Wide-Field Imaging: I

S. Bhatnagar
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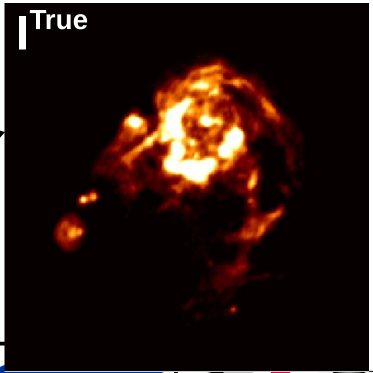
Followup issues: Deconvolution

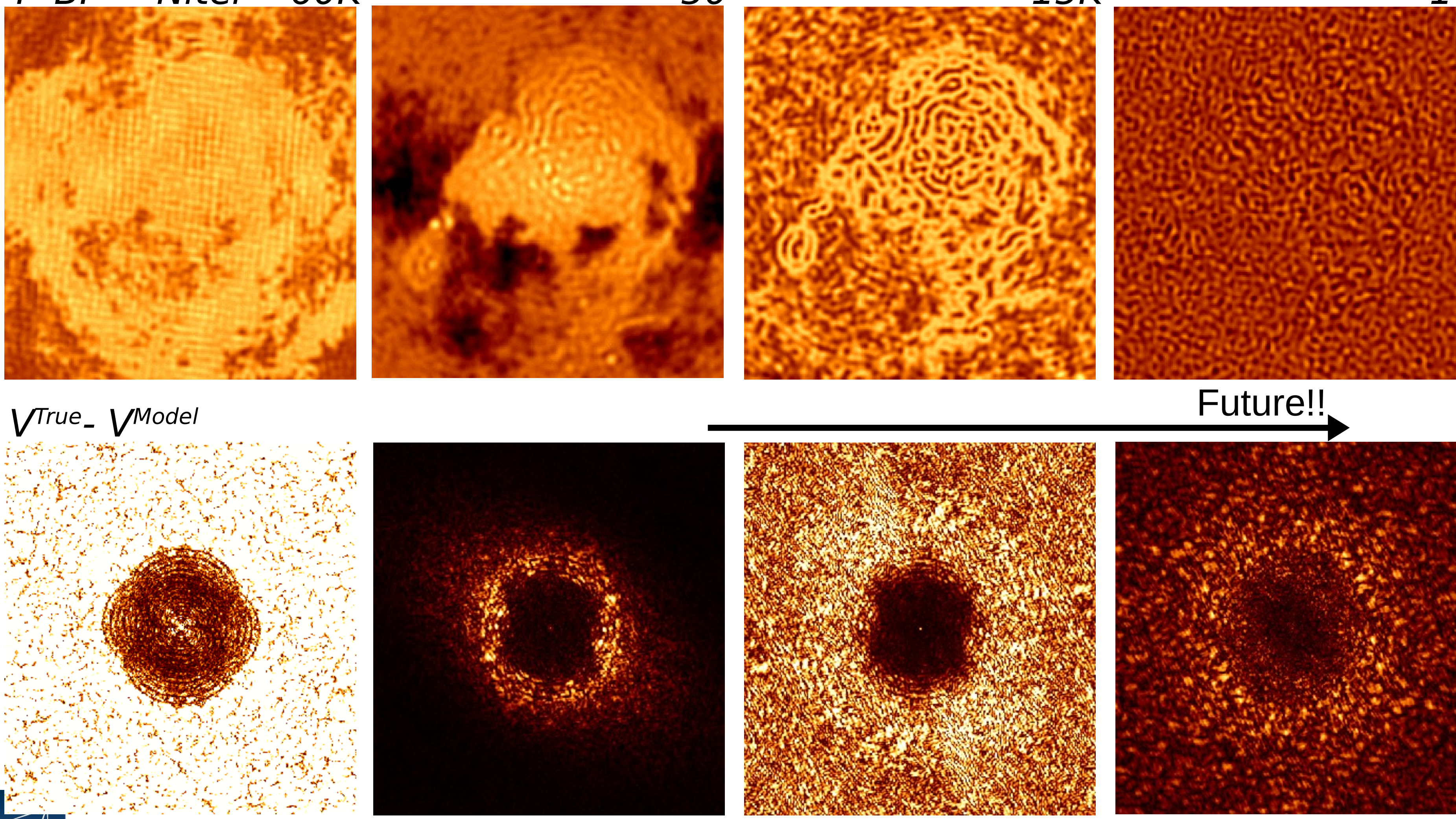
- "Clean components" are nothing *fundamental.* Just a compact representation of the Model Image (CASA holds an Image – not CC)
- What exactly is "loop gain"?
  - $I^M_{i+1} = I^M_i + \alpha \frac{\partial \chi^2}{\partial(Parameters)}$ *...just the step-size used in any iterative minimization*
  - Elements of the Hessian matrix – often the sum of its diagonal elements. In practice, often a constant < 1.0
- Schwarz('78): Deconvolution is iterative constrained χ^2 minimization
  - Clean constraints: finite support; MEM constraints: Smoothness
  - Both use the <u>same</u> basis-set (Pixel basis – <u>bad for modeling complex emission</u>)
- Clean is a POCS algorithm! (Cornewll, AIPS++ Note #184)
- Imaging performance of deconvolution algorithms...



Followup: Residuals for Clean, MEM, MS, Asp

I^d-BI^M Niter ~60K 50 ~15K ~1 K

V^{True}- V^{Model} Future!!



Imaging & Deconvolution lecture, Summer School, 2006

Theory re-cap: Measurement Eq.

$$V(u,v;\nu,t)=\int I(l,m)\ e^{\iota[ul+vm]}dl\ \ dm$$

- van-Cittert Zernike Theorem: Coherence function is a 2D Fourier Transform of the Sky Brightness distribution

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- Imaging $I(l,m)=\int V(u,v;\nu,t)\ e^{-\iota[ul+vm]}du\ \ dv$
- (u,v) are implicitly a function of time (HA) and frequency

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{\nu}{c} \begin{bmatrix} .. & .. & .. \\ .. & f(HA,\delta) & .. \\ .. & .. & .. \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- Standard continuum imaging corresponds to vector average of calibrated data over time and frequency (with the appropriate kernel). However...

Theory: Measurement Eq.

- Full ME for calibrated data:

$$V_{ij}(\nu)=\int J^S_{ij}(s,\nu,t)\;\; I(s,\nu)\;\; e^{\iota(ul+vm+w(\sqrt{1-l^2-m^2}-1))}\;\; ds$$

Data — Primary Beam — Sky — Geometry

- Visibility *explicitly* depends on time and frequency

- Imaging without correcting for time- and frequency-dependence of J^S_{ij} locks-in the errors after averaging
- The kernel in general is *not* even a Fourier transform kernel!

$$s=(l,m,n)=(l,m,\sqrt{1-l^2-m^2}-1):\quad \textit{Direction cosins}$$

Why wide-field imaging

- **Due to higher sensitivity, modern telescopes are sensitivity to emission farther out → need to image wider field of view (FoV) for noise-limited imaging**

With EVLA @L-Band, VLA sensitive achieved at the location of VLA-PB null!

- What do we mean by wide–field imaging

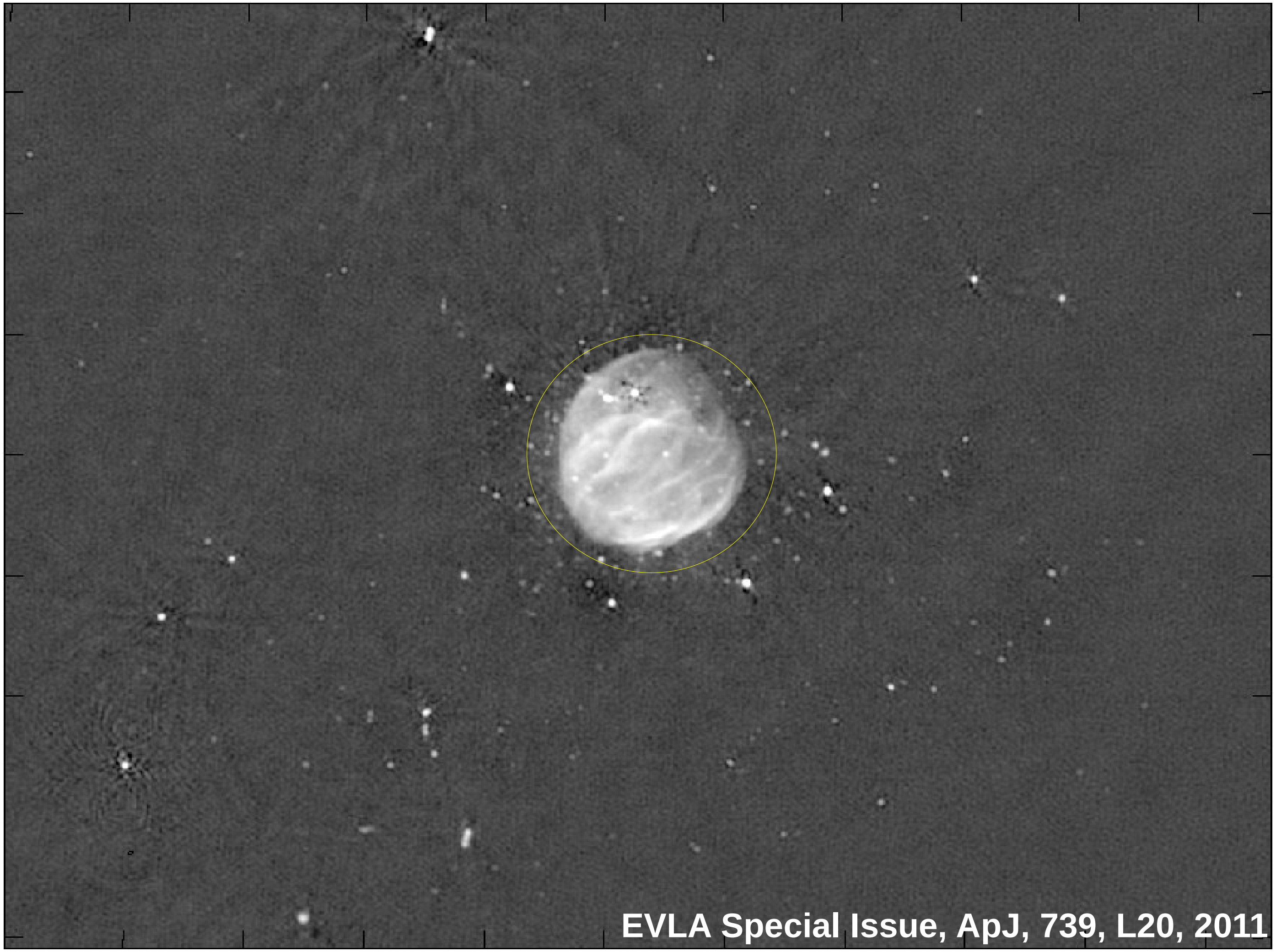
  When the ME has significant terms that make it increasingly deviate from the Standard ME *<u>with distance from the pointing/phase center.</u>*

- *The errors due to these terms scale with distance from the phase/pointing center.*

- Time and frequency dependence of the PB, Sky, and effects of 3D geometry are *<u>not accounted for in Standard ME kernel</u>*

$$e^{\iota[ul+vm+w(\sqrt{1-l^2-m^2}-1)]}$$

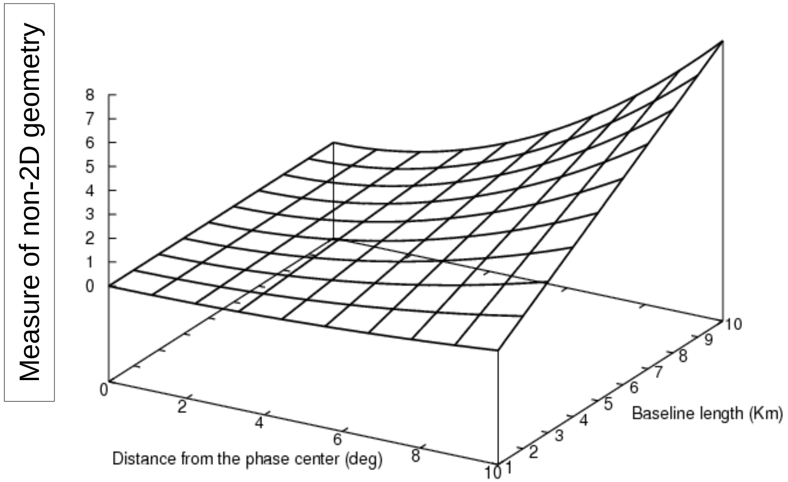
Example: JVLA Imaging @ L-Band

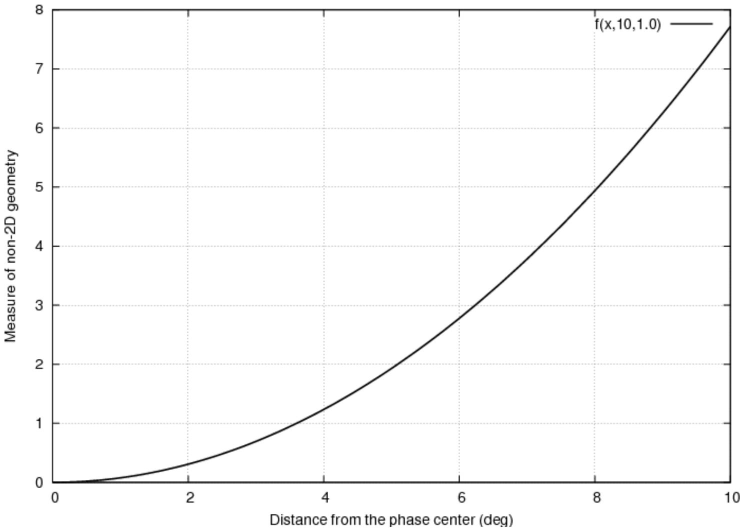


Wide-field imaging: W-term

- Wide-field: Deviation from 2D geometry increase with FoV and baseline length.
- W-Term: 2D Fourier transform approximation breaks down

$$e^{\iota[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} \neq \textit{2D FT Kernel}$$

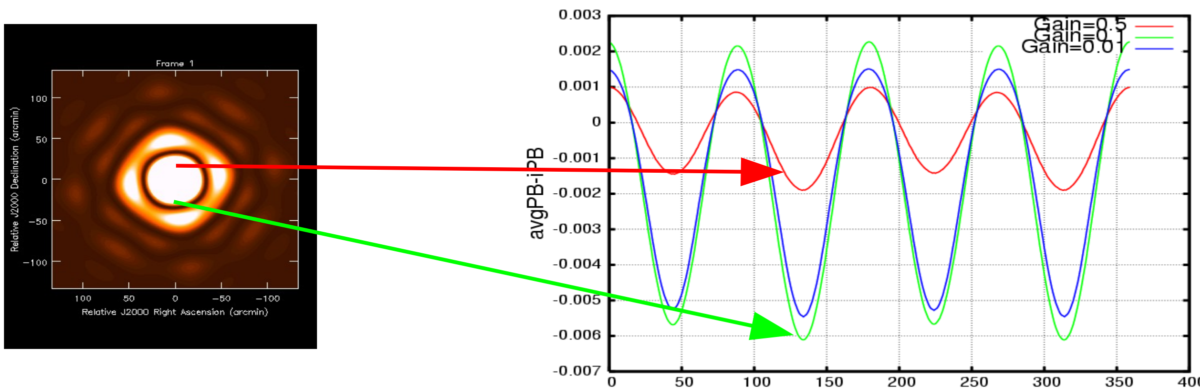


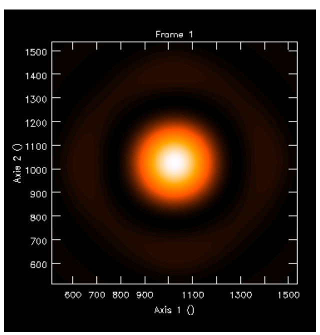


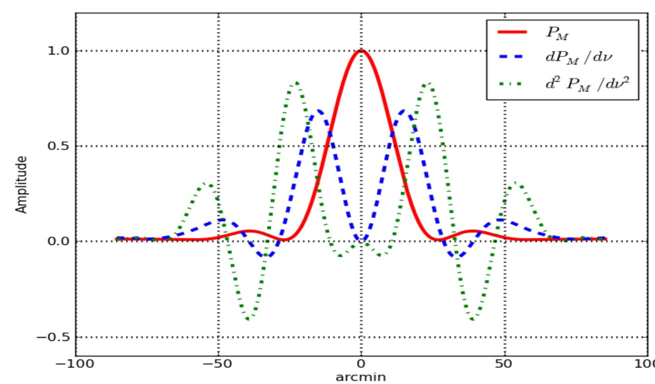
Wide-field imaging: Primary Beam

- Full-beam imaging: Antenna Primary Beam (PB) effects cannot be ignored

$$PB(\nu, t): \textit{Scales with frequency and changes with time}$$







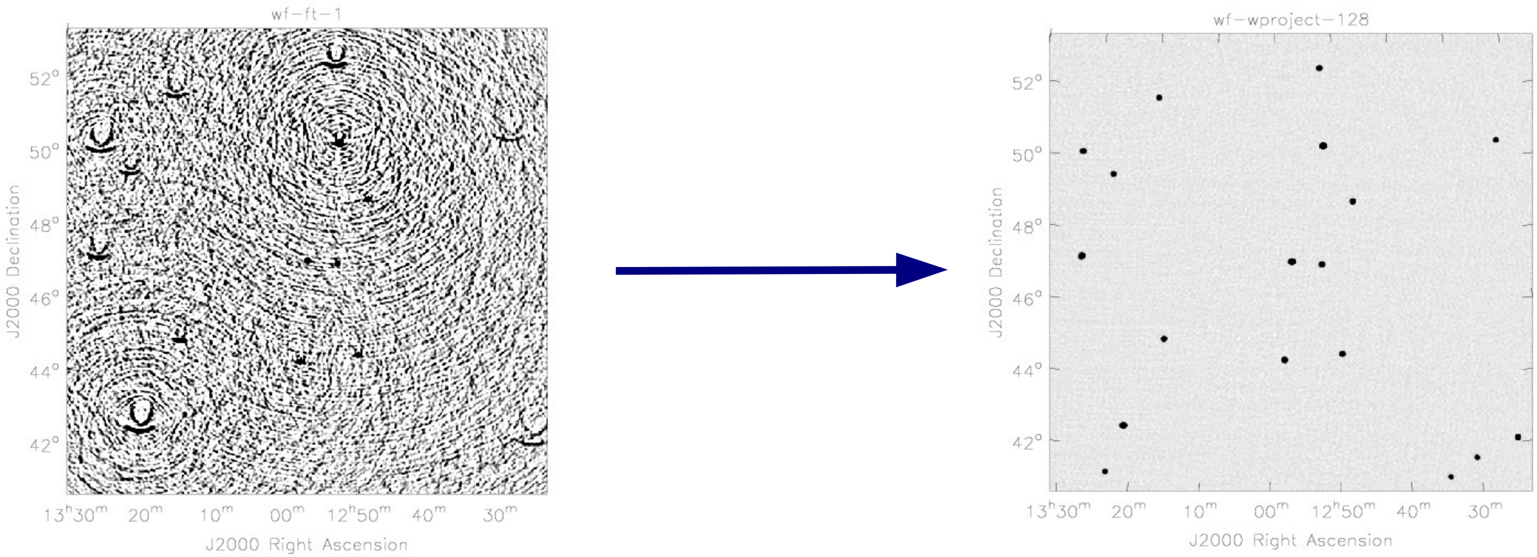
Wide-field imaging: Mosaicking & Pointing error

