Error Recognition and Image Analysis Greg Taylor (UNM)

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INTRODUCTION

• Why are these two topics – 'Error Recognition' and 'Image Analysis' in the same lecture?

Error recognition is used to determine defects in the (visibility) data and image during and after the 'best' calibration, editing, etc.
Image analysis describes the many ways in which useful insight, information and parameters can be extracted from the image.
non-imaging analysis describes how to extract information directly from the (u,v) data

- Perhaps these topics are related to the reaction one has when looking at an image after 'good' calibration, editing, self-calibration, etc.
- If the reaction is:

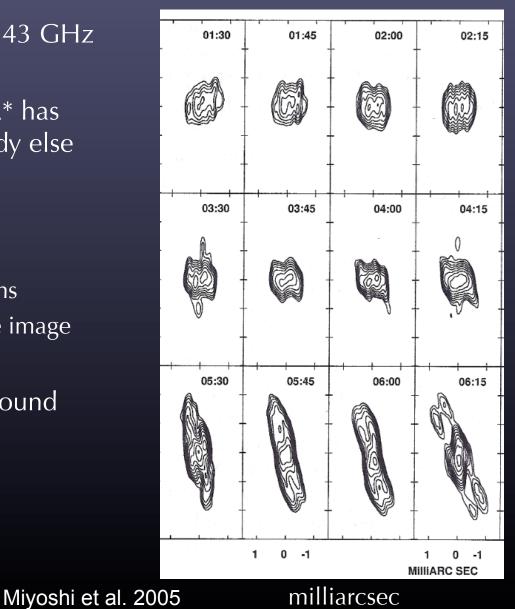
OBVIOUS IMAGE PROBLEMS

VLBA observations of SgrA* at 43 GHz

This can't be right. Either SgrA* has bidirectional jets that nobody else has ever seen or:

Clear signs of problems: Image rms > expected rms Unnatural features in the image

How can the problems be found and corrected?



HIGH QUALITY IMAGES

Reality

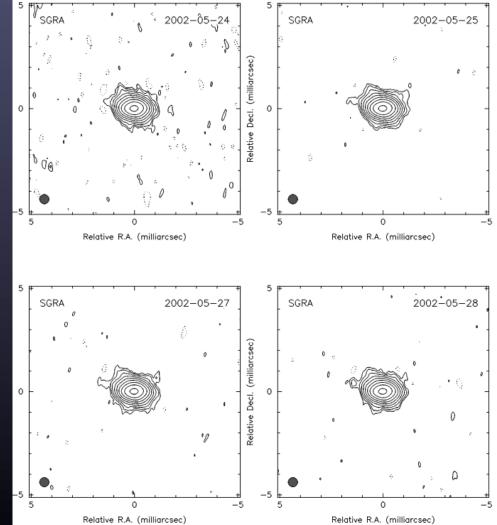
With care we can obtain good images.

What were defects?

Two antennas had ~30% calibration errors at low elevations.

This part of the lecture.

How to find the errors and remove them.



milliarcsec

GENERAL PROCEDURE

We assume that the data have been edited and calibrated reasonably successfully (earlier lectures) including self-calibration if necessary.

So, the first serious display of an image leads one-

- to inspect again and clean-up the data repeating some or all of the previous reduction steps.
 - removal of one type of problem can reveal next problem!
- once all is well, proceed to image-analysis and obtaining scientific results from the image.

But, first a digression on data and image display. First: Images

IMAGE DISPLAYS (1)

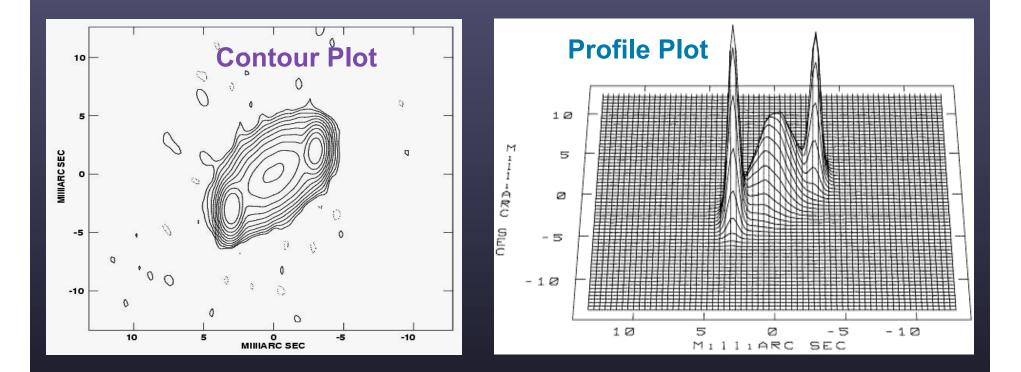
	Pixel values														lues														
				2	35				1	245				. 3	255				- 2	265				1	275				
287	0	0	0	0	û	0	0	0	0	0	0	٥	٥	Û	٥	٥	0	0	0	Ŷ	0	0	0	0	0	0	0	0	0
285	0	û	0	٥	0	û	0	٥	0	0	0	0	0	0	0	٥	0	0	0	0	1	1	0	0	٥	0	0	0	0
283	0	0	û	0	0	Û	û	0	0	0	٥	0	0	0	٥	û	٥	1	1	1	1	1	1	1	0	0	0	٥	0
281	0	Q	0	0	0	û	0	0	Û	0	0	Û	0	0	0	1	1	1	2	3	3	.3	4	3	1	0	0	0	0
279	0	ú	0	0	٥	Q	0	0	٥	0	Û	0	٥	1	1	2	3	4	- 4	5	δ	8	12	8	з	1	0	Û	0
277	0	0	0	0	0	0	0	0	0	0	٥	û	1	2	з	5	7	7	8	9	9	19	32	22	6	1	0	0	0
275	0	0	0	0	0	0	0	0	0	0	0	1	2	4	6	9	13	14	15	14	16	40	72	47	12	2	0	0	0
273	0	û	0	0	٥	û	0	0	1	1	1	2	4	8	12	17	22	23	24	22	27	77	136	87	19	2	0	٥	0
271	0	0	o	0	o	0	o	0	1	1	2	4	8	15	21	28	35	36	37	33	43	126	217:	132	28	3	0	0	0
269	0	Ô.	٥	٥	0	â	٥	0	- 1	з	4	8	15	25	34	44	54	84	53	48	611	173	298:	168	34	з	0	0	0
267	0	0	Q.	0	0	Û	Û	1	2	4	8	14	25	40	52	67	79	π	74	63	781	199	316:	177	34	з	0	٥	0
265	0	0	0	0	Q	0	0	1	3	. 7	14	24	40	60	77	97 :	1091	102	93	74	793	191	289:	155	29	з	0	0	0
263	0	Ó.	0	0	٥	Ó	1	2	δ	11	22	37	58	86:	108:	1301	137 1	123:	105	79	73:	154	220:	113	20	2	0	0	0
261	0	0	0	0	0	1	1	з	8	17	33	54	81:	116:	139:	1561	1561	133:	107	75	611	106	140	69	12	2	0	0	0
259	0	0	0	0	0	1	2	5	12	24	45	721	1051	431	62:	1701	611	131	99	66	47	64	78	36	6	1	0	0	0
257	0	Û	0	0	0	2	4	8	18	32	58	881	241	601	71:	1691	1521	118	86	85	36	36	36	16	з	1	0	٥	0
285	0	0	o	0	1	2	7	16	27	42	701	011	351	621	641	561	341	00	71	44	27	20	16	7	1	0	0	0	0
253	0	Ô.	0	0	1	4	1δ	34	43	51	771	051	1331	1501	461	351	12	81	56	34	19	11	7	3	1	0	0	0	0
251	0	0	û	0	1	8	34	73	70	59	791	001	201	1301	221	10	88	61	41	24	12	6	3	1	0	0	0	٥	0
249	0	0	0	1	2	14	69:	1411	12	65	73	871	001	106	96	83	64	43	27	14	7	3	1	1	0	0	0	0	0
247	0	Ű.	0	1	з	23:	121:	2381	67	69	62	69	77	81	70	59	42	26	16	8	3	1	0	0	0	٥	0	û	0
248	0	0	û	1	з	34:	1803	338:	217	69	48	52	86	57	47	36	25	15	8	3	1	0	0	0	0	0	0	0	0
243	0	Q	0	1	4	423	2224	022	42	68	36	37	38	37	29	21	14	7	4	1	0	0	0	0	0	0	0	0	0
241	0	ú	0	1	4	44;	2293	398:	28	56	26	25	28	22	16	11	7	з	1	0	0	0	Ū.	0	0	٥	0	û	0
239	0	0	0	1	з	39:	196:	3271	79	41	18	16	15	12	8	5	з	1	1	0	0	0	0	0	0	0	0	0	0
237	0	0	0	1	з	28:	139:	2231	18	26	11	9	8	6	4	2	1	1	0	0	0	0	0	0	0	0	0	0	0
235	0	Q.	0	Û	2	18	82:	127	64	14	6	5	4	з	1	1	1	0	0	0	0	0	Q	0	0	٥	0	٥	0
233	0	0	o	0	1	9	40	60	29	7	з	2	1	1	0	Ŷ	0	0	0	0	0	0	0	0	0	0	0	Q	0
231	0	û	0	٥	0	4	17	23	11	3	1	1	٥	0	Q	٥	0	0	û	٥	٥	0	Ô	٥	٥	0	0	0	0
229	0	Û	û	0	0	2	6	7	3	1	0	0	٥	0	0	٥	ø	0	Û	û	0	0	Û	û	0	0	0	٥	0
227	0	û	û	0	û	1	2	2	1	0	0	û	0	0	0	Û	0	0	Û	Û	0	û	Û	0	0	û	û	0	0
225	0	Ú.	0	0	0	Ū.	0	0	0	0	0	0	0	0	0	0	0	0	Ó	0	0	0	Ó	0	0	o	0	û	0
223	0	0	0	0	0	0	0	0	0	0	0	Û	0	0	0	Û	0	0	0	Û	0	0	0	Û	0	0	0	٥	0

