Astrometry

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"From little things, big things grow…"   Bruce Springsteen/Paul Kelly

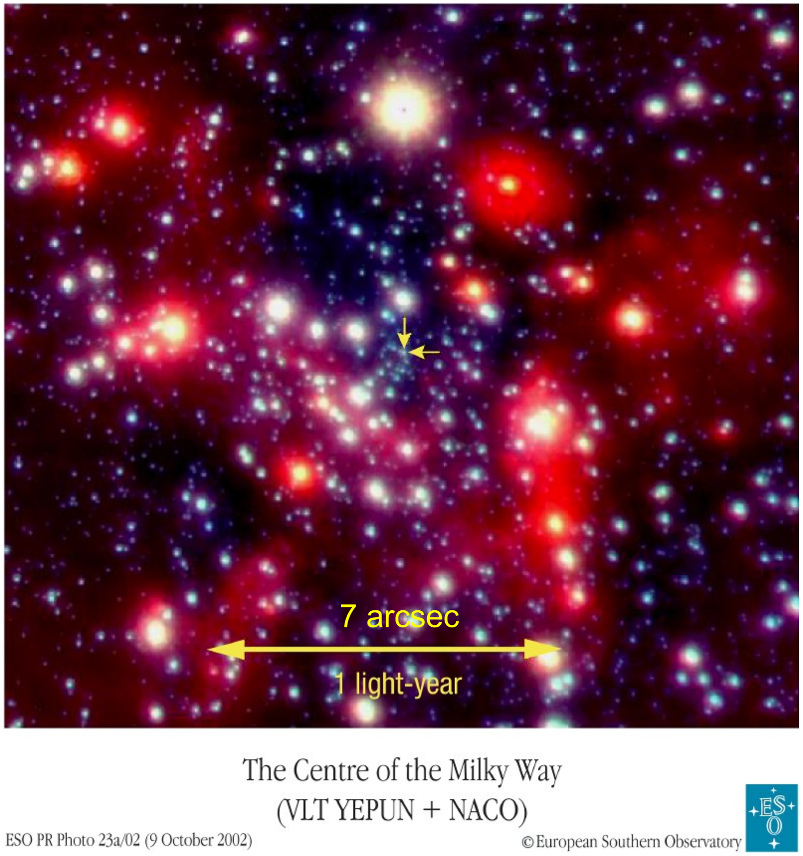
"Success is in the details"   Chinese fortune cookie

Where is the Galactic Center?

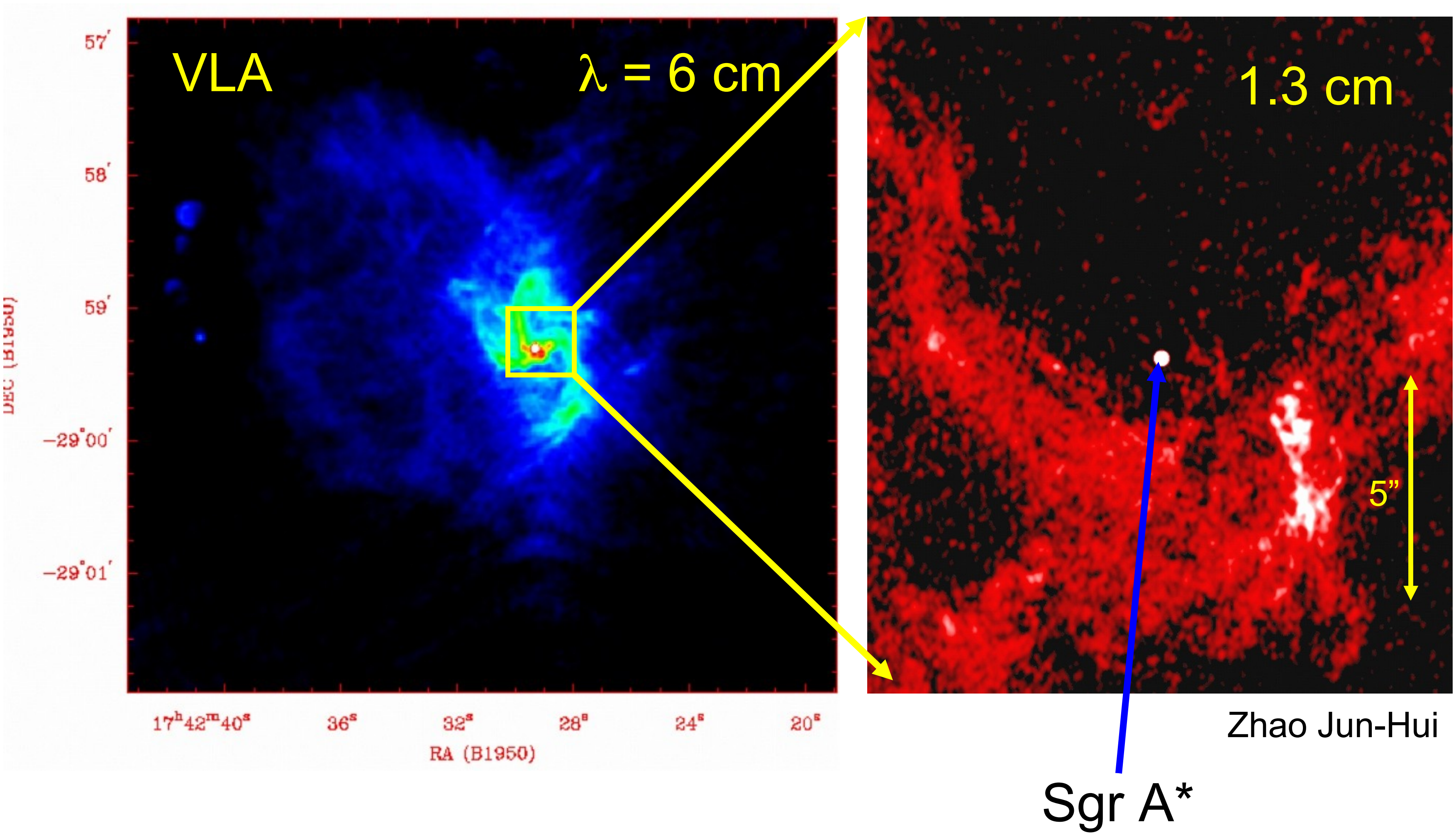
- A_v ~ 30 mag; A_K ~3 mag
- Many bright young stars
- But, no obvious AGN (Sgr A*)

Where is Sgr A*?

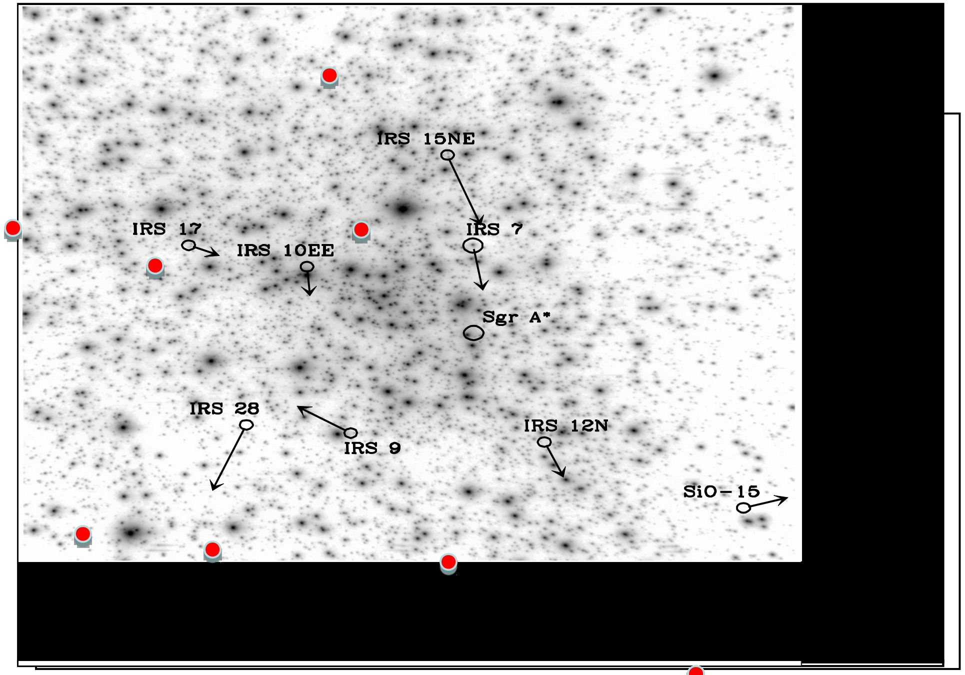
Infrared K-band (2 μm) image



Where is the Galactic Center?