- Mosaicking: Imaging fields with emission larger than the antenna Field-of-View (FoV):
- For most cases, a straight forward extension of full-beam single-pointing imaging (linear addition of single pointing case)
- However, pay attention to the next lecture

- The other important PB effect, not included in this lecture is corrections for time-varying antenna pointing errors
    - Limits mosaic imaging performance
    - Solutions:
        - Reference pointing (for single pointing)
        - Pointing SelfCal: New possibility, but not yet fully tested
    - Talk to me later if you are interested...

The W-Term

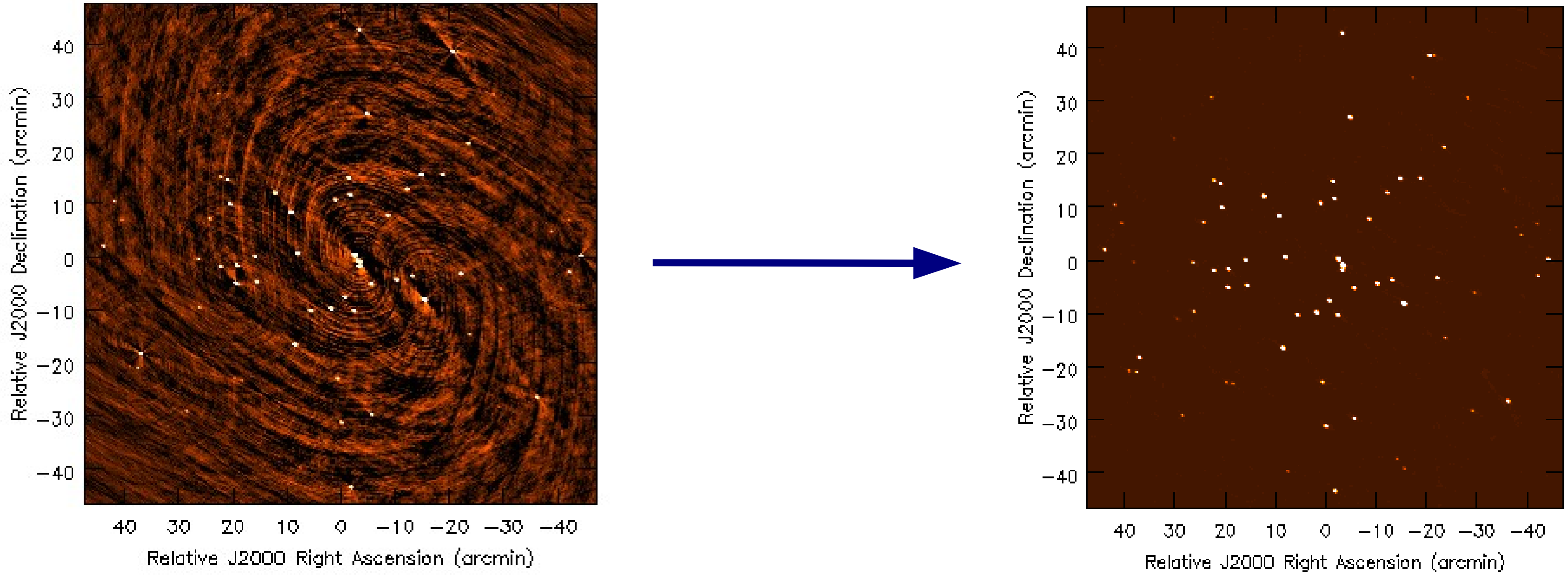
- 2D approximation of the measurement equation (ME) breaks down (“The W-term problem”).



- Imaging dynamic range throughout the image is limited by deconvolution errors due to the sources away from the (phase) center.

Primary Beam Effects

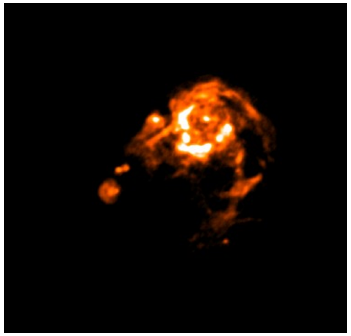
- Antenna Primary Beam (PB) pattern cannot be approximated by unity ("Full Beam Imaging").

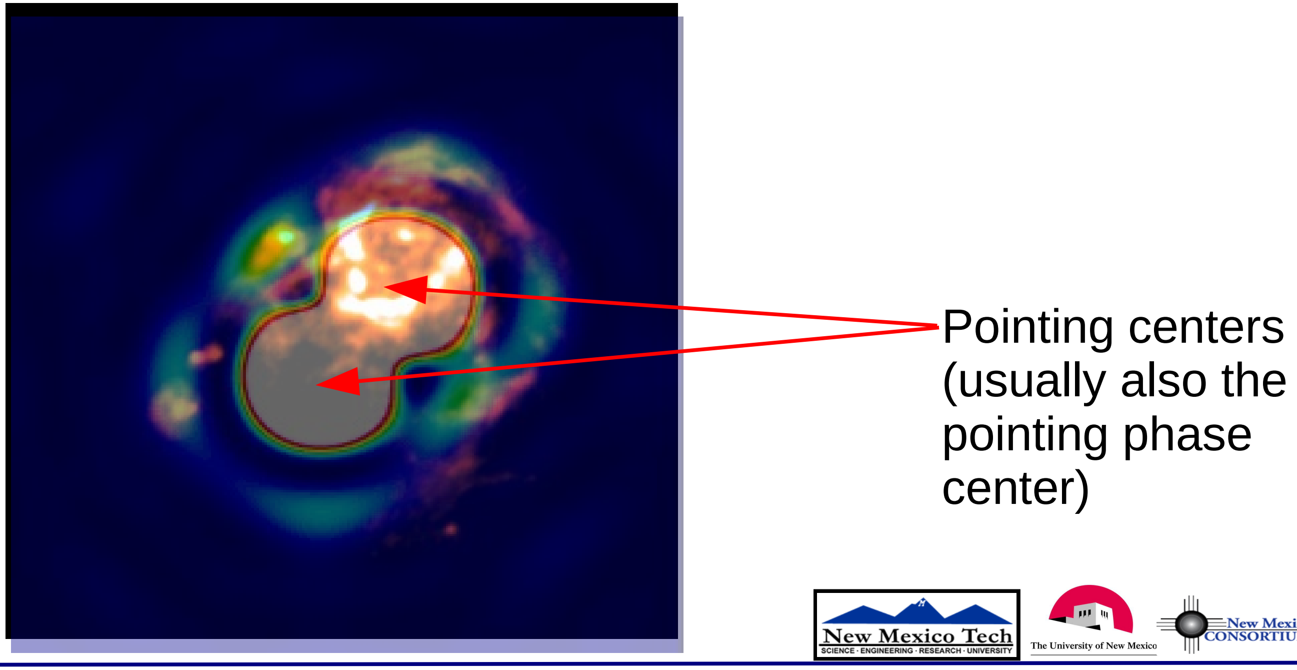


- Imaging dynamic range throughout the image is limited by the deconvolution errors due to the sources in the half-power points and the side lobes.

Mosaicking (see later lectures)

- Imaging emission wider than the Field-of-View (FoV) ("Mosaicking")
- Dominant sources of errors
    - Antenna Pointing
    - PB effects: rotation, multiple types of antenna in the array (ALMA)
    - Deconvolution errors for extended emission





The W-Term: Theory

- Assuming $J^s_{ij}=1$ (ignore effects of PB)

$$V_{ij}^{Obs}(\nu)=\int I(s,\nu)\ e^{\iota\left[u_{ij}l+v_{ij}m+w_{ij}\left(\sqrt{1-l^2-m^2}\cdot\cdot 1\right)\right]}\,d\,s$$

- FoV is small: $l^2+m^2\ll 1$

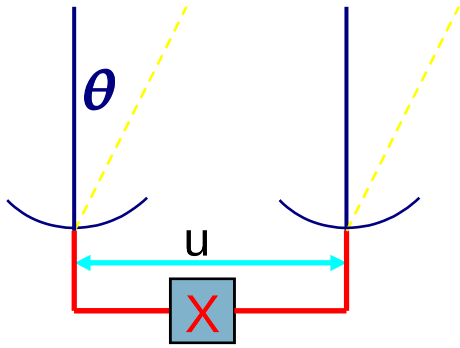
  Array is co-planar $w\ll\left(\sqrt{u^2_{max}+v^2_{max}}\right)$

  - vCZ: 2D Fourier transform works
  - $\dfrac{\lambda_{max}B_{max}}{[N_{lobes}D]^2}<1$ $\quad D\equiv Antenna\ diameter\ ;B_{max}=Max.\ baseline$

- When FoV **or** w_{ij} is "large", data and the image are not related by a simple 2D Fourier transform relationship.

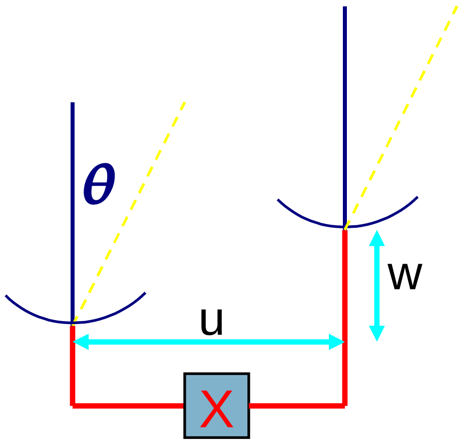
Ref: Chapter 19

The W-Term: Geometric interpretation



$$l = \sin(\theta)$$

$$n = \cos(\theta)$$



- Phase of the visibilities for direction $\boldsymbol{\theta}$
    - For the interferometer in a plane: $\phi = 2\pi u l$
    - For the interferometer not in a plane: $\phi = 2\pi \left[u l + w(n-1) \right]$
- 2D approximation valid only when: (1) w is small compared to u, or (2) $\theta \approx 0$

The W-Term: Optics interpretation

- We need to know

$$V^o_{12} = \langle \boldsymbol{E}'_1(u, v, w \neq 0)\, \boldsymbol{E}_2^*(0,0,0) \rangle$$

- We measure...

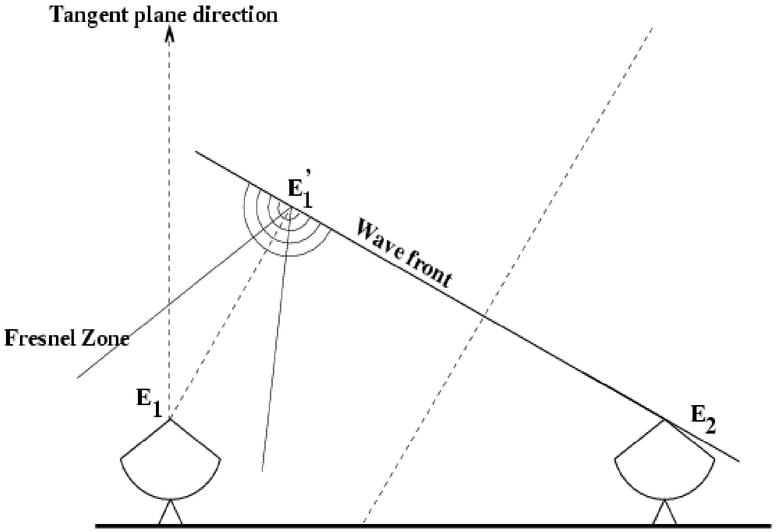
$$V_{12} = \langle \boldsymbol{E}_1(u, v, w = 0)\, \boldsymbol{E}_2^*(0,0,0) \rangle$$

- $\boldsymbol{E}_1$ is $\boldsymbol{E}'_1$ propagated using Fresnel diffraction theory

$$V^o_{12} = V_{12}(u, v, w=0) * G(u, v, w), where$$

$$G(u, v, w) = Fresnel\ Propagater = FT\left[e^{2\pi\iota w\sqrt{1-l^2-m^2}}\right]$$

$$V^o_{12} = \int I(l, m)\, e^{2\pi\iota\left[u_{12}l + v_{12}m\right]} e^{2\pi\iota w_{12}\sqrt{1-l^2-m^2}-1}\, dl\, dm$$

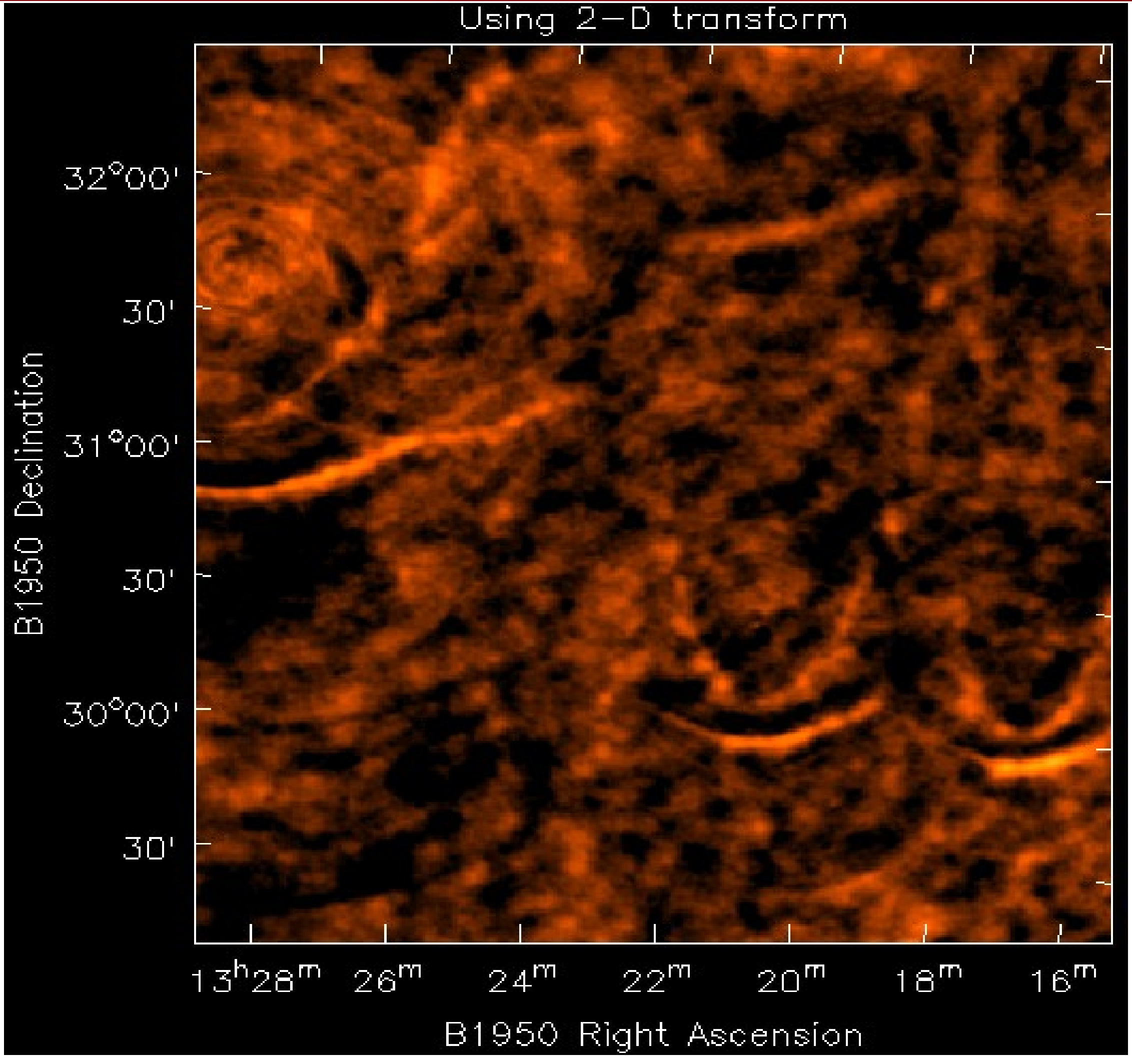


- A single interferometer is sensitive to <u>*multiple*</u> Fourier component

- Concept of redundant baselines is more restrictive than is usually thought!