Digital image

Numbers are proportional to the intensity

Old School

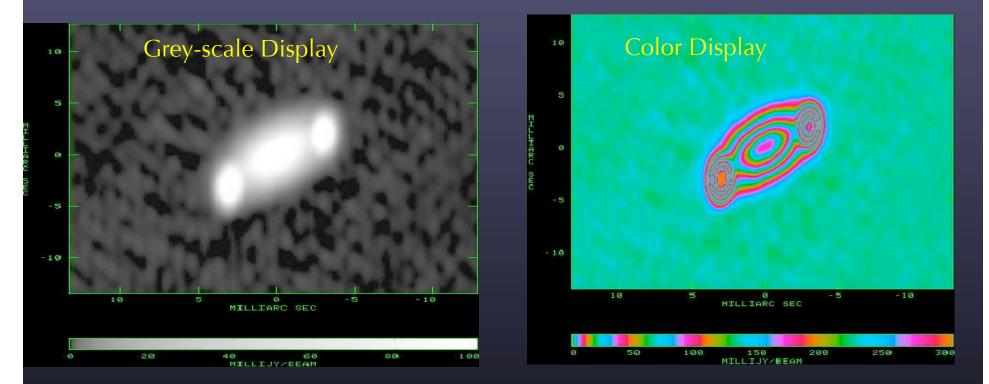
IMAGE DISPLAYS (2)



These plots are easy to reproduce and print

Contour plots give good representation of faint emission. Profile plots give a good representation of the bright emission.

IMAGE DISPLAYS (3)



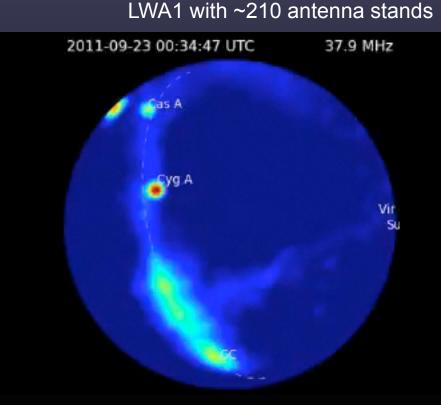
TV-based displays are most useful and interactive:

Grey-scale shows faint structure, but not good for high dynamic range and somewhat unbiased view of source Color displays more flexible; e.g. pseudo contours

Movies and Radio Frequency Interference (RFI)

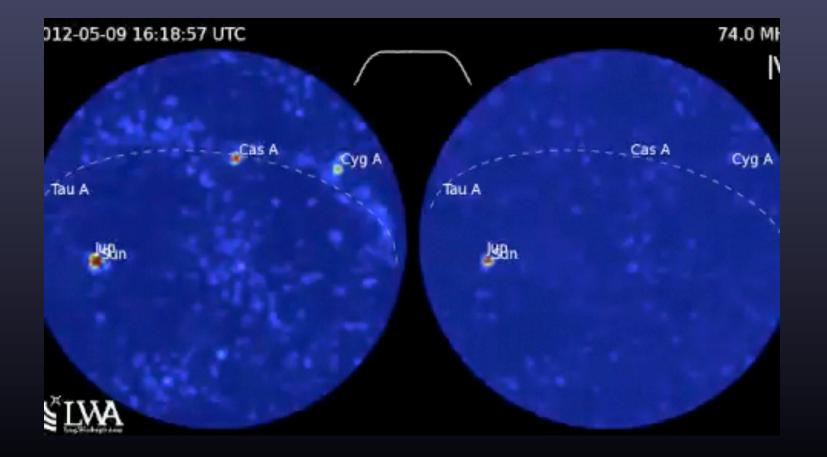
Great pressure from wireless devices Need to "free-up" 500 MHz of spectrum Gigabit networks Dynamic allocation/Shared use of spectrum Passive use is still useful!

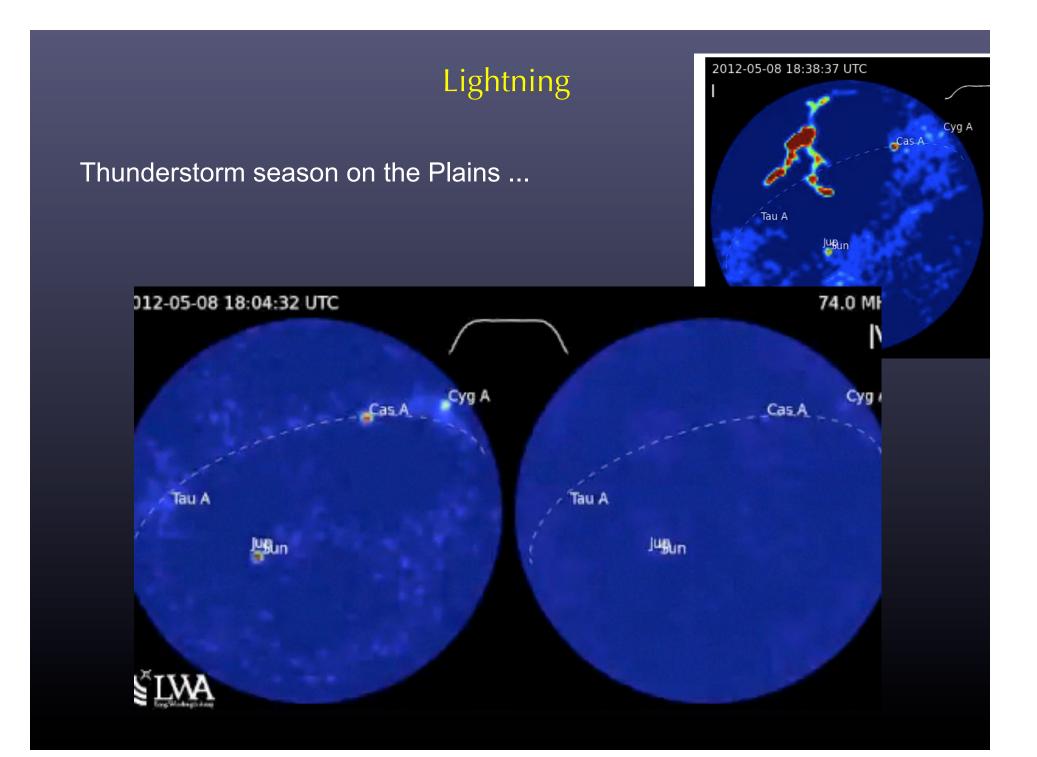
Likely changes: 1215 - 1300 MHz mobile comm. 1675 - 1710 MHz mobile comm. 1755 - 1850 MHz mobile comm. 2155 - 2200 MHz mobile comm. 4200 - 4220 MHz altimeters 4380 - 4440 MHz altimeters 5925 - 7250 level-probing-radar 14000 - 14500 air to ground 15400 - 15700 radar 76000 - 77000 automobile radar



