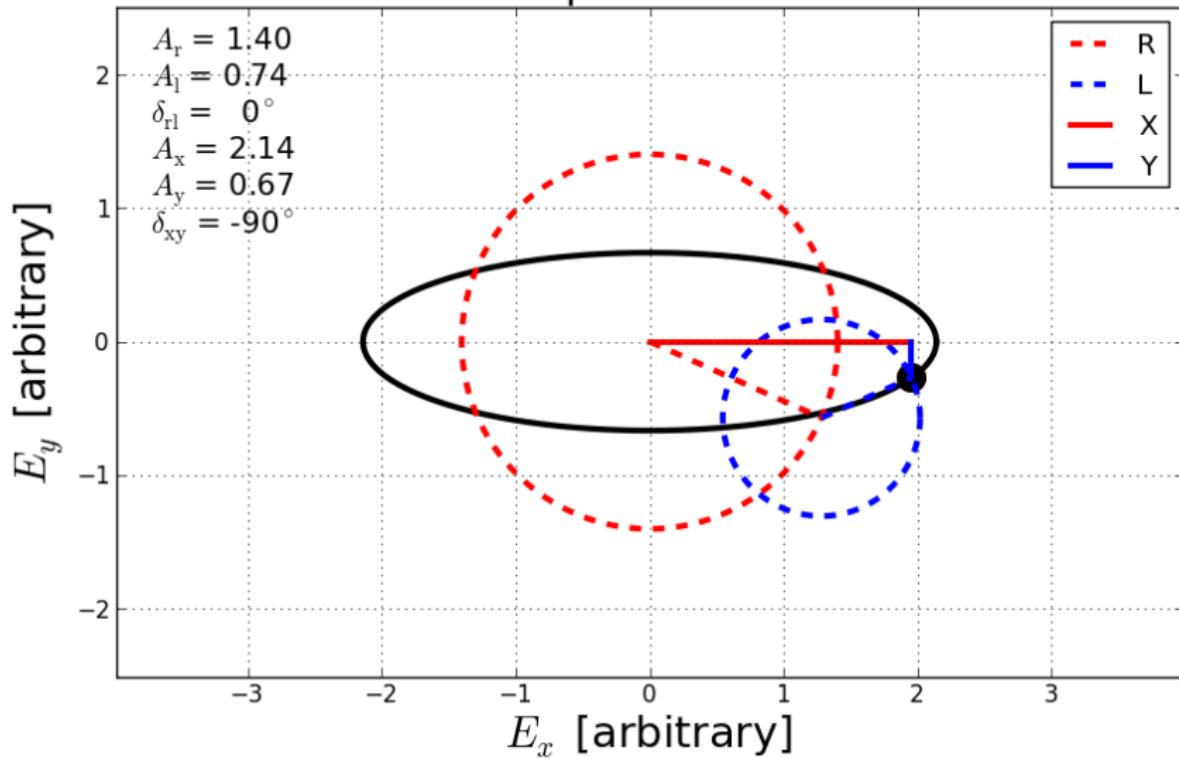
## Polarization ellipse: circular and linear



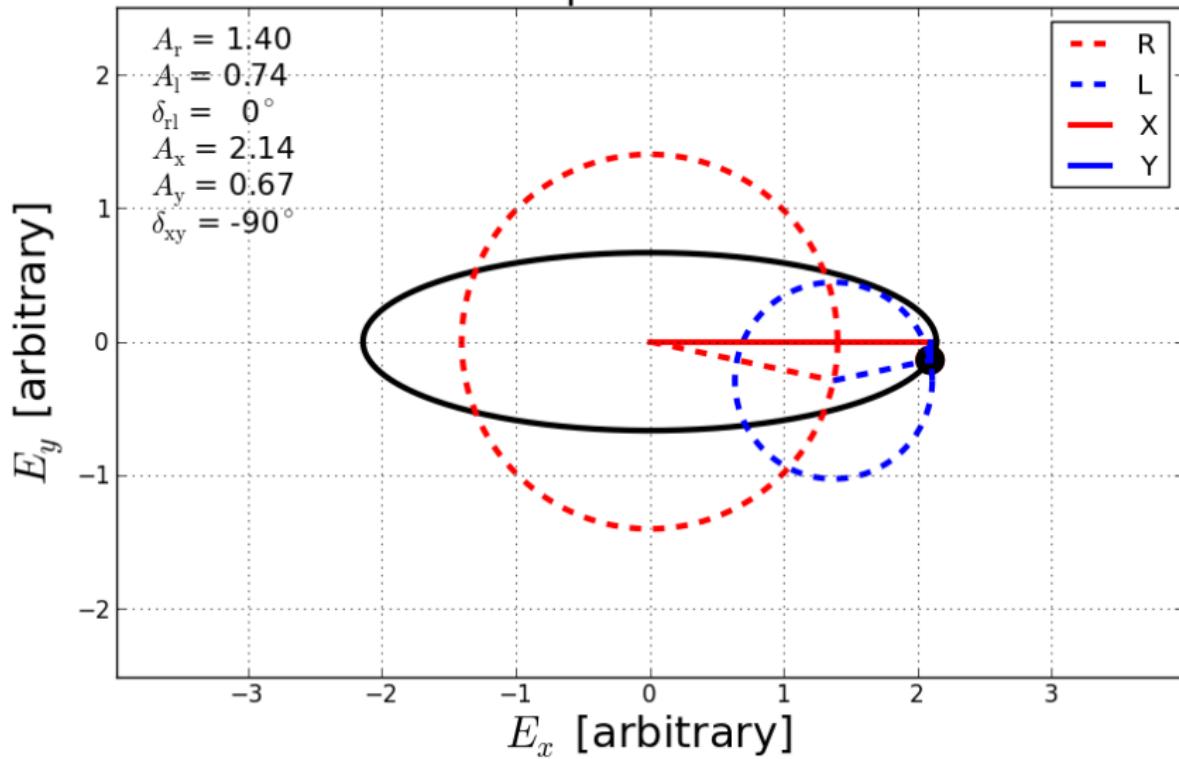
## Polarization ellipse: circular and linear



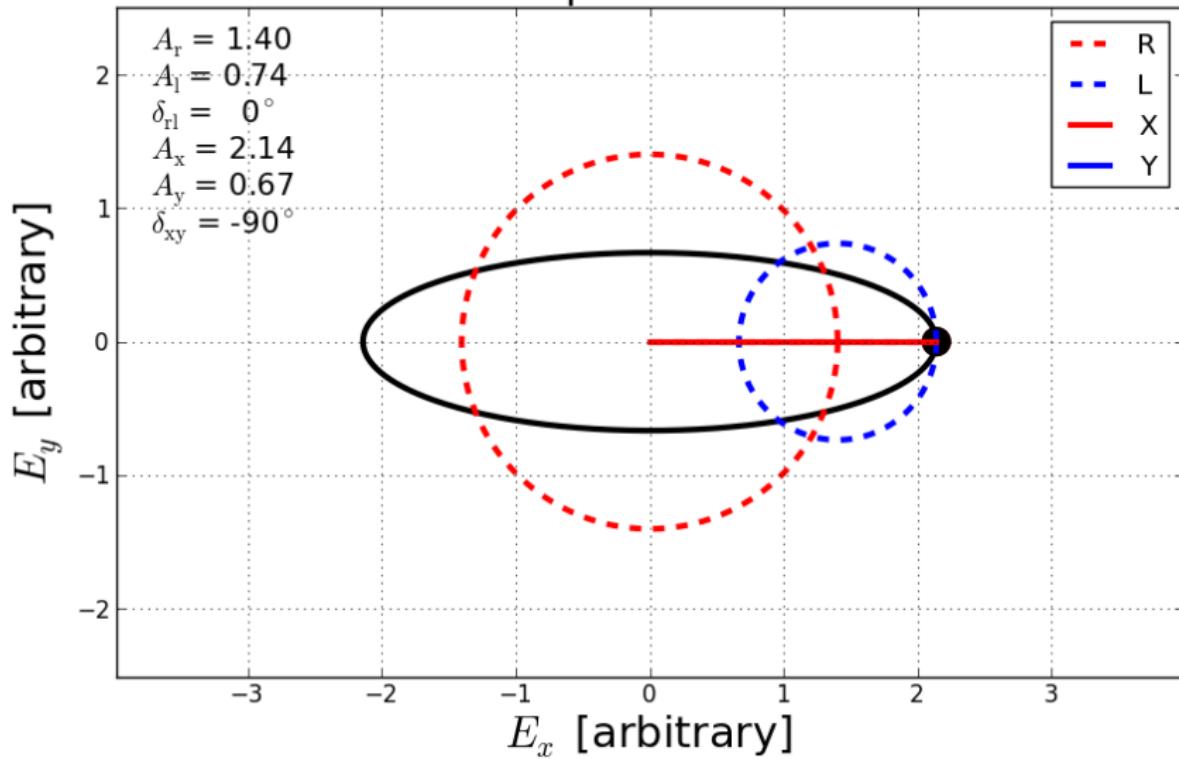
## Polarization ellipse: circular and linear



## Polarization ellipse: circular and linear



## Polarization ellipse: circular and linear



- Three parameters enough
- Same units is convenient
- George Stokes defined four parameters (1856)
- Chandrasekhar introduced them to astronomy (1946)

$$I = A_x^2 + A_y^2$$

$$I = A_r^2 + A_l^2$$

$$Q = A_x^2 - A_y^2$$

$$Q = 2A_r A_l \cos \delta_{rl}$$

$$U = 2A_x A_y \cos \delta_{xy}$$

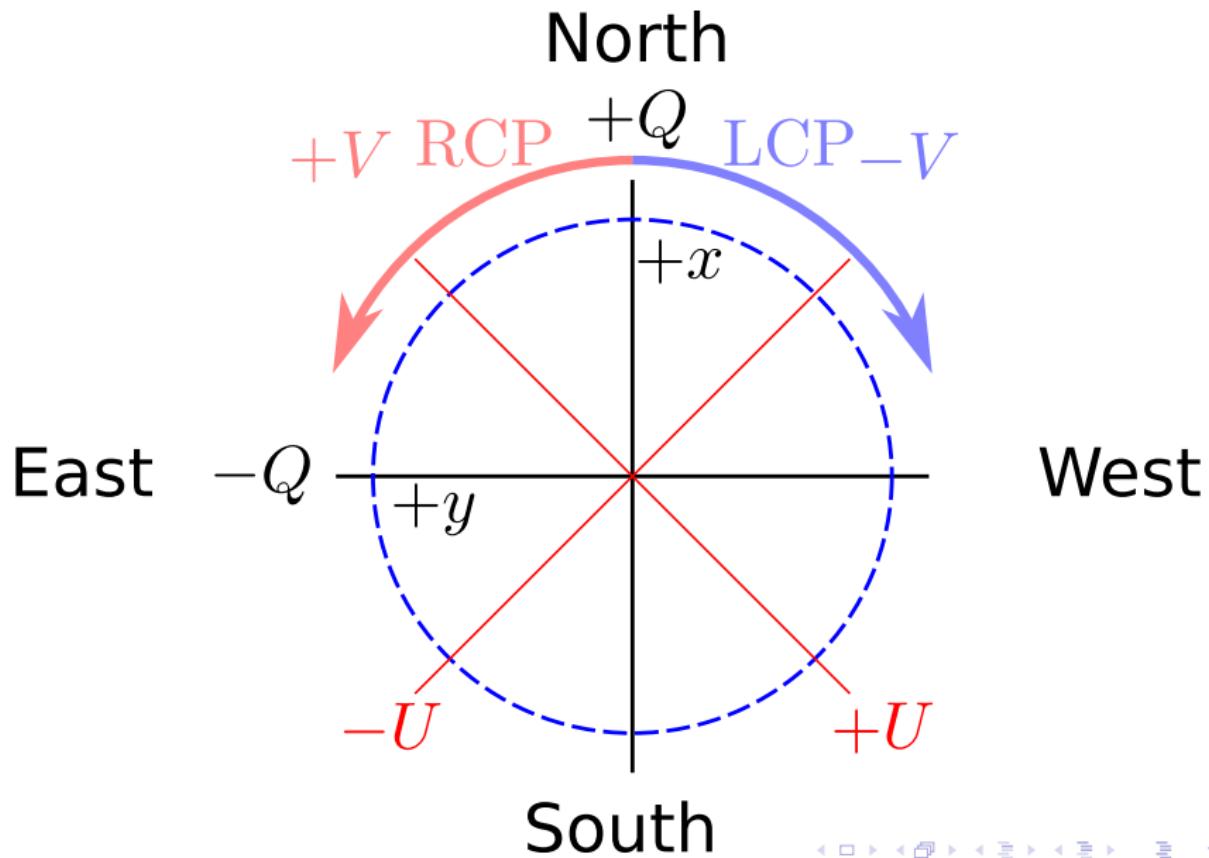
$$U = 2A_r A_l \sin \delta_{rl}$$

$$V = 2A_x A_y \sin \delta_{xy}$$

$$V = A_r^2 - A_l^2$$

- Monochromatic wave 100% polarized:

$$I^2 = Q^2 + U^2 + V^2$$



## Quasi-monochromatic approximation

- Monochromatic radiation does not exist
- Finite bandwidth  $\Delta\nu$ ; averaging time  $\tau \gg \Delta\nu^{-1}$

$$I = \langle A_x^2 \rangle + \langle A_y^2 \rangle$$

$$Q = \langle A_x^2 \rangle - \langle A_y^2 \rangle$$

$$U = \langle 2A_x A_y \cos \delta_{xy} \rangle$$

$$V = \langle 2A_x A_y \sin \delta_{xy} \rangle$$

$$I = \langle A_r^2 \rangle + \langle A_l^2 \rangle$$

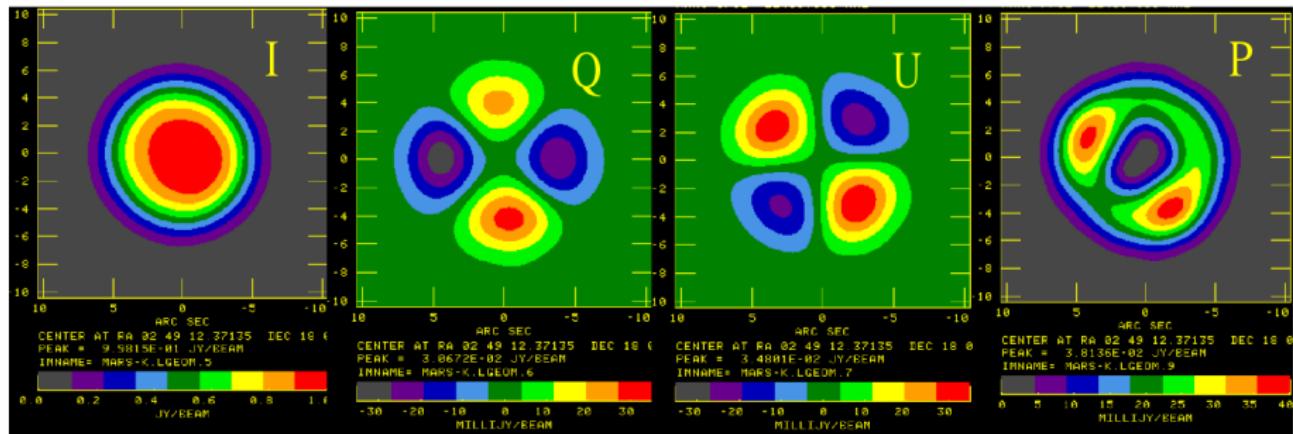
$$Q = \langle 2A_r A_l \cos \delta_{rl} \rangle$$

$$U = \langle 2A_r A_l \sin \delta_{rl} \rangle$$

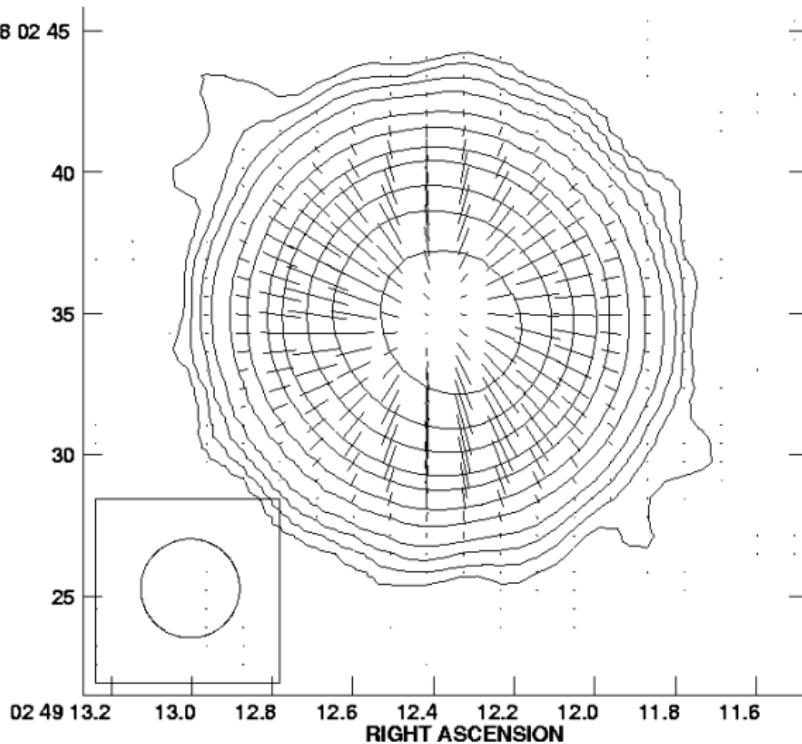
$$V = \langle A_r^2 \rangle - \langle A_l^2 \rangle$$

$$I^2 \geq Q^2 + U^2 + V^2$$

- Fractional linear pol:  $p = \sqrt{Q^2 + U^2}/I \leq 1$
- Fractional circular pol:  $v = \|V\|/I \leq 1$



- Thermal emission
- What do pol vectors look like?
- Why is it even polarized?



Peak contour flux = 9.9738E-01 JY/BEAM  
Levs = 9.974E-03 \* (-0.250, 0.250, 0.500, 1, 2,  
5, 10, 20, 30, 50, 70, 90)

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# Introducing Stokes visibilities



$$\begin{aligned}\mathcal{I}(u, v) &= \mathcal{F}^+(I(l, m)) \\ \mathcal{Q}(u, v) &= \mathcal{F}^+(Q(l, m)) \\ \mathcal{U}(u, v) &= \mathcal{F}^+(U(l, m)) \\ \mathcal{V}(u, v) &= \mathcal{F}^+(V(l, m)),\end{aligned}$$

where

$$\mathcal{F}^+(f) = \int_{Im} f e^{+2\pi i \nu(u l + v m)/c} dI dm$$

## Cartesian

$$E_x = \Re \left\{ A_x e^{2\pi i \nu t} \right\}$$

$$E_y = \Re \left\{ A_y e^{i\delta_{xy}} e^{2\pi i \nu t} \right\}$$

$$I = \langle A_x^2 \rangle + \langle A_y^2 \rangle$$

$$= \langle E_x E_x^* \rangle + \langle E_y E_y^* \rangle$$

$$Q = \langle A_x^2 \rangle - \langle A_y^2 \rangle$$

$$= \langle E_x E_x^* \rangle - \langle E_y E_y^* \rangle$$

$$U = \langle 2A_x A_y \cos \delta_{xy} \rangle$$

$$= \langle E_x E_y^* \rangle + \langle E_y E_x^* \rangle$$

$$V = \langle 2A_x A_y \sin \delta_{xy} \rangle$$

$$= i (\langle E_x E_y^* \rangle - \langle E_y E_x^* \rangle)$$

## Circular

$$E_r = \Re \left\{ A_r e^{2\pi i \nu t} \right\}$$

$$E_l = \Re \left\{ A_l e^{i\delta_{rl}} e^{2\pi i \nu t} \right\}$$

$$I = \langle A_r^2 \rangle + \langle A_l^2 \rangle$$

$$= \langle E_r E_r^* \rangle + \langle E_l E_l^* \rangle$$

$$Q = \langle 2A_r A_l \cos \delta_{rl} \rangle$$

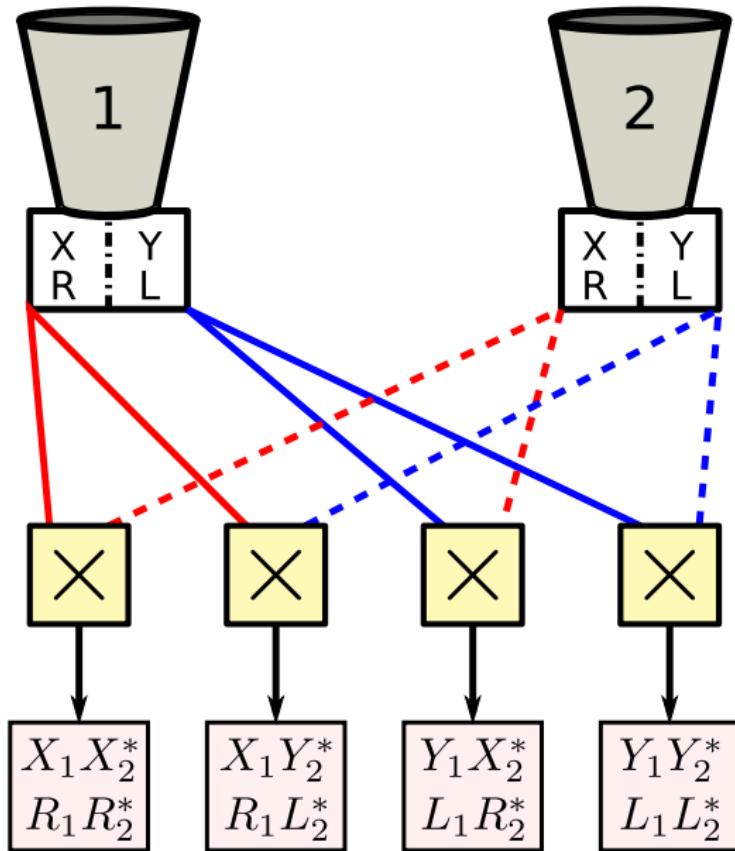
$$= \langle E_r E_l^* \rangle + \langle E_l E_r^* \rangle$$

$$U = \langle 2A_r A_l \sin \delta_{rl} \rangle$$

$$= i (\langle E_r E_l^* \rangle - \langle E_l E_r^* \rangle)$$

$$V = \langle A_r^2 \rangle - \langle A_l^2 \rangle$$

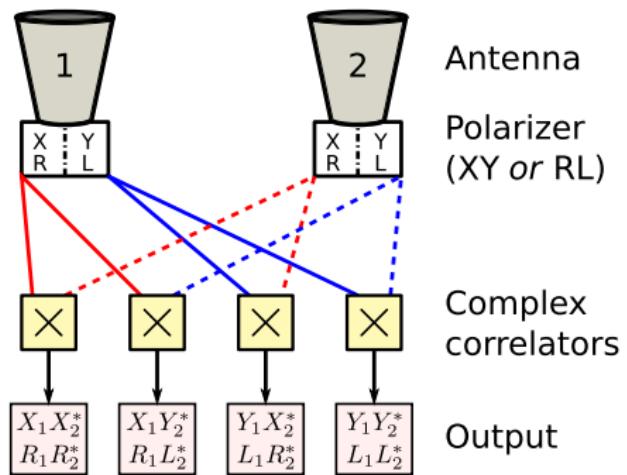
$$= \langle E_r E_r^* \rangle - \langle E_l E_l^* \rangle$$



Antenna

Polarizer  
(XY or RL)Complex  
correlators

Output

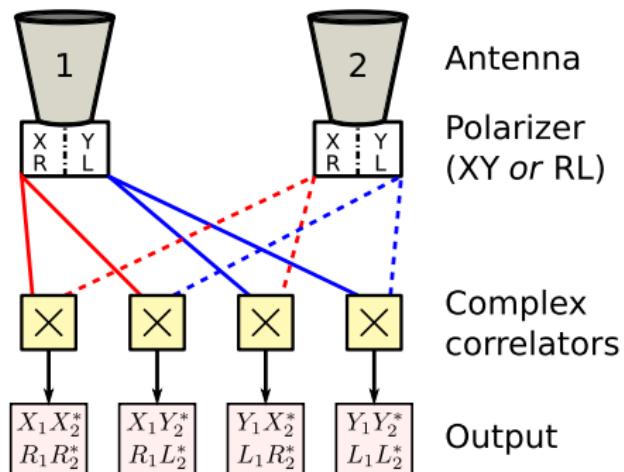
**Cartesian**

$$\begin{aligned}\mathcal{I} &= x_1 x_2^* + y_1 y_2^* \\ \mathcal{Q} &= x_1 x_2^* - y_1 y_2^* \\ \mathcal{U} &= x_1 y_2^* + y_1 x_2^* \\ \mathcal{V} &= i(x_1 y_2^* - y_1 x_2^*)\end{aligned}$$

**Circular**

$$\begin{aligned}\mathcal{I} &= r_1 r_2^* + l_1 l_2^* \\ \mathcal{Q} &= r_1 l_2^* + l_1 r_2^* \\ \mathcal{U} &= i(r_1 l_2^* - l_1 r_2^*) \\ \mathcal{V} &= r_1 r_2^* - l_1 l_2^*\end{aligned}$$

- From here on,  $\langle \cdot \rangle$  is implied for correlator outputs.



From here on,  $p$  and  $q$  designate either  $x$  and  $y$ , or  $r$  and  $l$ .

- Polarizers produce vector:

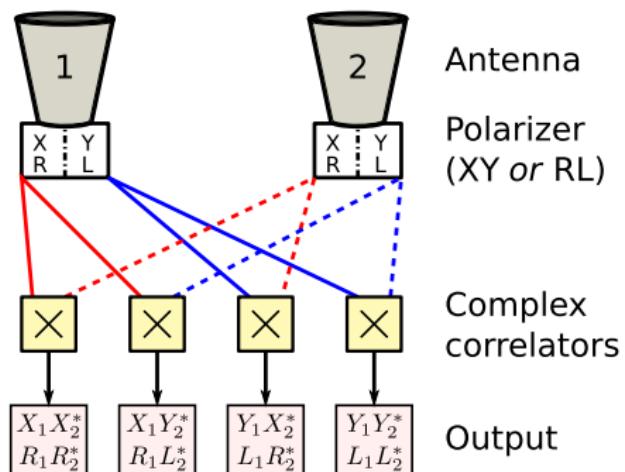
$$\mathbf{e}_i = \begin{pmatrix} p_i \\ q_i \end{pmatrix}$$

- Correlator multiplies:

$$\mathbf{E}_{ij} = \mathbf{e}_i \mathbf{e}_j^\dagger = \begin{pmatrix} p_i \\ q_i \end{pmatrix} \begin{pmatrix} p_j^* & q_j^* \end{pmatrix}$$

$$\mathbf{E}_{ij} = \begin{pmatrix} p_i p_j^* & p_i q_j^* \\ q_i p_j^* & q_i q_j^* \end{pmatrix}$$

- $\mathbf{E}_{ij}$  is the **coherency matrix**



Until now...

- Assumed all systems perfect

From now...

- Assume all systems linear:

$$\mathbf{e}'_i = \mathbf{J}_i \mathbf{e}_i$$

- $\mathbf{J}_i$  ( $2 \times 2$ ) is Jones matrix
- Cross correlation:

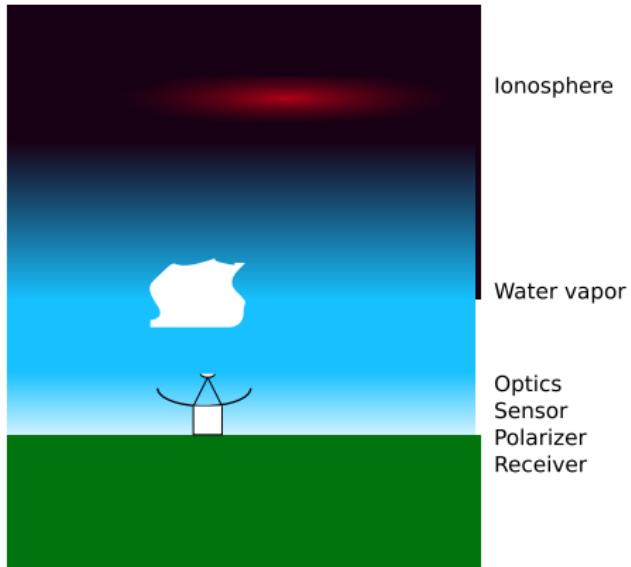
$$\mathbf{E}'_{ij} = \mathbf{e}'_i \mathbf{e}'_j^\dagger$$

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{e}_i (\mathbf{J}_j \mathbf{e}_j)^\dagger$$

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{e}_i \mathbf{e}_j^\dagger \mathbf{J}_j^\dagger$$

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j^\dagger$$

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- The measurement equation:

$$\mathbf{E}'_{ij} = \mathbf{J}_i \mathbf{E}_{ij} \mathbf{J}_j^\dagger$$

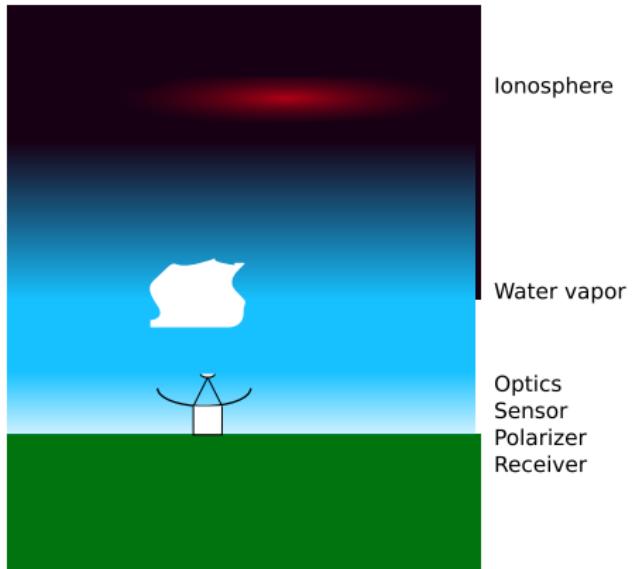
- Invertable!

$$\mathbf{E}_{ij} = \mathbf{J}_i^{-1} \mathbf{E}'_{ij} \mathbf{J}_j^{\dagger -1},$$

- where

$$\mathbf{J} = \mathbf{R}\mathbf{P}\mathbf{D}\mathbf{O}\mathbf{W}\mathbf{F}\mathbf{T}$$

- ... riiiiight...



- Perfect instrument:

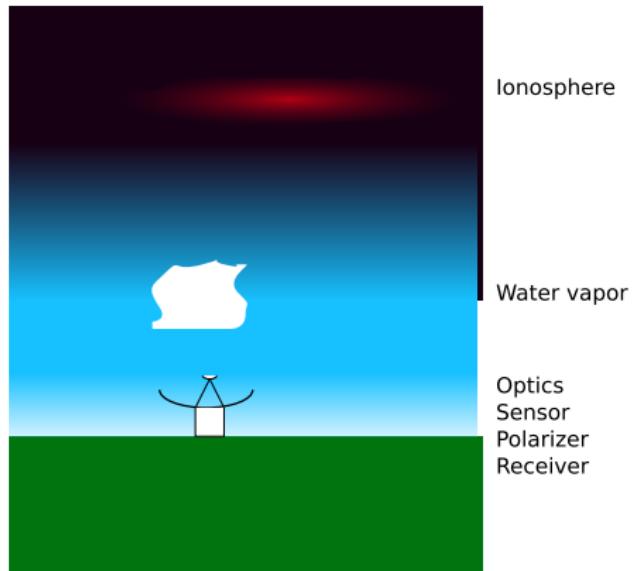
$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- Ionospheric time delay:

$$\mathbf{J} = \begin{pmatrix} e^{2\pi i \nu \tau} & 0 \\ 0 & e^{2\pi i \nu \tau} \end{pmatrix}$$

- Receiver gain:

$$\mathbf{J} = \begin{pmatrix} g_p & 0 \\ 0 & g_q \end{pmatrix}$$



- Polarization leakage:

$$\mathbf{J} = \begin{pmatrix} g_p & d_{qp} \\ d_{pq} & g_q \end{pmatrix}$$

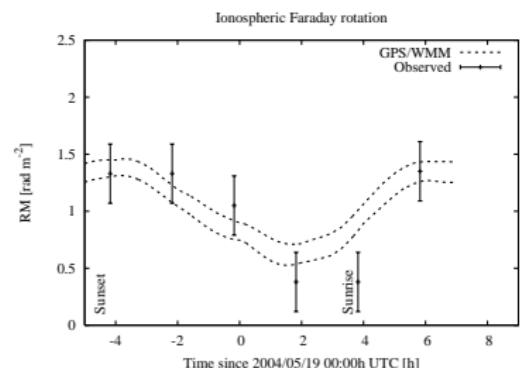
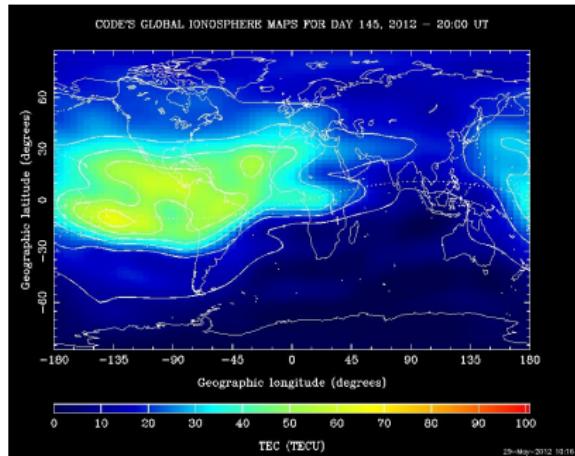
- Parallactic angle or feed rotation XY:

$$\mathbf{J} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$$

- Parallactic angle or feed rotation RL:

$$\mathbf{J} = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$$

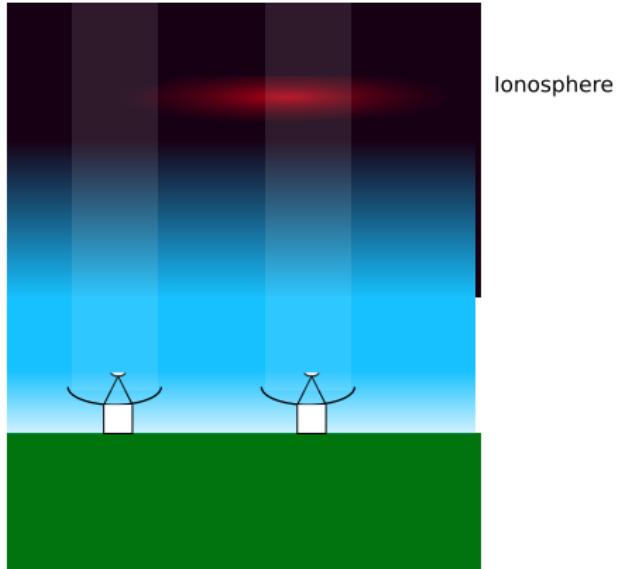
# Ionospheric Faraday rotation



- Remember:  $\Delta\chi = \chi_0 + \phi\lambda^2$
- Faraday depth

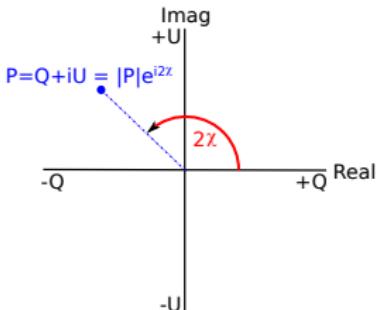
$$\phi = 0.812 \int_{\text{there}}^{\text{here}} n_e \mathbf{B} \cdot d\mathbf{l}$$

- ionosphere: plasma within Earth's magnetic field
- $\phi \approx -10 - +10 \text{ rad m}^{-2}$
- Very significant below 1 GHz
- Use TEC/IGRF models for correction, check with pulsar.