Ref: IEEE Special topics in SP, Vol. 2, No5, 2008

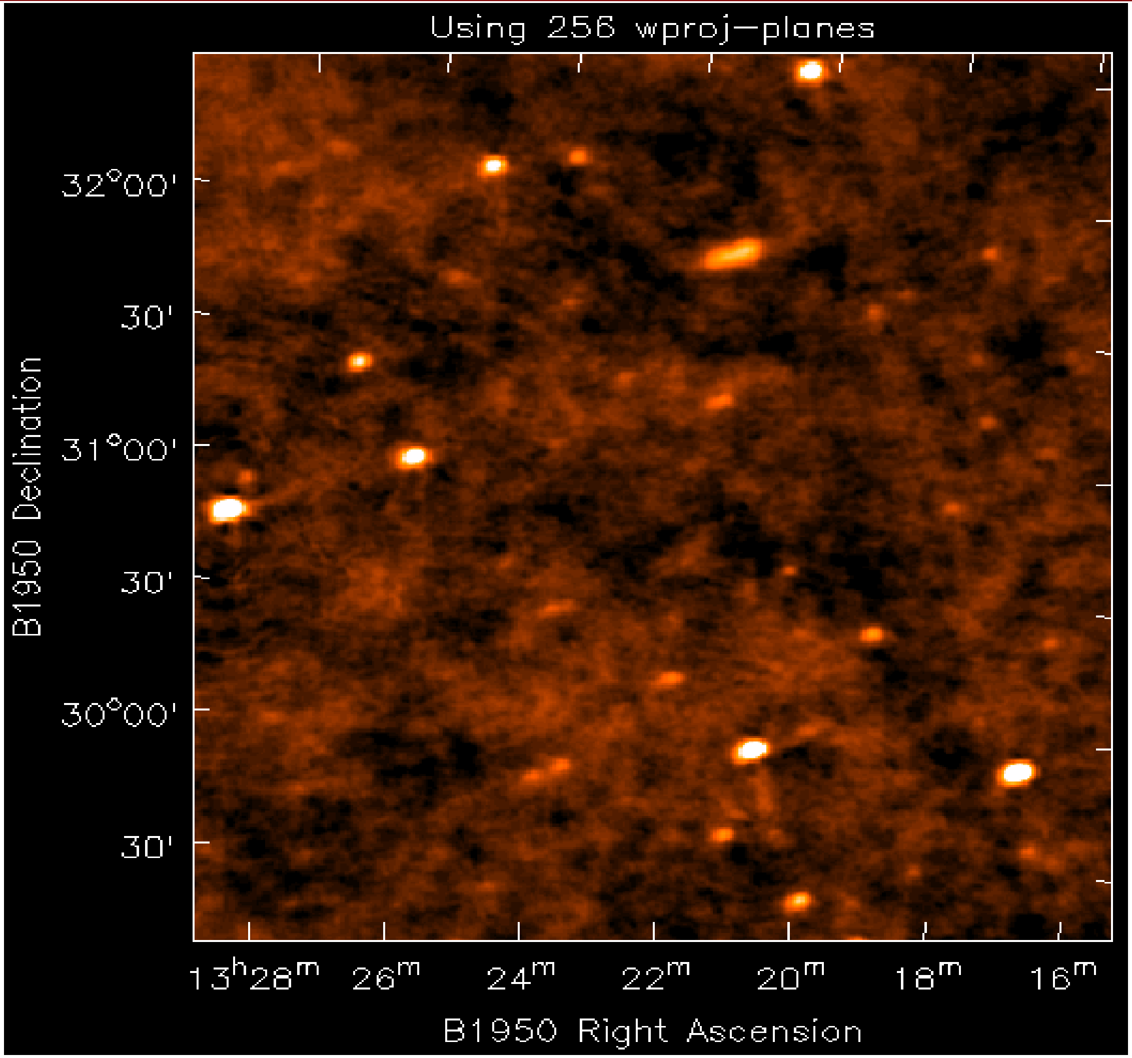
Example: No W-Term Correction



- W-term is a phase error
- Sources move in the image in a systematic way

- Hermitian but a "dispersive" effect

Example: After W-Term Correction



Solutions: 3D Imaging

- Do a 3D inversion of the ME to make a 3D "Image"

$$F(l,m,n)=\int V(u,v,w)\ e^{\iota\left[u_{ij}l+v_{ij}m+w_{ij}n\right]}\ du\,dv\,dw$$

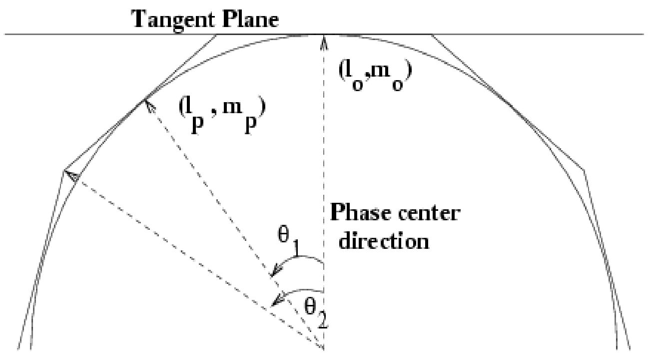
- Relate F(l,m,n) to the physical image as

$$I(l,m)=\frac{F(l,m,n)}{\sqrt{1-l^2-m^2}}\,\delta(l^2+m^2+n^2-1)$$

- Interpretation
  - Physical emission I(l,m) exists along the surface of a unit sphere inside the 3D-Image F(l,m,n)
- Resulting algorithm is not efficient
- Not used very often (read "never used" :-))

Solutions: Faceted Imaging

- Interpret *I(l,m)* as emission on the surface of the Celestial Sphere of unit radius: $l^2+m^2+n^2=1$
  - Approximate the celestial sphere by a set of tangent planes – a.k.a. “facets” – such that 2D geometry is valid per facet
  - Use 2D imaging on each facet
  - Re-project and stitch the facet-images to a single 2D plane



- Number of facets required

$$N_{Poly} = \theta_f^2 \frac{B_{max}}{\lambda} max$$

$$= \frac{B_{max} \lambda_{max}}{[N_{lobes} D]^2} \quad D \equiv Antenna\ diameter; \quad \theta_f = Antenna\ FoV$$

Solutions: UV plane equivalent

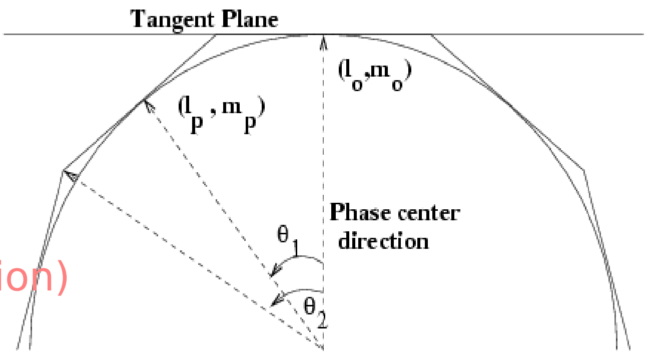
- Since the facet-images are related to the single-plane image by a linear co-ordinate transformation, there must exist a equivalent operation in the visibility plane.

$$\boldsymbol{I}(\boldsymbol{C}\, l) \rightarrow |det(\boldsymbol{C})|^{-1} \boldsymbol{V}(\boldsymbol{C}^{-1^T} u)$$

where ***C*** is the image domain co-ordinate transform

l and *u* are the image and visibility plane co-ordinates respectively

- Projection error: $\epsilon = \sin(\theta_1)(1-\cos(\theta_2)) \approx \frac{1}{2}\theta_1 \theta_2^2$
  - Error same as in image plane faceting!
  - Produces a single image (no edge effects)
  - Global deconvolution possible (extended emission)
  - Use of advanced algorithms for extended emission possible
  - Region definition as in the usual case



Available in CASA and possibly in AIPS

A small digression: Image deconvolution theory

- Function optimization view of deconvolution

$$\boldsymbol{V}^{o} = \boldsymbol{A}\,\boldsymbol{I}^{o} + \boldsymbol{N} \qquad N \text{ is Gaussian random (in the data domain)}$$

$$\boldsymbol{V}^{M} = \boldsymbol{A}\,\boldsymbol{I}^{M}$$

- χ^2 is the optimal estimator. Deconvolution is then equivalent to

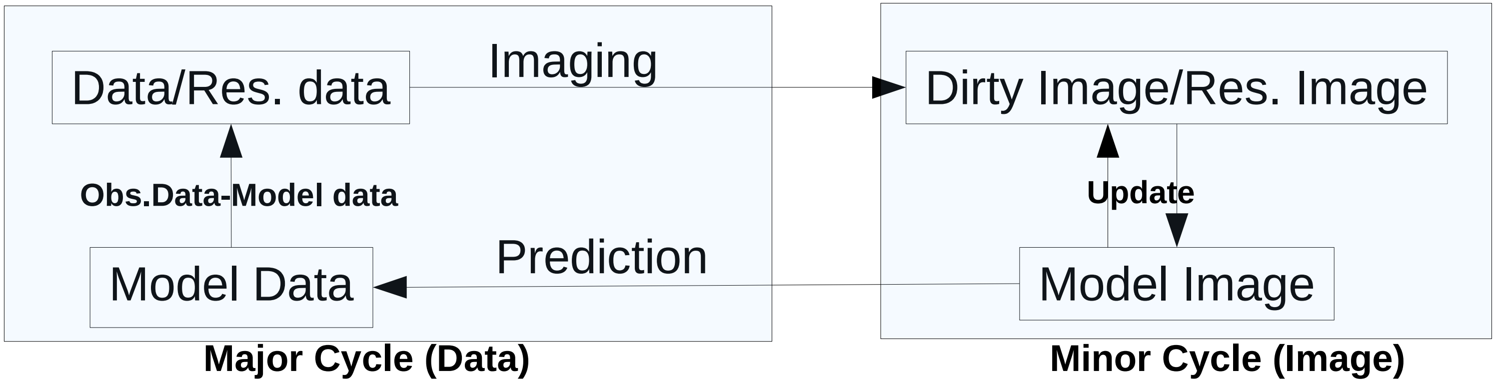
minimize: $\chi^2 = \left|\boldsymbol{V}^{o} - A I^{M}\right|^2 \quad \textit{where } I^{M} = \sum_k P_k; \quad P_k \text{ is the Pixel Model}$

$$\boxed{\frac{\partial \chi^2}{\partial\, \text{Pixel Model}} \equiv \text{Dirty Image}}$$

$$\boxed{I_i^M = I_{i-1}^M + \alpha\, \Delta \chi^2}$$

- Various minor cycle algorithms differ in (1) parametrization of P_k, (2) types of constraints, and (3) how the constraints are applied
- Projection algorithm use **A** different from FT kernel for forward and reverse transforms to included DD effects

A small digression: Structure of imaging algorithms



$$\frac{\partial \chi^2}{\partial \, \text{Pixel Model}} \equiv \text{Dirty Image}$$

$$I_i^M = I_{i-1}^M + \alpha \, \Delta \chi^2$$

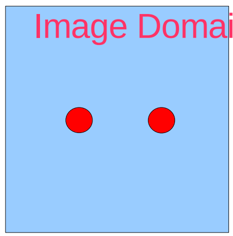
- Natural domain for modeling/correcting for antenna-based effects is the Data Domain
  - Accessible in Major Cycle
- Natural domain for image-plane (non-antenna based) effects is the Image Domain
  - Accessible in Minor Cycle

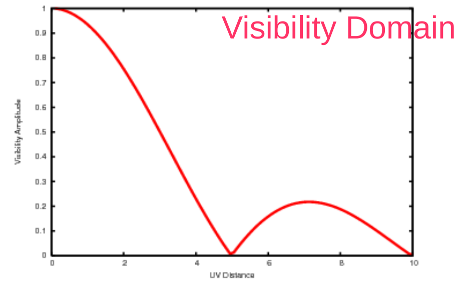
Ref: Imaging & Deconvolution Basics, Thur. Lecture Series
https://safe.nrao.edu/wiki/pub/Software/Algorithms/WebHome/LectureBasicsIntro.pdf

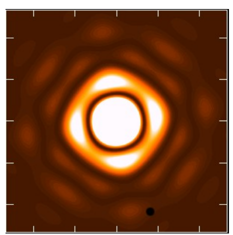
Natural domains and optimal algorithm design

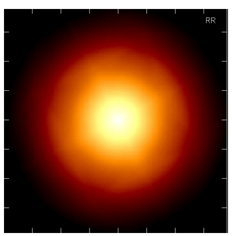
- Natural domain for modeling sky: Image plane

  Natural domain for modeling instrumental effects: Data domain









- Projection methods utilize the available data optimally
- Since DD effects are typically also vary with time & frequency, image domain based correction are non-optimal (and impossible for many cases)
  - Only average quantities are accessible in the image plane
- Corrections for DD effects cannot be decoupled from Imaging!

A small digression: Projection algorithms

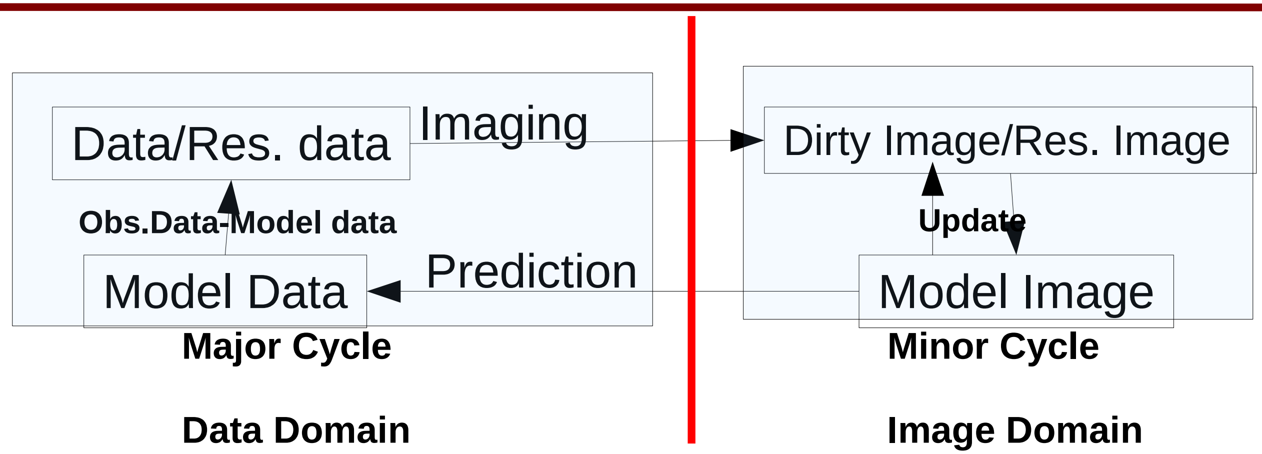
- ME entirely in the visibility domain: $V_{ij}^{Obs} = E_{ij}\left[V^o\right]$

  where E_{ij} represents a direction dependent (DD) effect

- Construct a $\boldsymbol{K}_{ij}$ *which models the desired DD effect*
- If $\boldsymbol{K}^T_{ij}\boldsymbol{E}_{ij}$ ~ **Constant** (Unitary Operator after normalization)
  - Use K_{ij}^T for making images — **Forward transform (approximate)**
  - Use K_{ij} computing model data — **Reverse transform (accurate)**

- Projection methods use modified forward and reverse transforms to iteratively correct for DD effects
- Iterations will converge – if the operator is least at approximately unitary

Full imaging algorithms



- Projection algorithms → Image plane algorithms

  Images corrected for DD effects

- W-Projection

  (WB) A-Projection

  Mosaicking

CS-Clean

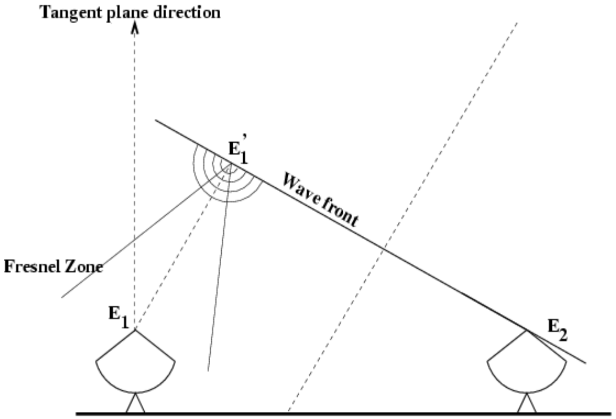
MS-Clean

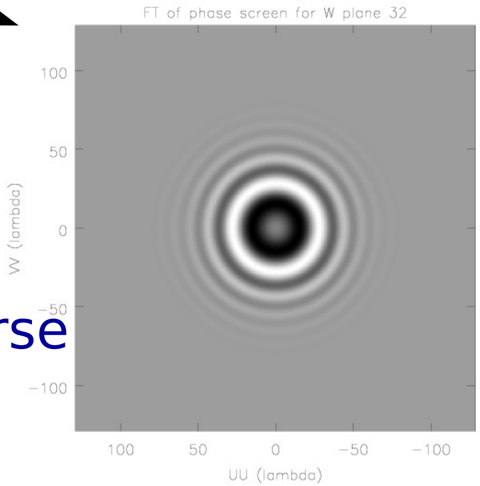
MT-MFS, MS-MFS

Solutions: W-Projection

- Optics interpretation
  - $V_{12}^{o} = \int I(l,m) e^{2\pi\iota[u_{12} l + v_{12} m]} e^{2\pi\iota w_{12}\sqrt{1-l^2-m^2}-1}\, dl\, dm$
  - $\boxed{\begin{array}{c} V^{o}(u,v,w) = V(u,v,w=0) * G(u,v,w),\ where \\ G(u,v,w) = FT\left[e^{2\pi\iota w\sqrt{1-l^2-m^2}}\right] \end{array}}$
  - Size of the Fresnel Zone:

