Calibration and Editing

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Synopsis

Why calibration and editing?

Idealistic formalism ☾ Realistic practice

Editing and RFI

Practical Calibration Considerations

Baseline-based vs. Antenna-based Calibration

Scalar Calibration Example

Full Polarization Generalization

A Dictionary of Calibration Effects

Calibration Heuristics

New Calibration Challenges

Summary

Why Calibration and Editing?

Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, gain stability, geometric model errors, etc.)

Need to accommodate deliberate engineering (e.g., frequency conversion, digital electronics, filter bandpass, etc.)

Hardware or control software occasionally fails or behaves unpredictably

Scheduling/observation errors sometimes occur (e.g., wrong source positions)

Atmospheric conditions not ideal

RFI

Determining *instrumental properties* (calibration)

is a prerequisite to

determining *radio source properties*

From Idealistic to Realistic

Formally, we wish to use our interferometer to obtain the visibility function:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl dm$$

….which we intend to invert to obtain an image of the sky:

$$I(l,m) = \int_{uv} V(u,v) e^{i2\pi(ul+vm)} du dv$$

V(u,v) set the amplitude and phase of 2D sinusoids that add up to an image of the sky

How do we measure *V(u,v)?*

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From Idealistic to Realistic

In practice, we correlate (multiply & average) the electric field (voltage) samples, *xi* & *xj*, received at pairs of telescopes (*i,j*) and processed through the observing system:

$$V_{ij}^{obs}(u_{ij}, v_{ij}) = \langle x_i(t) \cdot x_j^*(t) \rangle_{\Delta t}$$
$$= J_{ij} V_{ij}^{true}(u_{ij}, v_{ij})$$

xi & *xj* are delay-compensated for a specific point on the sky

Averaging duration is set by the expected timescales for variation of the correlation result (~seconds)

Jij is an *operator* characterizing the net effect of the observing process for baseline (*i,j*), which we must *calibrate*

Sometimes *Jij* corrupts the measurement irrevocably, resulting in data that must be *edited* or *"flagged"*

What Is Delivered by a Synthesis Array?

An *enormous* list of complex numbers! *(Enormous!)*

E.g., the EVLA:

At each timestamp (~1s intervals): 351 baselines (+ 27 auto-correlations)

For each baseline: 1-64 Spectral Windows ("subbands" or "IFs")

For each spectral window: tens to thousands of channels

For each channel: 1, 2, or 4 complex correlations

- RR or LL or (RR,LL), or (RR,RL,LR,LL)

With each correlation, a weight value

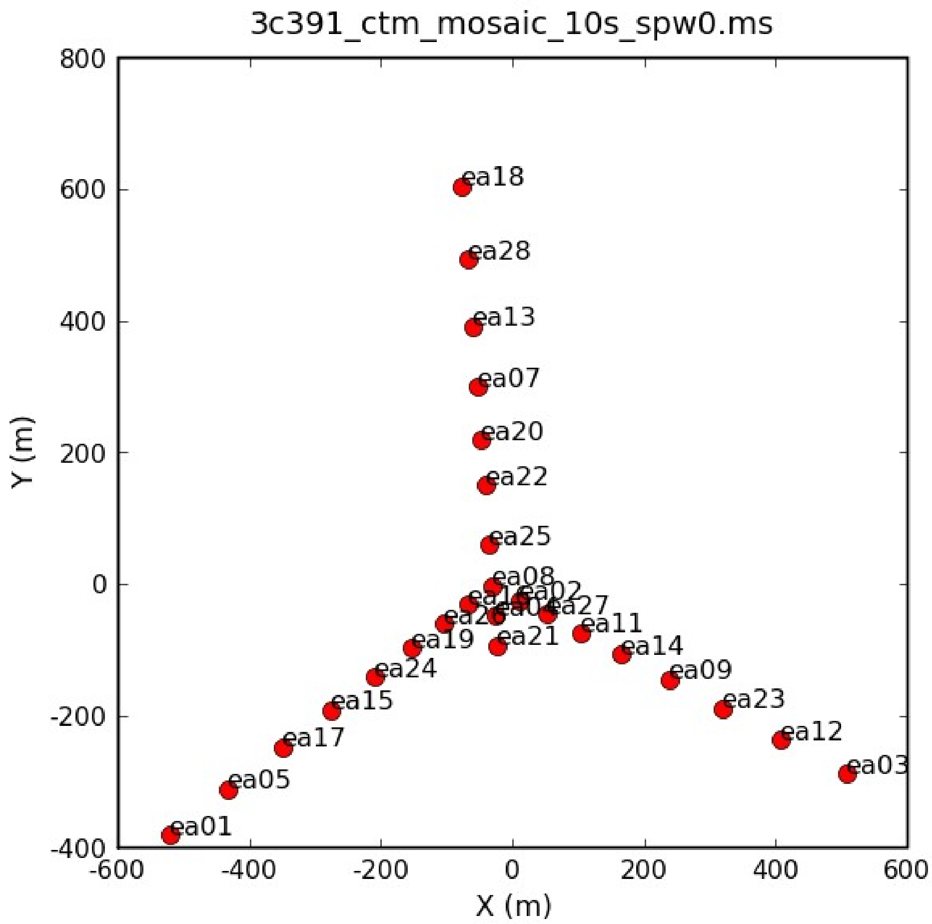
Meta-info: Coordinates, antenna, field, frequency label info

Ntotal = Nt x Nbl x Nspw x Nchan x Ncorr visibilities

EVLA: ~1300000 x Nspw x Nchan x Ncorr vis/hour (10s to 100s of GB per observation)

ALMA: ~3-5X more baselines than EVLA…

The Array


3c391_ctm_mosaic_10s_spw0.ms
Y (m)
X (m)
800
600
400
200
0
-200
-400
-600
-400
-200
0
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ea18
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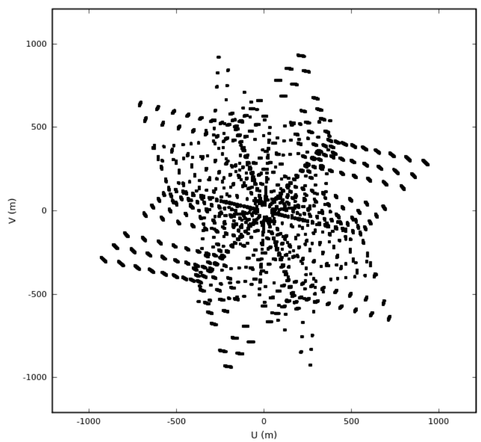
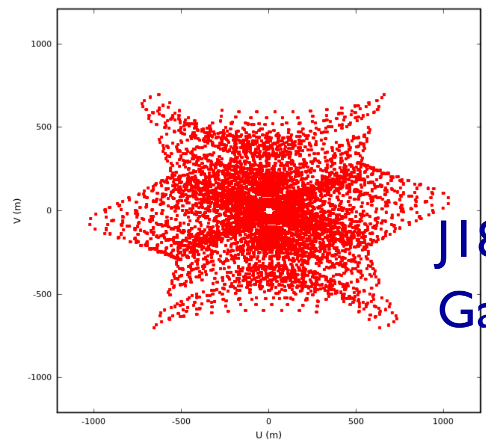
UV-coverages

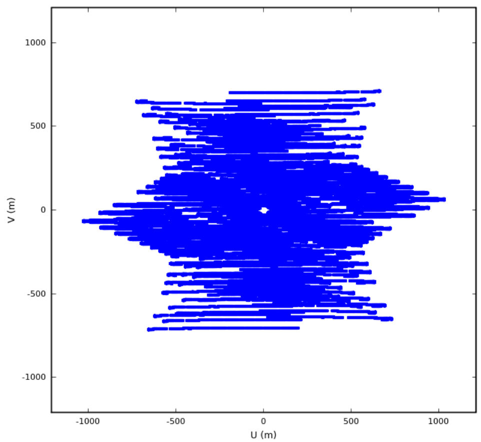
3C286
Flux Density



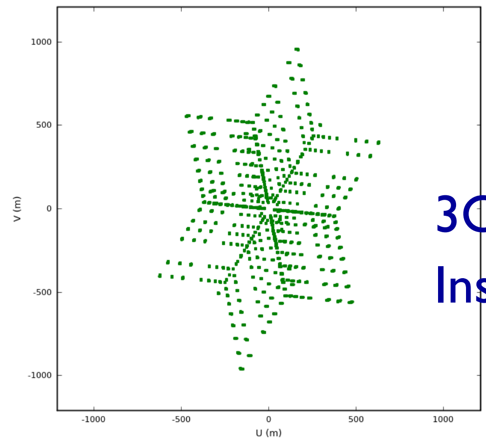
J1822-0938
Gain Calibrator



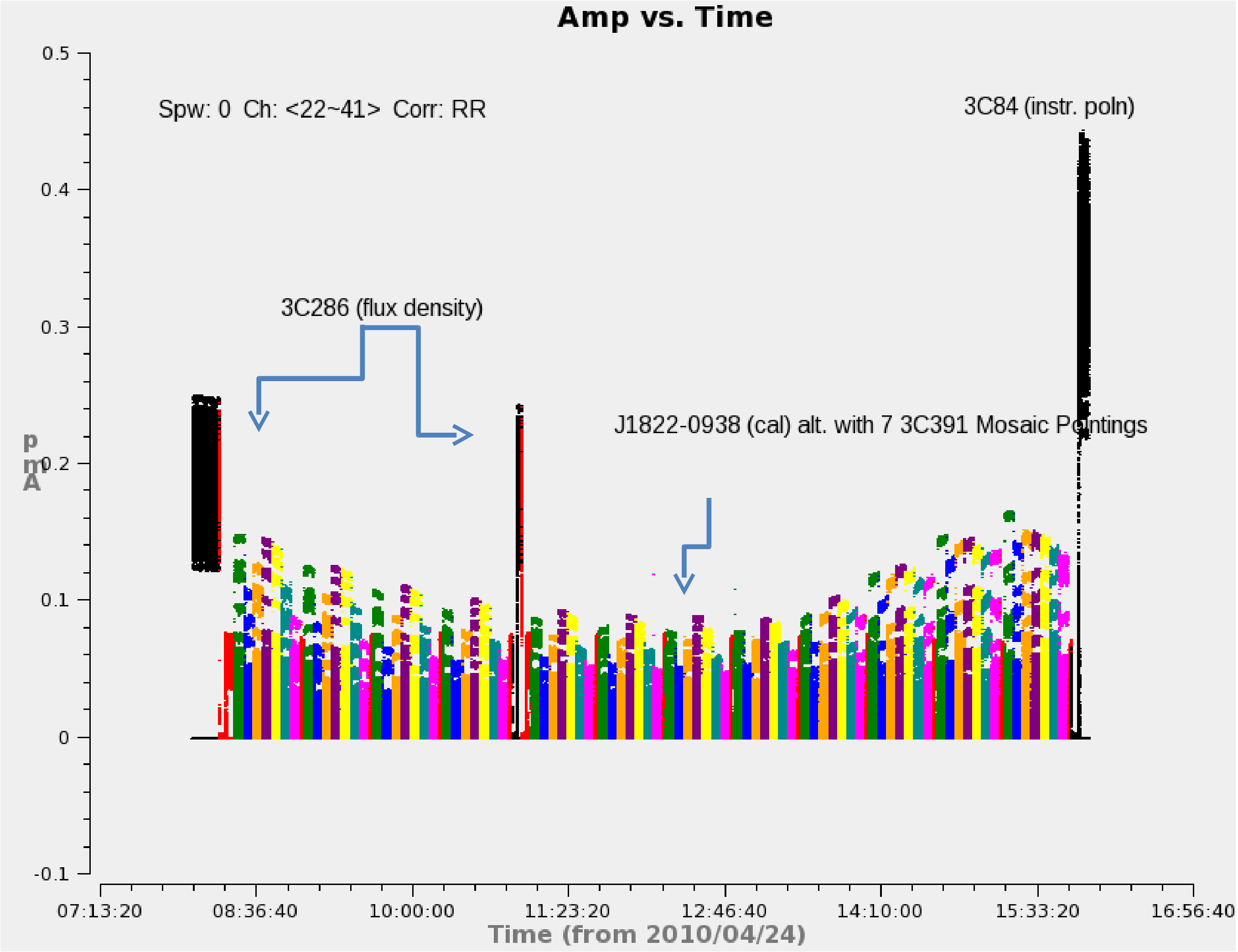
3C391
Science Target



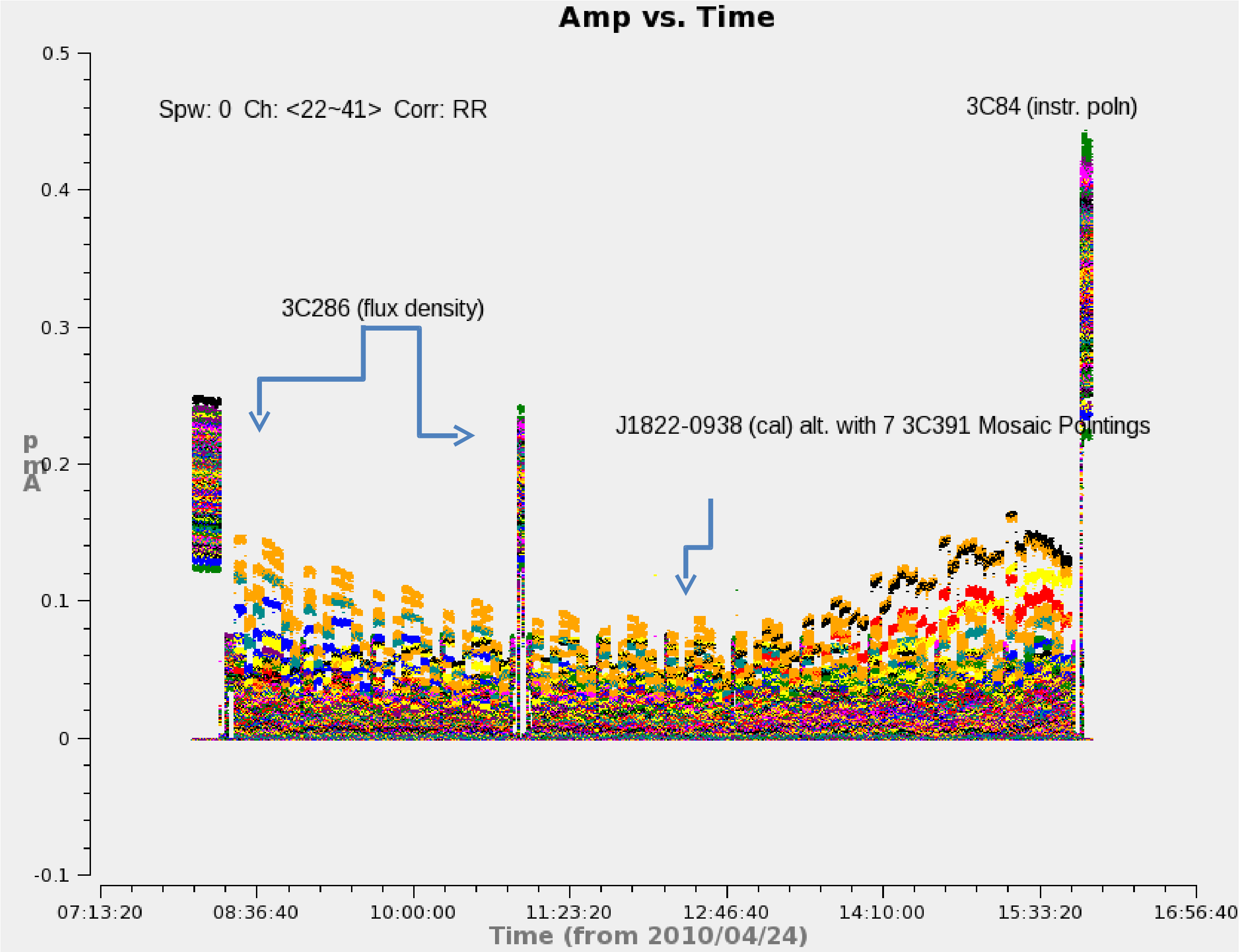
3C84
Instr. Poln Calibrator



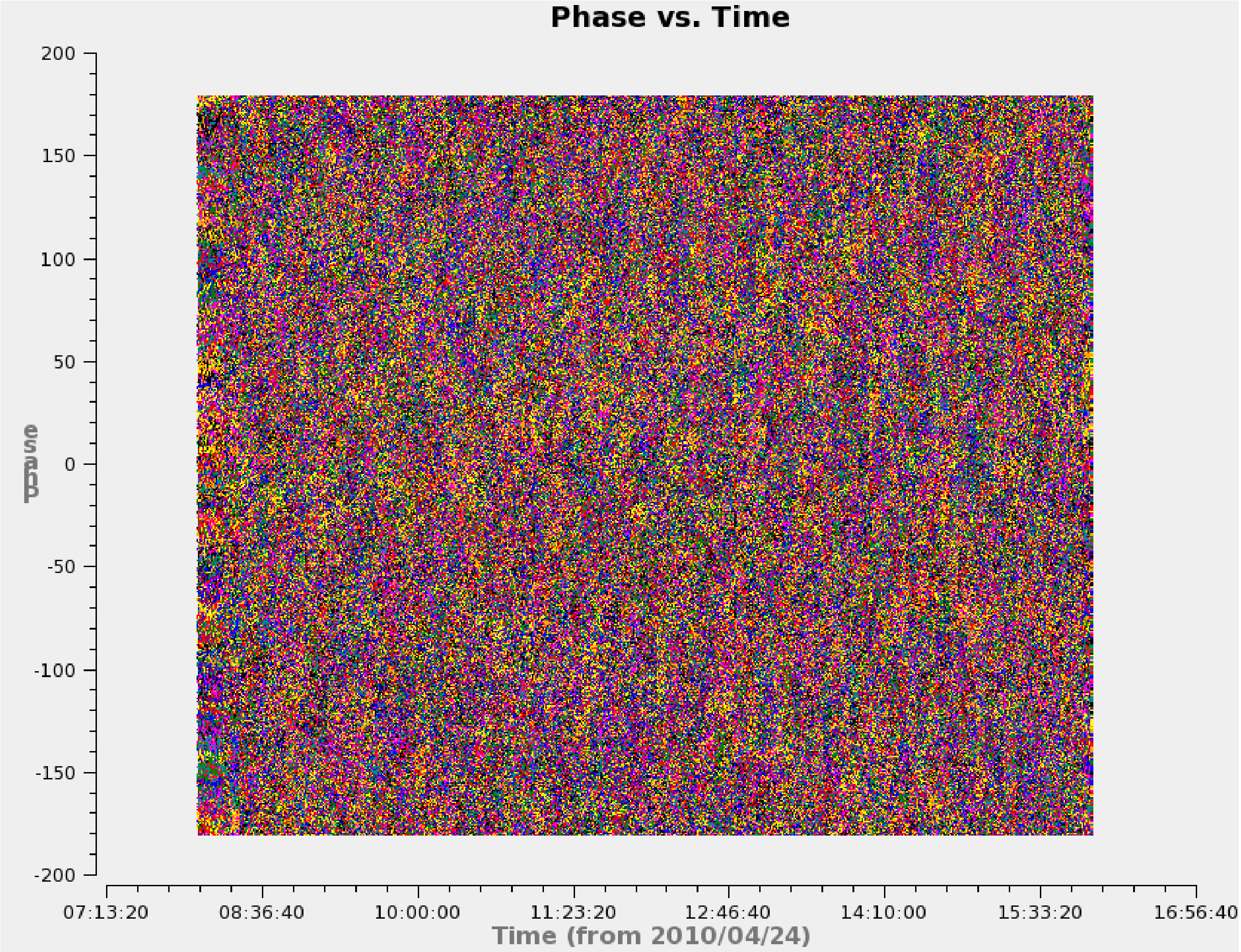
The Visibility Data   (source colors)