- $\Delta\chi = \chi_0 + \phi\lambda^2$
- Rotation of linear pol = delay between RCP and LCP
- Antennas see different ionosphere
- Leakage from LL to RR or v.v. after cross correlation
- Rotates  $\begin{pmatrix} I \\ V \end{pmatrix}$  vector
- Important below 300 MHz at baselines  $\geq 20$  km

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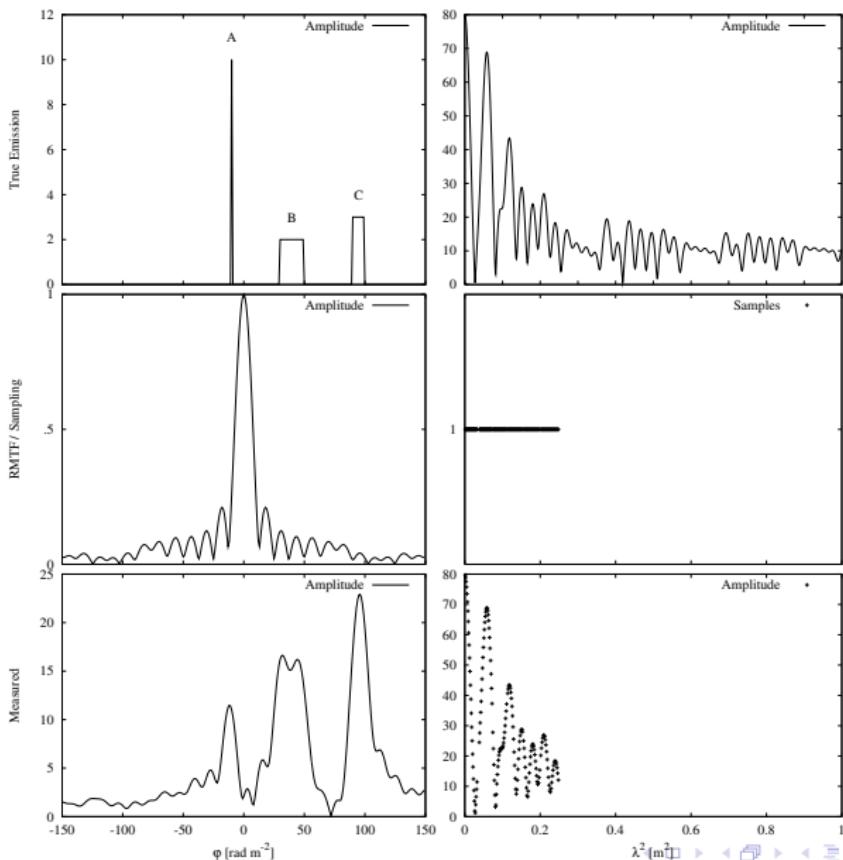


$$P(\lambda_c^2) = \int_{\lambda^2'} \int_{\Omega} \int_{-\infty}^{\infty} W(\Omega, \lambda_c^2 + \lambda^{2'}) f(\Omega, \phi, \lambda_c^2 + \lambda^{2'}) e^{2i\phi(\lambda_c^2 + \lambda^{2'})} d\phi$$

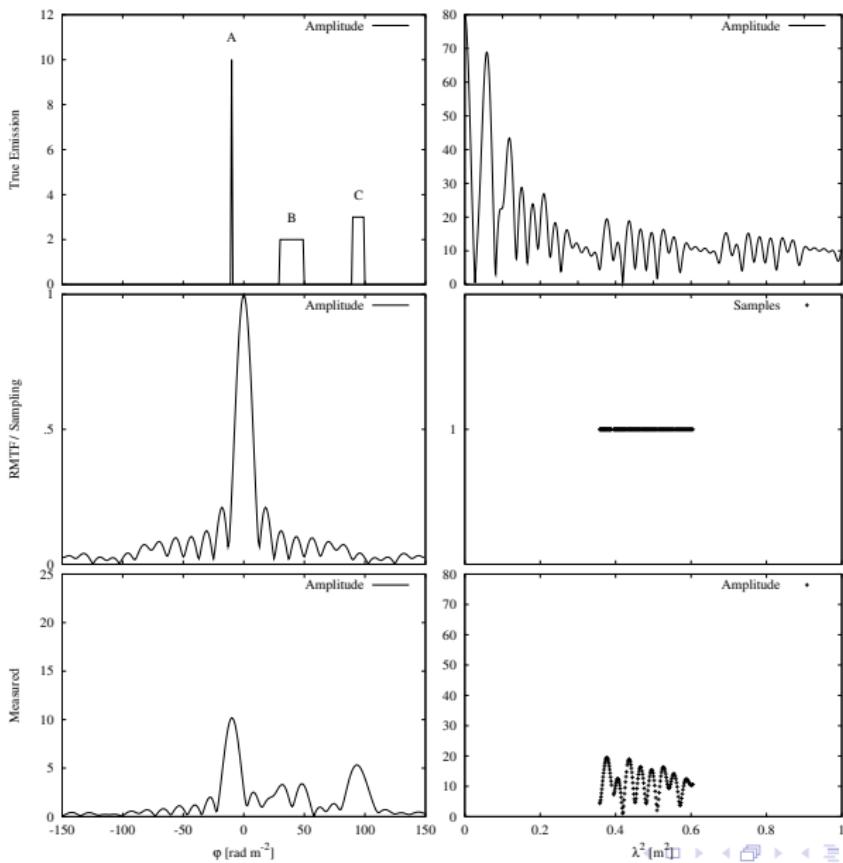
$$P(\lambda^2) = W(\lambda^2) \int_{-\infty}^{\infty} f(\phi) e^{2i\phi\lambda^2} d\phi$$

$$f(\phi) * R(\phi) = \int_{-\infty}^{\infty} P(\lambda^2) e^{-2i\phi\lambda^2} d\lambda^2$$

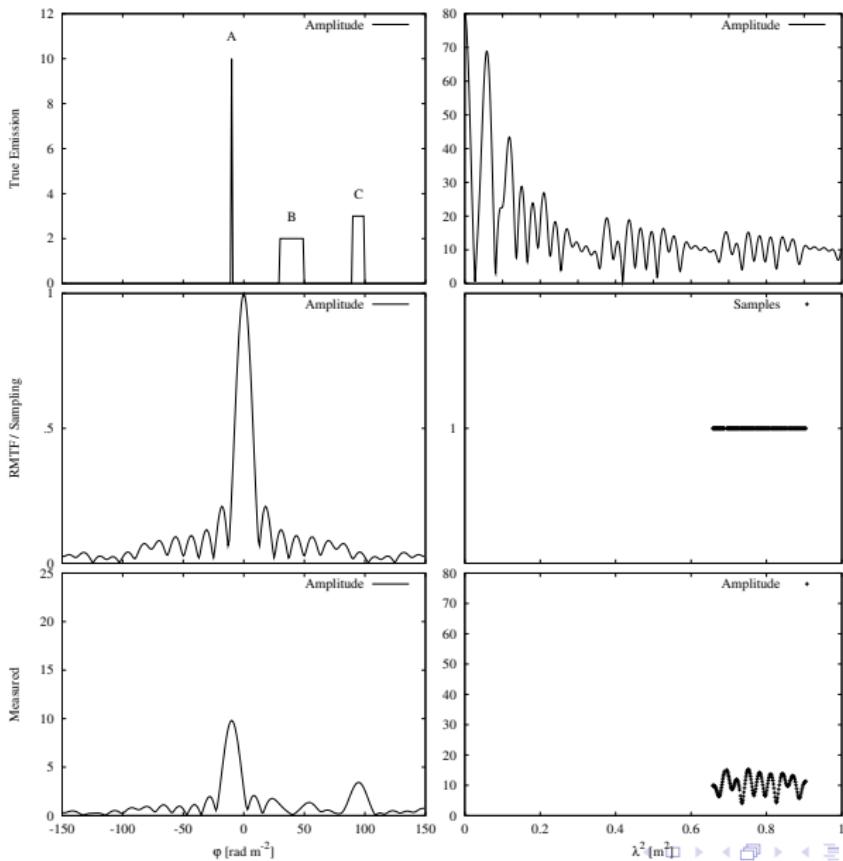
# Short wavelength



# Intermediate wavelength

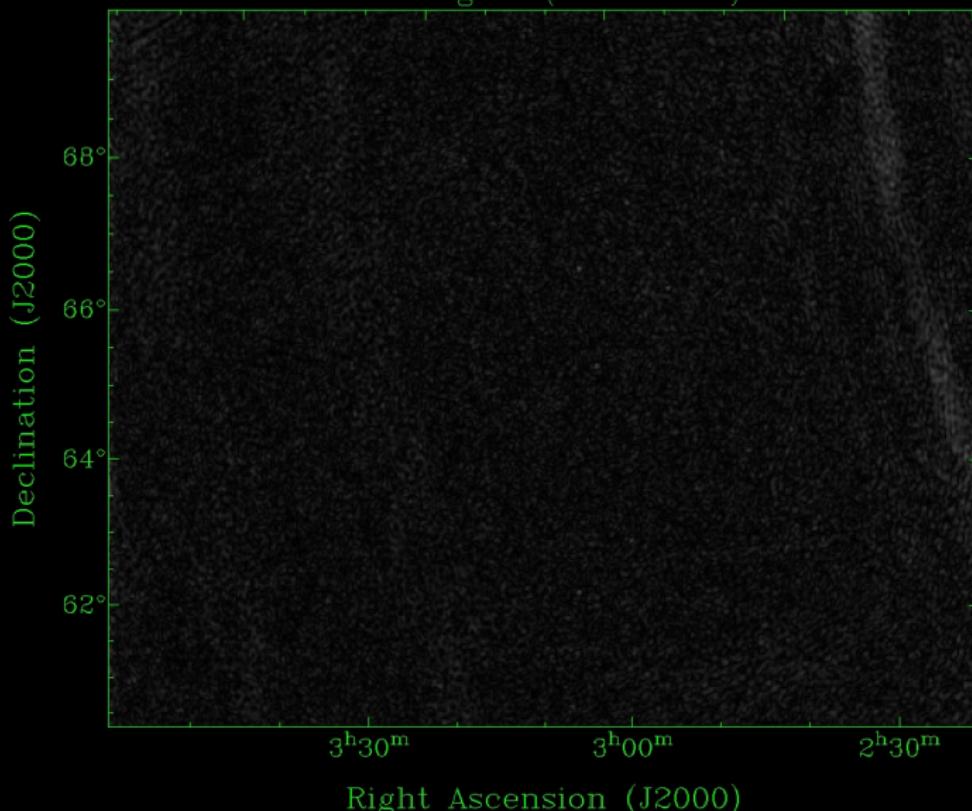


# Long wavelength



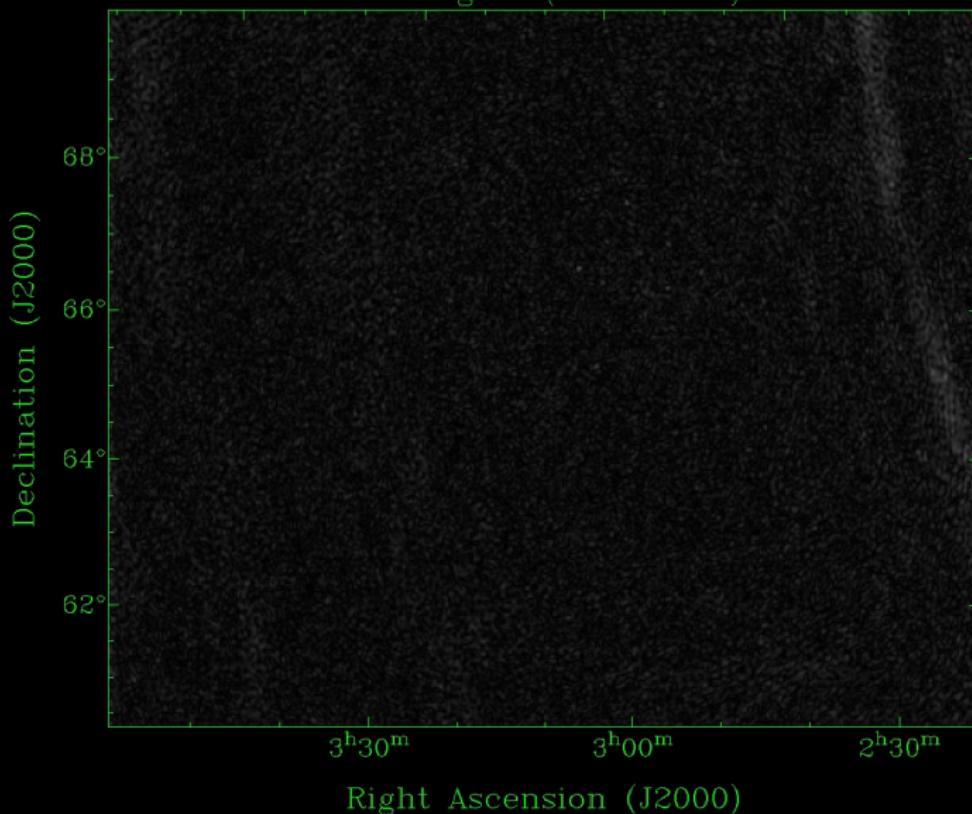
RM: -2.500000e+01

Fan region (WSRT LFFE)



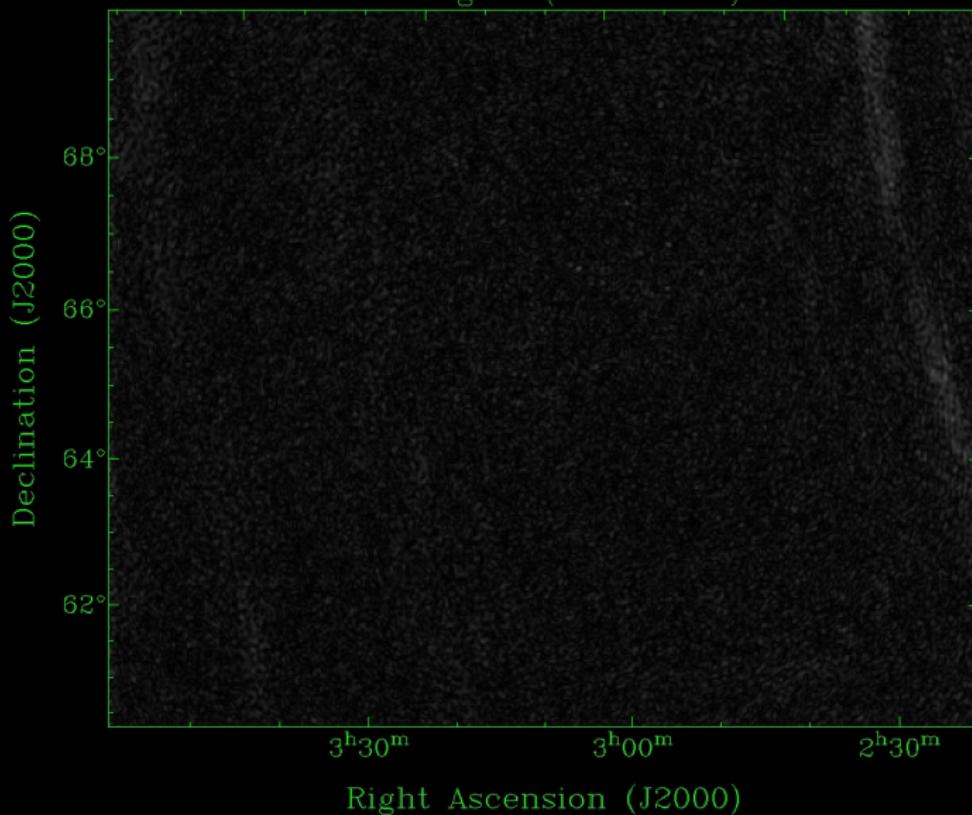
RM: -2.450000e+01

Fan region (WSRT LFFE)



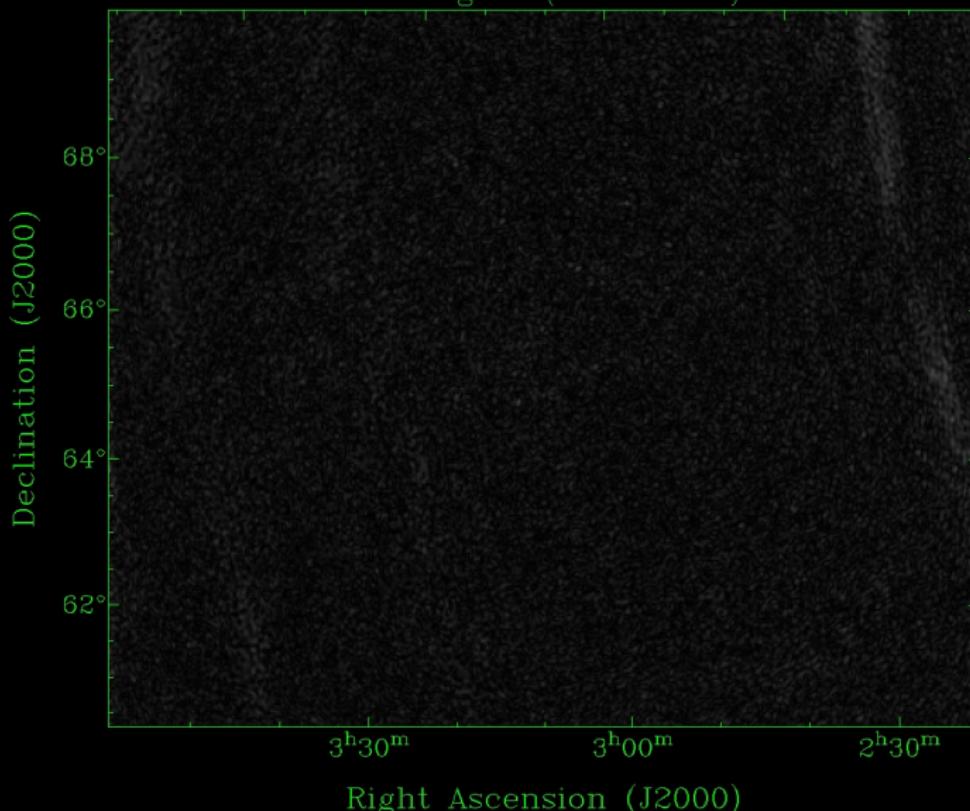
RM: -2.400000e+01

Fan region (WSRT LFFE)



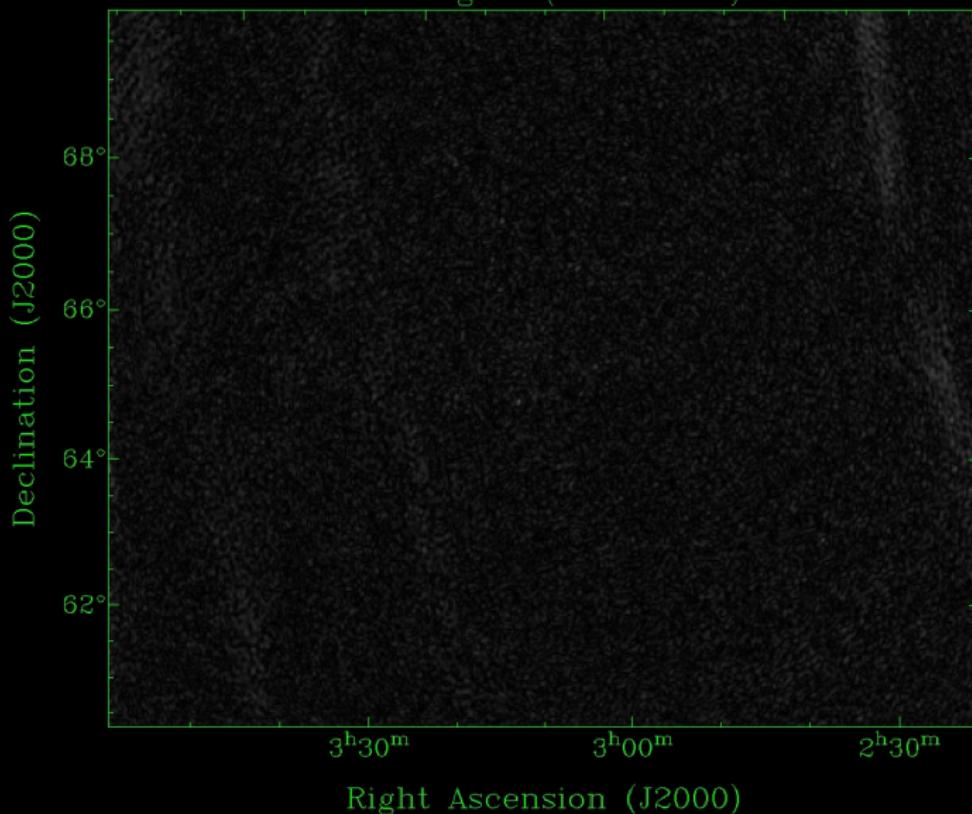
RM: -2.350000e+01

Fan region (WSRT LFFE)



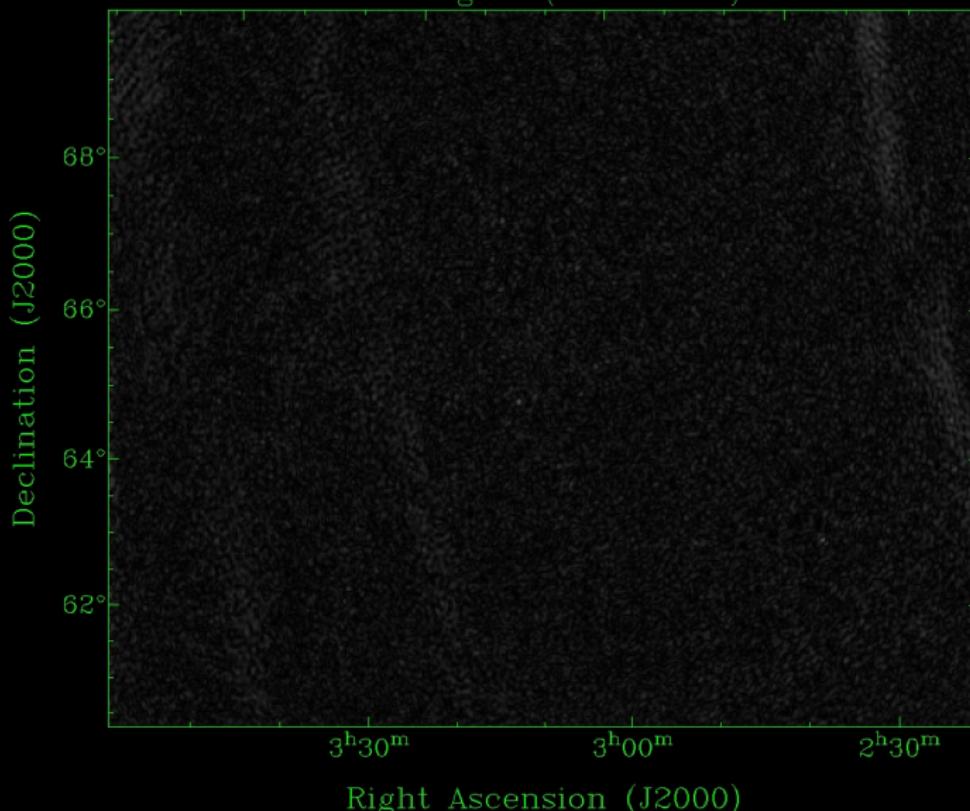
RM: -2.300000e+01

Fan region (WSRT LFFE)



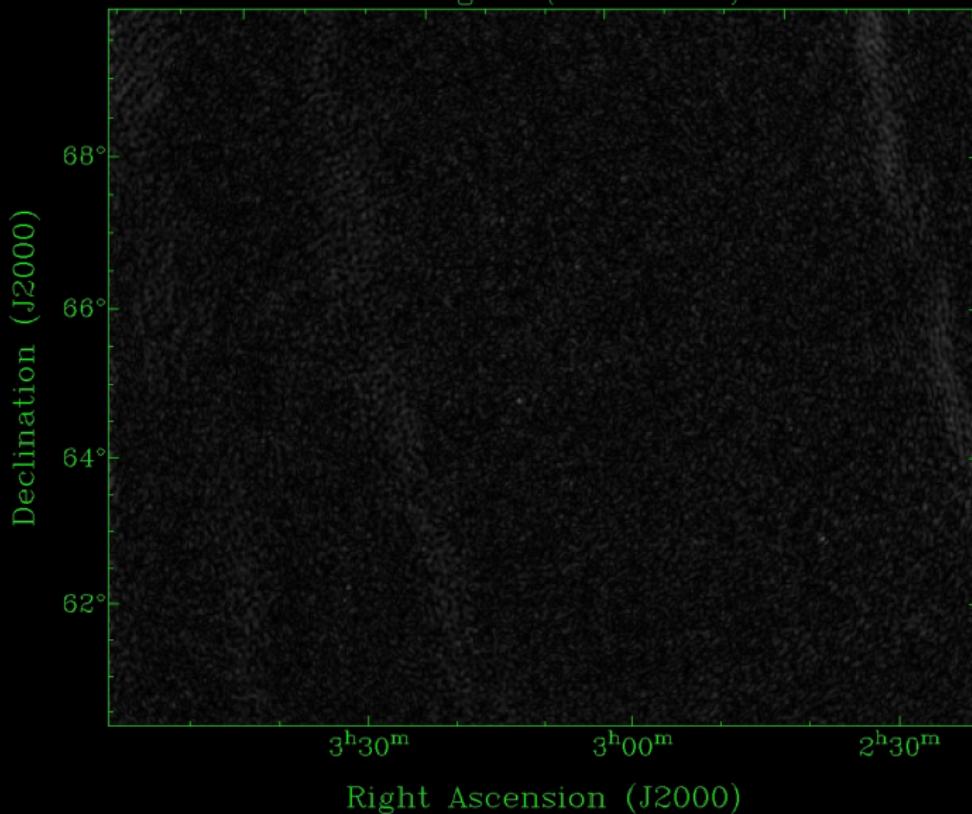
RM: -2.250000e+01

Fan region (WSRT LFFE)



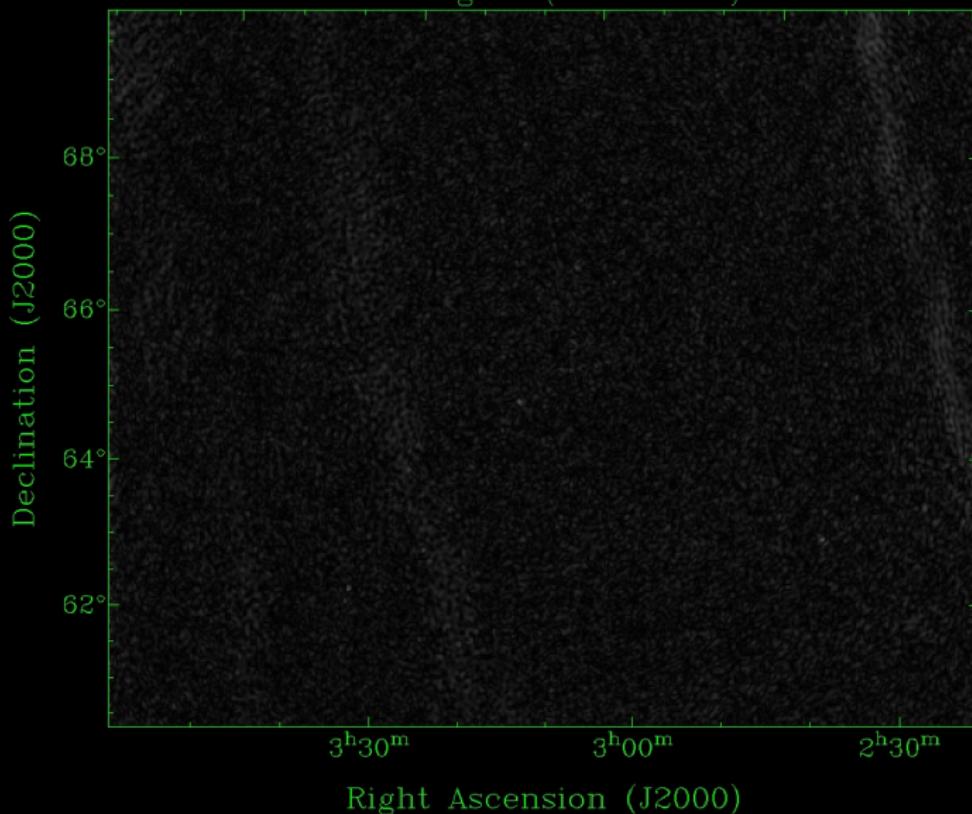
RM: -2.200000e+01

Fan region (WSRT LFFE)



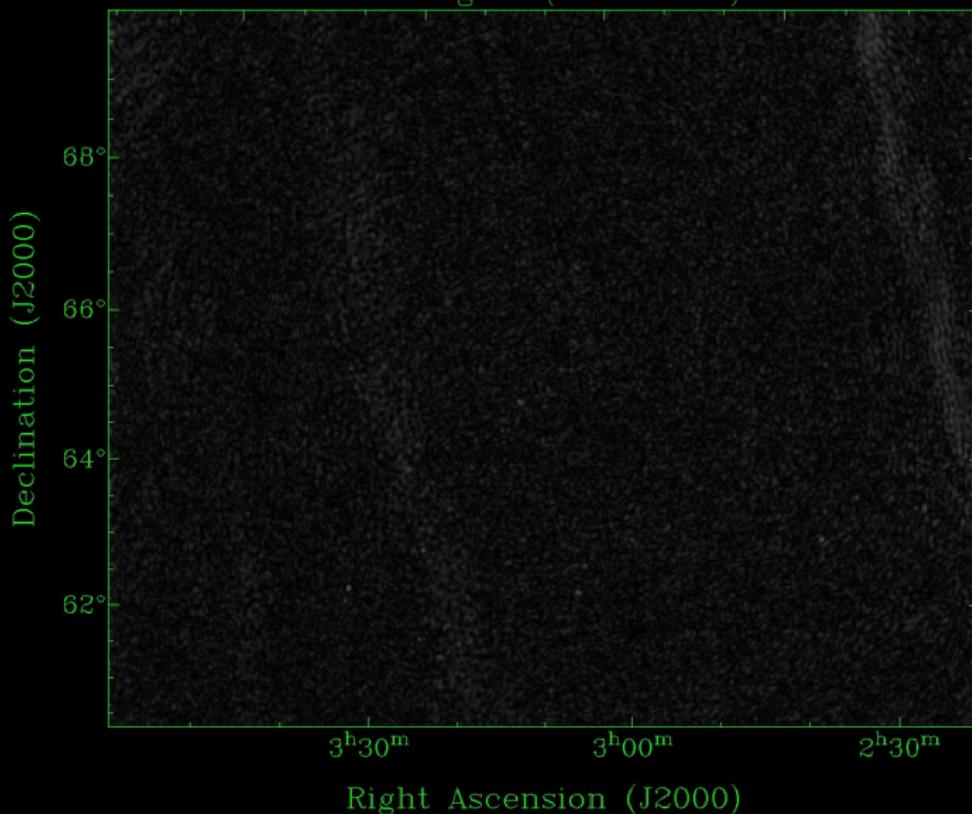
RM: -2.150000e+01

Fan region (WSRT LFFE)



RM: -2.100000e+01

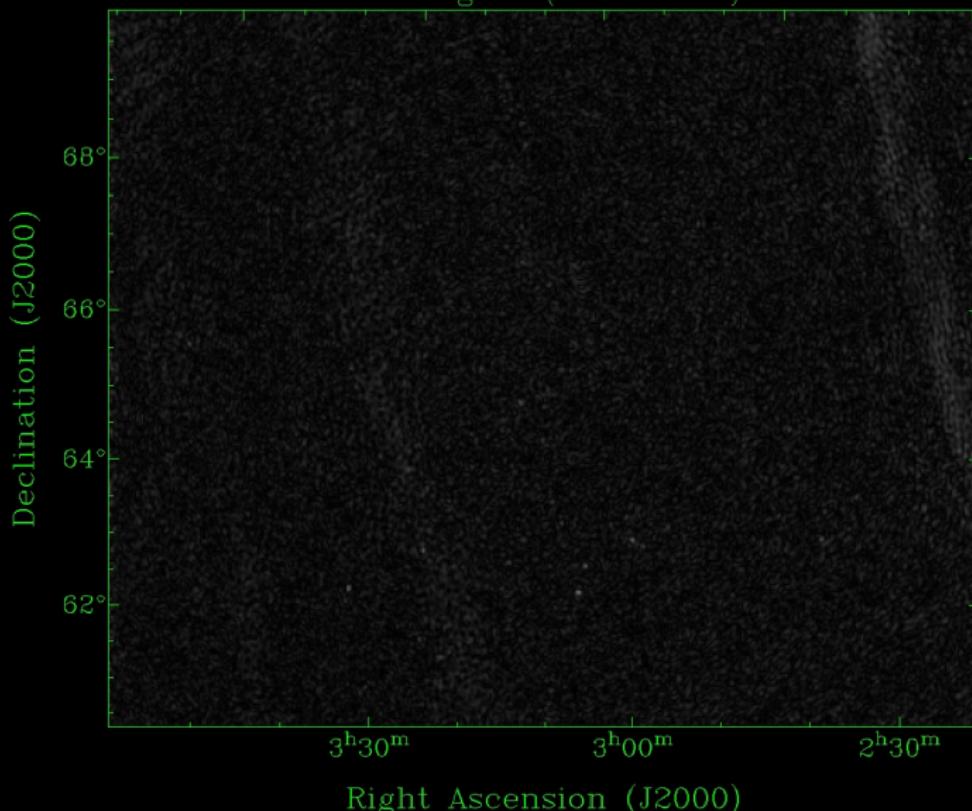
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: -2.050000e+01

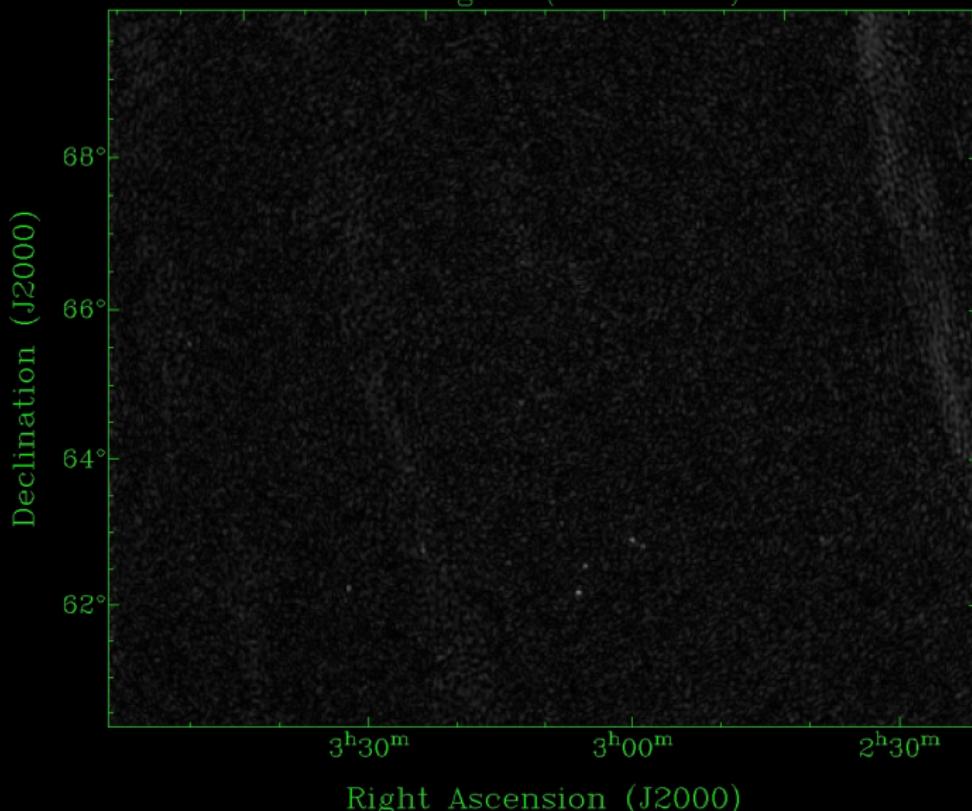
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

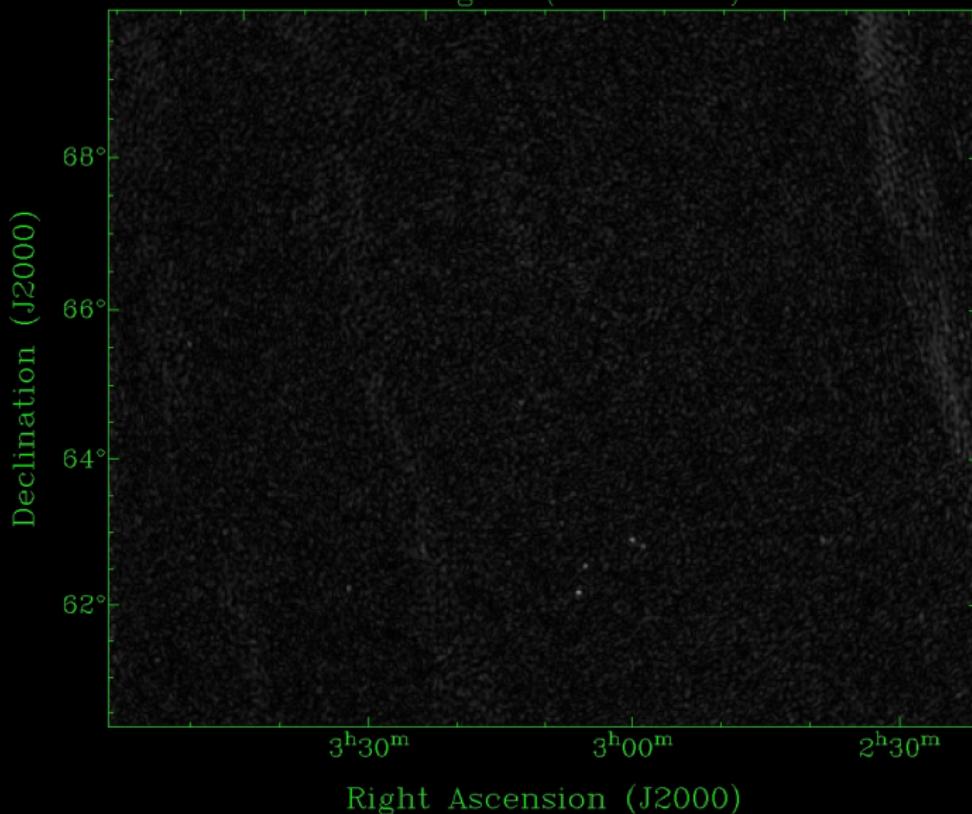
RM: -2.000000e+01

Fan region (WSRT LFFE)