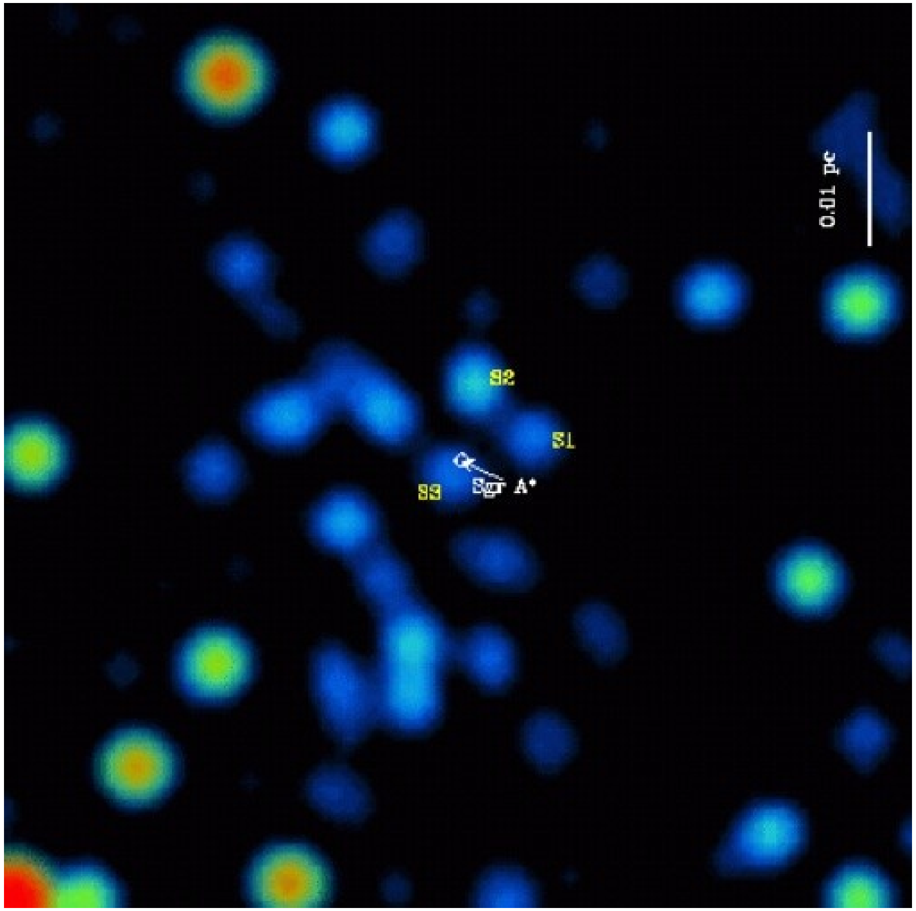
Red Giant SiO Maser Stars



Where is Sgr A* (IR)

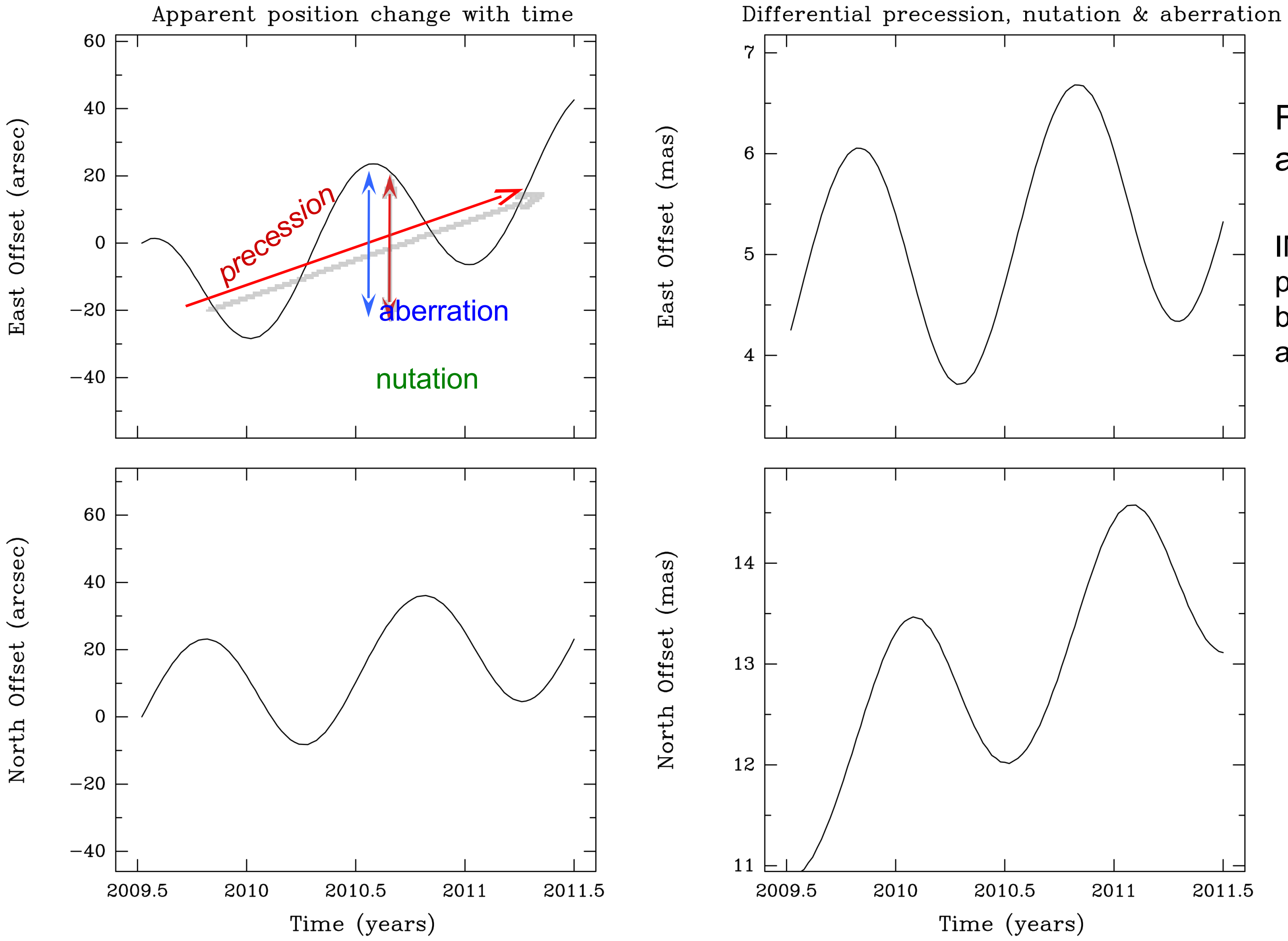
Use radio “grid”;solve for

- IR plate scale
- IR plate rotation
- IR (low order) distortions



- Sgr A* very dark
- Allows accurate orbit measurements

Subtle Effects



For source at 20^h,+45^o and offset of (10",0")

IMAGR maps corrected for precession to J2000 frame, but not for nutation or aberration

Micro-arcsec Astrometry with the VLBA

Comparable to GAIA & SIM

Fringe spacing:

$\theta_f \sim \lambda/D \sim 1$ cm / 8000 km = 250 μas

Centroid Precision:

$0.5\ \theta_f$ / SNR ~ 10 μas

Systematics:

path length errors ~ 2 cm (~2 λ)

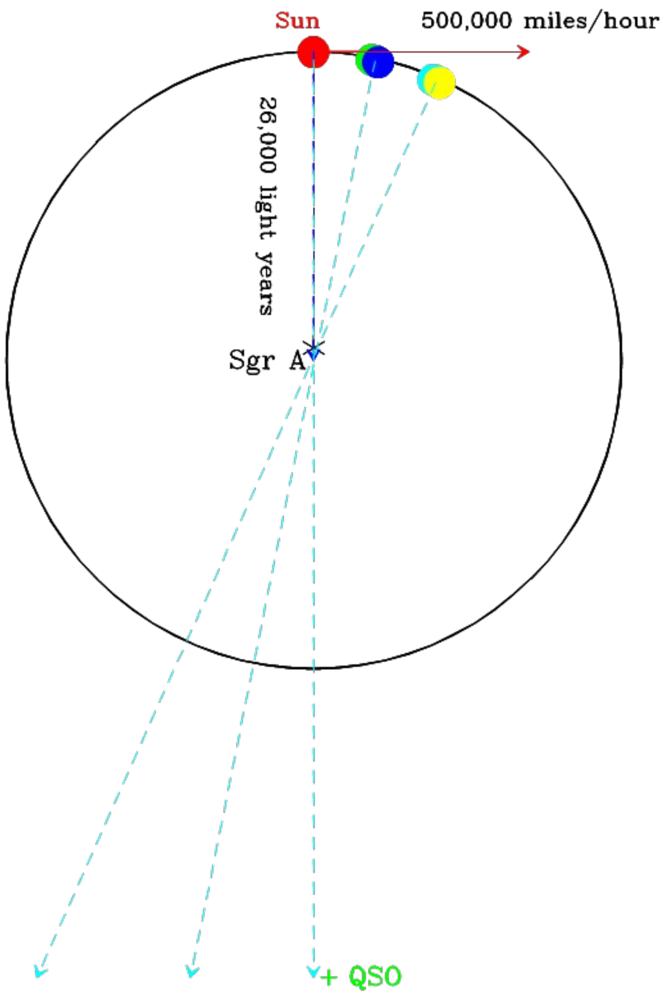
shift position by ~ $2\theta_f$

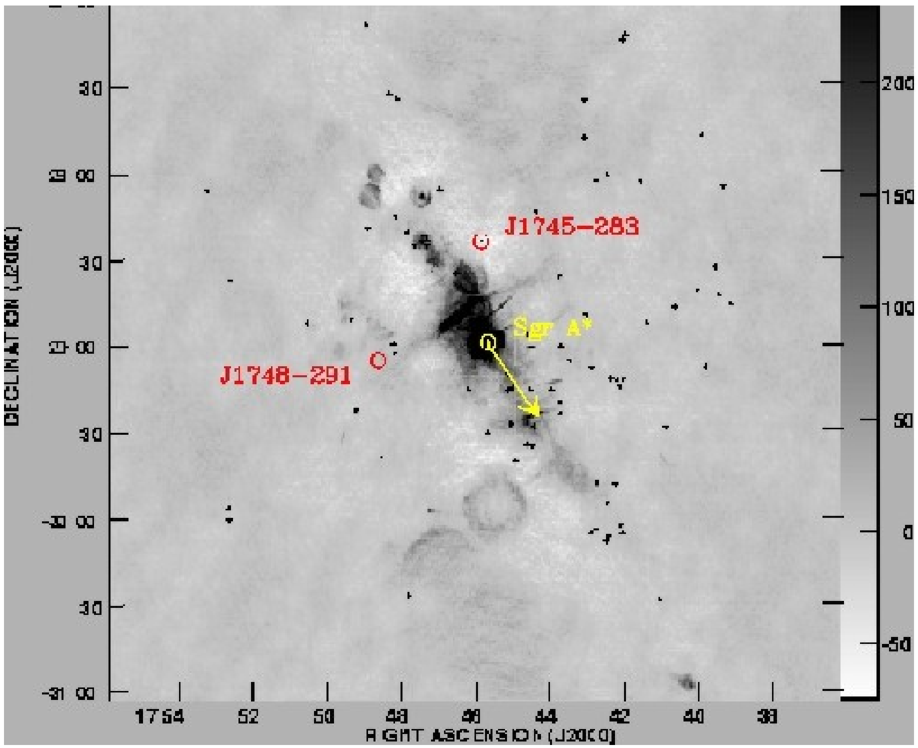
Relative positions (to QSOs):