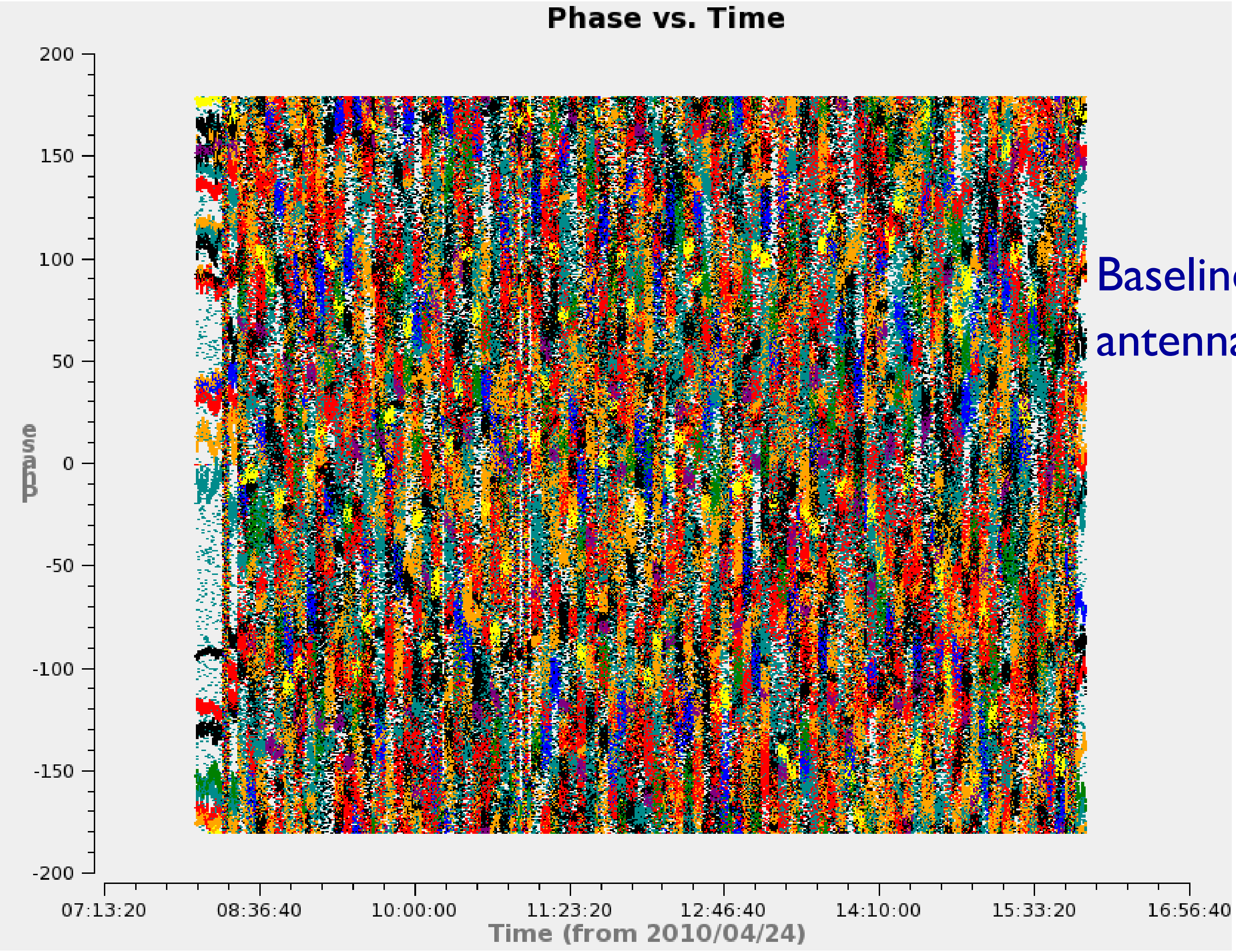
The Visibility Data (baseline colors)



The Visibility Data (baseline colors)

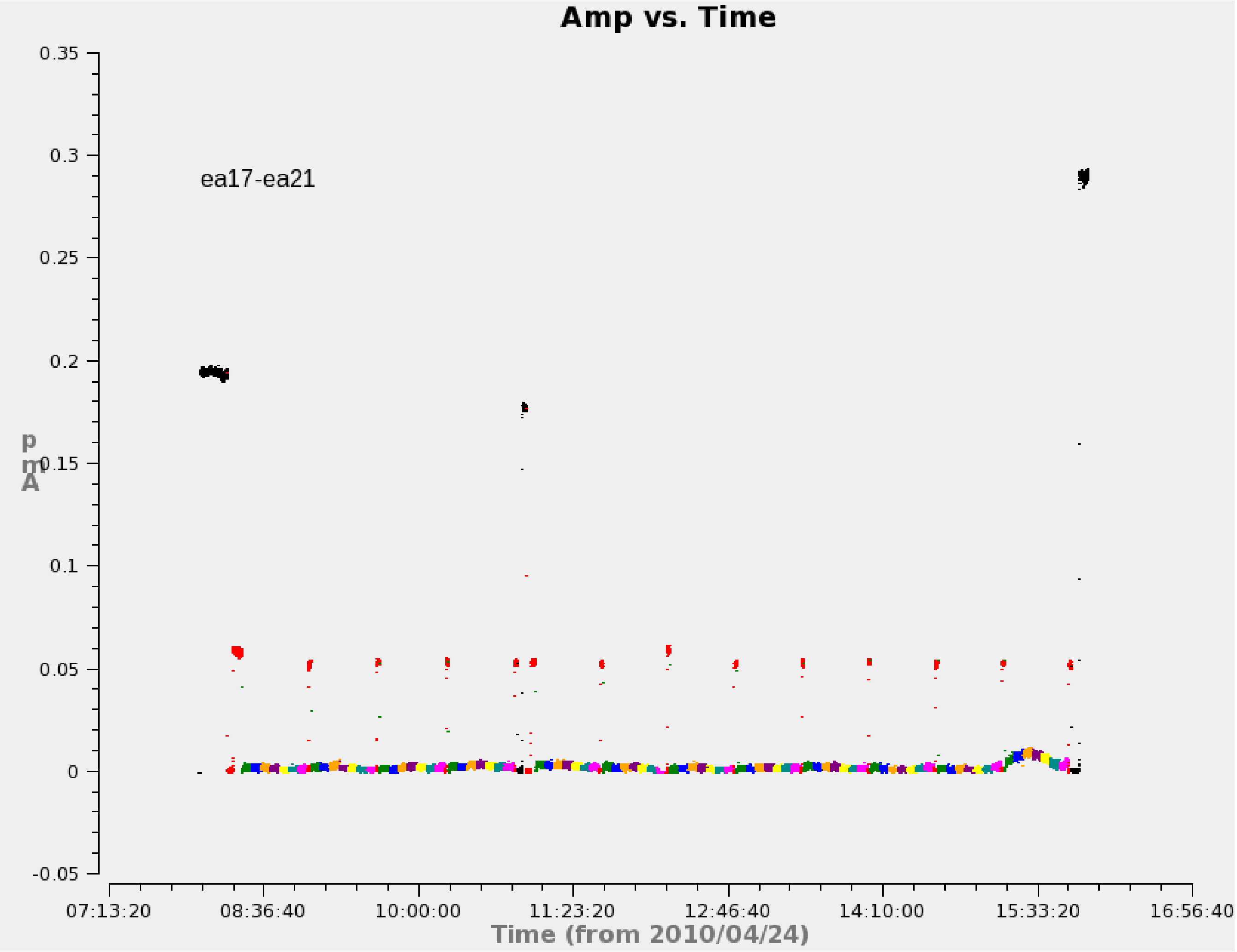


The Visibility Data (baseline colors)

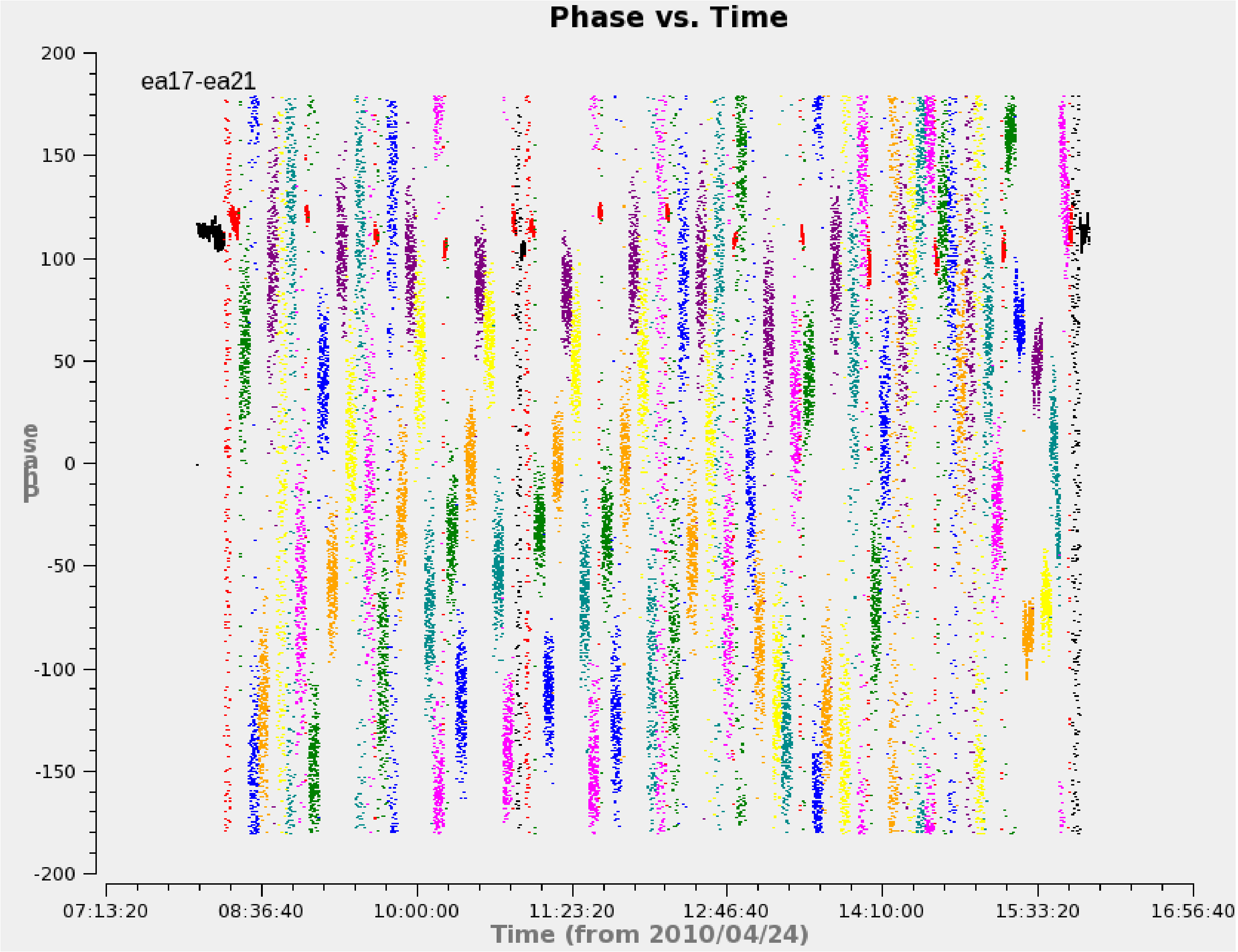


Baselines to antenna ea21

A Single Baseline - Amp



A Single Baseline - Phase



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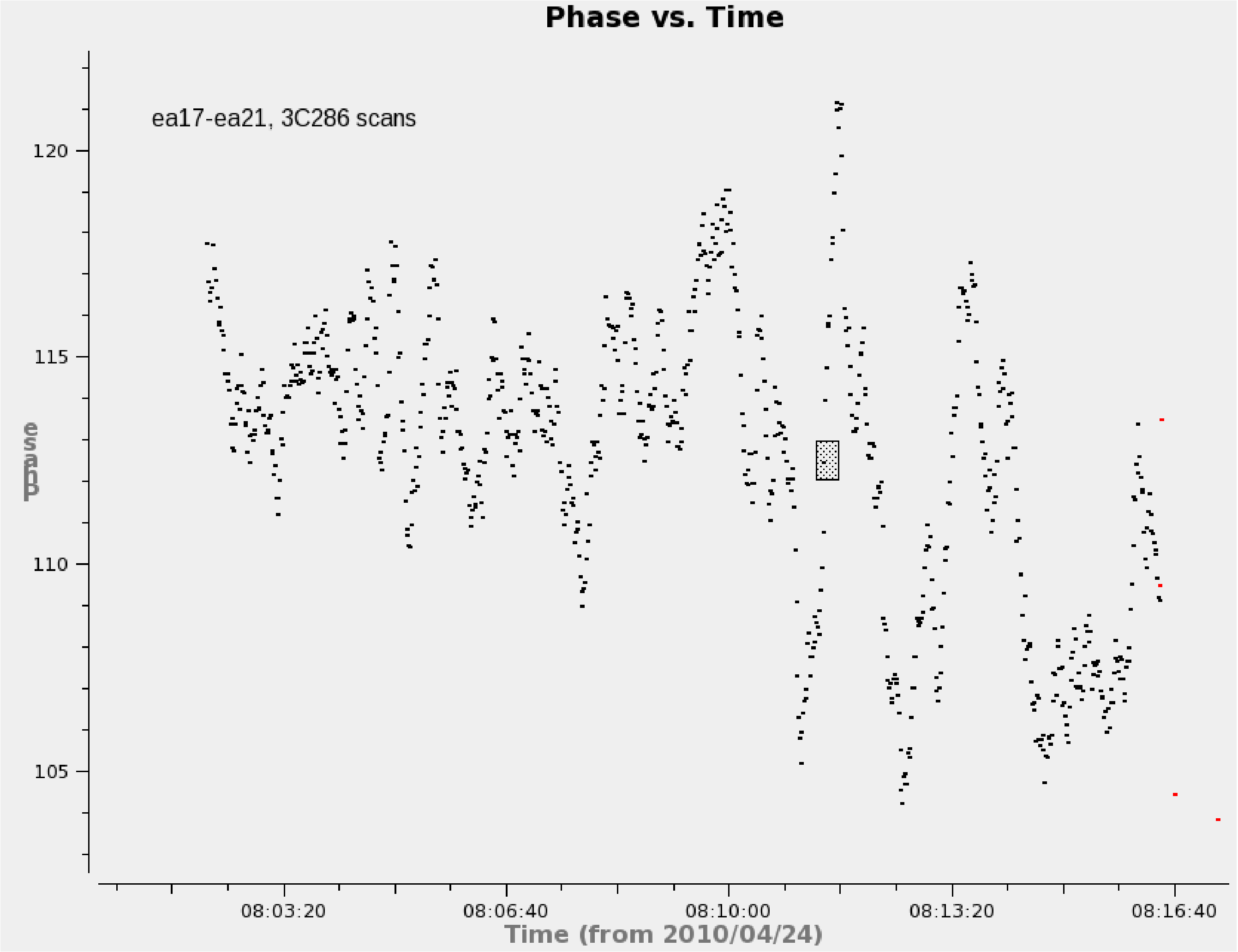
A Single Baseline – 2 scans on 3C286



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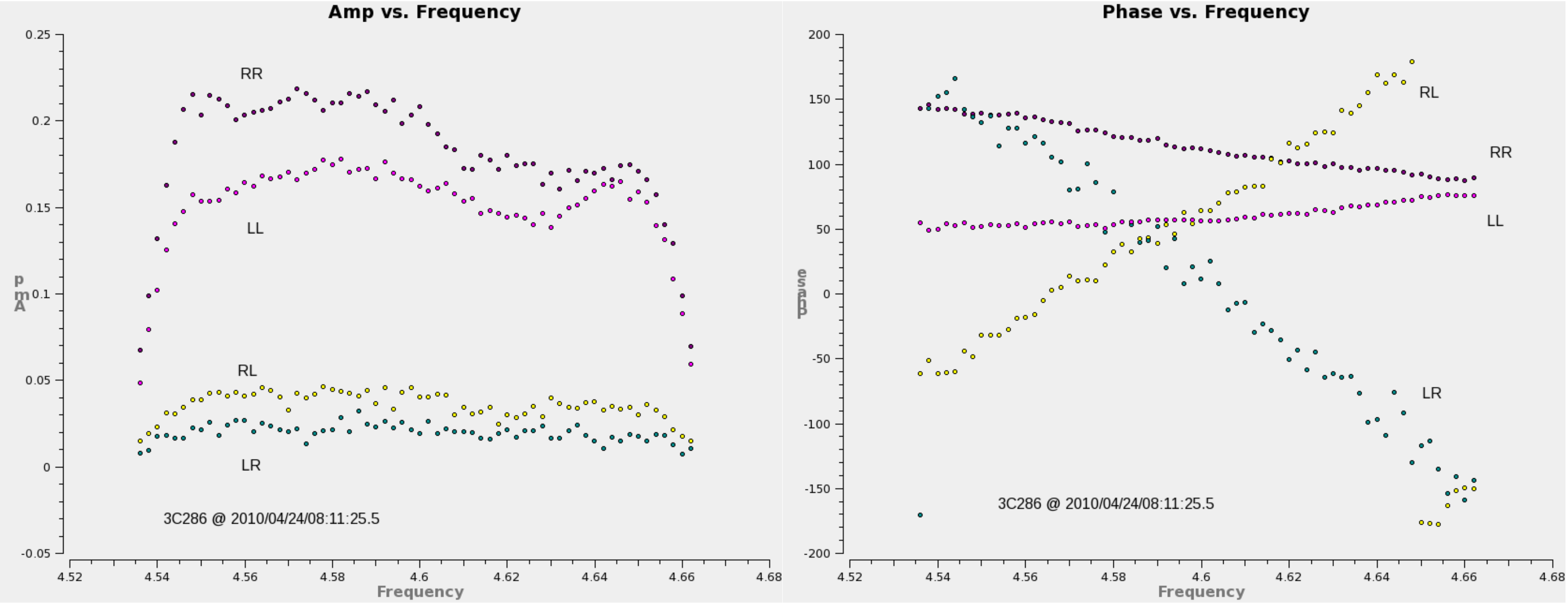
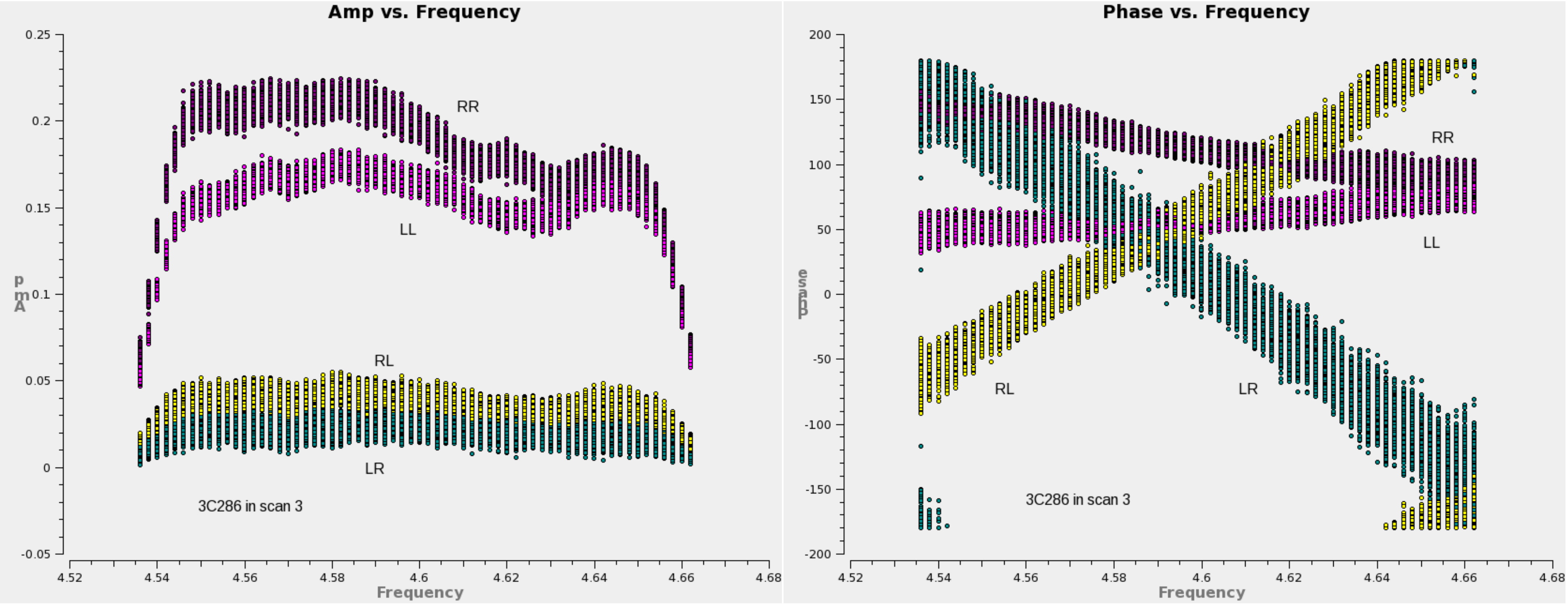
Single Baseline, Single Integration Visibility Spectra (4 correlations)