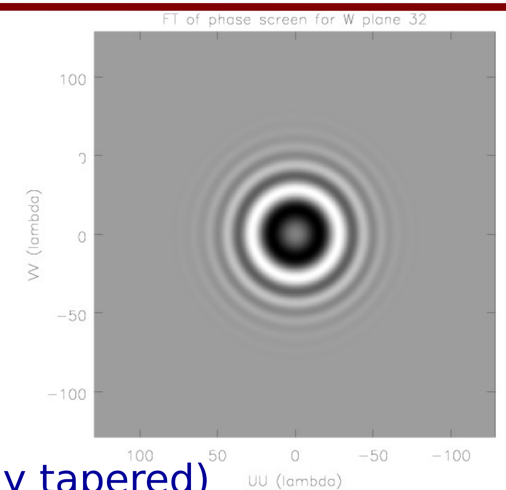
$$\frac{r_f}{\lambda} \approx \sqrt{w}$$





- If $G^T G$ is unitary (or even approximately so) G^T *and* G can be used in the forward and inverse transforms to produce distortion free images

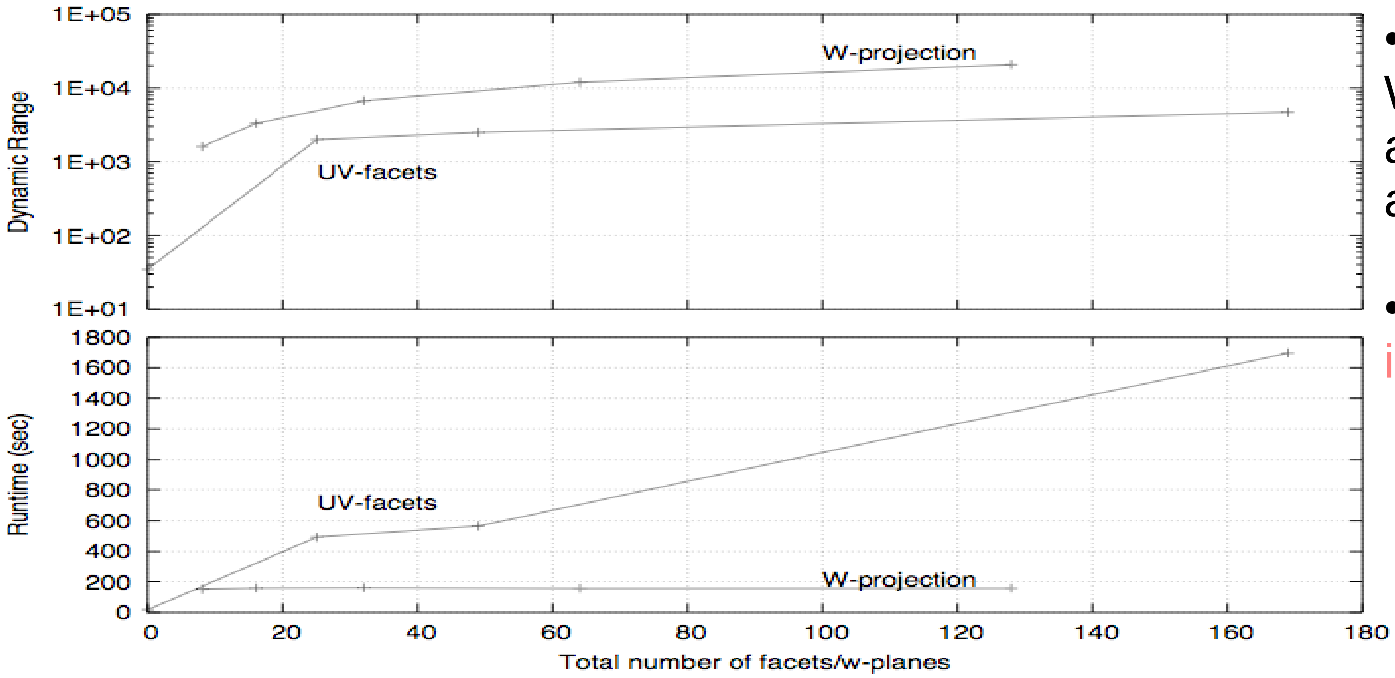
The W-Projection Algorithm



- Optics interpretation
  - $V^o(u,v,w) = V(u,v,w=0) * G(u,v,w)$
- Algorithm
  - Model prediction (major cycle) [Residual computation]
    - Perform a 2D FFT of the model image (appropriately tapered) (this is $V(u,v,w=0)$)
    - Evaluate the above convolution equation during de-Gridding to get $V^o(u,v,w)$
  - Compute the Dirty Image (minor cycle) [Deconvolution]
    - Use $G^T(u,v,w)$ on each $V^o(u,v,w)$ during gridding to evaluate $V(u,v)$
    - Perform a 2D FFT^{-1} of $V(u,v)$

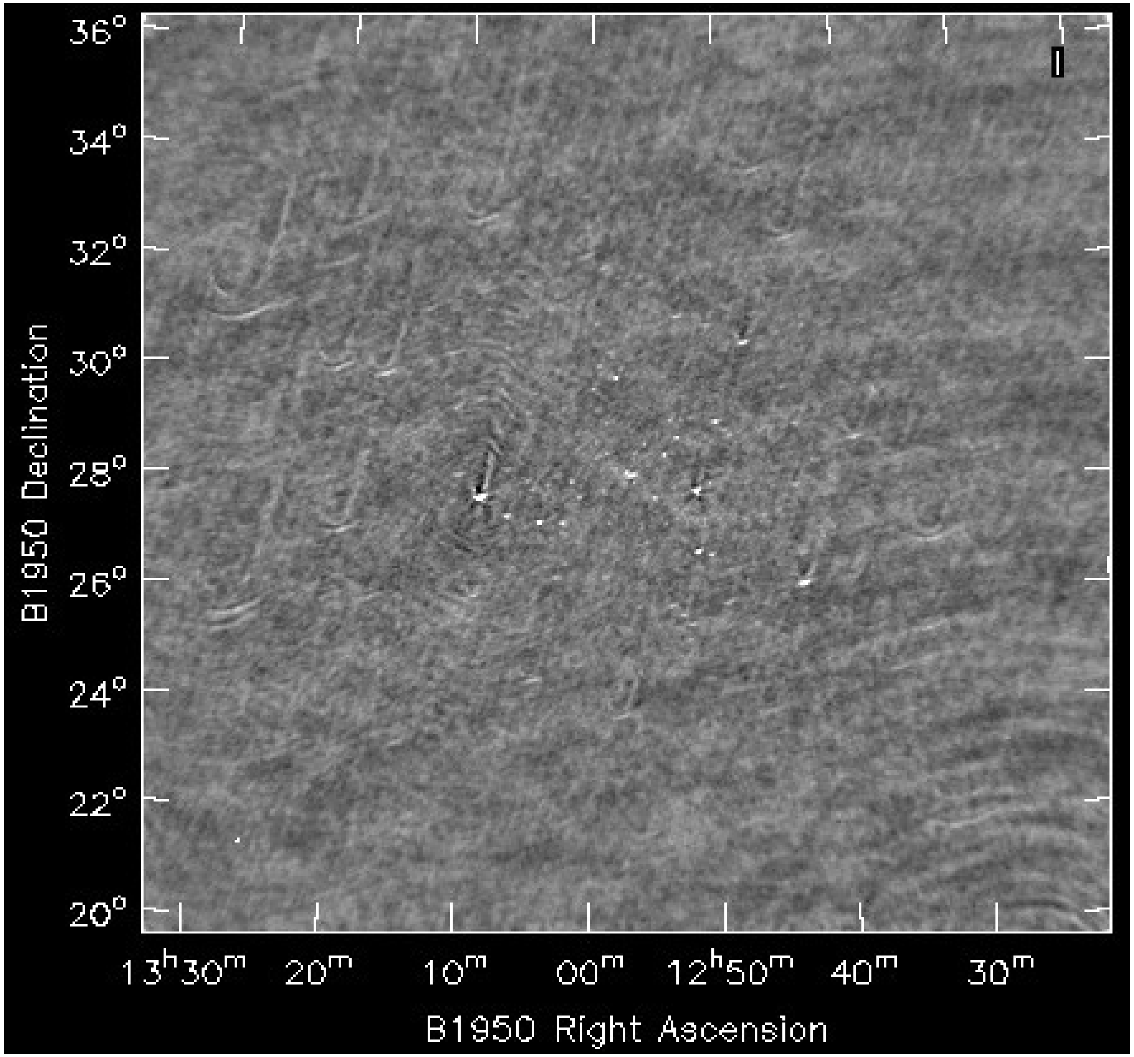
W-Projection: Performance

- Scaling laws:
  - Facet imaging: $(N^2_{Facets} N^2_{GCF}) N_{vis}$
  - W-Projection: $(N^2_{WPlanes} + N^2_{GCF}) N_{vis}$
  - Ratio: $\approx N^2_{GCF}$ *for large number of facets/WPlanes*



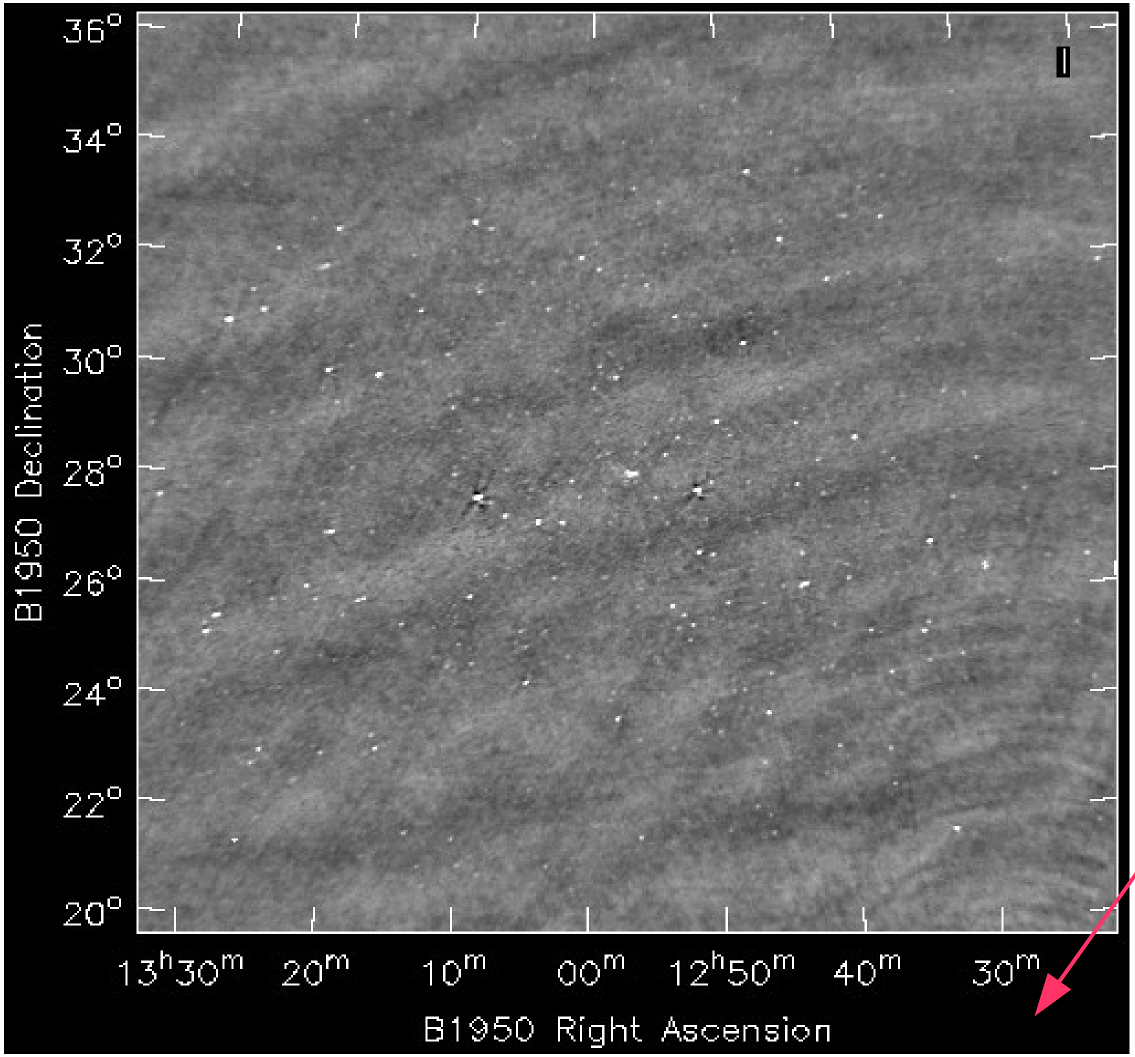
- In practice WProjection algorithm is about 10x faster
- Size of G(u,v,w) increases with W

