Calibration

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Thirteenth Synthesis Imaging Workshop

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Synopsis

- Why do we have to calibrate?
- Idealistic formalism → Realistic practice…. data!
- Editing
- Fundamental Calibration Principles
  - Practical Calibration Considerations
  - Baseline-based vs. Antenna-based Calibration
- Scalar Calibration Example
- Generalizations
  - Full Polarization
  - A Dictionary of Calibration Effects
  - Calibration Heuristics and 'Bootstrapping'
- New Calibration Challenges
- Summary

Why Calibration and Editing?

- Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, gain stability, geometric model errors, etc.)
- Need to accommodate deliberate engineering (e.g., frequency conversion, analog/digital electronics, filter bandpass, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- Radio Frequency Interference (RFI)

Determining *instrumental properties* (calibration)
is a prerequisite to
determining *radio source properties*

From Idealistic to Realistic

- Formally, we wish to use our interferometer to obtain the visibility function:

$$V(u,v) = \int_{sky} I(l,m)e^{-i2\pi(ul+vm)}dldm$$

- ….a Fourier transform which we intend to invert to obtain an image of the sky:

$$I(l,m) = \int_{uv} V(u,v)e^{i2\pi(ul+vm)}dudv$$

  – *V(u,v)* describes the amplitude and phase of 2D sinusoids that add up to an image of the sky
    - Amplitude: "~how concentrated?"
    - Phase: "~where?"
- How do we measure *V(u,v)?*

From Idealistic to Realistic

- In practice, we correlate (multiply & average) the electric field (voltage) samples, x_i & x_j, received at pairs of telescopes (*i,j*) and processed through the observing system:

$$V_{ij}^{obs}(u_{ij}, v_{ij}) = \left\langle x_i(t) \cdot x_j^{*}(t) \right\rangle_{\Delta t}$$

$$= J_{ij} V_{ij}^{true}(u_{ij}, v_{ij})$$

  – x_i & x_j are mutually delay-compensated for a specific point on the sky
  – Averaging duration is set by the expected timescales for variation of the correlation result (~seconds)
- J_{ij} is an *operator* characterizing the *net* effect of the observing process for antennas *i* and *j* on baseline *ij*, which we must *calibrate*
- Sometimes J_{ij} corrupts the measurement irrevocably, resulting in data that must be *edited* or *"flagged"*

What Is Delivered by a Synthesis Array?

- An *enormous* list of complex visibilities! *(Enormous!)*
  - At each timestamp (~1-10s intervals): N(N-1)/2 baselines
    - EVLA: 351baselines
    - VLBA: 45 baselines
    - ALMA: 1225-2016 baselines
  - For each baseline: up to 64 Spectral Windows ("spws", "subbands" or "IFs")
  - For each spectral window: tens to thousands of channels
  - For each channel: 1, 2, or 4 complex correlations (polarizations)
    - EVLA or VLBA: RR or LL or (RR,LL), or (RR,RL,LR,LL)
    - ALMA: XX or YY or (XX,YY) or (XX,XY,YX,YY)
  - With each correlation, a weight value and a flag (T/F)
  - Meta-info: Coordinates, antenna, field, frequency label info
- $N_{total} = N_t \times N_{bl} \times N_{spw} \times N_{chan} \times N_{corr}$ visibilities
  - ~few $10^6 \times N_{spw} \times N_{chan} \times N_{corr}$ vis/hour → 10s to 100s of GB per observation

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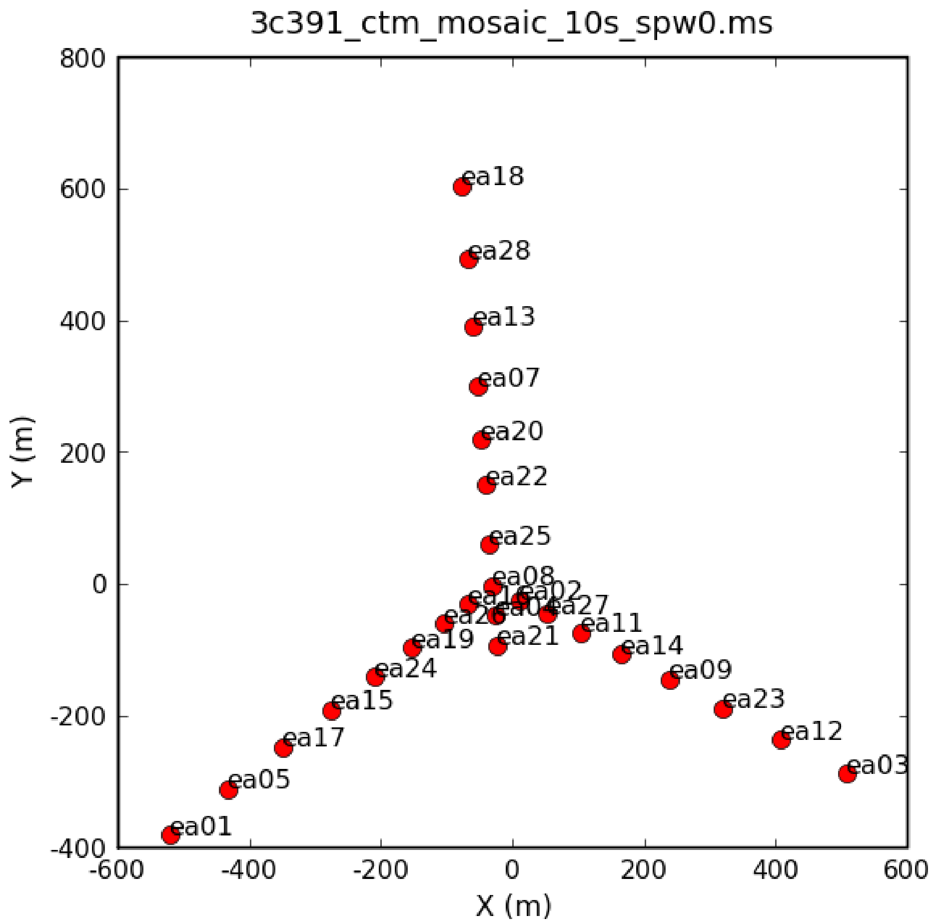
A Typical Dataset (Polarimetry)

- Array:
  - EVLA D-configuration (Apr 2010)
- Sources:
  - Science Target: 3C391 (7 mosaic pointings)
  - Near-target calibrator: J1822-0938 (~11 deg from target)
  - Flux Density calibrator: 3C286
  - Instrumental Polarization Calibrator: 3c84
- Signals:
  - RR,RL,LR,LL correlations
  - One spectral window centered at 4600 MHz, 128 MHz bandwidth, 64 channels

The Array



3c391_ctm_mosaic_10s_spw0.ms
Y (m)
X (m)
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600
400
200
0
-200
-400
-600
-400
-200
0
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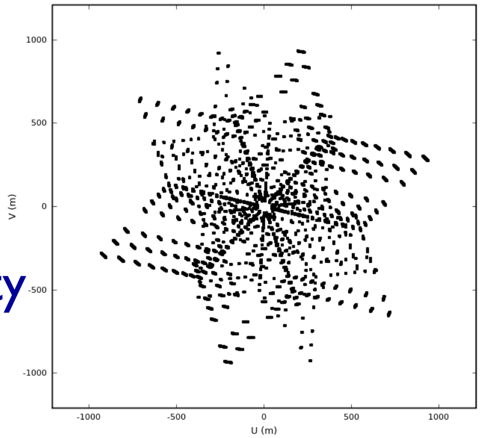
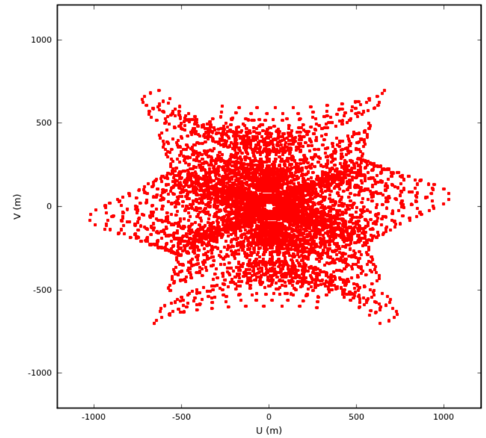
UV-coverages

3C286

Flux Density



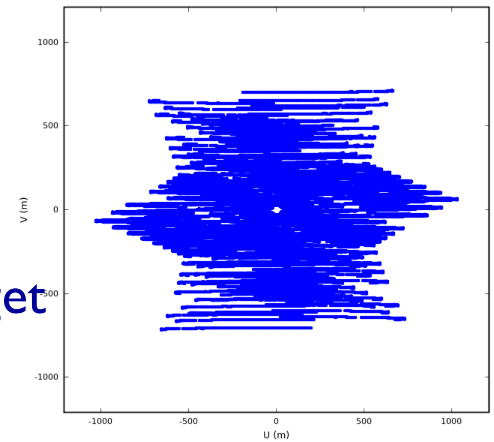


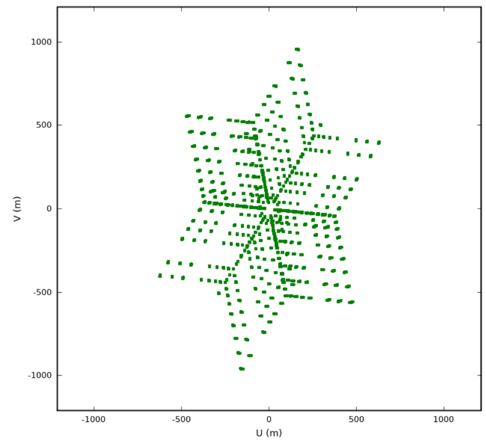
J1822-0938

Gain Calibrator

3C391

Science Target





3C84

Instr. Poln Calibrator

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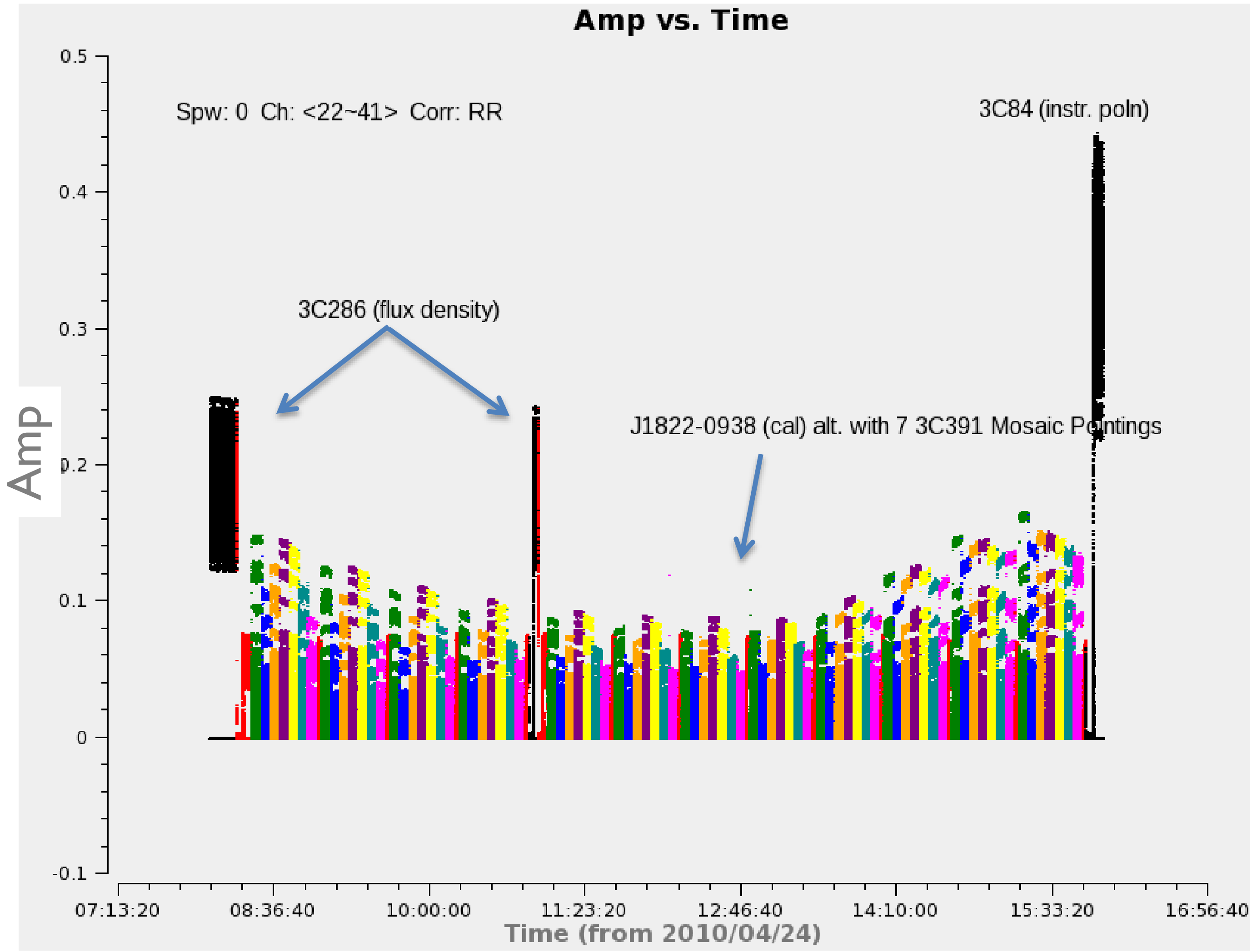
The Visibility Data (source colors)