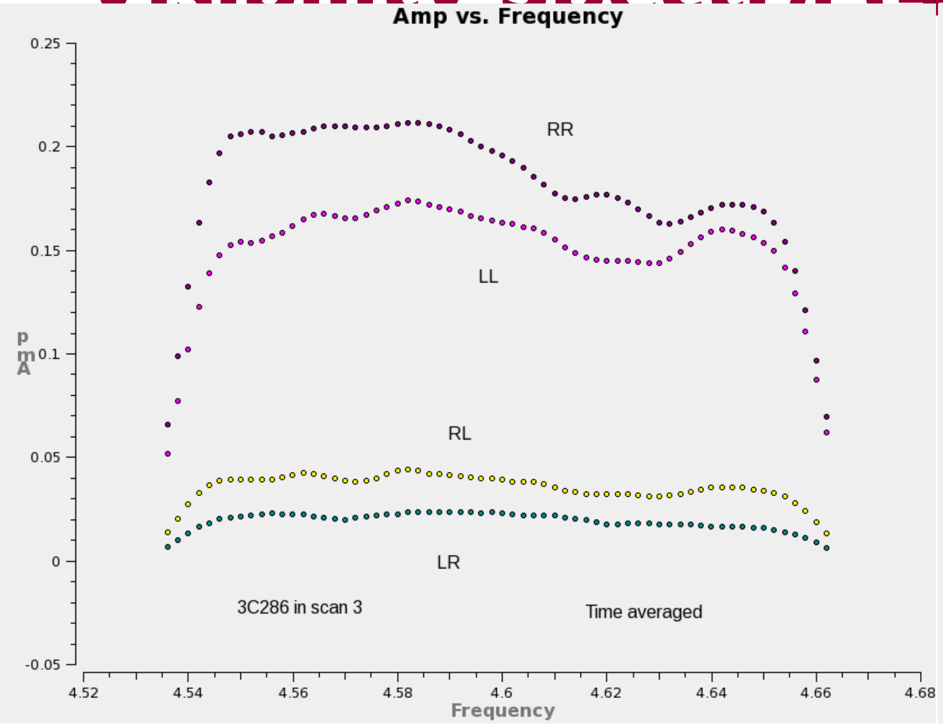
baseline ea17-ea21

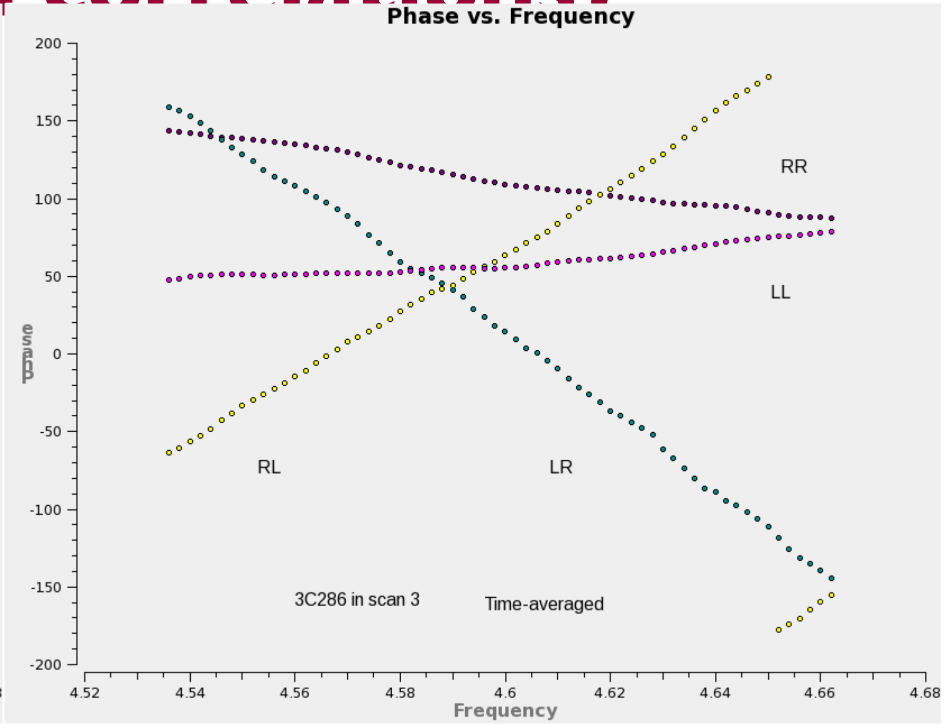
Single Baseline, Single Scan Visibility Spectra (4 correlations)



baseline ea17-ea21

Single Baseline, Single Scan (time-averaged)
Visibility Spectra (4 correlations)





baseline ea17-ea21

Data Examination and Editing

After observation, initial data examination and editing very important

Will observations meet goals for calibration and science requirements?

What to edit:

Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)

Any persistently 'dead' antennas (check operator's logs)

Periods of especially poor weather? (check operator's log)

Any antennas shadowing others? Edit such data.

Amplitude and phase should be continuously varying—edit outliers

Radio Frequency Interference (RFI)?

Caution:

Be careful editing noise-dominated data (noise bias).

Be conservative: those antennas/timeranges which are bad on calibrators are probably bad on weak target sources—edit them

Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have signifi cantly different amplitude response which may not be fatal—it may only need to be calibrated

Choose reference antenna wisely (ever-present, stable response)

Increasing data volumes increasingly demand automated editing algorithms…

Radio Frequency Interference (RFI)

RFI originates from man-made signals generated in the antenna electronics or by external sources (e.g., satellites, air traffic, cell-phones, radio and TV stations, automobile ignitions, microwave ovens, computers and other electronic devices, etc.)

Adds to total noise power in all observations, thus decreasing the fraction of desired natural signal passed to the correlator, thereby reducing sensitivity and possibly driving electronics into non-linear regimes

Can correlate between antennas if of common origin and baseline short enough (insufficient decorrelation via geometry compensation), thereby obscuring natural emission in spectral line observations

Least predictable, least controllable threat to a radio astronomy observation

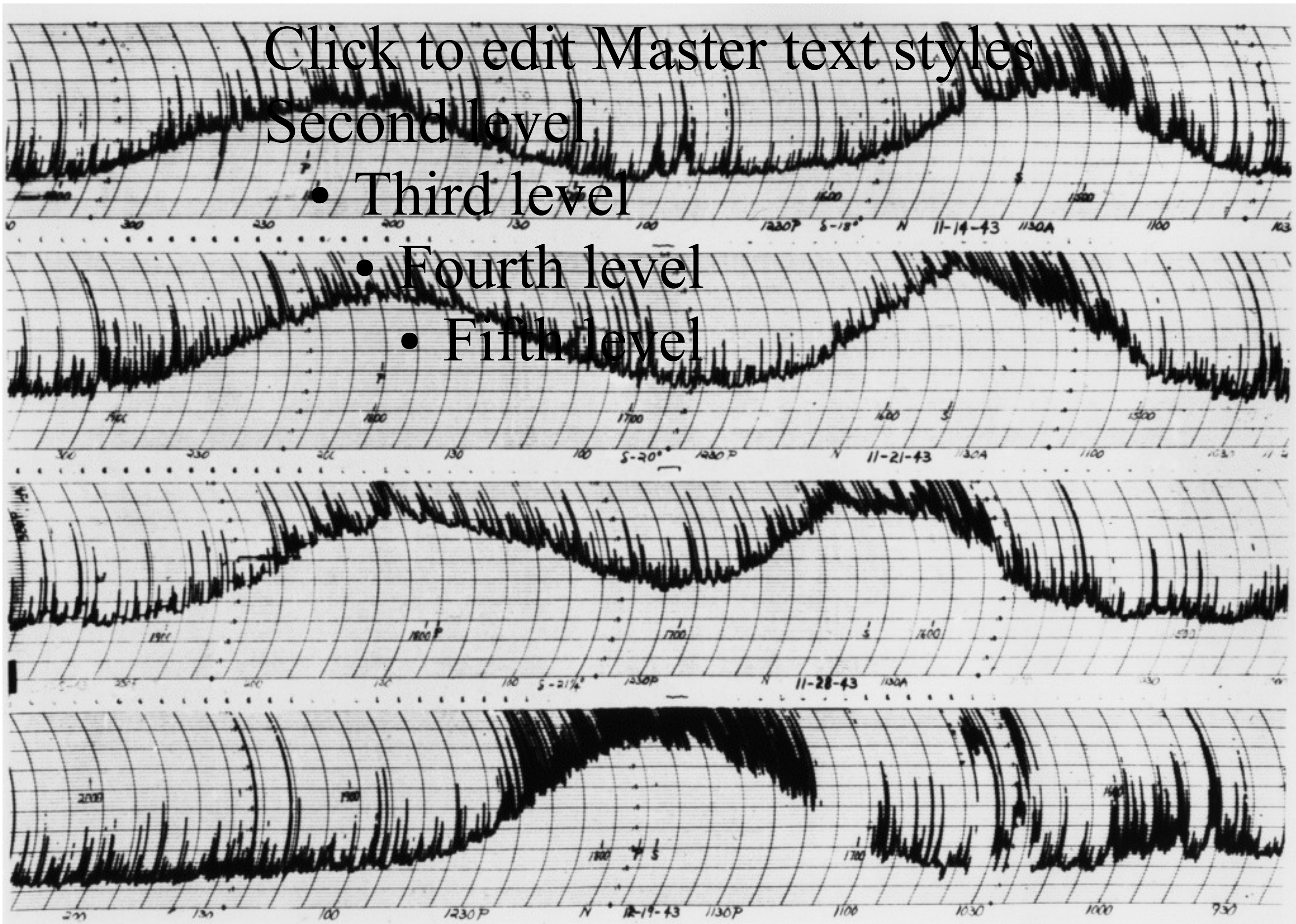
Radio Frequency Interference

Has always been a problem (Reber, 1944, in total power)!

Click to edit Master text styles

Second level

- Third level
  - Fourth level
    - Fifth level



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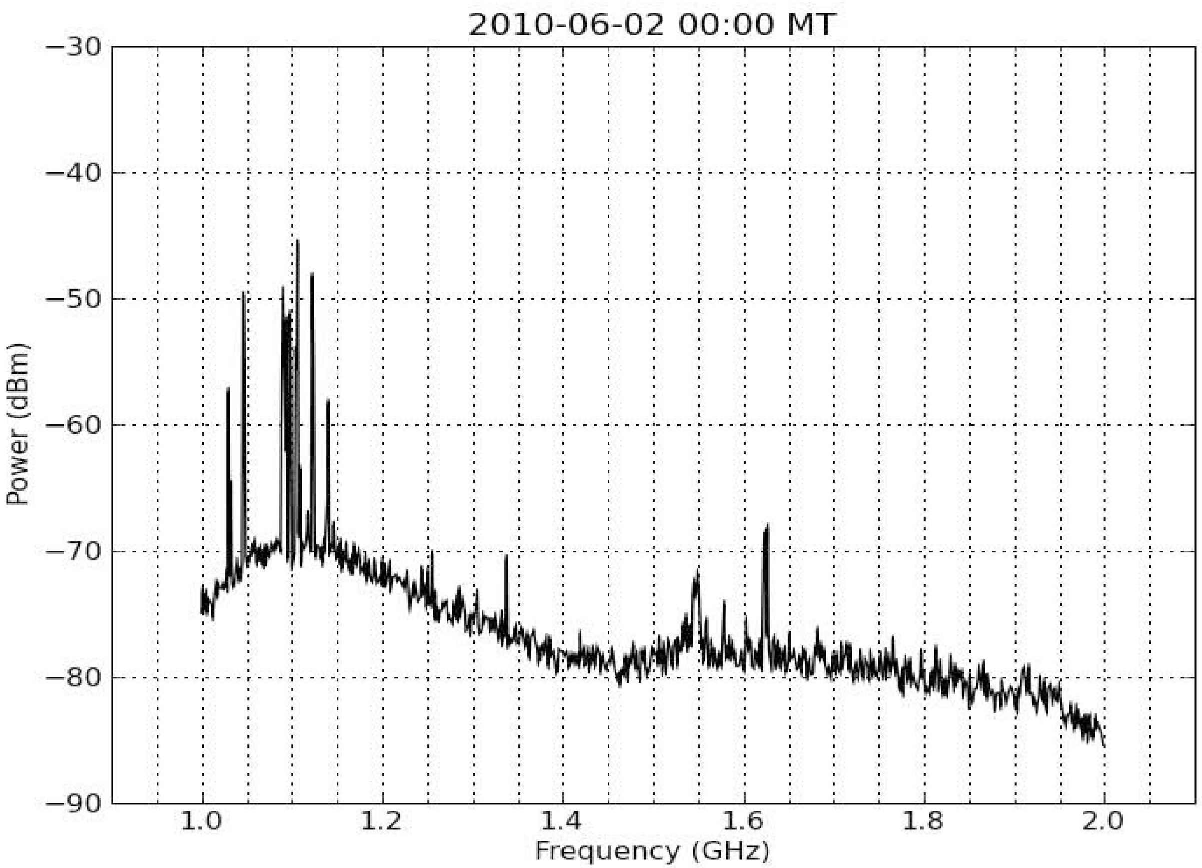
Radio Frequency Interference (cont)

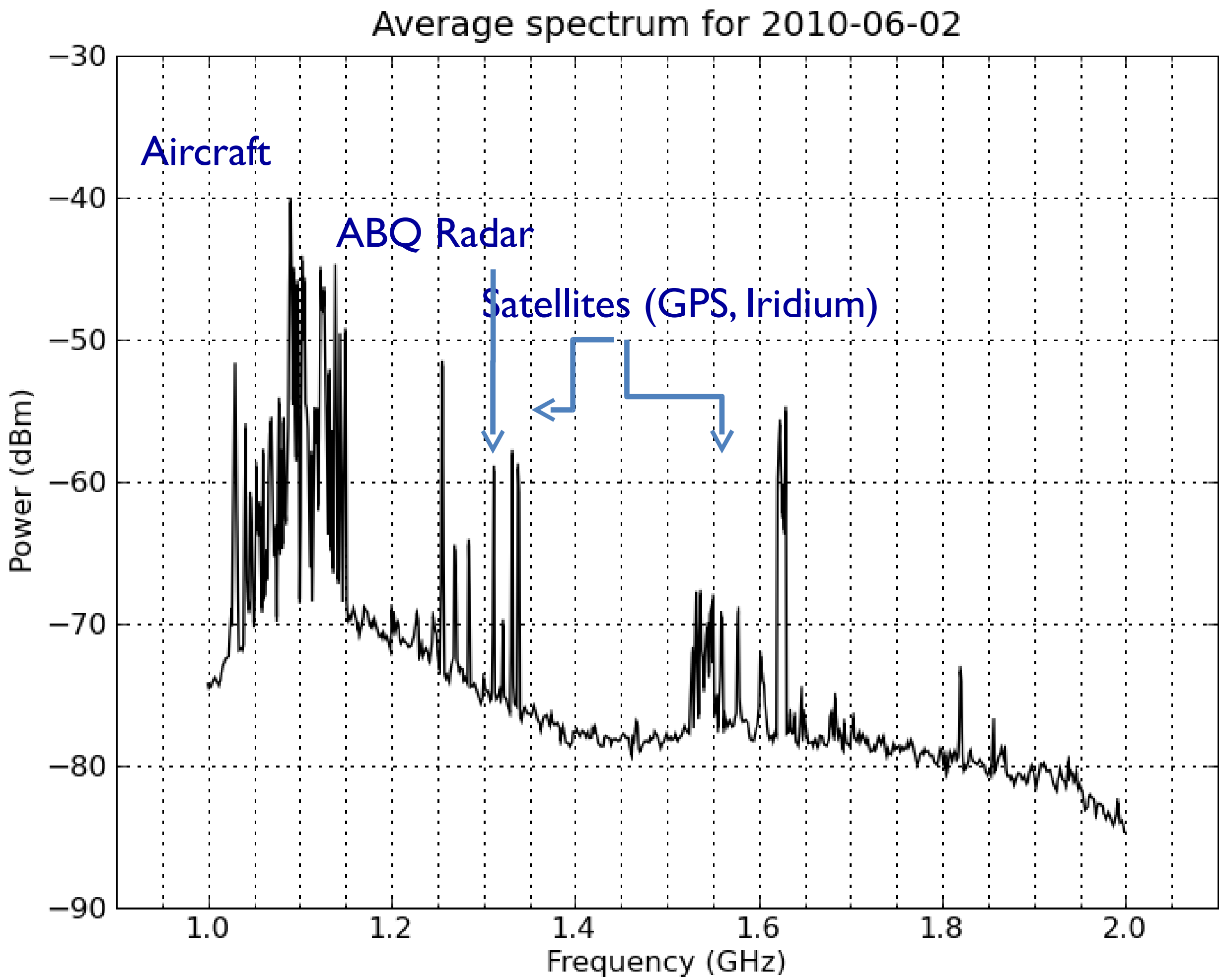
Growth of telecom industry threatening radioastronomy!