Examples: 74MHz, before correction



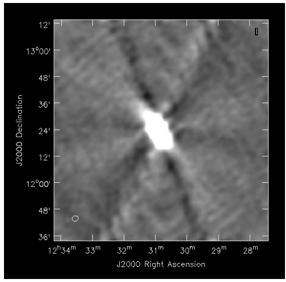
Courtesy:
K. Golap

Examples: W-Projection imaging

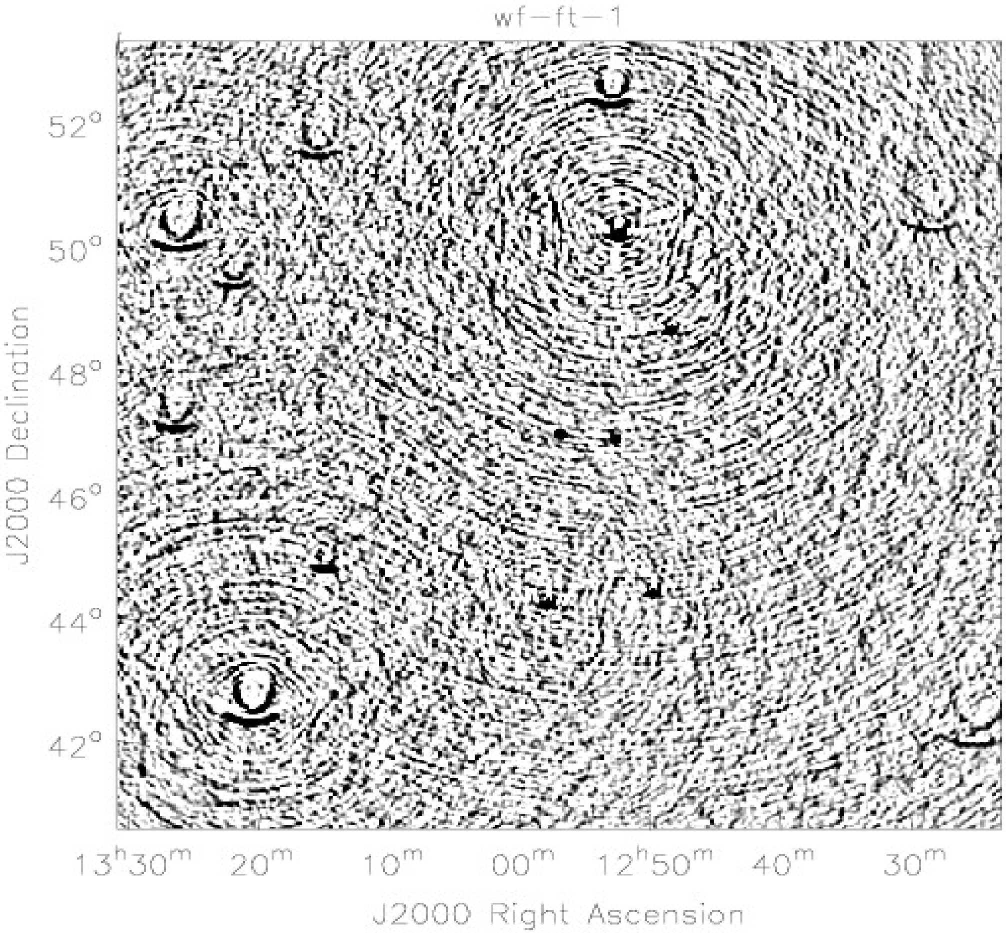


Courtesy:
K. Golap

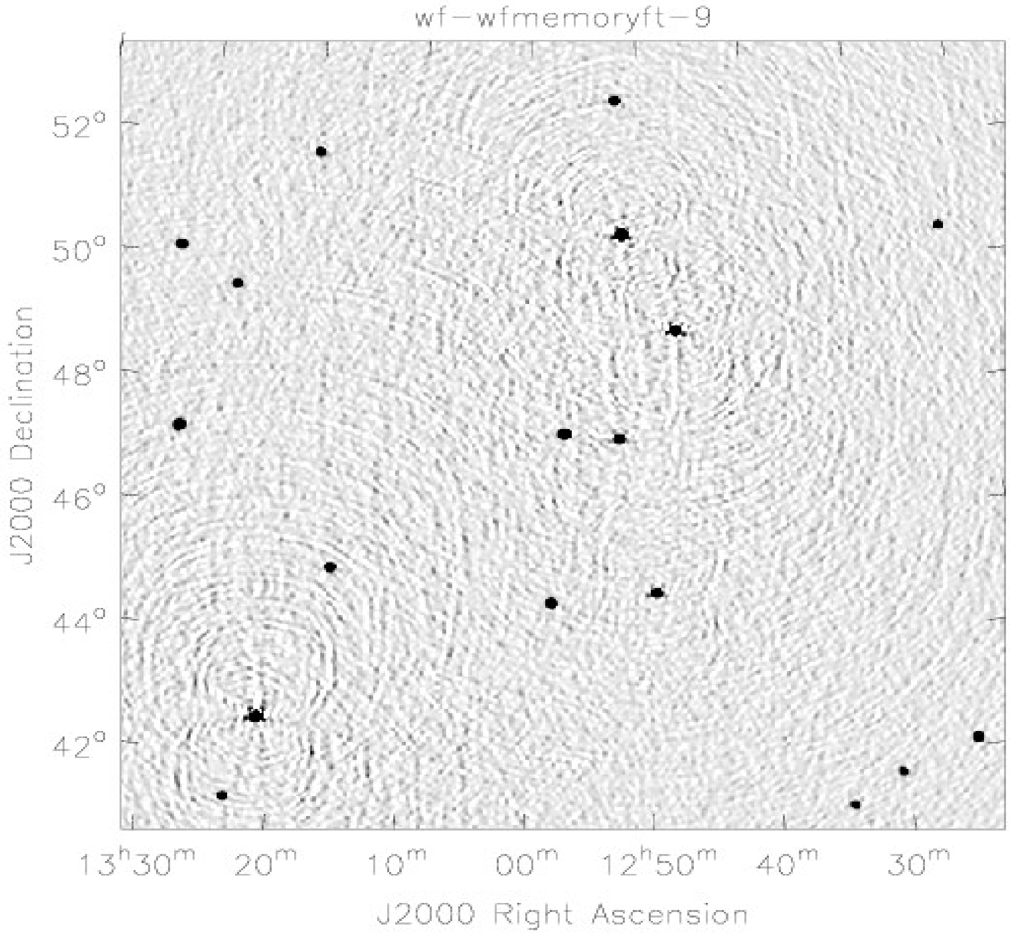
Sub-image of
an “outlier” field



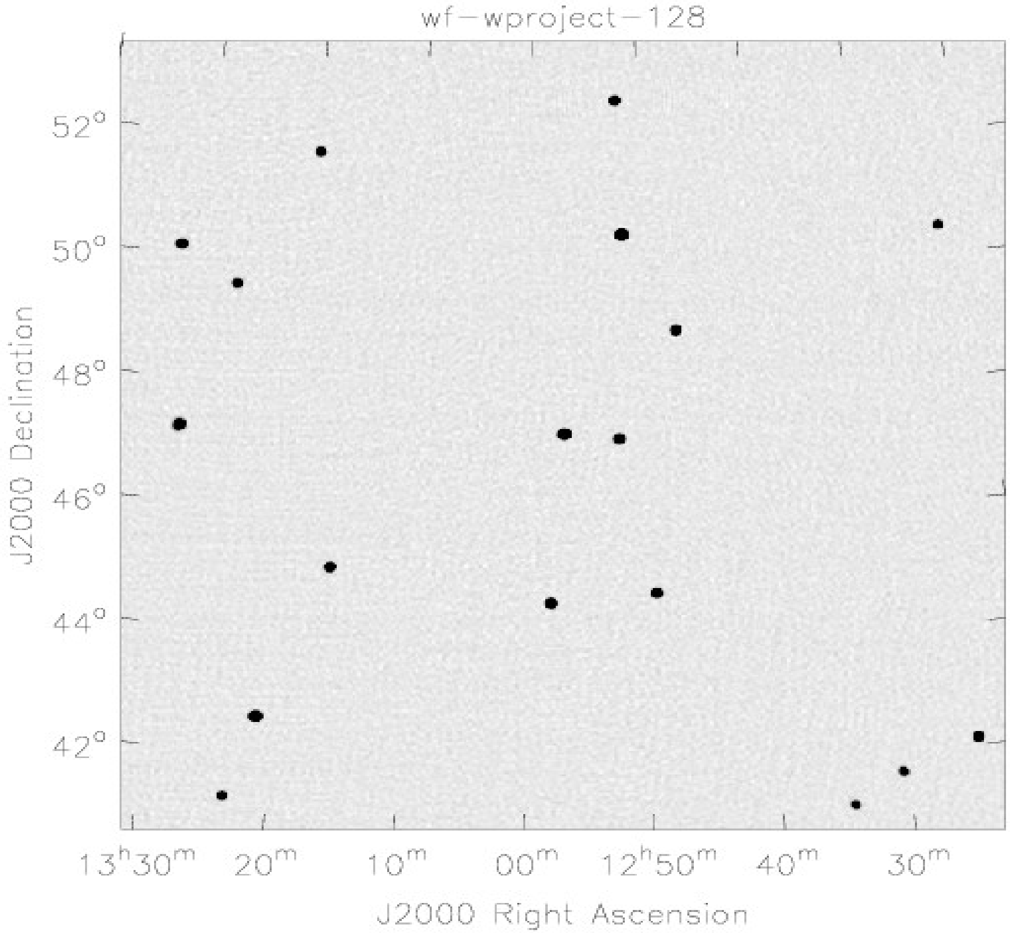
Examples: 2D Imaging



Examples: Facet Imaging



Examples: W-Projection Imaging

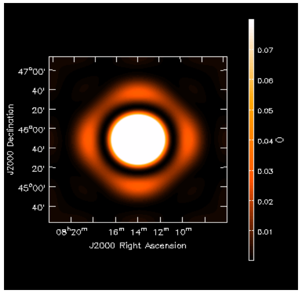


Full beam imaging

- Ignore the w-term for the moment:

$$V_{ij}^{Obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{\iota\left(u_{ij}l + v_{ij}m\right)}\ ds$$

- Some observations:
  - J_{ij}^{S} is direction dependent:

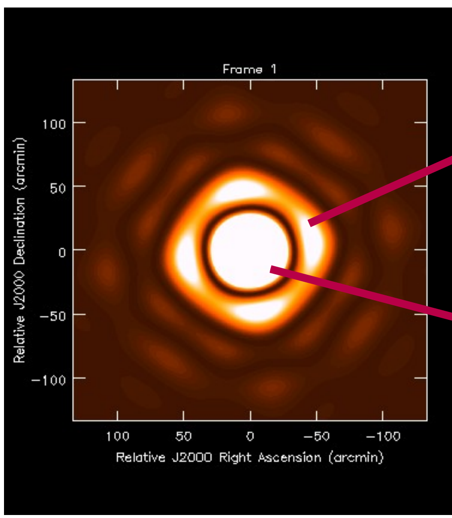


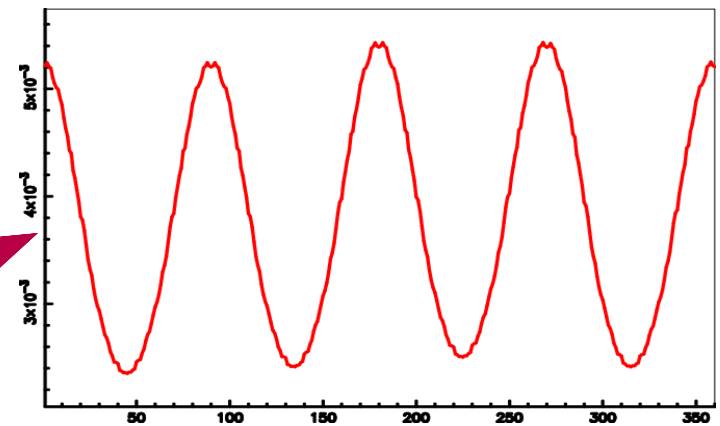
  - $J_{ij}^{S} = J_{i}^{S} \otimes J_{j}^{S*}$: This is true for most instrumental, atmospheric /ionospheric corruptions (all effects that obey “closure relationship”)
  - When $J_{i}^{S} = J_{j}^{S}$ and stationary in time (e.g. PB of ideal, identical antennas), it's effects can be corrected in the image domain

$$\frac{I^{Obs}}{J^{s}(s,\nu)} = I(s,\nu)$$

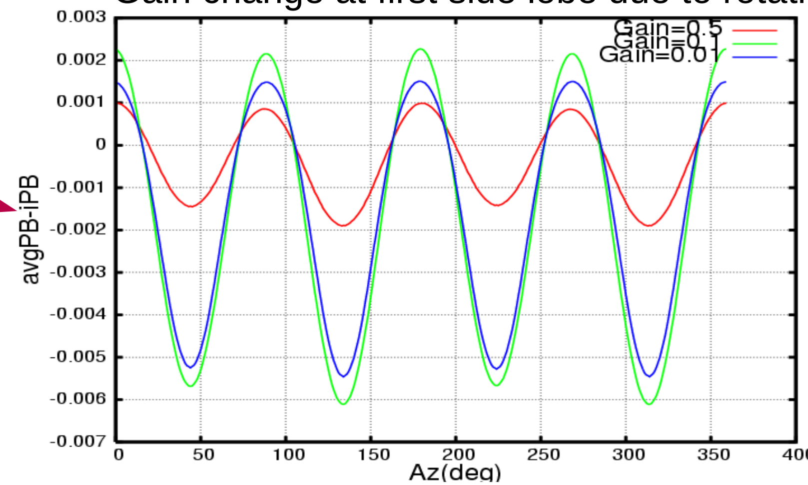
More observations...

- J^s_i are in general complex (complex Primary Beams!)
- In real life, $J^S_i \neq J^S_j$
- In real life, J^s_i vary with time...