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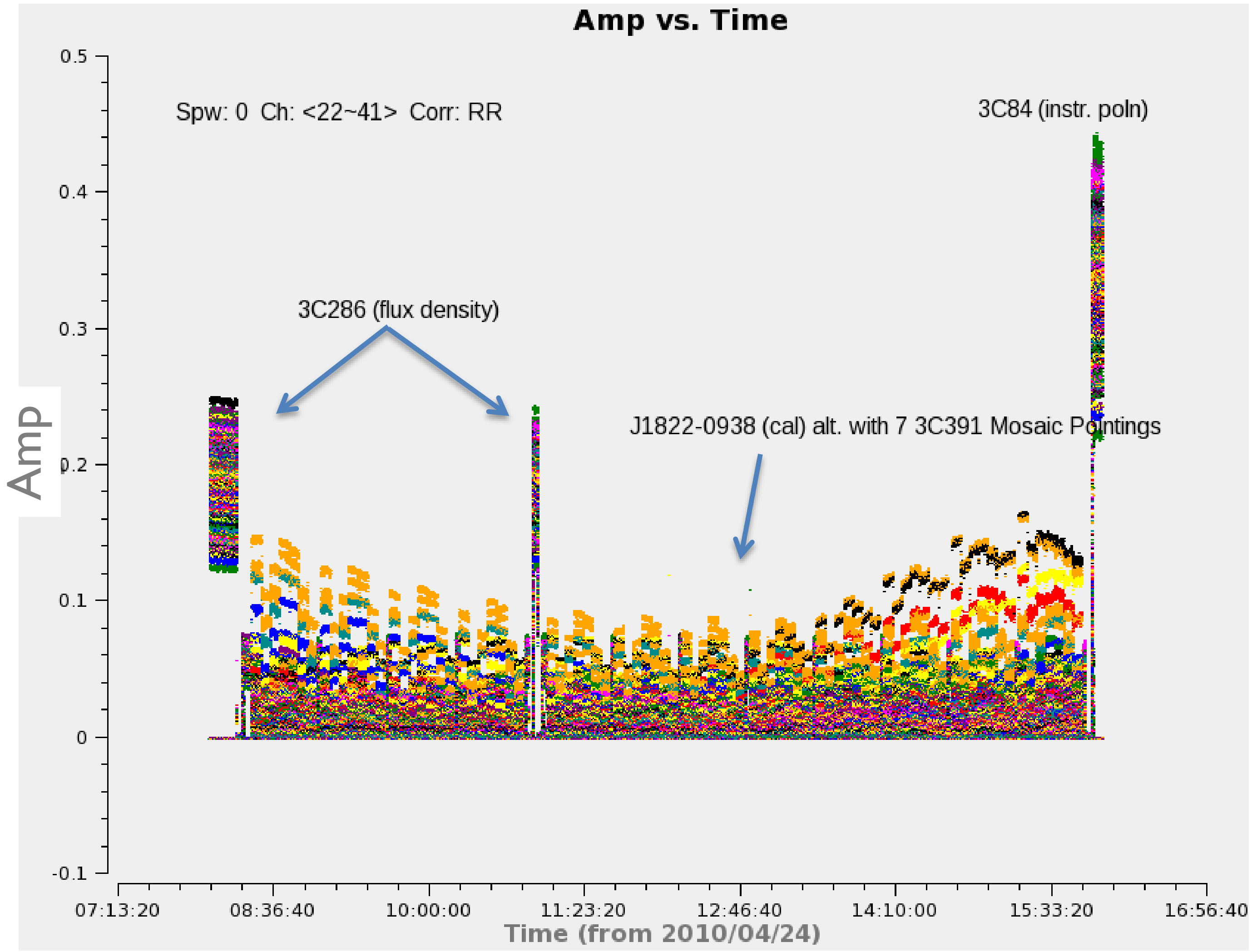
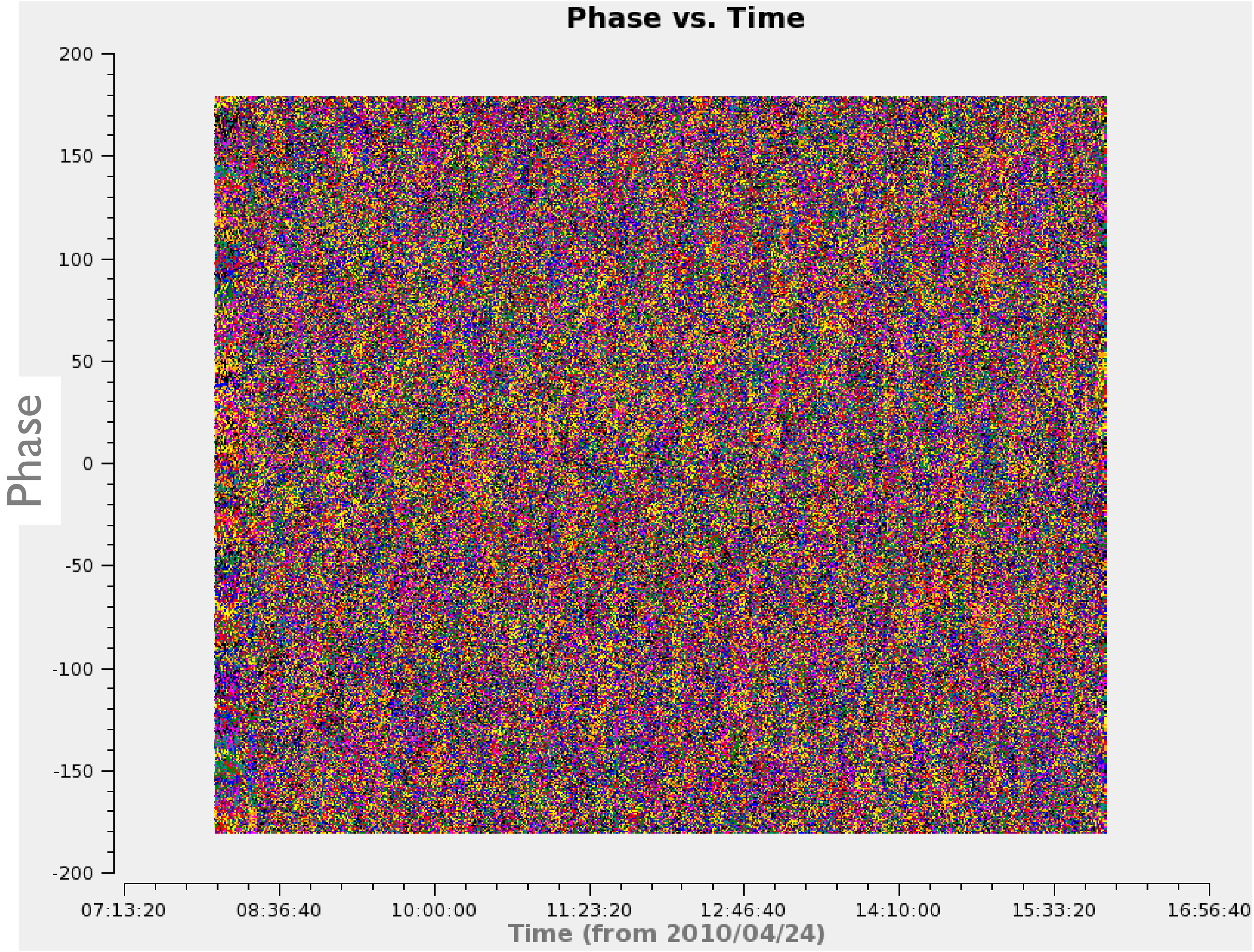
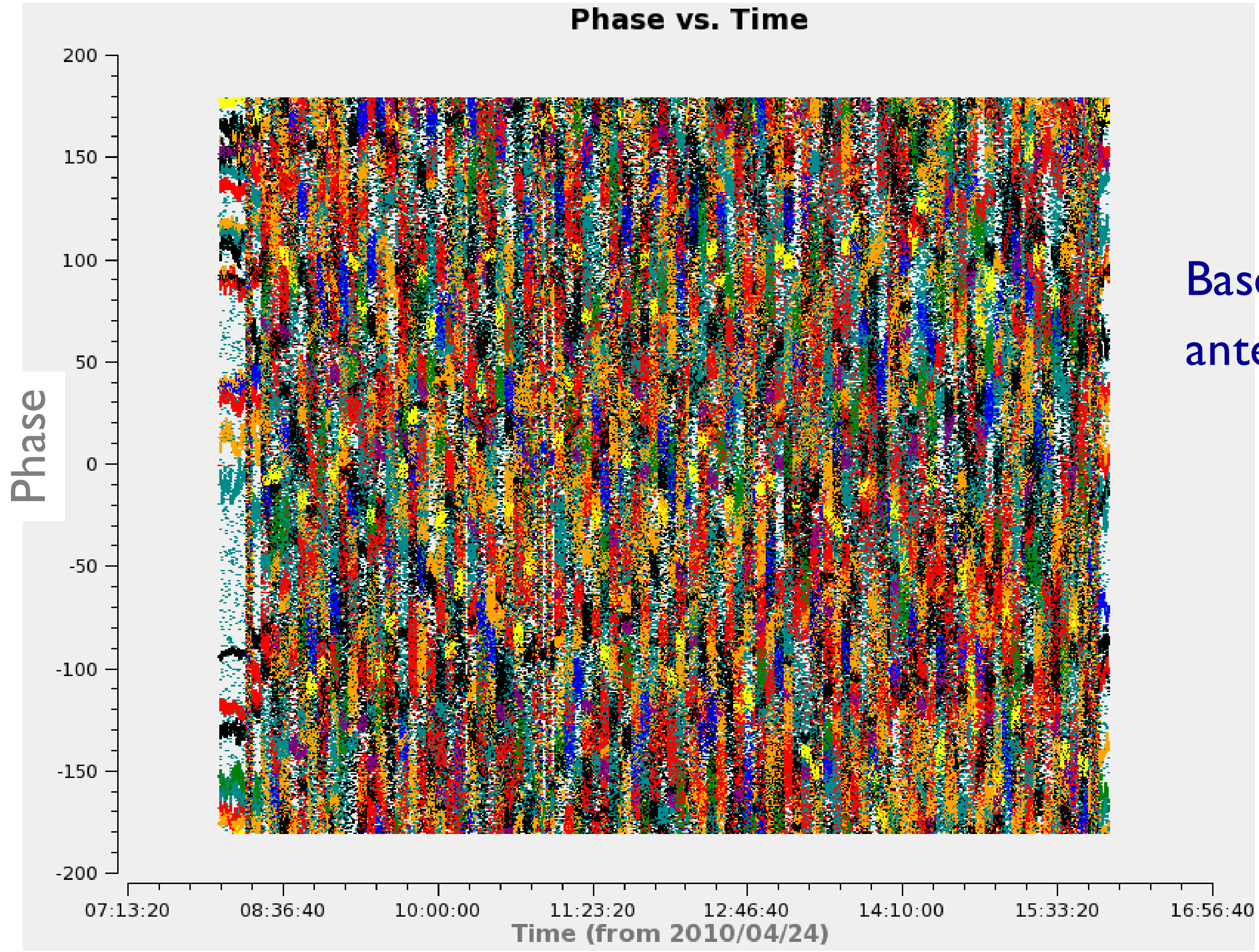
The Visibility Data (baseline colors)



The Visibility Data (baseline colors)

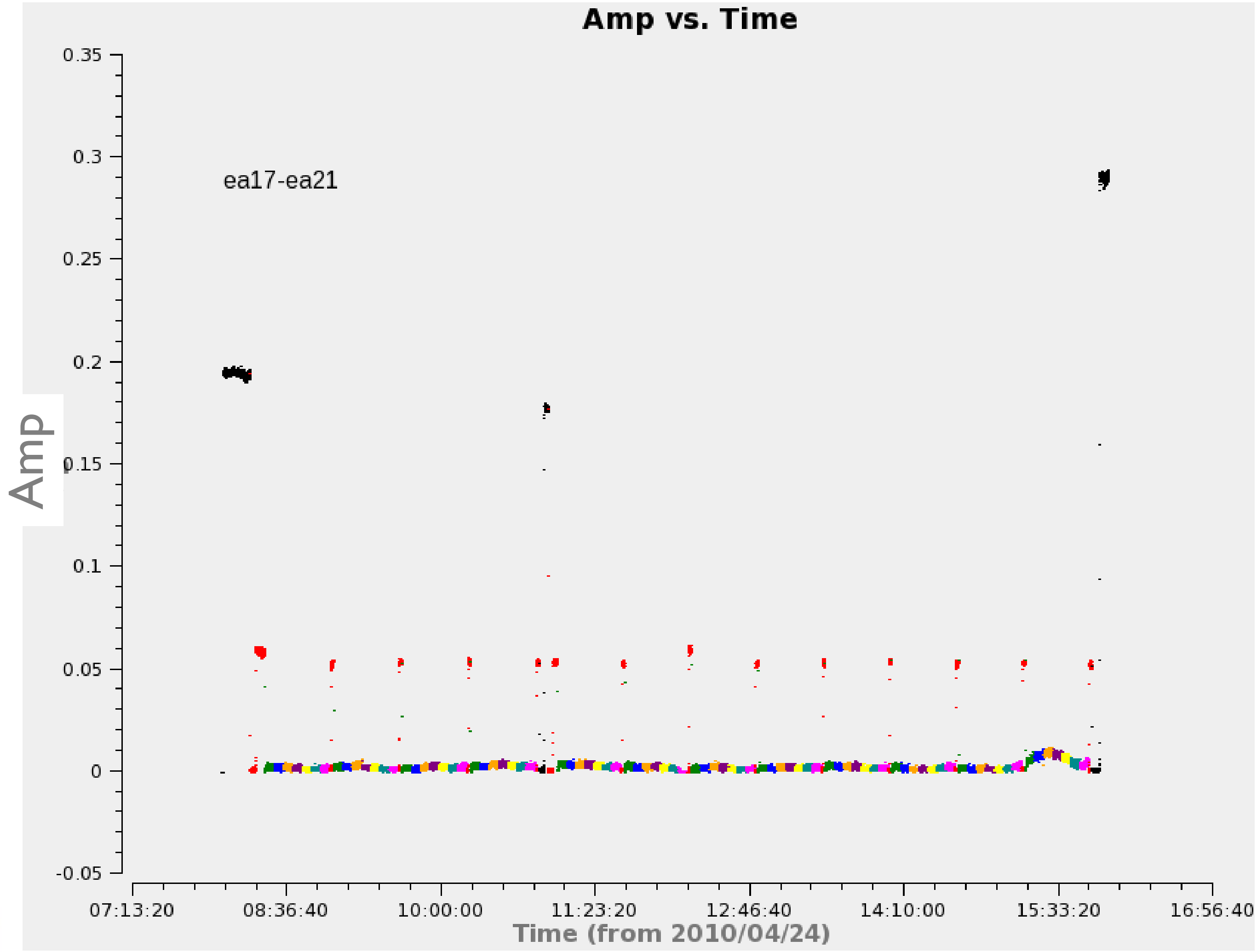


The Visibility Data (baseline colors)

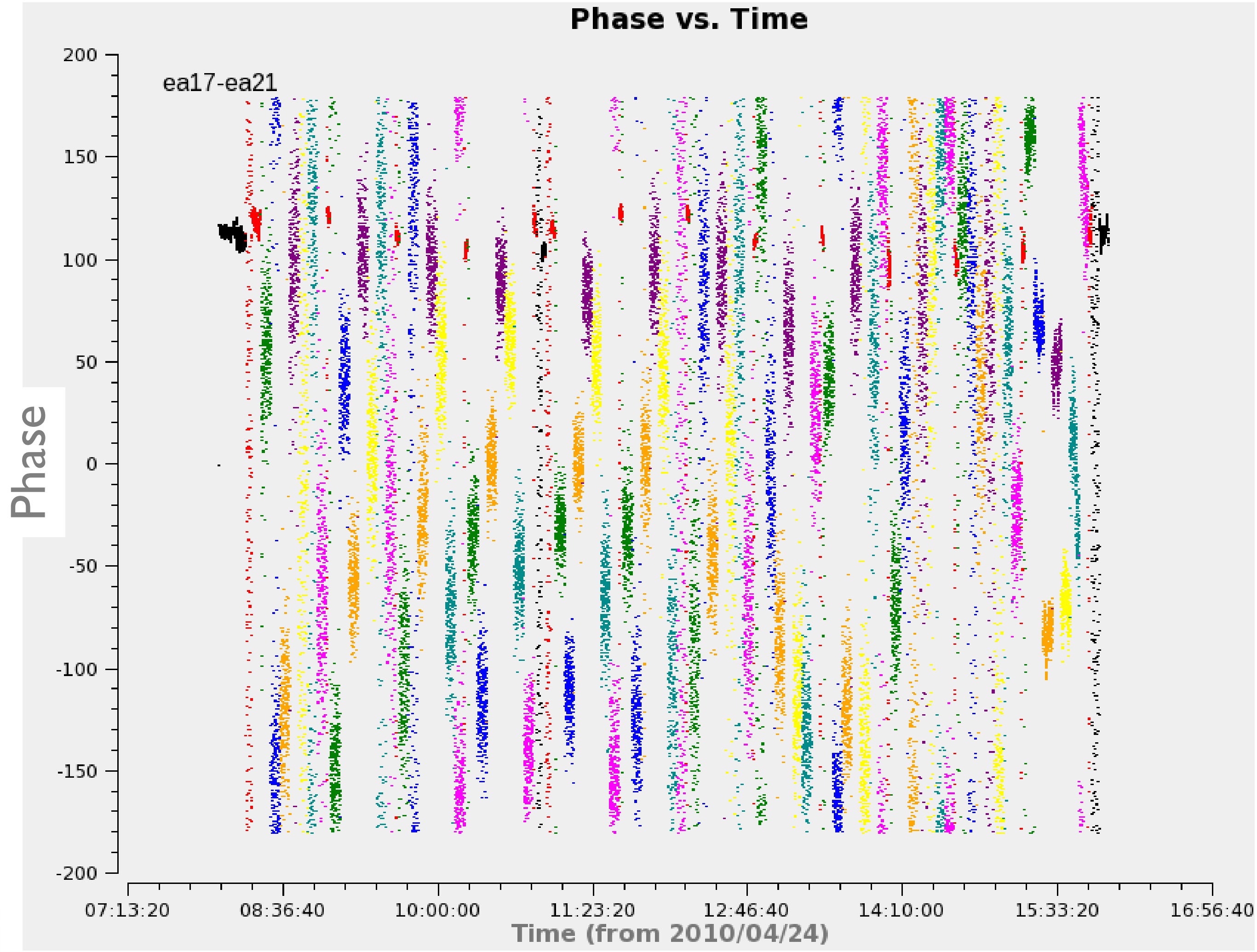


Baselines to antenna ea21

A Single Baseline - Amp



A Single Baseline - Phase



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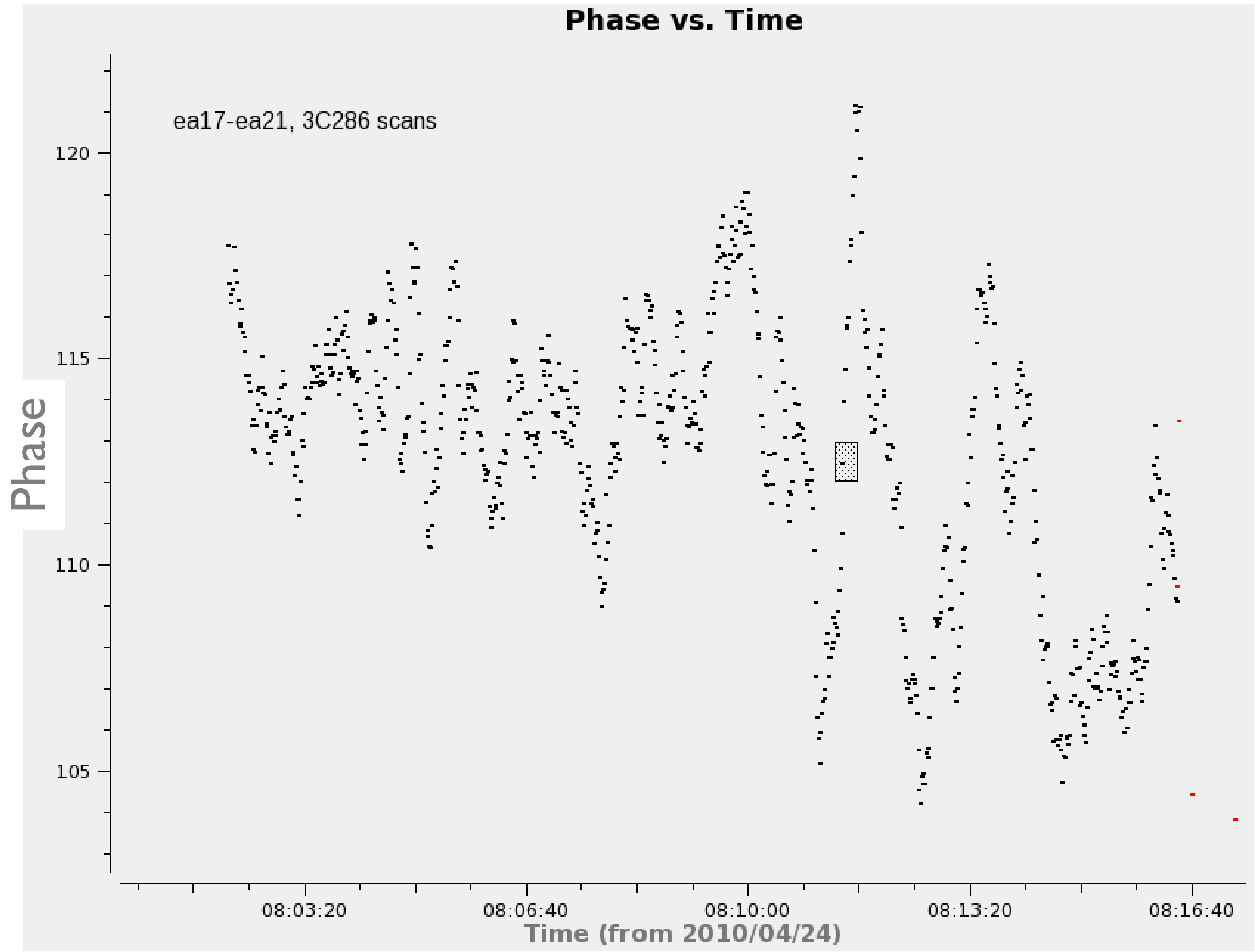
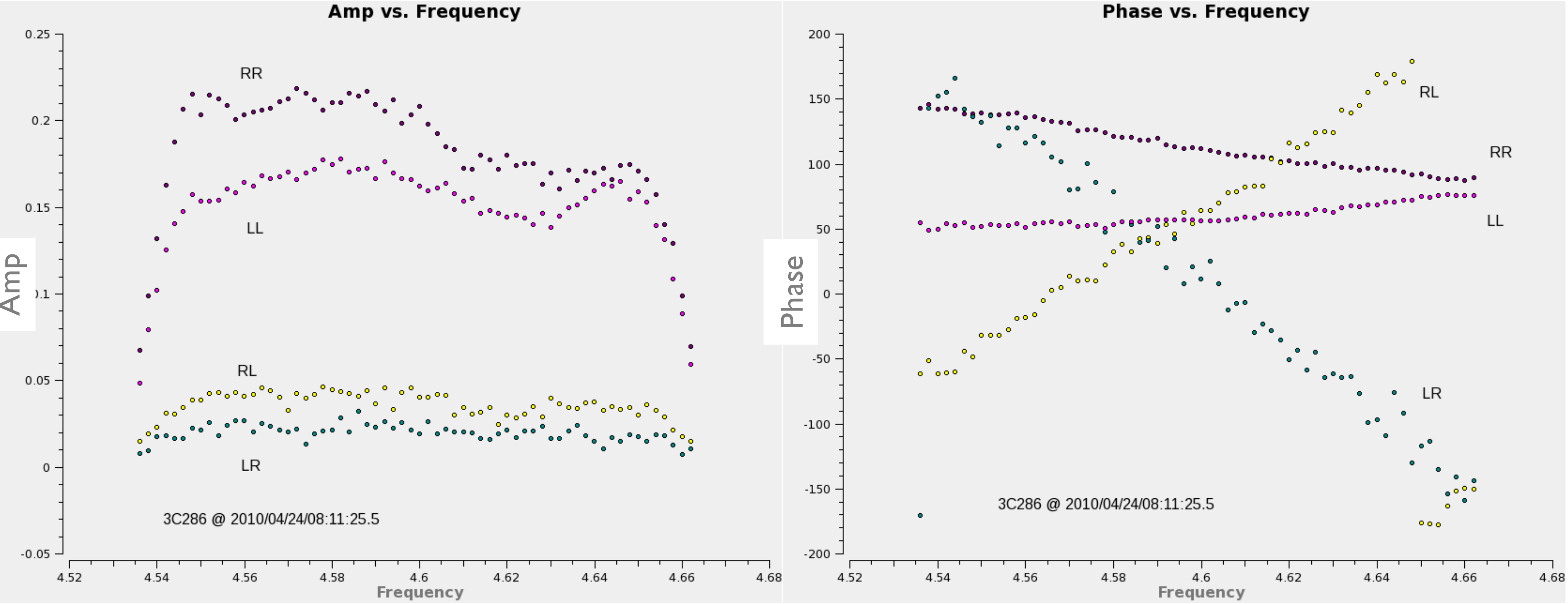
A Single Baseline – 2 scans on 3C286