Radio Frequency Interference (cont)



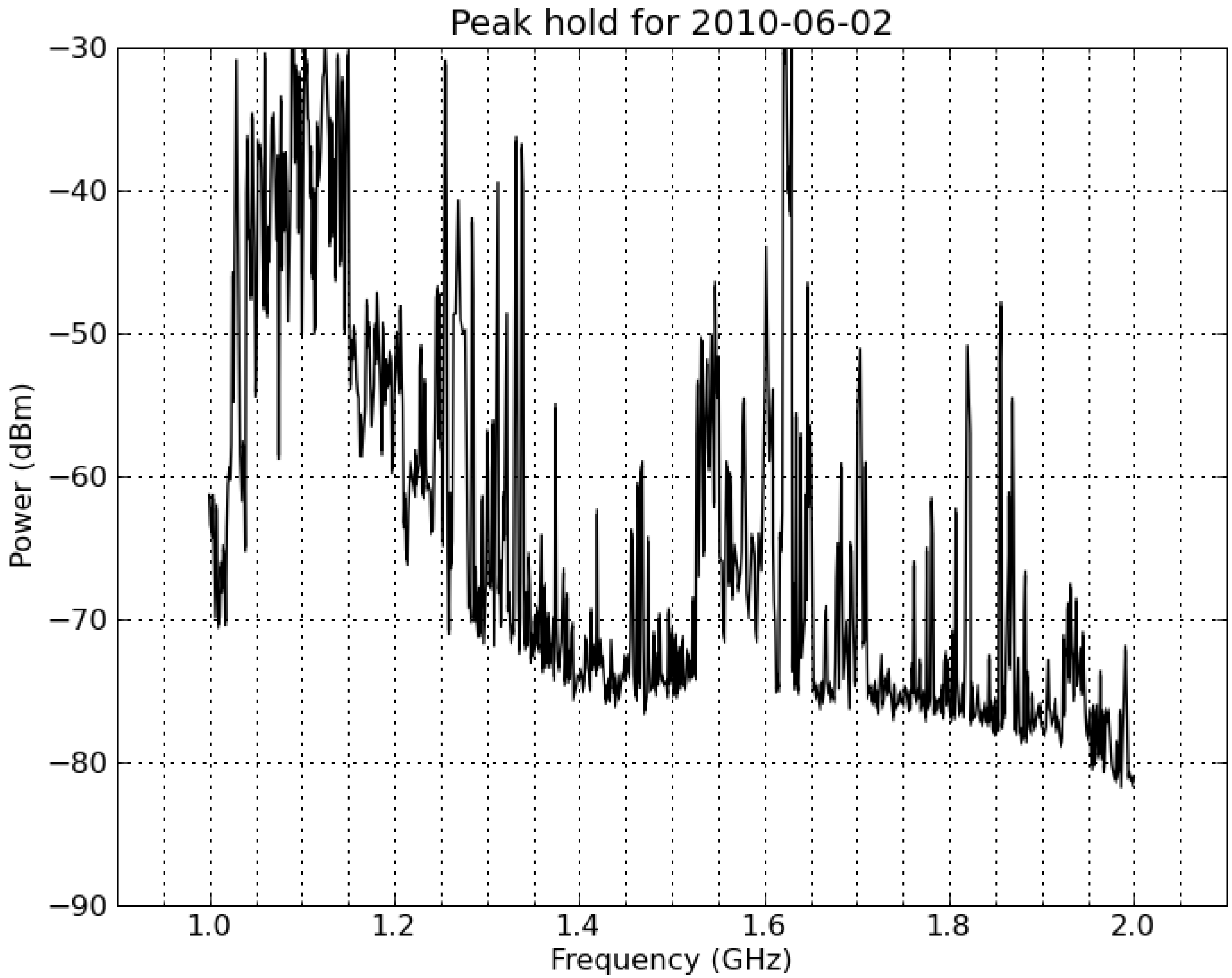
- Movie courtesy of T. Hunter.

Radio Frequency Interference (cont)



- Plot courtesy of T. Hunter.

Radio Frequency Interference (cont)



- Plot courtesy of T. Hunter.

Radio Frequency Interference (cont)

RFI Mitigation

Careful electronics design in antennas, including filters, shielding, blanking in the correlator

High-dynamic range digital sampling

Observatories world-wide lobbying for spectrum management

Choose interference-free frequencies (very difficult in EVLA 1-2 GHz band, for continuum bandwidths)

Observe continuum experiments in spectral-line modes so affected channels can be edited

Various off-line mitigation techniques under study

E.g., correlated RFI power that originates in the frame of the array appears at celestial pole (also stationary in array frame) in image domain…

Ue-Li Pen's lecture "Radio Frequency Interference Excision" (Thursday)

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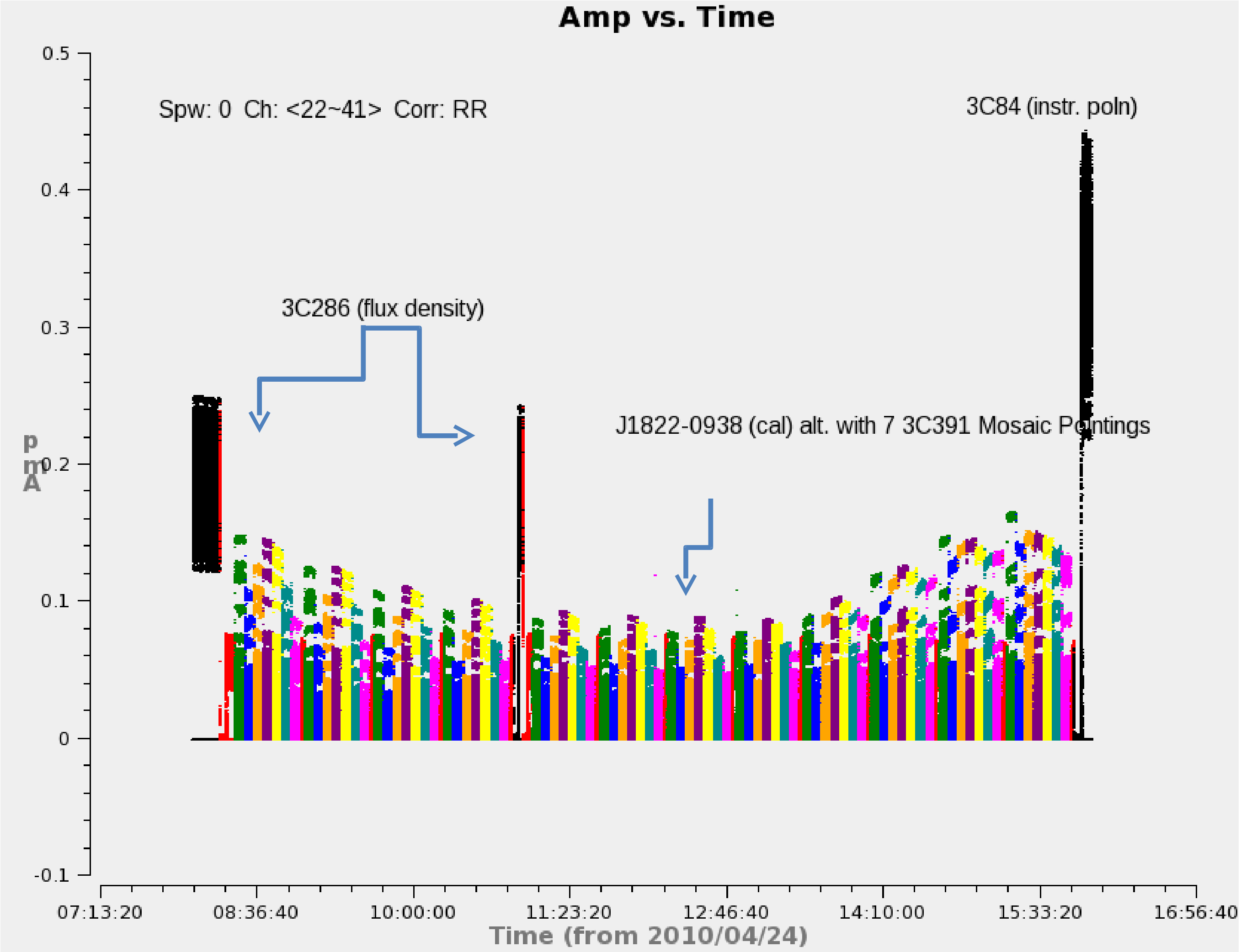
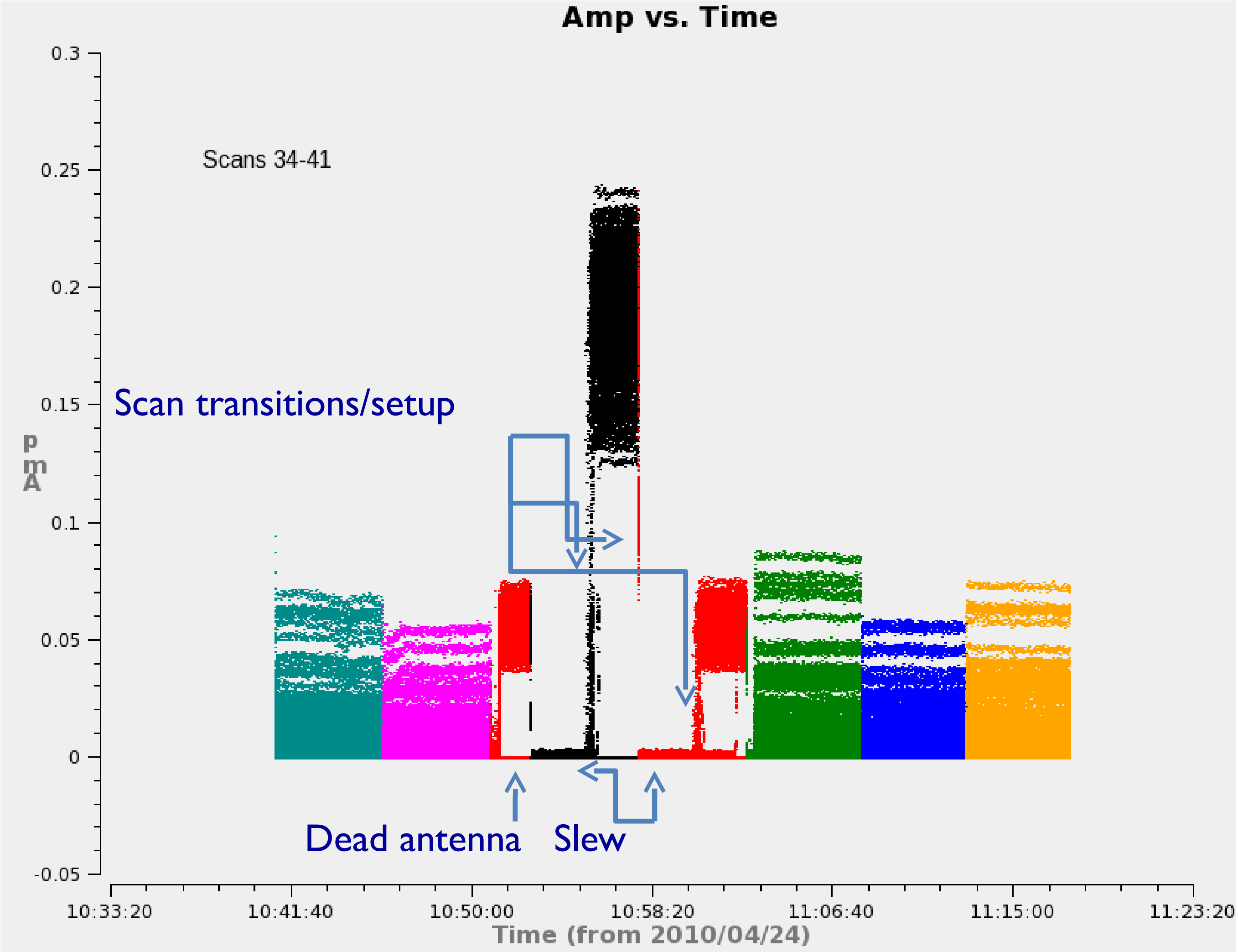
Editing Example



Editing Example



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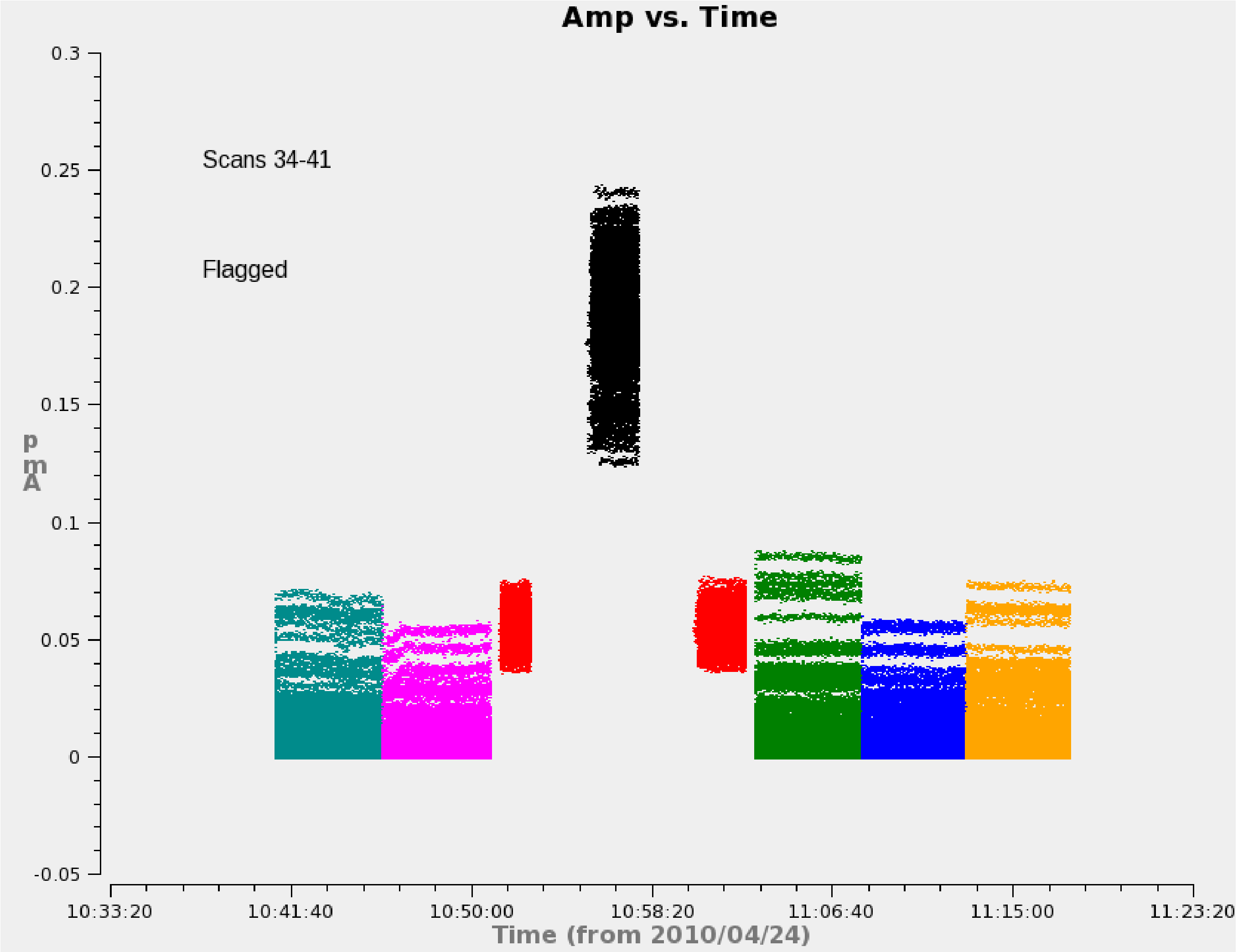
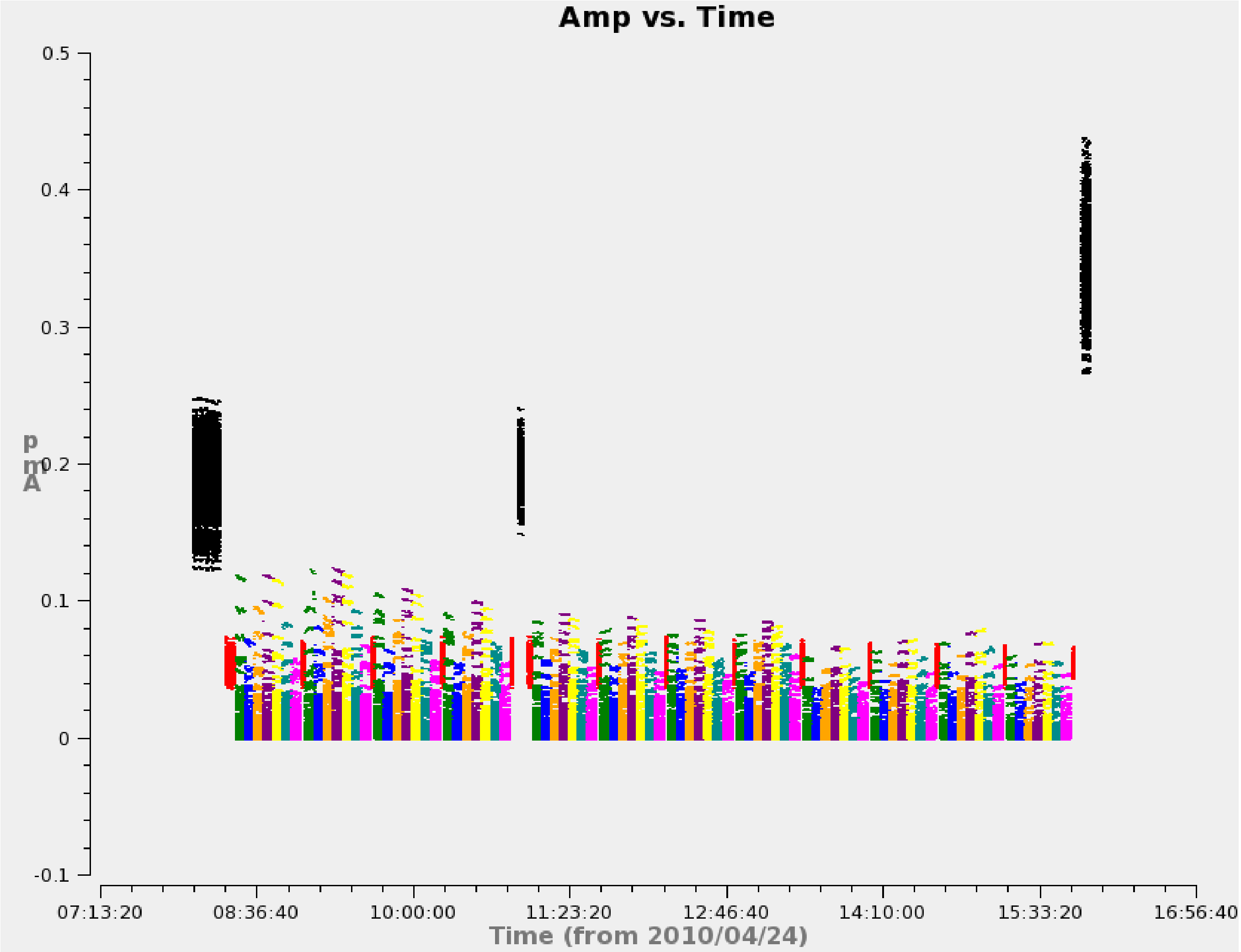
Editing Example



Editing Example



Practical Calibration Considerations

A priori "calibrations" (provided by the observatory)

Antenna positions, earth orientation and rate

Clocks

Antenna pointing, gain curve, voltage pattern

Calibrator coordinates, flux densities, polarization properties

Tsys, system gain (EVLA)

Absolute *engineering* calibration?

Very difficult, would require heroic efforts by observatory scientific and engineering staff (hot/cold loads, climbing ladders, etc.)