$\Delta\Theta \sim 1$ deg (0.02 rad)

cancel systematics: $\Delta\Theta \star 2\theta_f \sim 10\ \mu$as

Sgr A* Proper Motion





Sgr A* Proper Motion

IR Stellar Orbits:

$M_{IR} \sim 4 \times 10^6 M_{sun}$

R < 50 AU

Radio Observations:

Sgr A* motionless ➔

M > 10% of M_{IR}

Observed size:

R < 0.5 AU

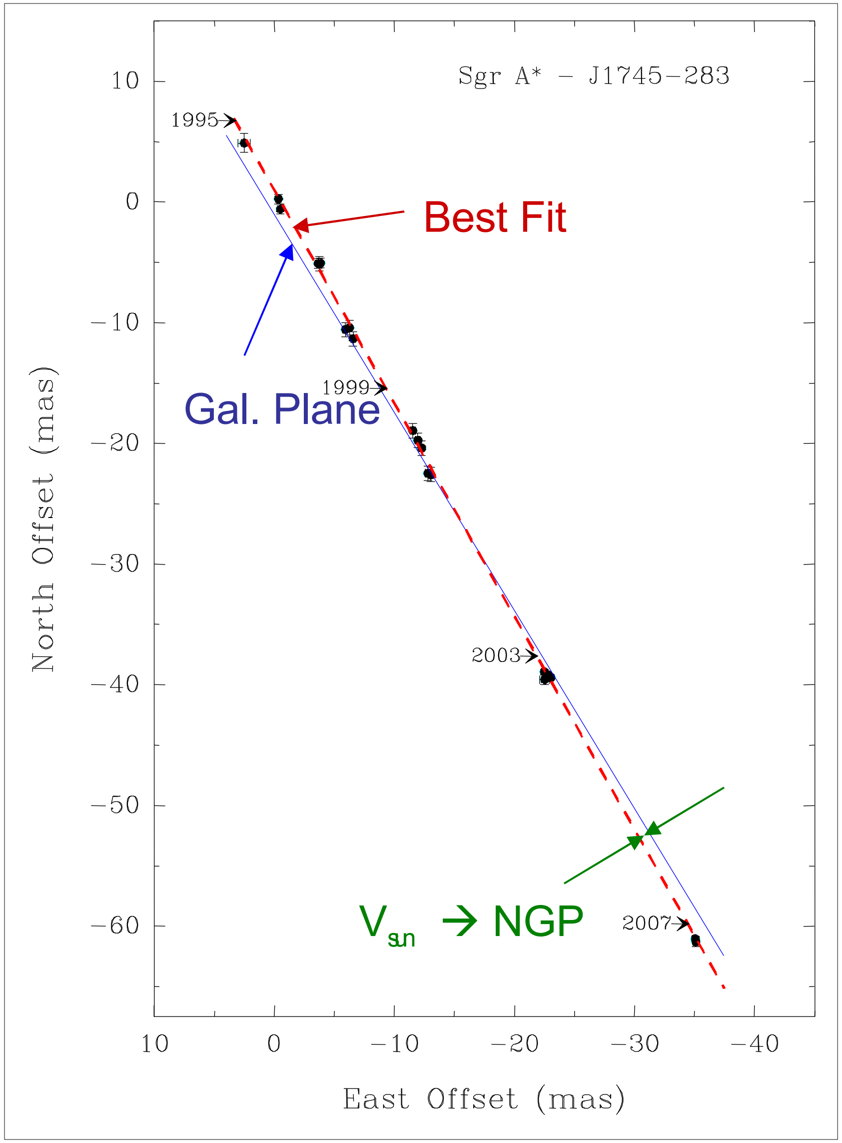
IR + Radio data combined:

Dark mass = luminous source

Density > 10^{22} M_{sun} /pc^3

Overwhelming evidence for a Super-Massive Black Hole

How do we make such measurements?



Calibration: Step 1

Fix phase errors in VLBA correlator model:

• Parallactic Angle (feed rotation effect)	CLCOR
• Atmospheric zenith delays ("geodetic" blocks)	DELZN/CLCOR
• Ionospheric zenith delays (global electron models)	TECOR
• Earth's Orientation Parameter errors	CLCOR
• Source coordinate errors (if known)	CLCOR

Calibration: Step 2

Calibrate amplitudes (correlation coefficient → flux density):

- Correct for clipper bias — CLCAL
- Apply system temperatures/gain curves — APCAL/CLCAL

Calibration: Step 3

Align electronic phase shifts among bands:

- Determine band phases on strong source FRING
- Correct all data CLCAL

Fix spectral drift (Doppler shift from Earth's rotation)

- Apply bandpass corrections (if necessary) BPASS
- Fourier transform to delays,

  Apply phase-slope across delay function,

  Inverse Fourier transform back CVEL

Calibration: Step 4

Phase reference data to 1 source / band / spectral channel :

- Calculate phase reference CALIB or FRING

- Apply phases to all data CLCAL

What limits positional accuracy? …

Signal to Noise Limitations

$$\sigma_S = \frac{b \ \mathrm{SEFD}}{\sqrt{2B\tau \ N(N-1)/2}} \quad \approx 0.2 \ \ \mathrm{mJy}$$

$$\begin{aligned} b &= 1.2 \\ 2B &= 512 \times 10^6 \ \ \mathrm{Hz} \\ \tau &= 3,600 \ \ \mathrm{sec} \\ N &= 10 \ \ \mathrm{antennas} \\ \mathrm{SEFD} &= 1500 \ \ \mathrm{Jy} \end{aligned}$$

$$\sigma_\theta = 0.5 \ \mathrm{FWHM/SNR} \quad \approx 0.05 \ (S/2 \ \mathrm{mJy})^{-1} \ \mathrm{mas}$$

$$\begin{aligned} \mathrm{FWHM} &\approx 1 \ \mathrm{mas} \\ \mathrm{SNR} &\approx 5S(\mathrm{mJy}) \qquad (= S / \sigma_S) \end{aligned}$$

Systematic Limitations

$$\sigma_\theta = \mathrm{FWHM}\ (c\ \Delta\tau/\lambda)\ \Delta\theta \quad \approx 0.05\ \mathrm{mas}$$

$$c\ \Delta\tau \approx c\ \Delta\tau_0\ \sec ZA \tan ZA$$

(will explain formula later)

$$\begin{aligned} \mathrm{FWHM} &\approx 1\ \mathrm{mas} \\ c\ \Delta\tau_0 &\approx 1\ \ \mathrm{cm} \\ ZA &\approx 60\ \ \mathrm{deg} \\ \lambda &\approx 1.3\ \ \mathrm{cm} \\ \Delta\theta &\approx 1\ \ \mathrm{deg} \end{aligned}$$

Note: σ_θ independent of λ, since FWHM ~ λ/D

Signal to Noise vs. Systematic Limitations

σ_θ (noise) $\approx 0.05\ (S/2\ \mathrm{mJy})^{-1}$ mas

σ_θ (systematics) $\approx 0.05\ (c\Delta\tau/4\ \mathrm{cm})$ mas

Typically, σ_θ (noise) $< \sigma_\theta$ (systematics)

for $S > 2$ mJy

If $S > 2$ mJy, use more observing time to calibrate.

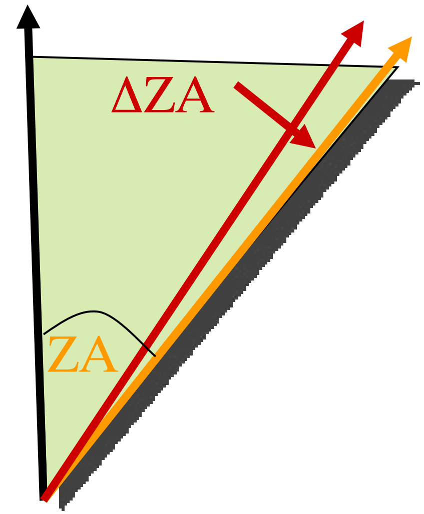
Atmospheric & Ionospheric Errors

Frequency (maser)		Un-modeled[1] zenith path length Atmosphere	Ionosphere[2]
43 GHz	(SiO)	5 cm	0.5 cm
22	(H_2O)	5	2
12	(CH_3OH)	5	6
6.7	(CH_3OH)	5	20
1.6	(OH)	5	300

[1] After removing VLBA correlator model

[2] Highly variable night-to-day and with solar cycle. Can be partially corrected with global models of total electron content.

Relative Atmospheric Delay Errors



$$\tau_A \approx \tau_0 \ \sec ZA$$

Difference between target and reference sources:

$$\Delta\tau_A = \left(\frac{\partial \tau_A}{\partial ZA}\right) \Delta ZA$$

$$\Delta\tau_A = \tau_0 \ \sec(ZA) \ \tan(ZA) \ \Delta ZA$$

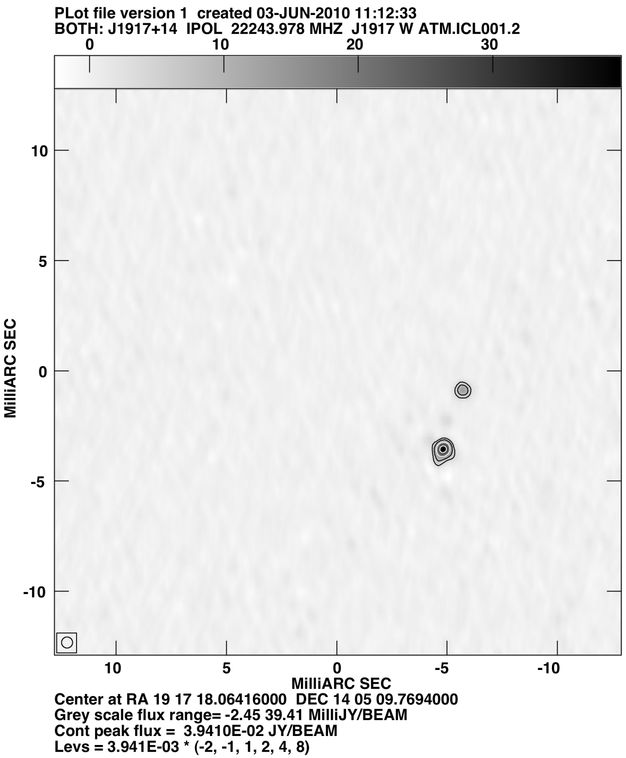
Note: $\sec(ZA) \ \tan(ZA) \approx 3.5 \quad \text{for } ZA = 60^\circ$