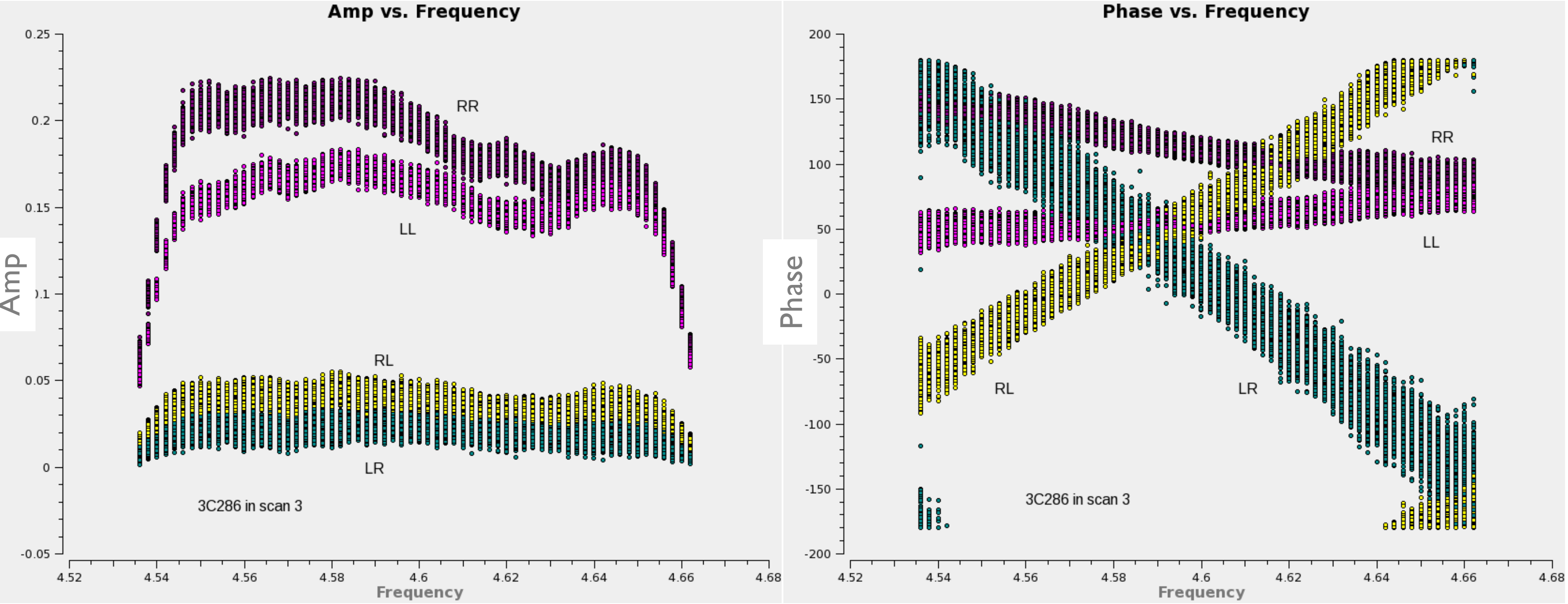


Single Baseline, Single Integration Visibility Spectra (4 correlations)



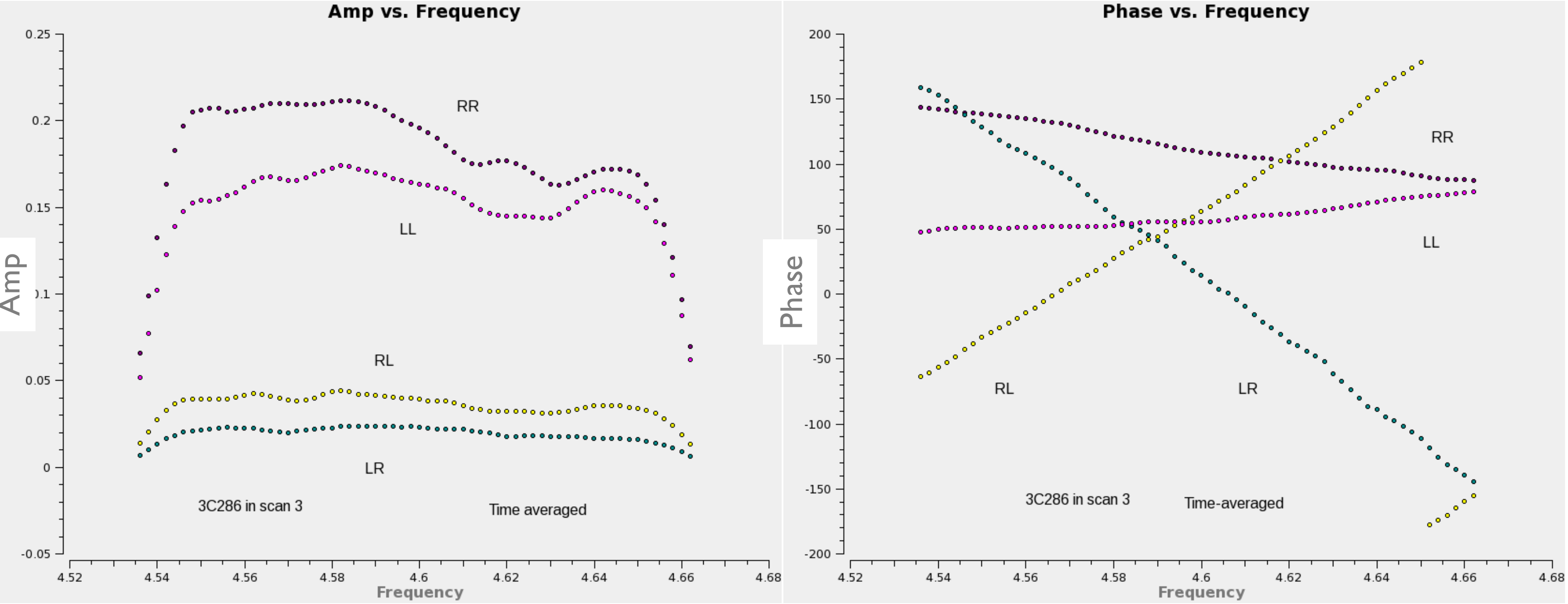
baseline ea17-ea21

Single Baseline, Single Scan Visibility Spectra (4 correlations)



baseline ea17-ea21

Single Baseline, Single Scan (time-averaged) Visibility Spectra (4 correlations)

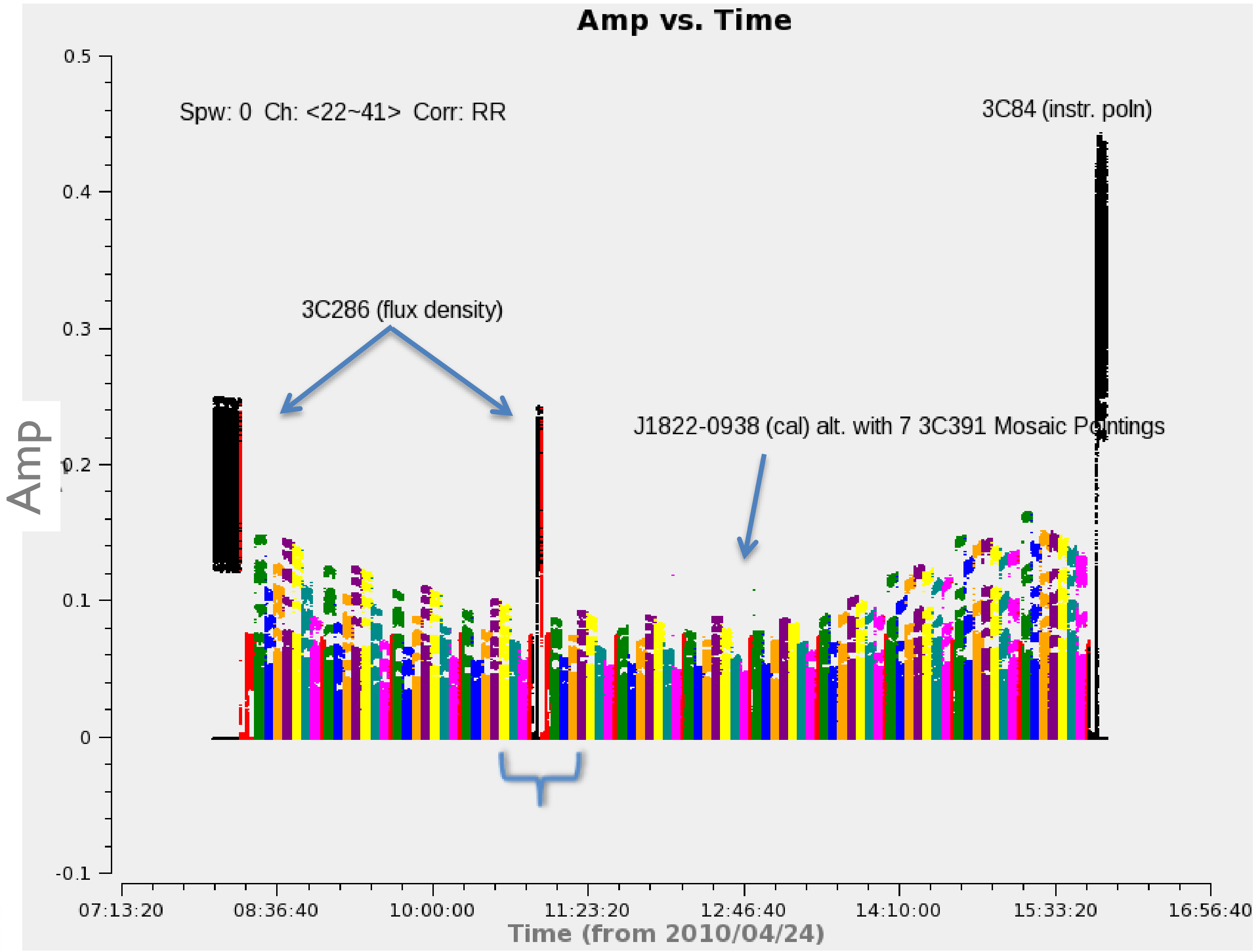


baseline ea17-ea21

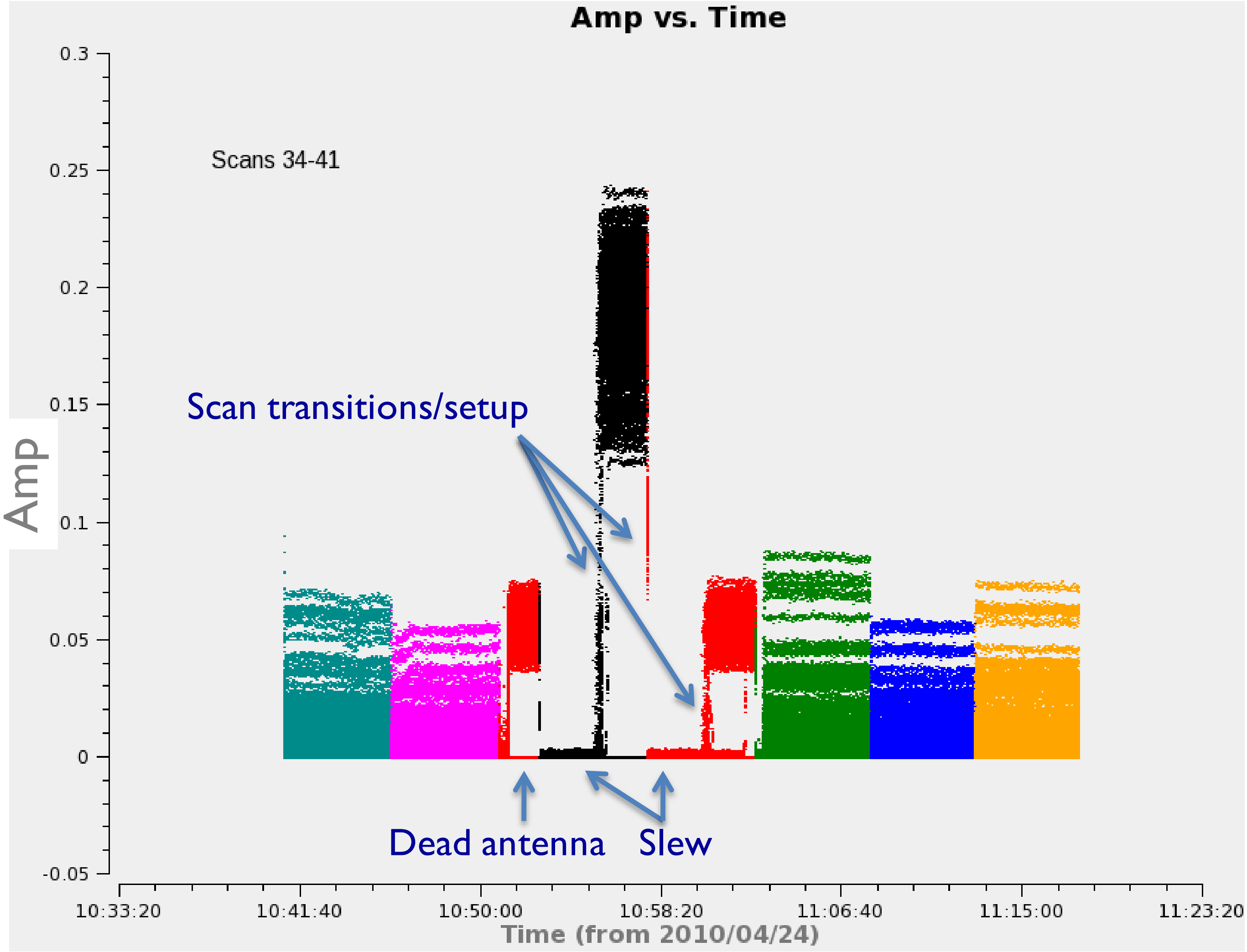
Data Examination and Editing

- After observation, initial data examination and editing very important
  - Will observations meet goals for calibration and science requirements?
- What to edit (much of this is automated):
  - Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)
  - Any persistently 'dead' antennas (check operator's logs)
  - Periods of especially poor weather? (check operator's log)
  - Any antennas shadowing others? Edit such data.
  - Amplitude and phase should be continuously varying—edit outliers
  - Radio Frequency Interference (RFI)?
- Caution:
  - Be careful editing noise-dominated data.
  - Be conservative: those antennas/timeranges which are bad on calibrators are probably bad on weak target sources—edit them
  - Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have significantly different amplitude response which may not be fatal—it may only need to be calibrated
  - Choose (phase) reference antenna wisely (ever-present, stable response)
- Increasing data volumes increasingly demand automated editing algorithms…