Concentrate instead on ensuring instrumental *stability* on adequate timescales

Cross-calibration a better choice

Observe nearby ***point sources*** against which calibration (*Jij*) can be solved, and transfer solutions to target observations

Choose appropriate calibrators; usually strong point sources because we can easily predict their visibilities

Choose appropriate timescales for calibration

"Absolute" Astronomical Calibrations

Flux Density Calibration

Radio astronomy flux density scale set according to several "constant" radio sources

Use resolved models where appropriate

Astrometry

Most calibrators come from astrometric catalogs; directional accuracy of target images tied to that of the calibrators

Beware of resolved and evolving structures, and phase transfer biases due to troposphere (especially for VLBI)

Linear Polarization Position Angle

Usual flux density calibrators also have significant stable linear polarization position angle for registration

Relative calibration solutions (and dynamic range) insensitive to errors in these "scaling" parameters

Baseline-based Cross-Calibration

$$V_{ij}^{obs} = J_{ij} V_{ij}^{mod}$$

Simplest, most-obvious calibration approach: measure complex response of *each baseline* on a standard source, and scale science target visibilities accordingly

"Baseline-based" Calibration

Only option for single baseline "arrays" (3-antenna arrays are effectively baseline-based for per-integration amplitude calibration)

Calibration precision same as calibrator visibility sensitivity (on timescale of calibration solution).

Calibration accuracy sensitive to departures of calibrator from known structure

Un-modeled calibrator structure transferred (in inverse) to science target!

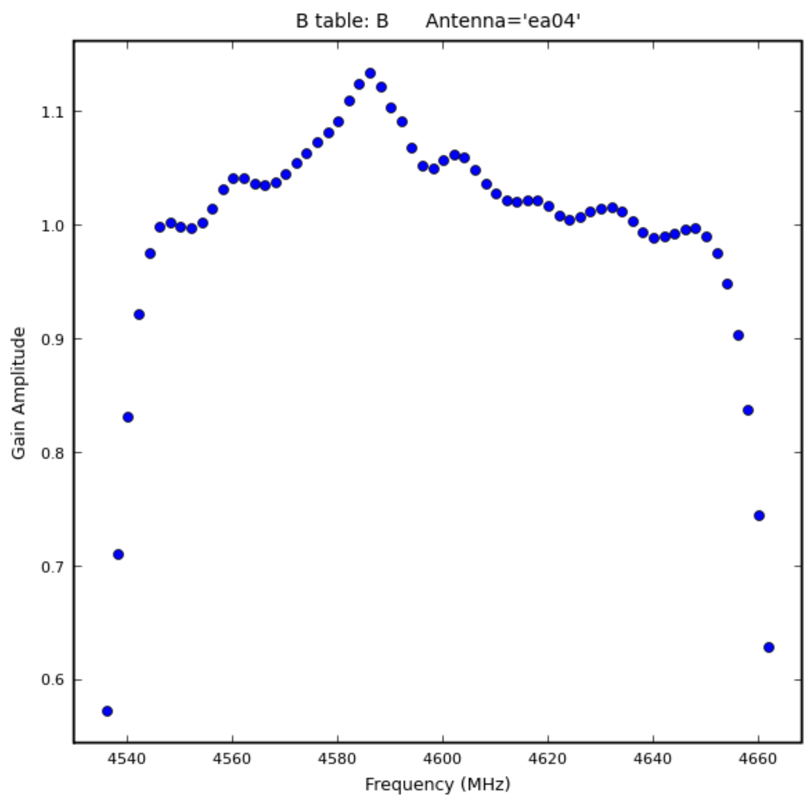
Antenna-based Cross Calibration

Measured visibilities are formed from a product of *antenna-based* signals. Can we take advantage of this fact?

e.g., bandpass…

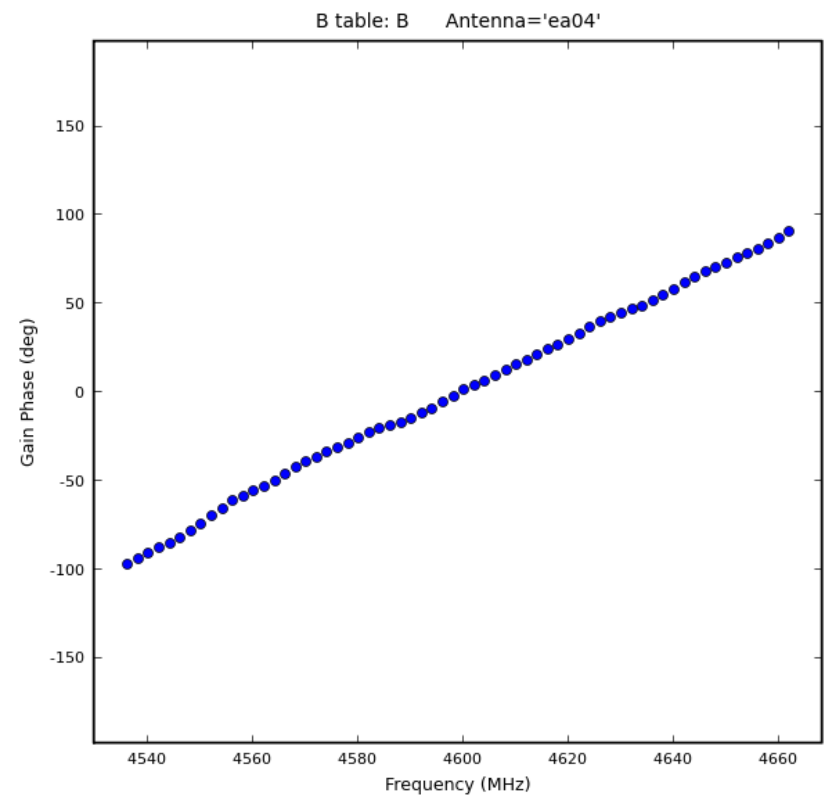
Rationale for Antenna-based Calibration

Amp vs. Frequency



Frequency

Phase vs. Frequency



Frequency

Antenna-based Cross Calibration

The net time-dependent (*t*) signal delivered by antenna *i, xi(t),* is a combination of the desired signal, *si(t,l,m),* corrupted by a factor *Ji(t,l,m)* and integrated over the sky (*l,m*), and diluted by noise, *ni(t)*:

$$x_i(t) = \int_{sky} J_i(t,l,m) s_i(t,l,m)\, dl dm + n_i(t)$$

$$= s_i'(t) + n_i(t)$$

xi(t) is sampled voltage at the correlator input

Ji(t,l,m) is the product of a series of effects encountered by the incoming signal

Ji(t,l,m) is an *antenna-based* complex number

Usually, |*ni* |>> |*si*|

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Correlation of Realistic Signals - I

The correlation of two realistic (aligned for a specific direction) signals from different antennas:

$$\left\langle x_i \cdot x_j^* \right\rangle_{\Delta t} = \left\langle \left(s_i' + n_i\right) \cdot \left(s_j' + n_j\right)^* \right\rangle_{\Delta t}$$

$$= \left\langle s_i' \cdot s_j'^* \right\rangle + \left\langle s_i' \cdot n_j^* \right\rangle + \left\langle n_i \cdot s_j'^* \right\rangle + \left\langle n_i \cdot n_j^* \right\rangle$$

- Noise correlations have zero mean—even if |*ni*|>> |*si*|, the correlation process isolates desired signals:

$$= \left\langle s_i' \cdot s_j'^* \right\rangle$$

$$= \left\langle \int_{sky} J_i s_i(t, l_i, m_i) dl_i dm_i \cdot \int_{sky} J_j^* s_j^*(t, l_j, m_j) dl_j dm_j \right\rangle_{\Delta t}$$

- In integral, only *si(t,l,m)* from the same directions correlate (i.e., when *li=lj, mi=mj*), so order of integration and signal product can be reversed:

$$= \left\langle \int_{sky} J_i J_j^* s_i(t, l, m) s_j^*(t, l, m) dl dm \right\rangle_{\Delta t}$$

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Correlation of Realistic Signals - II

The *si* & *sj* are the *common* radio source signals, and differ *only* by the relative arrival phase at each antenna, which varies with direction. This *is* the Fourier phase term (to a good approximation), which we factor out:

$$V_{ij} = \left\langle \int_{sky} J_i J_j^* \left| s_i(t,l,m) \right|^2 e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)} dldm \right\rangle_{\Delta t}$$

On the timescale of the averaging, the only meaningful average is of the *squared* source signal itself (in each direction), which is just the image of the source:

$$= \int_{sky} J_i J_j^* \left\langle \left| s_i(t,l,m) \right|^2 \right\rangle_{\Delta t} e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)} dldm$$

$$= \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)} dldm$$

If all *J*=*1*, we of course recover the ideal expression:

$$= \int_{sky} I(l,m) e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)} dldm$$

Aside: Auto-correlations and Single Dishes

The *auto*-correlation of a signal from a *single* antenna:

$$\left\langle x_i \cdot x_i^* \right\rangle_{\Delta t} = \left\langle (s_i' + n_i) \cdot (s_i' + n_i)^* \right\rangle_{\Delta t}$$
$$= \left\langle s_i' \cdot s_i'^* \right\rangle + \left\langle n_i \cdot n_i^* \right\rangle$$
$$= \left\langle \int_{sky} |J_i|^2 |s_i|^2 \, dl dm \right\rangle_{\Delta t} + \left\langle |n_i|^2 \right\rangle$$
$$= \int_{sky} |J_i|^2 I(l,m) \, dl dm + \left\langle |n_i|^2 \right\rangle$$

- This is an integrated (sky) power measurement plus *non-zero* noise
- Desired signal *not* isolated from noise
- Noise usually dominates

Single dish radio astronomy calibration strategies rely on switching schemes to isolate desired signal from the noise

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The Scalar Measurement Equation

$$V_{ij}^{obs} = \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dl dm$$

First, isolate non-direction-dependent effects, and factor them from the integral:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} \left(J_i^{sky} J_j^{sky*}\right) I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dl dm$$

Next, we recognize that over small fields of view, it is possible to assume *Jsky=1*, and we have a relationship between ideal and observed Visibilities:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dl dm$$

$$V_{ij}^{obs} = \left(J_i^{vis} J_j^{vis*}\right) V_{ij}^{true} = J_i J_j^* V_{ij}^{true}$$

Standard calibration of most existing arrays reduces to solving this last equation for the *Ji,* assuming a visibility model *Vijmod* for a calibrator

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Solving for the *Ji*

We can write:

$$V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} = 0$$

…and define chi-squared:

$$\chi^2 = \sum_{\substack{i,j \\ i \neq j}} \left| V_{ij}^{obs} - J_i J_j^* V_{ij}^{mod} \right|^2 w_{ij}$$

…and minimize chi-squared w.r.t. each *Ji**, yielding (iteration):

$$J_i = \sum_{\substack{j \\ i \neq j}} \left(\frac{V_{ij}^{obs}}{V_{ij}^{mod}} J_j w_{ij} \right) \Bigg/ \sum_{\substack{j \\ i \neq j}} \left(\left| J_j \right|^2 w_{ij} \right) \qquad \left(\frac{\partial \chi^2}{\partial J_i^*} = 0 \right)$$

- (…which we may recognize as a weighted average of the *Ji* contribution to the chi-squared equation)

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Solving for *Ji* (cont)

For a uniform array (~same sensitivity on all baselines, ~same calibration magnitude on all antennas), it can be shown that the error in the calibration solution is:

$$\sigma_{J_i} \approx \frac{\sigma_{V^{obs}}(\Delta t)}{V^{mod}\,\bar{J}\sqrt{N_{ant} - 1}}$$

Calibration error decreases with increasing calibrator strength and square-root of *Nant* (c.f. baseline-based calibration).

Other properties of the antenna-based solution:

Minimal degrees of freedom (*Nant* factors, *Nant(Nant-1)/2* measurements)

Net calibration for a baseline involves a phase difference, so absolute directional information is lost

Closure…

Antenna-based Calibration and Closure

Success of synthesis telescopes relies on antenna-based calibration

Fundamentally, any information that can be factored into antenna-based terms, could *be* antenna-based effects, and not source visibility

For *Nant* > 3, source visibility information cannot be *entirely* obliterated by any antenna-based calibration

Observables independent of antenna-based calibration:

Closure phase (3 baselines):

$$\varphi_{ij}^{obs} + \varphi_{jk}^{obs} + \varphi_{ki}^{obs} = \left(\varphi_{ij}^{true} + \theta_i - \theta_j\right) + \left(\varphi_{jk}^{true} + \theta_j - \theta_k\right) + \left(\varphi_{ki}^{true} + \theta_k - \theta_i\right)$$

Closure amplitude (4 baselines):

$$\left|\frac{V_{ij}^{obs} V_{kl}^{obs}}{V_{ik}^{obs} V_{jl}^{obs}}\right| = \left|\frac{J_i J_j V_{ij}^{true} J_k J_l V_{kl}^{true}}{J_i J_k V_{ik}^{true} J_j J_l V_{jl}^{true}}\right| = \left|\frac{V_{ij}^{true} V_{kl}^{true}}{V_{ik}^{true} V_{jl}^{true}}\right|$$

Baseline-based calibration formally violates closure!

Simple Scalar Calibration Example

Sources:

Science Target: 3C391 (7 mosaic pointings)

Near-target calibrator: J1822-0938 (~11 deg from target; unknown flux density, assumed 1 Jy)

Flux Density calibrators: 3C286 (7.747 Jy, essentially unresolved)

Signals:

RR correlation only for this illustration (total intensity only)

One spectral window centered at 4600 MHz, 128 MHz bandwidth

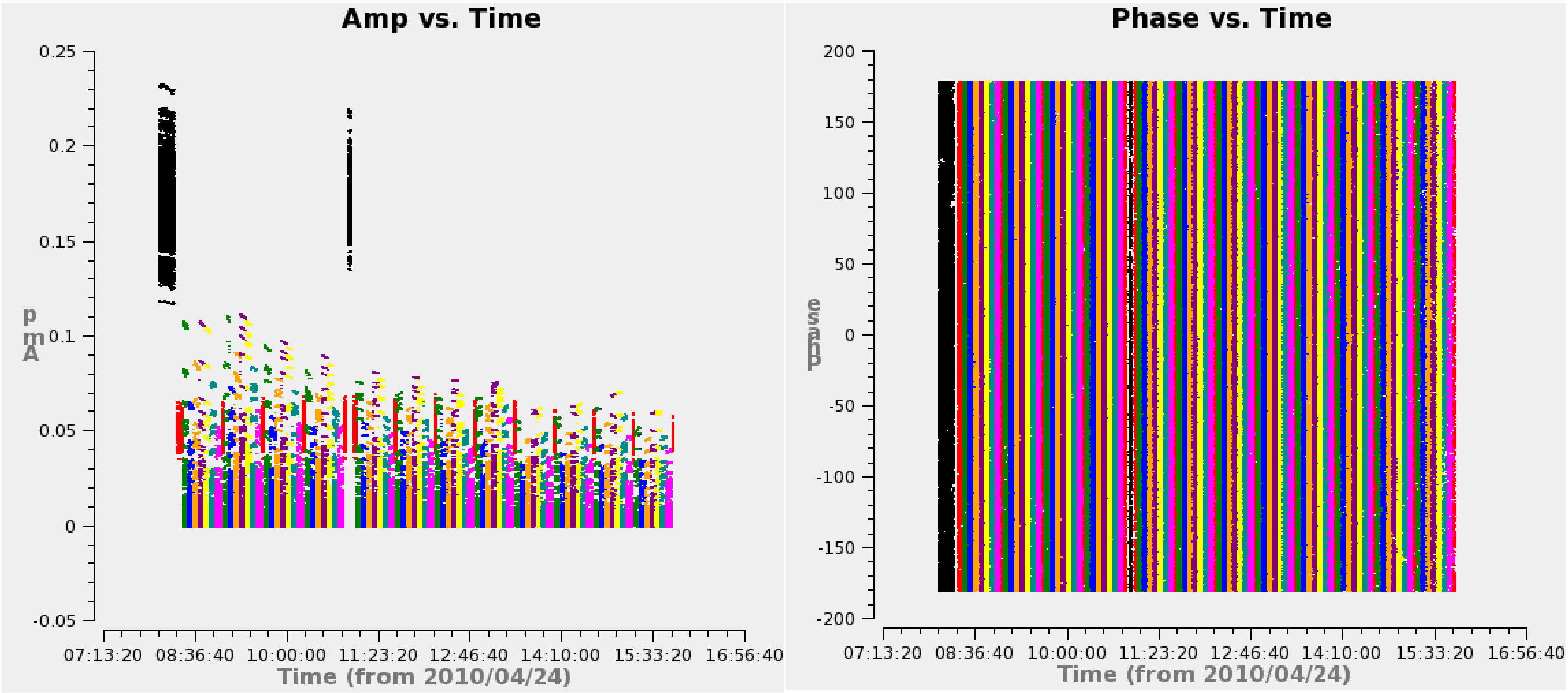
64 observed spectral channels averaged with normalized bandpass calibration applied (this illustration considers only the time-dependent 'gain' calibration)

(extracted from a continuum polarimetry mosaic observation)

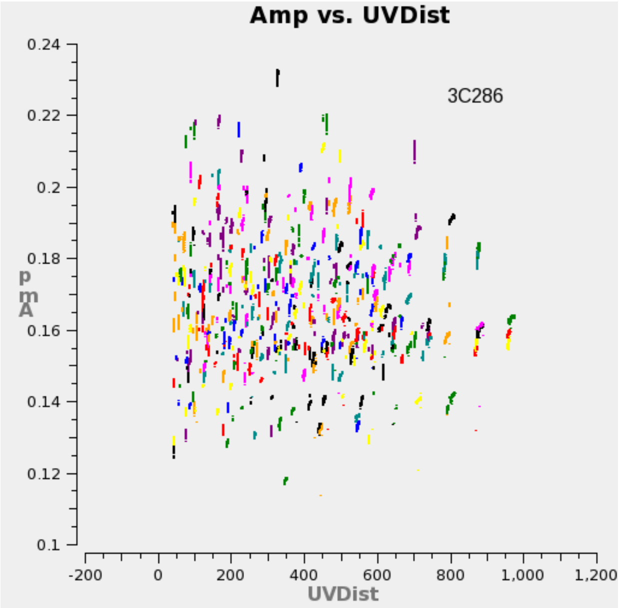
Array:

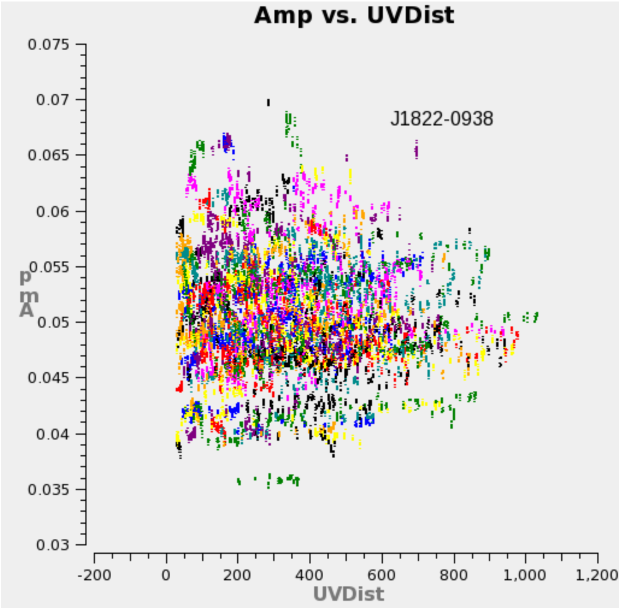
EVLA D-configuration (Apr 2010)