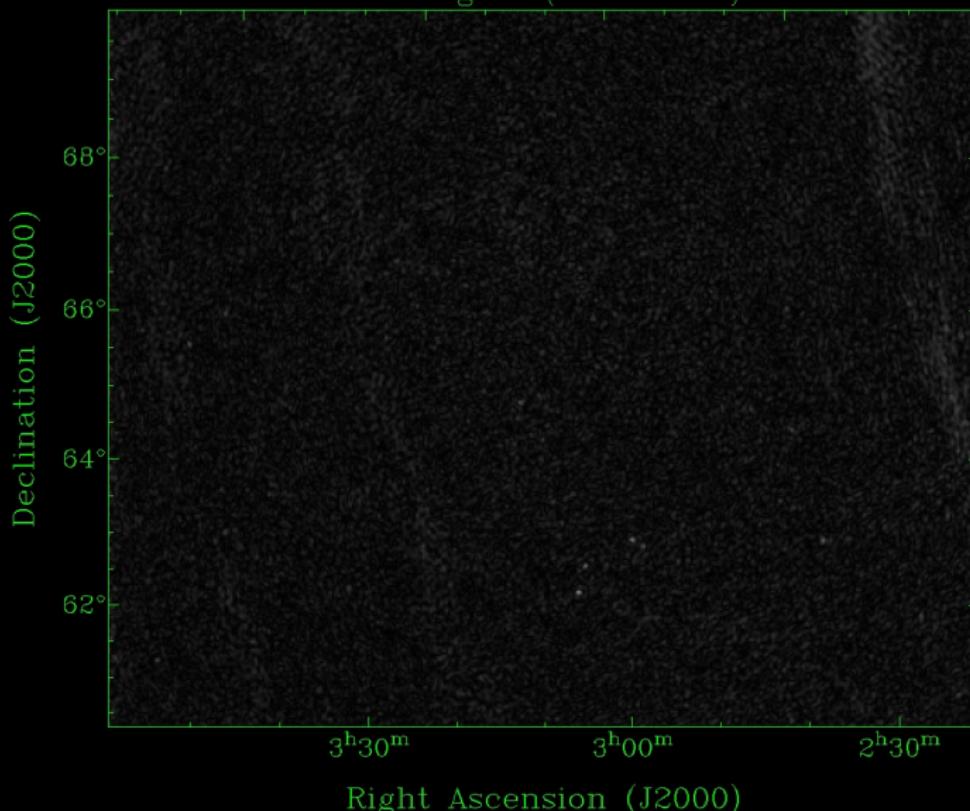
RM: -1.950000e+01

Fan region (WSRT LFFE)



RM: -1.900000e+01

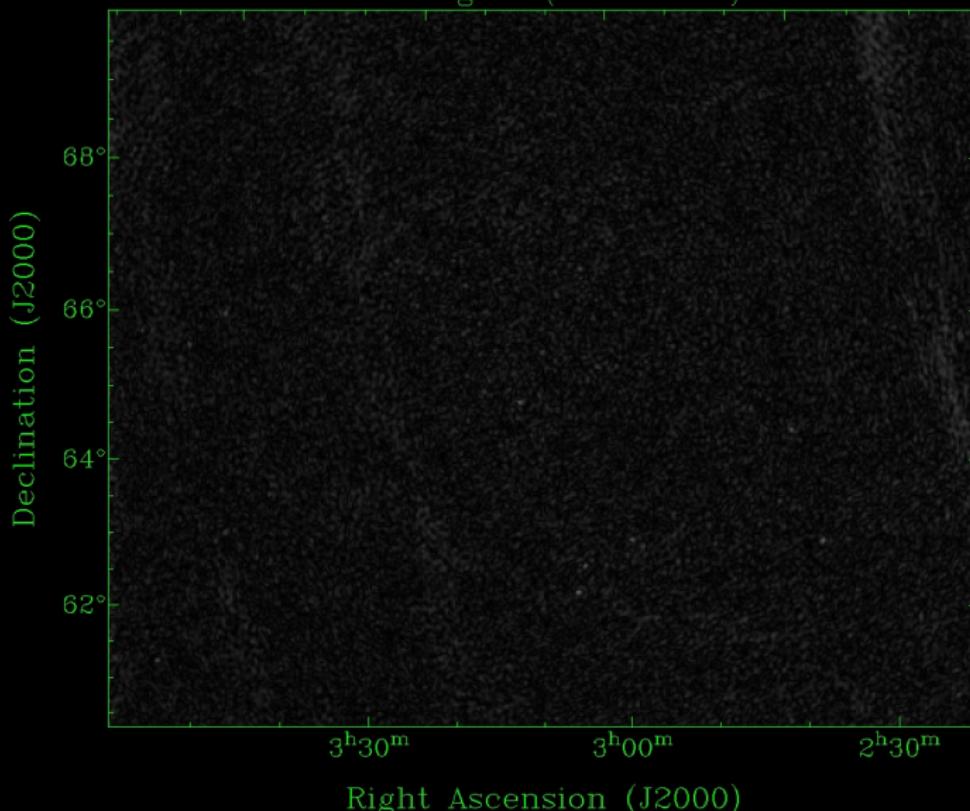
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

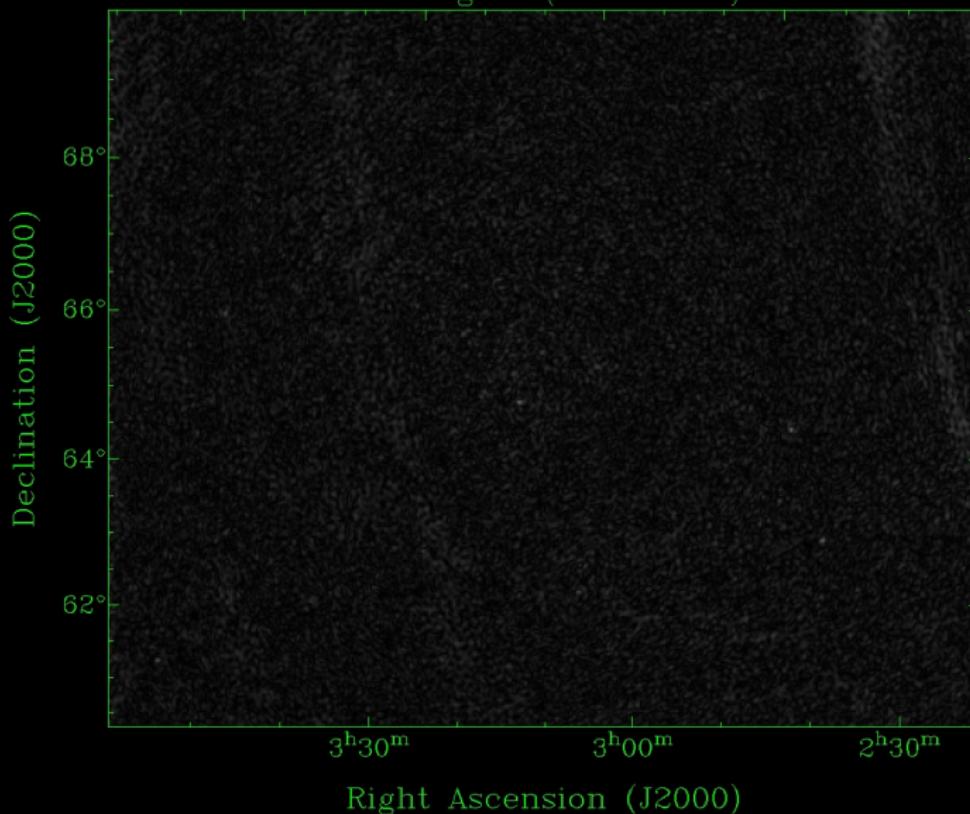
RM: -1.850000e+01

Fan region (WSRT LFFE)



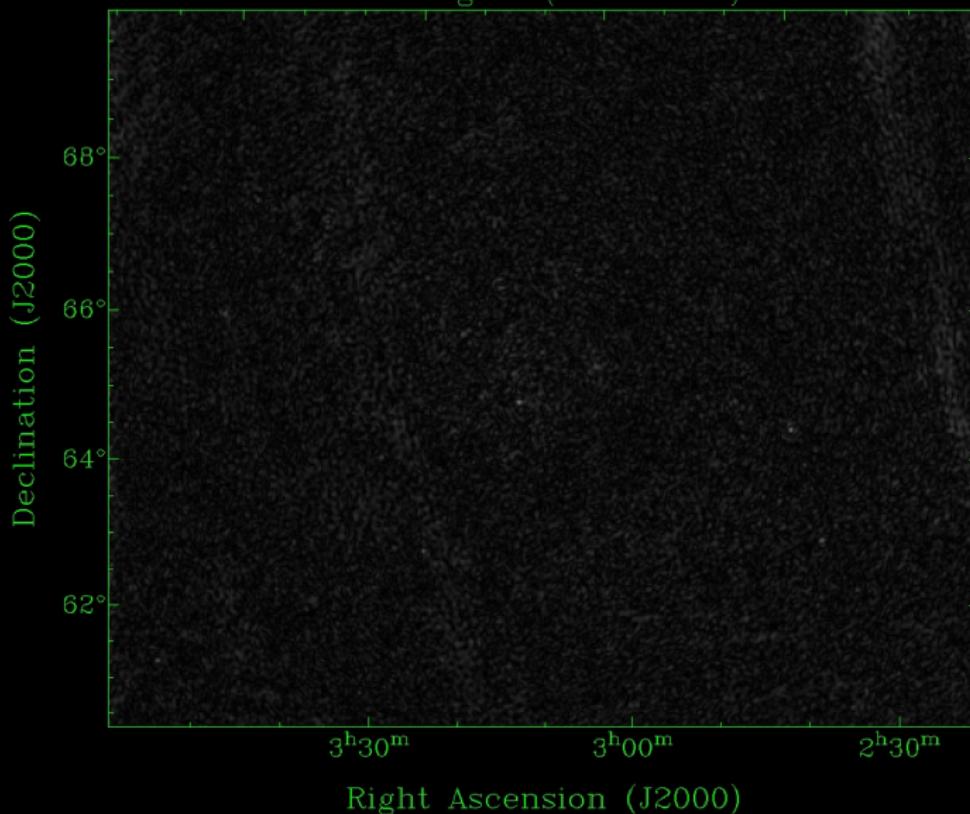
RM: -1.800000e+01

Fan region (WSRT LFFE)



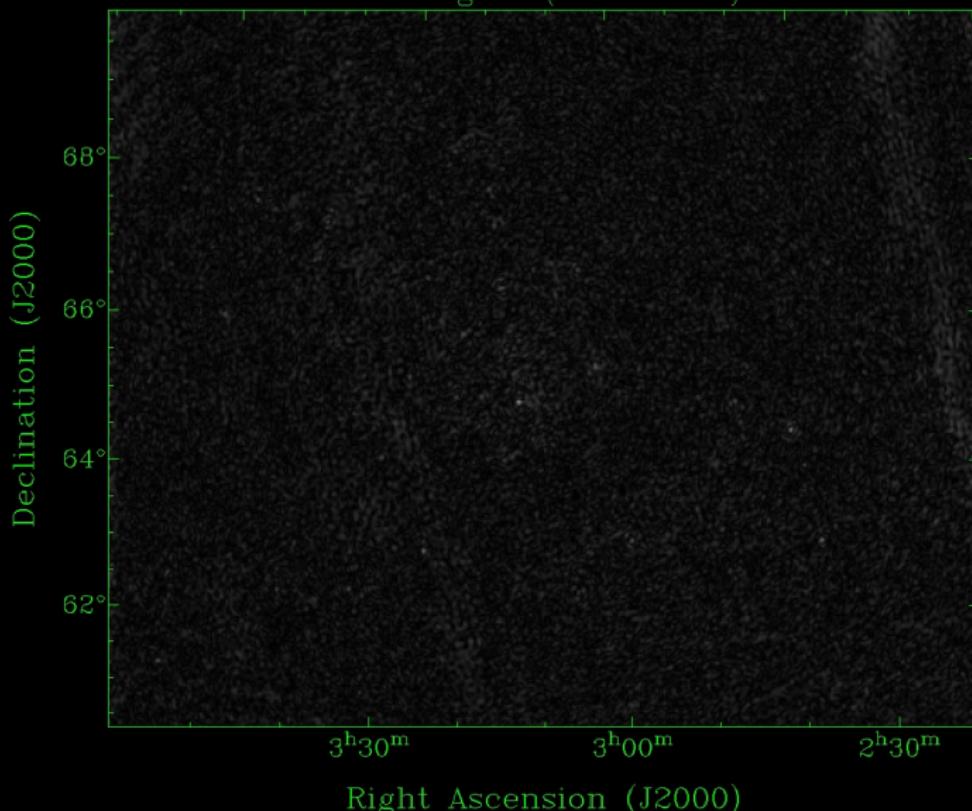
RM: -1.750000e+01

Fan region (WSRT LFFE)



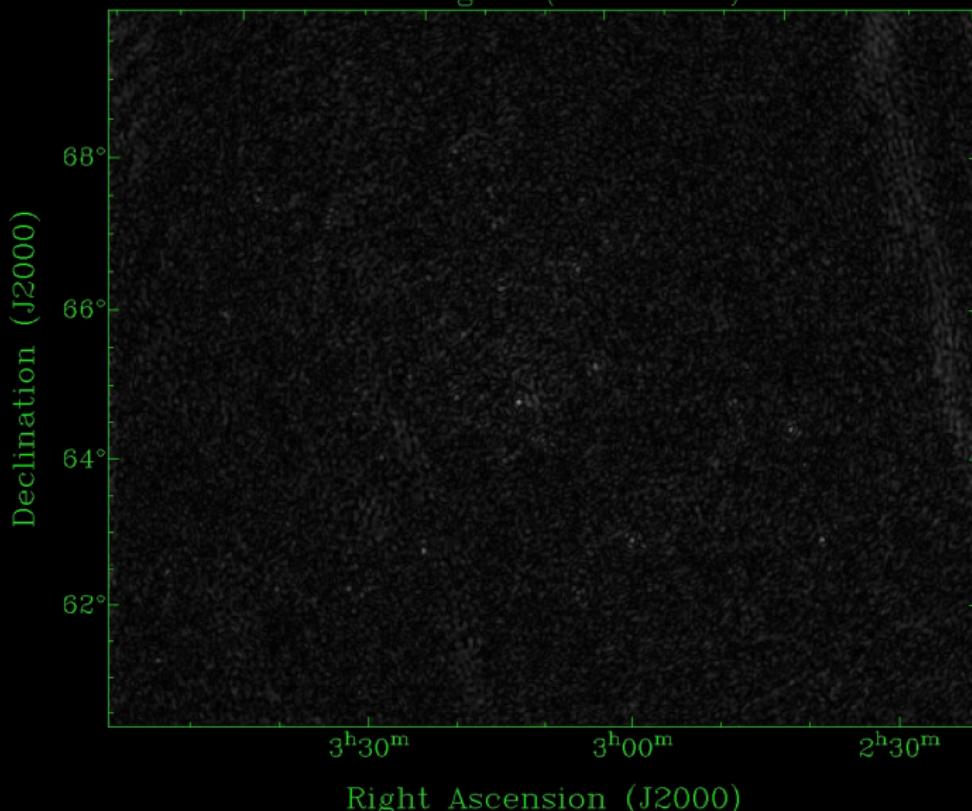
RM: -1.700000e+01

Fan region (WSRT LFFE)



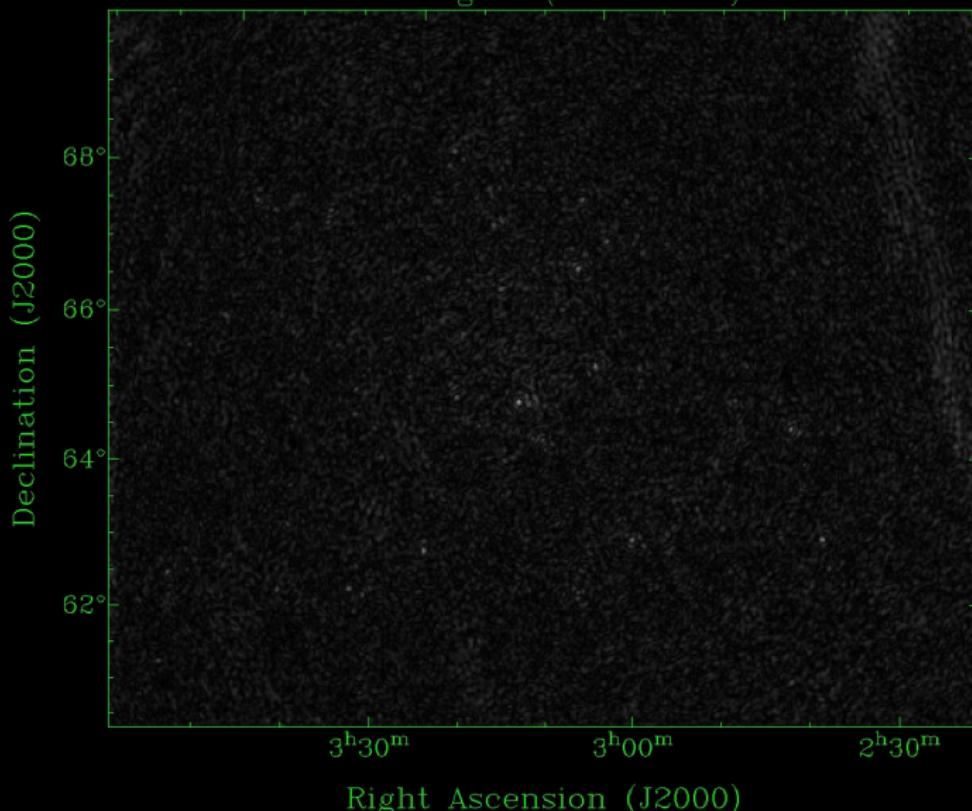
RM: -1.650000e+01

Fan region (WSRT LFFE)



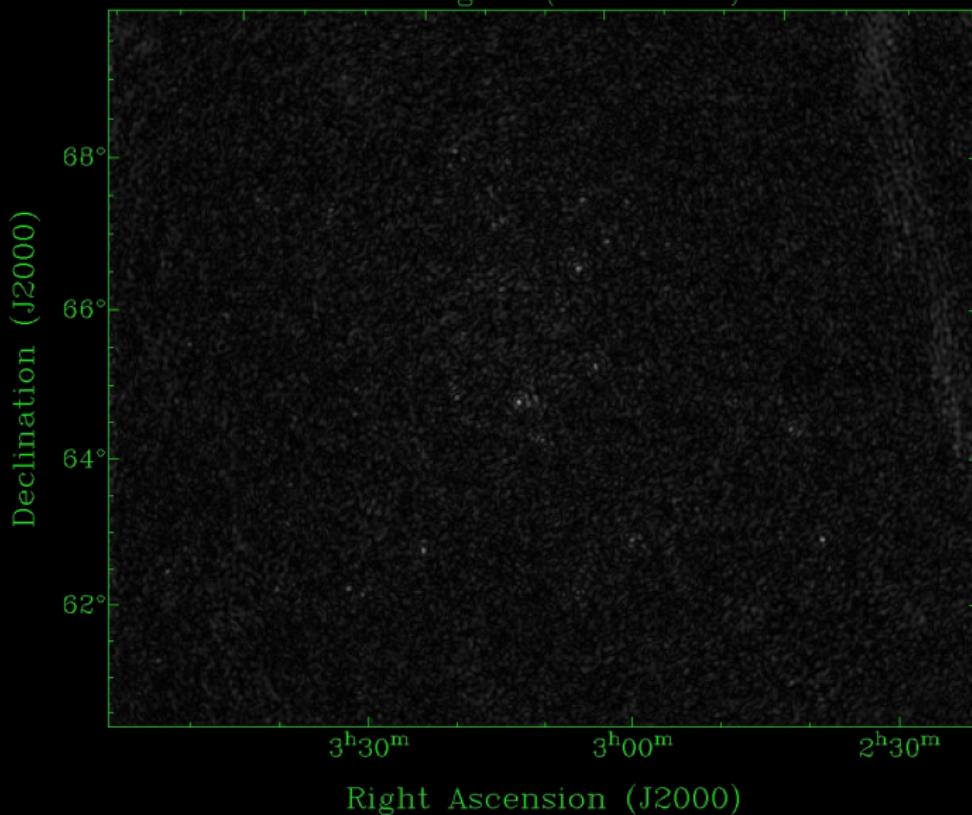
RM: -1.600000e+01

Fan region (WSRT LFFE)



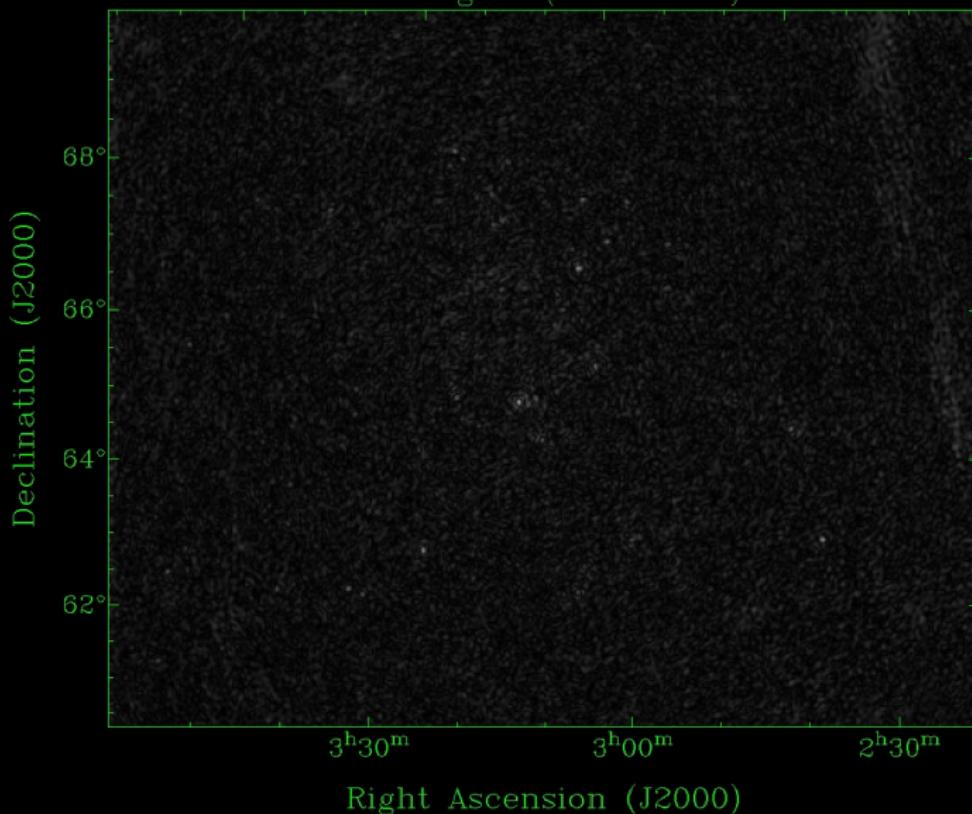
RM: -1.550000e+01

Fan region (WSRT LFFE)



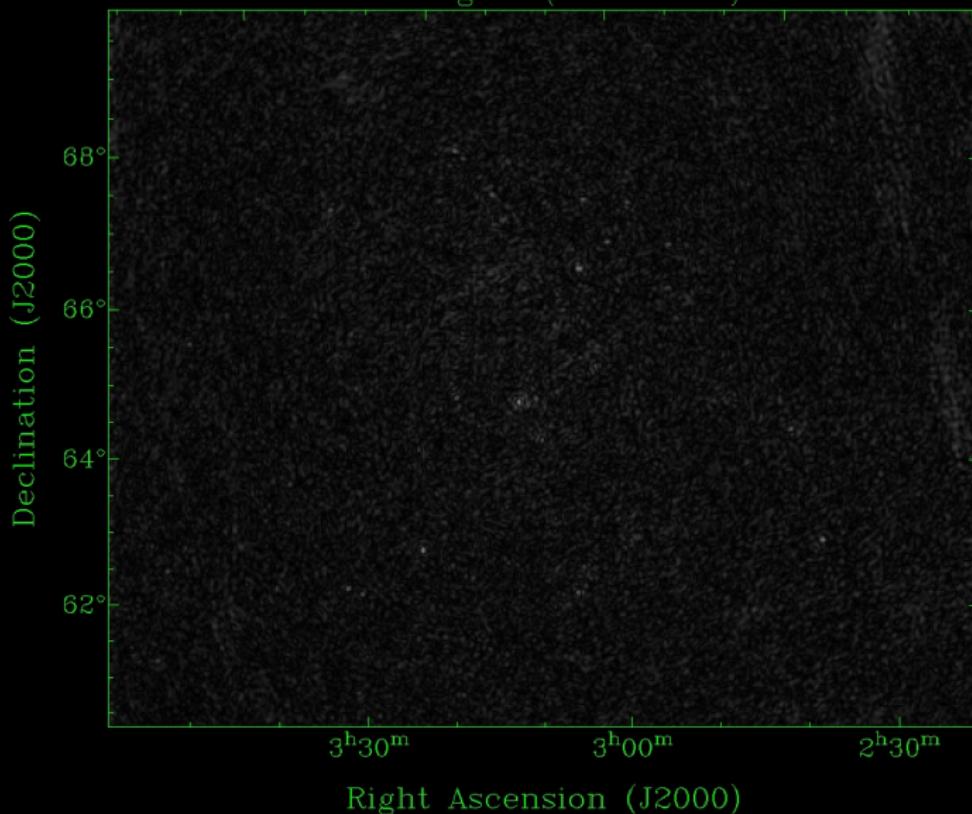
RM: -1.500000e+01

Fan region (WSRT LFFE)



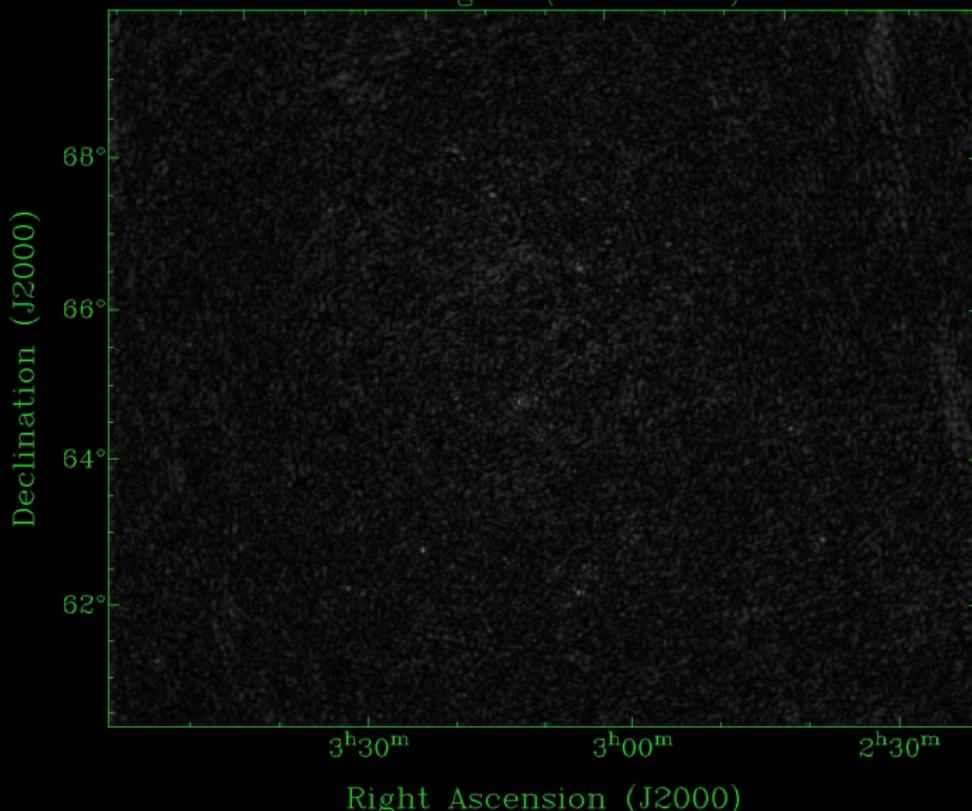
RM: -1.450000e+01

Fan region (WSRT LFFE)



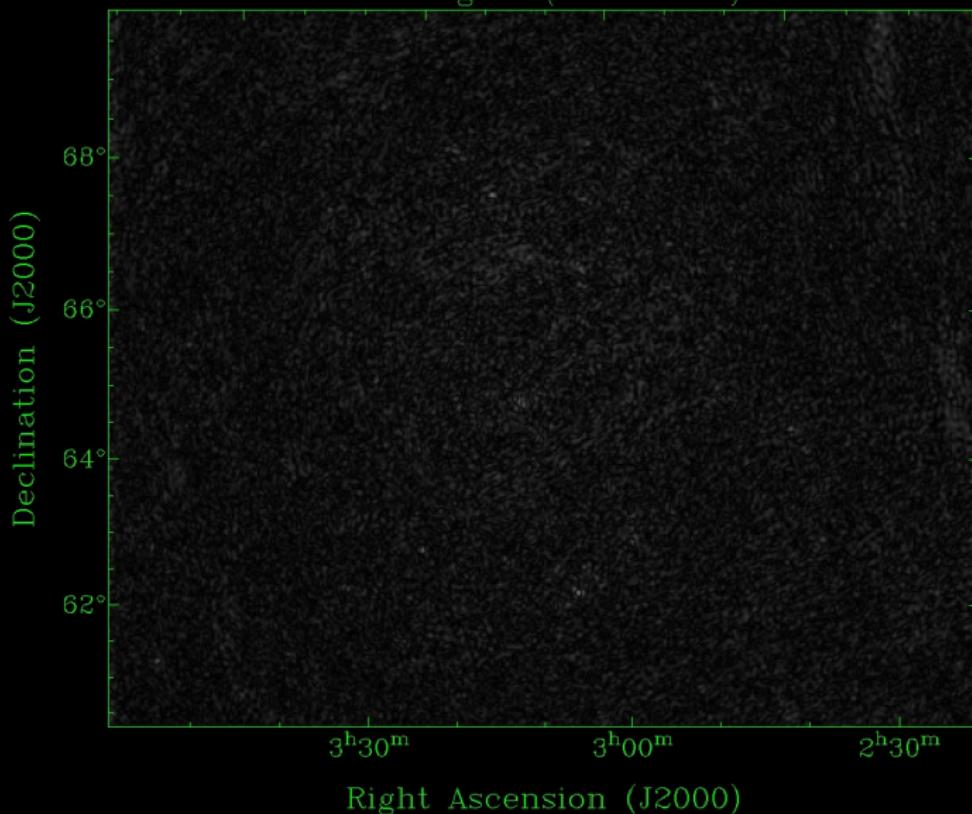
RM: -1.400000e+01

Fan region (WSRT LFFE)



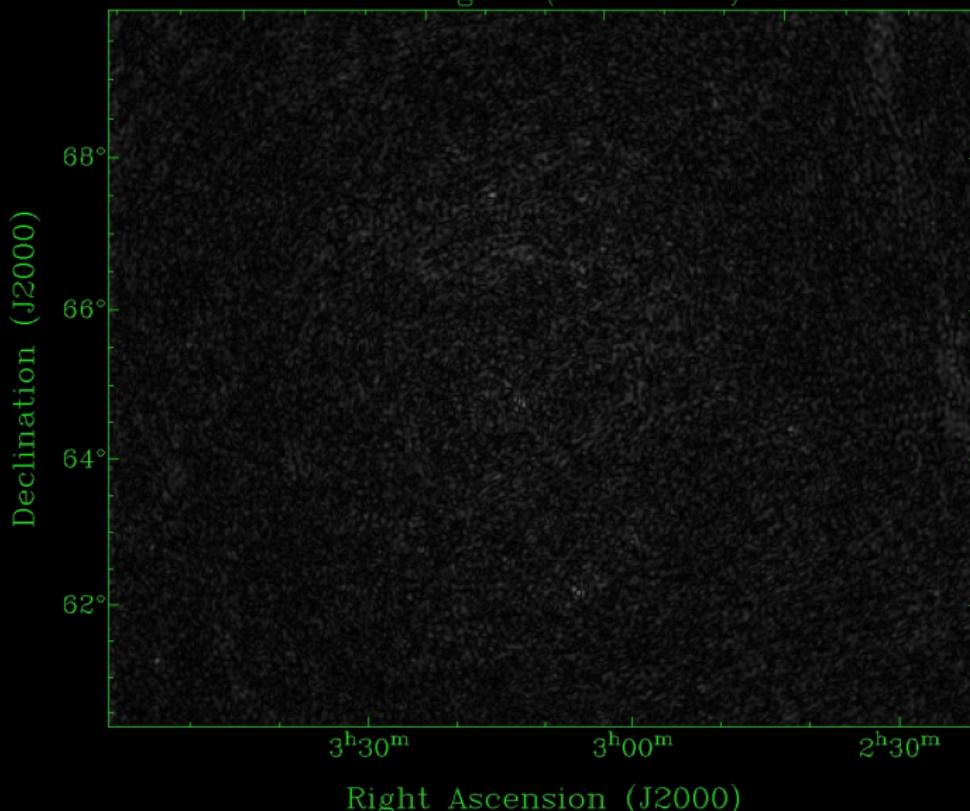
RM: -1.350000e+01

Fan region (WSRT LFFE)



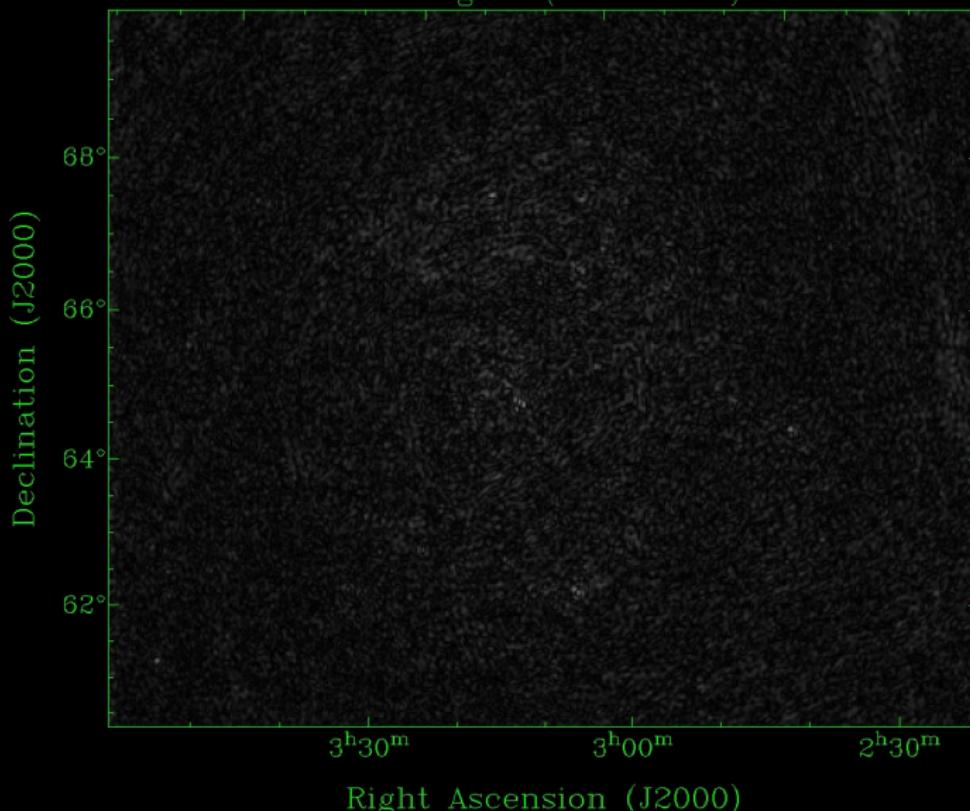
RM: -1.300000e+01

Fan region (WSRT LFFE)



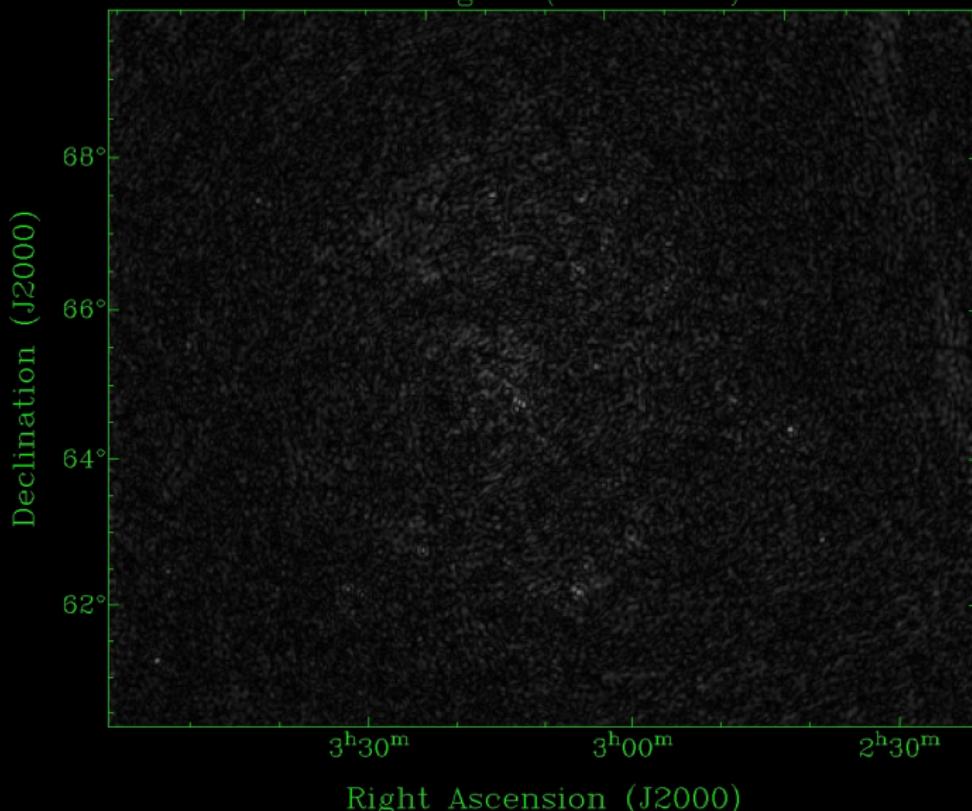
RM: -1.250000e+01

Fan region (WSRT LFFE)