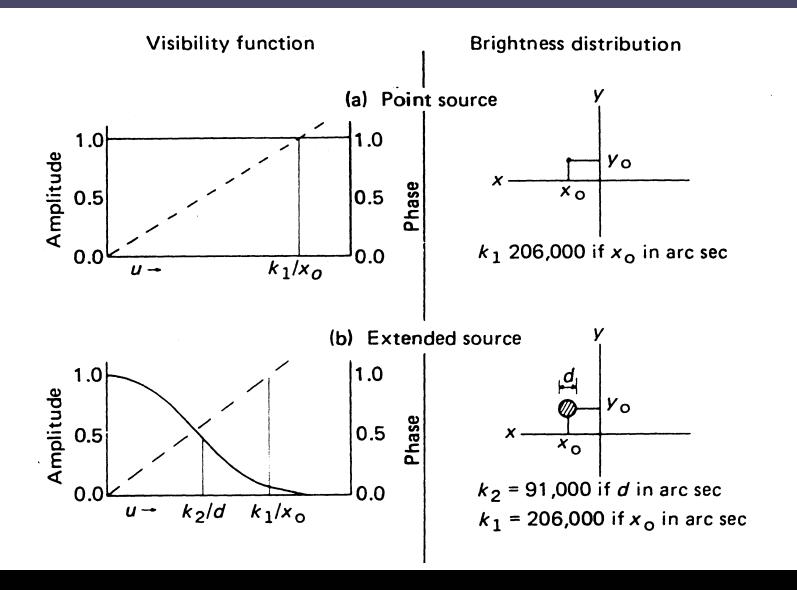
Movies and Solar Interference

Heading into Solar Maximum so watch out for the Active Sun

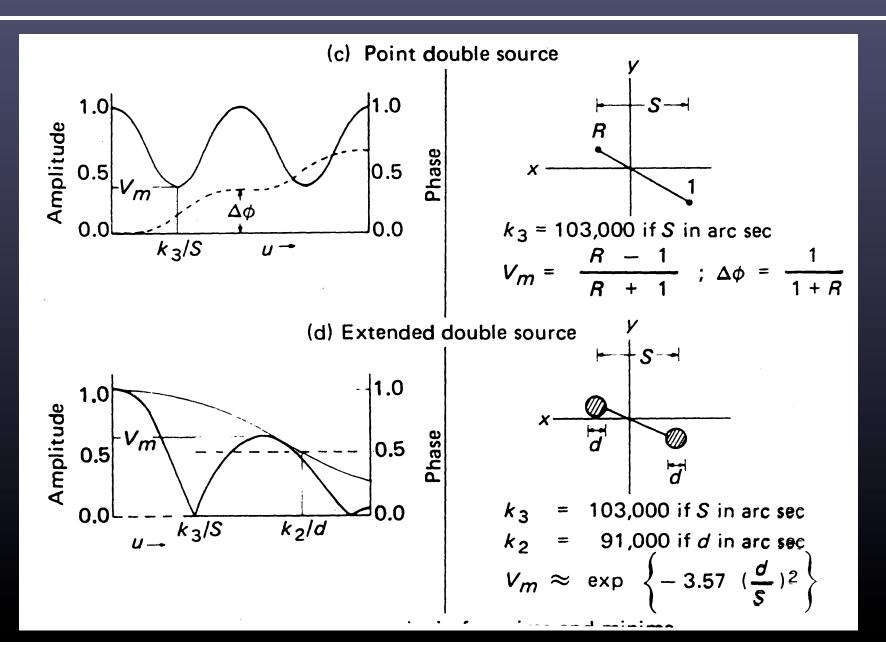




Visibility \leftarrow F \rightarrow Brightness



Visibility \leftarrow F \rightarrow Brightness

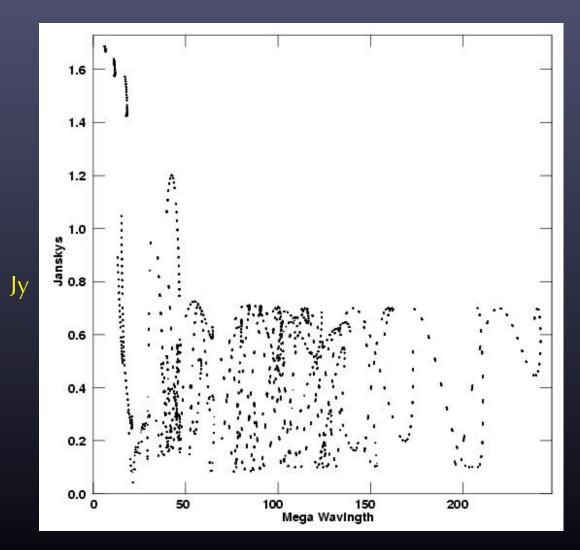


DATA DISPLAYS(1)

List of (u,v) Data

Source=	J0121+11	Freq=	8.	434	4858511	Sort	= TB	1	RR	
Vis #	IAT	Ant	Su	Fq	U(klam)	V(klam)	W(klam)	Amp	Phas	Wt
2191	0/22:35:08.2	2 5- 6	1	0	94220	23776	100371	0.614	-16	1.0000
3971	0/22:43:43.34	1 5-6	1	0	97659	24517	96844	0.508	-13	1.0000
6431	0/23:07:05.1	5 5-6	1	0	106307	26661	86632	0.154	17	1.0000
6611	0/23:07:14.94	8 5-6	1	0	106364	26677	86557	0.152	17	1.0000
6791	0/23:07:24.8	1 5-6	1	0	106421	26692	86483	0.150	18	1.0000
6971	0/23:07:34.64	1 5- 6	1	0	106477	26708	86408	0.148	19	1.0000
7151	0/23:07:44.4	7 5- 6	1	0	106534	26724	86333	0.146	19	1.0000
7331	0/23:07:54.30	0 5-6	1	0	106591	26739	86259	0.144	20	1.0000
7511	0/23:15:06.84	4 5- 6	1	0	109027	27438	82930	0.101	74	1.0000
7691	0/23:15:16.6	7 5-6	1	0	109081	27454	82854	0.101	75	1.0000
7871	0/23:15:26.5	0 5- 6	1	0	109135	27470	82777	0.102	77	1.0000
8051	0/23:15:36.3	3 5-6	1	0	109189	27486	82701	0.102	78	1.0000
8231	0/23:15:46.10	6 5-6	1	0	109243	27502	82624	0.103	79	1.0000
8411	0/23:15:55.99	9 5- 6	1	0	109297	27518	82547	0.104	81	1.0000
9701	0/23:31:02.30	5 5- 6	1	0	114020	29035	75322	0.260	134	1.0000
9791	0/23:31:06.25	9 5- 6	1	0	114040	29042	75290	0.261	134	1.0000
10301	0/23:31:29.84	8 5-6	1	0	114156	29082	75098	0.266	134	1.0000
10861	0/23:39:02.04	8 5-6	1	0	116320	29863	71379	0.348	139	1.0000
10951	0/23:39:06.0	1 5- 6	1	0	116339	29870	71346	0.348	139	1.0000
11171	0/23:39:15.84	4 5- 6	1	0	116384	29887	71264	0.350	139	1.0000

Very primitive display, but sometimes worth-while: e.g., can search on e.g. Amp > 1.0, or large weight. Often need precise times in order to flag the data appropriately.



DATA DISPLAYS(2)

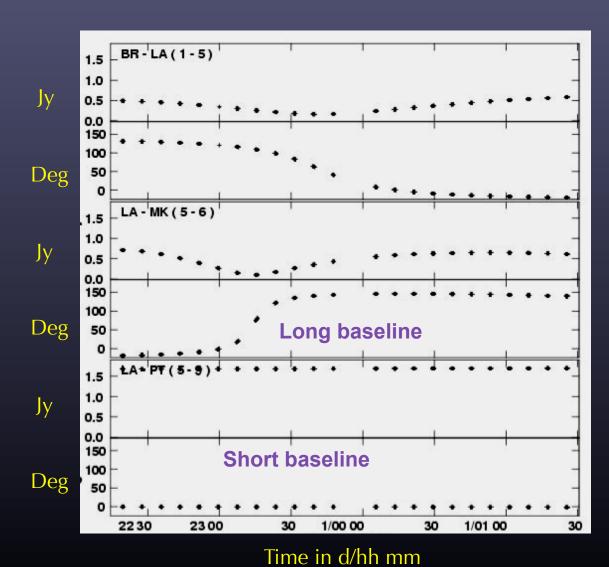
Visibility Amplitude versus Projected (u,v) spacing

General trend of data. Useful for relatively strong sources.

Triple source model. Large component cause rise at short spacings.

Oscillations at longer spacings suggest close double.

Mega Wavelength



DATA DISPLAYS(3)

Visibility amplitude and phase versus time for various baselines

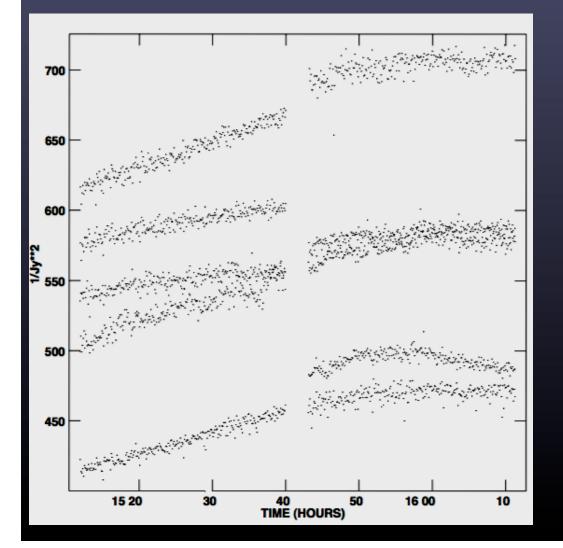
Good for determining the continuity of the data

Should be relatively smooth with time

Outliers are obvious.

DATA DISPLAYS(4)

Weights of antennas 4 with 5,6,7,8,9



All (u,v) data points have a weight. The weight depends on the antenna sensitivity, measured during the observations

The amplitude calibration values also modify the weights.

Occasionally the weight of the points become very large, often caused by subtle software bugs.

A large discrepant weight causes the same image artifacts as a large discrepant visibility value.

Please check weights to make sure they are reasonable.

DATA DISPLAYS(5) – Amplitude vs Phase

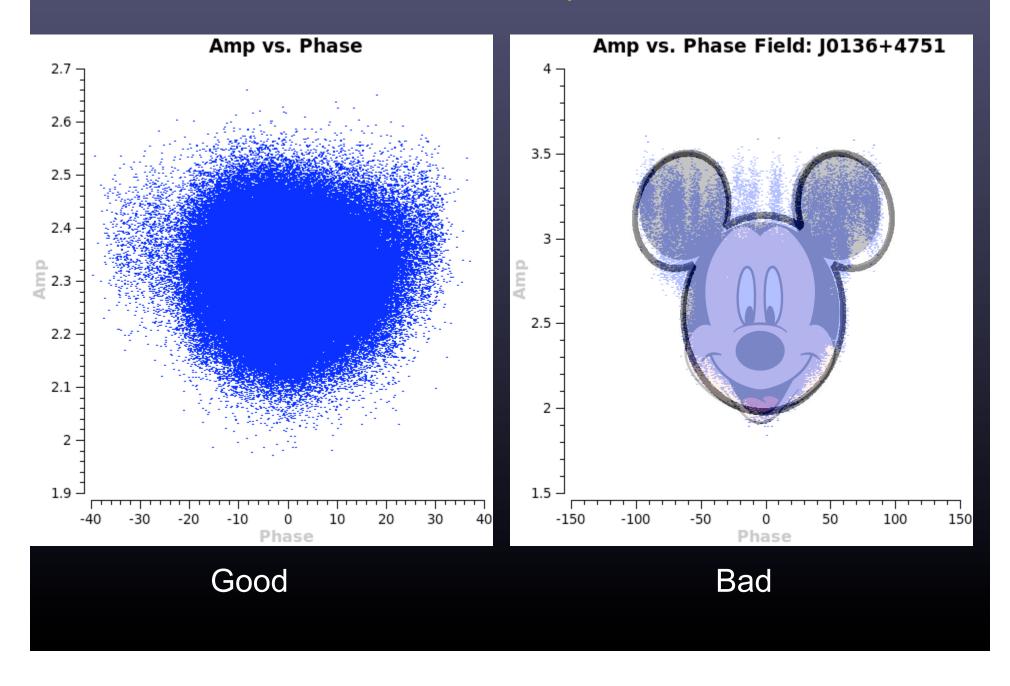


IMAGE PLANE OR DATA (U,V) PLANE INSPECTION?