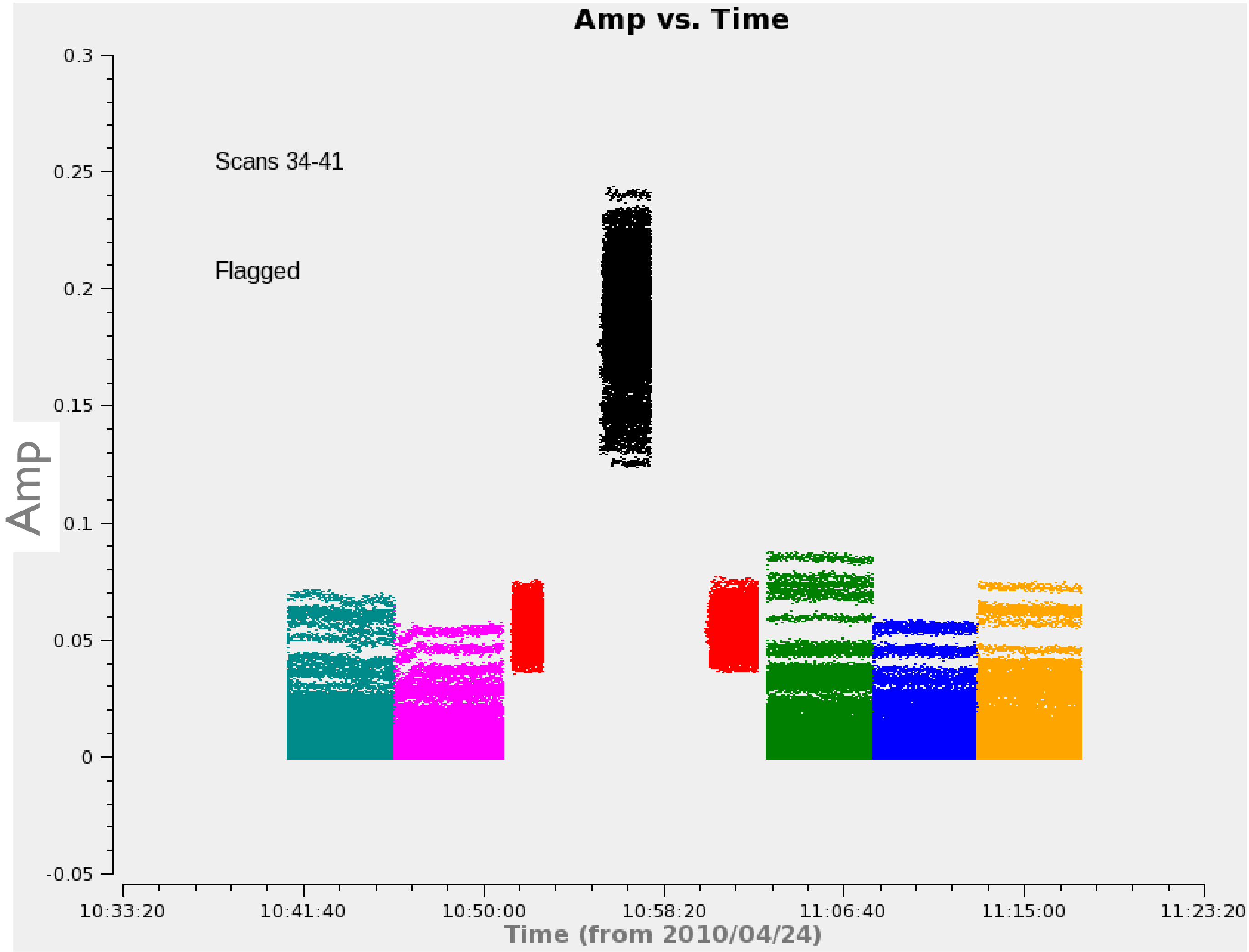
Editing Example



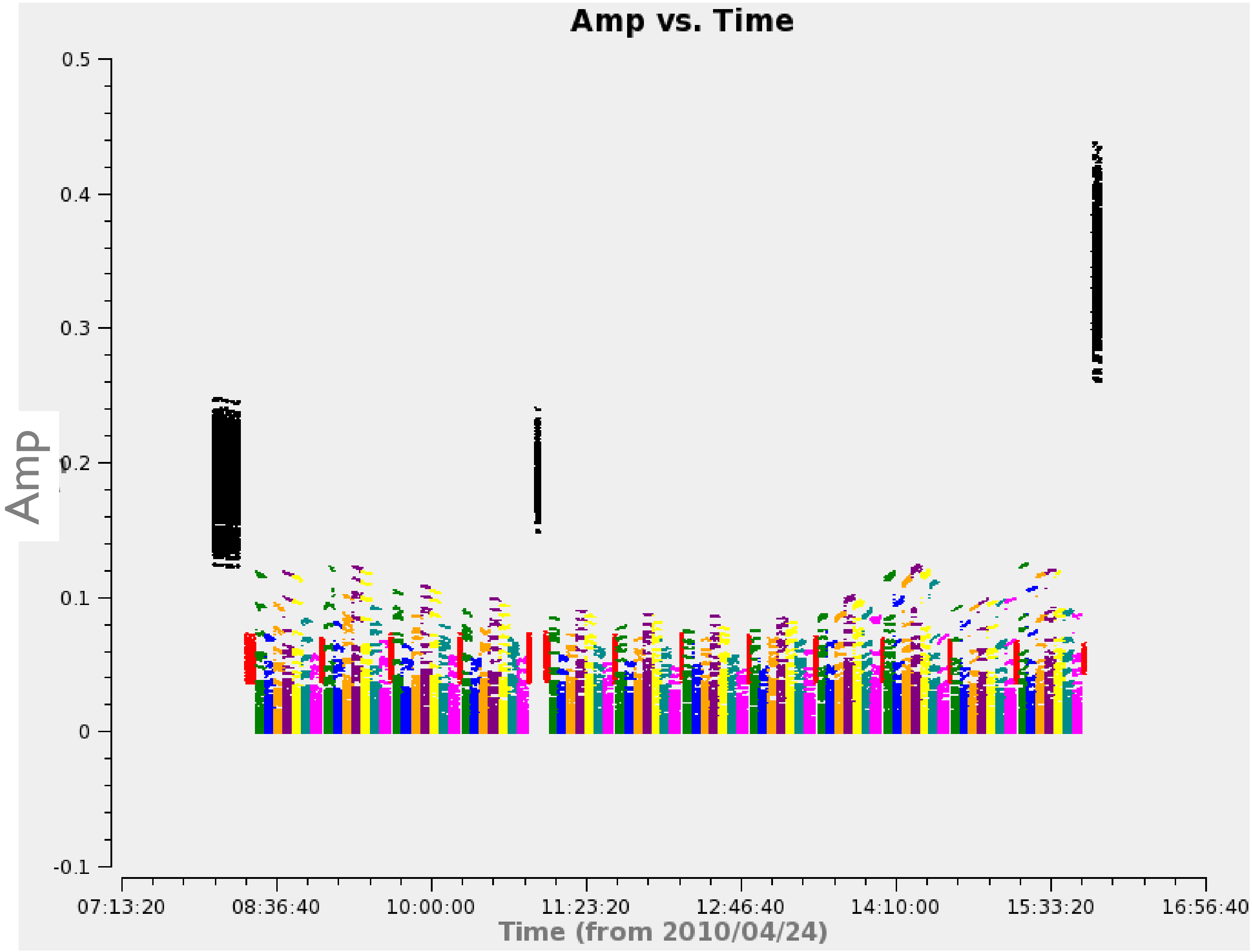
Editing Example



Editing Example



Editing Example



Practical Calibration Considerations

- A priori "calibrations" (provided by the observatory)
  - Antenna positions, earth orientation and rate, clocks
  - Antenna pointing, voltage pattern, gain curve
  - Calibrator coordinates, flux densities, polarization properties
- Absolute *engineering* calibration (dBm, K, volts)?
  - Amplitude: episodic (ALMA) or continuous (EVLA/VLBA) Tsys/switched-power monitoring to enable calibration to nominal K (or Jy, with antenna efficiency information)
  - Phase: practically impossible (relative antenna phase)
  - Traditionally, we concentrate instead on ensuring instrumental *stability* on adequate timescales
- ***Cross-calibration*** a better choice
  - Observe strong nearby sources against which calibration (J_{ij}) can be solved, and transfer solutions to target observations
  - Choose appropriate calibrators; usually ***point sources*** because we can easily predict their visibilities (Amp ~ constant, phase ~ 0)
  - Choose appropriate timescales for calibration

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“Absolute” Astronomical Calibrations

- Flux Density Calibration
  - Radio astronomy flux density scale set according to several “constant” radio sources, and planets/moons
  - Use resolved models where appropriate
- Astrometry
  - Most calibrators come from astrometric catalogs; sky coordinate accuracy of target images tied to that of the calibrators
  - Beware of resolved and evolving structures, and phase transfer biases due to troposphere (especially for VLBI)
- Linear Polarization Position Angle
  - Usual flux density calibrators also have significant stable linear polarization position angle for registration

- Relative calibration solutions (and dynamic range) insensitive to errors in these “scaling” parameters

Baseline-based Cross-Calibration

$$V_{ij}^{obs} = J_{ij} V_{ij}^{mod}$$

- Simplest, most-obvious calibration approach: measure complex response of *each baseline* on a standard source, and scale science target visibilities accordingly
  - "Baseline-based" Calibration: $$J_{ij} = \left\langle V_{ij}^{obs} / V_{ij}^{mod} \right\rangle_{\Delta t}$$
- Only option for single baseline "arrays"
- Calibration precision same as calibrator visibility sensitivity (on timescale of calibration solution). Improves only with calibrator strength.
- Calibration accuracy sensitive to departures of calibrator from assumed structure
  - Un-modeled calibrator structure transferred (in inverse) to science target!

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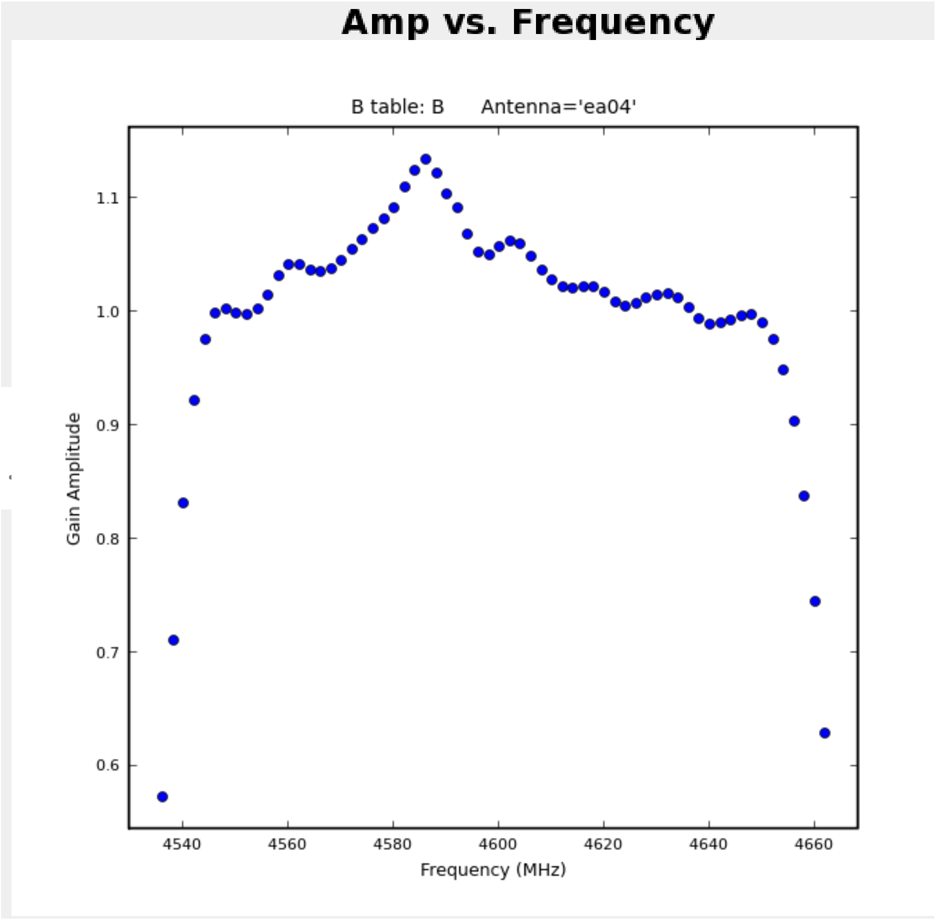
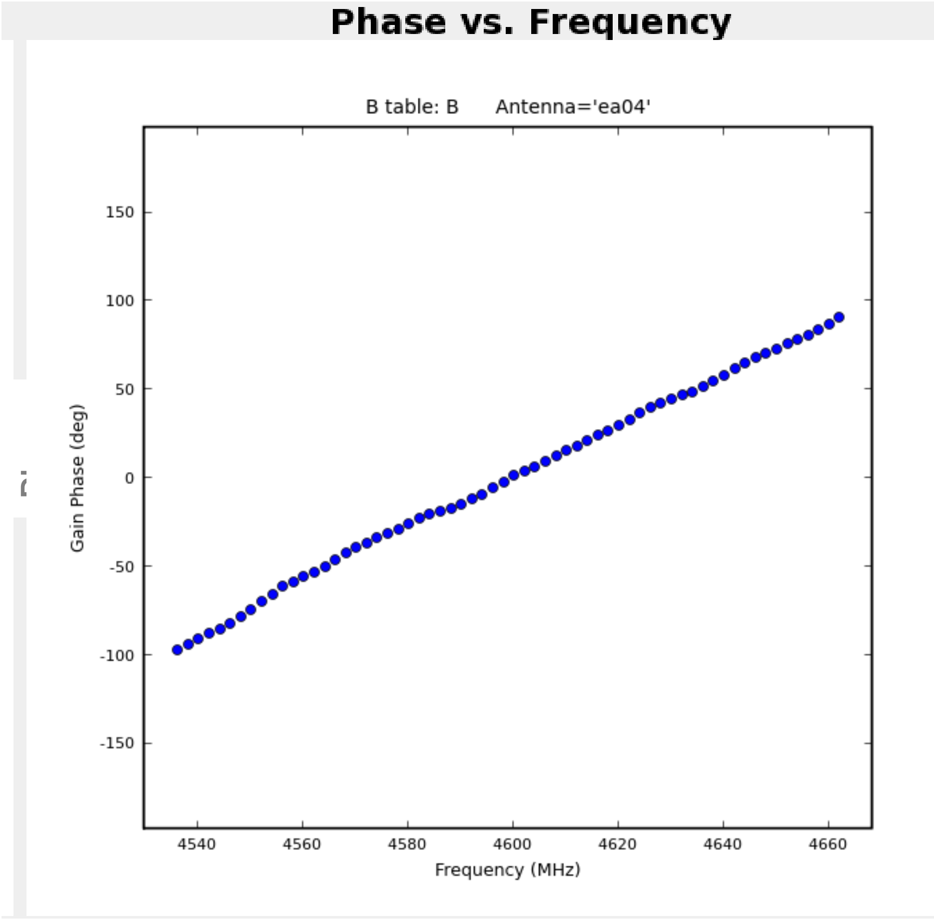
Antenna-based Cross Calibration

- Measured visibilities are formed from a product of *antenna-based* signals. Can we take advantage of this fact?
  - e.g., bandpass…

Rationale for Antenna-based Calibration





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Antenna-based Cross Calibration

- The net time-dependent E-field signal sampled by antenna *i*, $x_i(t)$, is a combination of the desired signal, $s_i(t,l,m)$, corrupted by a factor $J_i(t,l,m)$ and integrated over the sky (l,m), and diluted by noise, $n_i(t)$:

$$x_i(t) = \int_{sky} J_i(t,l,m)\, s_i(t,l,m)\, dl dm + n_i(t)$$

$$= s_i'(t) + n_i(t)$$

- $x_i(t)$ is sampled (complex) voltage provided to the correlator input
- $J_i(t,l,m)$ is the product of a series of effects encountered by the incoming signal
- $J_i(t,l,m)$ is an *antenna-based* complex number
- Usually, $|n_i| >> |s_i'|$ (i.e., noise dominates)

Correlation of Realistic Signals - I

- The correlation of two realistic (aligned for a specific direction) signals from different antennas:

$$\left\langle x_i \cdot x_j^* \right\rangle_{\Delta t} = \left\langle (s_i' + n_i) \cdot (s_j' + n_j)^* \right\rangle_{\Delta t}$$

$$= \left\langle s_i' \cdot s_j'^* \right\rangle_{\Delta t} + \left\langle s_i' \cdot n_j^* \right\rangle_{\Delta t} + \left\langle n_i \cdot s_j'^* \right\rangle_{\Delta t} + \left\langle n_i \cdot n_j^* \right\rangle_{\Delta t}$$

  - Noise correlations have zero mean—even if $|n_i| >> |s_i|$, the correlation process isolates desired signals:

$$= \left\langle s_i' \cdot s_j'^* \right\rangle_{\Delta t}$$

$$= \left\langle \int_{sky} J_i s_i \, dl_i \, dm_i \cdot \int_{sky} J_j^* s_j^* \, dl_j \, dm_j \right\rangle_{\Delta t}$$

  - In integral, only $s_i(t,l,m)$ from the same directions potentially correlate (i.e., when $l_i = l_j$, $m_i = m_j$), so order of integration and signal product can be reversed:

$$= \left\langle \int_{sky} J_i J_j^* s_i s_j^* \, dl dm \right\rangle_{\Delta t}$$

Correlation of Realistic Signals - II

- The s_i & s_j are the *common* radio source signals, and differ *only* by the relative arrival phase at each antenna, which varies with direction. This difference *is* the Fourier phase term (to a good approximation), which we factor out:

$$V_{ij} = \left\langle \int_{sky} J_i J_j^* |s_i|^2 e^{-i2\pi(u_{ij} l + v_{ij} m)} dldm \right\rangle_{\Delta t} \qquad s_j = s_i e^{i2\pi(u_{ij} l + v_{ij} m)}$$

- On the timescale of the averaging, the only meaningful average is of the *squared* source signal itself (in each direction), which is just the brightness distribution of the source *I(l,m)*:

$$= \int_{sky} J_i J_j^* \left\langle |s_i(t, l, m)|^2 \right\rangle_{\Delta t} e^{-i2\pi(u_{ij} l + v_{ij} m)} dldm$$

$$= \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij} l + v_{ij} m)} dldm$$

- If all *J=1.0*, we of course recover the ideal expression:

$$= \int_{sky} I(l,m) e^{-i2\pi(u_{ij} l + v_{ij} m)} dldm$$

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Aside: Auto-correlations and Single Dishes

- The *auto*-correlation of a signal from a *single* antenna:

$$\left\langle x_i \cdot x_i^* \right\rangle_{\Delta t} = \left\langle (s_i' + n_i)(s_i' + n_i)^* \right\rangle_{\Delta t}$$

$$= \left\langle s_i' \cdot s_i'^* \right\rangle + \left\langle n_i \cdot n_i^* \right\rangle$$

$$= \left\langle \int_{sky} |J_i|^2 |s_i|^2 \, dldm \right\rangle_{\Delta t} + \left\langle |n_i|^2 \right\rangle$$

$$= \int_{sky} |J_i|^2 I(l,m) \, dldm + \left\langle |n_i|^2 \right\rangle$$

  - This is an integrated (sky) power measurement plus *non-zero-mean* noise
  - Desired signal *not* simply isolated from noise
  - Noise usually dominates
- Single dish radio astronomy calibration strategies rely on switching (differencing) schemes to isolate desired signal from the noise

The Scalar Measurement Equation

$$V_{ij}^{obs} = \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi\left(u_{ij} l + v_{ij} m\right)} dldm$$

- First, isolate non-direction-dependent effects, and factor them from the integral:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} \left(J_i^{sky} J_j^{sky*}\right) I(l,m) e^{-i2\pi\left(u_{ij} l + v_{ij} m\right)} dldm$$

- Next, we recognize that over small fields of view, it is possible to assume $J^{sky}=1.0$, and we have a relationship between ideal and observed Visibilities:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} I(l,m) e^{-i2\pi\left(u_{ij} l + v_{ij} m\right)} dldm$$

$$V_{ij}^{obs} = \left(J_i^{vis} J_j^{vis*}\right) V_{ij}^{true} = J_i J_j^* V_{ij}^{true}$$