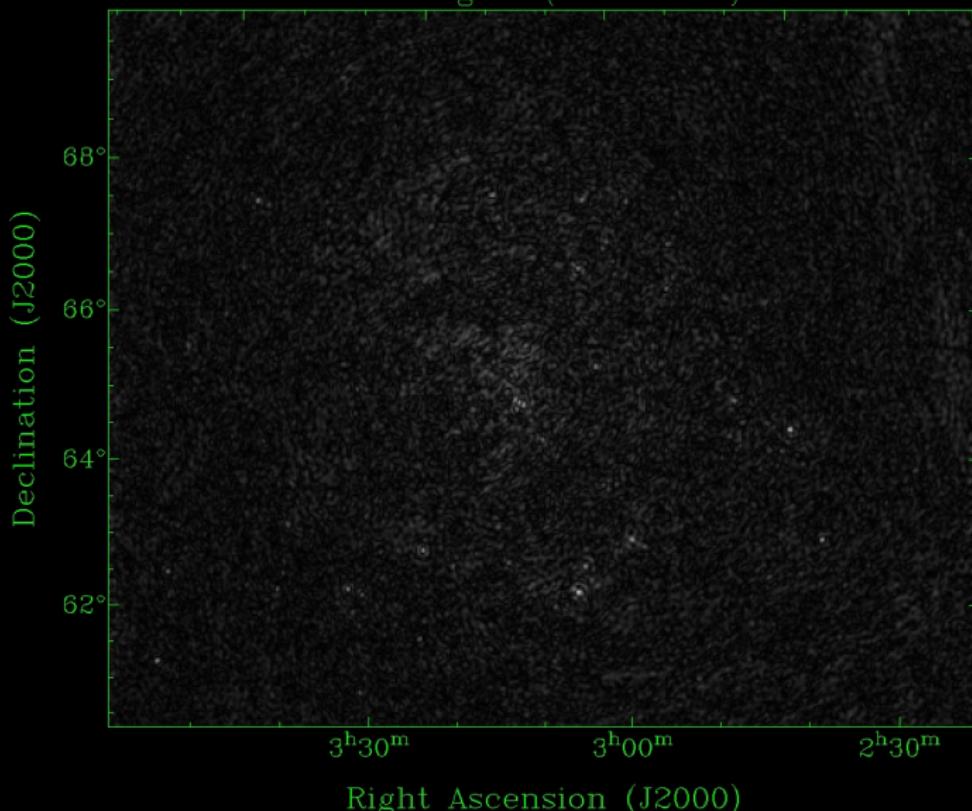
RM: -1.200000e+01

Fan region (WSRT LFFE)



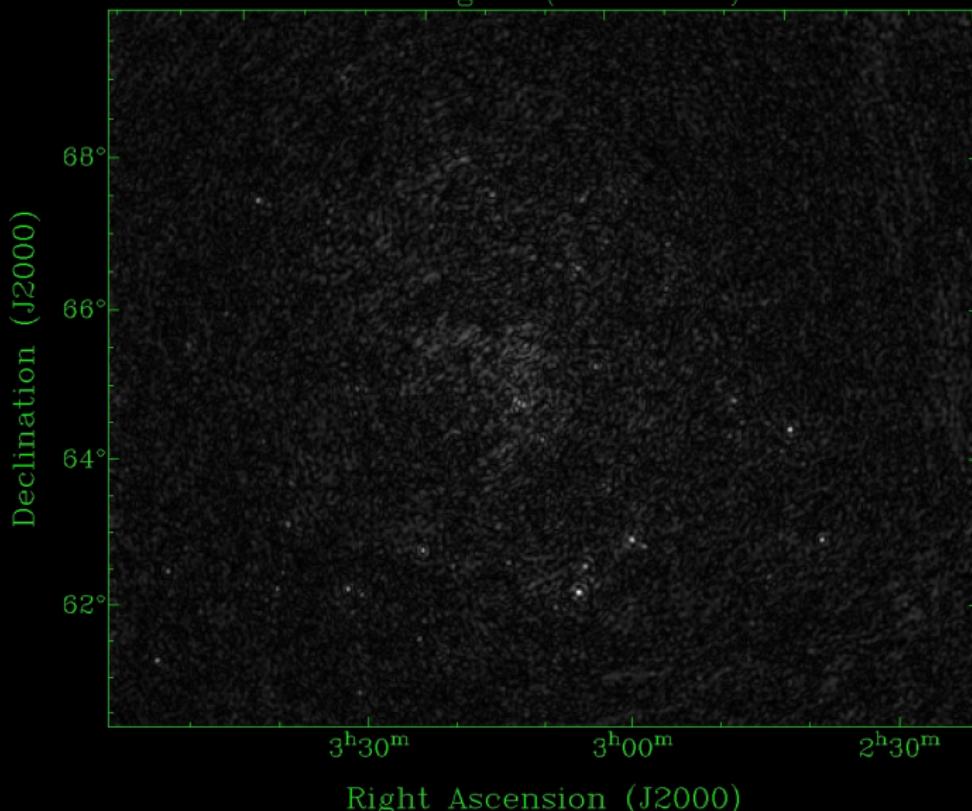
RM: -1.150000e+01

Fan region (WSRT LFFE)



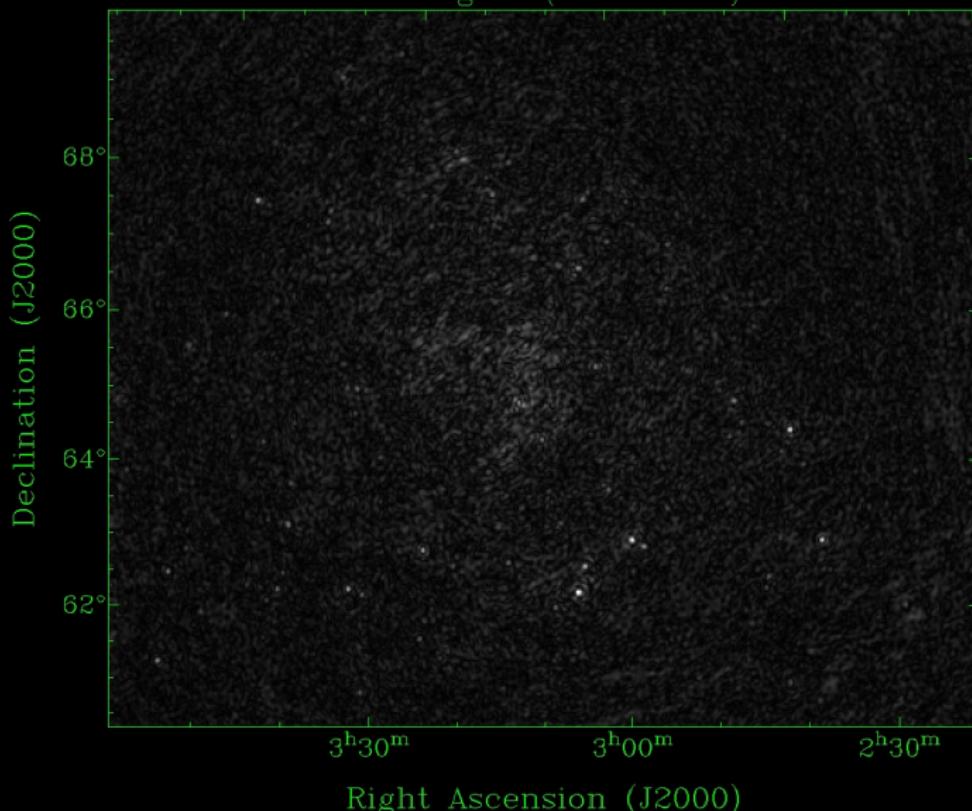
RM: -1.100000e+01

Fan region (WSRT LFFE)



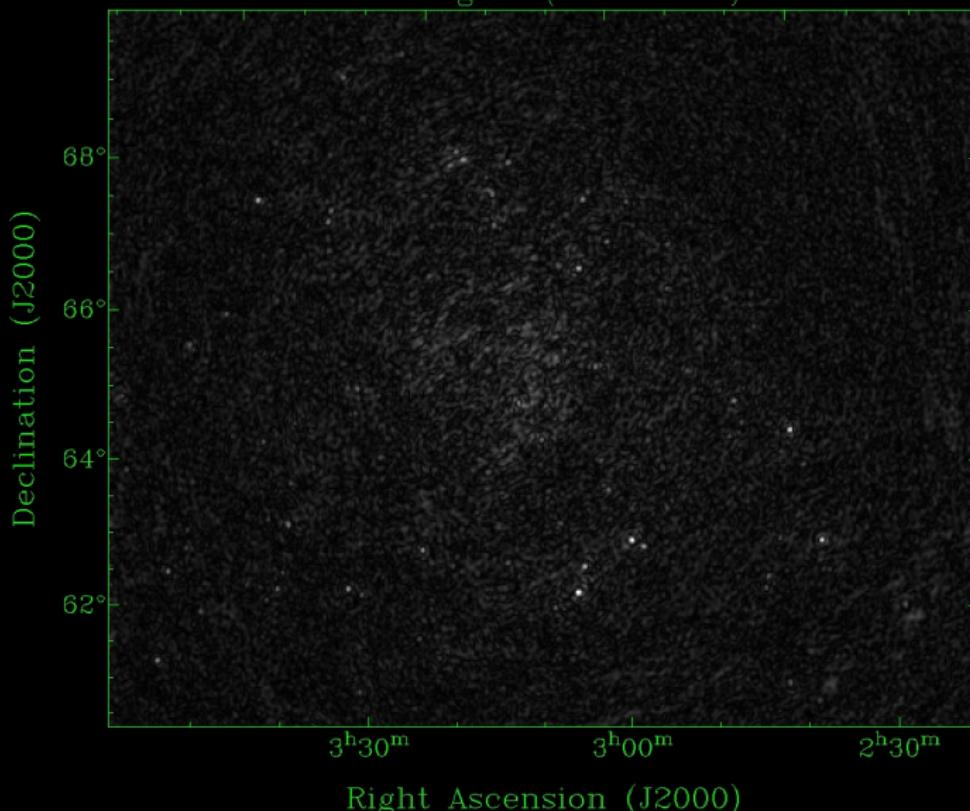
RM: -1.050000e+01

Fan region (WSRT LFFE)



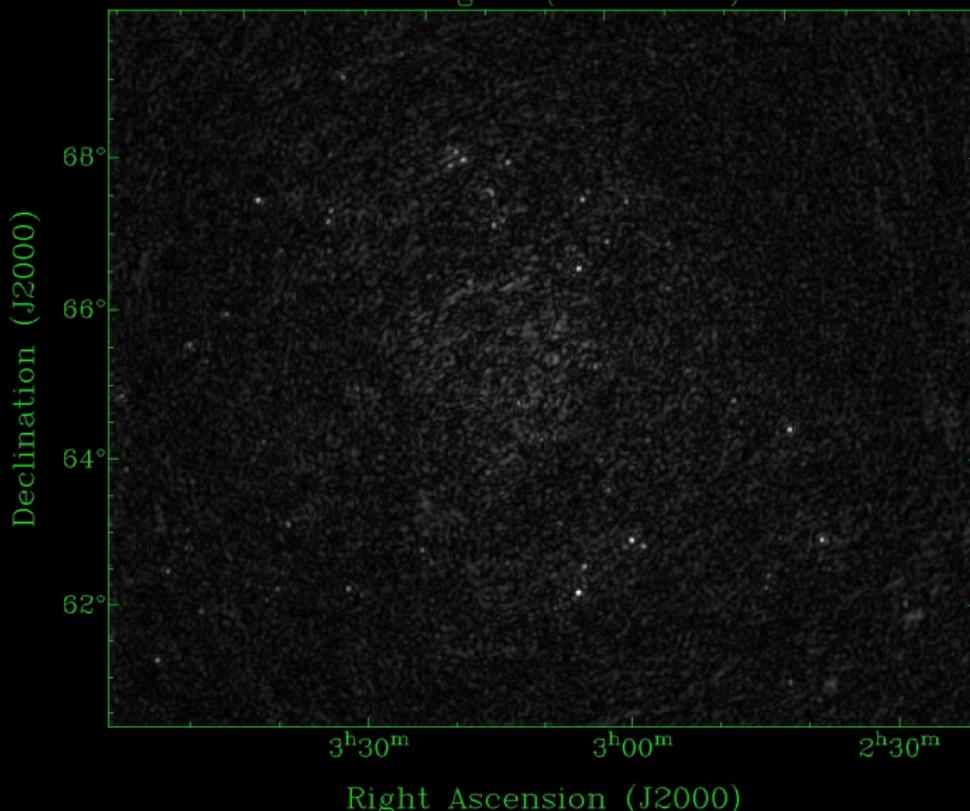
RM: -1.000000e+01

Fan region (WSRT LFFE)



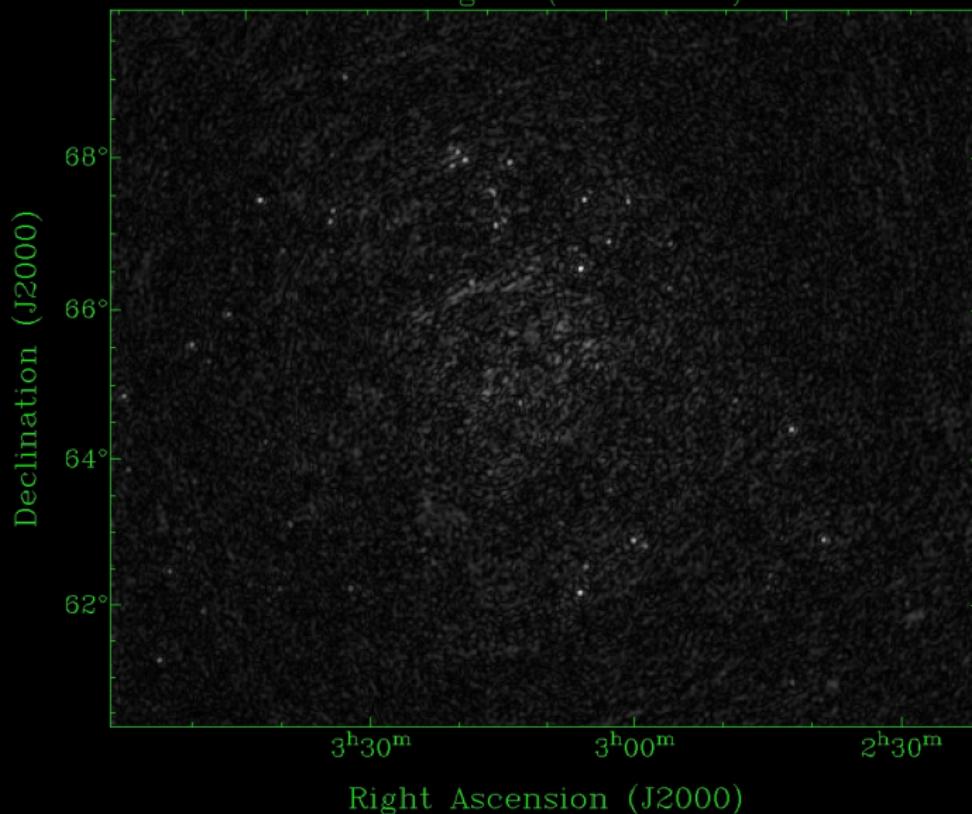
RM: -9.500000e+00

Fan region (WSRT LFFE)



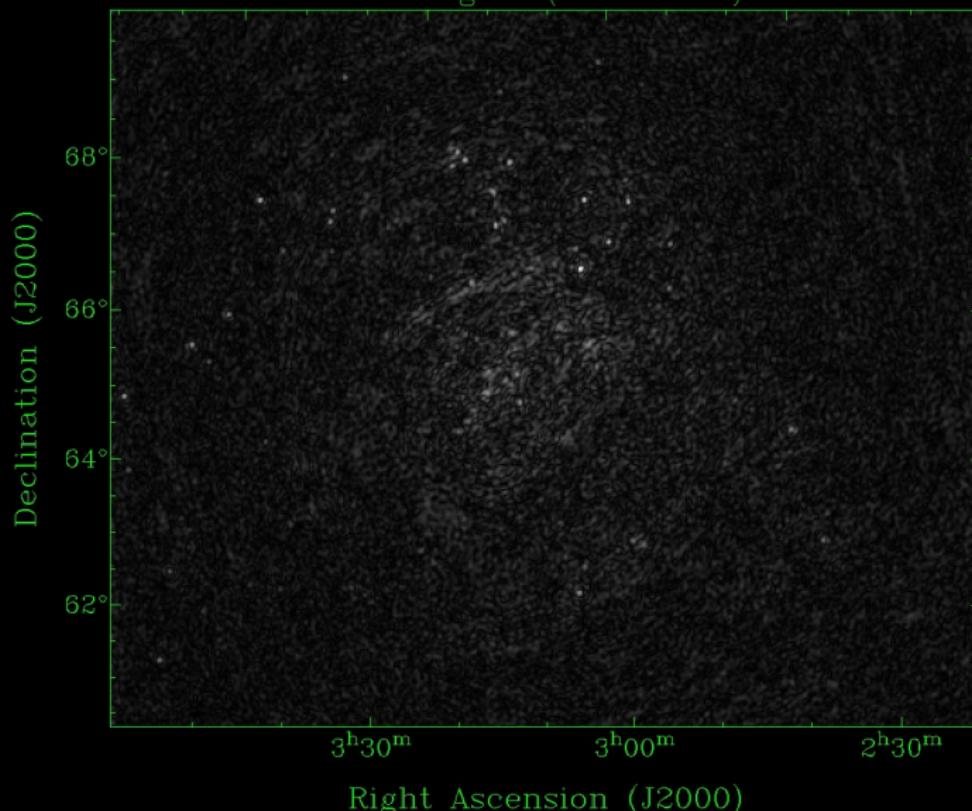
RM: -9.000000e+00

Fan region (WSRT LFFE)



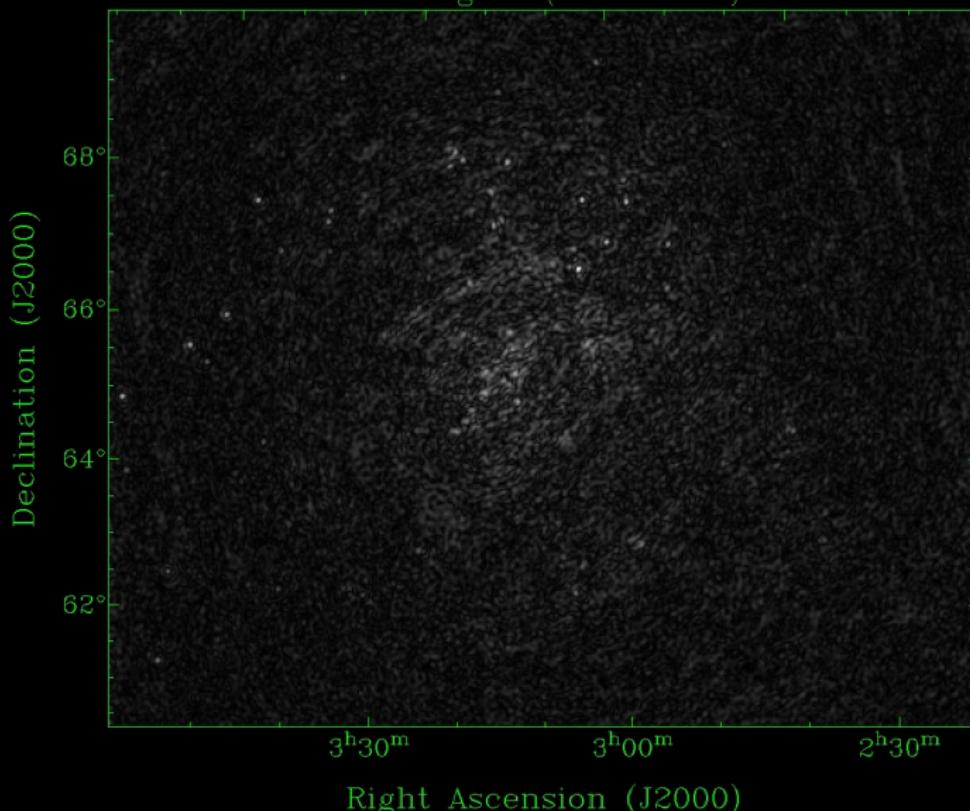
RM: -8.500000e+00

Fan region (WSRT LFFE)



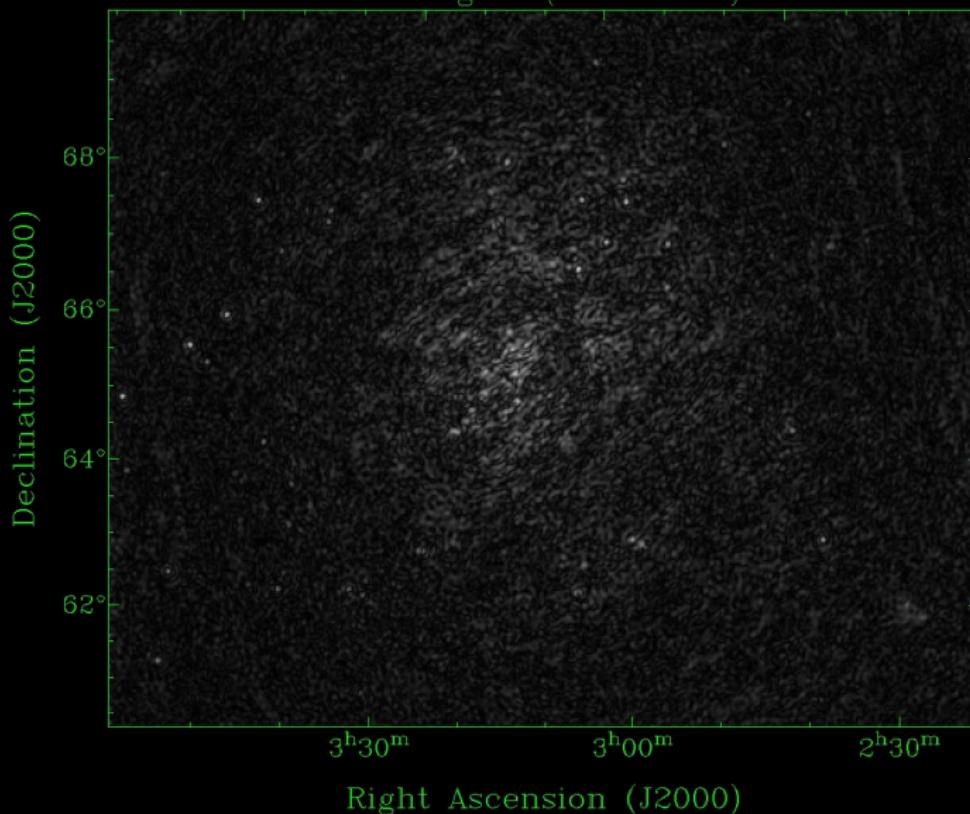
RM: -8.000000e+00

Fan region (WSRT LFFE)



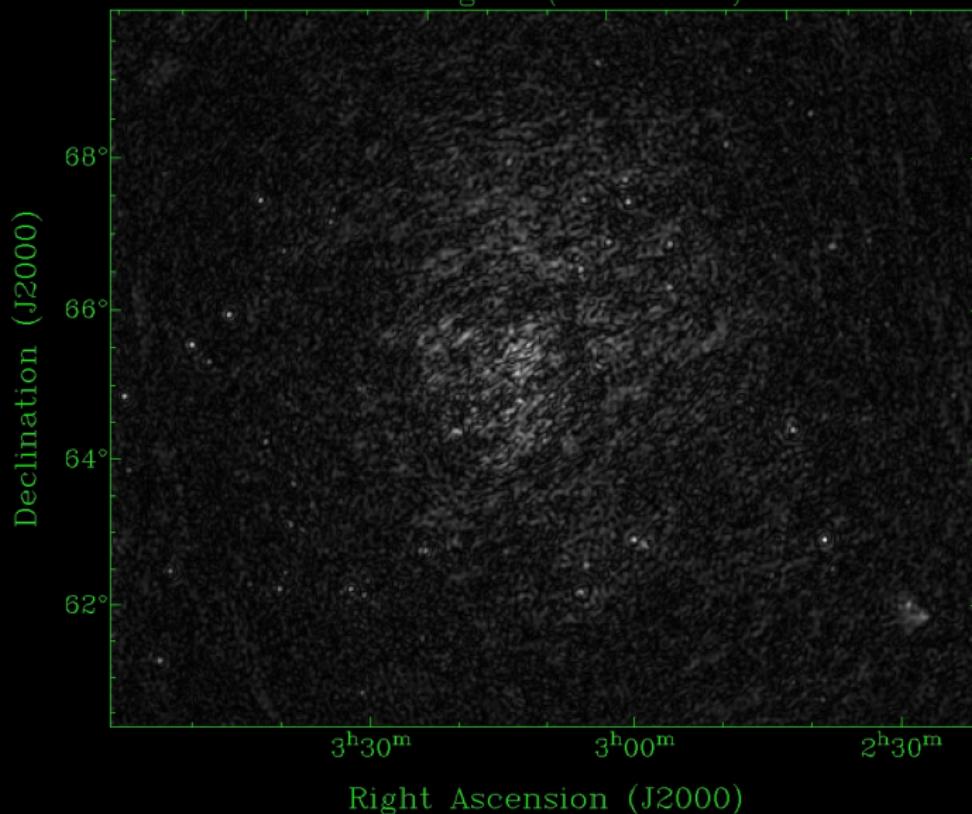
RM: -7.500000e+00

Fan region (WSRT LFFE)



RM: -7.000000e+00

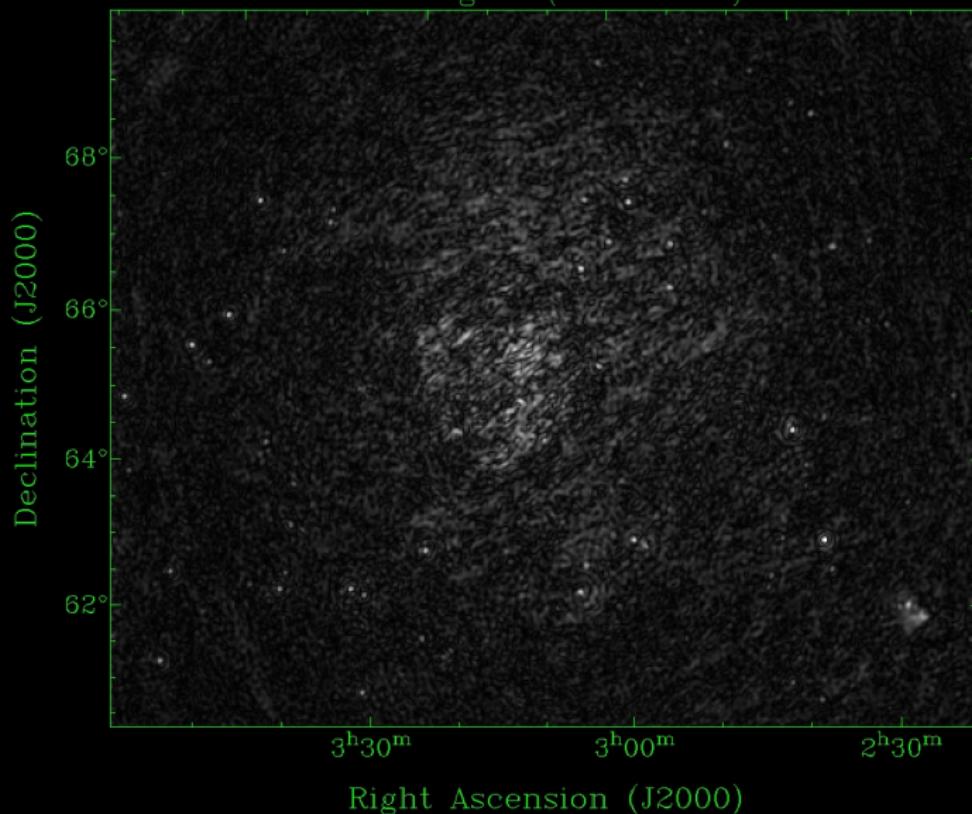
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

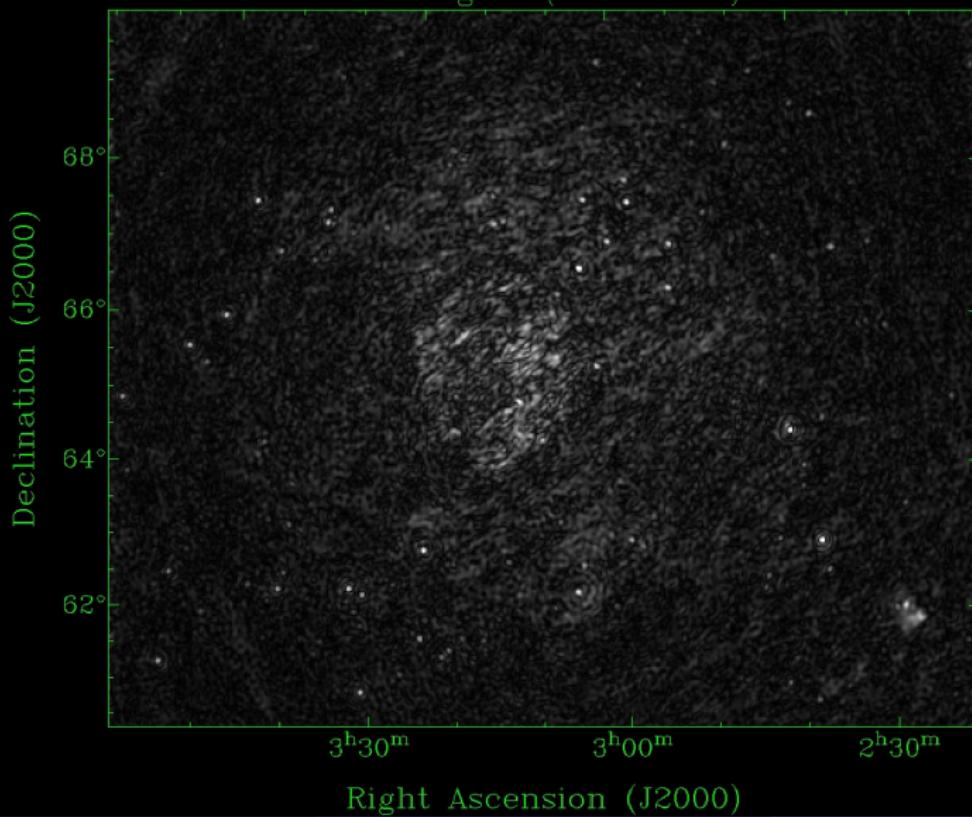
RM: -6.500000e+00

Fan region (WSRT LFFE)



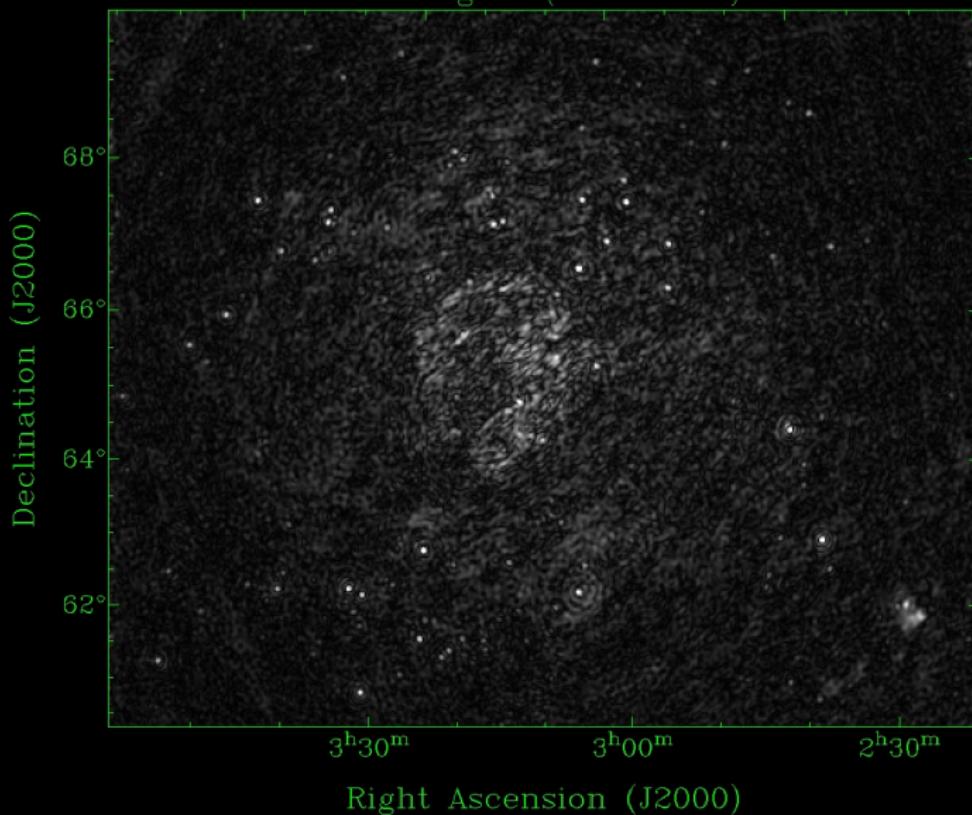
RM: -6.000000e+00

Fan region (WSRT LFFE)



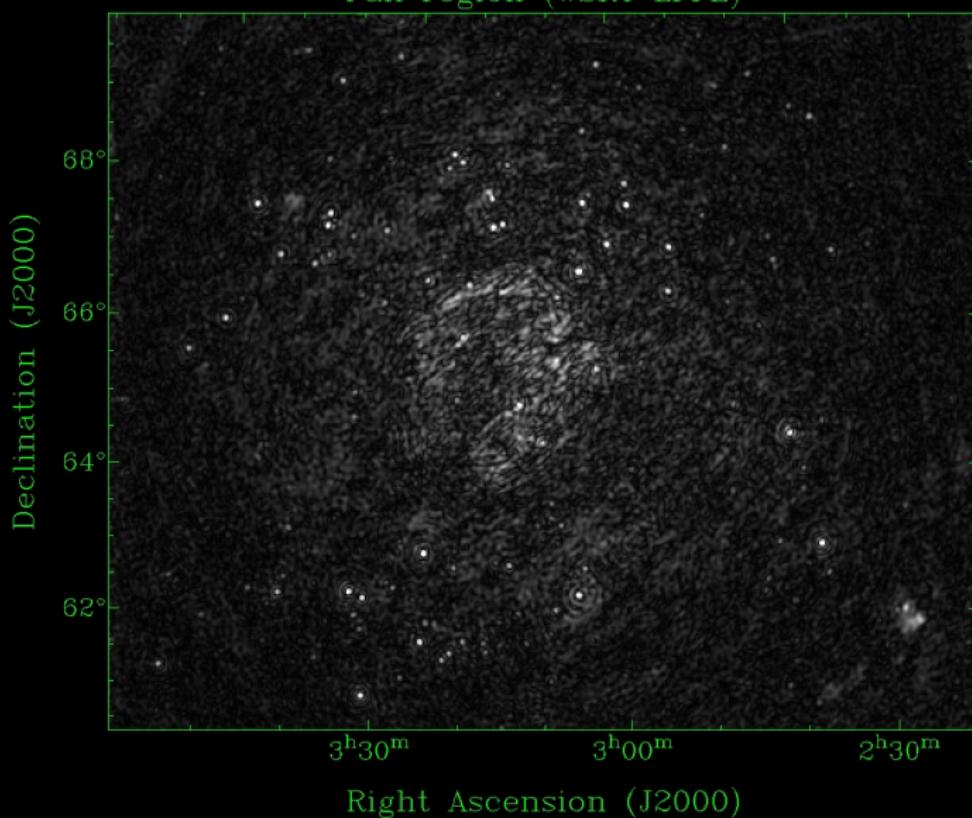
RM: -5.500000e+00

Fan region (WSRT LFFE)



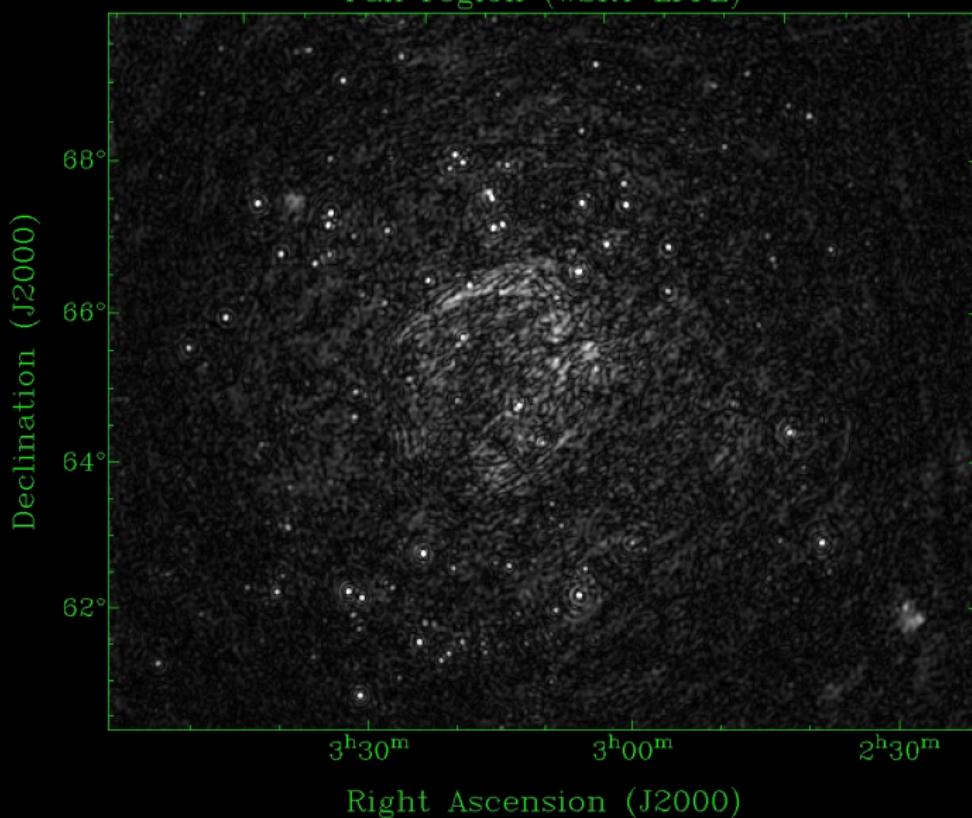
RM: -5.000000e+00

Fan region (WSRT LFFE)