Errors obey Fourier transform relationship

Narrow feature in (u,v) plane $\langle -\rangle$ wide feature in image plane Wide feature in (u,v) plane <-> narrow feature in image plane Note: easier to spot narrow features Data (u,v) amplitude errors <->symmetric image features Data (u,v) phase errors <-> asymmetric image features An obvious defect may be hardly visible in the transformed plane A small, almost invisible defect may become very obvious in the transformed plane Noise bumps can have sidelobes!

FINDING ERRORS

---Obvious outlier data (u,v) points:

100 bad points in 100,000 data points gives an 0.1% image error (unless the bad data points are 1 million Jy)

LOOK at DATA to find gross problem (you'd be hard pressed to find it in the image plane other than a slight increase in noise) ---Persistent small data errors:

e.g. a 5% antenna gain calibration error is difficult to see
in (u,v) data (not an obvious outlier), but will produce a
1% effect in image with specific characteristics (more later).
USE IMAGE to discover problem

---Non-Data Problems:

Data ok, but algorithms chosen aren't up to the task.

ERROR RECOGNITION IN THE (u,v) PLANE

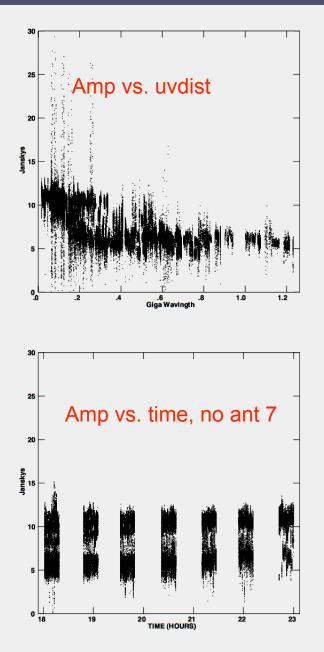
Editing obvious errors in the (u,v) plane

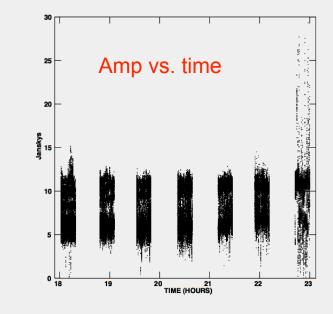
---Mostly consistency checks assume that the visibility cannot change much over a small change in (u,v) spacing

---Also, look at gains and phases from calibration processes. These values should be relatively stable.

See Summer school lecture notes in 2002 by Myers See ASP Vol 180, Ekers, Lecture 15, p321

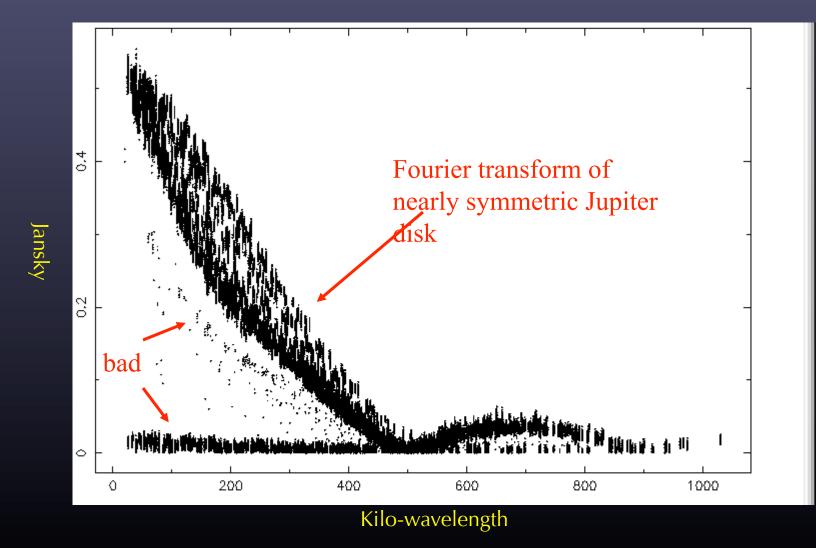
VISIBILITY AMPLITUDE PLOTS





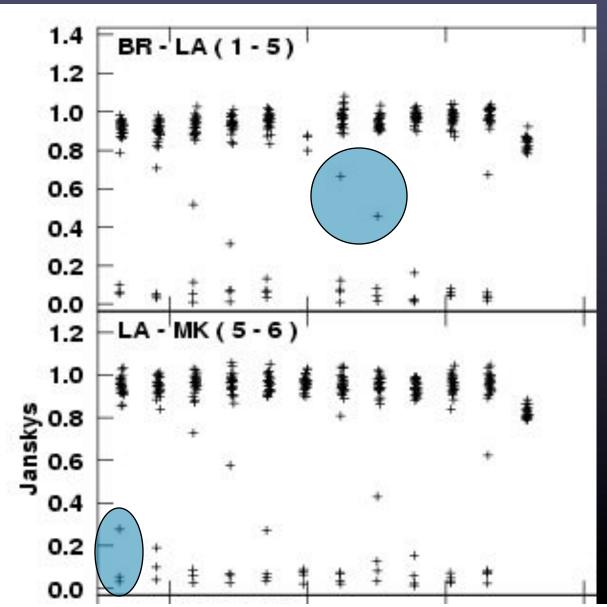
Amp vs. uvdist shows outliers Amp vs. time shows outliers in last scan Amp vs. time without ant 7 shows good data (3C279 VLBA data at 43 GHz)

Example Edit – plotms (2)



Butler lecture: Solar System Objects

Drop-outs at Scan Beginnings



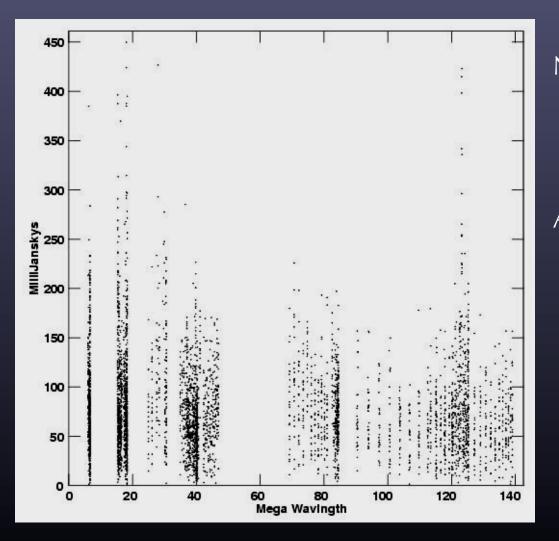
Often the first few points of a scan are low. E.g. antenna not on source.

Software can remove these points (aips,casa 'quack')

Flag extension:

Should flag all sources in the same manner even though you cannot see dropout for weak sources

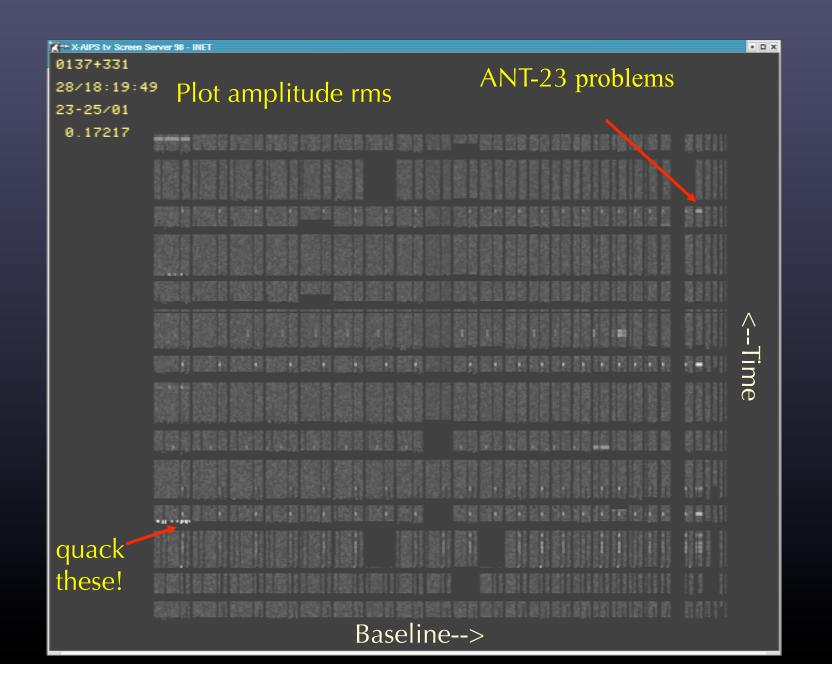
Editing Noise-dominated Sources



No source structure information is detected. Noise dominated.