Gain change at first side lobe due to rotation

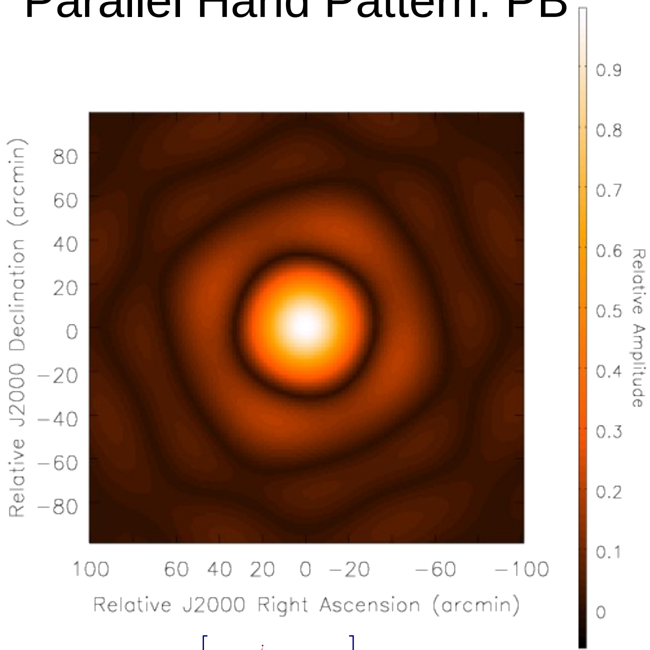


Source of time variability
- Pointing errors
- Geometric distortions

Polarization dependence

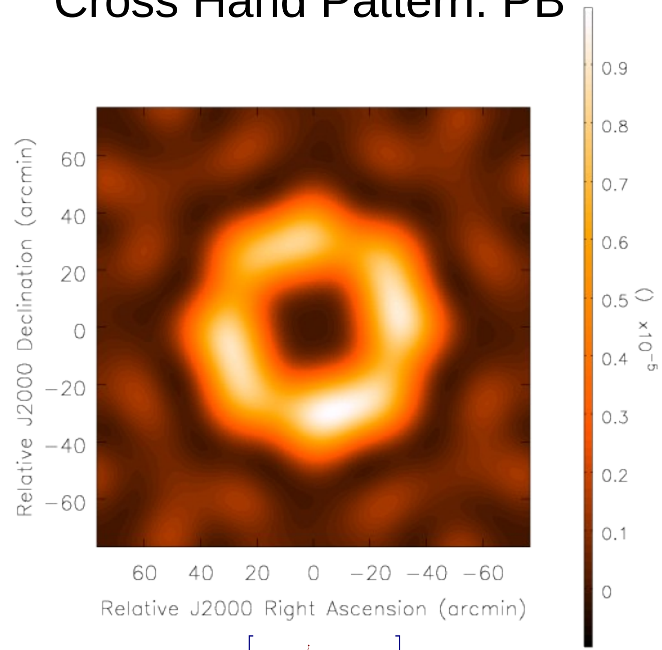
- ... J^s_i vary with polarization

Parallel Hand Pattern: PB^{RR}



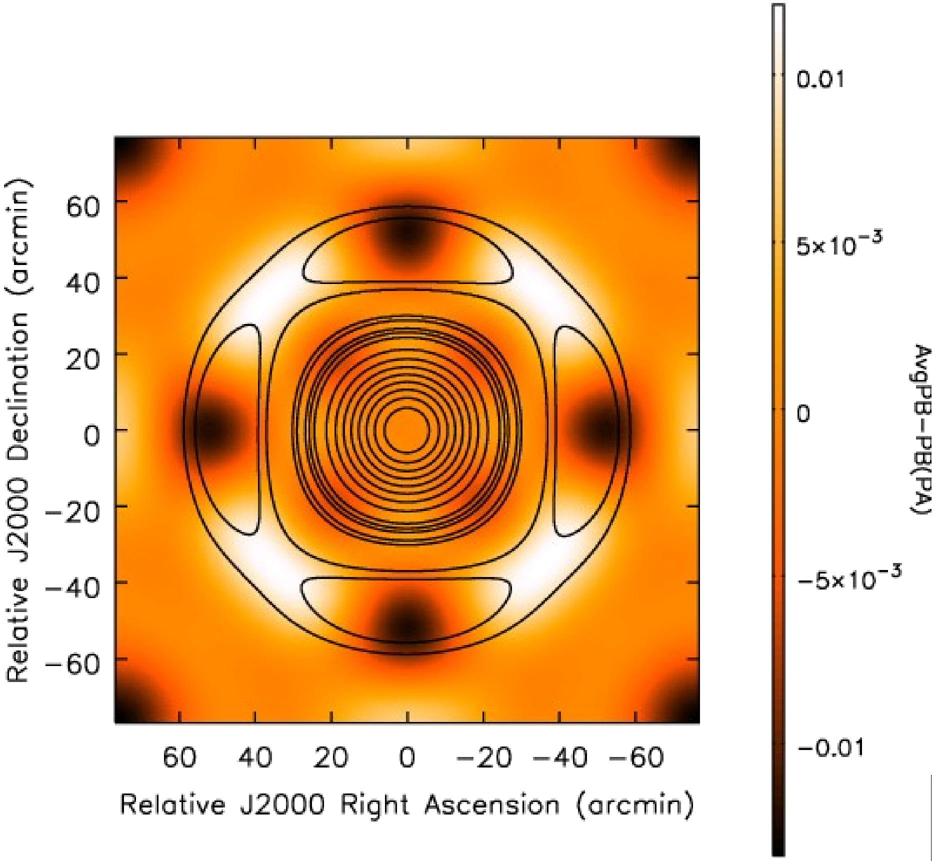
$$FT\left[E^{R^{*}} * E^{R}\right]$$

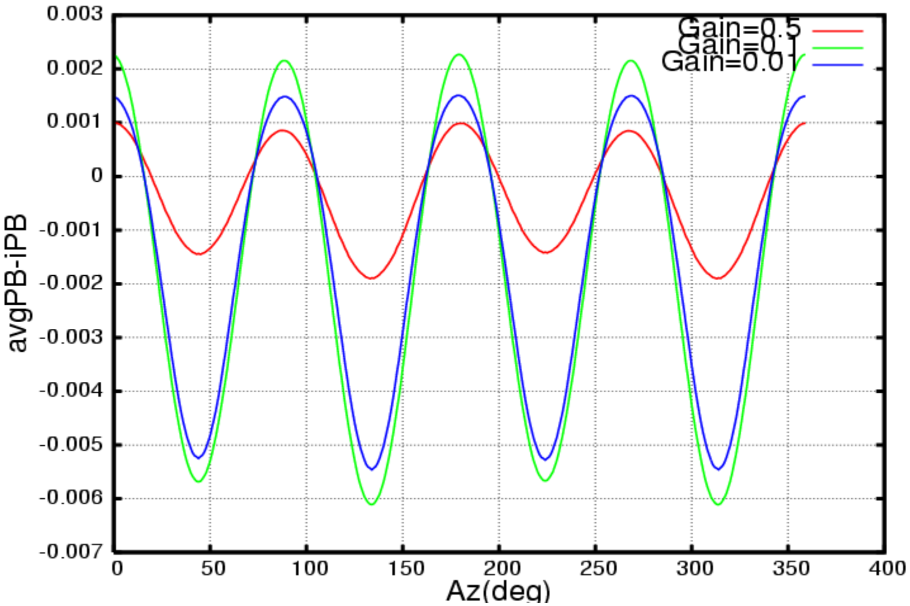
Cross Hand Pattern: PB^{RL}



$$FT\left[E^{R^{*}} * E^{L}\right]$$

Error propagation

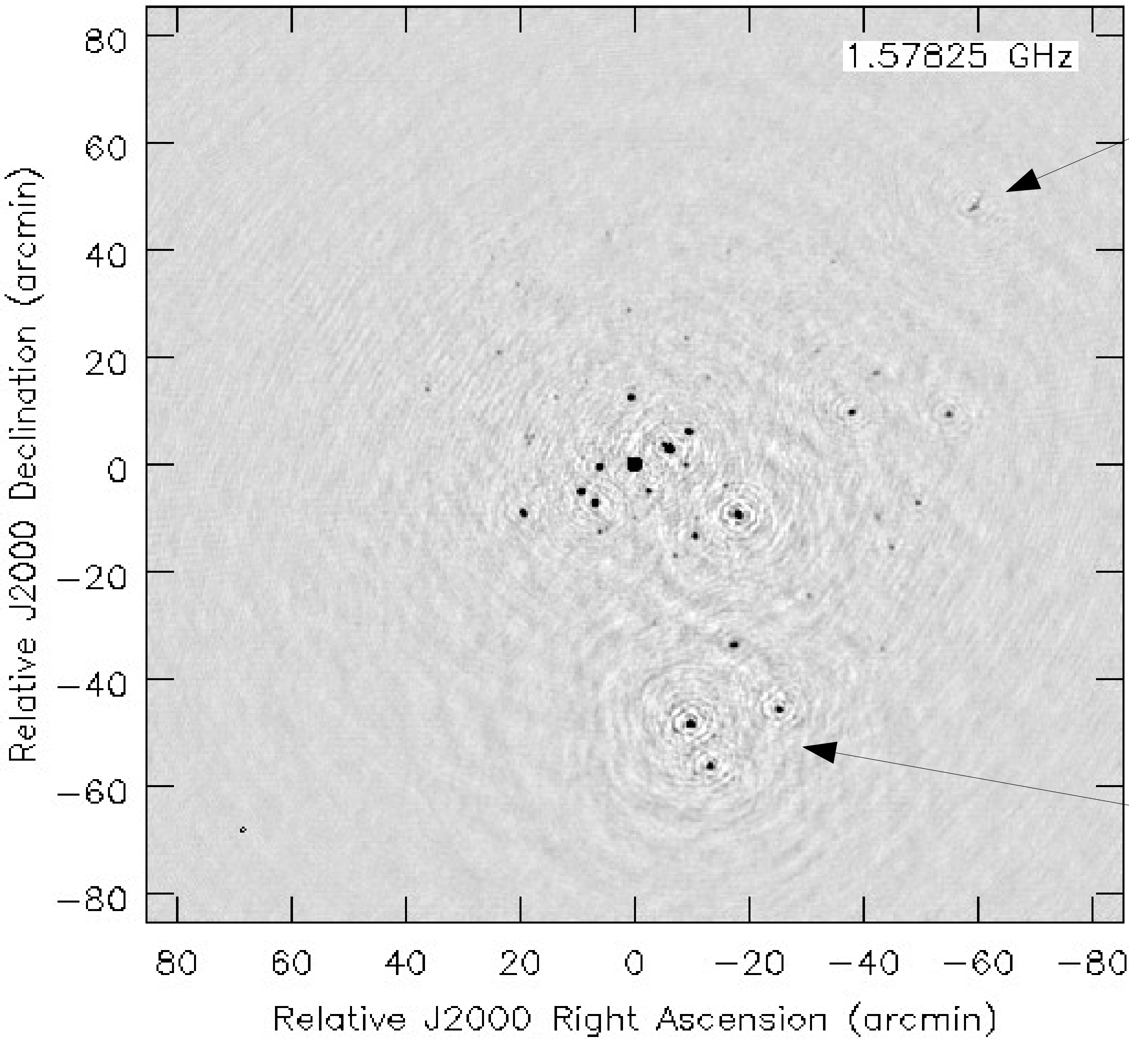




Azimuthal cuts at 50%, 10% and 1% of the Stokes-I error pattern AvgPB - PB(t_o)

$$\Delta I^R = \sum_{\psi} PSF(\psi) * [\delta PB(\psi) I^o]$$

Examples: Stokes-I and -V imaging



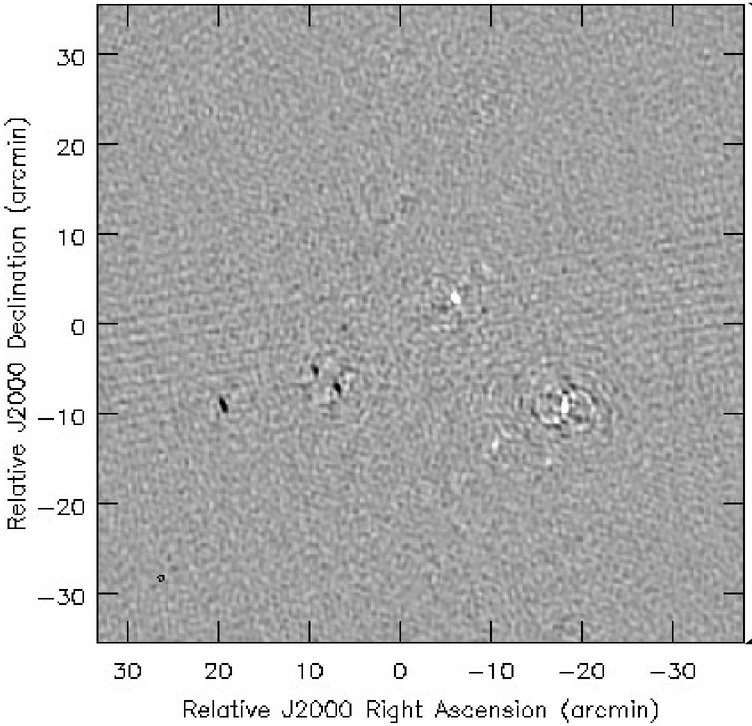
W-Term errors!

- 3C147, EVLA, L-Band
- High DR "700,000:1"

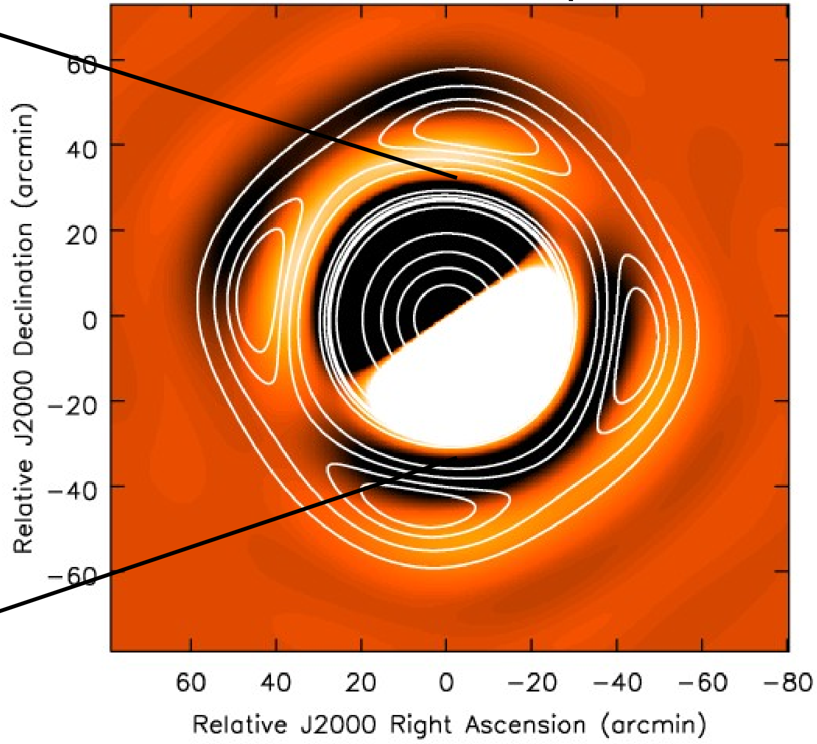
Errors due PB side-lobes?

Examples: Stokes-V

Stokes-V = I^{RR}-I^{LL}



Stokes-V Power pattern



Theory: Full-beam imaging

- Re-cap – the simplified ME

$$V_{ij}^{Obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\; I(s,\nu)\; e^{\iota\left(u_{ij} l + v_{ij} m\right)}\; ds$$

- Re-write it as

$$V_{ij}^{Obs} = E_{ij} * \left[V\right] = E_i^{*} * E_j * \left[V^{o}\right]$$

E_i: *Antenna Aperture Illumination Pattern*

$E_i = FT\left[J_i^{s}\right]$

V^{obs} is equal to true visibilities convolved with the correlation of the two antenna Aperture Illumination patterns.

- If there exists a function K_{ij} such that $K^{T}_{ij} * E_{ij} \sim Constant$
  - Gridding: $V_{ij}^{G} = K_{ij}^{T} * V_{ij}^{Obs} \quad = \quad K_{ij}^{T} * E_{ij} * \left[V^{o}\right] \quad \approx \quad \left[V^{o}\right]_{ij}$
  - Imaging: $FFT\left[V^{o}\right] \rightarrow I^{d}$
  - Prediction: $V_{ij}^{M} = K_{ij} * FFT\left[I^{M}\right]$
    - K_{ij} is the coherence field filter for the baseline i-j

Ref: A&A, 487, 419, 2009 (arXiv:0808.0834)

NRAO

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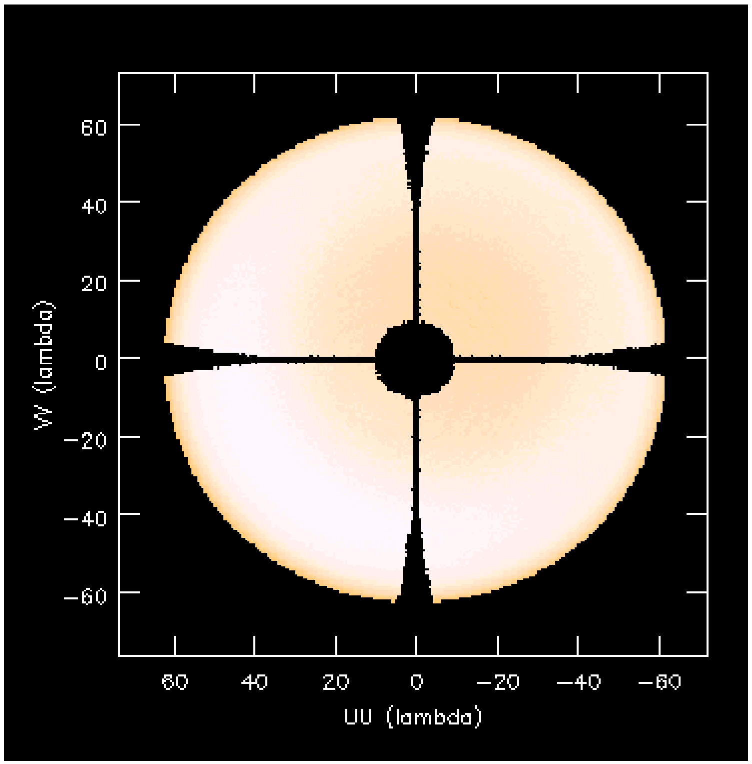
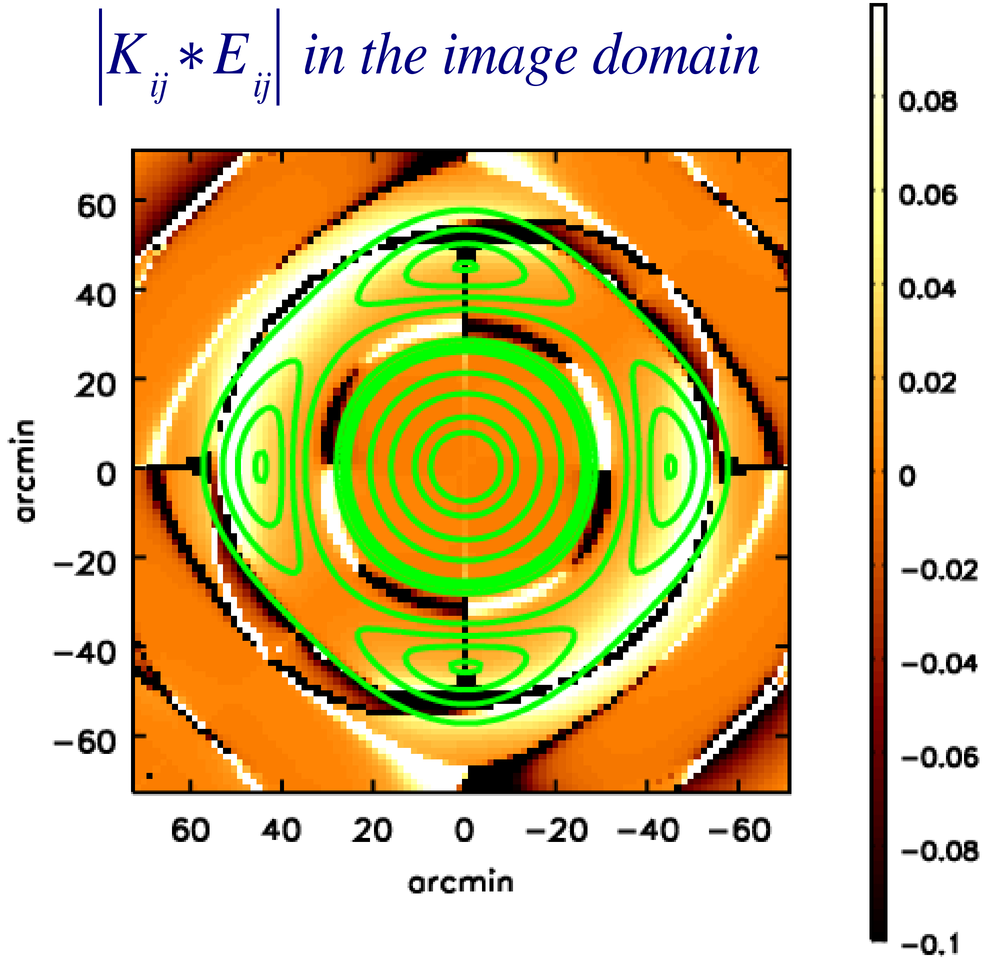
Solution: A-Projection algorithm

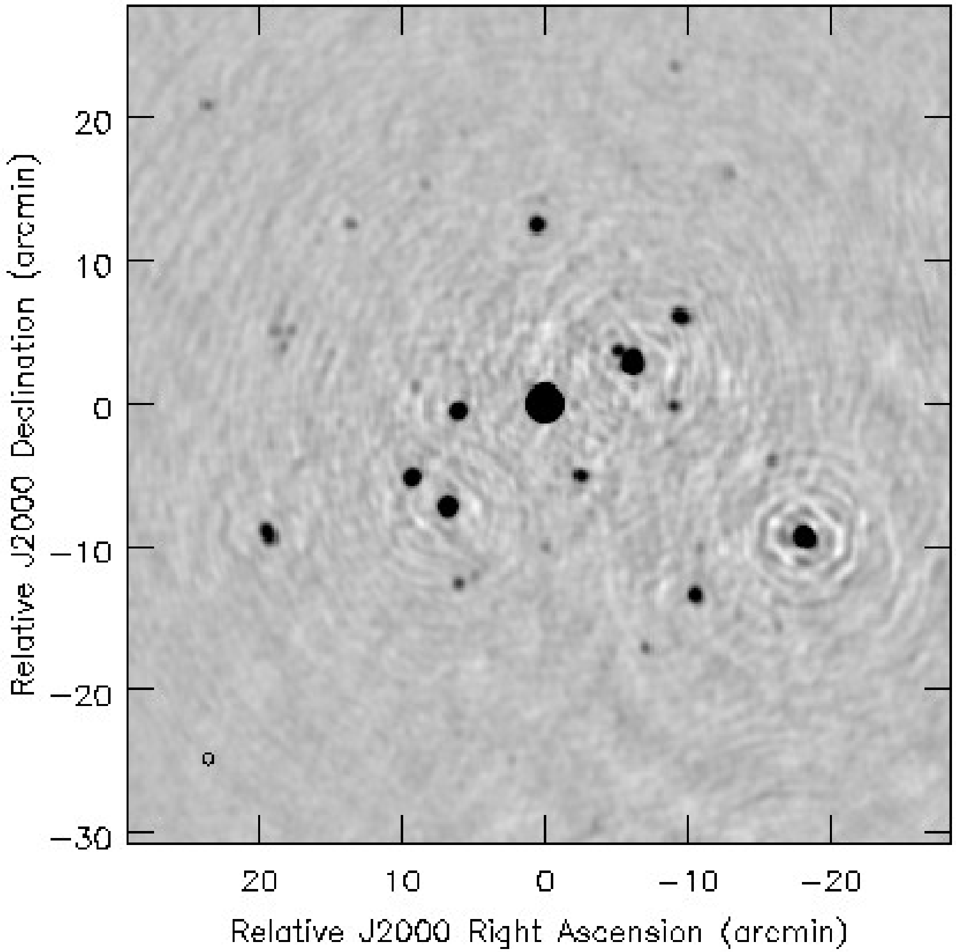
$\Re(\boldsymbol{K}_i^R)$: *Antenna Aperture Illumination Pattern*