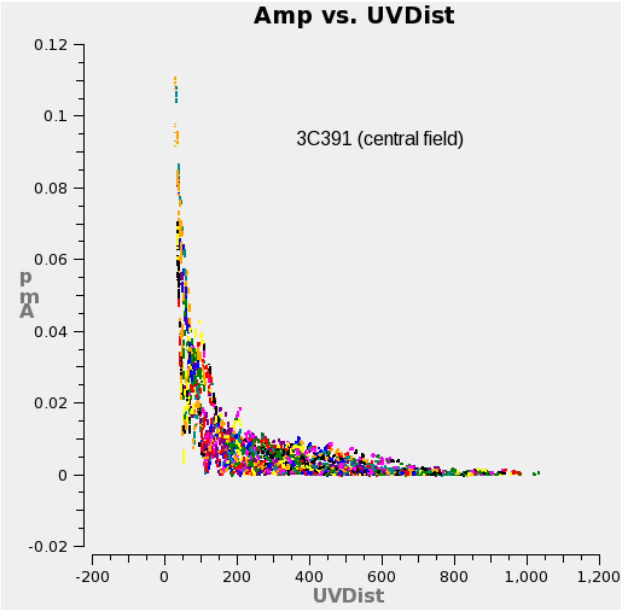
Views of the Uncalibrated Data



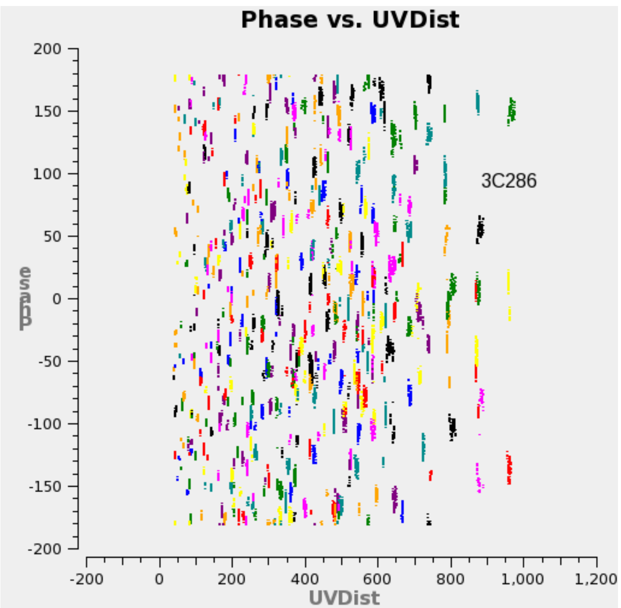
Views of the Uncalibrated Data

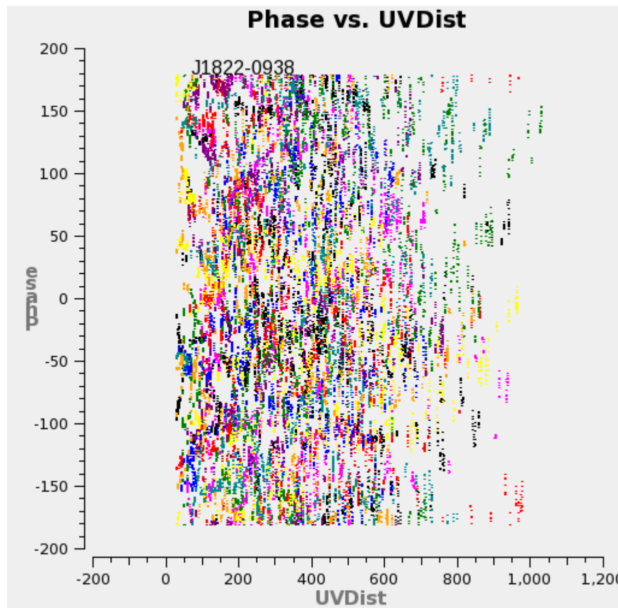


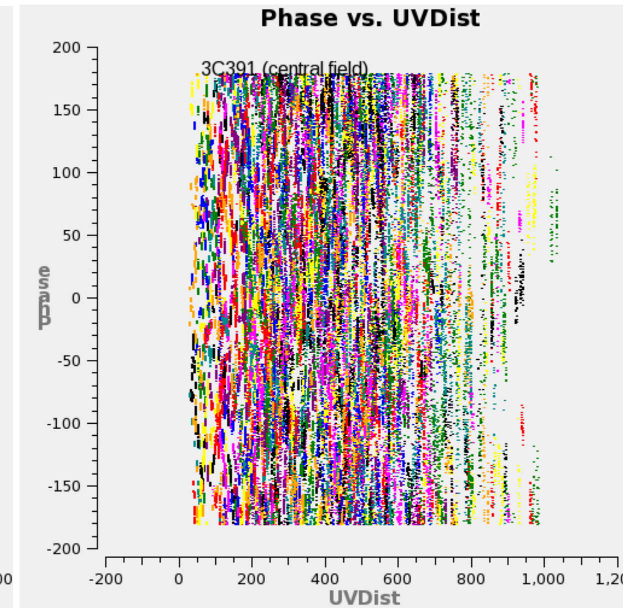




Views of the Uncalibrated Data







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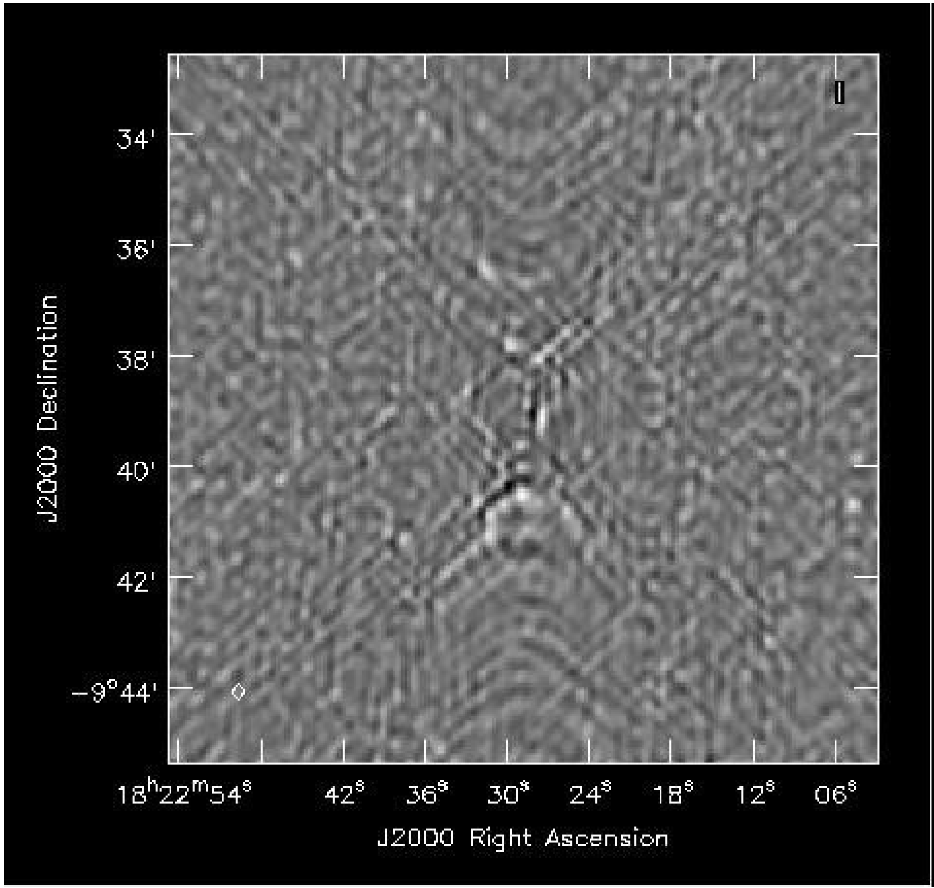
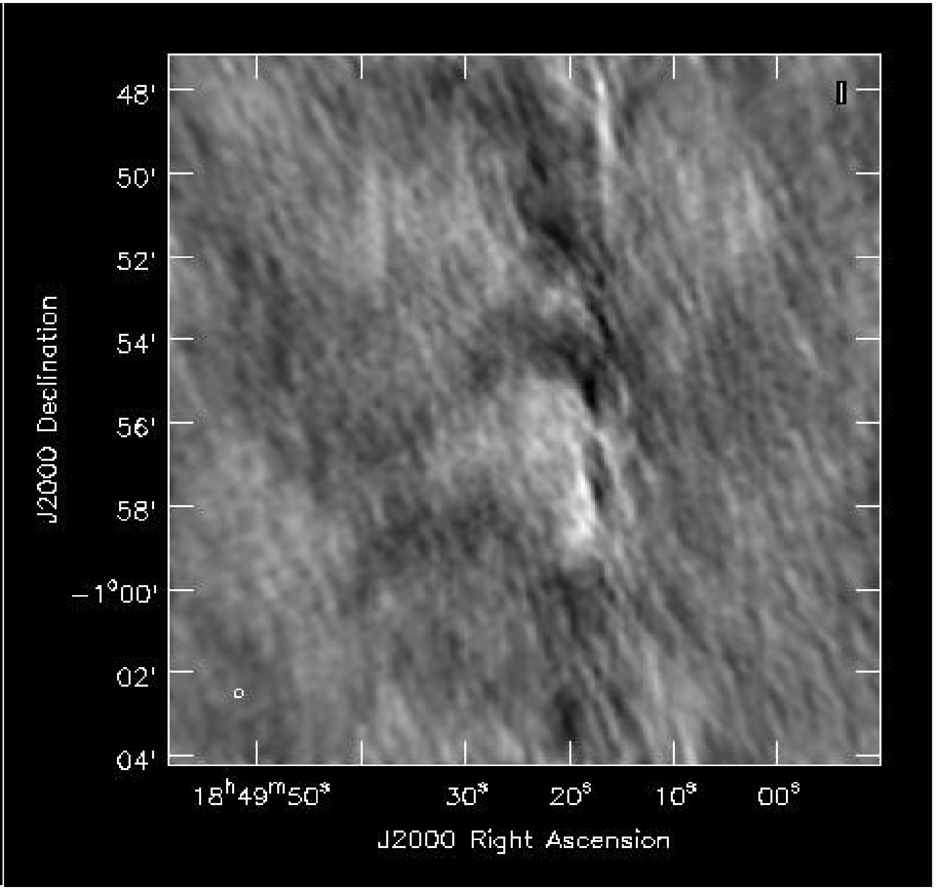
Uncalibrated Images



J1822-0938
(calibrator)



3C391
(science)

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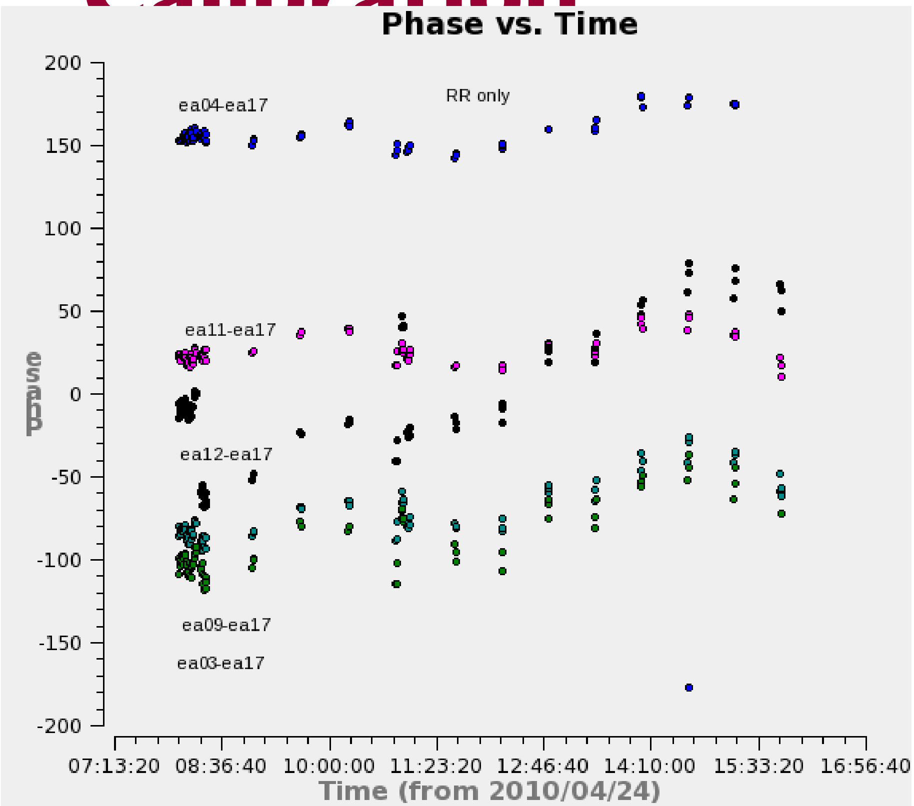
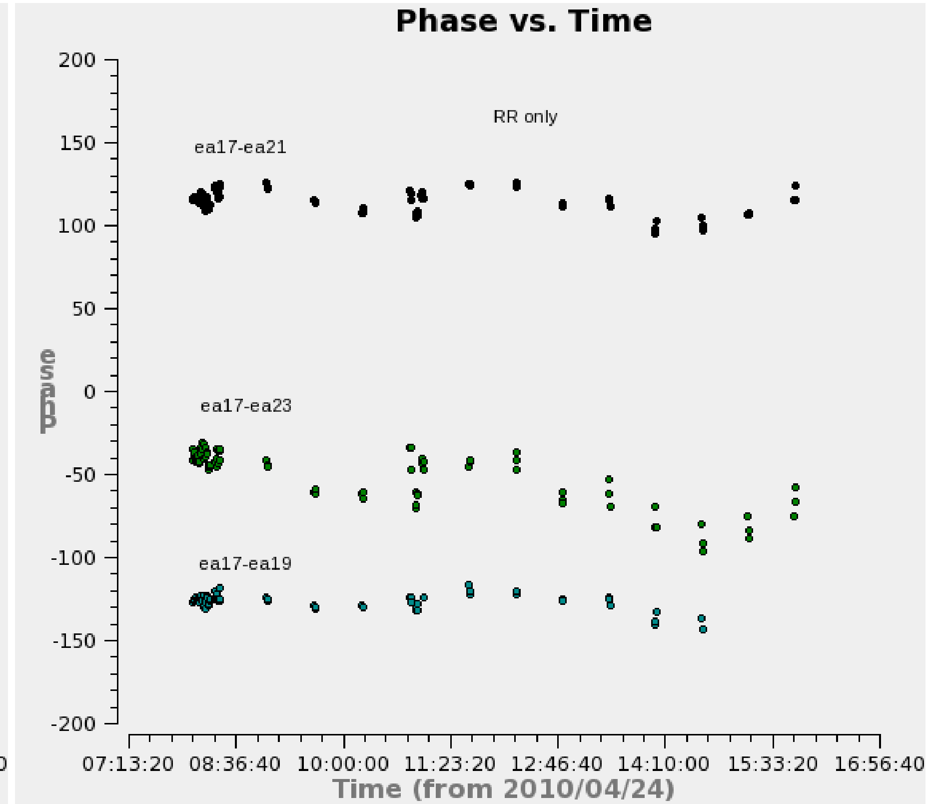
Rationale for Antenna-based Calibration





The Calibration Process

Solve for antenna-based gain factors for each scan on all calibrators:

$$V_{ij}^{obs} = \left(G_i G_j^*\right) V_{ij}^{mod}$$

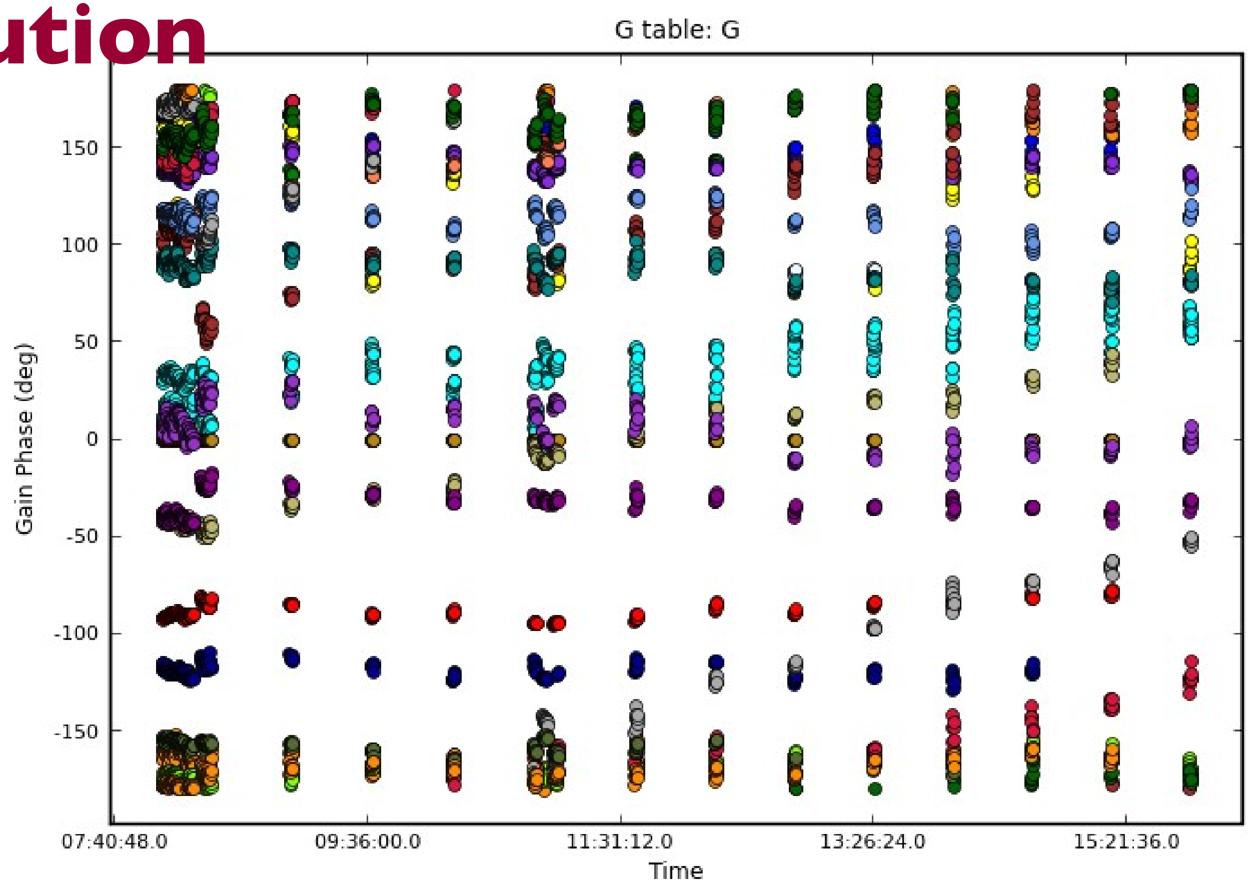
Bootstrap flux density scale by enforcing gain consistency over all calibrators:

$$\left\langle G_i\left(all\ cals\right)\right\rangle = \left\langle G_i\left(fd\ cal\right)\right\rangle$$

Correct data (interpolate, as needed):