All you can do is quack and remove outlier points above ~3sigma (0.3 Jy). Precise level not important as long as large outliers are removed.

USING TVFLG (VIEWER) DISPLAY on a source



35 km 12 km 3 km baseline Frequency **AIPS: SPFLG**

Time

after before

RFI environment worse on short baselines

RFI Excision

Several 'types': narrow band, wandering, wideband, ...

Wideband interference hard for automated routines

AIPS tasks FLGIT, FLAGR, SPFLG and CASA flagdata, mode='rfi'

Automation is crucial for WIDAR (wide band, lots of data)

ERROR RECOGNITION IN THE IMAGE PLANE Some Questions to ask:

Noise properties of image:

Is the rms noise about that expected from integration time? Is the rms noise much larger near bright sources? Are there non-random noise components (faint waves and ripples)?

Funny looking Structure:

Non-physical features; stripes, rings, symmetric or anti-symmetric Negative features well-below 4xrms noise Does the image have characteristics that look like the dirty beam?

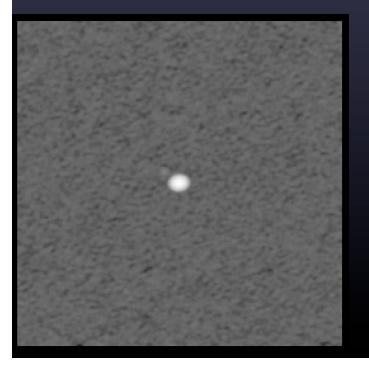
Image-making parameters:

Is the image big enough to cover all significant emission? Is cell size too large or too small? ~4 points per beam okay Is the resolution too high to detect most of the emission?

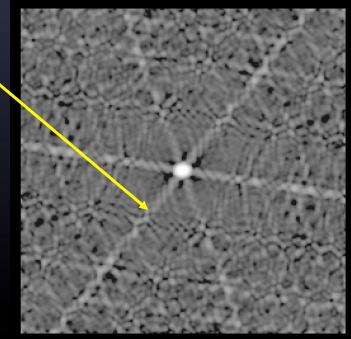
EXAMPLE 1 Data bad over a short period of time

Results for a point source using VLA. 13 x 5min observation over 10 hr. Images shown after editing, calibration and deconvolution.

no errors: max 3.24 Jy rms 0.11 mJy



6-fold symmetric pattern due to VLA "Y". Image has properties of dirty beam. 10% amp error for all antennas for 1 time period rms 2.0 mJy



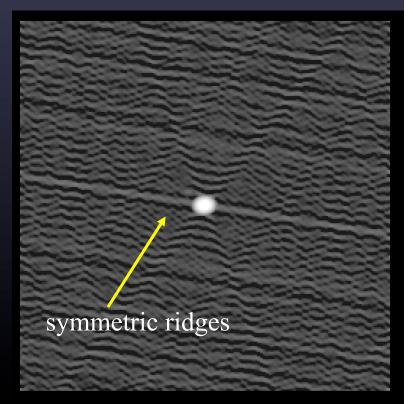
EXAMPLE 2 Short burst of bad data

Typical effect from one bad antenna

10 deg phase error for one antenna at one time rms 0.49 mJy

anti-symmetric ridges

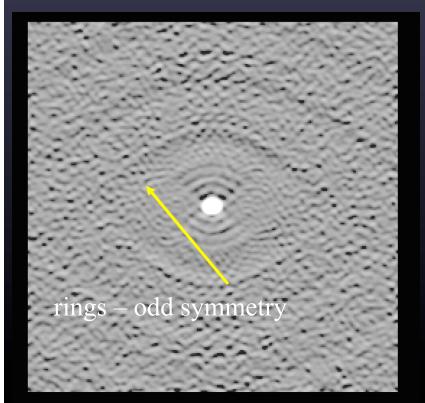
20% amplitude error for one antenna at one time rms 0.56 mJy (self-cal)



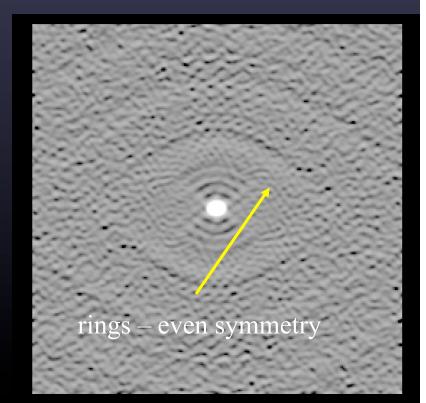
EXAMPLE 3 Persistent errors over most of observations

NOTE: 10 deg phase error to 20% amplitude error cause similar sized artifacts

10 deg phase error for one antenna all times rms 2.0 mJy



20% amp error for one antenna all times rms 2.3 mJy



EXAMPLE 4 Spurious Correlator Offset Signals

Occasionally correlators produce ghost signals or cross talk signals Occurred during change-over from VLA to EVLA system

Symptom: Garbage near phase center, dribbling out into image

Image with correlator offsets

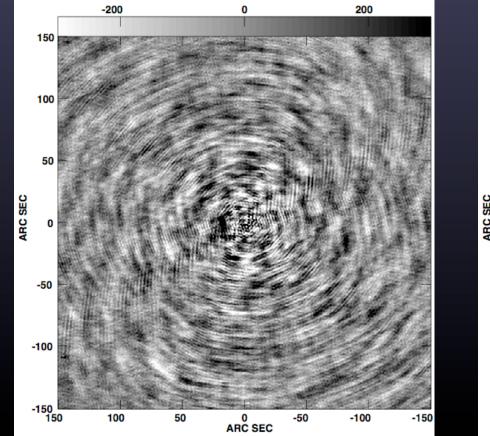
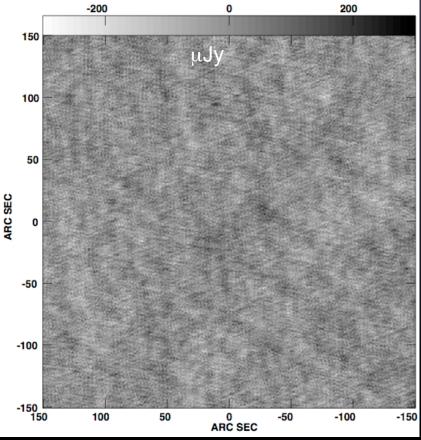
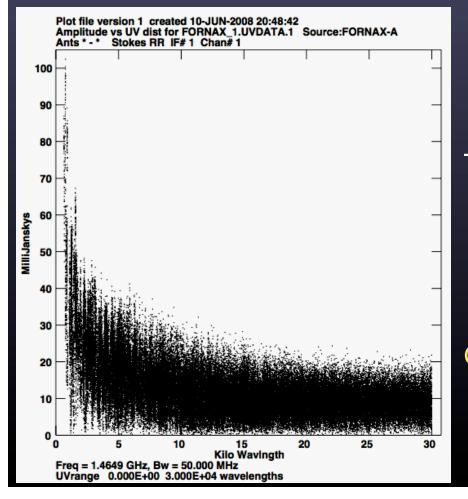


Image after correction of offsets



DECONVOLUTION ERRORS

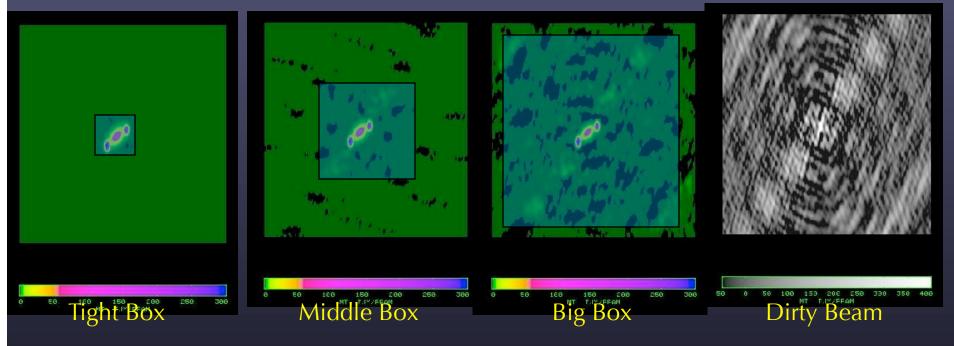
Even if the data are perfect, image errors and uncertainties will occur because the (u,v) coverage is not adequate to map the source structure.



The extreme rise of visibility at the short spacings makes it impossible to image the extended structure. You are better of imaging the source with a cutoff below about 2 kilo-wavelengths

Get shorter spacing or single-dish data

CLEANING WINDOW SENSITIVITY



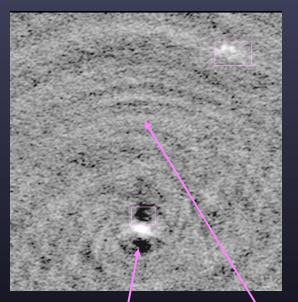
One small clean box One clean box around all emission

Clean entire inner map quarter

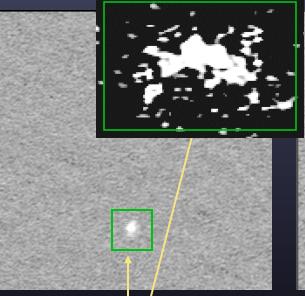
Make box as small as possible to avoid cleaning noise interacting with sidelobes

How Deep to Clean?