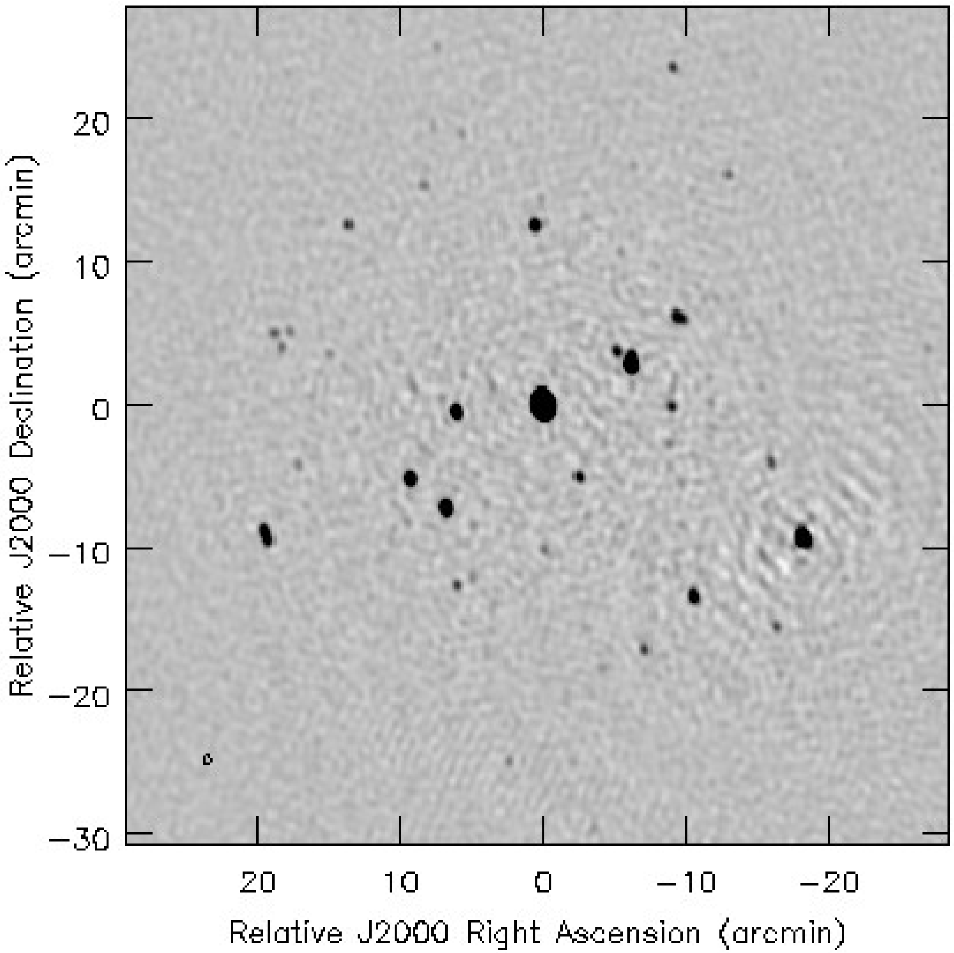
$\left|K_{ij} * E_{ij}\right|$ *in the image domain*



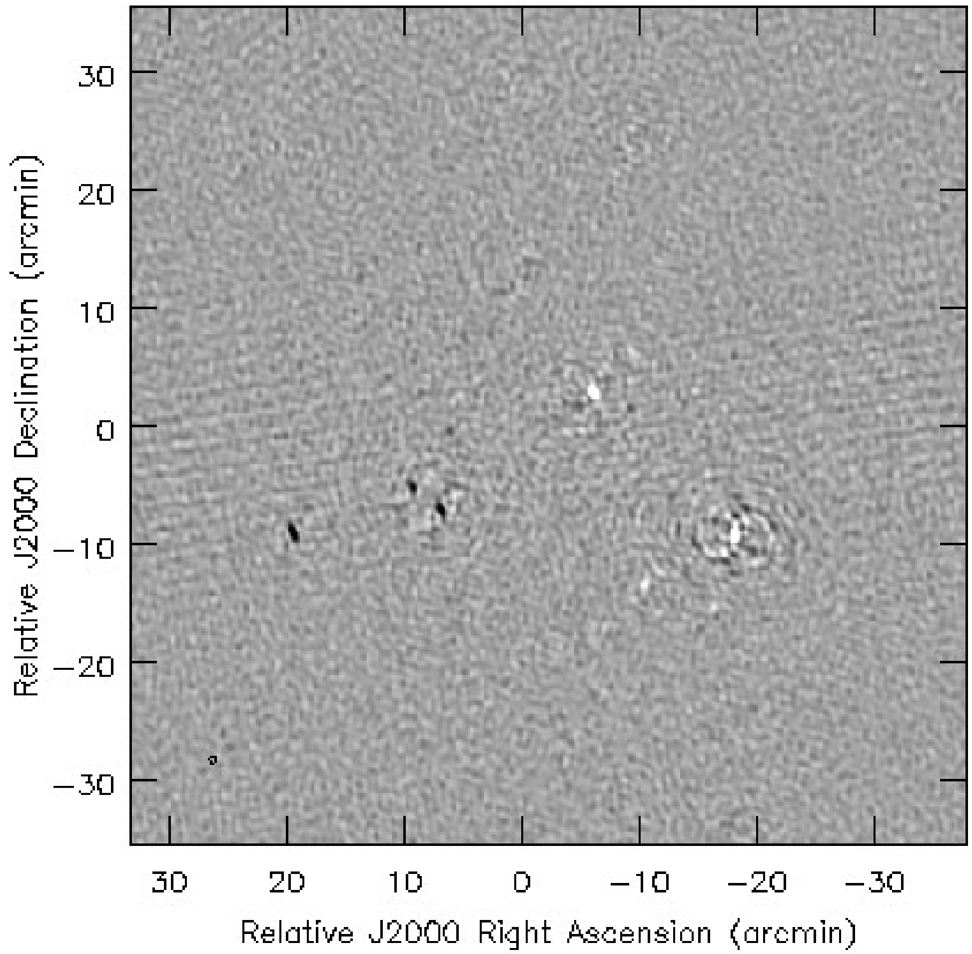
Stokes-I: Before correction



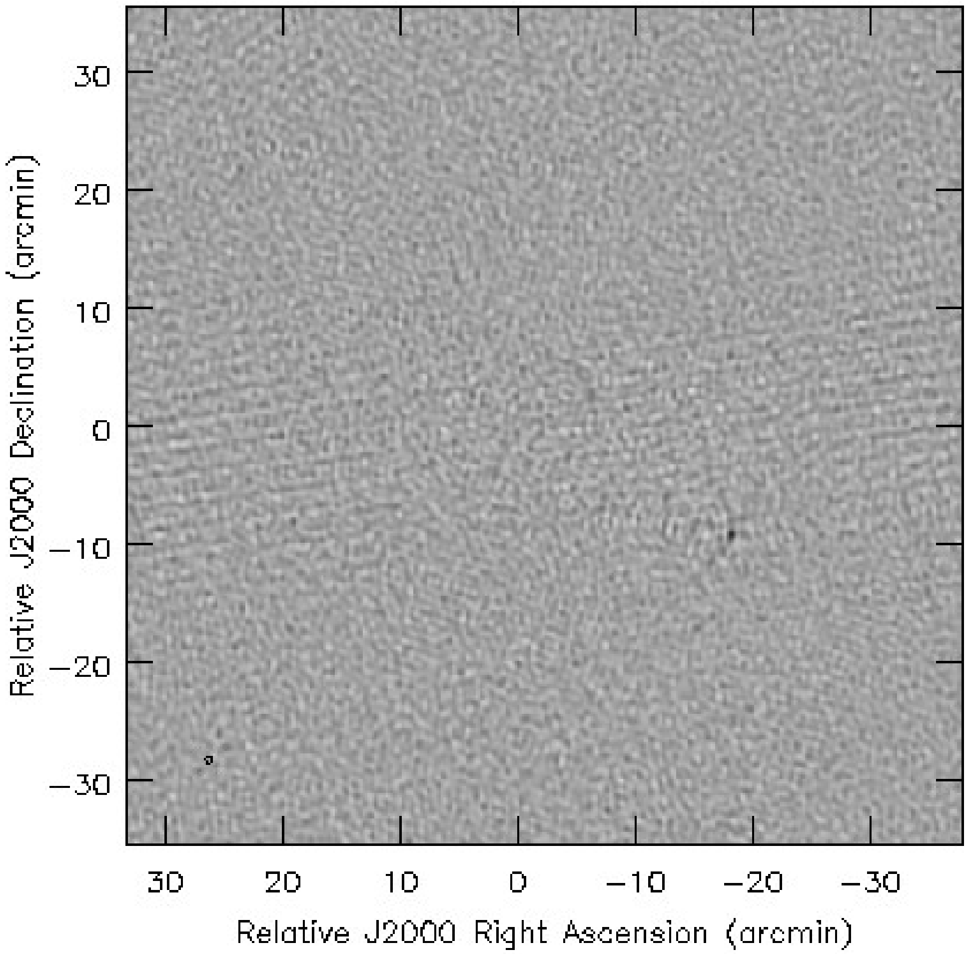
Stokes-I: After correction



Stokes-V: Before correction



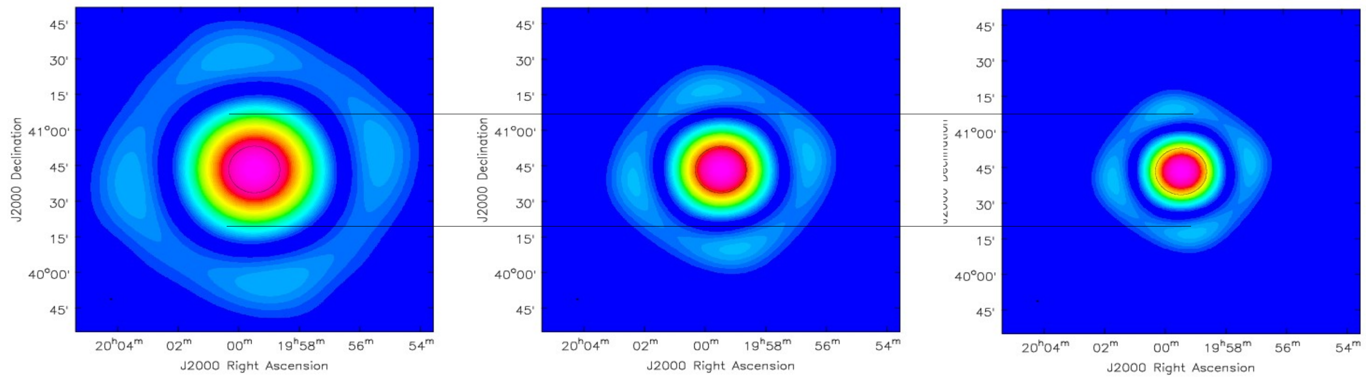
Stokes-V: After correction

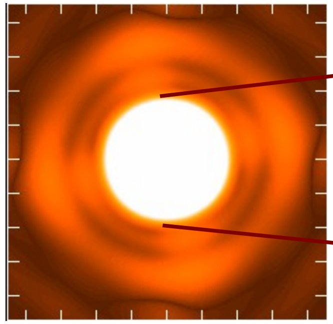


WB effects: Frequency dependent PB

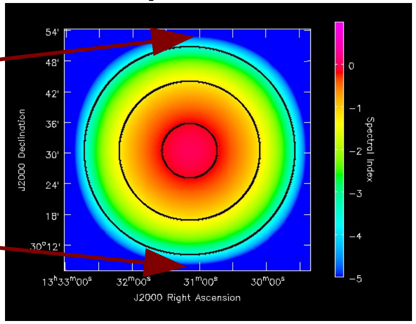
$$V_{ij}^{Obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{\iota\left(u_{ij}l + v_{ij}m\right)}\ ds$$

- ... J^s_i vary with frequency...



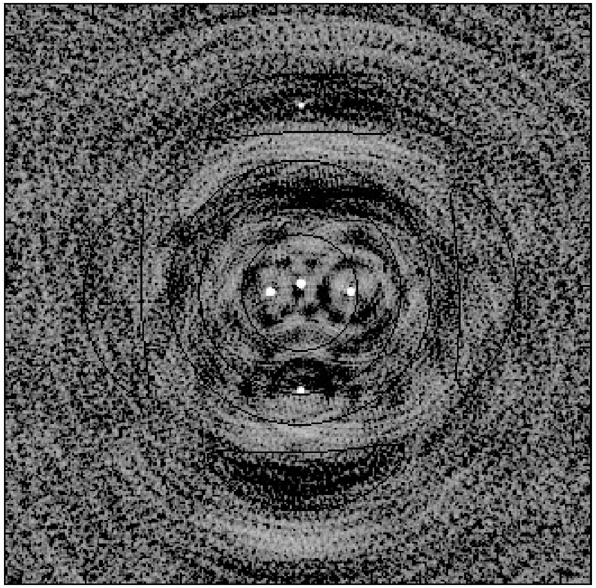


PB “Spectral Index”

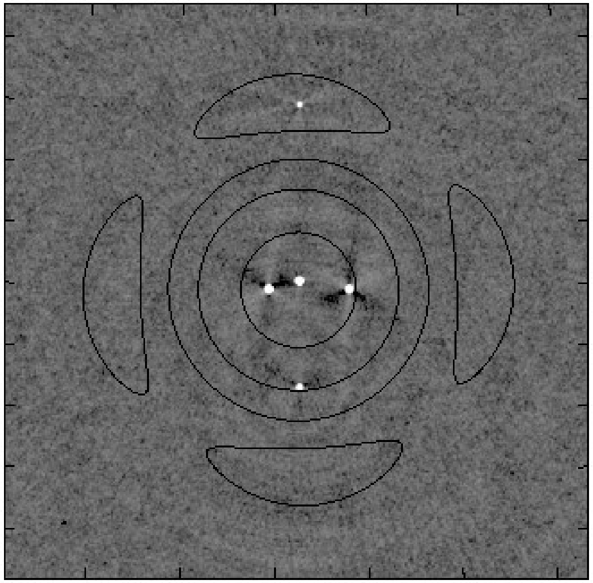


WB Effects: Frequency dependent PB

- L-band imaging with BW = 1GHz



Standard Imaging



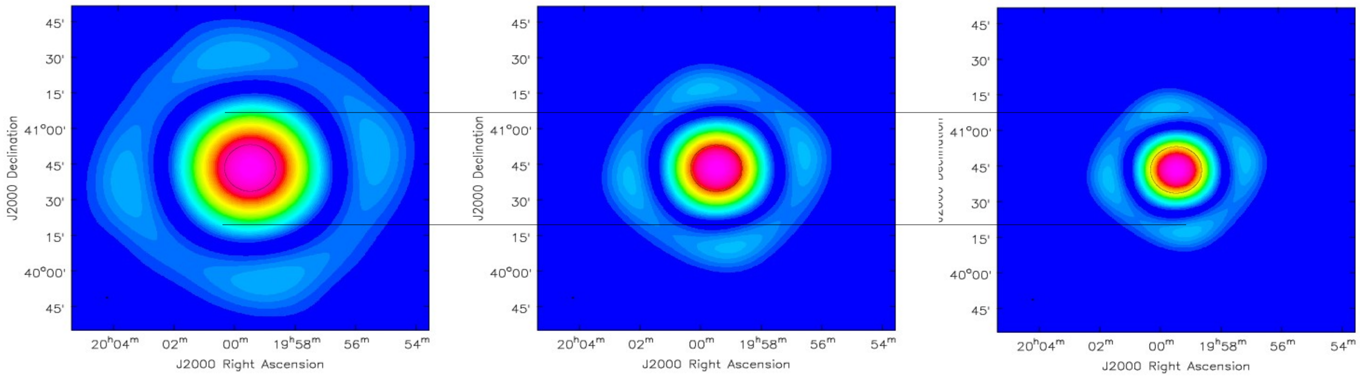
FT + MT-MFS

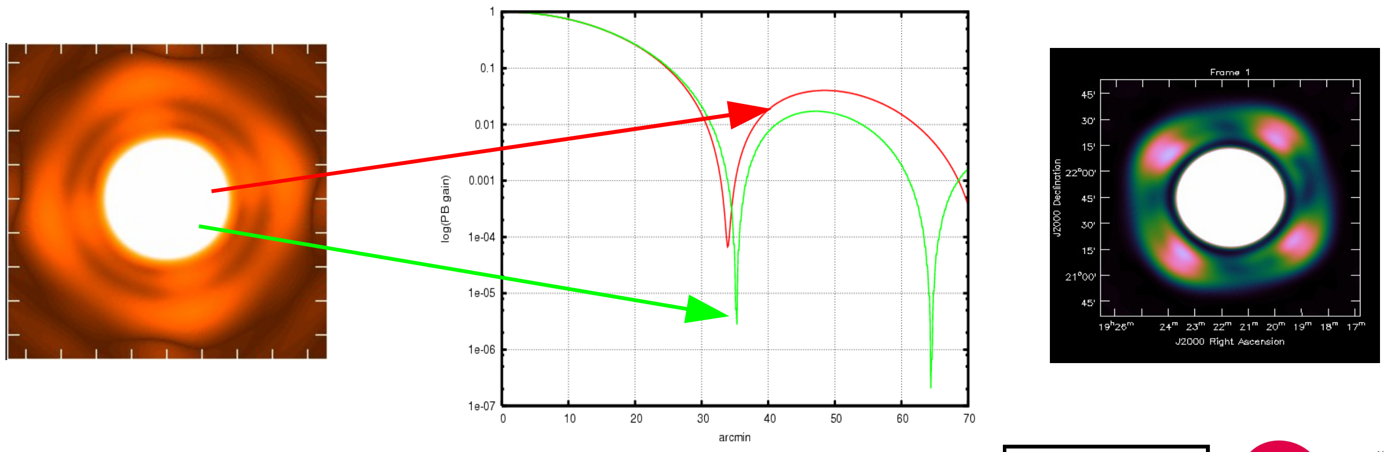
- *FT + MS-MFS: May work for static case*
  - *Requires more Taylor Terms ==> Higher memory footprint*

WB effects: Frequency dependent PB

$$V_{ij}^{Obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\; I(s,\nu)\; e^{\iota\left(u_{ij} l + v_{ij} m\right)}\; ds$$

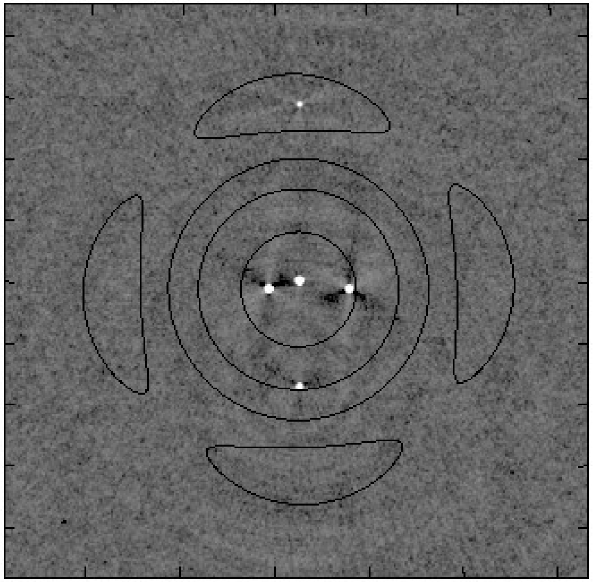
- ... J^s_i vary with frequency...frequency dependence also varies with time!