- Standard calibration of most existing arrays reduces to solving this last equation for the J_i, assuming a visibility model V_{ij}^{mod} for a calibrator
- NB: visibilities corrupted by *difference of antenna-based phases*, and *product of antenna-based amplitudes*

Solving for the J_i

- We can write: $V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$

- …and define chi-squared:

$$\chi^2 = \sum_{\substack{i,j \\ i \neq j}} \left| V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} \right|^2 w_{ij} \qquad \left(w_{ij} = \frac{1}{\sigma_{ij}^2} \right)$$

- …and minimize chi-squared w.r.t. each J_i^*, yielding (iteration):

$$J_i = \sum_{\substack{j \\ i \neq j}} \left(\frac{V_{ij}^{obs}}{V_{ij}^{mod}} J_j w_{ij} \right) \Bigg/ \sum_{\substack{j \\ i \neq j}} \left(\left| J_j \right|^2 w_{ij} \right) \qquad \left(\frac{\partial \chi^2}{\partial J_i^*} = 0 \right)$$

  - (…which we may be gratified to recognize as a peculiarly weighted average of the implicit J_i contribution to V^{obs}:)

$$\left[J_i = \sum_{\substack{j \\ i \neq j}} \left(J_i' \left| J_j \right|^2 w_{ij} \right) \Bigg/ \sum_{\substack{j \\ i \neq j}} \left| J_j \right|^2 w_{ij} \right]$$

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Solving for J_i (cont)

- Formal errors:

$$\sigma_{J_i} = \sqrt{\frac{1}{\sum_j \left|V_{ij}^{mod}\right|^2 \left|J_j\right|^2 / \sigma_{ij,\Delta t}^2}}$$

- For a ~uniform array (~same sensitivity on all baselines, ~same calibration magnitude on all antennas) and point-like calibrator:

$$\sigma_{J_i} \approx \frac{\sigma_{ij,\Delta t}}{\left|V^{mod}\right| \left\langle \left|J_j\right| \right\rangle \sqrt{N_{ant} - 1}}$$

- Calibration error decreases with increasing calibrator strength *and* square-root of N_{ant} (c.f. baseline-based calibration).
- Other properties of the antenna-based solution:
  - Minimal degrees of freedom (N_{ant} factors, $N_{ant}(N_{ant}-1)/2$ measurements)
  - Net calibration for a baseline involves a phase difference, so *absolute* directional information is lost
  - Closure…

Antenna-based Calibration and Closure

- Success of synthesis telescopes relies on antenna-based calibration
  - Fundamentally, any information that can be factored into antenna-based terms, could *be* antenna-based effects, and not source visibility
  - For $N_{ant} > 3$, source visibility information cannot be *entirely* obliterated by any antenna-based calibration
- Observables independent of antenna-based calibration:
  - Closure phase (3 baselines):

$$\phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} = \left(\phi_{ij}^{true} + \theta_i - \theta_j\right) + \left(\phi_{jk}^{true} + \theta_j - \theta_k\right) + \left(\phi_{ki}^{true} + \theta_k - \theta_i\right)$$

$$= \phi_{ij}^{true} + \phi_{jk}^{true} + \phi_{ki}^{true}$$

  - Closure amplitude (4 baselines):

$$\left|\frac{V_{ij}^{obs} V_{kl}^{obs}}{V_{ik}^{obs} V_{jl}^{obs}}\right| = \left|\frac{J_i J_j V_{ij}^{true} J_k J_l V_{kl}^{true}}{J_i J_k V_{ik}^{true} J_j J_l V_{jl}^{true}}\right| = \left|\frac{V_{ij}^{true} V_{kl}^{true}}{V_{ik}^{true} V_{jl}^{true}}\right|$$

- Baseline-based calibration formally violates closure!

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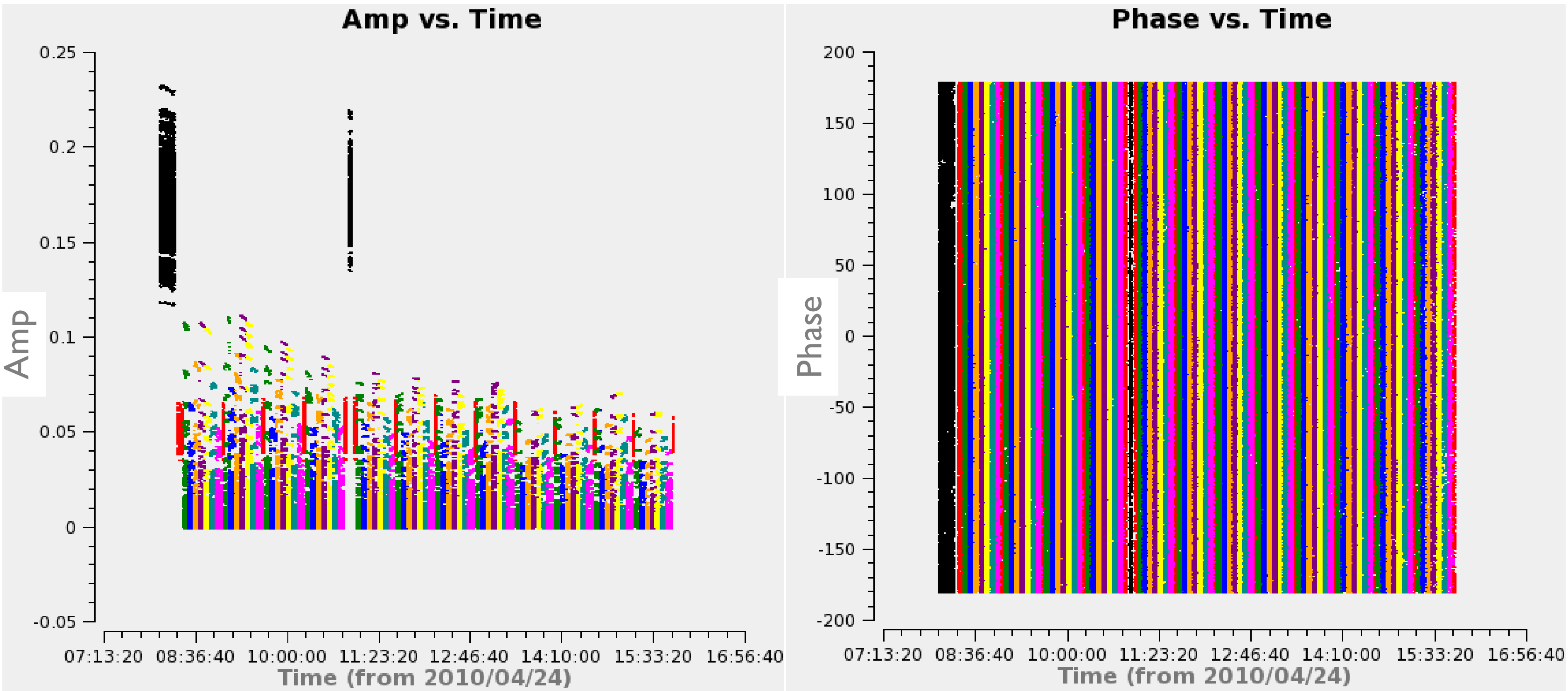
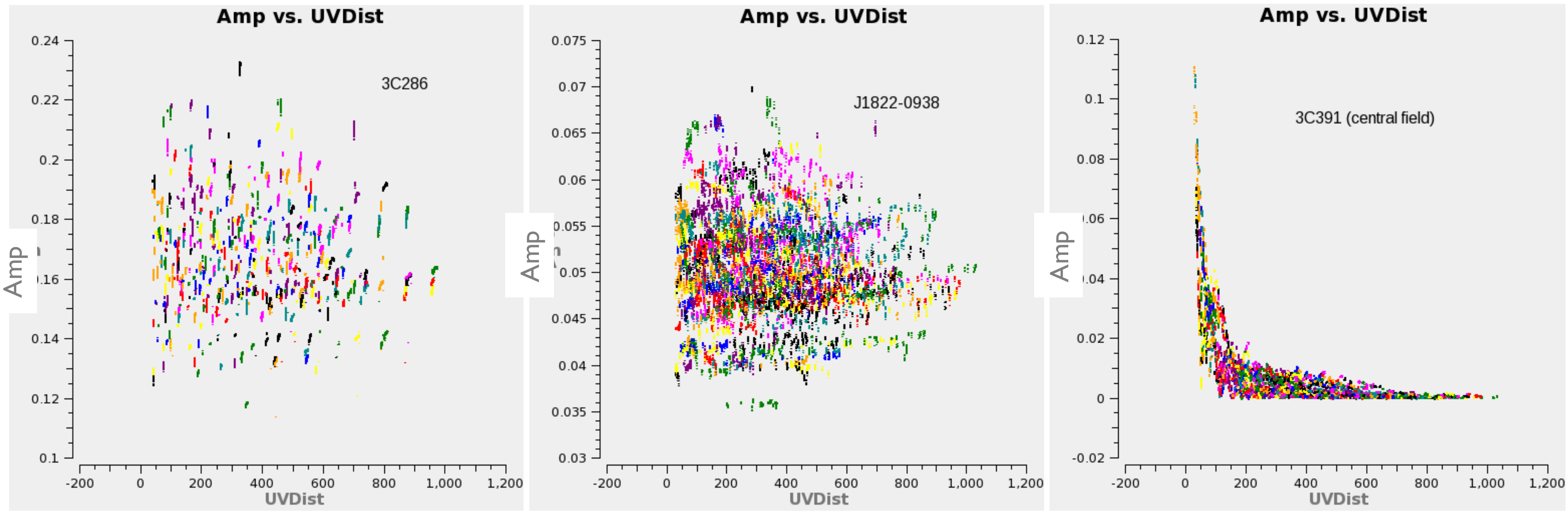
Simple Scalar Calibration Example

- Array:
  - EVLA D-configuration (Apr 2010)
- Sources:
  - Science Target: 3C391 (7 mosaic pointings)
  - Near-target calibrator: J1822-0938 (~11 deg from target; unknown flux density, assumed 1 Jy)
  - Flux Density calibrator: 3C286 (7.747 Jy, essentially unresolved)
- Signals:
  - RR correlation only for this illustration (total intensity only)
  - One spectral window centered at 4600 MHz, 128 MHz bandwidth
  - 64 observed spectral channels averaged with normalized bandpass calibration applied (this illustration considers only the time-dependent 'gain' calibration)
  - (extracted from a continuum polarimetry mosaic observation)

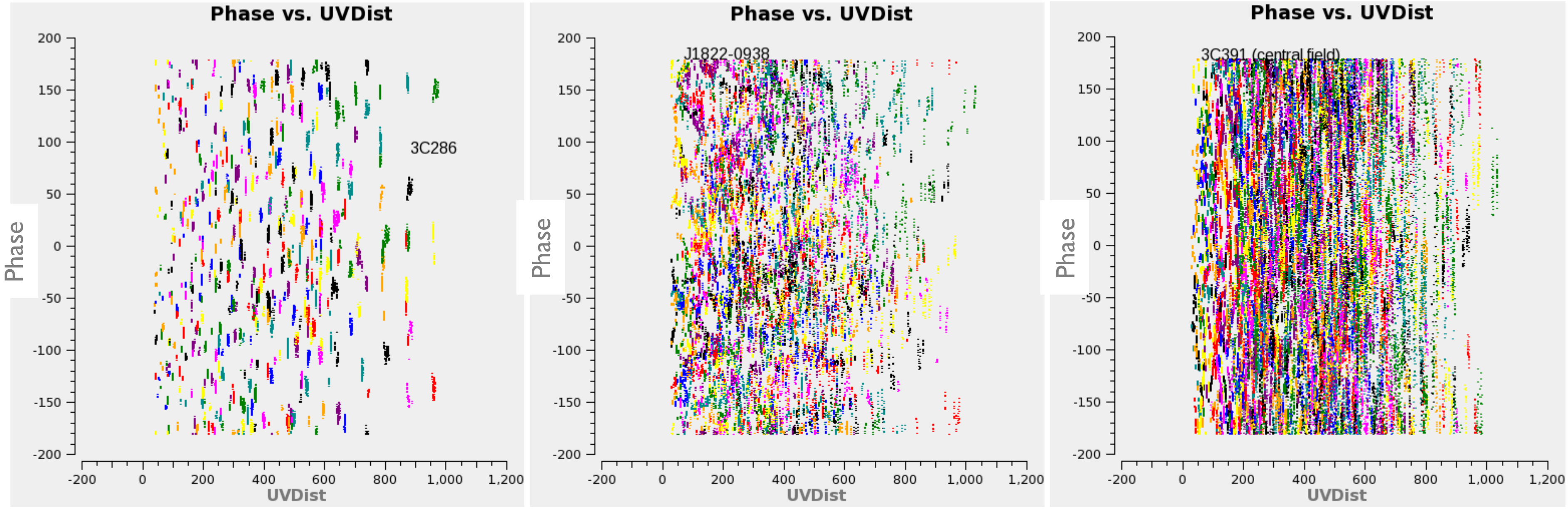
Views of the Uncalibrated Data



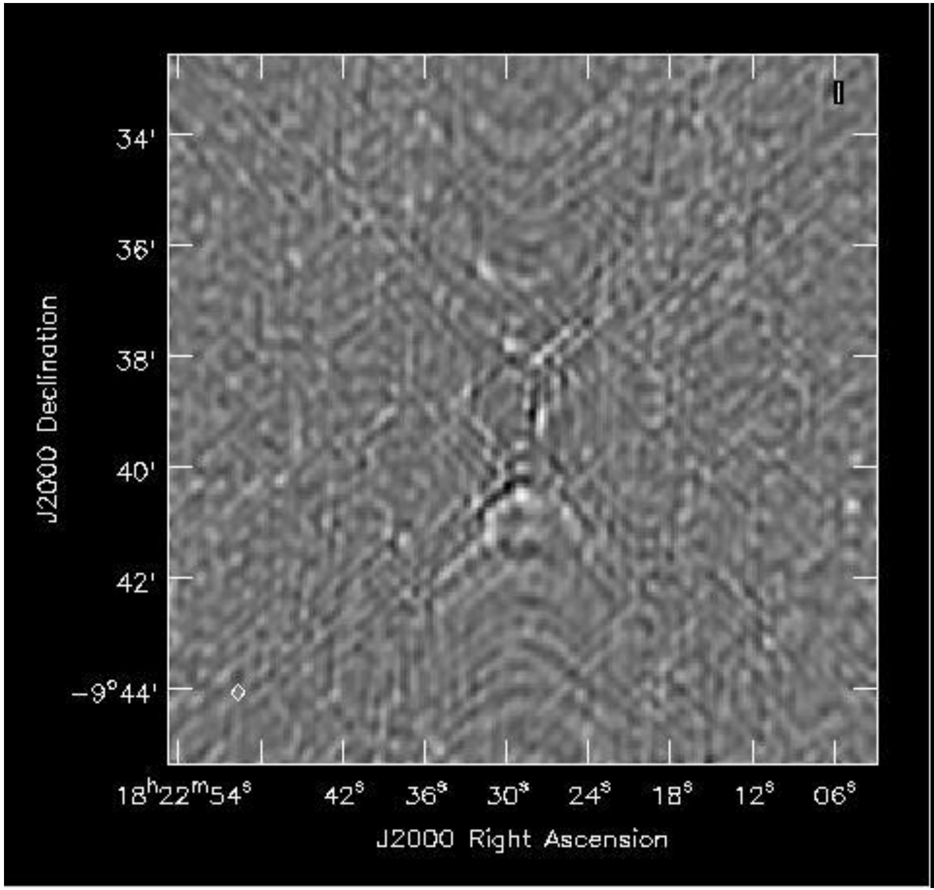
Views of the Uncalibrated Data



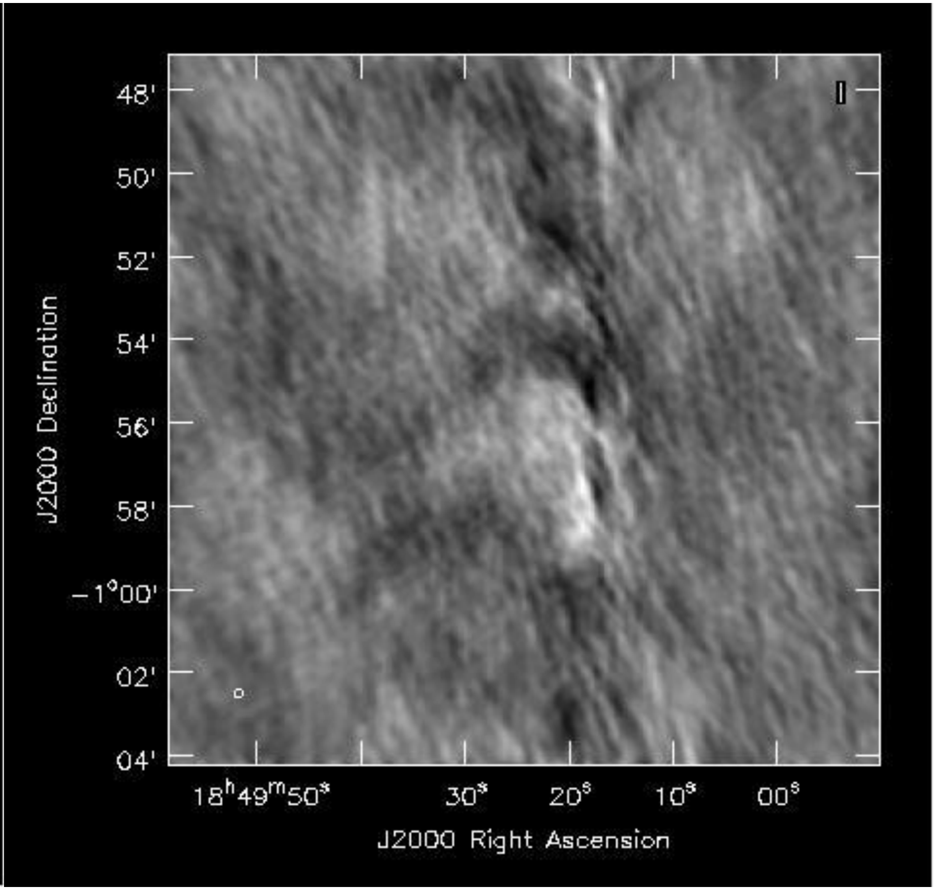
Views of the Uncalibrated Data



Uncalibrated Images



J1822-0938
(calibrator)



3C391
(science)