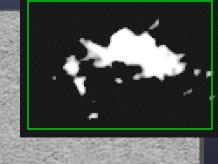
Under-cleaned



Over-cleaned



Properly cleaned





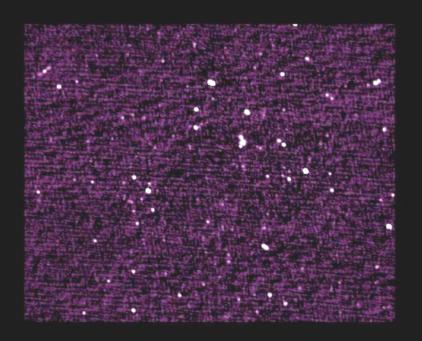
Residual sidelobes dominate the noise

Emission from second source sits atop a negative "bowl" Regions within clean boxes appear "mottled" Background is thermal noise-dominated; no "bowls" around sources.

Improvement of Image

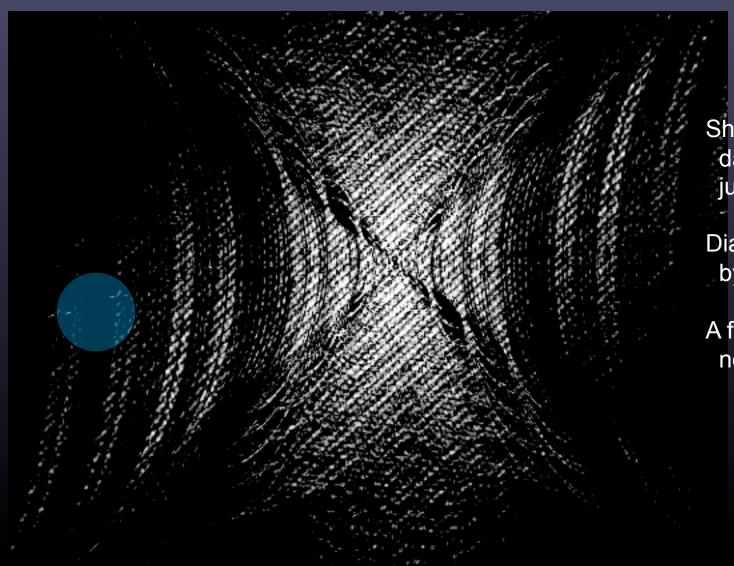
Removal of low level ripple improves detectability of faint sources

Before editing



After editing

Fourier Transform Dirty Image

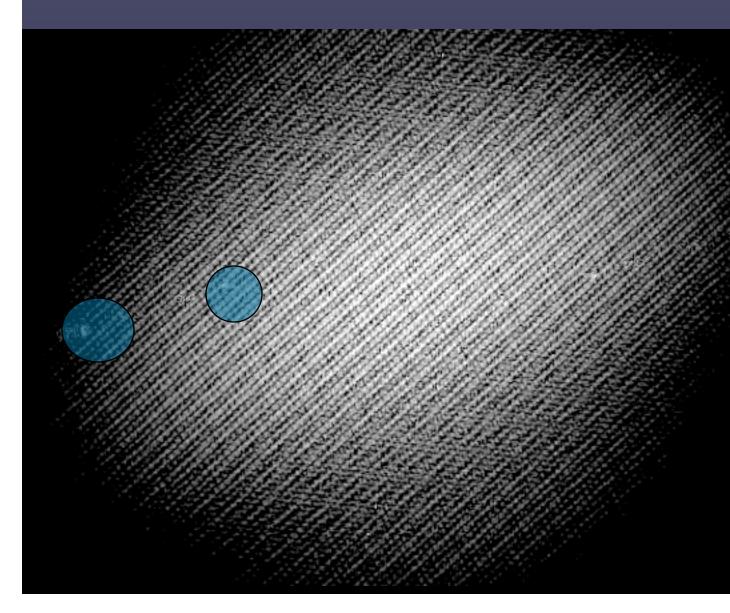


Shows the (u,v) data as gridded just before imaging

Diagonal lines caused by structure in field

A few odd points are not very noticeable

Fourier Transform Clean Image

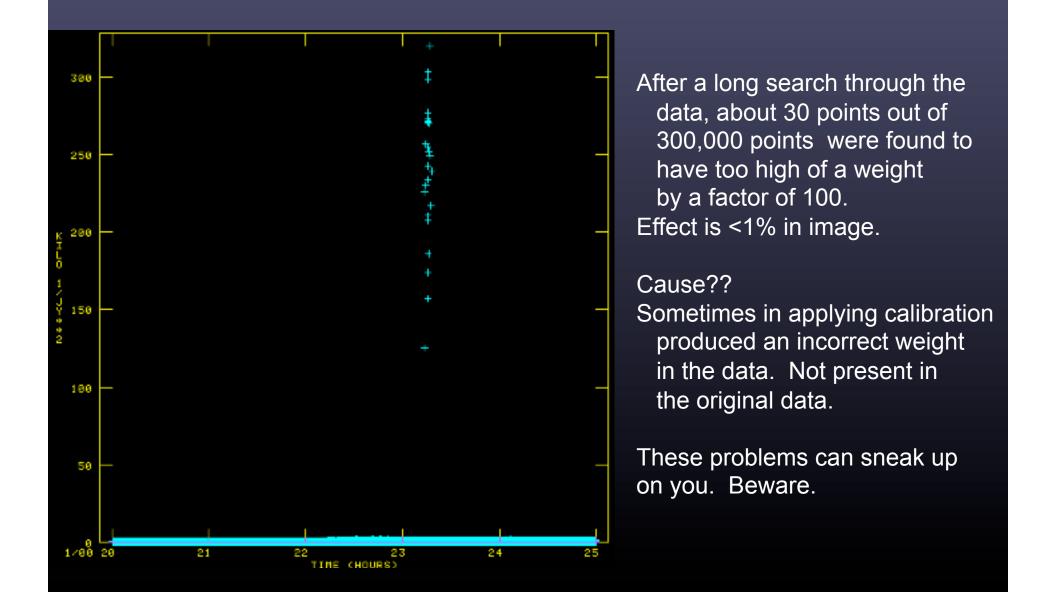


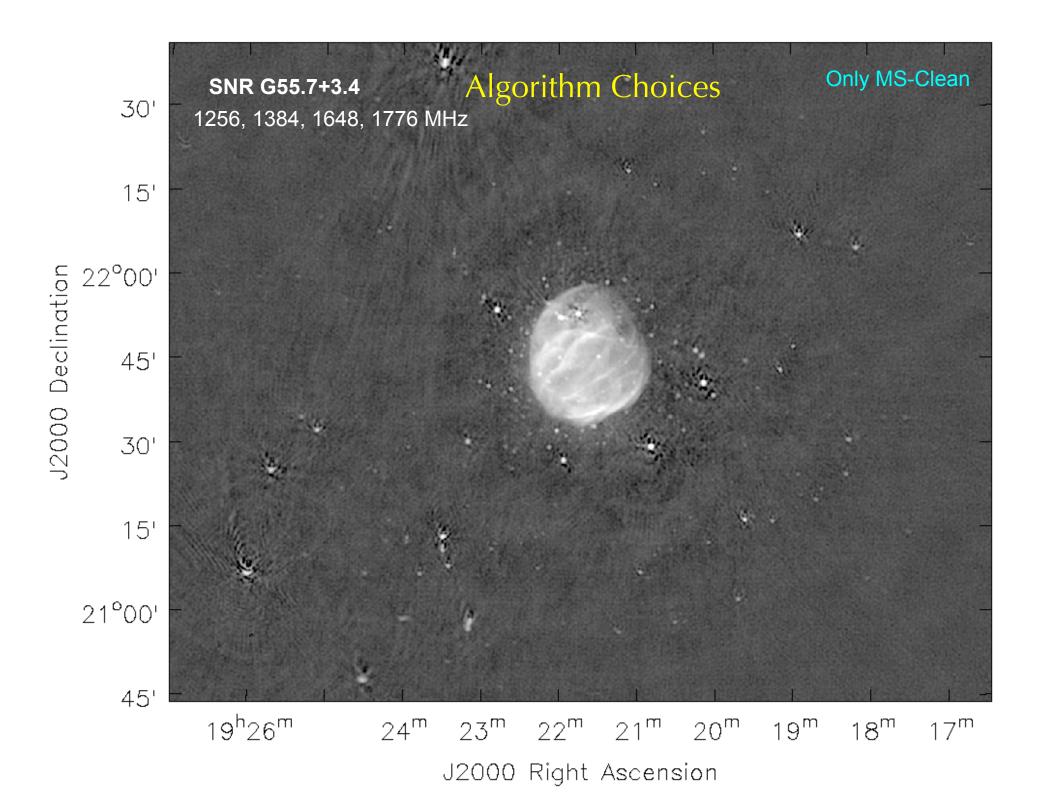
Shows the (u,v) data from clean image.