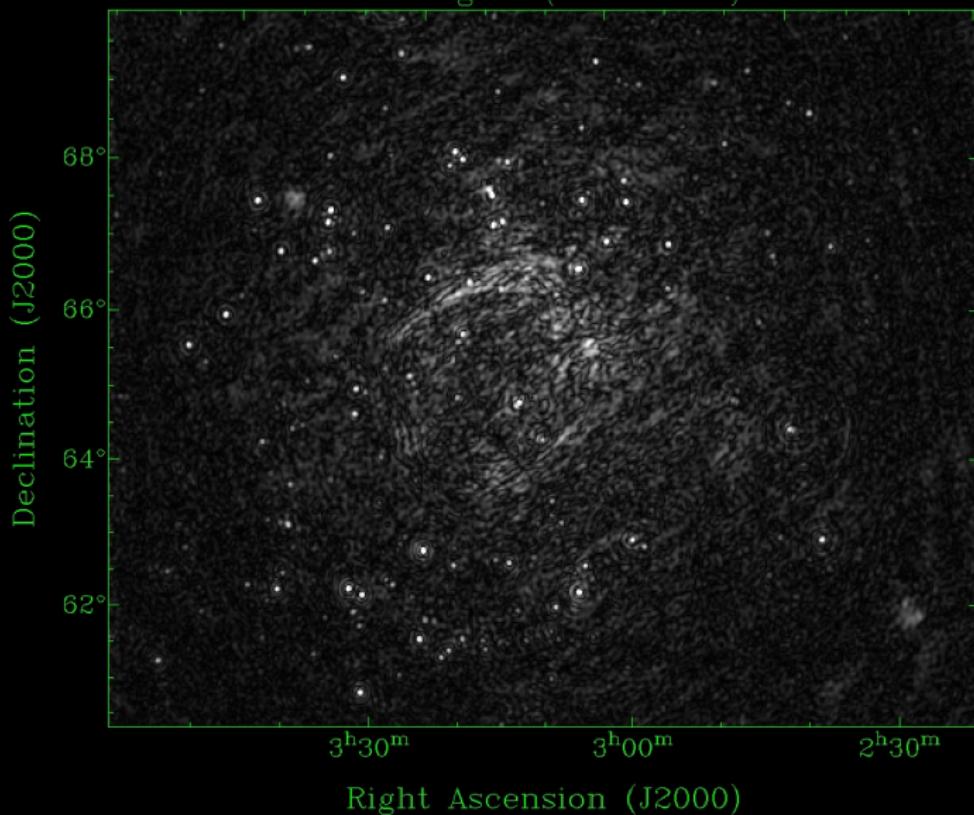
RM: -4.500000e+00

Fan region (WSRT LFFE)



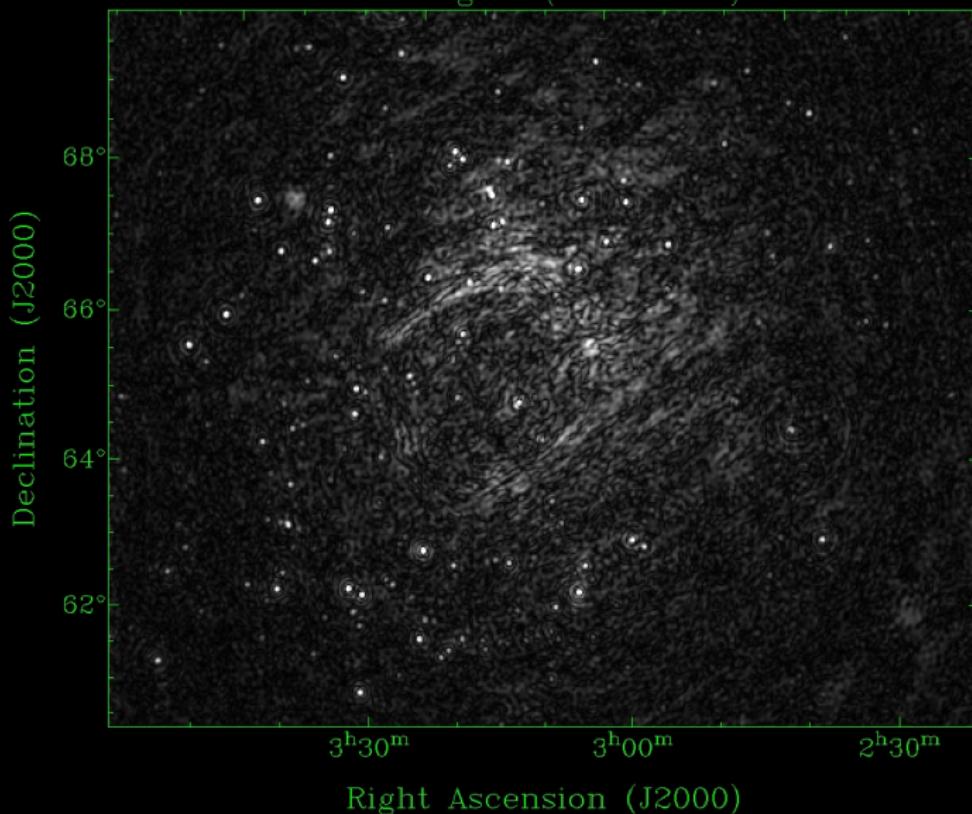
RM: -4.000000e+00

Fan region (WSRT LFFE)



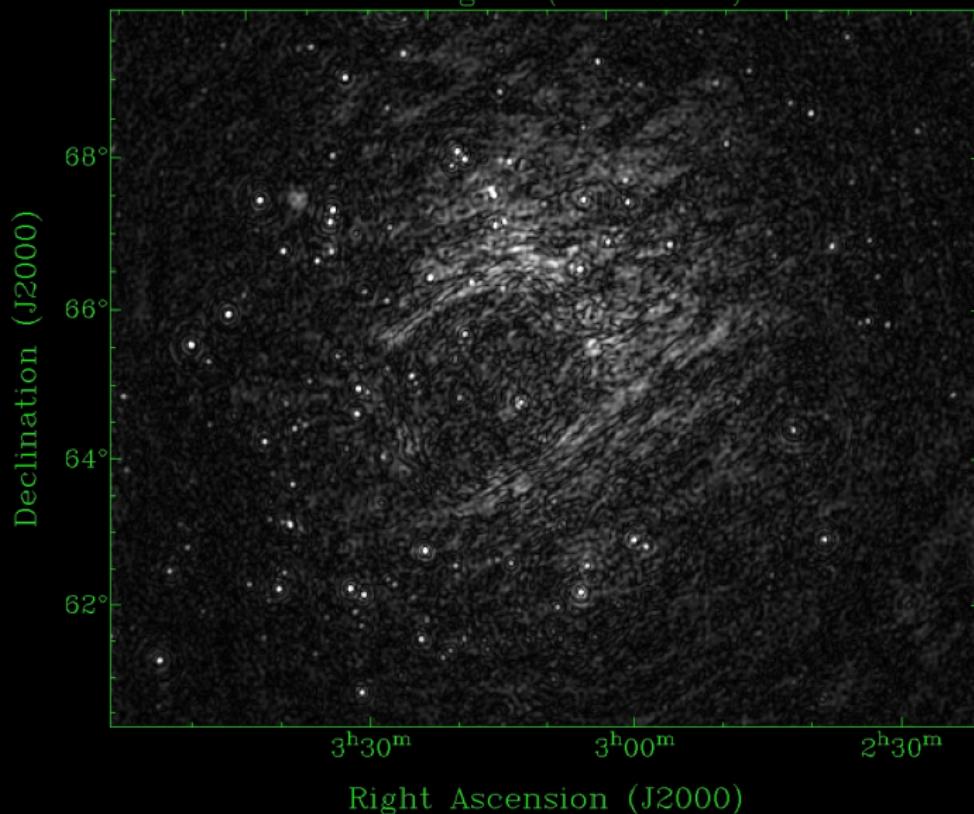
RM: -3.500000e+00

Fan region (WSRT LFFE)



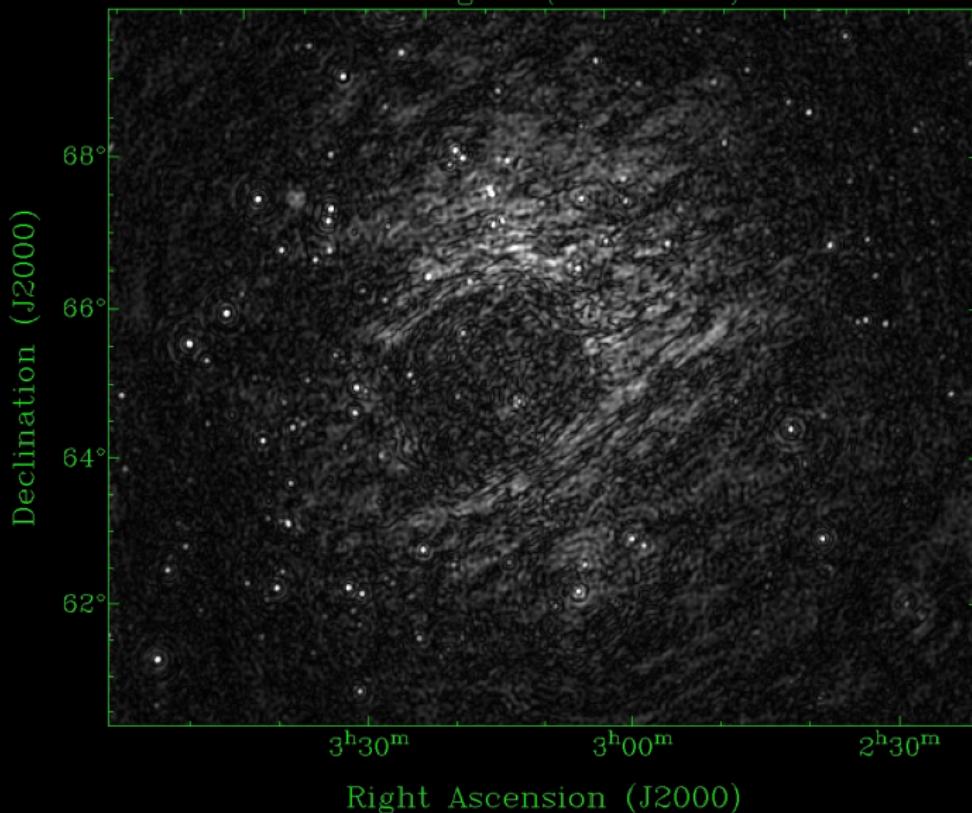
RM: -3.000000e+00

Fan region (WSRT LFFE)



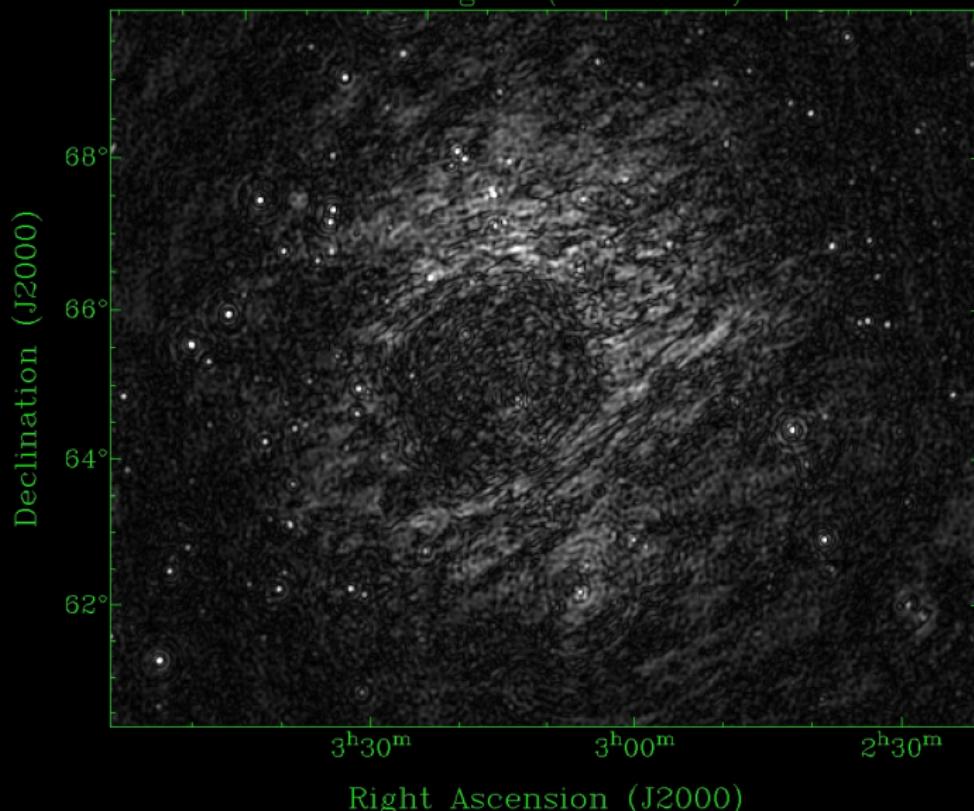
RM: -2.500000e+00

Fan region (WSRT LFFE)



RM: -2.000000e+00

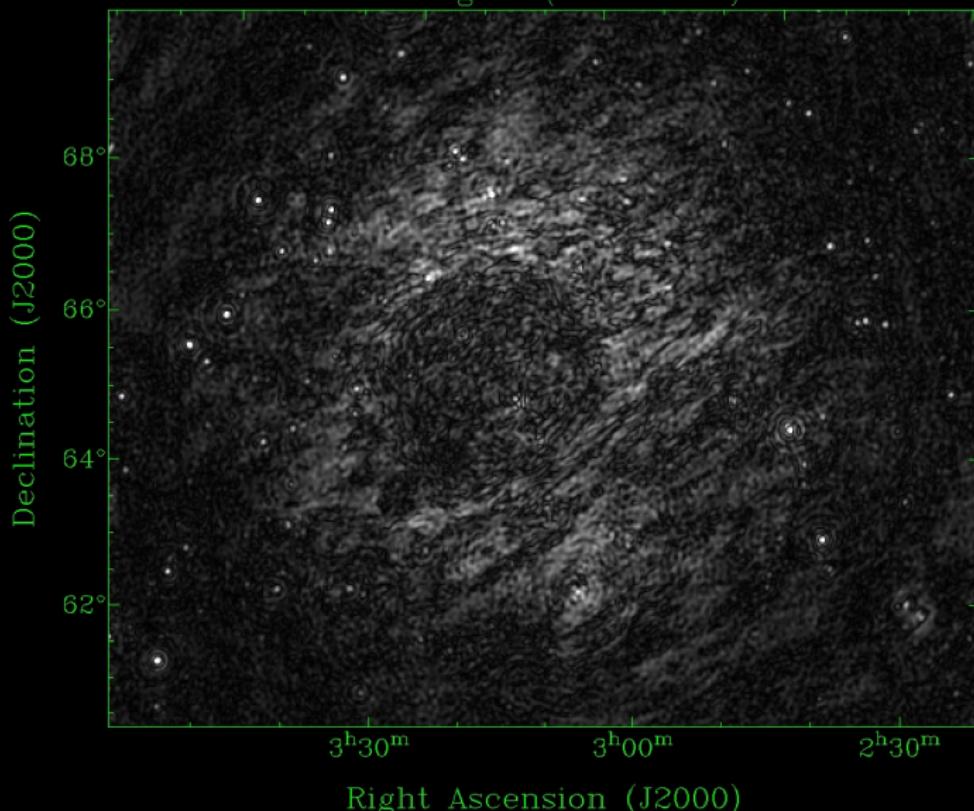
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: -1.500000e+00

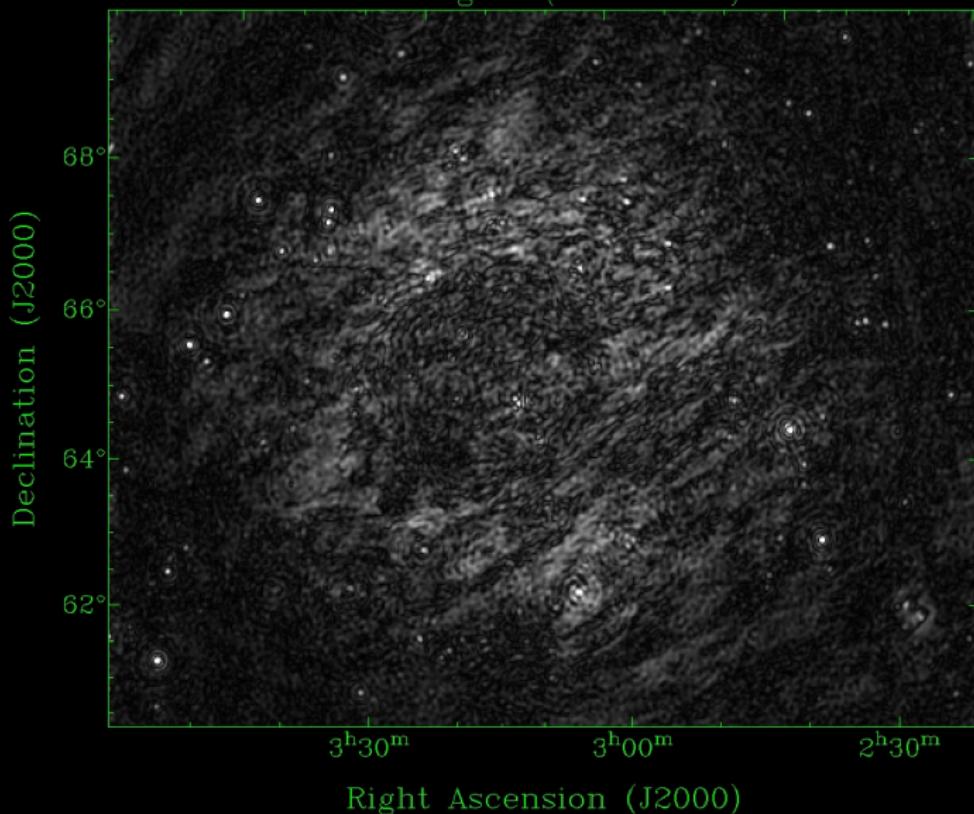
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: -1.000000e+00

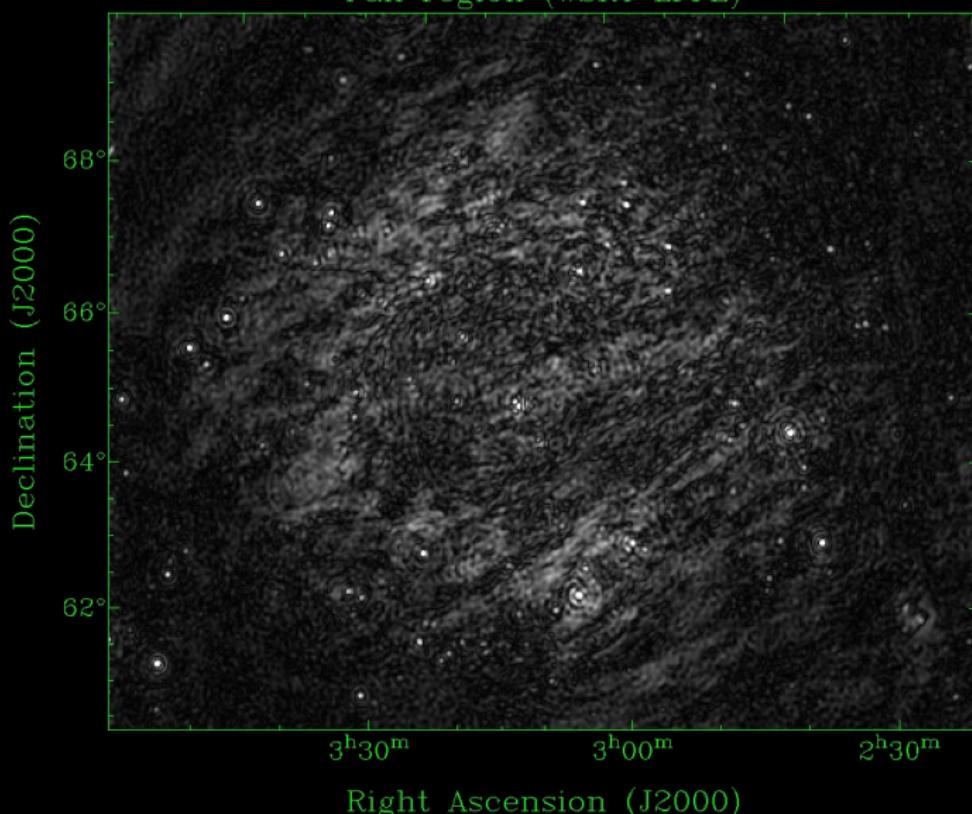
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: -5.000000e-01

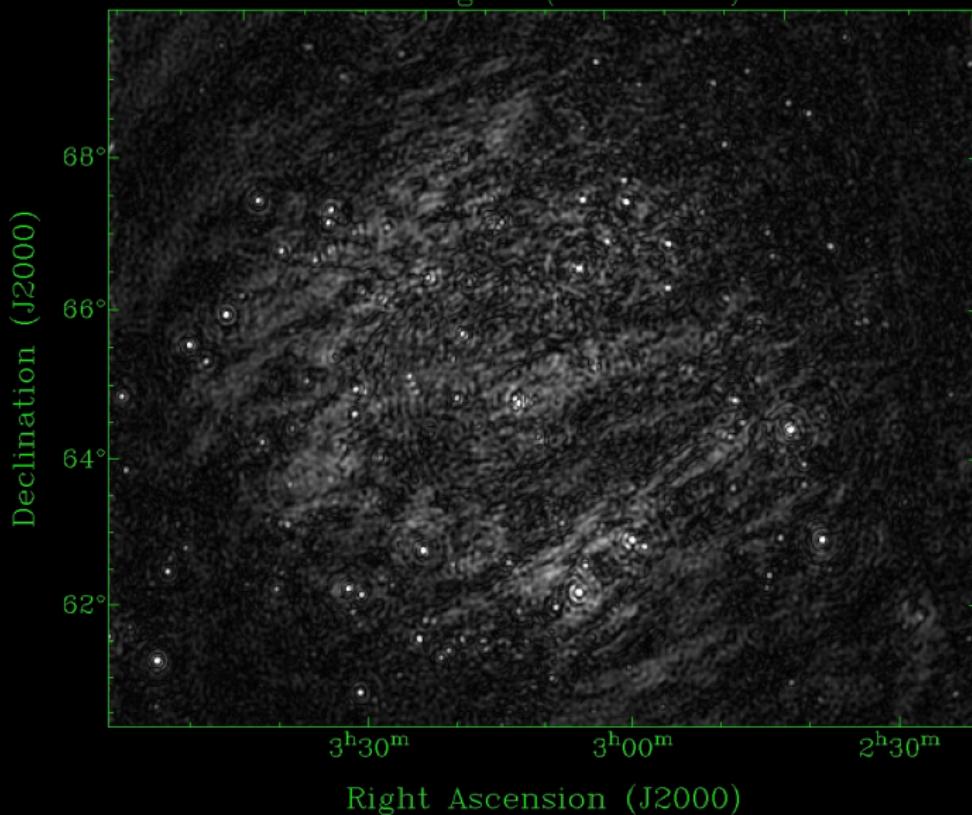
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

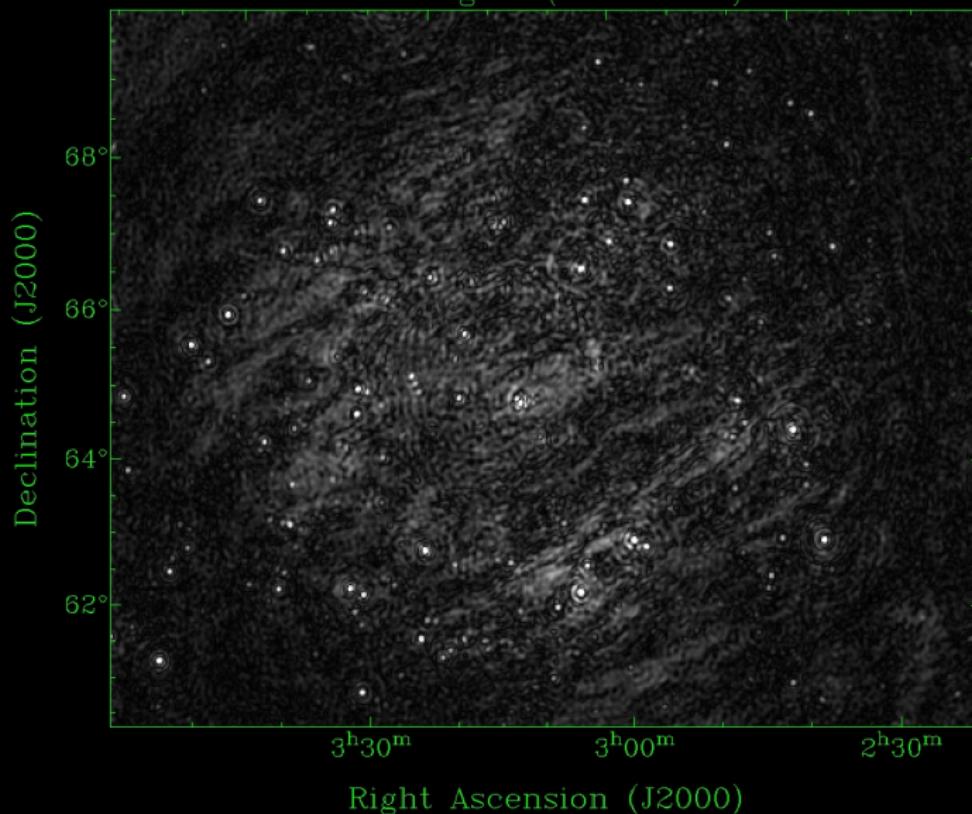
RM: 0.000000e+00

Fan region (WSRT LFFE)



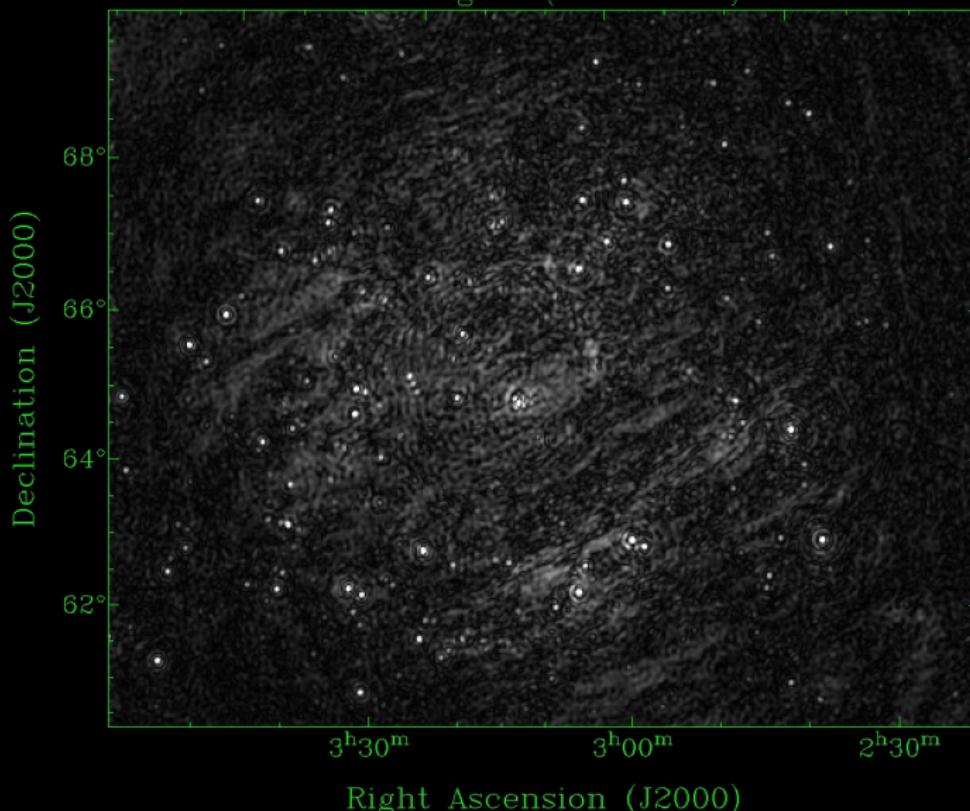
RM: 5.000000e-01

Fan region (WSRT LFFE)



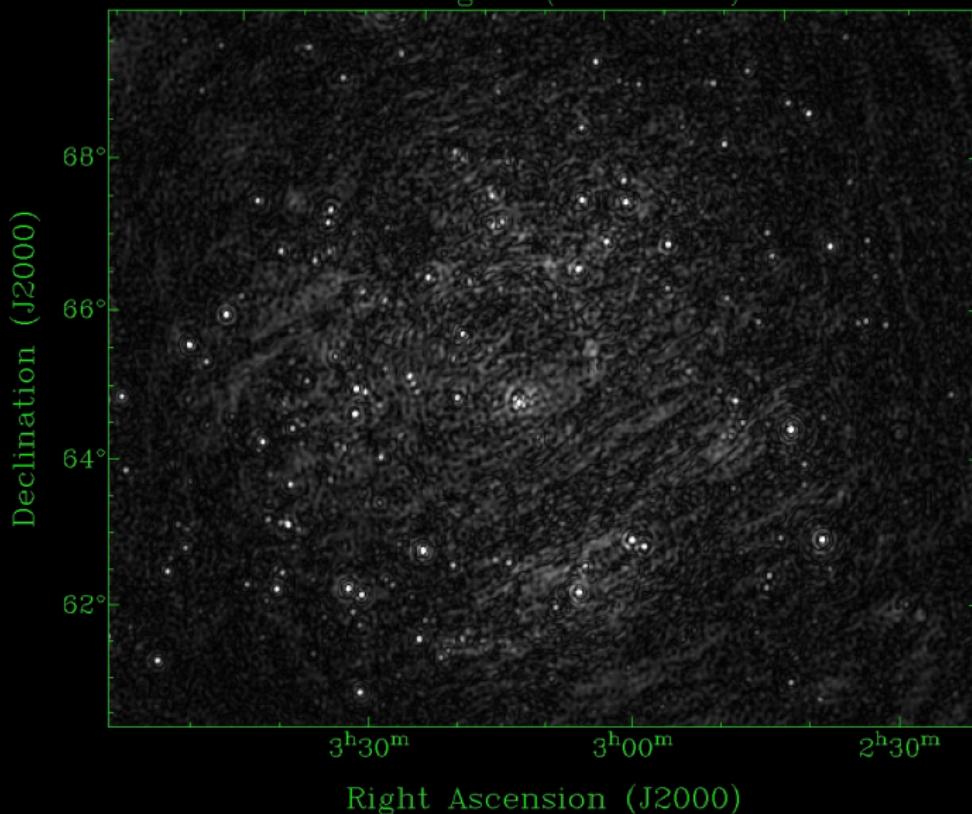
RM: 1.000000e+00

Fan region (WSRT LFFE)



RM: 1.500000e+00

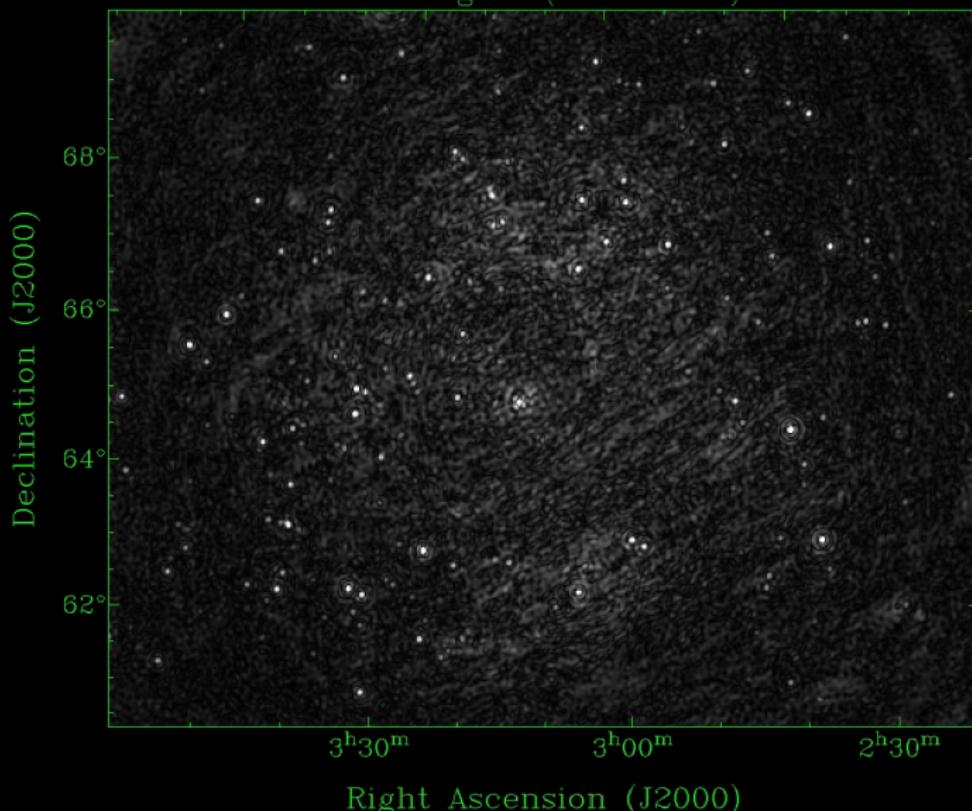
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

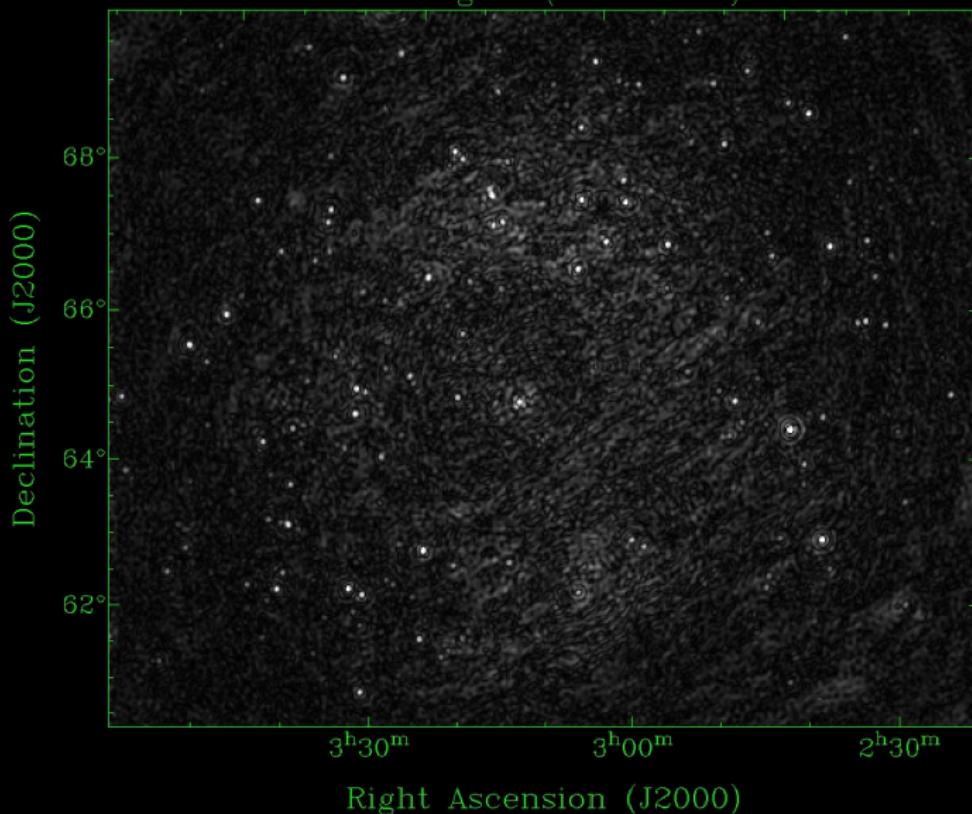
RM: 2.000000e+00

Fan region (WSRT LFFE)



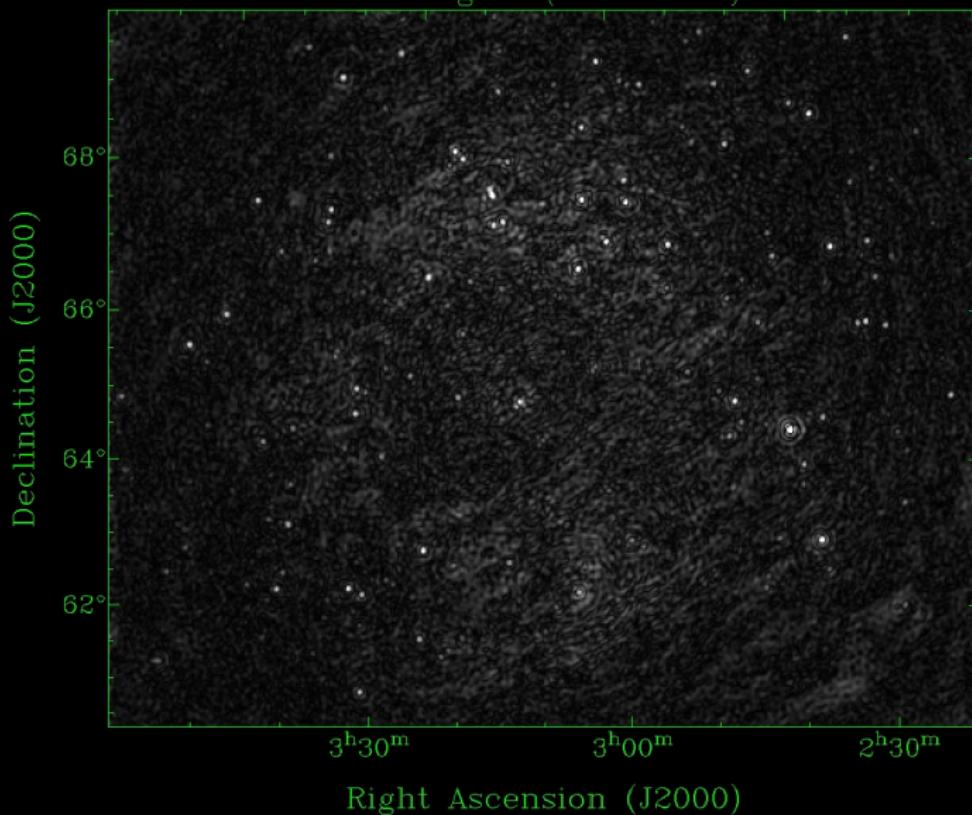
RM: 2.500000e+00

Fan region (WSRT LFFE)