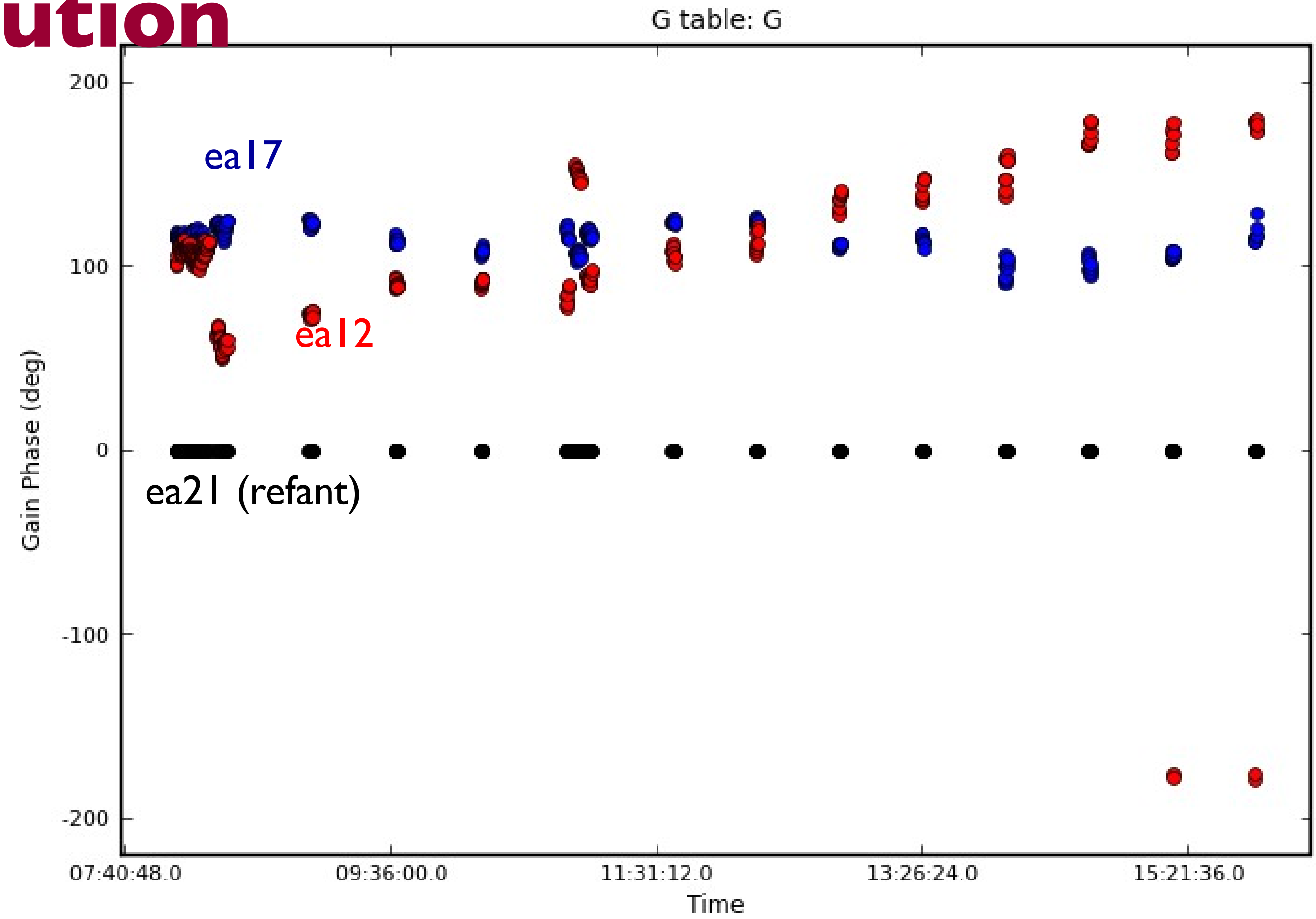
$$V_{ij}^{cor} = \left(G_i^{-1} G_j^{*-1}\right) V_{ij}^{obs}$$

The Antenna-based Calibration Solution

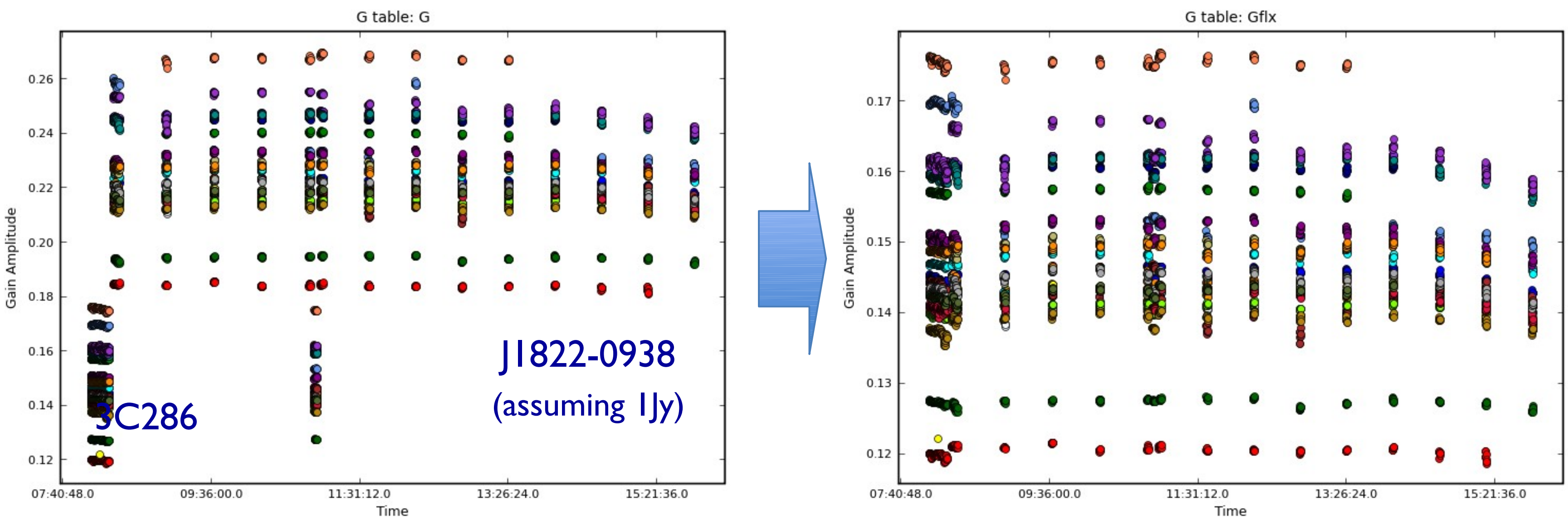


- Reference antenna: ea21 (phase = 0)

The Antenna-based Calibration Solution

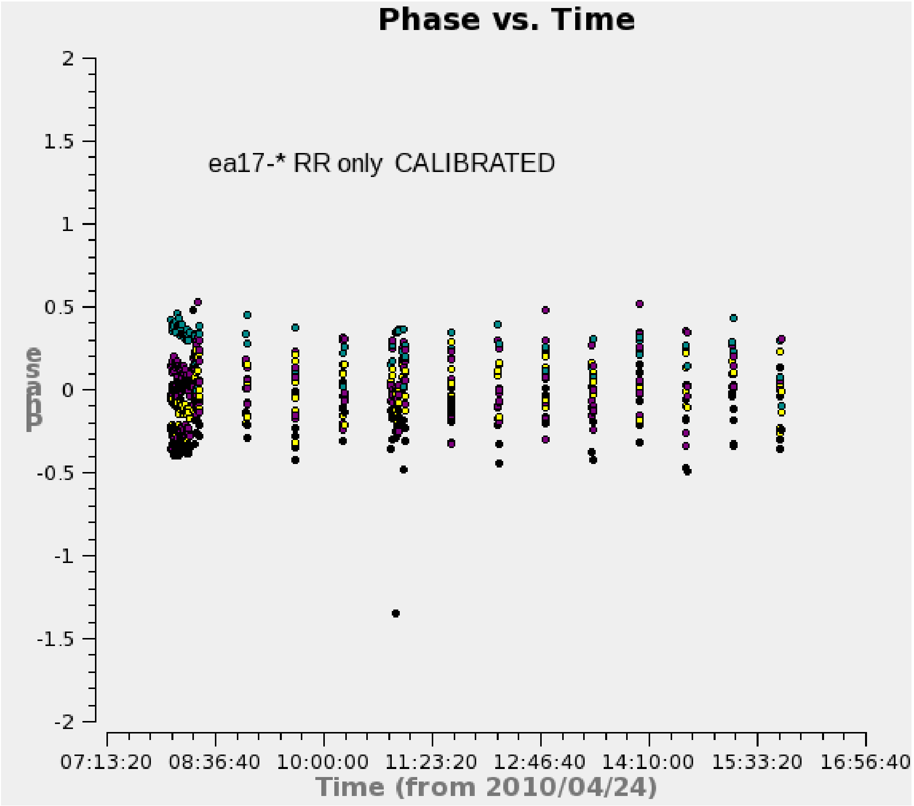


The Antenna-based Calibration

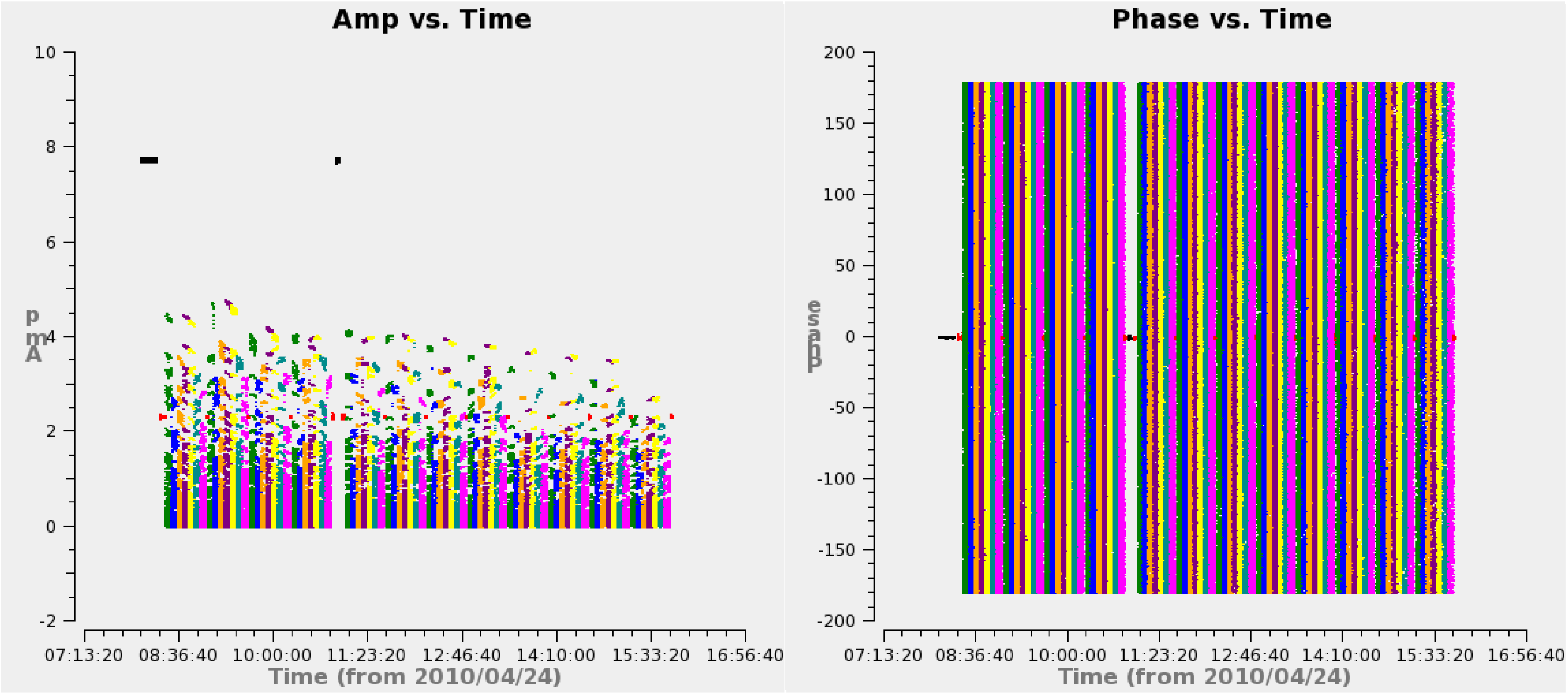


- 3C286's gains have correct scale
- Thus, J1822-0938 is 2.32 Jy (not 1 Jy, as assumed)

Effect of Antenna-based Calibration

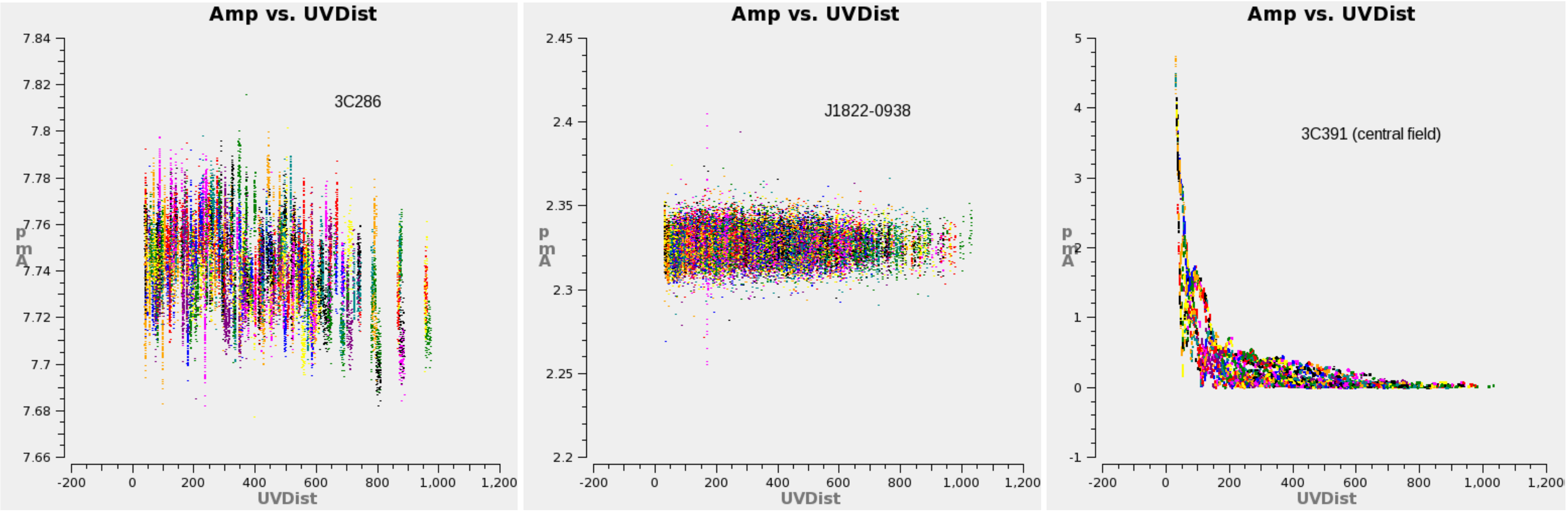


Effect of Antenna-based Calibration



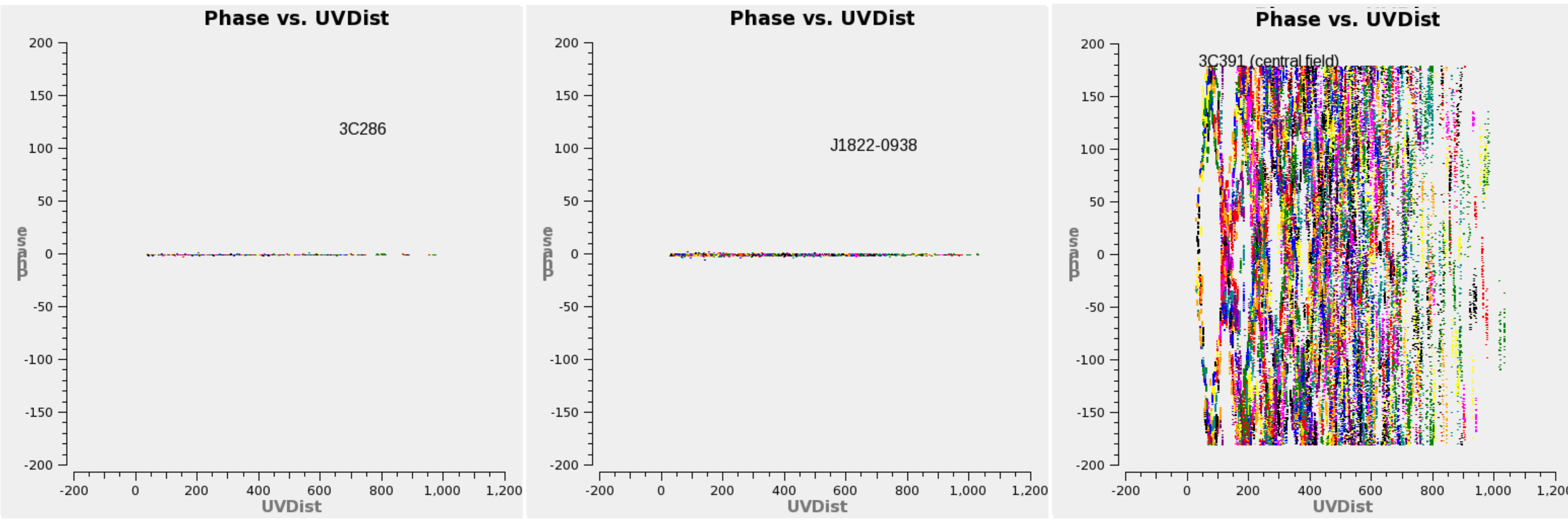
CALIBRATED

Effect of Antenna-based Calibration



CALIBRATED

Effect of Antenna-based Calibration



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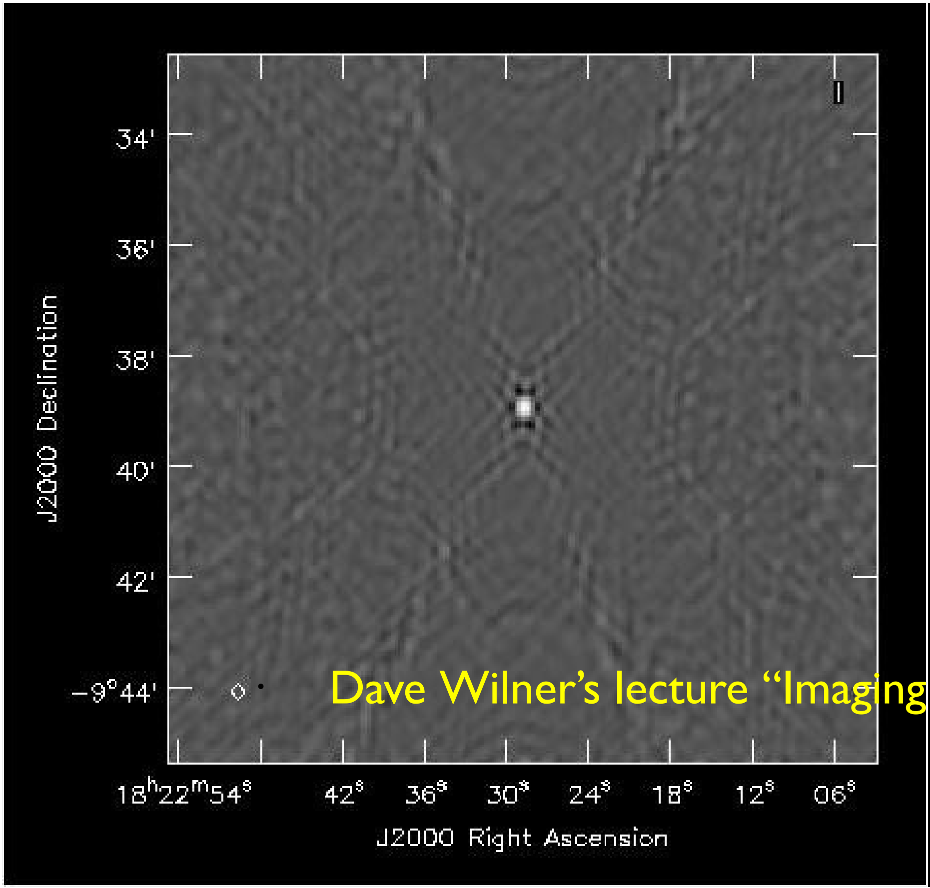
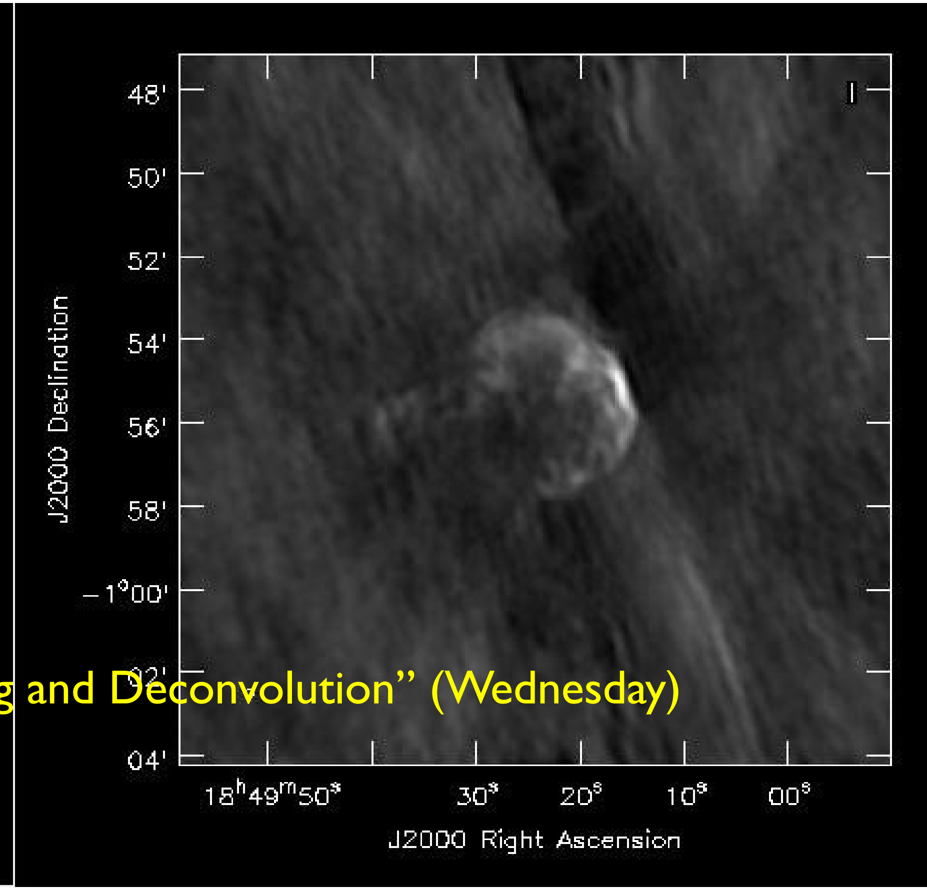
Calibration Effect on Imaging



J1822-0938
(calibrator)



3C391
(science)

Dave Wilner's lecture "Imaging and Deconvolution" (Wednesday)

Evaluating Calibration Performance

Are solutions continuous?