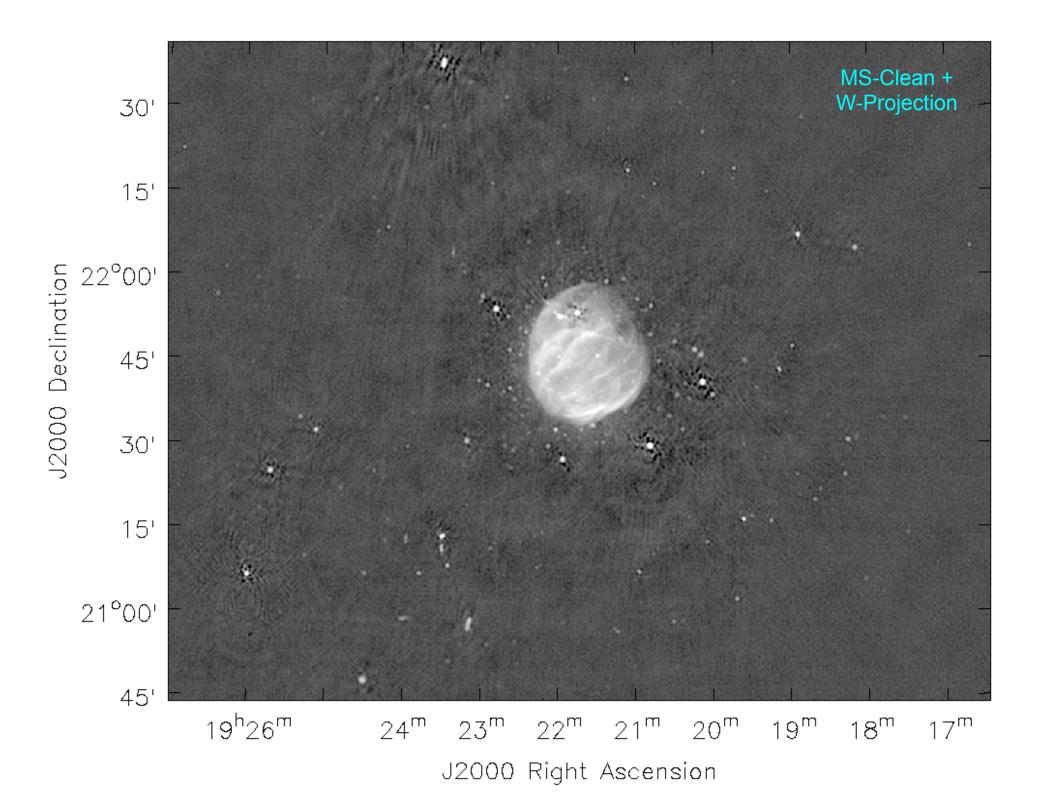
Diagonal lines still present. Notice that clean does an interpolation in the u,v plane between u,v tracks.

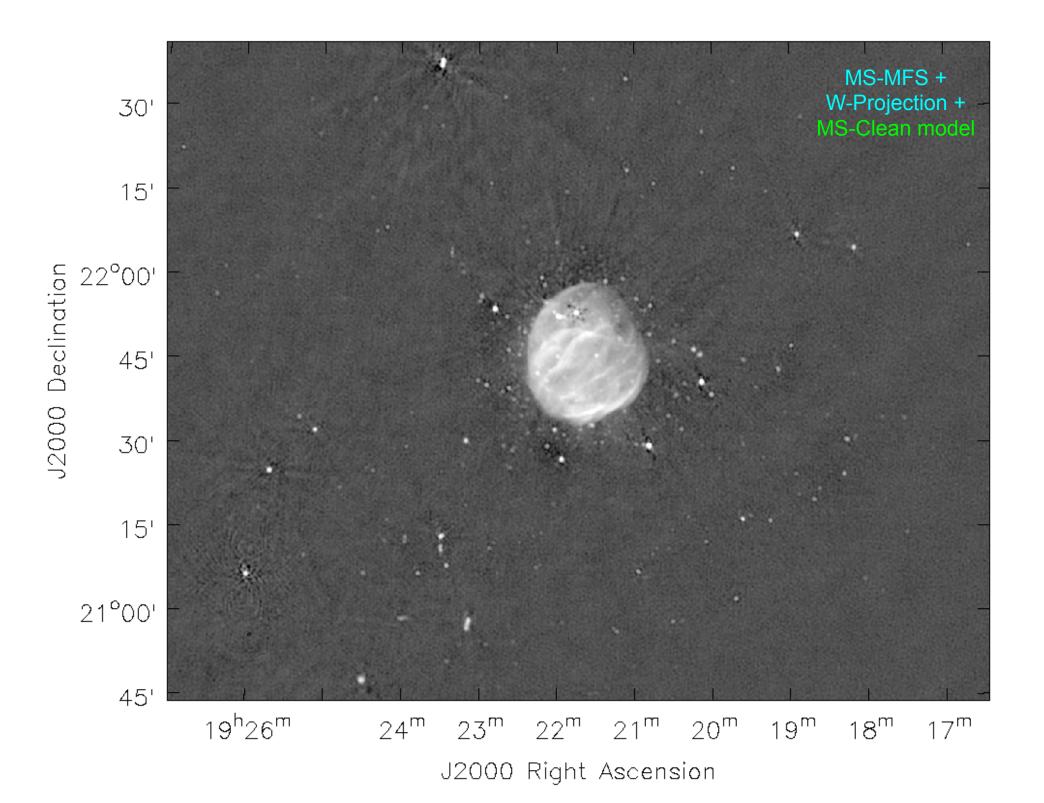
The odd points are smeared, but still present. These produce the low level ripples.

Bad weighting of a few (u,v) points

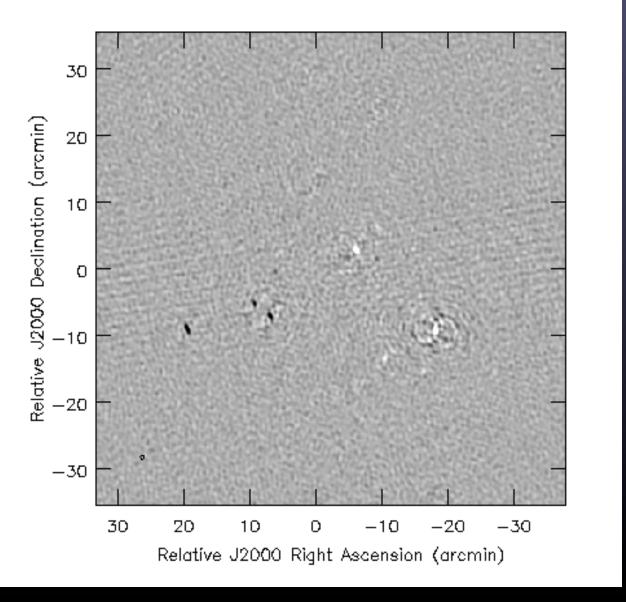








Stokes V wide-field imaging



SUMMARY OF ERROR RECOGNITION

Source structure should be 'reasonable', the rms image noise as expected, and the background featureless. If not,

(u,v) data

Look for outliers in (u,v) data using several plotting methods. Check calibration gains and phases for instabilities. Look at residual data (u,v data - clean components)

IMAGE plane

Do defects resemble the dirty beam? Are defect properties related to possible data errors? Are defects related to possible deconvolution problems? Are other corrections/calibrations needed? Does the field-of-view encompass all emission?

IMAGE ANALYSIS

- Input: Well-calibrated data-base producing a high quality image
- Output: Parameterization and interpretation of image or a set of images

This is very open-ended

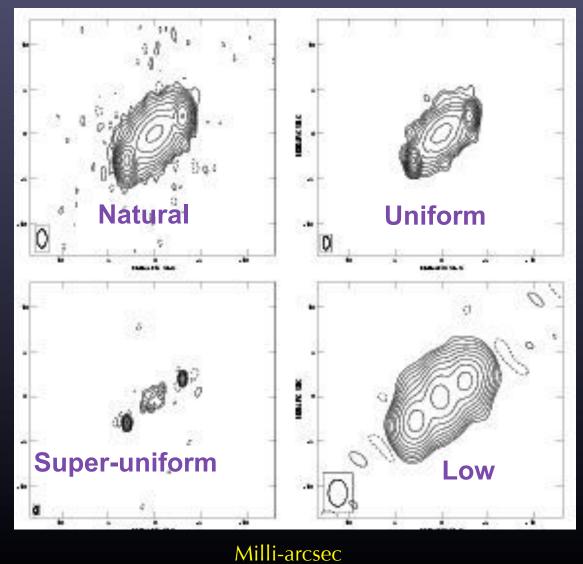
Depends on source emission complexity Depends on the scientific goals

Examples and ideas are given. Many software packages, besides AIPS and Casa (e.g., IDL, DS-9) are available.

IMAGE ANALYSIS OUTLINE

- Multi-Resolution of radio source.
- Parameter Estimation of Discrete Components
- Image Comparisons
- Positional Registration

IMAGE AT SEVERAL RESOLUTIONS



Different aspect of source structure can be see at various resolutions, shown by the ellipse in the lower left corner of each box.