RM: 3.000000e+00

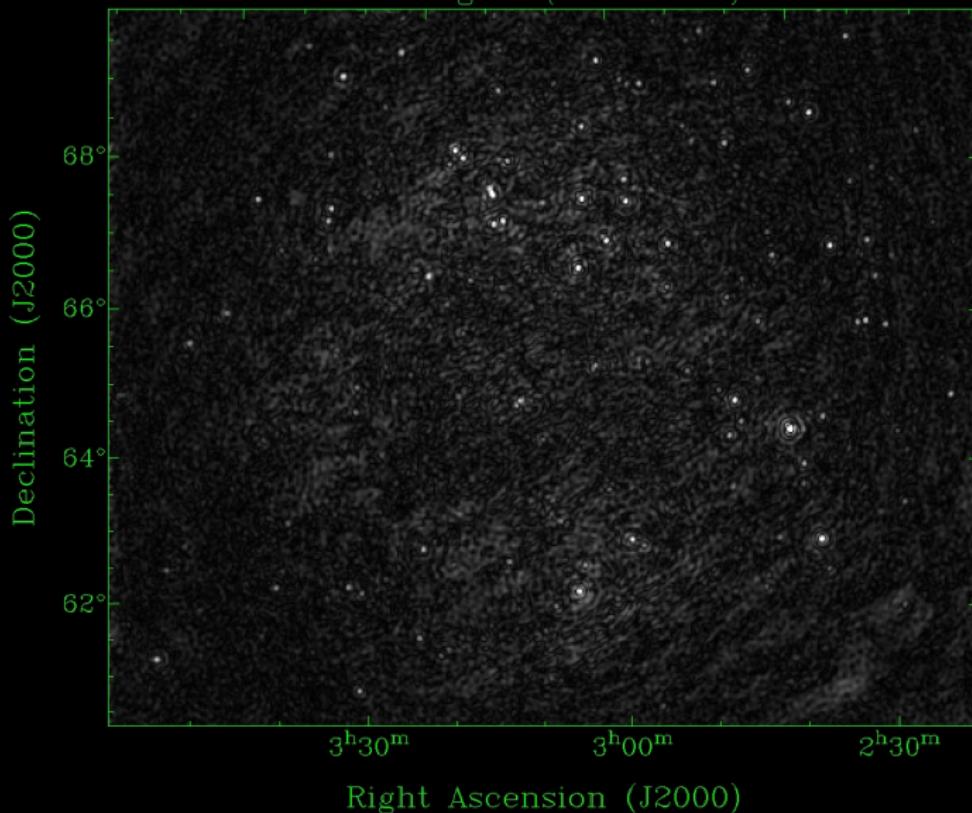
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: 3.500000e+00

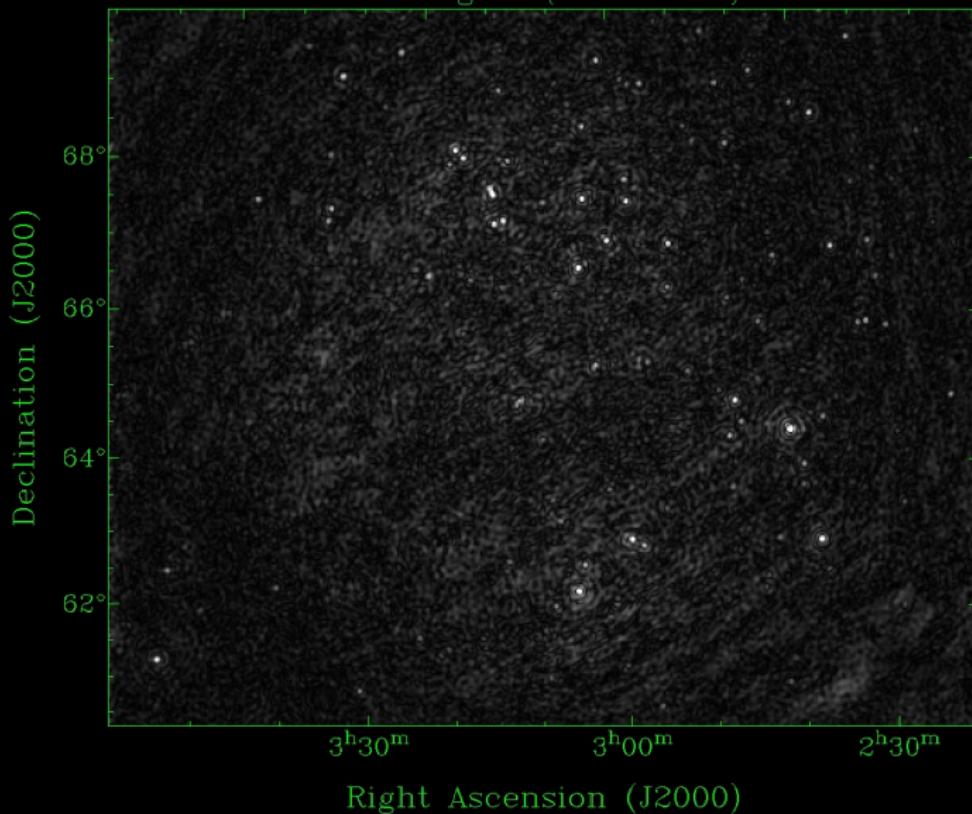
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

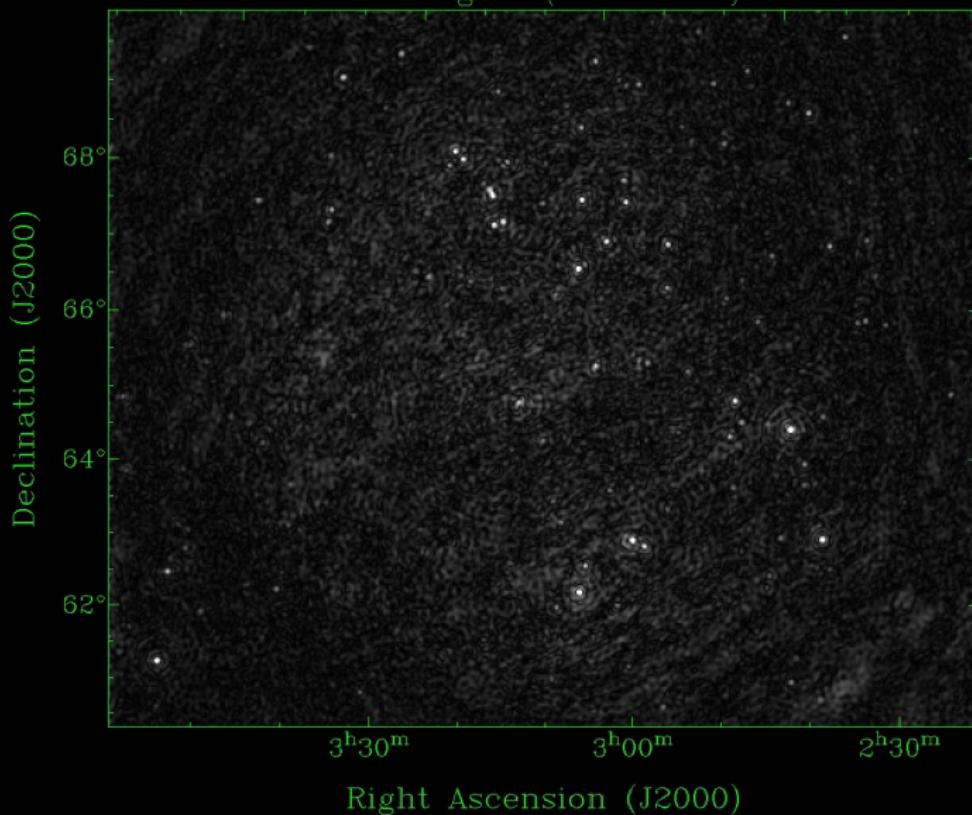
RM: 4.000000e+00

Fan region (WSRT LFFE)



RM: 4.500000e+00

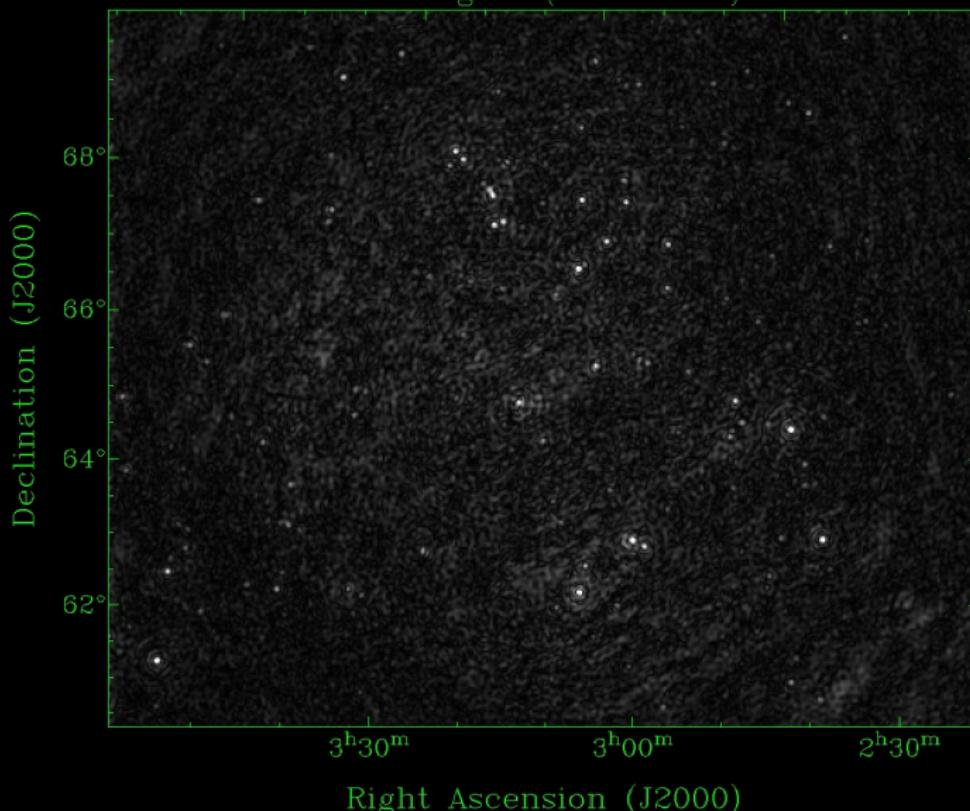
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: 5.000000e+00

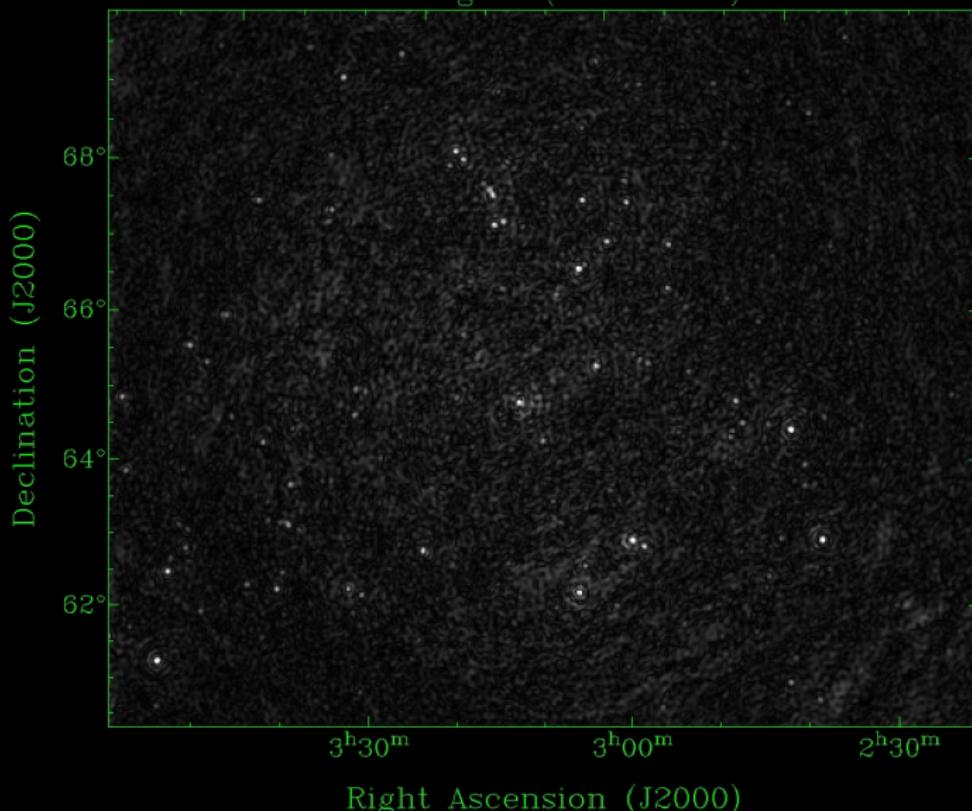
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

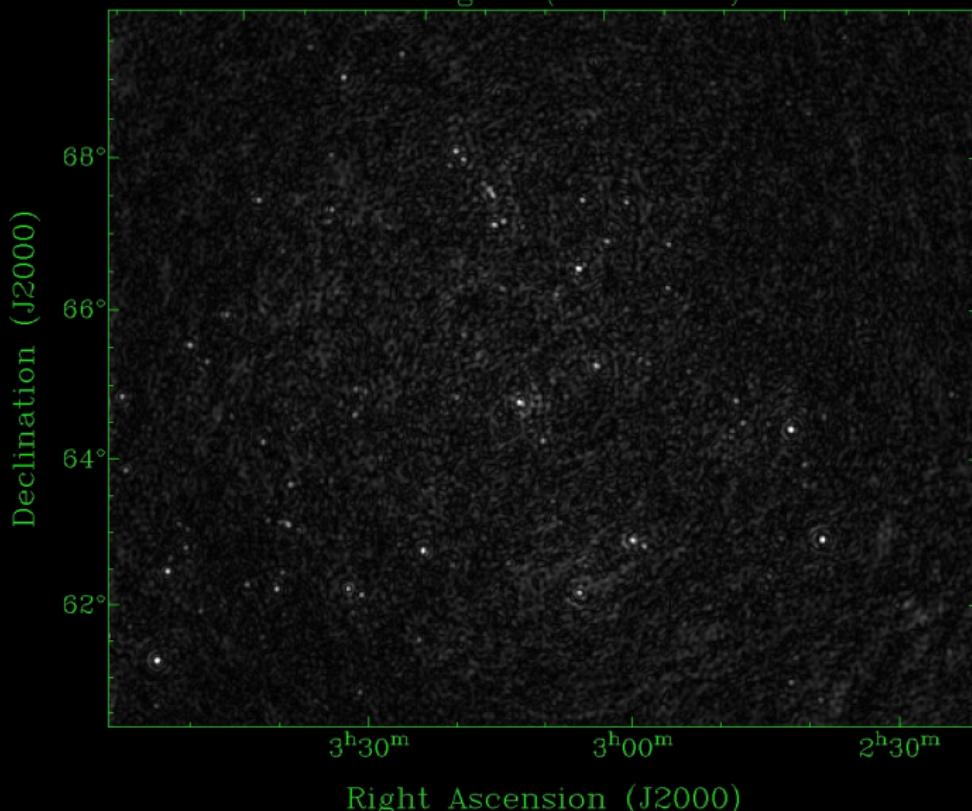
RM: 5.500000e+00

Fan region (WSRT LFFE)



RM: 6.000000e+00

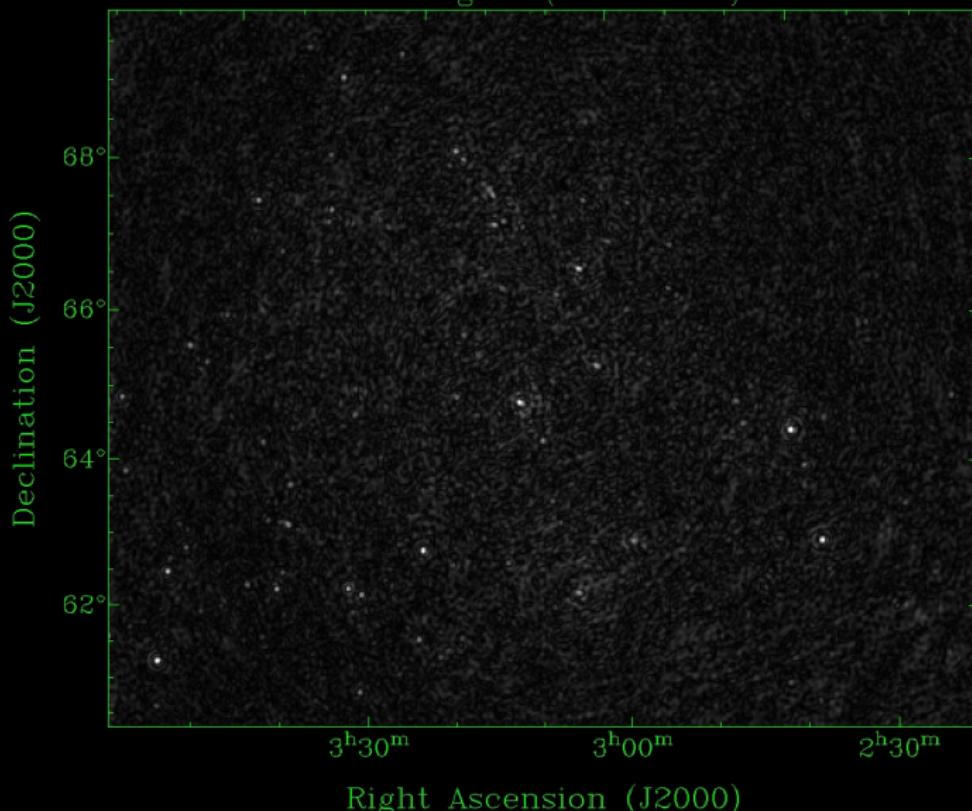
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: 6.500000e+00

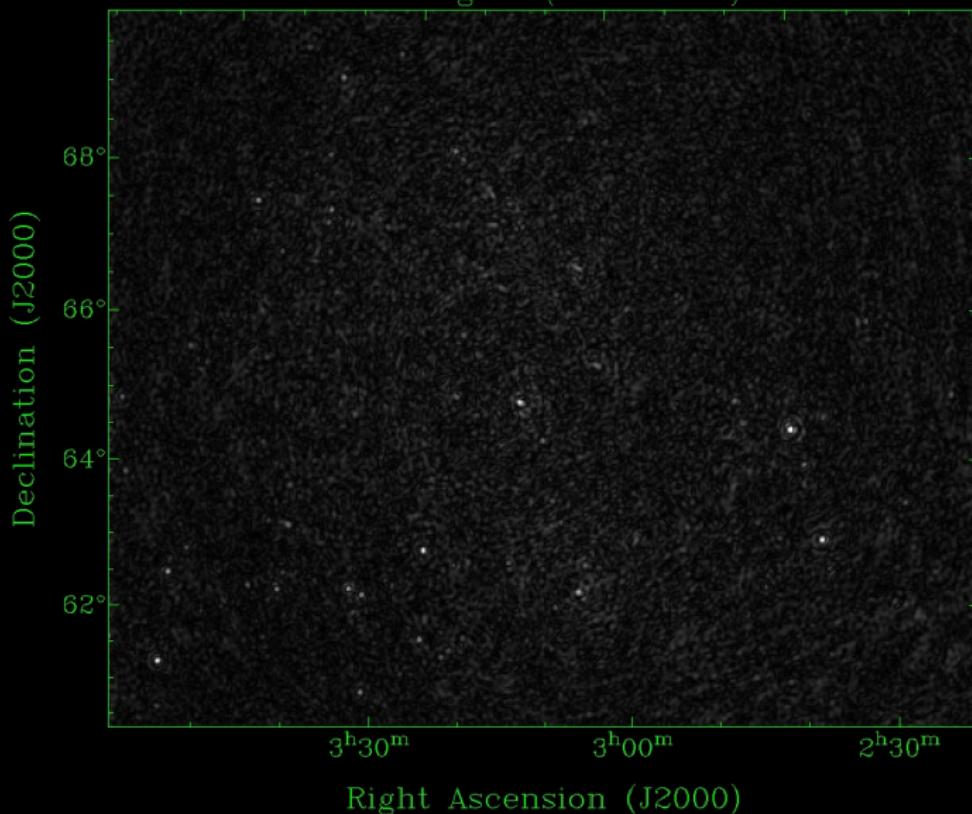
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

RM: 7.000000e+00

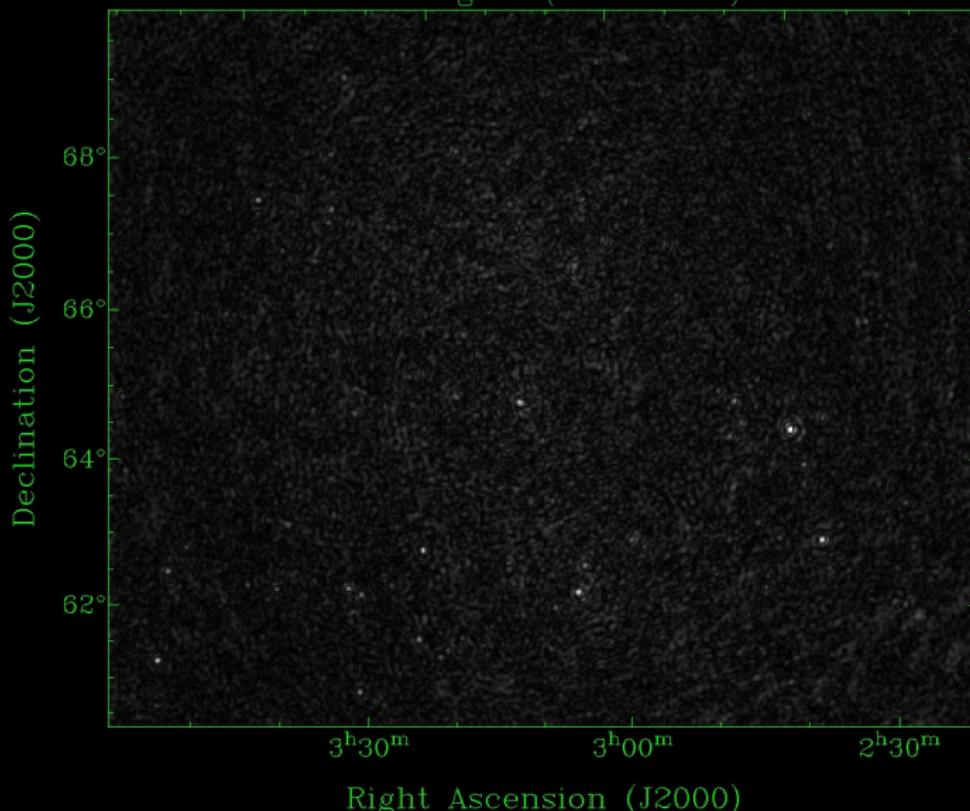
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

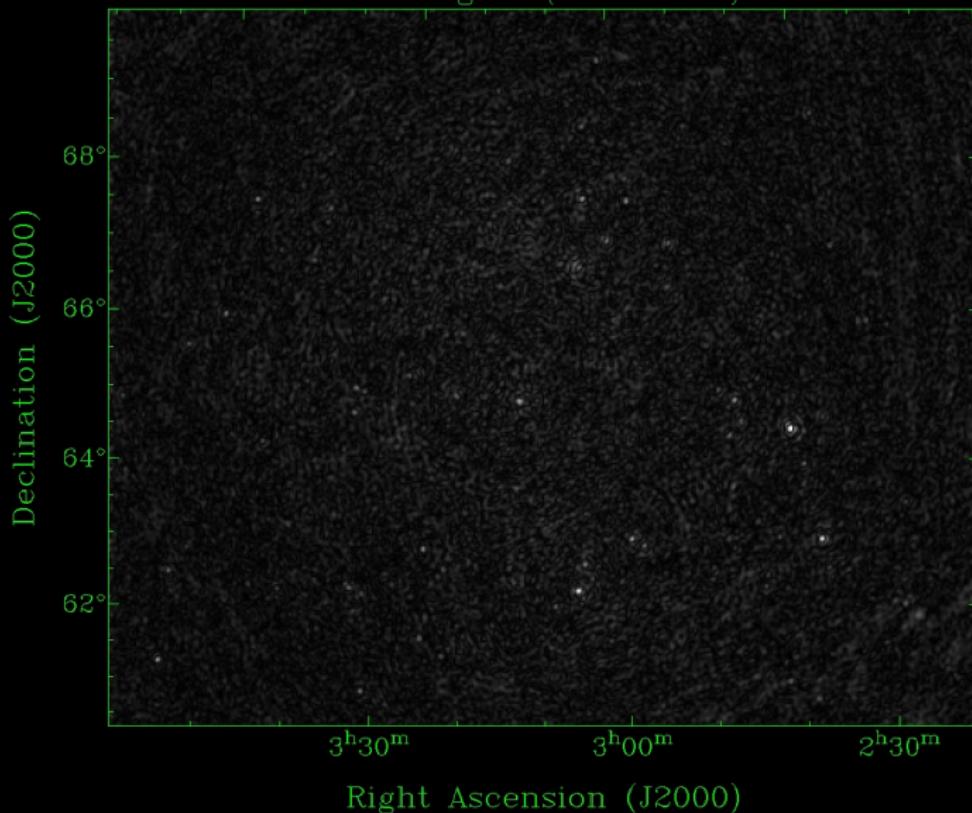
RM: 7.500000e+00

Fan region (WSRT LFFE)



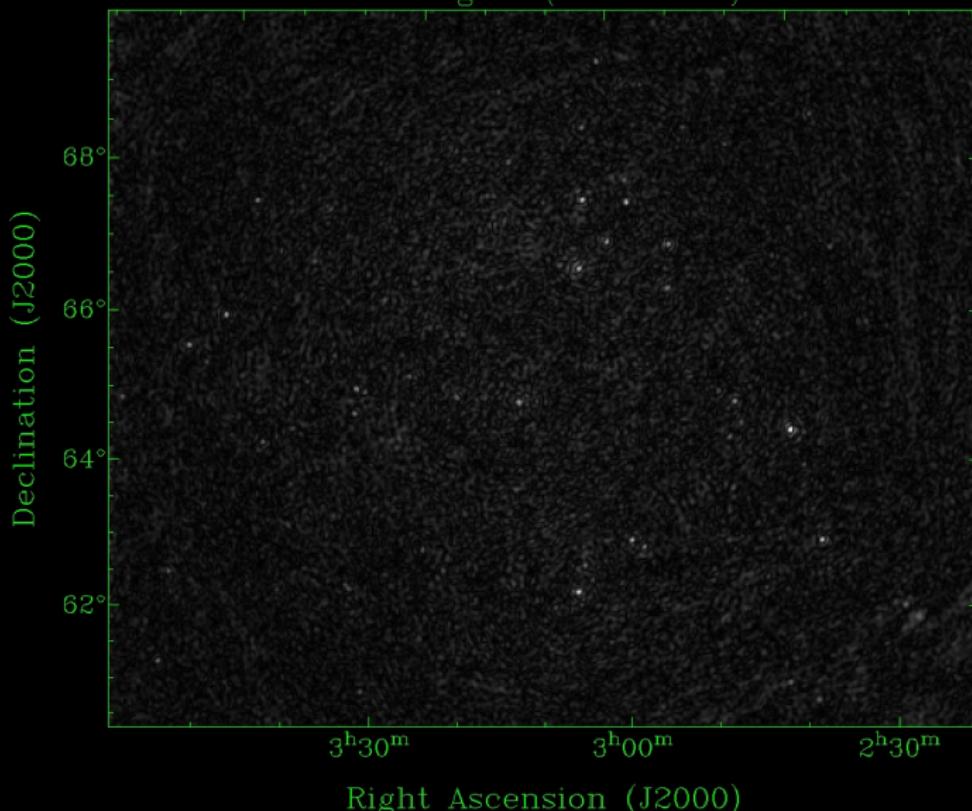
RM: 8.000000e+00

Fan region (WSRT LFFE)



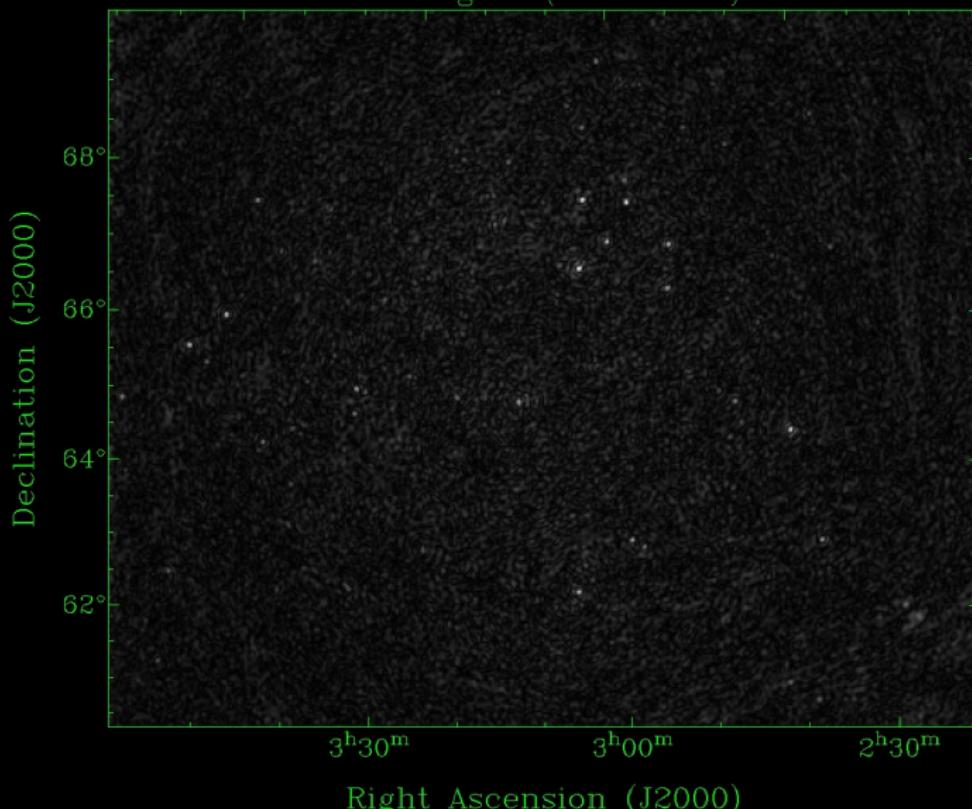
RM: 8.500000e+00

Fan region (WSRT LFFE)



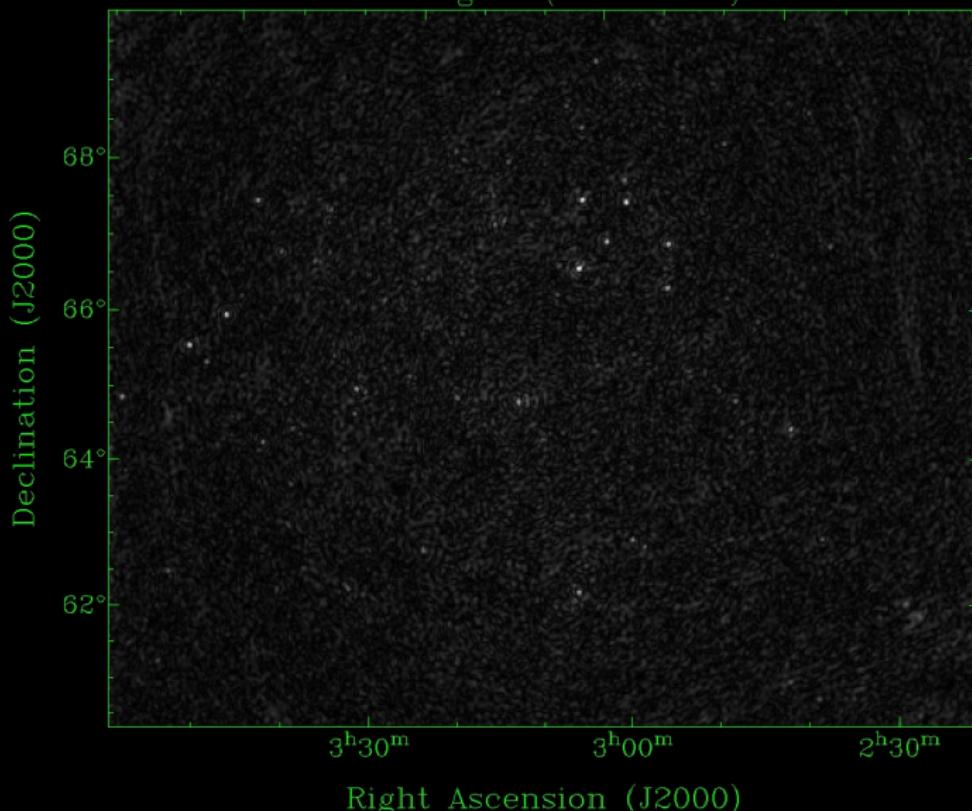
RM: 9.000000e+00

Fan region (WSRT LFFE)



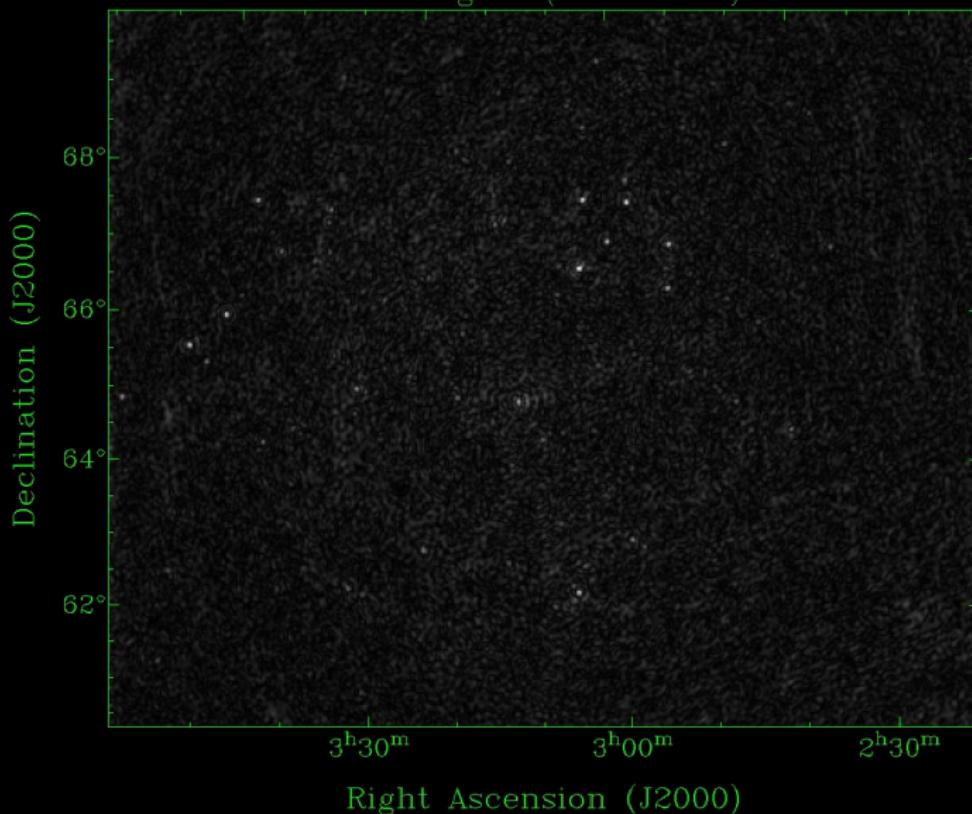
RM: 9.500000e+00

Fan region (WSRT LFFE)



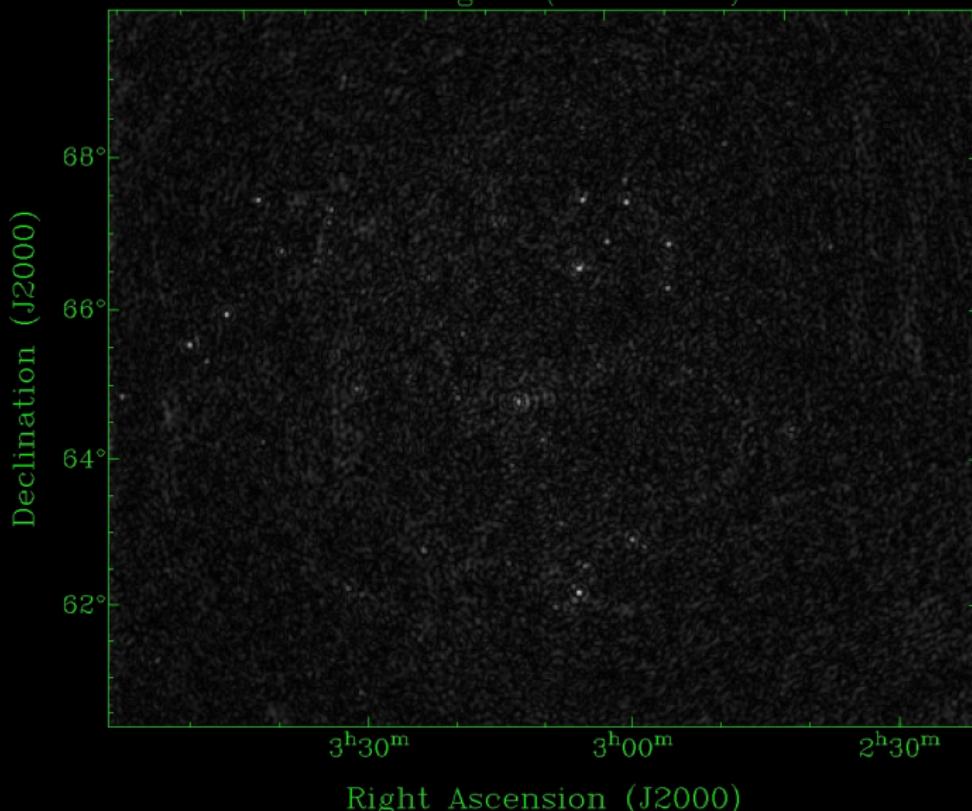
RM: 1.000000e+01

Fan region (WSRT LFFE)