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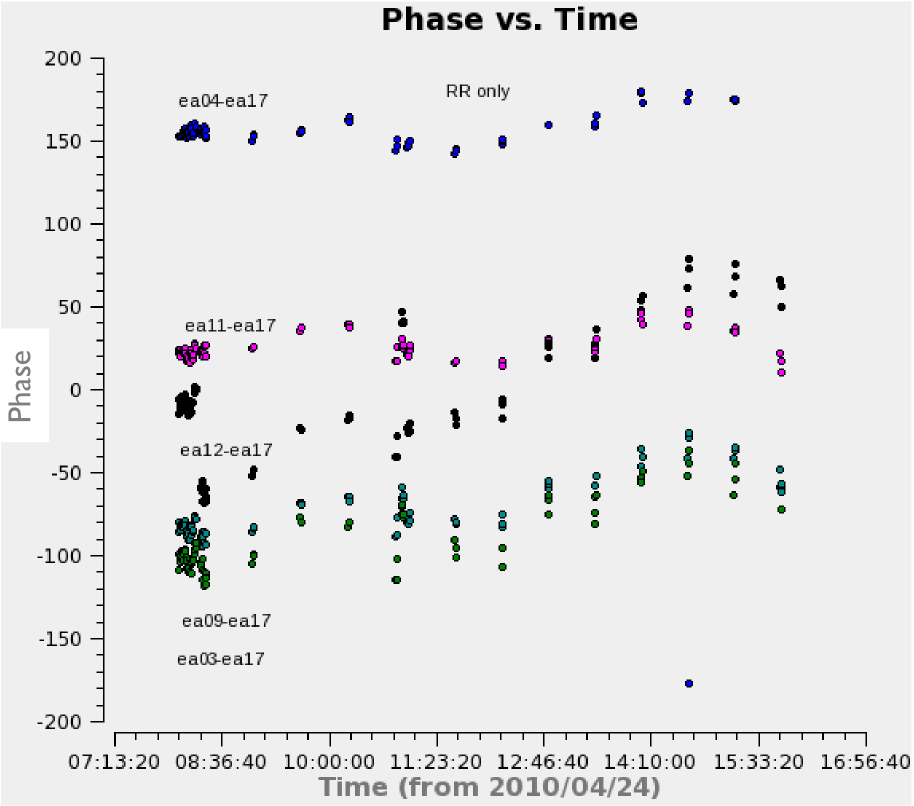
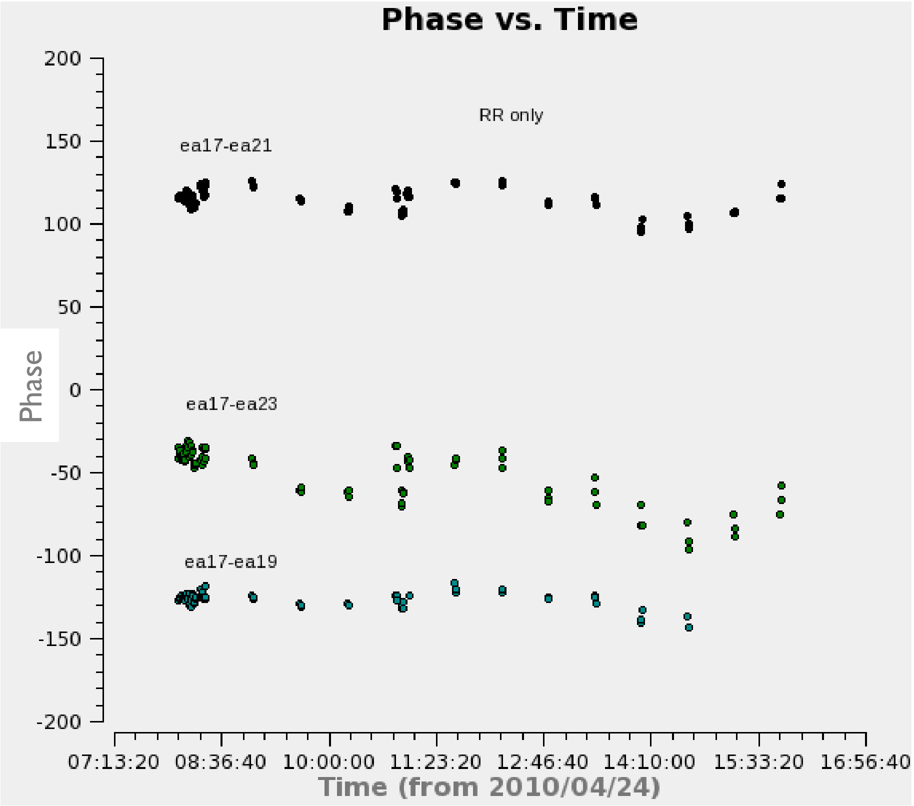
Rationale for Antenna-based Calibration





The Calibration Process

- Solve for antenna-based gain factors for each scan on all calibrators *(V^{mod}=S* for f.d. calibrator; *V^{mod}=1.0* for others) :

$$V_{ij}^{obs} = G_i G_j^* V_{ij}^{mod}$$

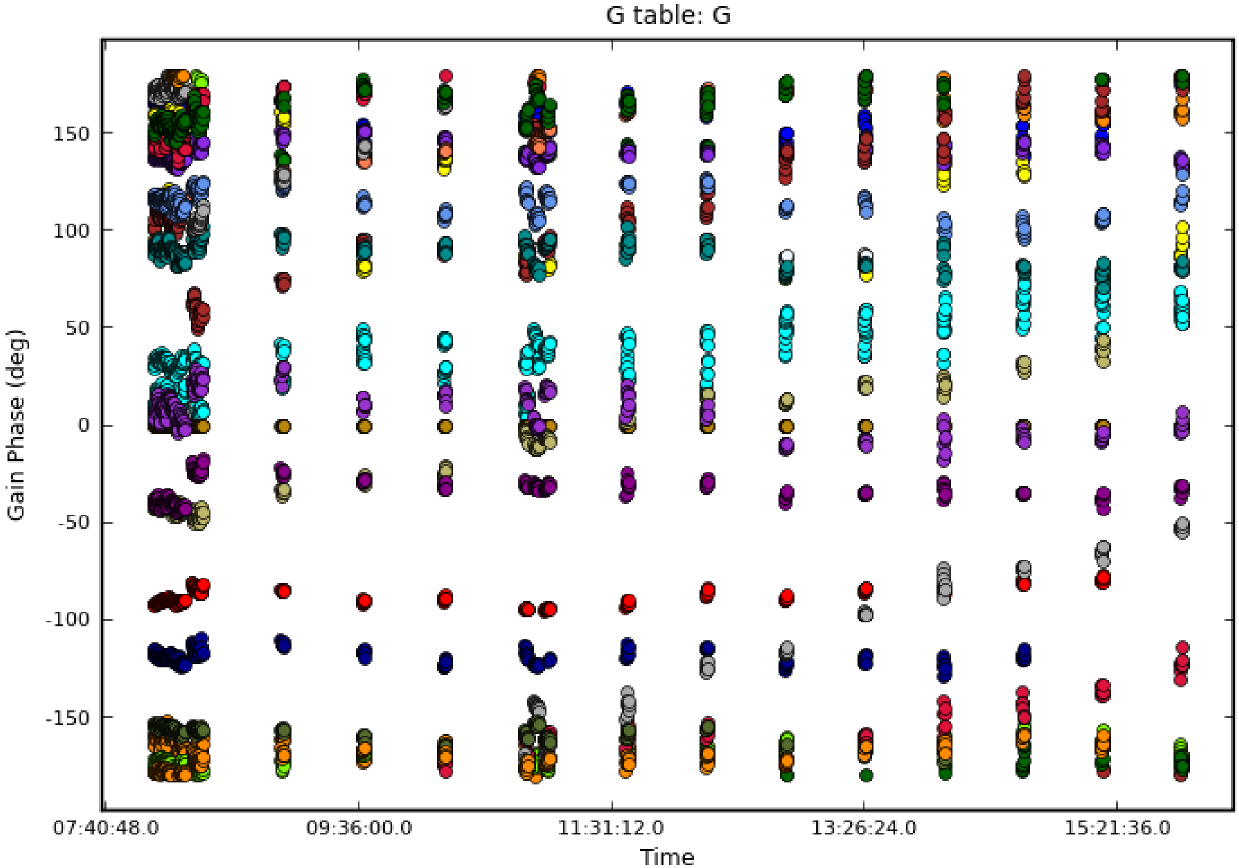
- Bootstrap flux density scale by enforcing gain consistency over all calibrators:

$$\left\langle G_i / G_i(fd\ cal) \right\rangle_{time, antennas} = 1.0$$

- Correct data (interpolate, as needed):

$$V_{ij}^{cor} = G_i^{-1} G_j^{*-1} V_{ij}^{obs}$$

The Antenna-based Calibration Solution



- Reference antenna: ea21 (phase = 0)

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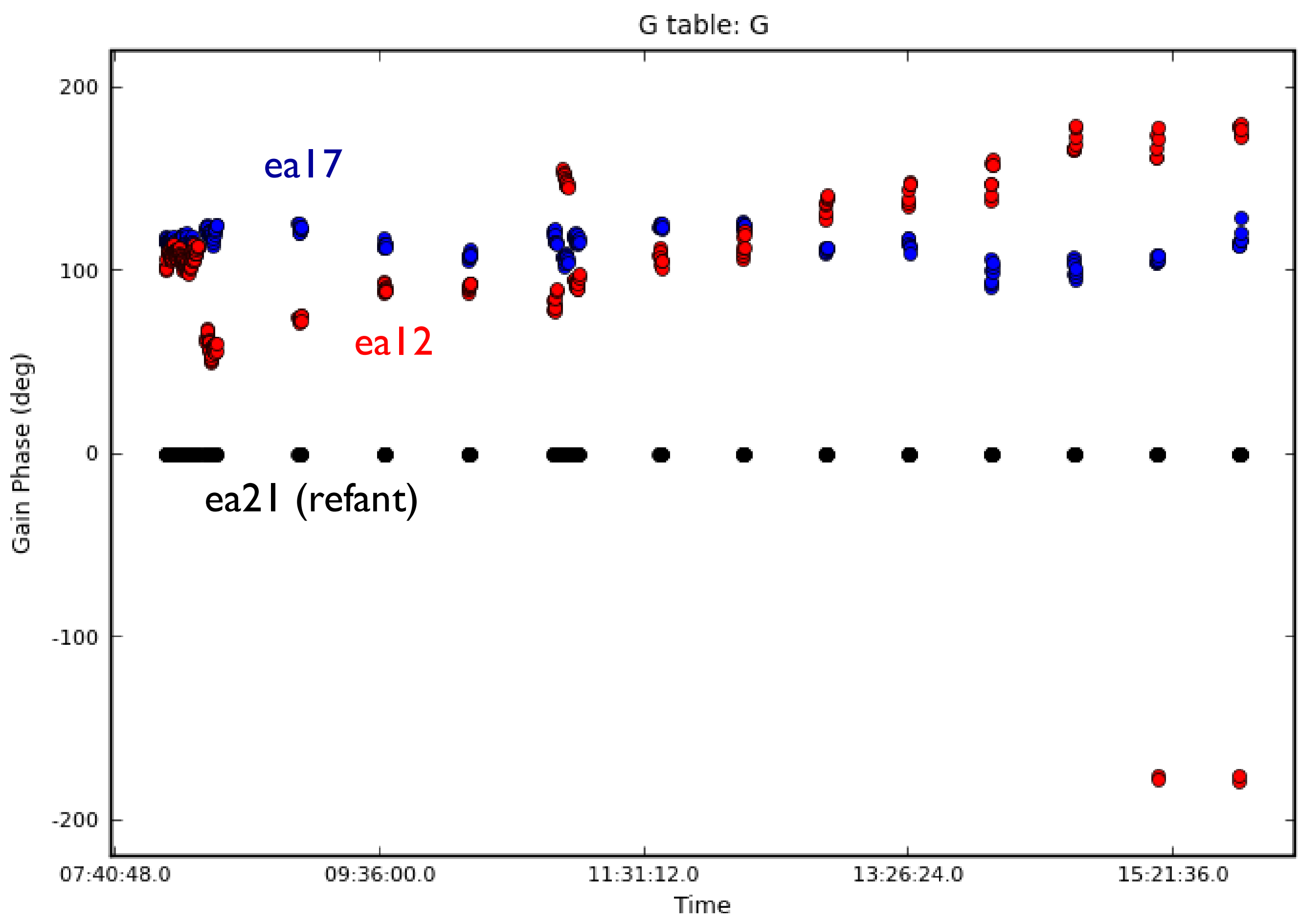
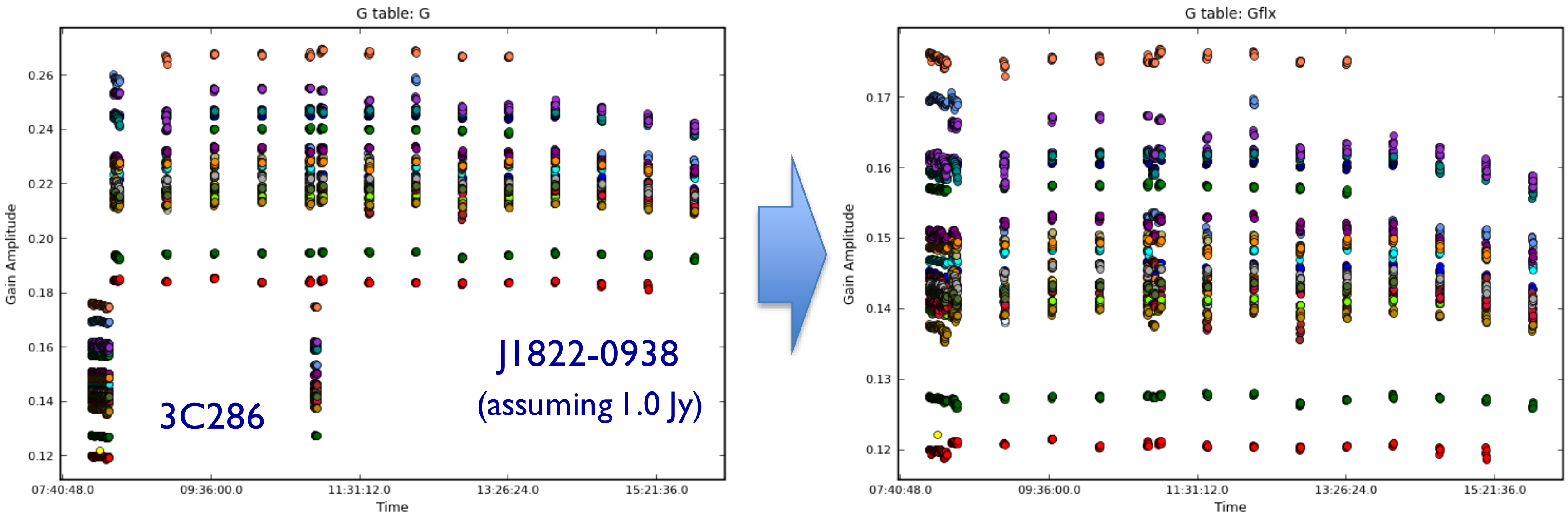
The Antenna-based Calibration Solution

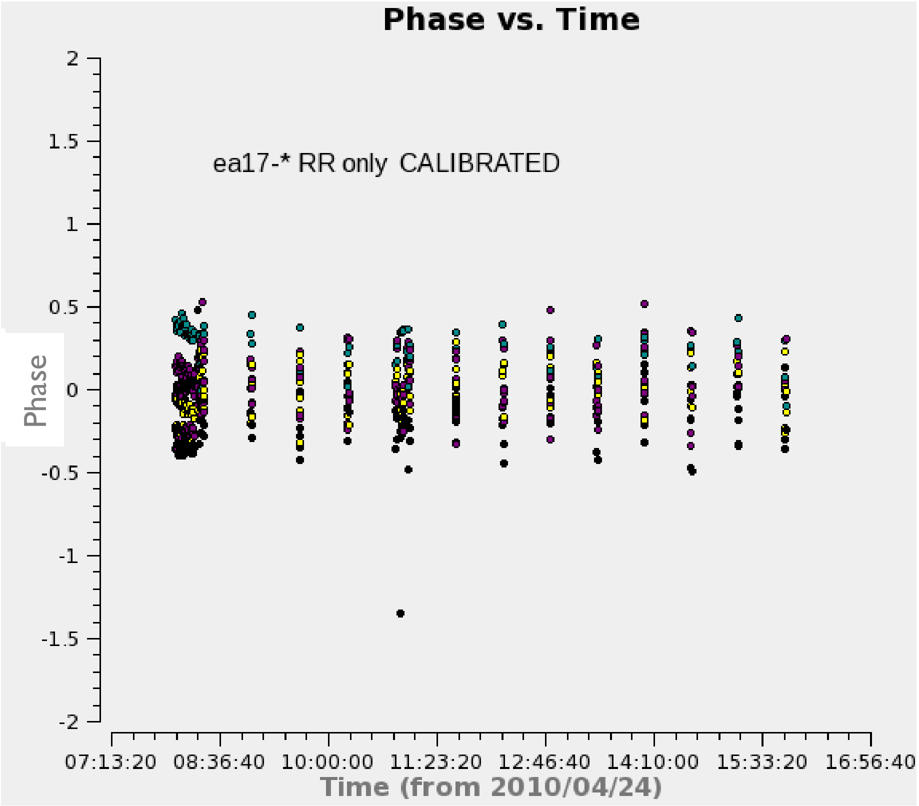


The Antenna-based Calibration Solution



- 3C286's gains have correct scale
- Thus, J1822-0938 is 2.32 Jy (not 1.0 Jy, as assumed)