$\sec(ZA) \ \tan(ZA) \approx 8.0 \quad \text{for } ZA = 70^\circ$

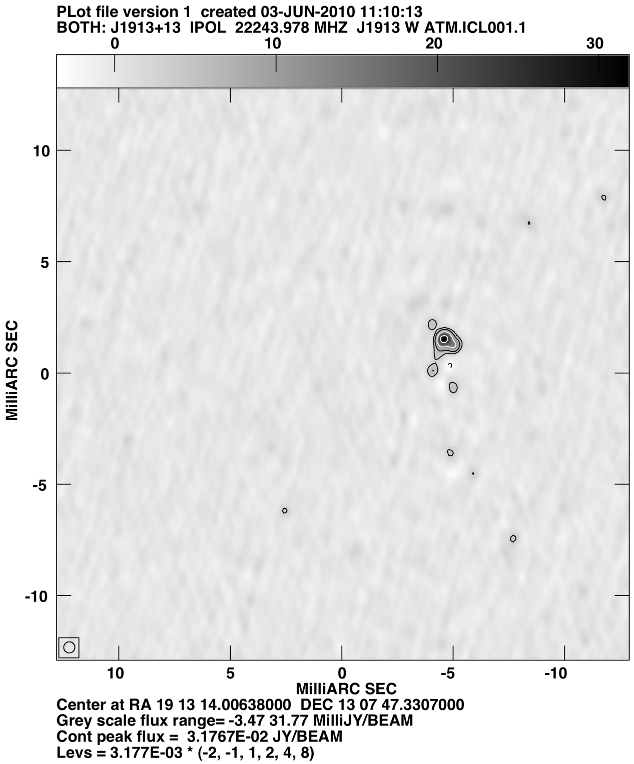
Effect of Separation of Target and Reference Source

G048.61+0.02 maser phase reference

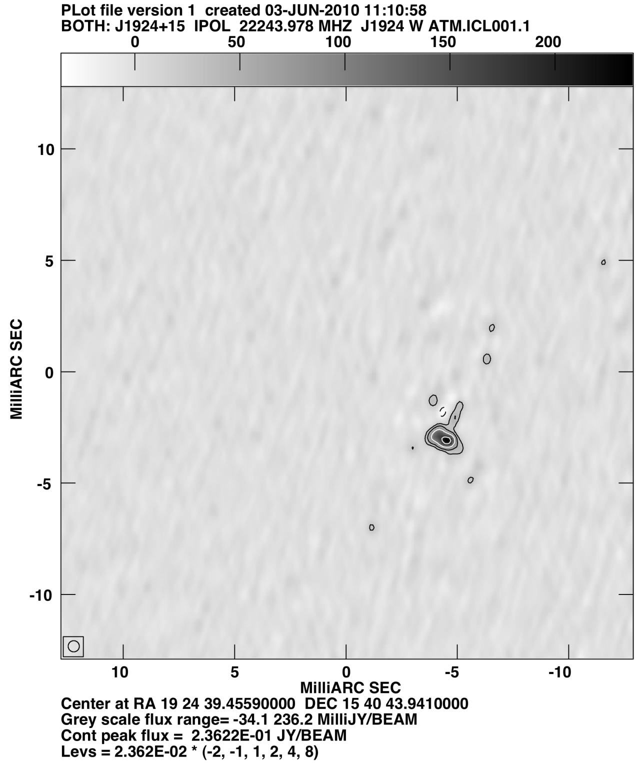
J1917+1405: $(\Delta\theta_x, \Delta\theta_y)$ = (-0.8,+0.2) deg



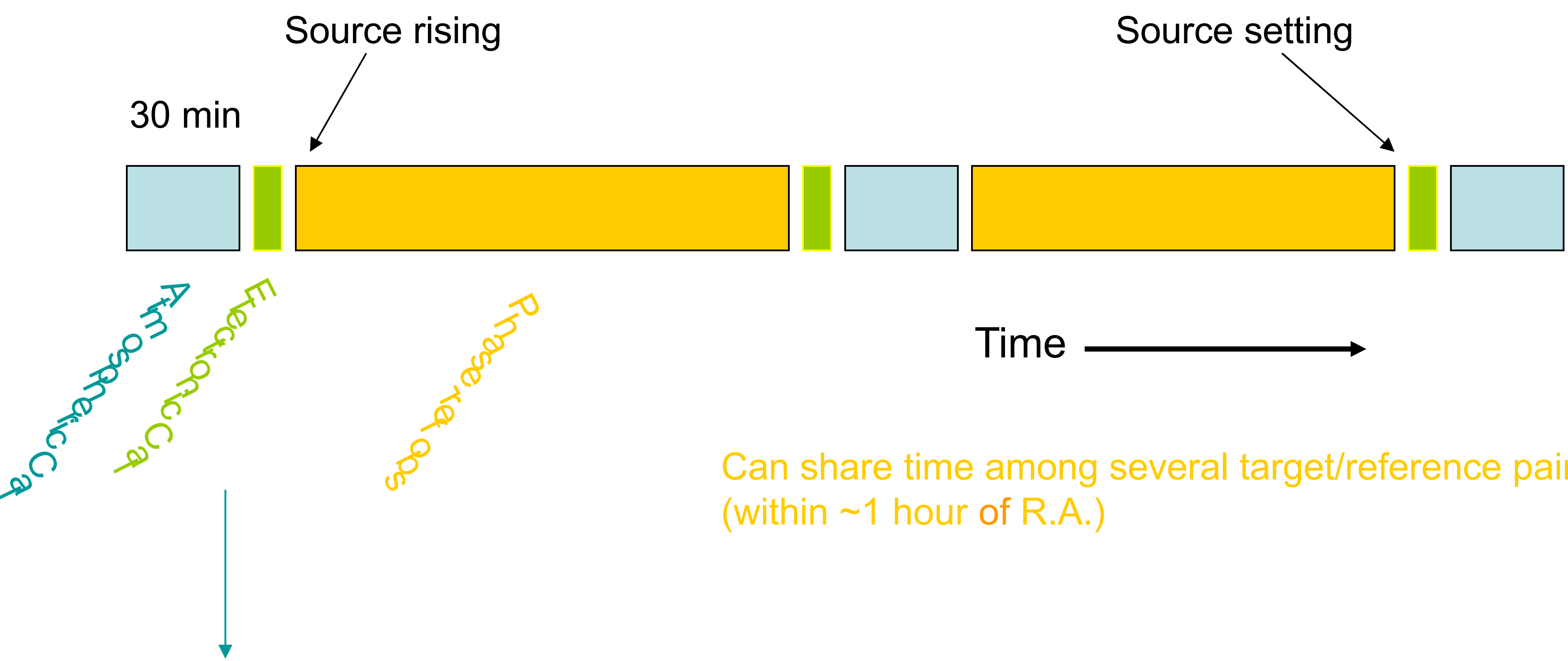
J1913+1307: $(\Delta\theta_x, \Delta\theta_y)$ = (-1.8,-0.8) deg



J1924+1540: $(\Delta\theta_x, \Delta\theta_y)$ = (+1.0,+1.8) deg



Typical Observing Sequence



Can share time among several target/reference pairs
(within ~1 hour of R.A.)

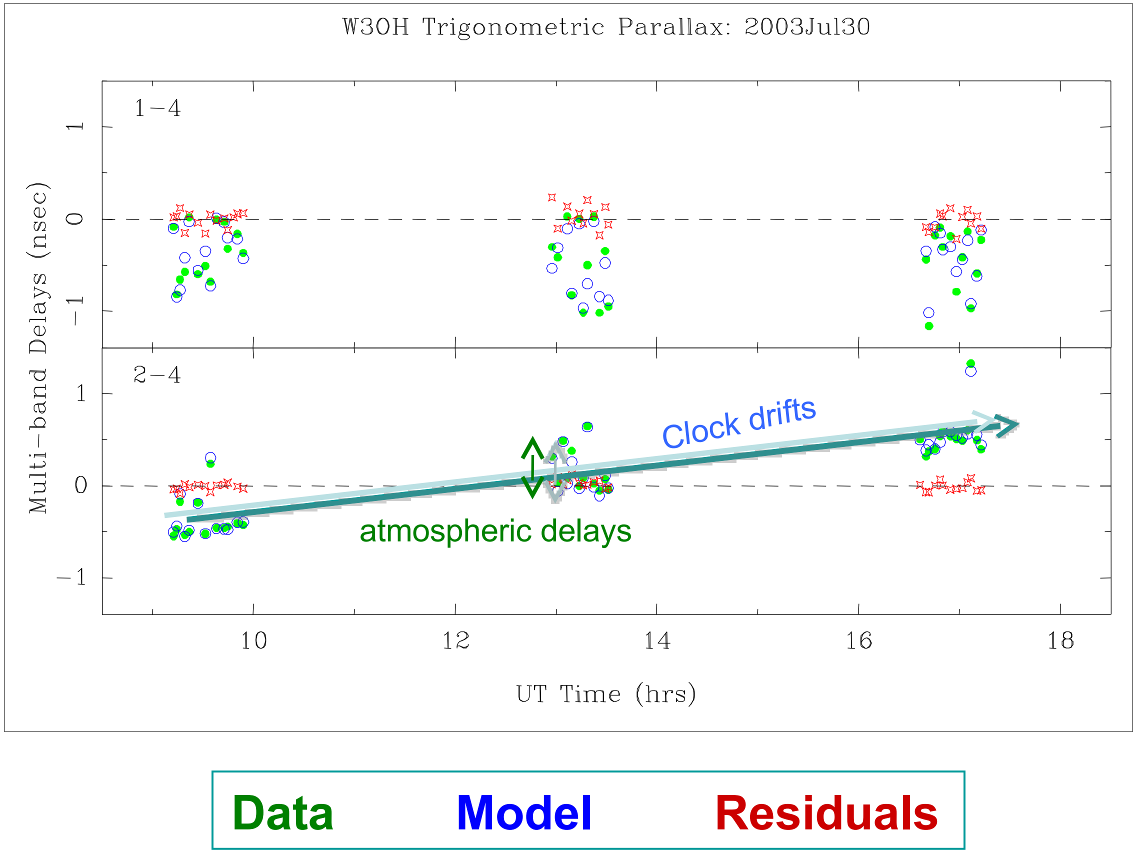
QSO-1(ZA=10^0), QSO-2(ZA=50^0), … QSO-12(ZA=30^0)