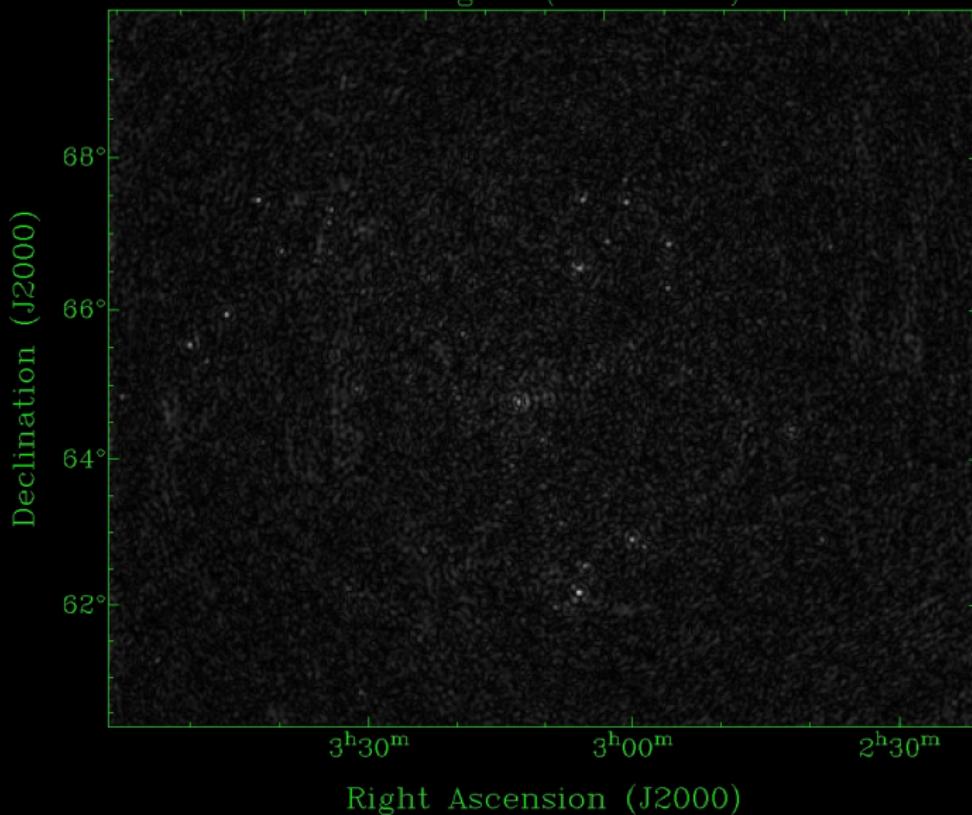
RM: 1.050000e+01

Fan region (WSRT LFFE)



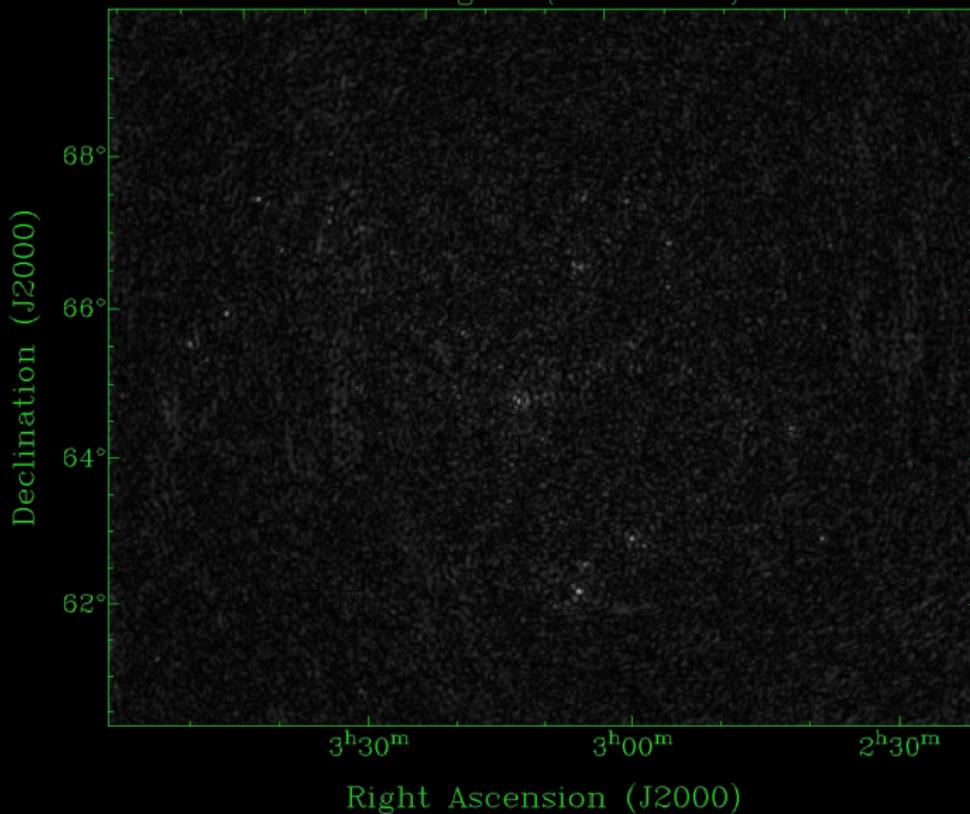
RM: 1.100000e+01

Fan region (WSRT LFFE)



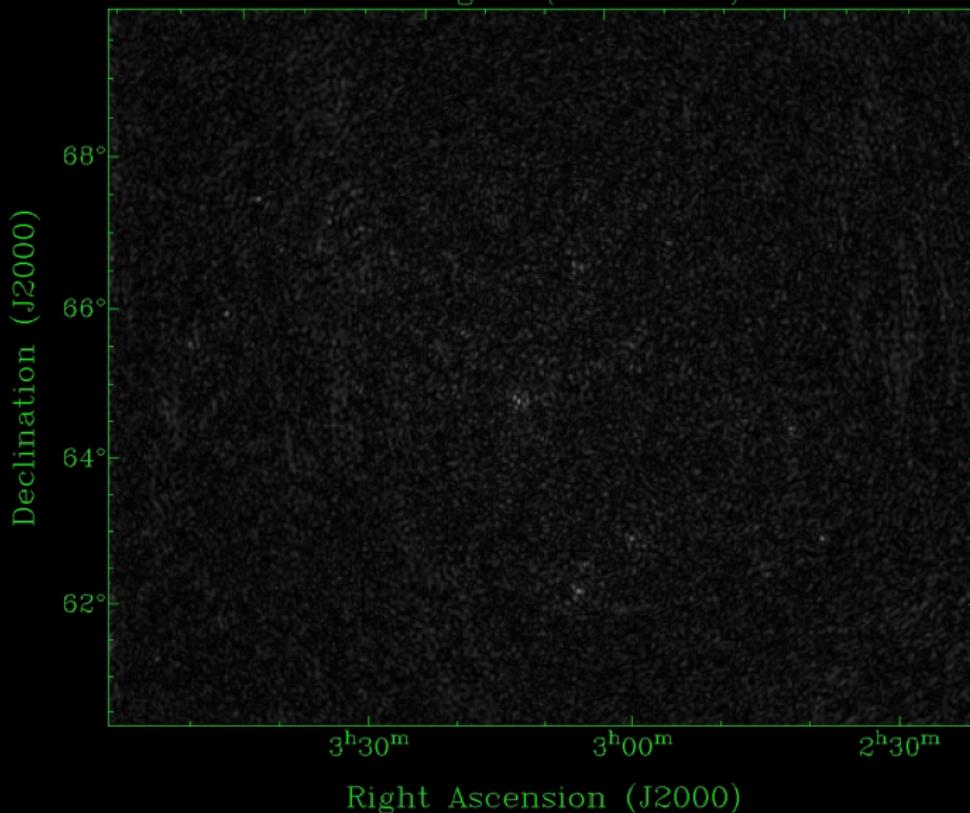
RM: 1.150000e+01

Fan region (WSRT LFFE)



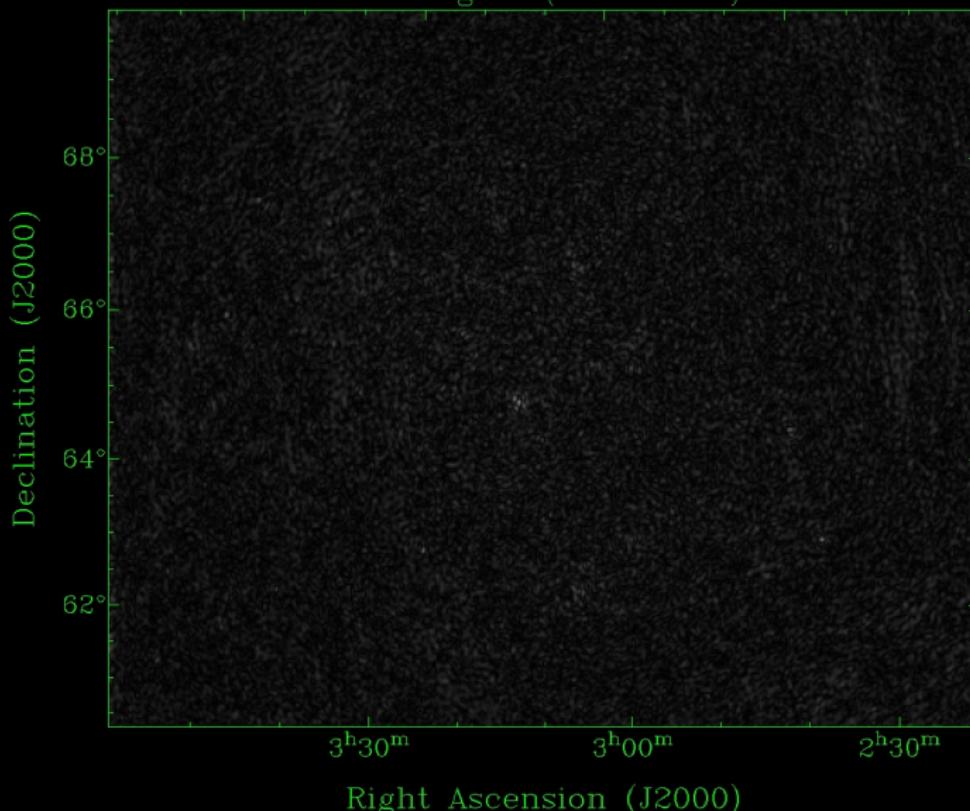
RM: 1.200000e+01

Fan region (WSRT LFFE)



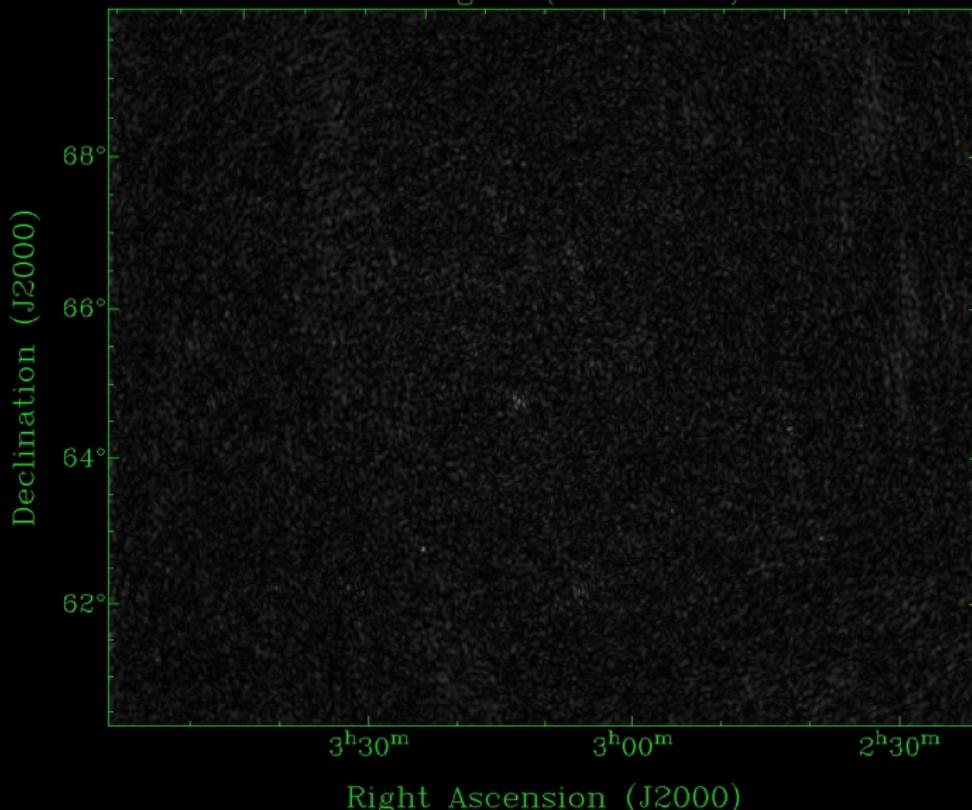
RM: 1.250000e+01

Fan region (WSRT LFFE)



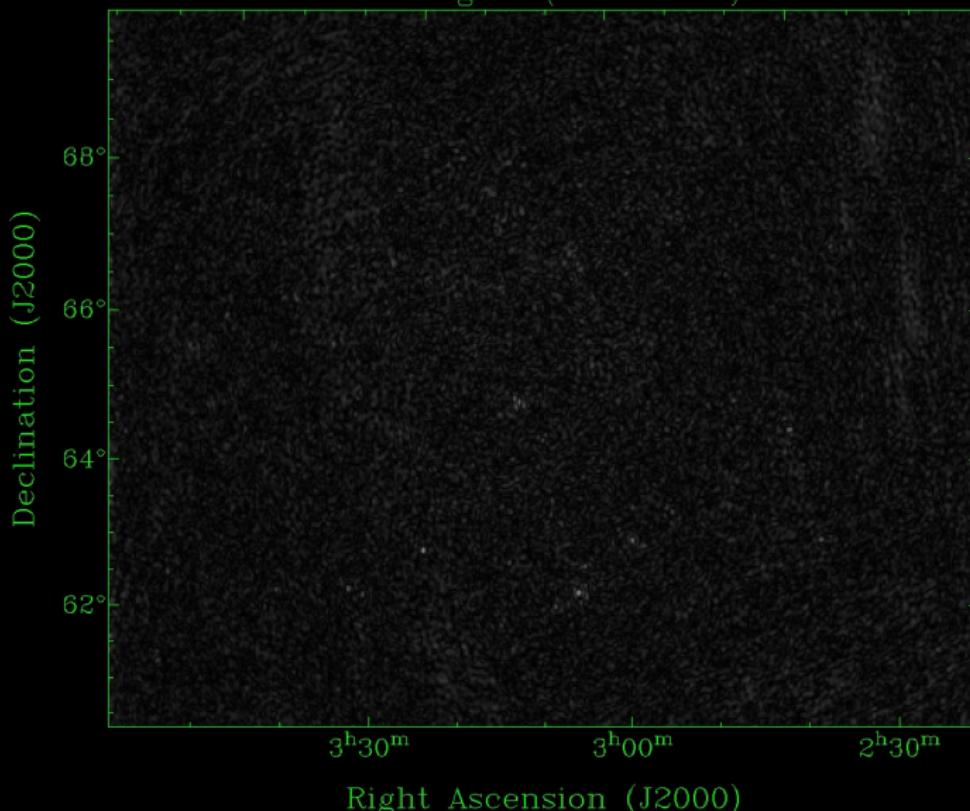
RM: 1.300000e+01

Fan region (WSRT LFFE)



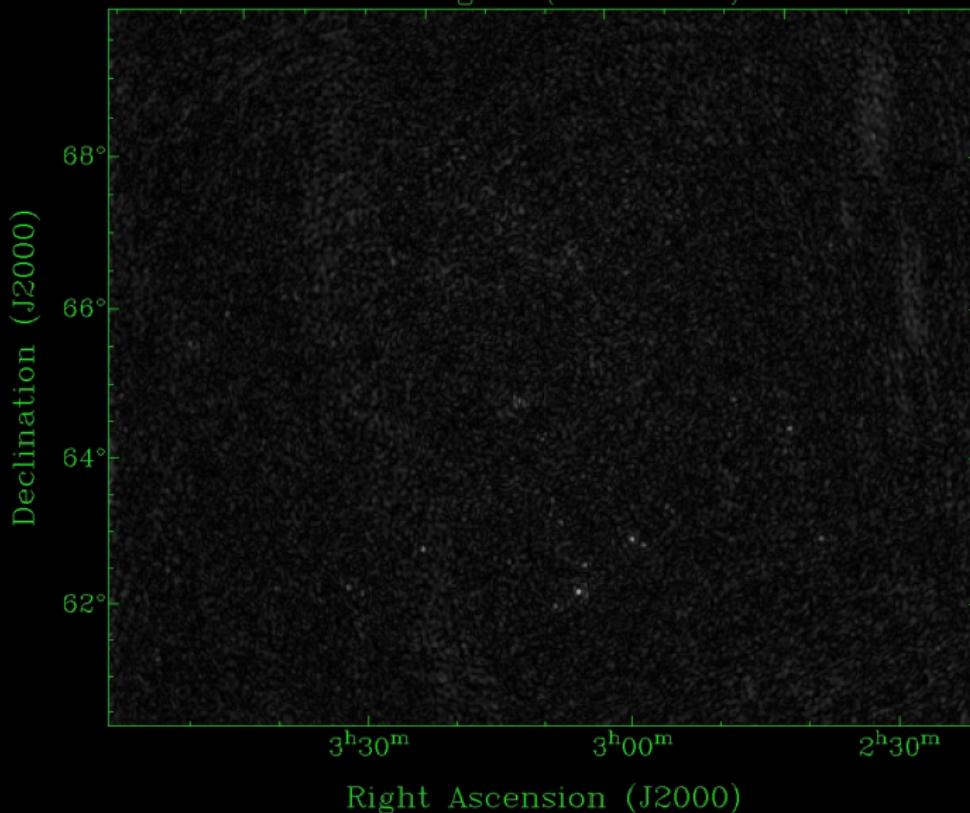
RM: 1.350000e+01

Fan region (WSRT LFFE)



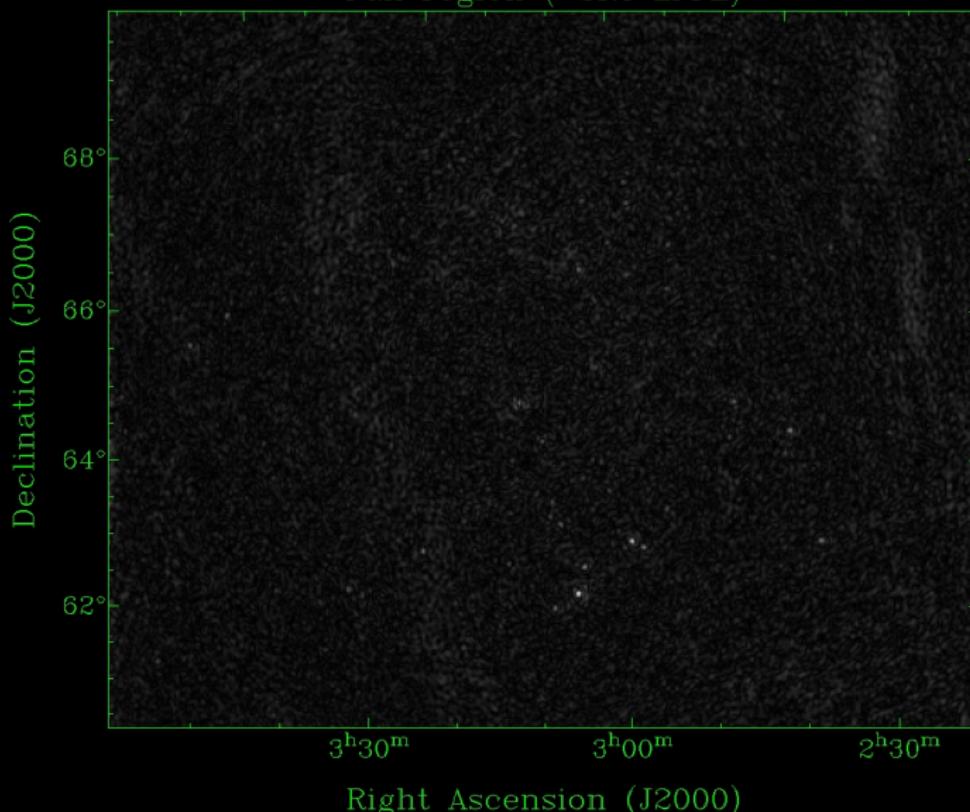
RM: 1.400000e+01

Fan region (WSRT LFFE)



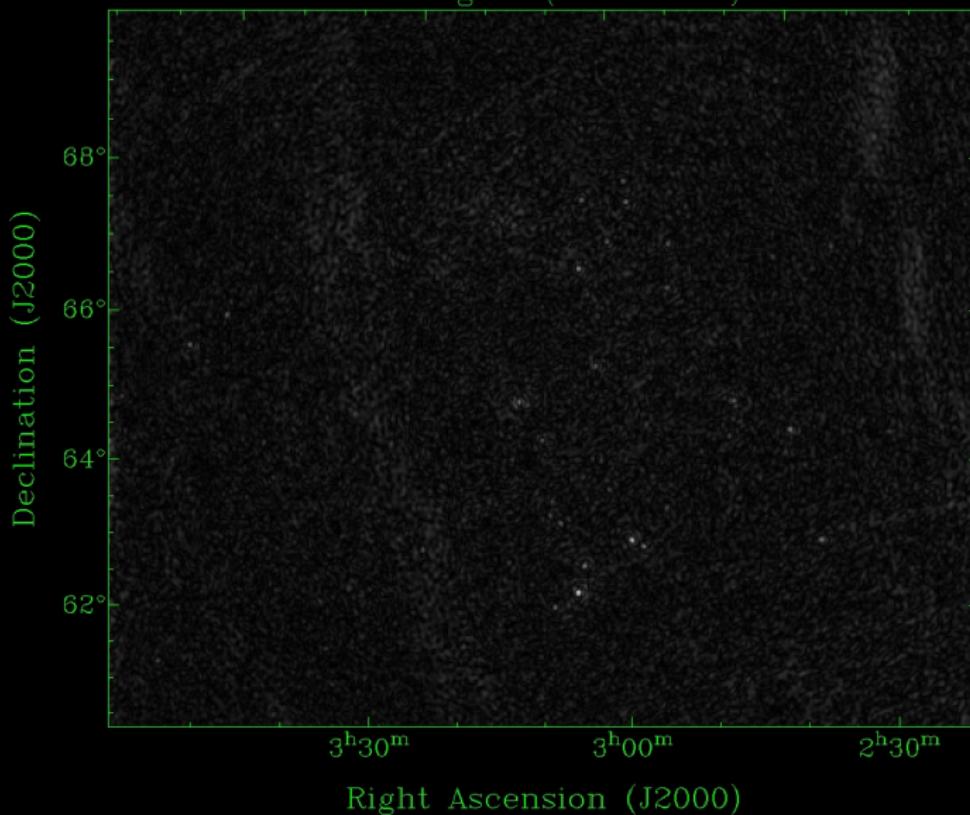
RM: 1.450000e+01

Fan region (WSRT LFFE)



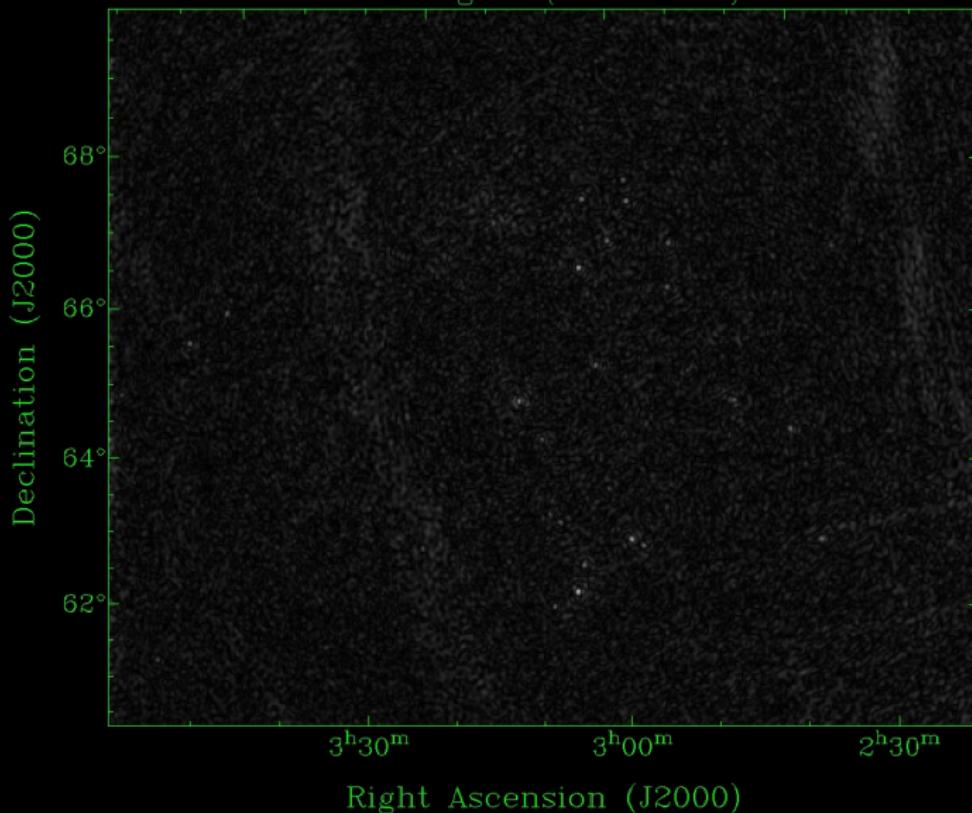
RM: 1.500000e+01

Fan region (WSRT LFFE)



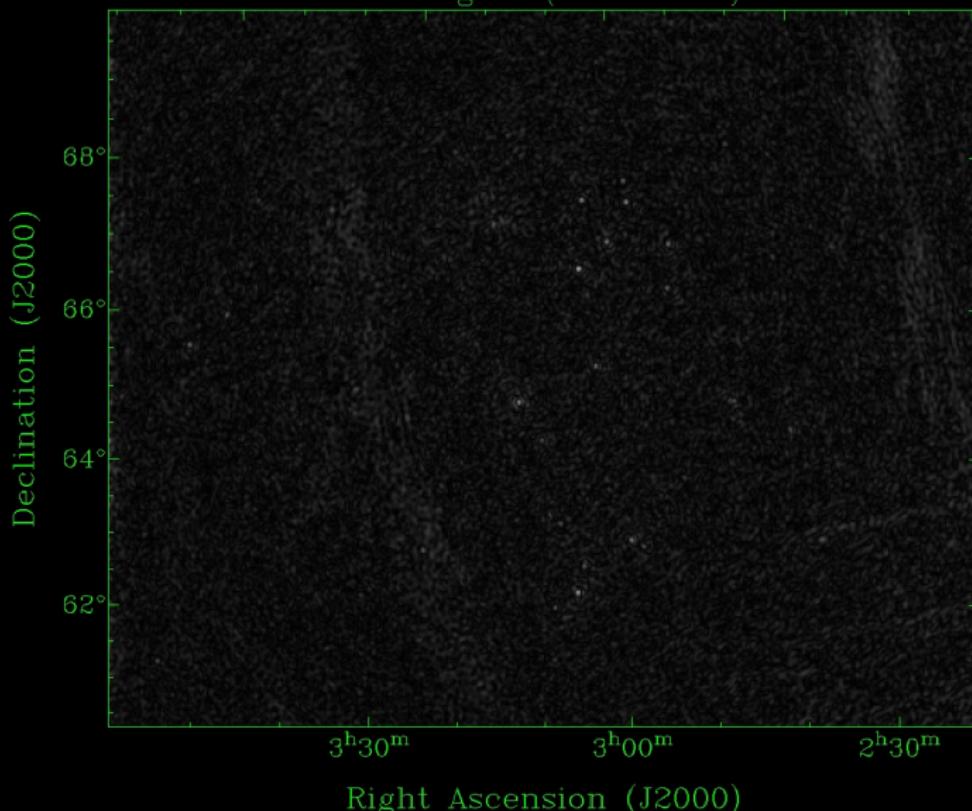
RM: 1.550000e+01

Fan region (WSRT LFFE)



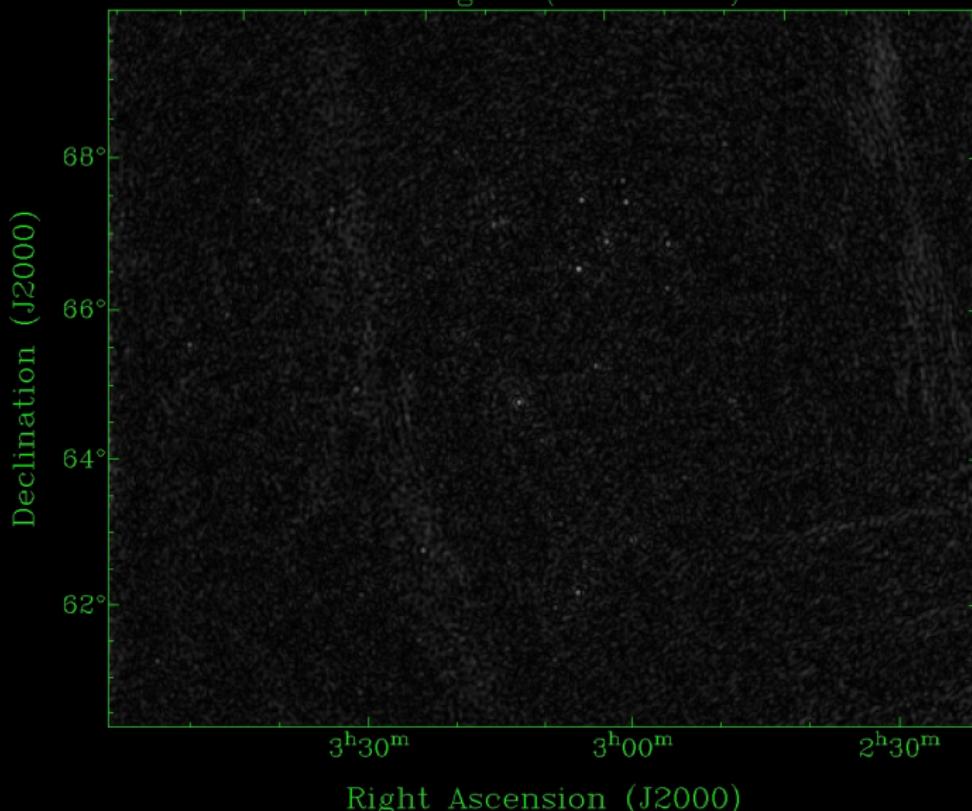
RM: 1.600000e+01

Fan region (WSRT LFFE)



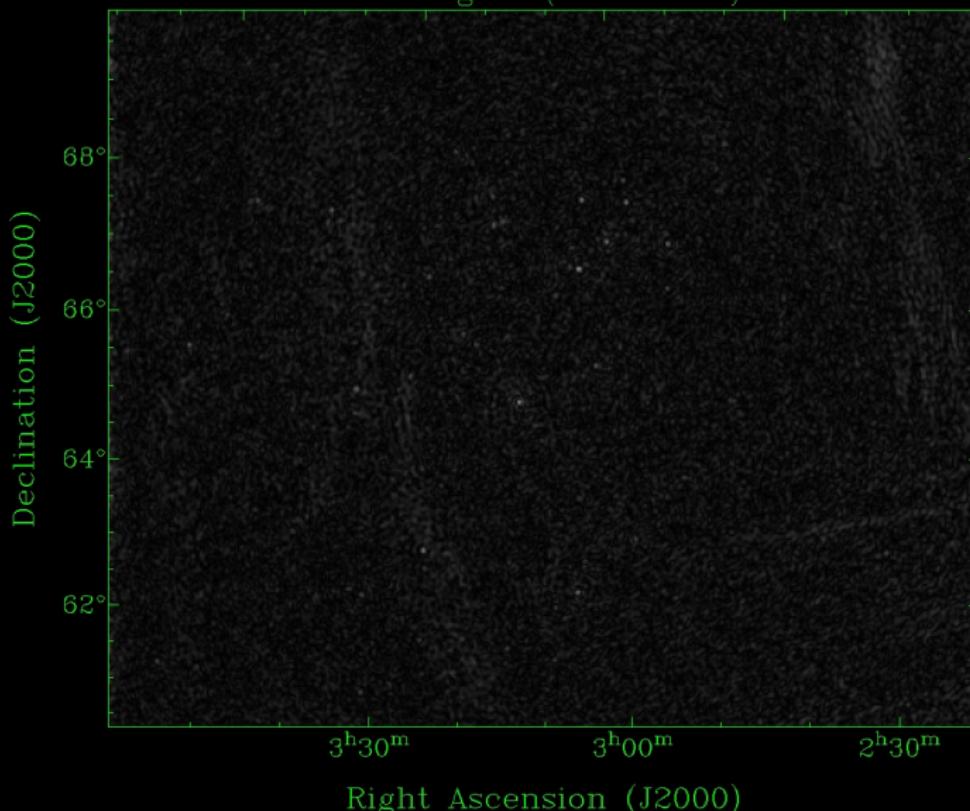
RM: 1.650000e+01

Fan region (WSRT LFFE)



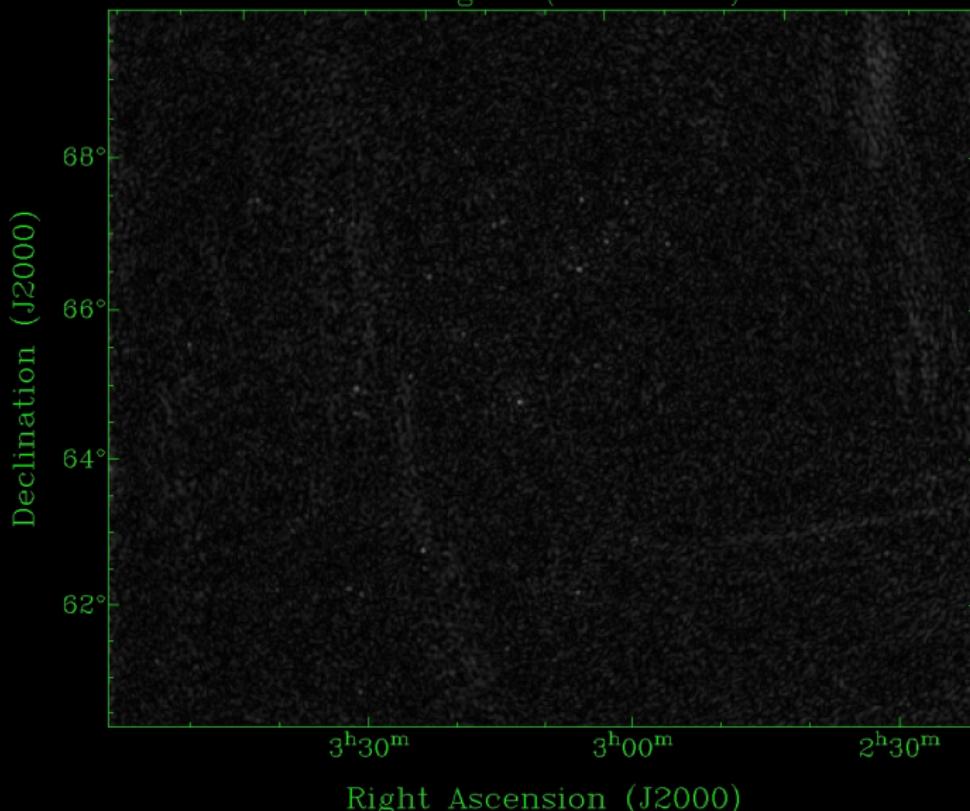
RM: 1.700000e+01

Fan region (WSRT LFFE)