Effect of Antenna-based Calibration



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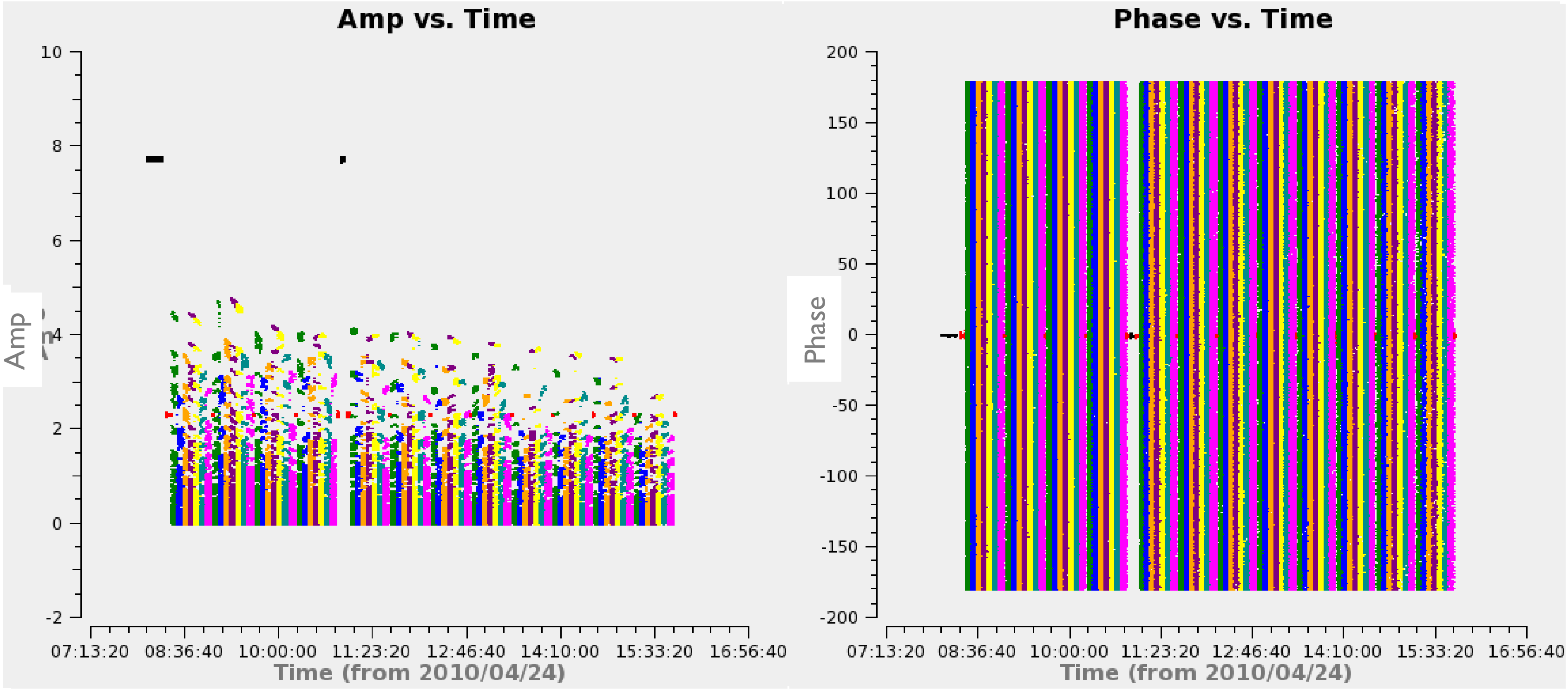
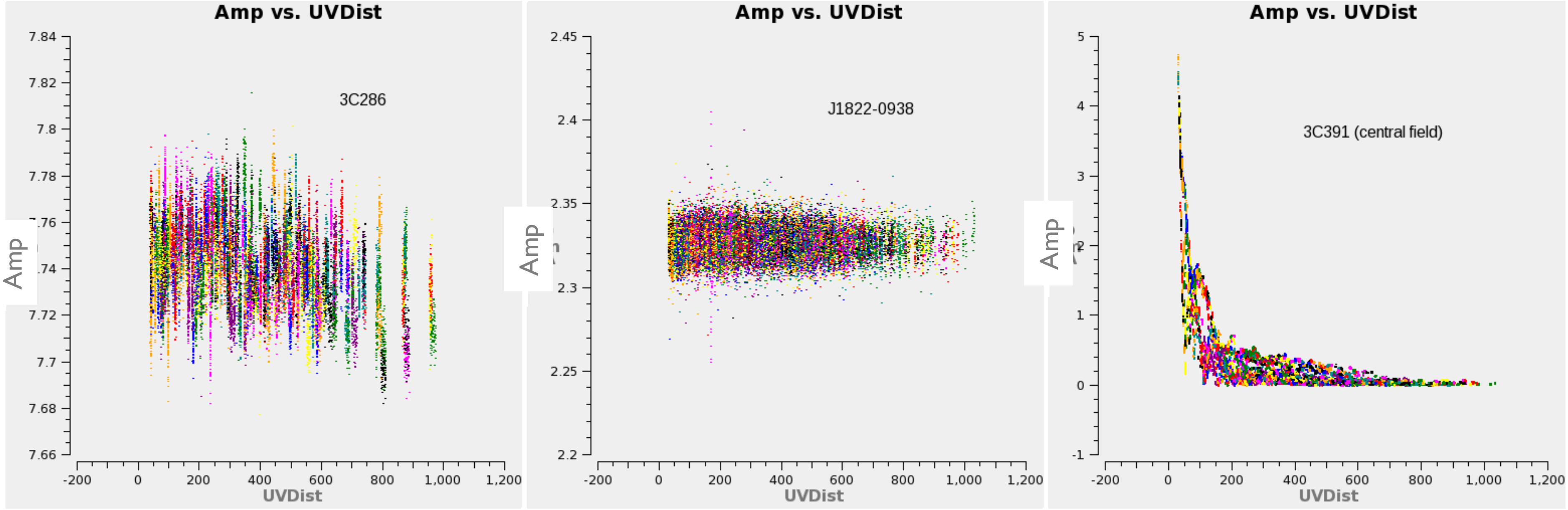
Effect of Antenna-based Calibration



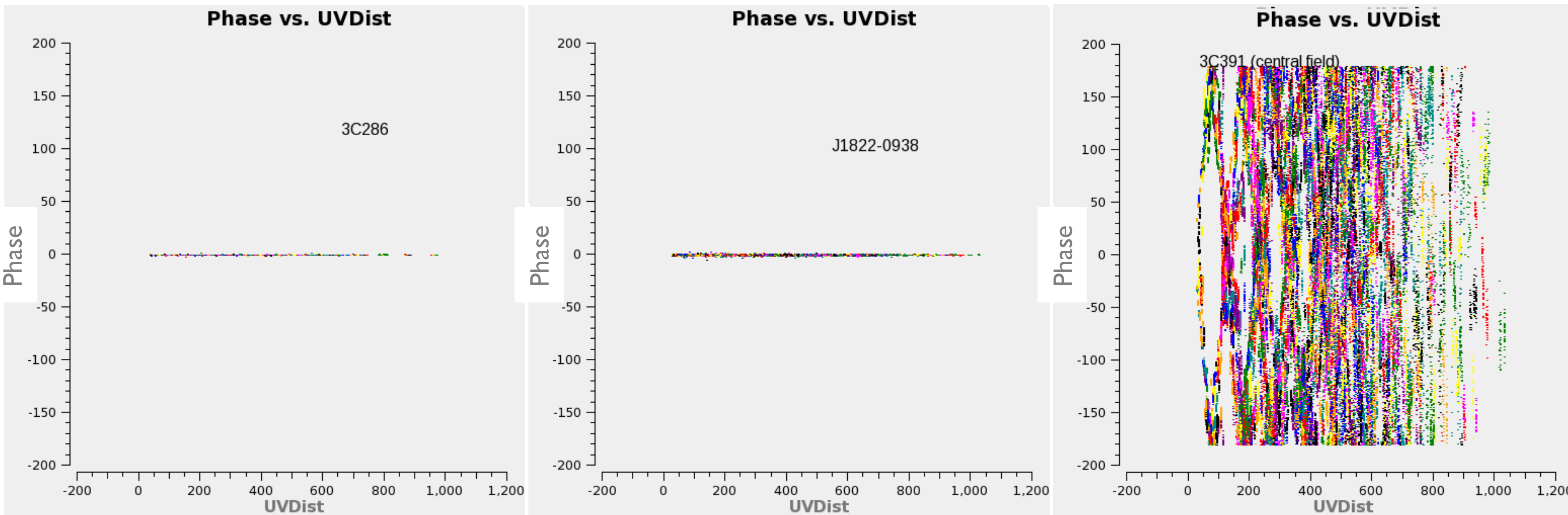
CALIBRATED

Effect of Antenna-based Calibration



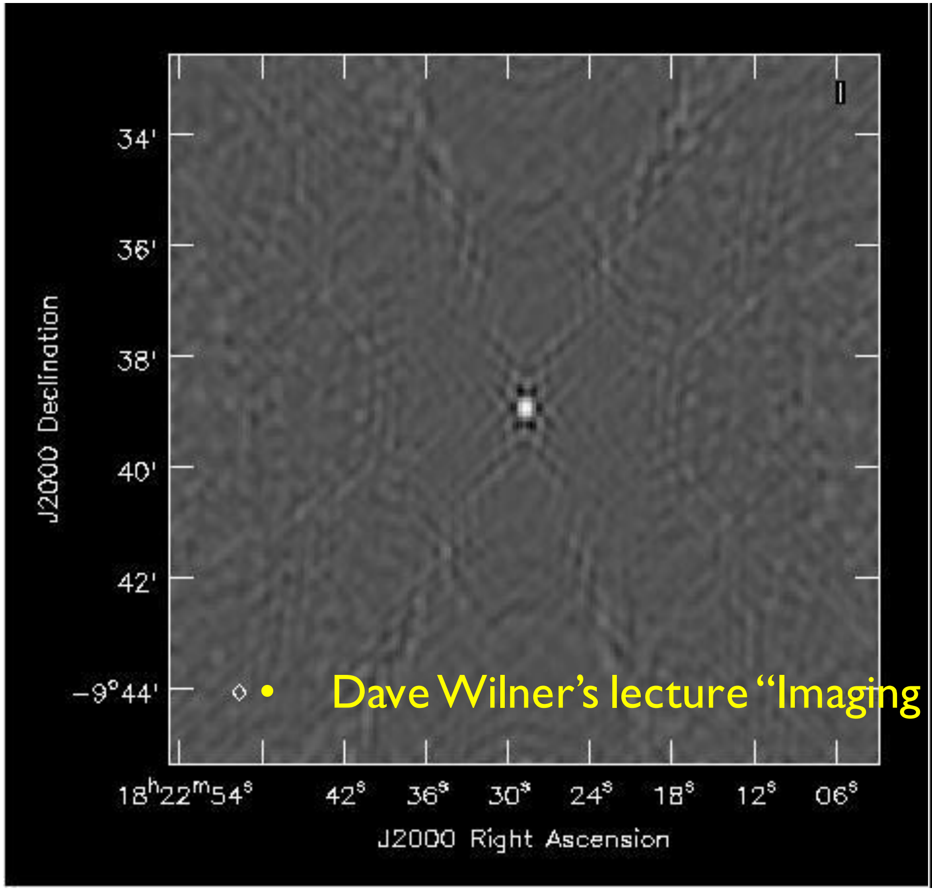
CALIBRATED

Effect of Antenna-based Calibration

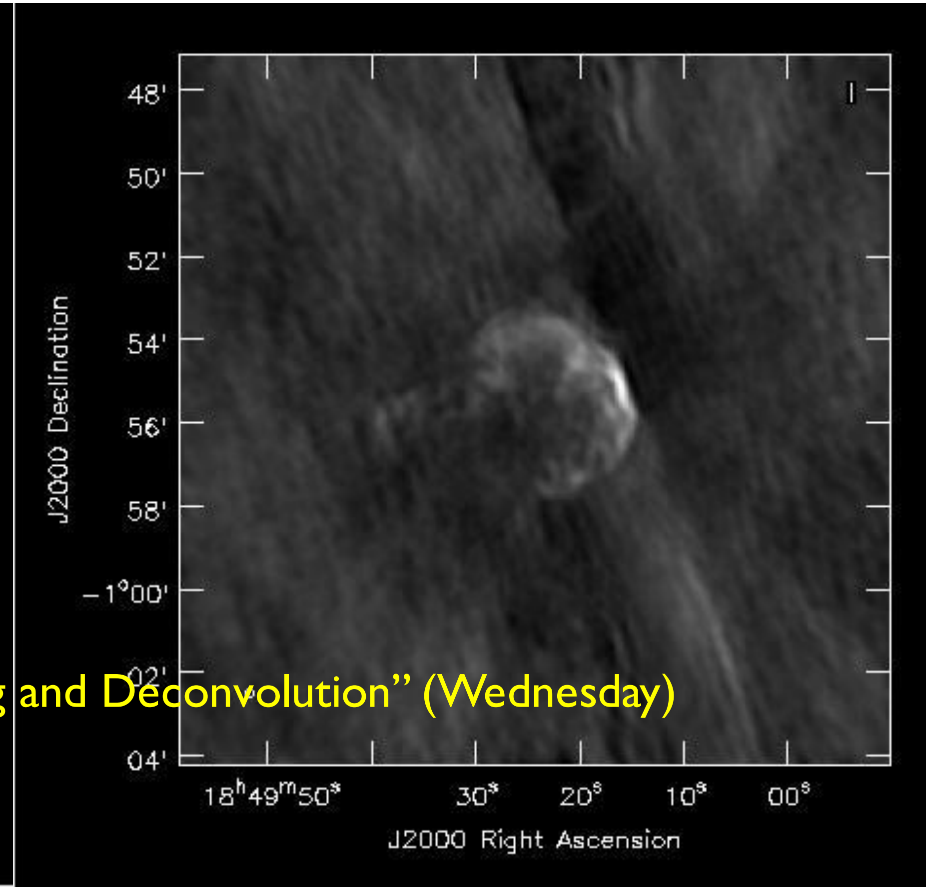


CALIBRATED

Calibration Effect on Imaging



J1822-0938
(calibrator)



3C391
(science)

- Dave Wilner's lecture "Imaging and Deconvolution" (Wednesday)

Evaluating Calibration Performance

- Are solutions continuous?
  - Noise-like solutions are just that—noise (beware calibration of pure noise generates a spurious point source)
  - Discontinuities indicate instrumental glitches (interpolate with care)
  - Any additional editing required?
- Are calibrator data fully described by antenna-based effects?
  - Phase and amplitude *closure errors* are the baseline-based residuals
  - Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration components
    - Crystal Brogan's lecture: "Advanced Calibration" (Wednesday)
- Any evidence of unsampled variation? Is interpolation of solutions appropriate?
  - Reduce calibration timescale, if SNR permits
- Greg Taylor's lecture: "Error Recognition" (Wednesday)

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Summary of Scalar Example

- Dominant calibration effects are ***antenna-based***
  - Minimizes degrees of freedom
  - More precise
  - Preserves closure
  - Permits higher dynamic range *safely*!

- Point-like calibrators effective
- Flux density bootstrapping

Generalizations

- Full-polarization Matrix Formalism
- Calibration Effects Factorization
- Calibration Heuristics and 'Bootstrapping'

Full-Polarization Formalism (Matrices!)

- Need dual-polarization basis (*p*,*q*) to fully sample the incoming EM wave front, where *p*,*q* = *R*,*L* (circular basis) or *p*,*q* = *X*,*Y* (linear basis):

$$\vec{I}_{circ} = \overleftrightarrow{S}_{circ} \vec{I}_{Stokes}$$

$$\begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

$$\vec{I}_{lin} = \overleftrightarrow{S}_{lin} \vec{I}_{Stokes}$$

$$\begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

- Devices can be built to sample these circular (R,L) or linear (X,Y) basis states in the signal domain (Stokes Vector is defined in "power" domain)
- Some components of J_i involve mixing of basis states, so dual-polarization matrix description desirable or even required for proper calibration

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Full-Polarization Formalism: Signal Domain

- Substitute:

$$\vec{s}_i = \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i \qquad \overleftrightarrow{J}_i = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_i$$

- The *Jones matrix* thus corrupts the vector wavefront signal as follows:

$$\vec{s}\,'_i = \overleftrightarrow{J}_i \vec{s}_i \qquad \text{(sky integral omitted)}$$

$$\begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_i \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i$$

$$= \begin{pmatrix} J^{p\to p} s^p + J^{q\to p} s^q \\ J^{p\to q} s^p + J^{q\to q} s^q \end{pmatrix}_i$$

Full-Polarization Formalism: Correlation - I

- Four correlations are possible from two polarizations. The *outer product* represents correlation in the matrix formalism:

$$\vec{V}_{ij}^{true} = \left\langle \vec{s}_i \otimes \vec{s}_j^{\,*} \right\rangle = \left\langle \begin{pmatrix} s^p \\ s^q \end{pmatrix} \otimes \begin{pmatrix} s^{*p} \\ s^{*q} \end{pmatrix} \right\rangle = \begin{pmatrix} \left\langle s_i^p \cdot s_j^{*p} \right\rangle \\ \left\langle s_i^p \cdot s_j^{*q} \right\rangle \\ \left\langle s_i^q \cdot s_j^{*p} \right\rangle \\ \left\langle s_i^q \cdot s_j^{*q} \right\rangle \end{pmatrix}$$

- Observed visibilities (note outer product identity):

$$\vec{V}_{ij}^{obs} = \left\langle \vec{s}_i^{\,\prime} \otimes \vec{s}_j^{\,\prime *} \right\rangle = \left\langle \overleftrightarrow{J}_i \vec{s}_i \otimes \overleftrightarrow{J}_j^{*} \vec{s}_j^{\,*} \right\rangle = \left\langle \overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^{*} \right\rangle \left\langle \vec{s}_i \otimes \vec{s}_j^{\,*} \right\rangle = \overleftrightarrow{J}_{ij} \vec{V}_{ij}^{true}$$

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Full-Polarization Formalism: Correlation - II

- The outer product for the Jones matrix:

$$\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^{*} = \begin{pmatrix} J_i^{p\to p} & J_i^{q\to p} \\ J_i^{p\to q} & J_i^{q\to q} \end{pmatrix} \otimes \begin{pmatrix} J_j^{*p\to p} & J_j^{*q\to p} \\ J_j^{*p\to q} & J_j^{*q\to q} \end{pmatrix}$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} & J_i^{p\to p} J_j^{*q\to p} & J_i^{q\to p} J_j^{*p\to p} & J_i^{q\to p} J_j^{*q\to p} \\ J_i^{p\to p} J_j^{*p\to q} & J_i^{p\to p} J_j^{*q\to q} & J_i^{q\to p} J_j^{*p\to q} & J_i^{q\to p} J_j^{*q\to q} \\ J_i^{p\to q} J_j^{*p\to p} & J_i^{p\to q} J_j^{*q\to p} & J_i^{q\to q} J_j^{*p\to p} & J_i^{q\to q} J_j^{*q\to p} \\ J_i^{p\to q} J_j^{*p\to q} & J_i^{p\to q} J_j^{*q\to q} & J_i^{q\to q} J_j^{*p\to q} & J_i^{q\to q} J_j^{*q\to q} \end{pmatrix}$$

  - J_{ij} is a 4x4 *Mueller matrix*
  - This is starting to get ugly…..
  - Synthesis array design driven by minimizing off-diagonal terms!