Atmospheric Delay Calibration

- Goal: measure zenith delay (τ_0) above each antenna
- Spread observing bands to cover 500 MHz

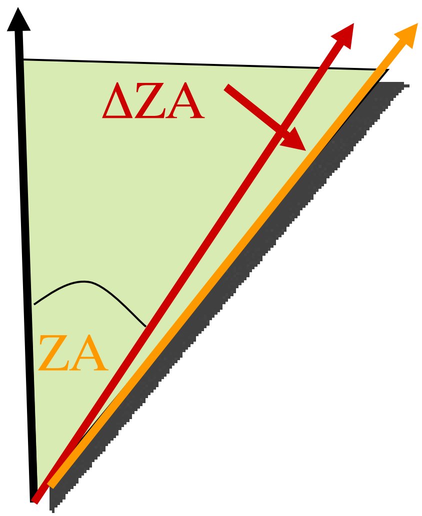
  $\sigma_\tau \sim (1/BW) * (1/SNR)$
- Observe QSOs over range of elevations
- Fit multi-band delays to atmospheric model:

  $\sigma_{\tau 0} \sim 1$ cm accuracy



Position Errors

Effects of position error of phase reference source:

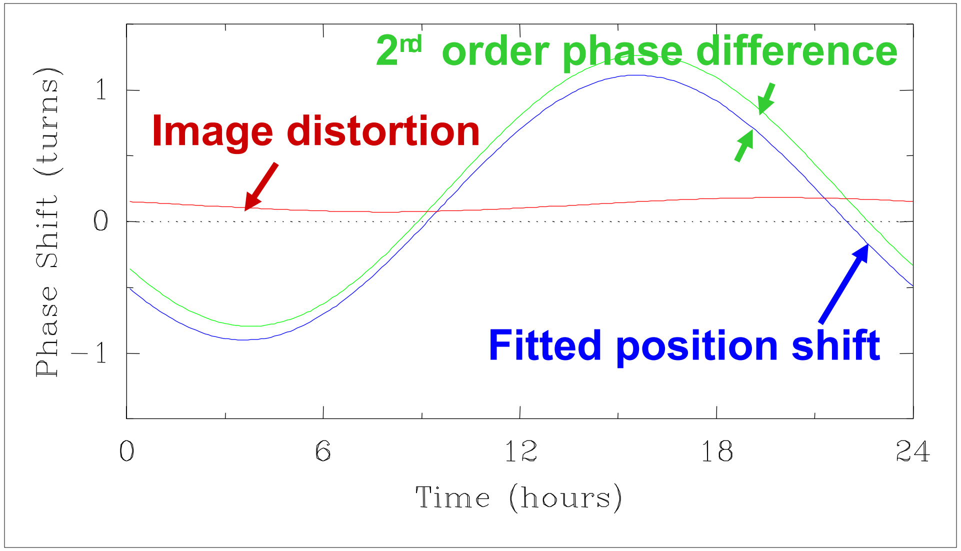


Target → Reference ΔΘ = 1 degree

Position Errors

Effects on target of position error of phase reference source:

- 1^{st} order correction:
  position shift of Target
- 2^{nd} order corrections:
  small shift of Target
  distorts image



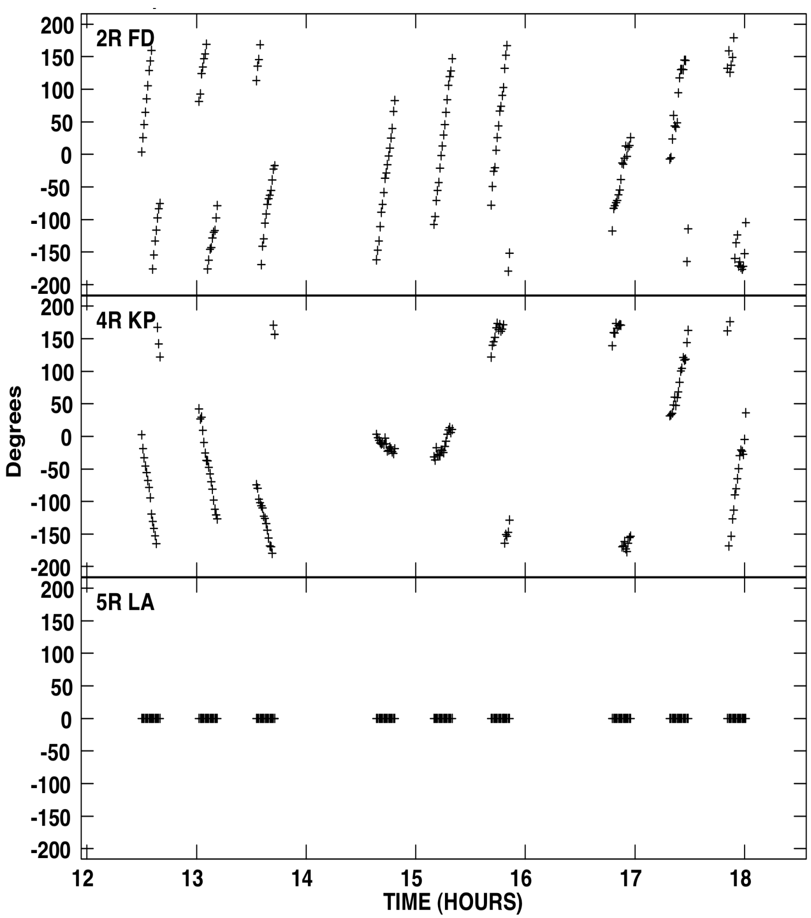
Target → Reference $\Delta\Theta$ = 1 degree

Reference pos. err σ_θ = 0.1 arcsec

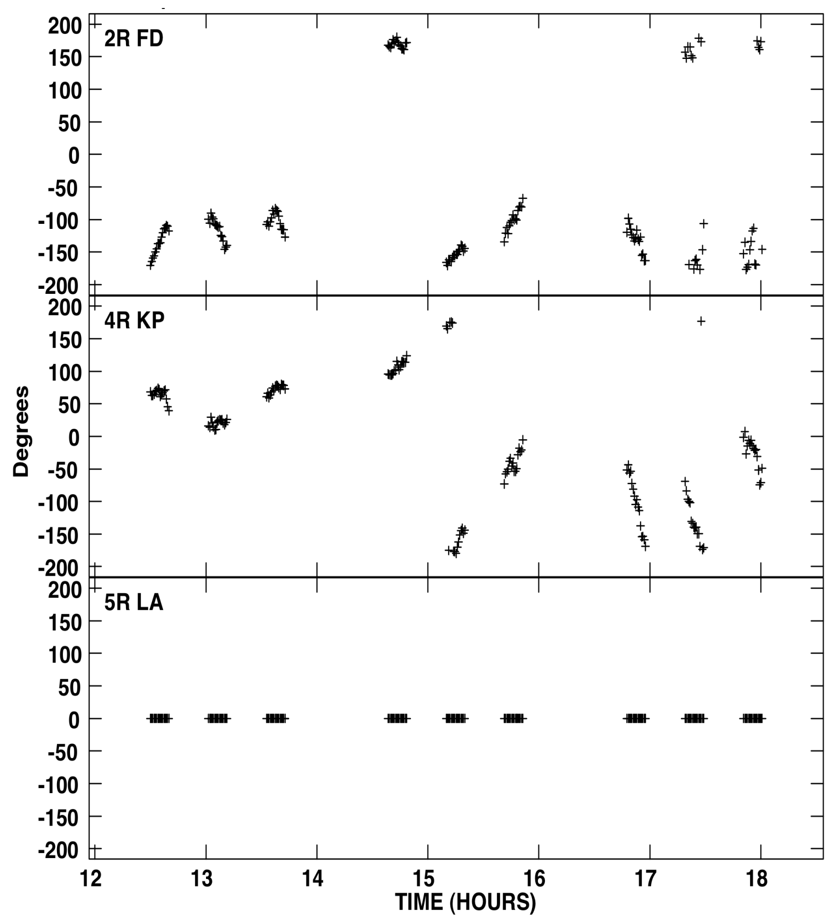
Need reference position accurate to ~10 mas