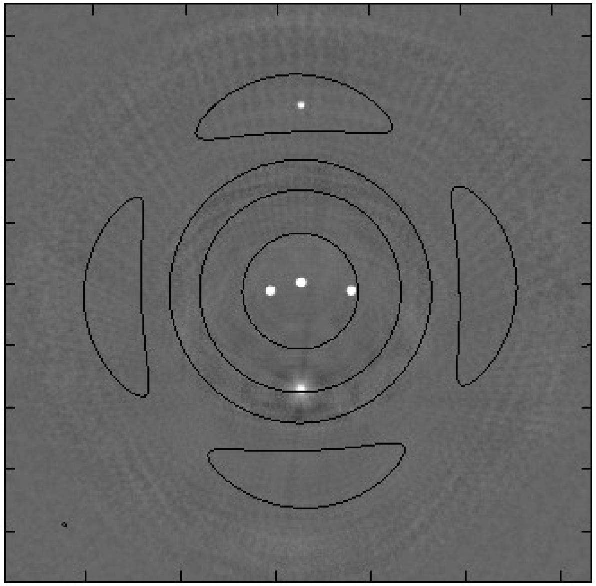


WB effects: Frequency dependent PB

- *A-Projection with MS-MFS: Account for <u>narrow-band PB</u> time variability*
  - *Only time-averaged quantities available in image domain*
  - *Cannot correct for time-varying WB effects*
  - *Require more Taylor Terms (even higher memory footprint)*



FT + MT-MFS

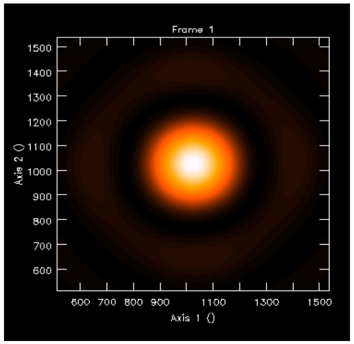


A-Proj. + MT-MFS

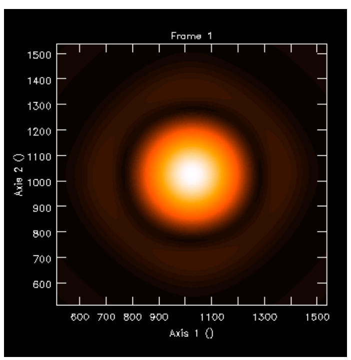
Solution: WB A-Projection

- Construct $A(\nu_*)$ such that $A(\nu_*)A^o(\nu)$ is *independent of frequency*

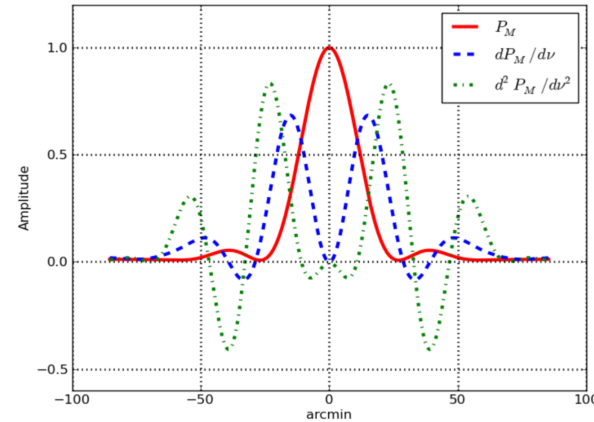
Frequency along the animation axis

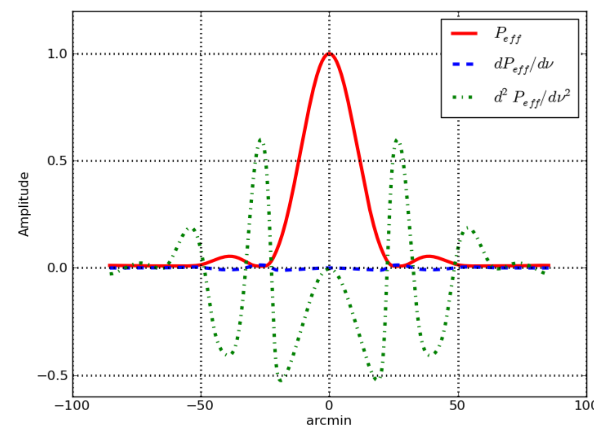


$PB(\nu)$



$\sqrt{PB(\nu)\,PB(\nu_*)}$

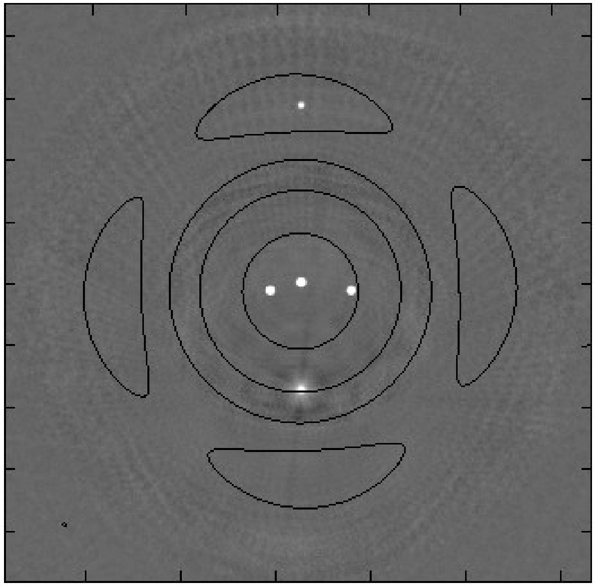




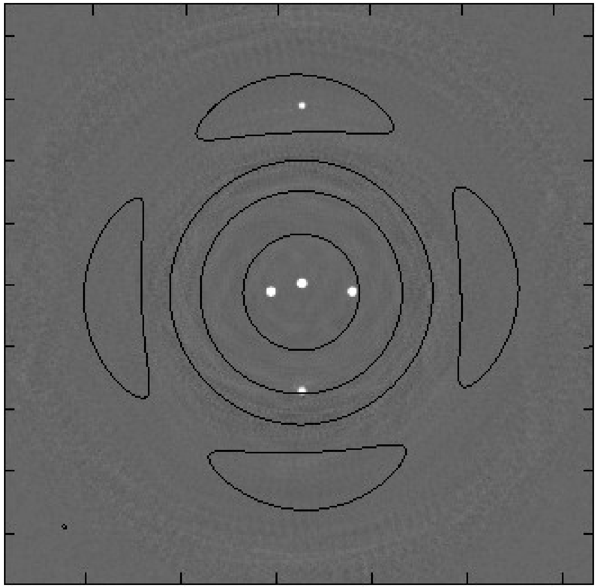
Bhatnagar & Rau, in prep.

Solution: WB A-Projection

- *Example of WB-AWP + MT-MFS (or any other appropriate minor-cycle algorithm)*



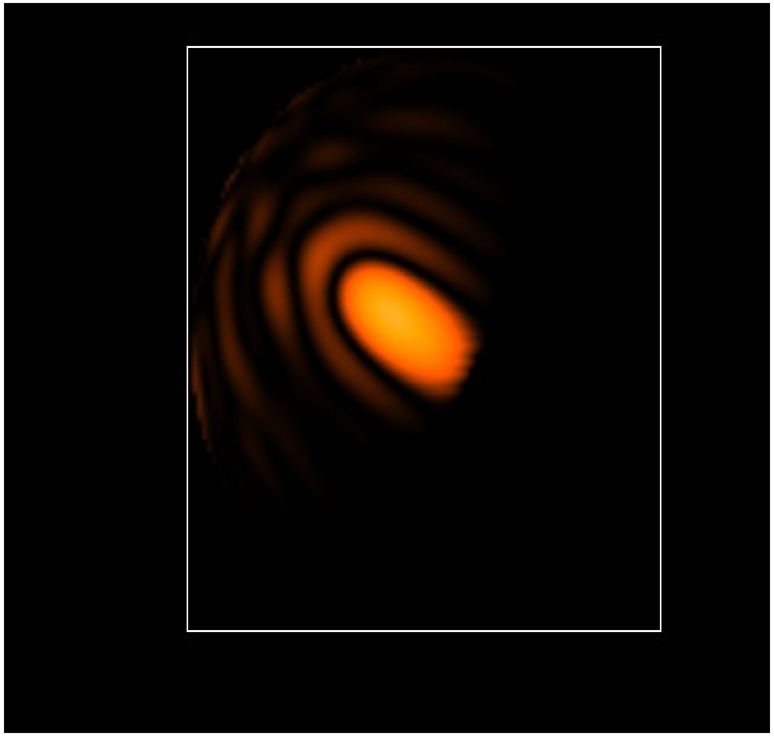
A-Proj. + MT-MFS



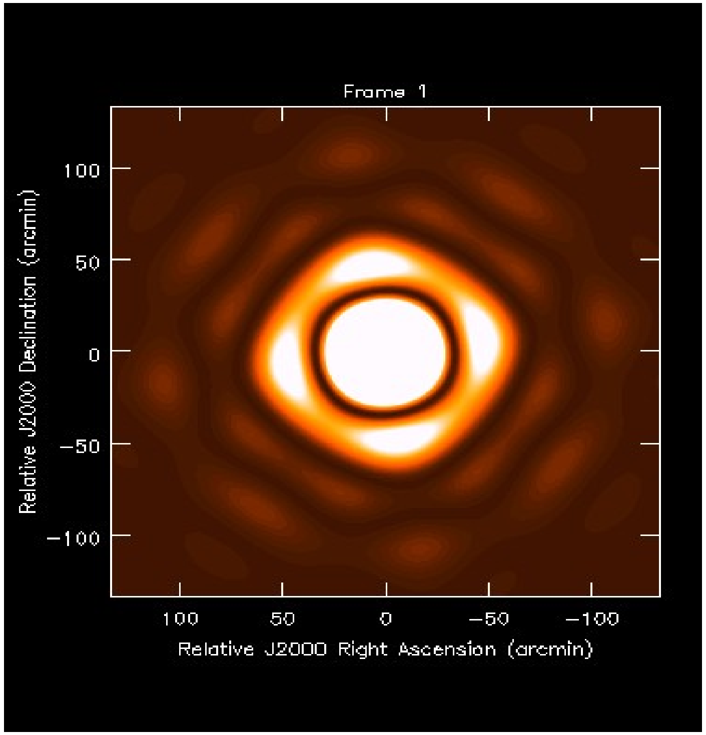
WB A-Proj. + MT-MFS

Bhatnagar & Rau, in prep.

Examples: Time varying PBs

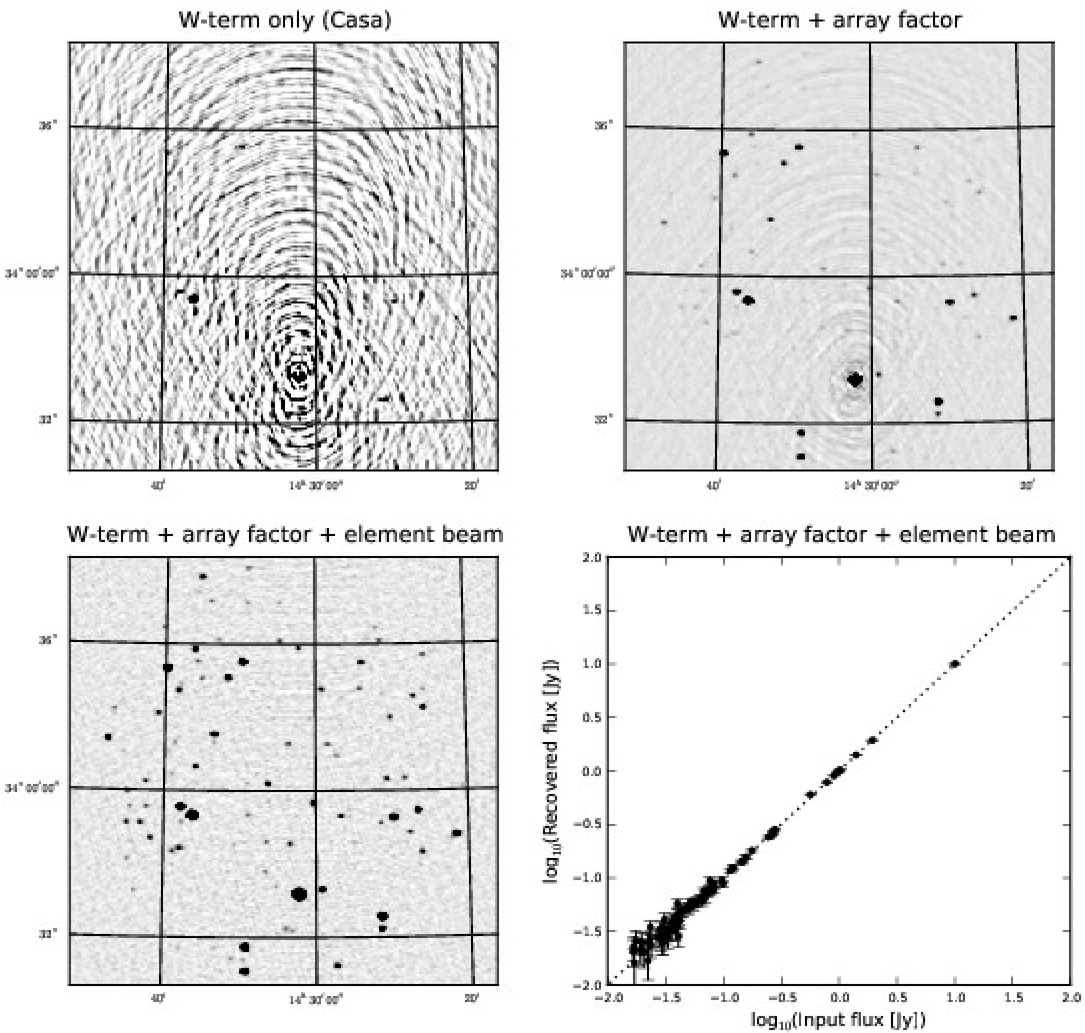


Simulations for LWA @50MHz
(Masaya Kuniyoshi (LWA/NRAO))



Model for EVLA PB at L-Band

Imaging with Aperture-array telescope



- Application of AW-Projection for imaging with Aperture-array
- Using R&D code modified for application to LOFAR

Image courtesy: C. Tasse, LOFAR

References:

Books

- Interferometry and Synthesis in Radio Astronomy, 2nd Ed.: Thompson, Moran and Swenson
- Synthesis Imaging in Radio Astronomy: II – The "White Book"

Papers

- W-Projection: IEEE Journal of Selected Topics in Signal Processing, Vol. 2, No. 5, 2008
- A-Projection: A&A, 487, 419, 2008 (arXiv:0808.0834)
- Scale sensitive deconvolution of astronomical images: A&A, 426, 747, 2004 (astro-ph/0407225)
- MS-Clean: IEEE Journal of Selected Topics in Signal Processing, Vol.2, No.5,2008

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- Advances in Calibration and Imaging in Radio Interferometry: Proc. IEEE, Vol. 97, No. 8, 2008
- Calibration and Imaging challenges at low frequencies: ASP Conf. Series, Vol. 407, 2009

Thesis

- High Fidelity Imaging of Moderately Resolved Source; PhD Thesis, Briggs, NMT, 1995
- Parametrized Deconvolution for Wide-band Radio Synthesis Imaging; PhD Thesis, Rao Venkata; NMT, 2010

Web Pages

- http://www.aoc.nrao.edu/~sbhatnag
- NRAO Algo. R&D Page: https://safe.nrao.edu/wiki/bin/view/Software/Algorithms/WebHome
- Home pages of SKA Calibration and Imaging Workshops (CALIM), 2005, 2006, 2008, 2009
- Home Pages of: EVLA, ALMA, ATA, LOFAR, ASKAP, SKA, MeerKat