Full-Polarization Formalism: Correlation - III

- And finally, for fun, expand the correlation of corrupted signals:

$$\vec{V}_{ij}^{\,obs} = \overleftrightarrow{J}_i \vec{s}_i \otimes \overleftrightarrow{J}_j^{*} \vec{s}_j^{\,*}$$

$$= \begin{pmatrix}
J_i^{p\to p} J_j^{*p\to p} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + & J_i^{p\to p} J_j^{*q\to p} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + & J_i^{q\to p} J_j^{*p\to p} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + & J_i^{q\to p} J_j^{*q\to p} \left\langle s_i^q \cdot s_j^{*q} \right\rangle \\
J_i^{p\to p} J_j^{*p\to q} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + & J_i^{p\to p} J_j^{*q\to q} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + & J_i^{q\to p} J_j^{*p\to q} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + & J_i^{q\to p} J_j^{*q\to q} \left\langle s_i^q \cdot s_j^{*q} \right\rangle \\
J_i^{p\to q} J_j^{*p\to p} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + & J_i^{p\to q} J_j^{*q\to p} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + & J_i^{q\to q} J_j^{*p\to p} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + & J_i^{q\to q} J_j^{*q\to p} \left\langle s_i^q \cdot s_j^{*q} \right\rangle \\
J_i^{p\to q} J_j^{*p\to q} \left\langle s_i^p \cdot s_j^{*p} \right\rangle + & J_i^{p\to q} J_j^{*q\to q} \left\langle s_i^p \cdot s_j^{*q} \right\rangle + & J_i^{q\to q} J_j^{*p\to q} \left\langle s_i^q \cdot s_j^{*p} \right\rangle + & J_i^{q\to q} J_j^{*q\to q} \left\langle s_i^q \cdot s_j^{*q} \right\rangle
\end{pmatrix}$$

- UGLY, but we rarely, if ever, need to worry about algebraic detail at this level---just let this occur "inside" the matrix formalism, and work with the matrix short-hand notation

The Matrix Measurement Equation

- We can now write down the Measurement Equation in matrix notation:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^* \right) \overleftrightarrow{S} \vec{I}(l,m) e^{-i2\pi\left(u_{ij}l + v_{ij}m\right)} dl dm$$

  – S maps Stokes parameters onto observed basis

- …and consider how the J_i are products of many effects.

A Dictionary of Calibration Components

- J_i contains many components, in principle:
  - F = ionospheric effects
  - T = tropospheric effects
  - P = parallactic angle
  - X = linear polarization position angle
  - E = antenna voltage pattern
  - D = polarization leakage
  - G = electronic gain
  - B = bandpass response
  - K = geometric compensation
  - M,A = baseline-based corrections

$$\overleftrightarrow{J}_i = \overleftrightarrow{K}_i \overleftrightarrow{B}_i \overleftrightarrow{G}_i \overleftrightarrow{D}_i \overleftrightarrow{E}_i \overleftrightarrow{X}_i \overleftrightarrow{P}_i \overleftrightarrow{T}_i \overleftrightarrow{F}_i$$

- Order of terms follows signal path (right to left)
- Each term has matrix form of J_i with terms embodying its particular algebra (on- vs. off-diagonal terms, etc.)
- Direction-dependent terms must stay inside FT integral
- 'Full' calibration is traditionally a bootstrapping process wherein relevant terms (usually a minority of above list) are considered in decreasing order of dominance, relying on approximate separability

Ionospheric Effects, *F*

$$\overset{\leftrightarrow}{F}^{RL} = e^{i\Delta\phi}\begin{pmatrix} e^{-i\varepsilon} & 0 \\ 0 & e^{i\varepsilon} \end{pmatrix}; \quad \overset{\leftrightarrow}{F}^{XY} = e^{i\Delta\phi}\begin{pmatrix} \cos\varepsilon & \sin\varepsilon \\ -\sin\varepsilon & \cos\varepsilon \end{pmatrix}$$

- The ionosphere introduces a dispersive path-length offset: $\Delta\phi \propto \frac{N_e\,[\mathrm{m}^{-2}]}{\nu}$
  - More important at lower frequencies (<5 GHz)
  - Varies more at solar maximum and at sunrise/sunset, when ionosphere is most active and variable
  - Direction-dependent within wide field-of-view
- The ionosphere is *birefringent*: Faraday rotation: $\varepsilon \propto \frac{\int B_{\parallel}\, n_e\,[\mathrm{m}^{-3}]}{\nu^2}$
  - as high as 20 rad/m^2 during periods of high solar activity will rotate linear polarization position angle by $\varepsilon = 50$ degrees at 1.4 GHz
  - Varies over the array, and with time as line-of-sight magnetic field and electron density vary, violating the usual assumption of stability in position angle calibration
- Book: Chapter 5, sect. 4.3,4.4,9.3; Chapter 6, sect. 6; Chapter 29, sect.3
- Lincoln Greenhill's lecture: "Low Frequency Interferometry" (Monday)

Tropospheric Effects, *T*

$$\overleftrightarrow{T} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- The troposphere causes polarization-independent amplitude and phase effects due to emission/opacity and refraction, respectively
    - Up to 2.3m excess path length at zenith compared to vacuum
    - Higher noise contribution, less signal transmission: Lower SNR
    - Most important at $\nu > 20$ GHz where water vapor and oxygen absorb/emit
    - Zenith-angle-dependent (more troposphere path nearer horizon)
    - Clouds, weather = variability in phase and opacity; may vary across array
    - Water vapor radiometry (estimate phase from power measurements)
    - Phase transfer from low to high frequencies (delay calibration)
- Book: Chapter 5: sect. 4.3,4.4; Chapter 28, sect. 3
- ALMA!

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Parallactic Angle, *P*

$$\overleftrightarrow{P}^{RL} = \begin{pmatrix} e^{-i\chi} & 0 \\ 0 & e^{i\chi} \end{pmatrix}; \quad \overleftrightarrow{P}^{XY} = \begin{pmatrix} \cos\chi & \sin\chi \\ -\sin\chi & \cos\chi \end{pmatrix}$$

- Visibility phase variation due to changing orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos l \sin h(t)}{\sin l \cos\delta - \cos l \sin\delta \cos h(t)}\right)$$

$$l = \text{latitude}, \quad h(t) = \text{hour angle}, \quad \delta = \text{declination}$$

  - Rotates the position angle of linearly polarized radiation
  - Analytically known, and its variation provides leverage for determining polarization-dependent effects

- Book: Chapter 6, sect. 2.1
- Michiel Brentjens' lecture: "Polarization in Interferometry" (today!)

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Linear Polarization Position Angle, X

$$\overleftrightarrow{X}^{RL} = \begin{pmatrix} e^{-i\Delta\chi} & 0 \\ 0 & e^{i\Delta\chi} \end{pmatrix}; \quad \overleftrightarrow{X}^{XY} = \begin{pmatrix} \cos\Delta\chi & \sin\Delta\chi \\ -\sin\Delta\chi & \cos\Delta\chi \end{pmatrix}$$

- Configuration of optics and electronics causes a linear polarization position angle offset
- Can be treated as an offset to the parallactic angle, P
- Calibrated by registration with a strongly polarized source with known polarization position angle (e.g., flux density calibrators)
- For circular feeds, this is a phase difference between the R and L polarizations, which is frequency-dependent (a R-L phase bandpass)
- For linear feeds, this is the orientation of the dipoles in the frame of the telescope

- Michiel Brentjens' lecture: "Polarization in Interferometry" (today!)

Antenna Voltage Pattern, E

$$\overleftrightarrow{E}^{pq} = \begin{pmatrix} E^p(l,m) & 0 \\ 0 & E^q(l,m) \end{pmatrix}$$

- Antennas of all designs have direction-dependent gain within field-of-view
  - Important when region of interest on sky comparable to or larger than λ/D
  - Important at lower frequencies where radio source surface density is greater and wide-field imaging techniques required
  - Beam squint: E^p and E^q offset, yielding spurious polarization
  - Sky rotates within field-of-view for alt-az antennas, so off-axis sources move through the pattern
  - Direction dependence of polarization leakage (*D*) may be included in *E* (off-diagonal terms then non-zero)
- Shape and efficiency of the voltage pattern may change with zenith angle: 'gain curve'

- Book: Chapters 19, 20
- Sanjay Bhatnagar's lecture: "Wide Field Imaging I" (Thursday)
- Juergen Ott's lecture: "Wide Field Imaging II" (Thursday)

Polarization Leakage, *D*

$$\overleftrightarrow{D} = \begin{pmatrix} 1 & d^{p} \\ d^{q} & 1 \end{pmatrix}$$

- Antenna & polarizer are not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed feeds have *d* ~ a few percent or less
  - A geometric property of the optics design, so frequency-dependent
  - For *R,L* systems, total-intensity imaging affected as *~dQ, dU,* so only important at high dynamic range (*Q,U,d each ~few %*, typically)
  - For *R,L* systems, linear polarization imaging affected as *~dI,* so almost always important
  - For small arrays (no *differential* parallactic angle coverage), only relative D solution is possible from standard linearized solution, so parallel-hands cannot be corrected absolutely (closure errors)
- Best calibrator: Strong, point-like, observed over large range of parallactic angle (to separate source polarization from *D*)

- Book: Chapter 6
- Michiel Brentjens' lecture: "Polarization in Interferometry" (today!)

"Electronic" Gain, G

$$\overleftrightarrow{G}^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

- Catch-all for most amplitude and phase effects introduced by antenna electronics and other generic effects
    - Most commonly treated calibration component
    - Dominates other effects for most standard observations
    - Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
    - Includes any internal system monitoring, like EVLA switched power calibration
    - Often also includes tropospheric and (on-axis) ionospheric effects which are typically difficult to separate uniquely from the electronic response
    - Excludes frequency dependent effects (see *B*)
- Best calibrator: strong, point-like, near science target; observed often enough to track expected variations
    - Also observe a flux density standard
- Book: Chapter 5

Bandpass Response, *B*

$$\overleftrightarrow{B}^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

- *G*-like component describing frequency-dependence of antenna electronics, etc.
  - Filters used to select frequency passband not square
  - Optical and electronic reflections introduce ripples across band
  - Often assumed time-independent, but not necessarily so
  - Typically (but not necessarily) normalized
  - ALMA Tsys is a "bandpass"
- Best calibrator: strong, point-like; observed long enough to get sufficient per-channel SNR, and often enough to track variations

- Book: Chapter 12, sect. 2
- David Meier's lecture: "Analysis of Data Cubes" (Wednesday)

Geometric Compensation, *K*

$$\overleftrightarrow{K}^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

- Must get geometry right for Synthesis Fourier Transform relation to work in real time
  - Antenna positions (geodesy)
  - Source directions (time-dependent in topocenter!) (astrometry)
  - Clocks
  - Electronic path-lengths introduce delays (polarization, spw differences)
  - Longer baselines generally have larger relative geometry errors, especially if clocks are independent (VLBI)
  - Importance scales with frequency
- *K* is a clock- & geometry-parameterized version of *G* (see chapter 5, section 2.1, equation 5-3 & chapters 22, 23)
  - All-sky observations used to isolate geometry parameters
- Book: Chapter 5, sect. 2.1; Chapters 22, 23
- Matt Lister's lecture: "Very Long Baseline Interferometry" (Wednesday)

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Non-closing Effects: *M, A*

- Baseline-based errors which do not decompose into antenna-based components
  - Digital correlators designed to limit such effects to well-understood and ***uniform*** (not dependent on baseline) scaling laws (absorbed in *f.d.* calibration)
  - Simple noise (additive)
  - Additional errors can result from averaging in time and frequency over variation in antenna-based effects and visibilities (practical instruments are finite!)
  - Instrumental polarization effects in parallel hands
  - Correlated "noise" (e.g., RFI)
  - Difficult to distinguish from source structure (visibility) effects
  - Geodesy and astrometry observers consider determination of radio source structure—a baseline-based effect—as a required *calibration* if antenna positions are to be determined accurately
  - Diagonal 4x4 matrices, M_{ij} multiplies, A_{ij} adds

Decoupling Calibration Effects

- Multiplicative gain (G) term will soak up many different effects; known priors should be compensated for separately, especially when direction-dependent differences (e.g., between calibrator and target) will limit the accuracy of calibration transfer:
  - Zenith angle-dependent atmospheric opacity, refraction (T,F)
  - Zenith angle-dependent gain curve (E)
  - Antenna position errors (K)

- Early calibration solves (e.g., G) are always subject to more subtle, uncorrected effects
  - E.g., instrumental polarization (D), which introduces gain calibration errors and causes apparent closure errors in *parallel-hand* correlations
  - When possible, iterate and alternate solves to decouple effects…

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The Full Matrix Measurement Equation

- The total general *Measurement Equation* has the form:

$$\vec{V}_{ij}^{\,obs} = \overleftrightarrow{M}_{ij}\overleftrightarrow{K}_{ij}\overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\int_{sky}\overleftrightarrow{D}_{ij}\overleftrightarrow{E}_{ij}\overleftrightarrow{X}_{ij}\overleftrightarrow{P}_{ij}\overleftrightarrow{T}_{ij}\overleftrightarrow{F}_{ij}\overleftrightarrow{S}\vec{I}(l,m)\,e^{-2\pi i(u_{ij}l+v_{ij}m)}\,dl\,dm + \vec{A}_{ij}$$

  - *S* maps the Stokes vector, *I*, to the polarization basis of the instrument, all calibration terms cast in this basis

- Suppressing the direction-dependence (on-axis calibration):

$$\vec{V}_{ij}^{\,obs} = \overleftrightarrow{M}_{ij}\overleftrightarrow{K}_{ij}\overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\overleftrightarrow{D}_{ij}\overleftrightarrow{E}_{ij}\overleftrightarrow{X}_{ij}\overleftrightarrow{P}_{ij}\overleftrightarrow{T}_{ij}\overleftrightarrow{F}_{ij}\vec{V}_{ij}^{\,true} + \vec{A}_{ij}$$

- Generally, only a subset of terms are considered, though highest-dynamic range observations may require more
- Solve for terms in decreasing order of dominance, iterate to isolate
- (Non-trivial direction-dependent solutions involve convolutional treatment of the visibilities, and is coupled to the imaging and deconvolution process)

Solving the Measurement Equation

- Formally, solving for any antenna-based visibility calibration component is always the same general non-linear fitting problem:

$$\vec{V}_{ij}^{\,corrected\ \cdot obs} = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^{\,*} \right) \vec{V}_{ij}^{\,corrupted\ \cdot mod}$$

  - Observed and Model visibilities are corrected/corrupted by available prior calibration solutions
  - Resulting solution used as prior in subsequent solves, as necessary
  - Each solution is relative to priors and assumed source model
  - Iterate sequences, as needed → generalized self-calibration
- Viability and accuracy of the overall calibration depends on isolation of different effects using *proper calibration observations*, and *appropriate solving strategies*
- Heuristic mnemonics….

Calibration Heuristics – Spectral Line

Total Intensity Spectral Line (B=bandpass, G=gain):

$$V^{obs} = B\,G\,V^{true}$$

1. Preliminary Gain solve on B-calibrator:

   $V^{obs} = G_B\,V^{mod}$

2. Bandpass Solve (using G_B) on B-calibrator (then discard G_B):

   $V^{obs} = B\,(G_B\,V^{mod})$

3. Gain solve (using inverse of B) on calibrators:

   $(B'\,V^{obs}) = G\,V^{mod}$

4. Flux Density scaling:

   $G \rightarrow G_f$ (enforce gain consistency)

5. Correct with inverted solutions:

   $V^{cor} = G_f'\,B'\,V^{obs}$

6. Image!

Heuristic notation!

Rigorous math notation (antenna-basedness, subscripts, etc.) omitted.

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Calibration Heuristics – Polarimetry

Polarimetry (B=bandpass, G=gain, D=instr. poln, X=pos. ang., P=parallactic ang.):

$$V^{obs} = B\ G\ D\ X\ P\ V^{true}$$

1. Preliminary Gain solve on B-calibrator:

   $$V^{obs} = G_B\ V^{mod}$$

2. Bandpass (B) Solve (using G_B) on B-calibrator (then discard G_B):

   $$V^{obs} = B\ (G_B\ V^{mod})$$

3. Gain (G) solve (using parallactic angle P, inverse of B) on calibrators:

   $$(B'\ V^{obs}) = G\ (P V^{mod})$$

4. Instrumental Polarization (D) solve (using P, inverse of G,B) on instrumental polarization calibrator:

   $$(G'\ B'\ V^{obs}) = D\ (P\ V^{mod})$$

Calibration Heuristics – Polarimetry

5. Polarization position angle solve (using P, inverse of D,G,B) on position angle calibrator:

   $(D' G' B' V^{obs}) = X\ (P\, V^{mod})$

6. Flux Density scaling:

   $G \rightarrow G_f$ (enforce gain consistency)

7. Correct with inverted solutions:

   $V^{cor} = P' X' D' G_f' B' V^{obs}$

8. Image!

- To use external priors, e.g., T (opacity), K (ant. position errors), E (gaincurve), revise step 3 above as:

3. $(B' K' V^{obs}) = G\ (E\, P\, T\, V^{mod})$

– and carry T, K, and E forward to subsequent steps

New Calibration Challenges (EVLA, ALMA)

- Bandpass Calibration
  - Parameterized solutions (narrow-bandwidth, high resolution regime)
  - Spectrum of calibrators, incl. structure (wide absolute bandwidth regime)
- 'Delay-aware' gain (self-) calibration
  - Troposphere and Ionosphere introduce time-variable phase effects which are easily parameterized in frequency and should be (c.f. merely sampling the calibration in frequency)
- Frequency-dependent Instrumental Polarization
  - Contribution of geometric optics is wavelength-dependent (standing waves)
- Frequency-dependent voltage pattern
- Wide-field voltage pattern accuracy (sidelobes, rotation)
- Direction-dependent components
  - E.g., Instrumental Polarization (polarized voltage pattern)
  - Couples to the imaging process
- Increased sensitivity: Can implied dynamic range be reached by conventional calibration and imaging techniques?

Summary

- Determining calibration is as important as determining source structure—can't have one without the other
- Data examination and editing an important part of calibration
- Calibration dominated by antenna-based effects, permits efficient, accurate and defensible separation of calibration from astronomical information (satisfies closure)
- Full calibration formalism algebra-rich, but is *modular*
- Calibration an iterative process, improving various components in turn, as needed
- Point sources are the best calibrators
- Observe calibrators according requirements of calibration components