Position Errors: Reference Phases

0.2 arcsec position error



0 arcsec position error



Improving Reference Source Position

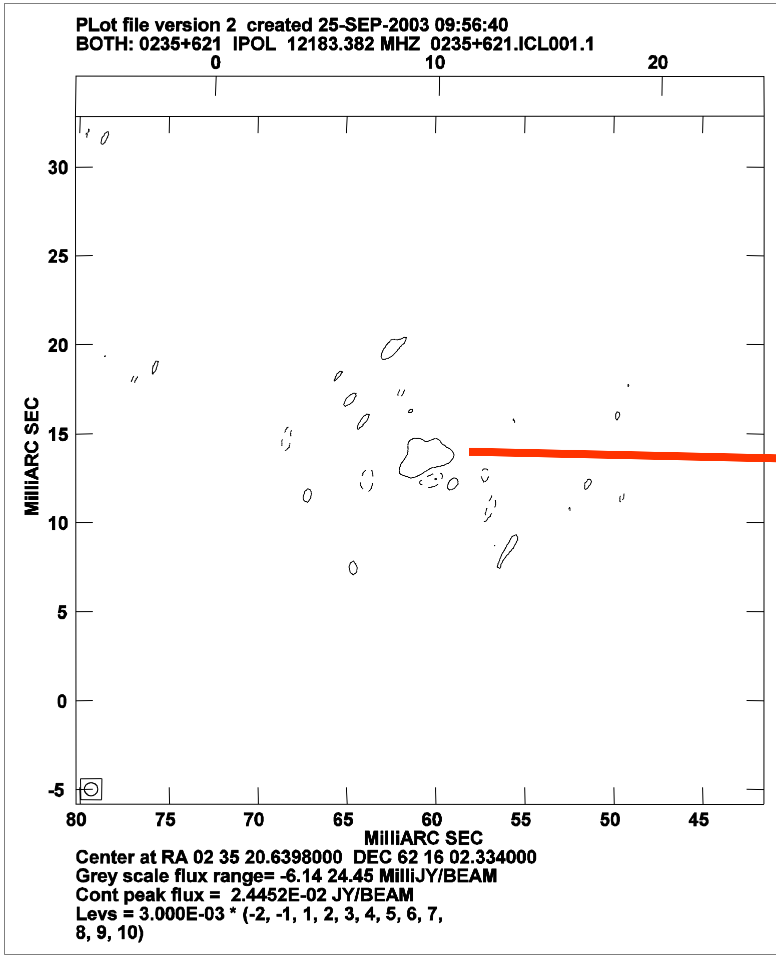
Measure with VLA:

- need largest (A) configuration
- ~10 mas accuracy

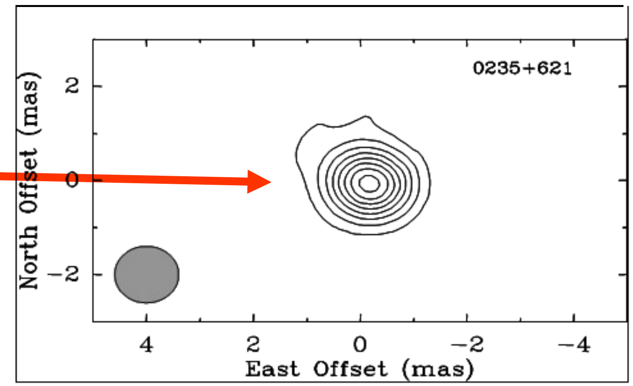
Measure with VLBA snapshots:

- Fit multi-band delays (QSOs)
  - ~1 mas acuracy
- Fit fringe rates (masers)
  - ~50 mas accuracy

Before and After Images



with ~0.3 arcsec error in reference position



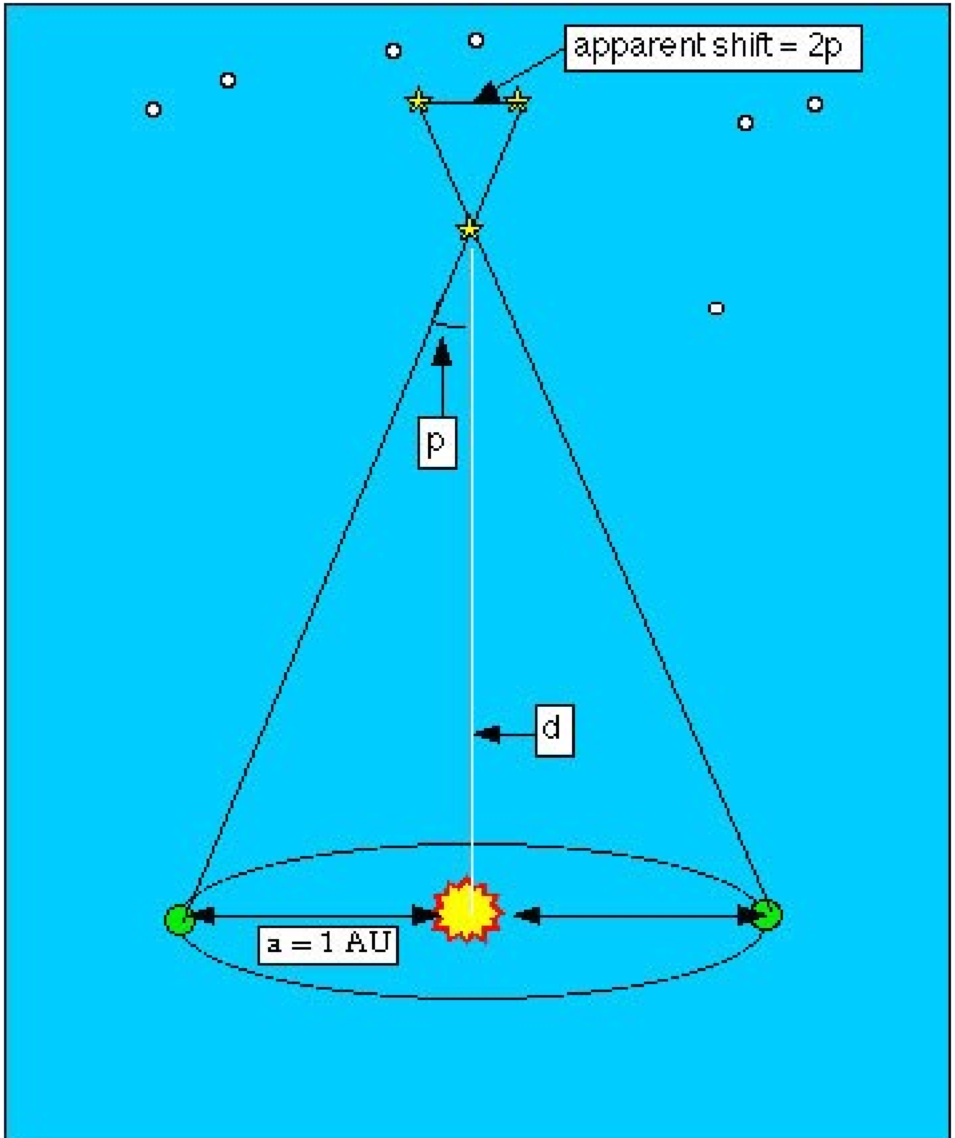
with no error in reference position

Lunar Parallax

Hipparchus (189 BC)

Pete Lawrence's Digitalsky: http://www.digitalsky.org.uk

Stellar (Annual) Parallax



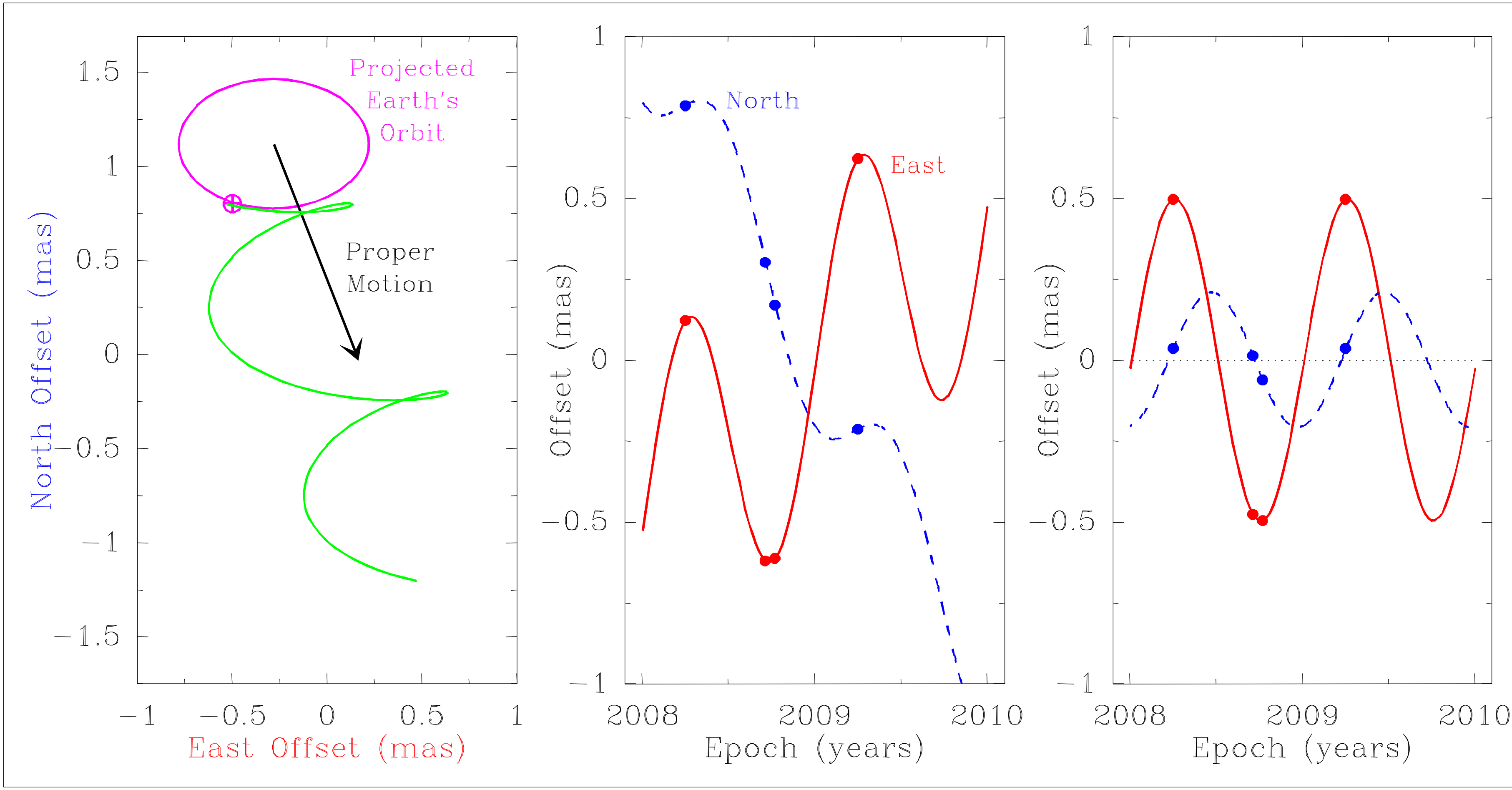
d(pc) = 1 / p(arcsec)