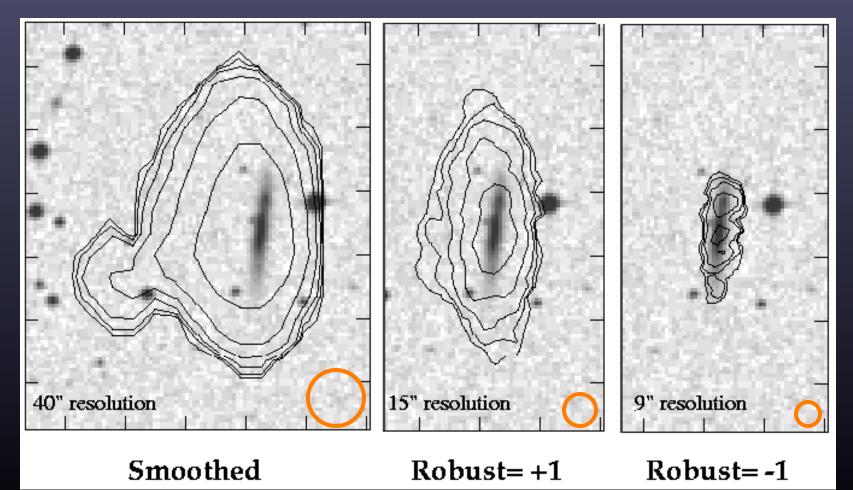
SAME DATA USED FOR ALL IMAGES

For example,

Outer components are small from SU resolution There is no extended emission from low resolution

Imaging and Deconvolution of Spectral Line Data:

Type of weighting in imaging



HI contours overlaid on optical images of an edge-on galaxy

PARAMETER ESTIMATION

Parameters associated with discrete components

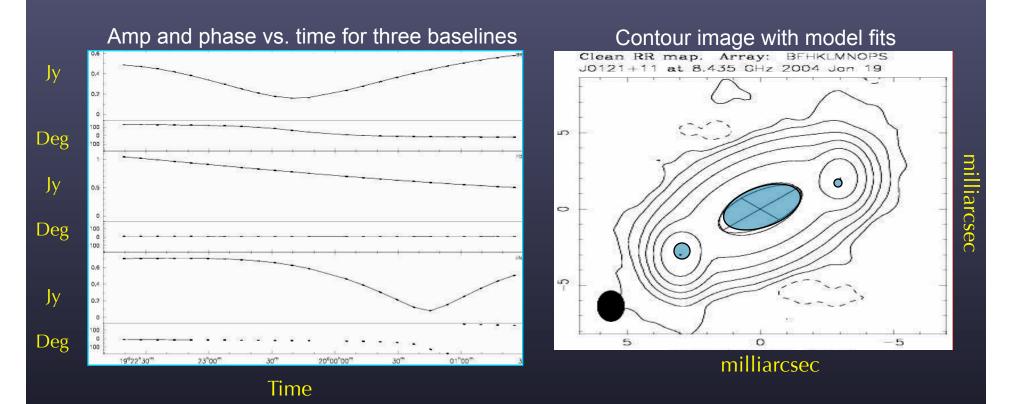
• Fitting in the image

- Assume source components are Gaussian-shaped
- Deep cleaning restores image intensity with Gaussian-beam
- True size * Beam size = Image size, if Gaussian-shaped. Hence, estimate of true size is relatively simple.
- Fitting in (u,v) plane (aka model-fitting)
 - Better estimates of parameters for simple sources
 - May be possible even when imaging is not
 - Can fit to more source models (e.g. Gaussian, ring, disk)
- Error estimates of parameters
 - Simple ad-hoc error estimates
 - Estimates from fitting programs
 - Monte Carlo simulations if model-fitting

IMAGE FITTING

Component 2-Gan	ssian							
Peak intensity	= 0.104 +/- 0.005	JY/BEAN						
Integral intens	ity= 0.998 +/- 9,47	JANSKYS						
X-position Y-position Major ax Minor ax Pos ang		0.0032 0.02						
			0	omponent 1-Gau				
5 -			0	omponent 1-Gan Peak intensity	.300 +/-	0.005	IV/REAL	4
M				Integral intens				1990 - C
				X-position				¹³ Comparison of the second seco
				Y-position	267.018			pixels
C S				Major ax	0.53	+/-	0.01	pixels
Ĕ				Minor ax	0.00			pixels
-5 -				Pos ang	21.6	+/-	1.1	deg
Component 3-Gaussian						AIPS	task:	JMFIT
Peak intensi	ty = 0.393 + / -	0.004 J	Y/BEAD	4		Casa	tool	
Integral int	ensity= 0.403 +/-	0.008 J	IANSKY:	5				
X-position	= 241.00	7 +/-	0.001	pixels		Ir	nfit	
Y-position	= 241.98	B +/-	0.001	pixels				
Major ax	1.54	+/-	0.01	pixels				
Minor ax	0.21	+/-	0.01	pixels				
Pos ang	3.6	+/-	0.2	deg				

(u,v) DATA FITTING



DIFMAP has good (u,v) fitting algorithm

Fit model directly to (u,v) data Compare model to data Contour display of image Ellipses show true component size. (SNR dependent resolution)

COMPONENT ERROR ESTIMATES

- **P** = Component Peak Flux Density
- σ = Image rms noise
- **B** = Synthesized beam size
- θ_{ι} = Component image size

 $\Delta P = \text{Peak error} = \sigma$ $\Delta X = \text{Position error} = B / 2S$ $\Delta \theta_{\iota} = \text{Component image size error} = B / 2S$ $\theta_{t} = \text{True component size} = (\theta_{\iota}^{2} - B^{2})^{1/2}$ $\Delta \theta_{t} = \text{Minimum component size} = B / S^{1/2}$

eg. S=100 means can determine size of B/10

 P/σ = signal/noise = S

Comparison and Combination of Images of Many Types

FORNAX-A Radio/Optical field

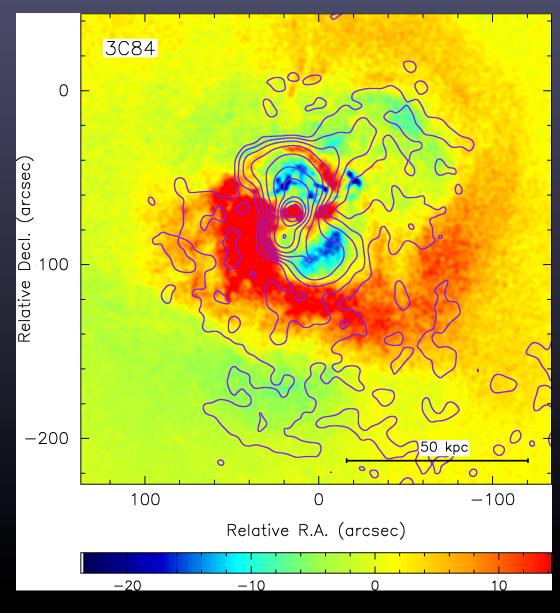
Radio is red Faint radio core in center of NGC1316

Optical in blue-white

Frame size is 60' x 40'



COMPARISON OF RADIO/X-RAY IMAGES



Contours of radio intensity at 1.4 GHz

Color intensity represents Xray intensity smoothed to radio resolution

IMAGE REGISTRATION AND ACCURACY

• Separation Accuracy of Components on One Image due to residual phase errors, regardless of signal/noise:

Limited to 1% of resolution

Position errors of 1:10000 for wide fields, i.e. 0.1" over 1.4 GHz PB

• Images at Different Frequencies:

Multi-frequency. Use same calibrator for all frequencies. Watch out at frequencies < 2 GHz when ionosphere can produce displacement. Minimize calibrator-target separation

• Images at Different Times (different configuration):

Use same calibrator for all observations. Daily troposphere changes can produce position changes up to 25% of the resolution.

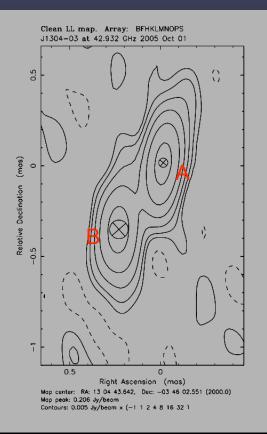
• Radio versus non-Radio Images:

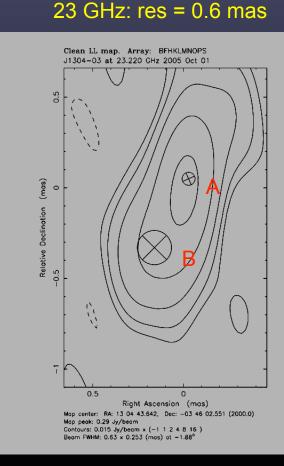
Header-information of non-radio images often much less accurate than for radio. For accuracy <1", often have to align using coincident objects.

Radio Source Alignment at Different Frequencies

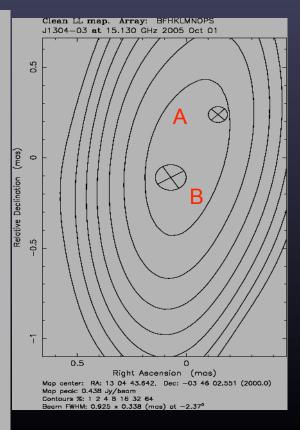
Self-calibration at each frequency aligns maximum at (0,0) point Frequency-dependent structure causes relative position of maximum to change Fitting of image with components can often lead to proper registration

43 GHz: res = 0.3 mas





15 GHz: res = 0.8 mas



ANALYSIS: SUMMARY

- Analyze and display data in several ways
 - Adjust resolution to illuminate desired interpretation, analysis
- Parameter fitting useful, but be careful of error estimates
 - Fitting in u,v plane and/or image plane
- Registration of a field at different frequencies or wave-bands can be subtle.
 - Whenever possible use the same calibrator
 - May be able to align using 'known' counterparts

Further Reading

- http://www.nrao.edu/whatisra/
- <u>www.nrao.edu</u>
- 2010 Lecture on Non-Imaging Analysis
- Synthesis Imaging in Radio Astronomy
- ASP Vol 180, eds Taylor, Carilli & Perley
- Numerical Recipes, Press et al. 1992