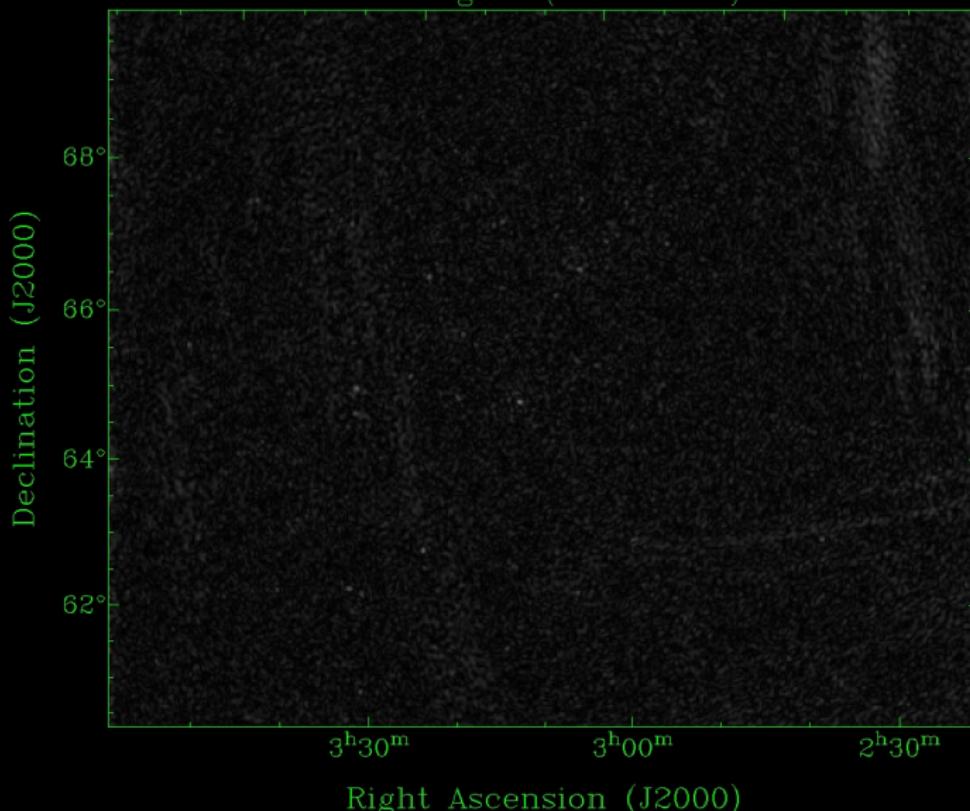
RM: 1.750000e+01

Fan region (WSRT LFFE)



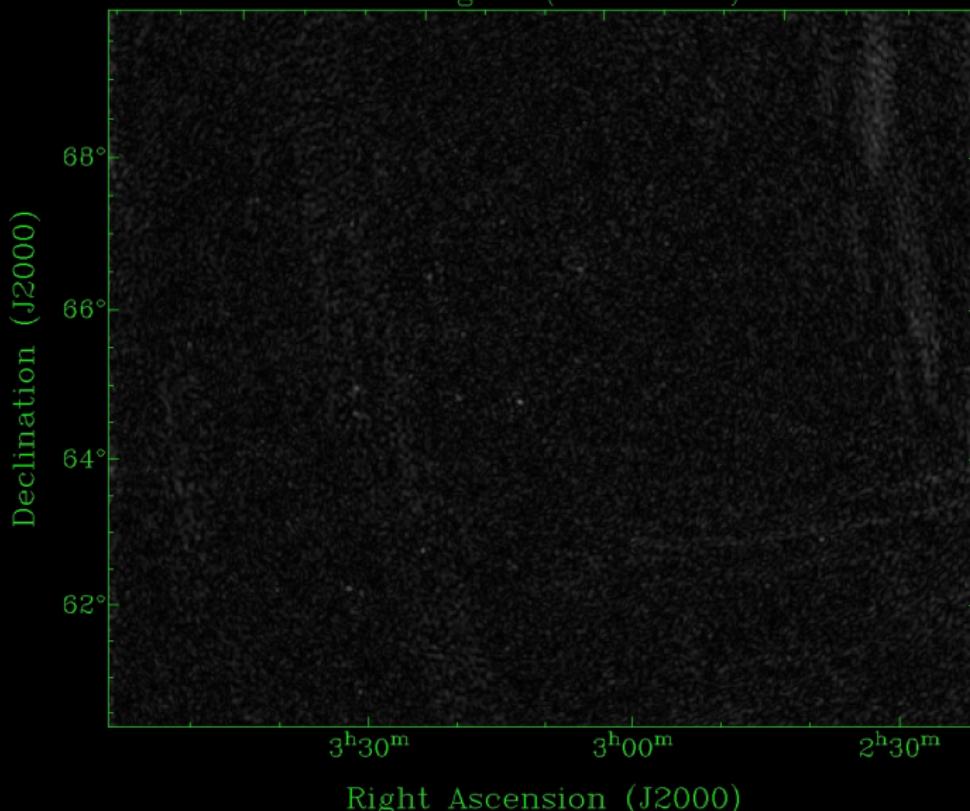
RM: 1.800000e+01

Fan region (WSRT LFFE)



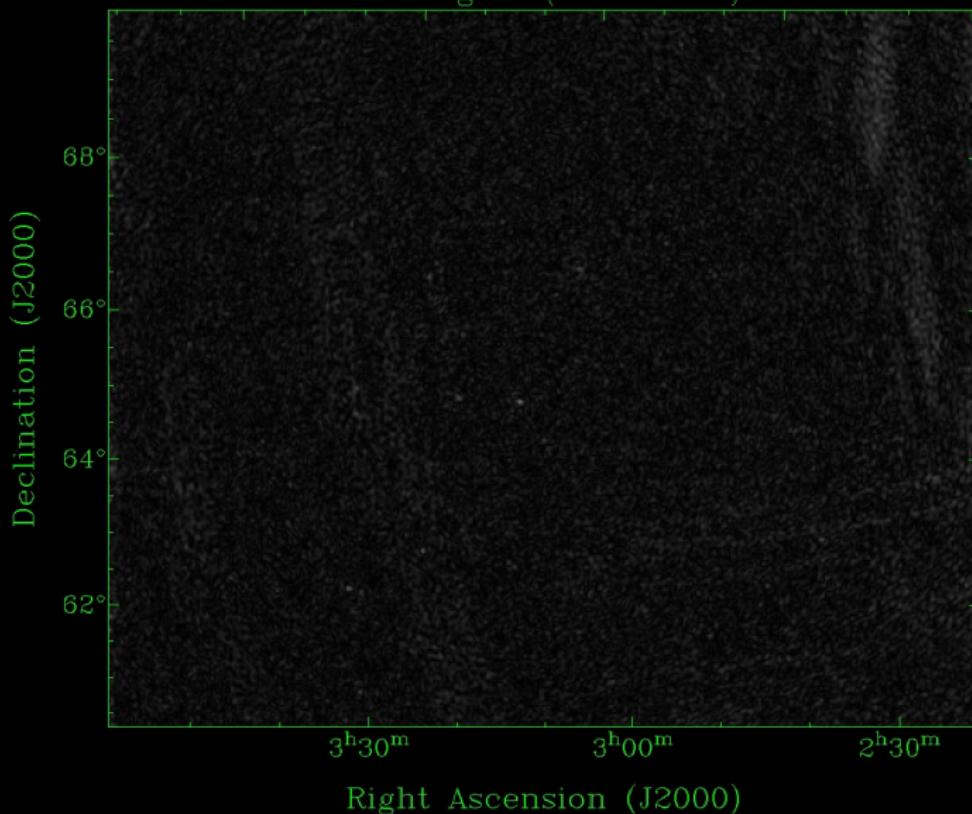
RM: 1.850000e+01

Fan region (WSRT LFFE)



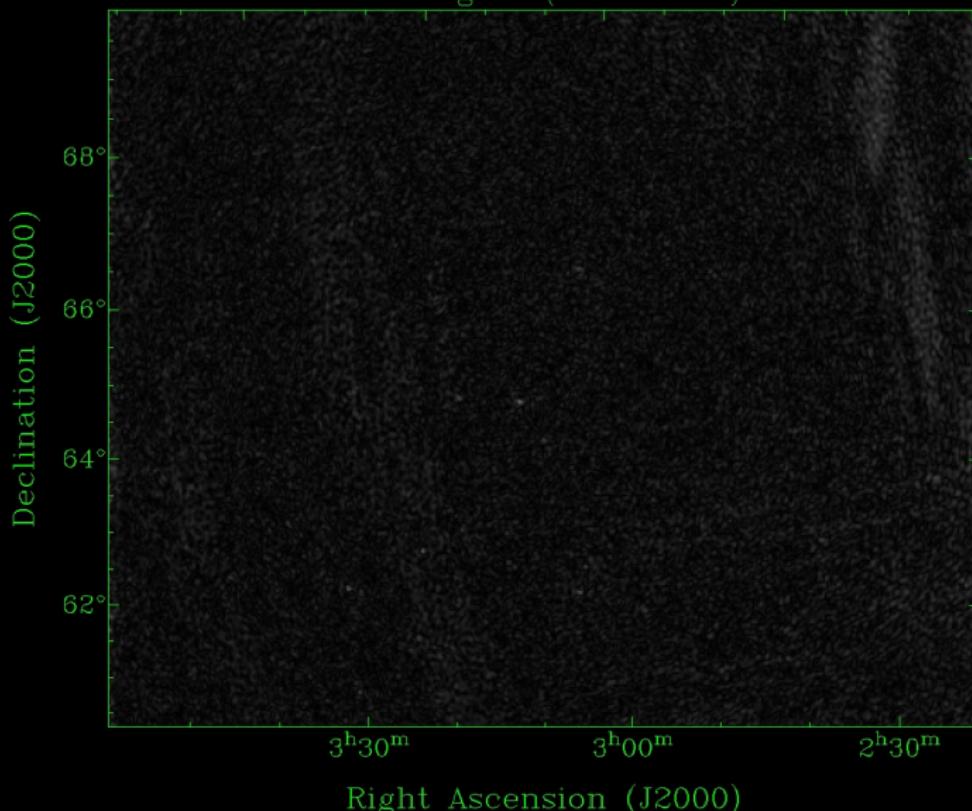
RM: 1.900000e+01

Fan region (WSRT LFFE)



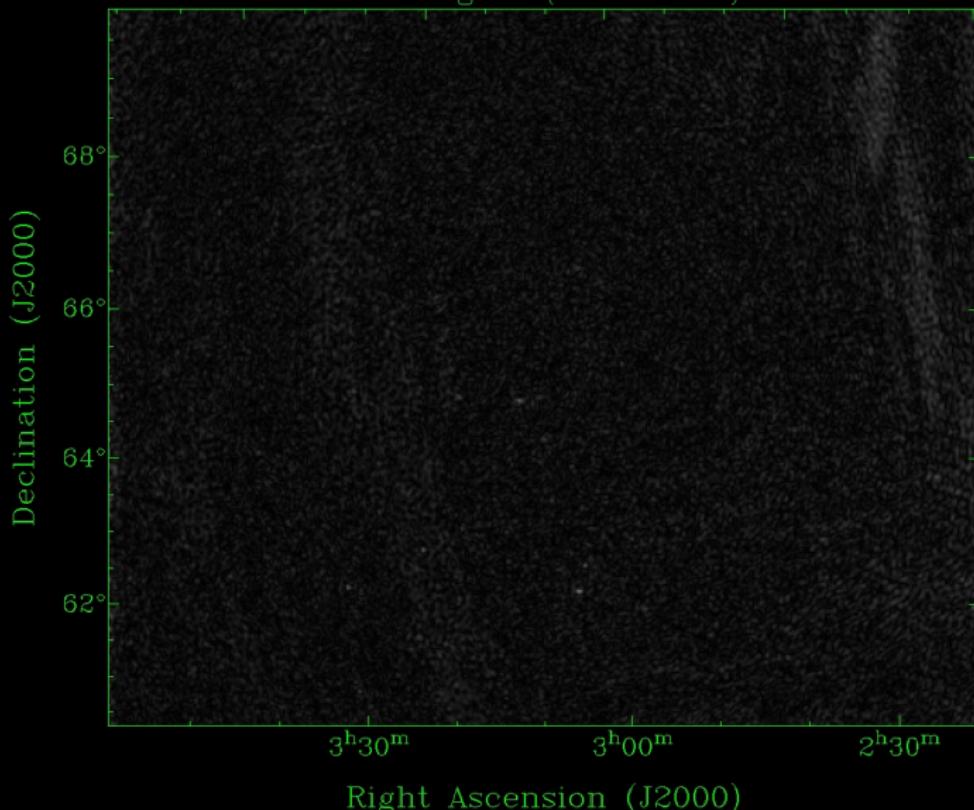
RM: 1.950000e+01

Fan region (WSRT LFFE)



RM: 2.000000e+01

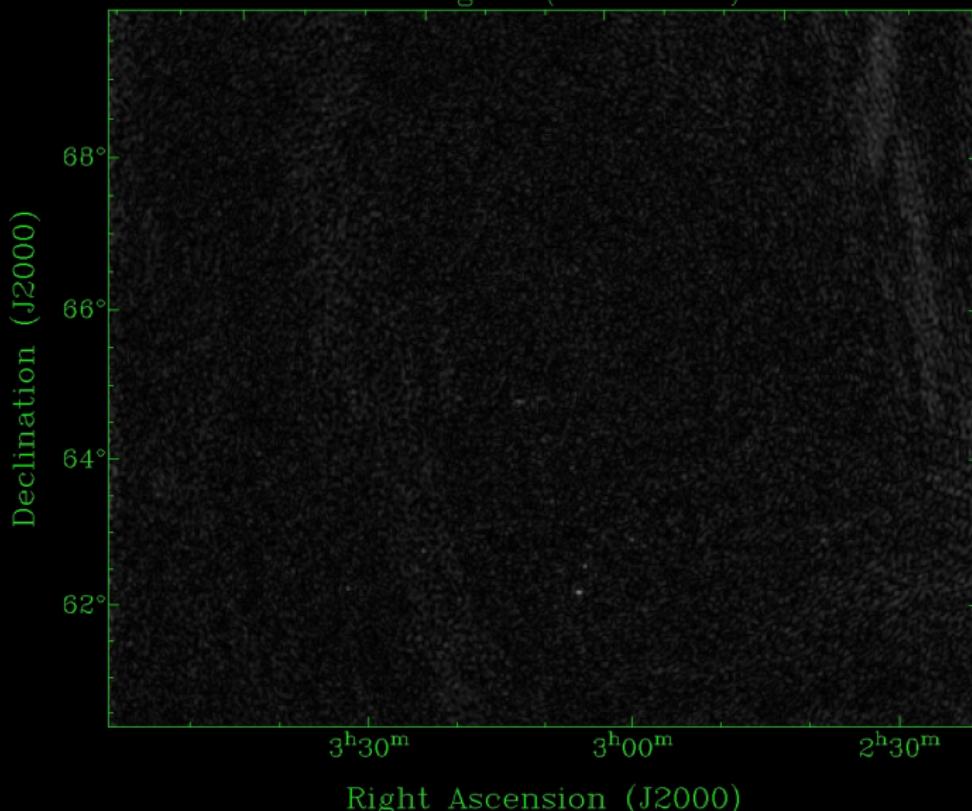
Fan region (WSRT LFFE)



# Fan region *G. Bernardi*

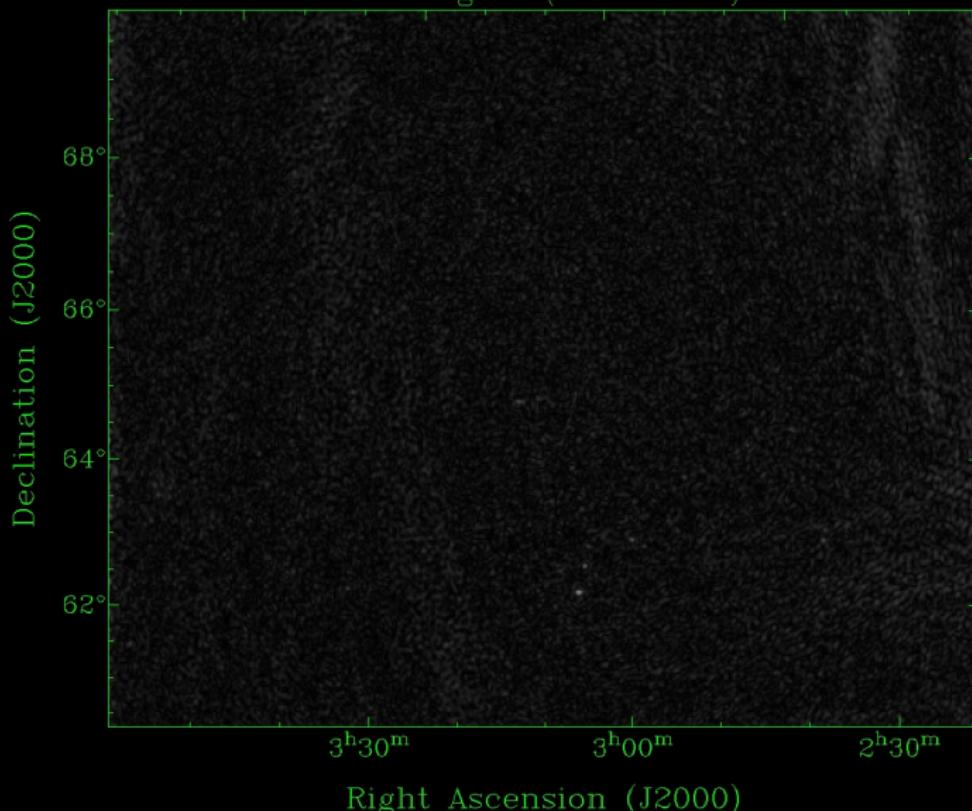
RM: 2.050000e+01

Fan region (WSRT LFFE)



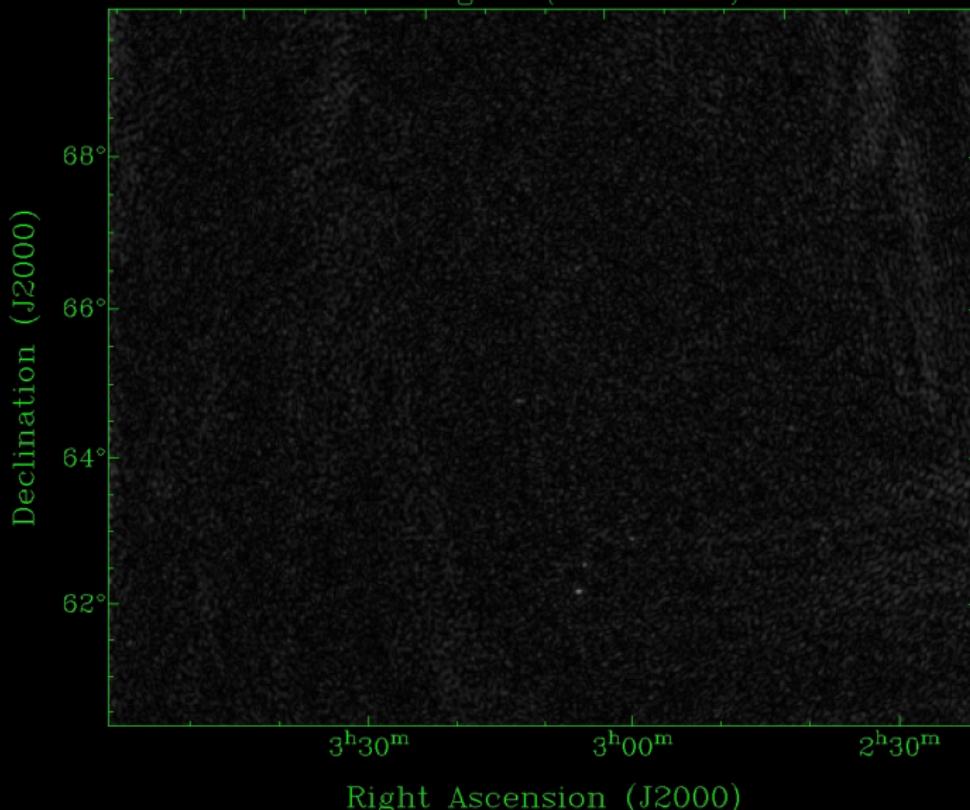
RM: 2.100000e+01

Fan region (WSRT LFFE)



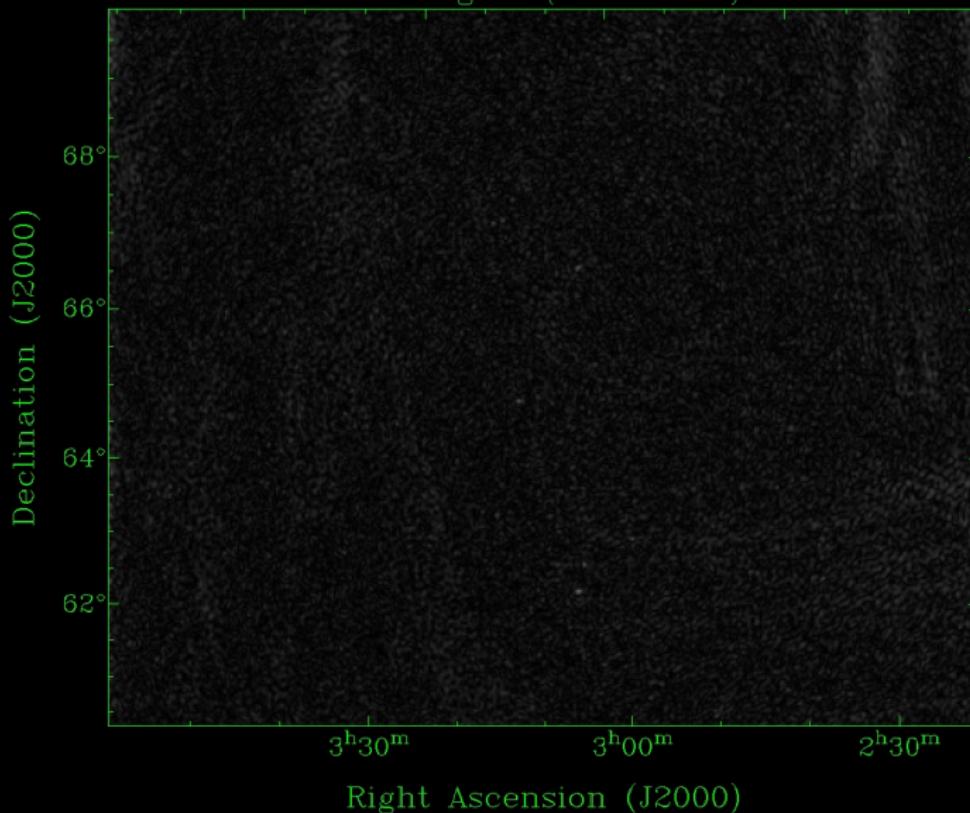
RM: 2.150000e+01

Fan region (WSRT LFFE)



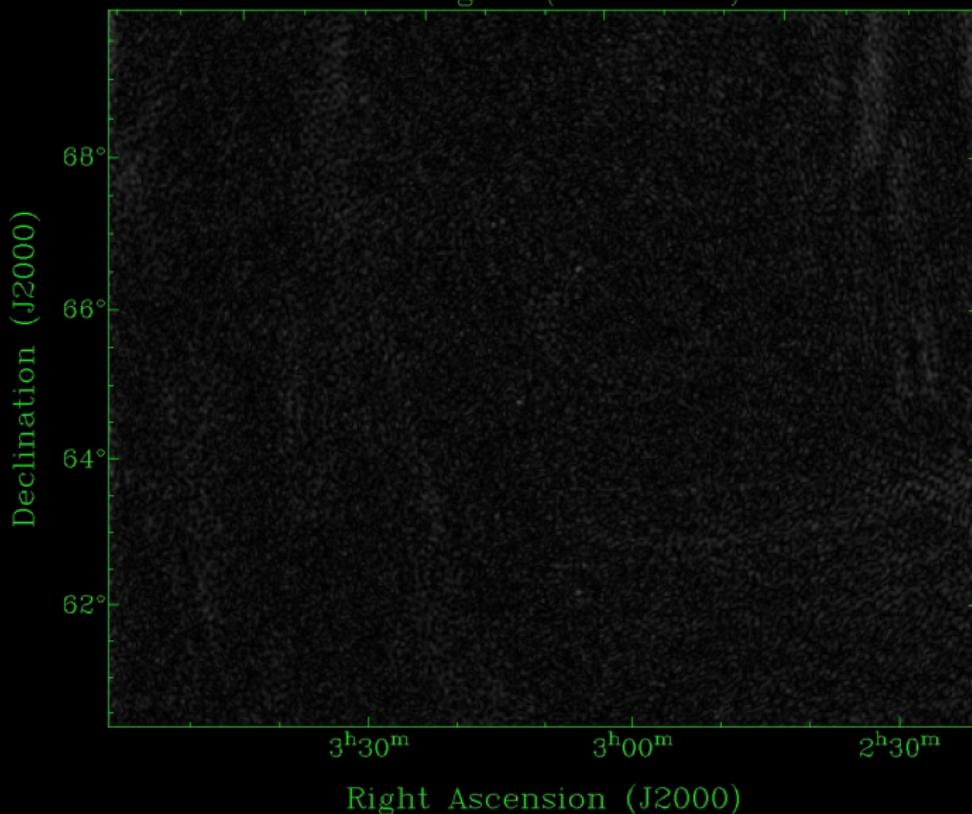
RM: 2.200000e+01

Fan region (WSRT LFFE)



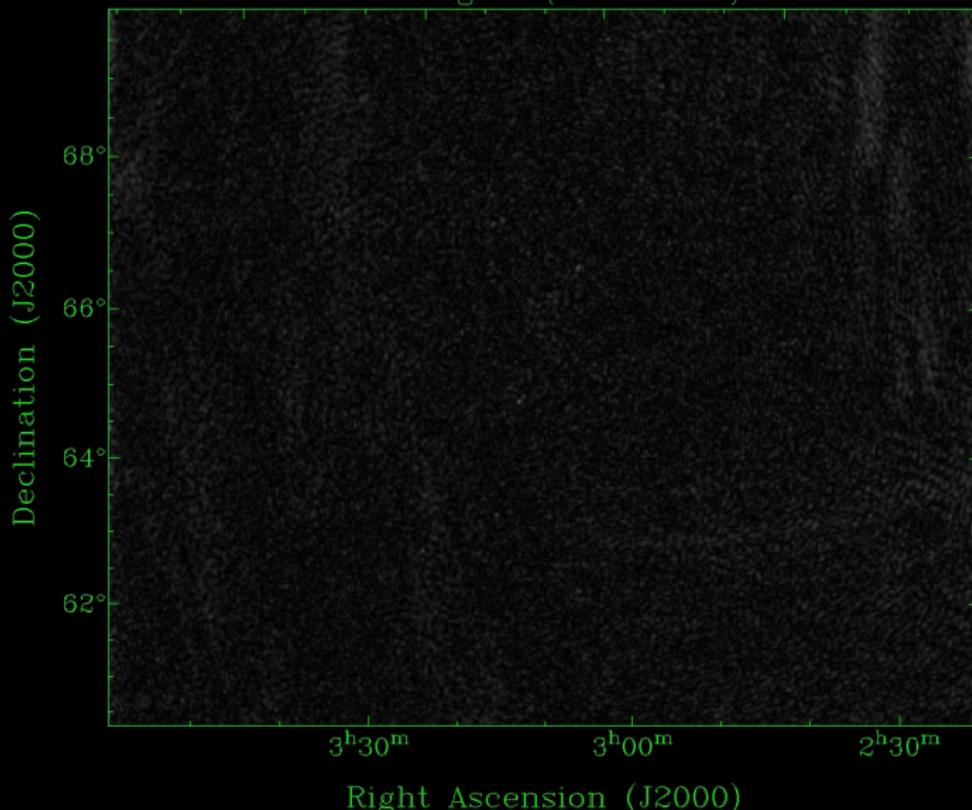
RM: 2.250000e+01

Fan region (WSRT LFFE)



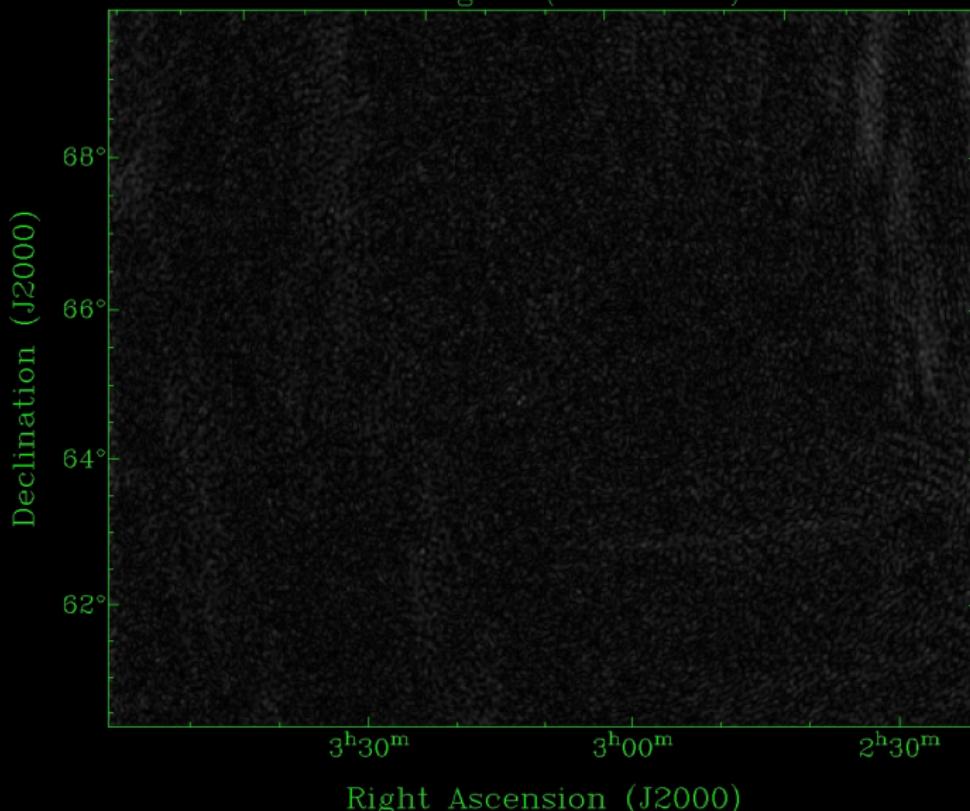
RM: 2.300000e+01

Fan region (WSRT LFFE)



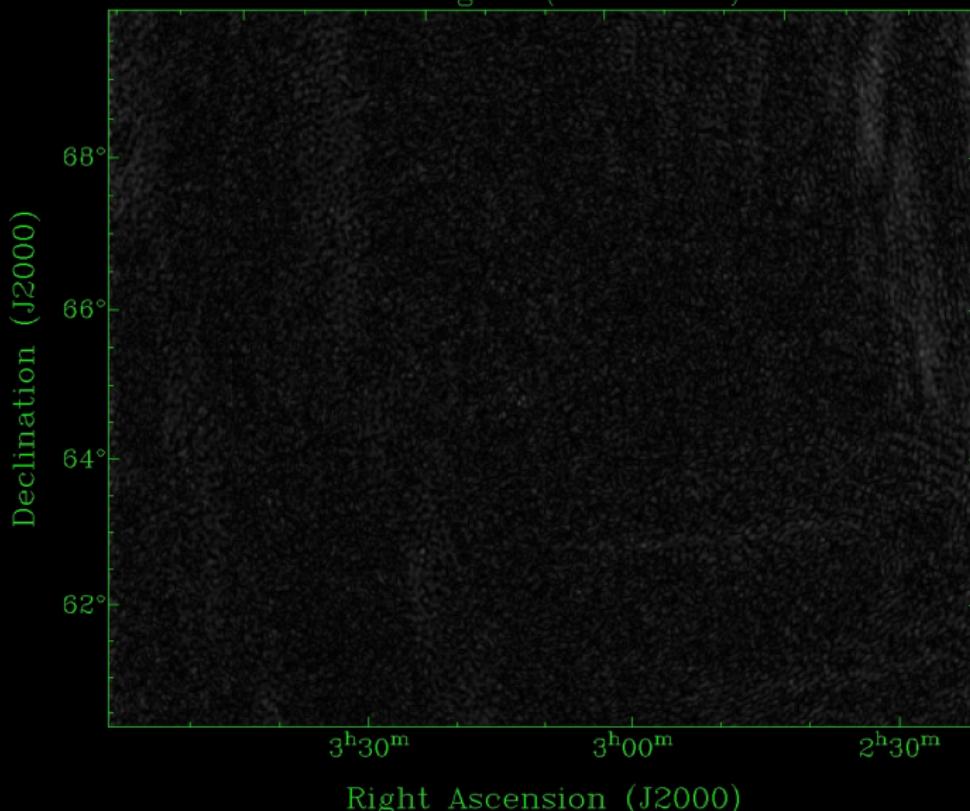
RM: 2.350000e+01

Fan region (WSRT LFFE)



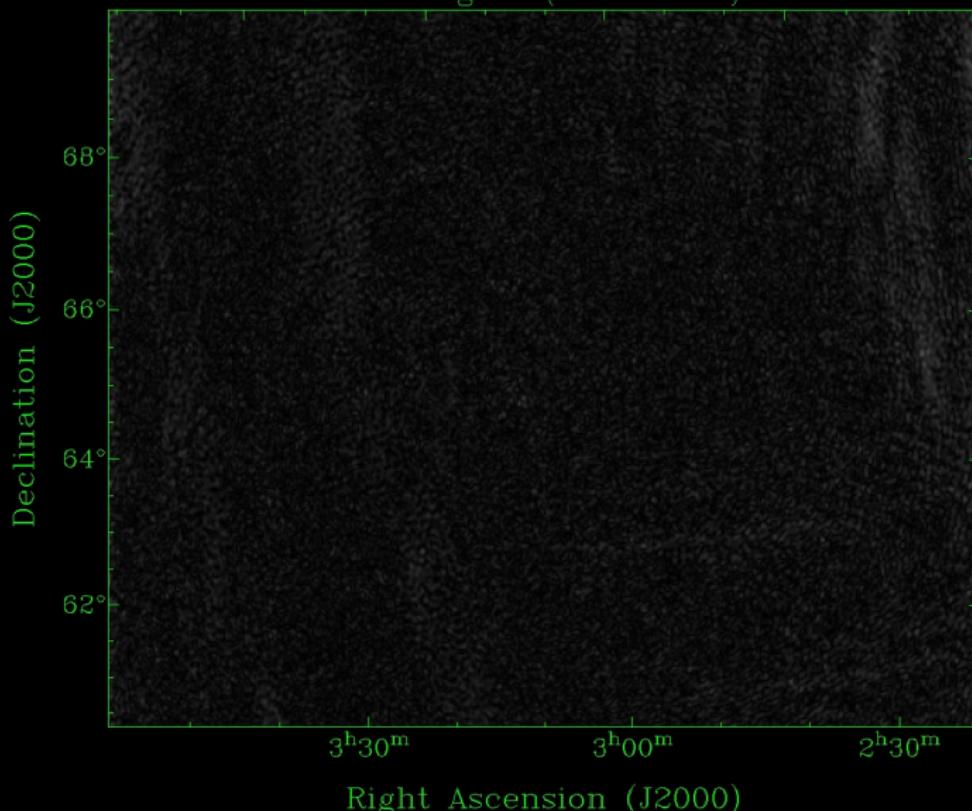
RM: 2.400000e+01

Fan region (WSRT LFFE)

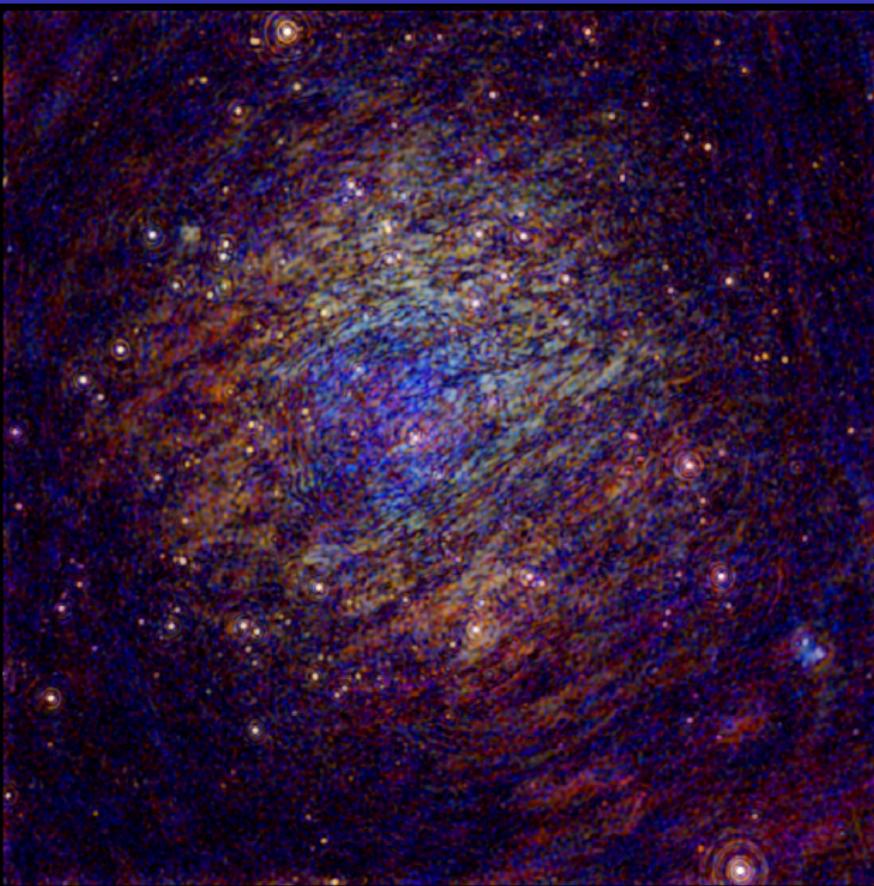


RM: 2.450000e+01

Fan region (WSRT LFFE)



# Fan region (combined RGB)



- Born & Wolf *Principles of optics*
- Thompson, Moran & Swenson *Interferometry and Synthesis in Radio Astronomy*
- Taylor, Carilli & Perley *Synthesis Imaging in Radio Astronomy II*
- Bracewell *The Fourier Transform & Its Applications*
- Hamaker, Bregman & Sault *Understanding radio polarimetry: paper I*(1996)
- Sault, Hamaker& Bregman *paper II*(1996)
- Hamaker & Bregman *paper III* (1996)
- Hamaker *paper IV* (2000)
- Hamaker *paper V* (2006)
- Brentjens & de Bruyn *Faraday rotation measure synthesis* (2005)