1 pc = 206,000 AU
= 3 x 10^{13} km
= 3.26 light-years

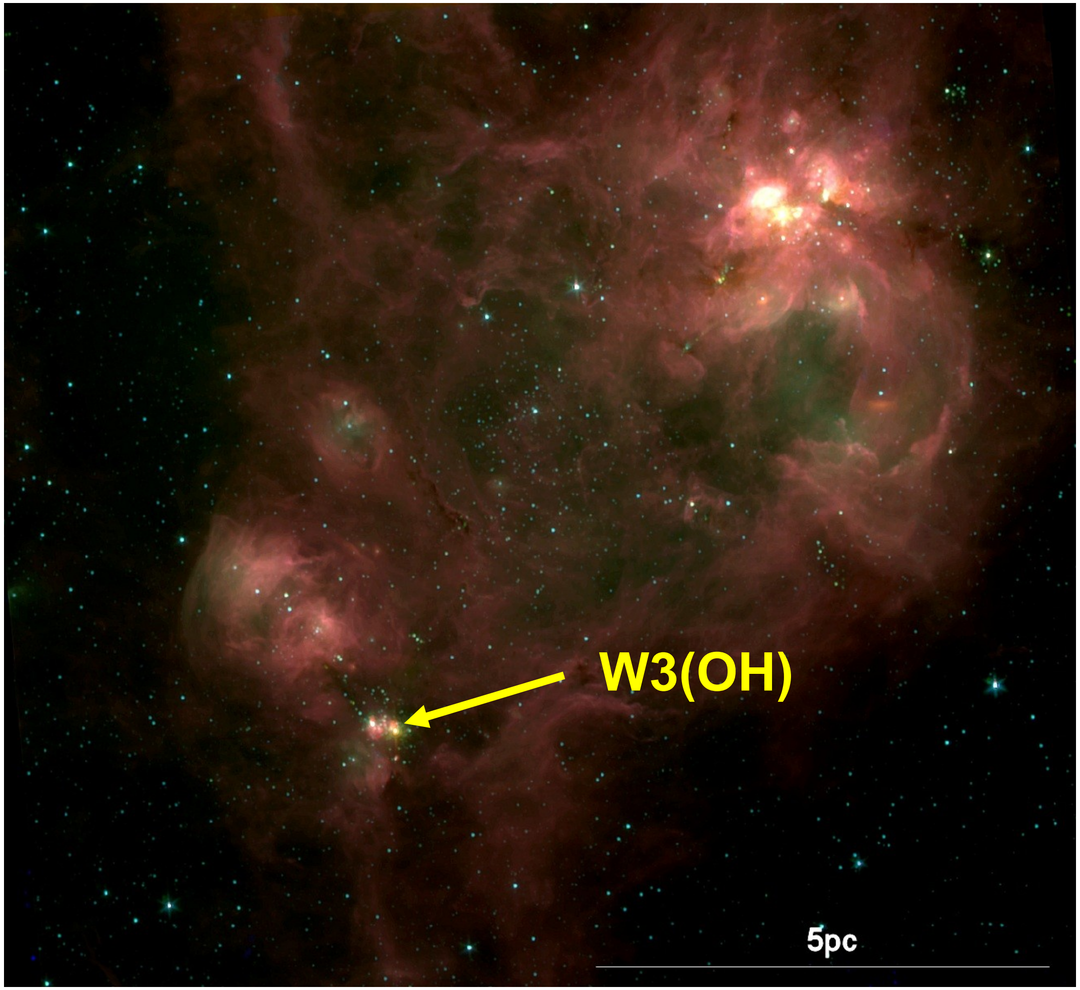
Some parallax values:

Moon: p ~ 0.1 deg
Nearest stars: p ~ 1 arcsec
Gal. Center: p ~ 0.1 mas
Nearby galaxy: p ~ 1 μas

Parallax 1.01



Milky Way Parallaxes



T. Megeath (Spitzer Space Telescope)

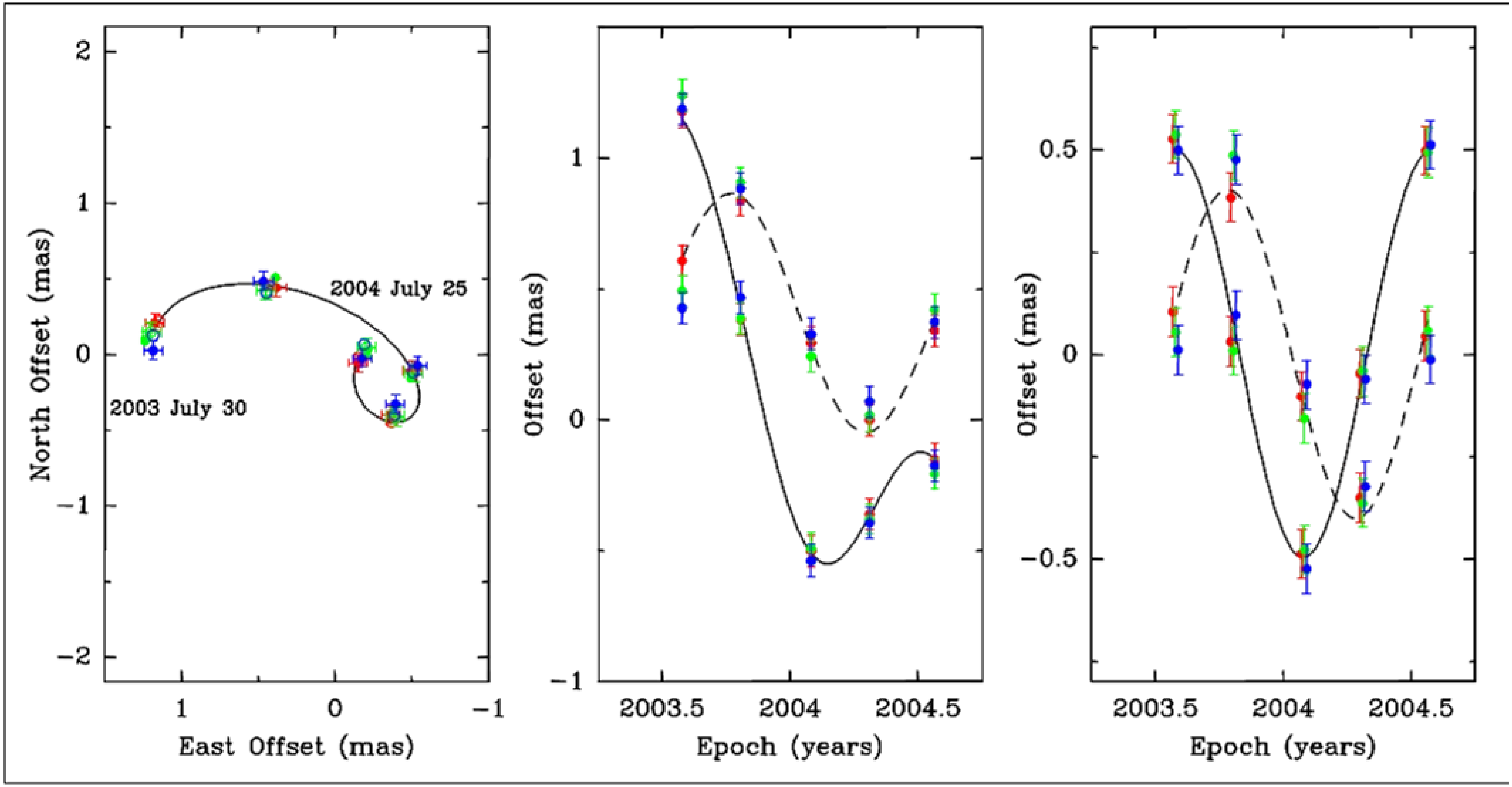
Distance estimates:

Kinematic = 4.3 kpc

Photometric = 2.2 kpc

(R. Humphreys 1970's)

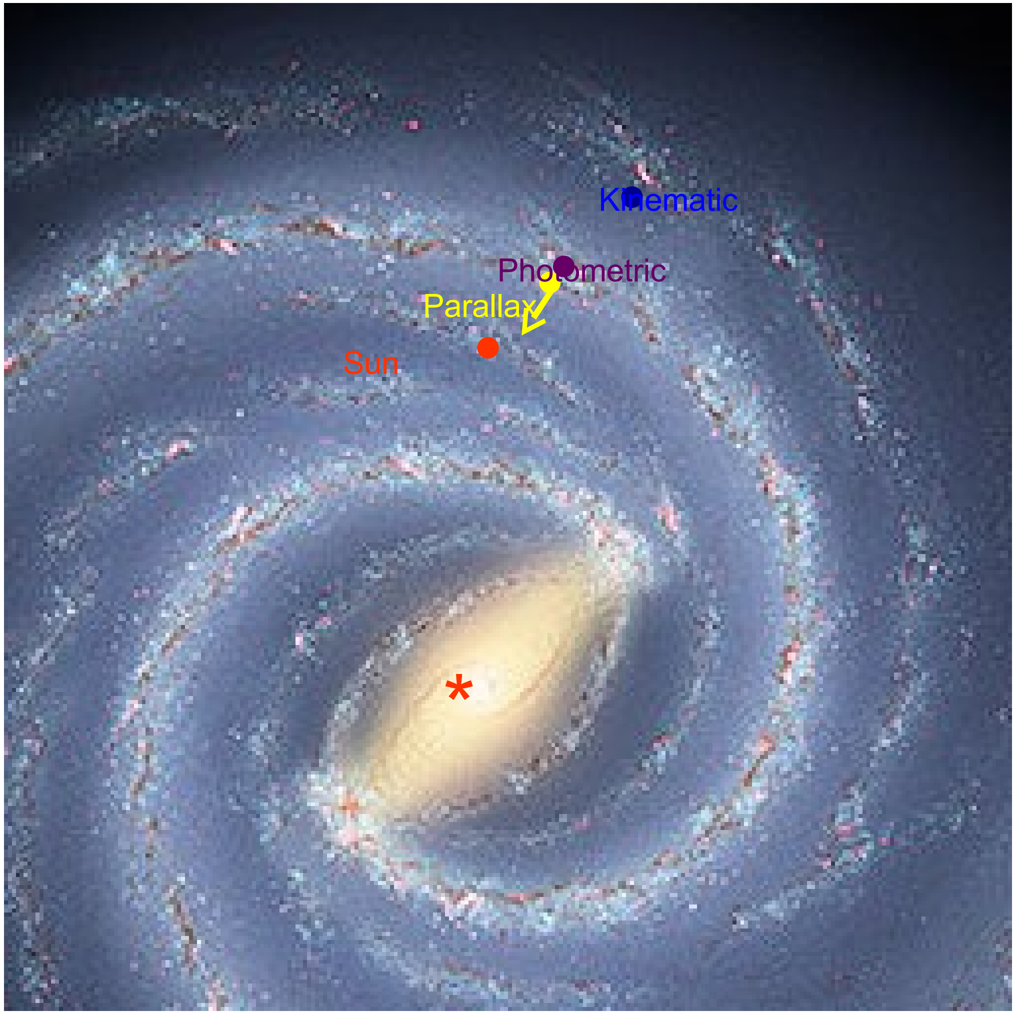
W3OH Parallax



Xu, Reid, Zheng & Menten (2006)

π = 0.512 +/- 0.010 mas

W3OH Parallax



- $D_{photo} \sim D_{parallax}$
- D_k way off
- In Perseus Arm, not in Outer Arm
- Large peculiar V

Least–Squares Fitting

Goal: Minimize $\chi^2 = \sum_{i=1}^{N}(d_i - m_i)^2$

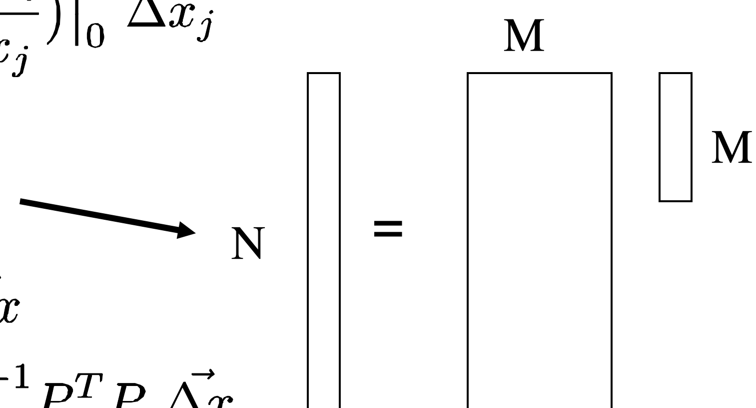
where d_i is i^{th} datum and m_i is its model value.

Taylor expand model about M parameters, x_j:

$$m_i = m_i\big|_0 + \sum_{j=1}^{M}\left(\frac{\partial m_i}{\partial x_j}\right)\Big|_0 \,\Delta x_j$$

$$r_i = d_i - m_i\big|_0 = \sum_{j=1}^{M}\left(\frac{\partial m_i}{\partial x_j}\right)\Big|_0 \,\Delta x_j$$

$$\vec{r} = P\ \vec{\Delta x}$$



$$P^T\vec{r} = P^T P\ \vec{\Delta x}$$

$$\left(P^T P\right)^{-1} P^T\vec{r} = \left(P^T P\right)^{-1} P^T P\ \vec{\Delta x}$$

$$\vec{\Delta x} = \left(P^T P\right)^{-1} P^T\vec{r}$$

Experiment Design

Least–squares solution: $\vec{\Delta x} = \left(P^T P\right)^{-1} P^T \vec{r}$

where $P = \text{Matrix}\left[\frac{\partial m_i}{\partial x_j}\right]$

"Design matrix" $\left(P^T P\right)^{-1}$

diagonal elements give parameter uncertainties (variances)

off-diagonals give parameter co-variances (correlations)

Note: don't need data to estimate parameter uncertainties!

Parallax Design

"Design matrix" $\left(P^T P\right)^{-1}$

where $P = \text{Matrix}\left[\frac{\partial m_i}{\partial x_j}\right]$

Parallax model: $m_i = \Pi \cos(\omega \Delta t_i) + \alpha_0 + \mu_\alpha \Delta t_i$

where $x_1 = \Pi$, $x_2 = \alpha_0$, and $x_3 = \mu_\alpha$

$$P_{i1} = \frac{\partial m_i}{\partial \Pi} = \cos(\omega \Delta t_i)$$

$$P_{i2} = \frac{\partial m_i}{\partial \alpha_0} = 1$$

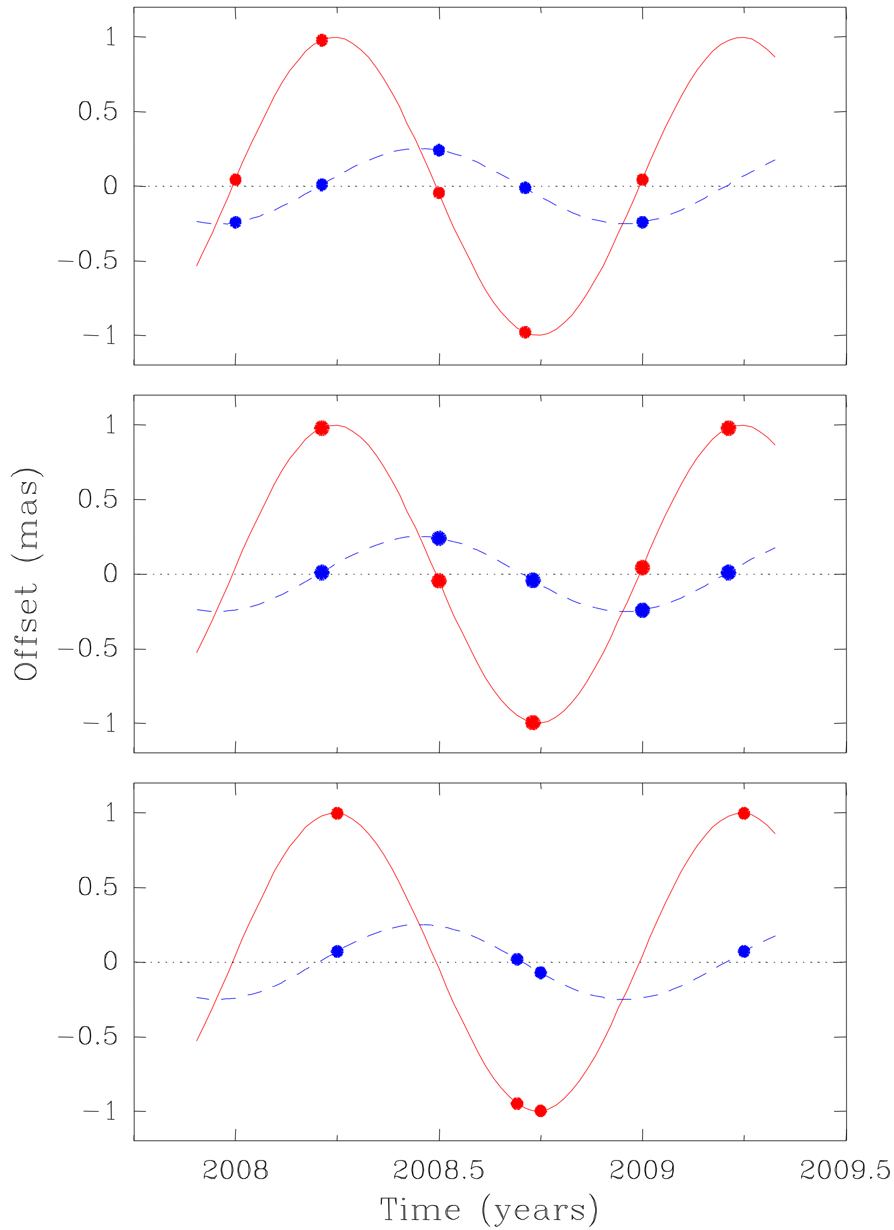
$$P_{i3} = \frac{\partial m_i}{\partial \mu_\alpha} = \Delta t_i$$

$$P = \begin{bmatrix} \cos(\omega\Delta t_1) & 1 & \Delta t_1 \\ \cos(\omega\Delta t_2) & 1 & \Delta t_2 \\ ... & ... & ... \\ ... & ... & ... \\ \cos(\omega\Delta t_N) & 1 & \Delta t_N \end{bmatrix}$$

$$P^T P = \begin{bmatrix} \sum \cos^2(\omega\Delta t_i) & \sum \cos(\omega\Delta t_i) & \sum \cos(\omega\Delta t_i)\Delta t_i \\ \sum \cos(\omega\Delta t_i) & \sum 1 & \sum \Delta t_i \\ \sum \cos(\omega\Delta t_i)\Delta t_i & \sum \Delta t_i & \sum {\Delta t_i}^2 \end{bmatrix}$$

If data symmetric about $t = 0$, off-diagonal terms $\rightarrow 0$

$$\left(P^T P\right)^{-1} = \begin{bmatrix} 1/\sum \cos^2(\omega\Delta t_i) & 0 & 0 \\ 0 & 1/\sum 1 & 0 \\ 0 & 0 & 1/\sum {\Delta t_i}^2 \end{bmatrix}$$



$$\sigma_\pi = 1 / \mathrm{sqrt}(\Sigma \cos^2 \omega t)$$

$N = 5$

$$\sigma_\pi = 1 / \mathrm{sqrt}(2) = 0.7$$

$N = 5$

$$\sigma_\pi = 1 / \mathrm{sqrt}(3) = 0.6$$

$N = 4$

$$\sigma_\pi = 1 / \mathrm{sqrt}(4) = 0.5$$

Extragalactic Proper Motions

Parallax accuracy:

10% at 10 kpc

not good enough for galaxies

Proper motion:

same techniques as Parallax,
but accuracy ~ $T^{-3/2}$

M33 project (A. Brunthaler thesis)

1) see spin (van Maanen)

2) see motion