Noise-like solutions are just that—noise (beware calibration of pure noise generates a spurious point source)

Discontinuities indicate instrumental glitches

Any additional editing required?

Are calibrator data fully described by antenna-based effects?

Phase and amplitude *closure errors* are the baseline-based residuals

Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration components

- Mark Claussen's lecture: "Advanced Calibration" (Wednesday)

Any evidence of unsampled variation? Is interpolation of solutions appropriate?

Reduce calibration timescale, if SNR permits

Gustaaf van Moorsel's lecture: "Error Recognition" (Wednesday)

Summary of Scalar Example

Dominant calibration effects are ***antenna-based***

- Minimizes degrees of freedom
- More precise
- Preserves closure
- Permits higher dynamic range *safely*!

Point-like calibrators effective

Flux density bootstrapping

Full-Polarization Formalism (Matrices!)

Need dual-polarization basis (*p*,*q*) to fully sample the incoming EM wave front, where *p*,*q* = *R*,*L* (circular basis) or *p*,*q* = *X*,*Y* (linear basis):

$$\vec{I}_{circ} = \hat{S}_{circ}\vec{I}_{Stokes} \qquad \vec{I}_{lin} = \hat{S}_{lin}\vec{I}_{Stokes}$$

$$\begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix} \qquad \begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

Devices can be built to sample these circular (R,L) or linear (X,Y) basis states in the signal domain (Stokes Vector is defined in "power" domain)

Some components of *Ji* involve mixing of basis states, so dual-polarization matrix description desirable or even required for proper calibration

Full-Polarization Formalism: Signal Domain

Substitute:

$$\vec{s}_i = \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i \qquad J_i = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_i$$

The *Jones matrix* thus corrupts the vector wavefront signal as follows:

$$\vec{s}_i{}' = J_i \vec{s}_i \qquad \text{(sky integral omitted)}$$

$$\begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_i \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i$$

$$= \begin{pmatrix} J^{p\to p} s^p + J^{q\to p} s^q \\ J^{p\to q} s^p + J^{q\to q} s^q \end{pmatrix}_i$$

Full-Polarization Formalism: Correlation - I

Four correlations are possible from two polarizations. The *outer product* (a 'bookkeeping' product) represents correlation in the matrix formalism:

$$\vec{V}_{ij}^{\,true} = \left\langle \vec{s}_i \otimes \vec{s}_j^{\,*} \right\rangle = \left\langle \begin{pmatrix} s^p \\ s^q \end{pmatrix} \otimes \begin{pmatrix} s^{*p} \\ s^{*q} \end{pmatrix} \right\rangle = \begin{pmatrix} \left\langle s_i^p \cdot s_j^{*p} \right\rangle \\ \left\langle s_i^p \cdot s_j^{*q} \right\rangle \\ \left\langle s_i^q \cdot s_j^{*p} \right\rangle \\ \left\langle s_i^q \cdot s_j^{*q} \right\rangle \end{pmatrix}$$

A very useful property of outer products:

$$\vec{V}_{ij}^{\,obs} = \left(\vec{s}_i^{\,\prime} \otimes \vec{s}_j^{\,\prime *}\right) = \left(\mathbf{J}_i \vec{s}_i\right) \otimes \left(\mathbf{J}_j^* \vec{s}_j^{\,*}\right) = \left(\mathbf{J}_i \otimes \mathbf{J}_j^*\right)\left(\vec{s}_i \otimes \vec{s}_j^{\,*}\right) = \mathbf{J}_{ij} \vec{V}_{ij}^{\,true}$$

Full-Polarization Formalism: Correlation - II

The outer product for the Jones matrix:

$$\vec{J}_i \otimes \vec{J}_j^* = \begin{pmatrix} J_i^{p\to p} & J_i^{q\to p} \\ J_i^{p\to q} & J_i^{q\to q} \end{pmatrix} \otimes \begin{pmatrix} J_j^{*p\to p} & J_j^{*q\to p} \\ J_j^{*p\to q} & J_j^{*q\to q} \end{pmatrix}$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} & J_i^{p\to p} J_j^{*q\to p} & J_i^{q\to p} J_j^{*p\to p} & J_i^{q\to p} J_j^{*q\to p} \\ J_i^{p\to p} J_j^{*p\to q} & J_i^{p\to p} J_j^{*q\to q} & J_i^{q\to p} J_j^{*p\to q} & J_i^{q\to p} J_j^{*q\to q} \\ J_i^{p\to q} J_j^{*p\to p} & J_i^{p\to q} J_j^{*q\to p} & J_i^{q\to q} J_j^{*p\to p} & J_i^{q\to q} J_j^{*q\to p} \\ J_i^{p\to q} J_j^{*p\to q} & J_i^{p\to q} J_j^{*q\to q} & J_i^{q\to q} J_j^{*p\to q} & J_i^{q\to q} J_j^{*q\to q} \end{pmatrix}$$

Jij is a 4x4 *Mueller matrix*

This is starting to get ugly…..

Antenna and array design driven by minimizing off-diagonal terms!

Full-Polarization Formalism: Correlation - III

And finally, for fun, the correlation of corrupted signals:

$$\overleftrightarrow{J}_i \vec{s}_i \otimes \overleftrightarrow{J}^*_j \vec{s}^{\,*}_j = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}^*_j\right)\left(\vec{s}_i \otimes \vec{s}^{\,*}_j\right)$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} & J_i^{p\to p} J_j^{*q\to p} & J_i^{q\to p} J_j^{*p\to p} & J_i^{q\to p} J_j^{*q\to p} \\ J_i^{p\to p} J_j^{*p\to q} & J_i^{p\to p} J_j^{*q\to q} & J_i^{q\to p} J_j^{*p\to q} & J_i^{q\to p} J_j^{*q\to q} \\ J_i^{p\to q} J_j^{*p\to p} & J_i^{p\to q} J_j^{*q\to p} & J_i^{q\to q} J_j^{*p\to p} & J_i^{q\to q} J_j^{*q\to p} \\ J_i^{p\to q} J_j^{*p\to q} & J_i^{p\to q} J_j^{*q\to q} & J_i^{q\to q} J_j^{*p\to q} & J_i^{q\to q} J_j^{*q\to q} \end{pmatrix} \begin{pmatrix} \langle s_i^p \cdot s_j^{*p} \rangle \\ \langle s_i^p \cdot s_j^{*q} \rangle \\ \langle s_i^q \cdot s_j^{*p} \rangle \\ \langle s_i^q \cdot s_j^{*q} \rangle \end{pmatrix}$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p\to p} J_j^{*q\to p} \langle s_i^p \cdot s_j^{*q} \rangle + J_i^{q\to p} J_j^{*p\to p} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q\to p} J_j^{*q\to p} \langle s_i^q \cdot s_j^{*q} \rangle \\ J_i^{p\to p} J_j^{*p\to q} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p\to p} J_j^{*q\to q} \langle s_i^p \cdot s_j^{*q} \rangle + J_i^{q\to p} J_j^{*p\to q} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q\to p} J_j^{*q\to q} \langle s_i^q \cdot s_j^{*q} \rangle \\ J_i^{p\to q} J_j^{*p\to p} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p\to q} J_j^{*q\to p} \langle s_i^p \cdot s_j^{*q} \rangle + J_i^{q\to q} J_j^{*p\to p} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q\to q} J_j^{*q\to p} \langle s_i^q \cdot s_j^{*q} \rangle \\ J_i^{p\to q} J_j^{*p\to q} \langle s_i^p \cdot s_j^{*p} \rangle + J_i^{p\to q} J_j^{*q\to q} \langle s_i^p \cdot s_j^{*q} \rangle + J_i^{q\to q} J_j^{*p\to q} \langle s_i^q \cdot s_j^{*p} \rangle + J_i^{q\to q} J_j^{*q\to q} \langle s_i^q \cdot s_j^{*q} \rangle \end{pmatrix}$$

UGLY, but we rarely, if ever, need to worry about detail at this level---just let this occur “inside” the matrix formalism, and work with the matrix short-hand notation

The Matrix Measurement Equation

We can now write down the Measurement Equation in matrix notation:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^{*} \right) \overleftrightarrow{S} \vec{I}(l,m) e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)} dldm$$

S maps Stokes parameters onto observed basis

…and consider how the *Ji* are products of many effects.

A Dictionary of Calibration Components

J_i contains many components, in principle:

- F = ionospheric effects
- T = tropospheric effects
- P = parallactic angle
- X = linear polarization position angle
- E = antenna voltage pattern
- D = polarization leakage
- G = electronic gain
- B = bandpass response
- K = geometric compensation
- M, A = baseline-based corrections

$$J_i = K_i B_i G_i D_i E_i X_i P_i T_i F_i$$

Order of terms follows signal path (right to left)

Each term has matrix form of J_i with terms embodying its particular algebra (on- vs. off-diagonal terms, etc.)

Direction-dependent terms must stay inside FT integral

'Full' calibration is traditionally a bootstrapping process wherein relevant terms (usually a minority of above list) are considered in decreasing order of dominance, relying on approximate separability

Ionospheric Effects, *F*

$$\overset{\leftrightarrow}{F}{}^{RL} = e^{i\Delta\varphi}\begin{pmatrix} e^{i\varepsilon} & 0 \\ 0 & e^{-i\varepsilon} \end{pmatrix}; \quad \overset{\leftrightarrow}{F}{}^{XY} = e^{i\Delta\varphi}\begin{pmatrix} \cos\varepsilon & -\sin\varepsilon \\ \sin\varepsilon & \cos\varepsilon \end{pmatrix}$$

The ionosphere introduces a dispersive path-length offset:

$$\Delta\varphi \propto \frac{N_e\left[\mathrm{cm}^{-2}\right]}{\nu}$$

- More important at lower frequencies (<5 GHz)
- Varies more at solar maximum and at sunrise/sunset, when ionosphere is most active and variable
- Direction-dependent within wide field-of-view

$$\varepsilon \propto \frac{\int B_{\parallel}\, n_e\left[\mathrm{cm}^{-3}\right]}{\nu^2}$$

The ionosphere is *birefringent*: Faraday rotation:

- as high as 20 rad/m2 during periods of high solar activity will rotate linear polarization position angle by $e = 50$ degrees at 1.4 GHz
- Varies over the array, and with time as line-of-sight magnetic field and electron density vary, violating the usual assumption of stability in position angle calibration

Tracy Clark's lecture: "Low Frequency Interferometry" (Monday)

Tropospheric Effects, *T*

$$\overset{\diamond}{T} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The troposphere causes polarization-independent amplitude and phase effects due to emission/opacity and refraction, respectively

- Up to 2.3m excess path length at zenith compared to vacuum
- Higher noise contribution, less signal transmission: Lower SNR
- Most important at $\nu > 20$ GHz where water vapor and oxygen absorb/emit
- Zenith-angle-dependent (more troposphere path nearer horizon)
- Clouds, weather = variability in phase and opacity; may vary across array
- Water vapor radiometry (estimate phase from power measurements)
- Phase transfer from low to high frequencies (delay calibration)

Crystal Brogan's lecture: "Millimeter Interferometry and ALMA" (Thursday)

Parallactic Angle, *P*

$$\overset{\leftrightarrow}{P}^{RL} = \begin{pmatrix} e^{i\chi} & 0 \\ 0 & e^{-i\chi} \end{pmatrix}; \quad \overset{\leftrightarrow}{P}^{XY} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}$$

Visibility phase variation due to changing orientation of sky in telescope's field of view

- Constant for equatorial telescopes
- Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos l \sin h(t)}{\sin l \cos\delta - \cos l \sin\delta \cos h(t)}\right)$$

$$l = \text{latitude}, \; h(t) = \text{hour angle}, \; \delta = \text{declination}$$

- Rotates the position angle of linearly polarized radiation
- Analytically known, and its variation provides leverage for determining polarization-dependent effects

Steve Myers' lecture: "Polarization in Interferometry" (today!)

Linear Polarization Position Angle, X

$$\overset{\leftrightarrow}{X}^{RL} = \begin{pmatrix} e^{i\Delta\chi} & 0 \\ 0 & e^{-i\Delta\chi} \end{pmatrix}; \quad \overset{\leftrightarrow}{X}^{XY} = \begin{pmatrix} \cos\Delta\chi & -\sin\Delta\chi \\ \sin\Delta\chi & \cos\Delta\chi \end{pmatrix}$$

Configuration of optics and electronics causes a linear polarization position angle offset

Can be treated as an offset to the parallactic angle, P

Calibrated by registration with a strongly polarized source with known polarization position angle (e.g., flux density calibrators)

For circular feeds, this is a phase difference between the R and L polarizations, which is frequency-dependent (a R-L phase bandpass)

For linear feeds, this is the orientation of the dipoles in the frame of the telescope

Steve Myers' lecture: "Polarization in Interferometry" (today!)

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Antenna Voltage Pattern, *E*

$$E^{pq} = \begin{pmatrix} E^{p}(l,m) & 0 \\ 0 & E^{q}(l,m) \end{pmatrix}$$

Antennas of all designs have direction-dependent gain within field-of-view

- Important when region of interest on sky comparable to or larger than l/D
- Important at lower frequencies where radio source surface density is greater and wide-field imaging techniques required
- Beam squint: *Ep* and *Eq* offset, yielding spurious polarization
- Sky rotates within field-of-view for alt-az antennas, so off-axis sources move through the pattern
- Direction dependence of polarization leakage (*D*) may be included in *E* (off-diagonal terms then non-zero)

Shape and efficiency of the voltage pattern may change with zenith angle: ‘gain curve’

Sanjay Bhatnagar’s lecture: “Wide Field Imaging I” (Thursday)

Juergen Ott’s lecture: “Wide Field Imaging II” (Thursday)

Polarization Leakage, *D*

$$\overset{\leftrightarrow}{D} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

Antenna & polarizer are not ideal, so orthogonal polarizations not perfectly isolated

- Well-designed feeds have *d* ~ a few percent or less
- A geometric property of the optical design, so frequency-dependent
- For *R,L* systems, total-intensity imaging affected as ~*dQ, dU,* so only important at high dynamic range (*Q,U,d each ~few* %, typically)
- For *R,L* systems, linear polarization imaging affected as ~*dI,* so almost always important
- For small arrays (no *differential* parallactic angle coverage), only relative D solution is possible from standard linearized solution, so parallel-hands cannot be corrected (closure errors)

Best calibrator: Strong, point-like, observed over large range of parallactic angle (to separate source polarization from *D*)

Steve Myers' lecture: "Polarization in Interferometry" (today!)

"Electronic" Gain, *G*

$$\overset{\leftrightarrow}{G}^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

Catch-all for most amplitude and phase effects introduced by antenna electronics and other generic effects

- Most commonly treated calibration component
- Dominates other effects for standard EVLA observations
- Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
- Often also includes tropospheric and (on-axis) ionospheric effects which are typically difficult to separate uniquely from the electronic response
- Excludes frequency dependent effects (see *B*)

Best calibrator: strong, point-like, near science target; observed often enough to track expected variations

Also observe a flux density standard

Bandpass Response, *B*

$$\overset{\leftrightarrow}{B}{}^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

G-like component describing frequency-dependence of antenna electronics, etc.

- Filters used to select frequency passband not square
- Optical and electronic reflections introduce ripples across band
- Often assumed time-independent, but not necessarily so
- Typically (but not necessarily) normalized

Best calibrator: strong, point-like; observed long enough to get sufficient per-channel SNR, and often enough to track variations

Ylva Pihlstrom's lecture: "Calibration, Imaging, and Analysis of Data Cubes" (Wednesday)

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Geometric Compensation, *K*

$$\vec{K}^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

Must get geometry right for Synthesis Fourier Transform relation to work in real time; residual errors here require "Fringe-fitting"

- Antenna positions (geodesy)
- Source directions (time-dependent in topocenter!) (astrometry)
- Clocks
- Electronic pathlengths (polarization, spw differences)
- Longer baselines generally have larger relative geometry errors, especially if clocks are independent (VLBI)
- Importance scales with frequency

K is a clock- & geometry-parameterized version of *G* (see chapter 5, section 2.1, equation 5-3 & chapters 22, 23)

- All-sky observations used to isolate geometry parameters

Adam Deller's lecture: "Very Long Baseline Interferometry" (Thursday)

Mark Reid's lecture "Astrometry" (Thursday)

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Non-closing Effects: *M*, *A*

Baseline-based errors which do not decompose into antenna-based components

Digital correlators designed to limit such effects to well-understood and ***uniform*** (not dependent on baseline) scaling laws (absorbed in *G*)

Simple noise (additive)

Additional errors can result from averaging in time and frequency over variation in antenna-based effects and visibilities (practical instruments are finite!)

Instrumental polarization effects in parallel hands

Correlated "noise" (e.g., RFI)

Difficult to distinguish from source structure (visibility) effects

Geodetic observers consider determination of radio source structure—a baseline-based effect—as a required *calibration* if antenna positions are to be determined accurately

Diagonal 4x4 matrices, M_{ij} multiplies, A_{ij} adds

Rick Perley's lecture "High Dynamic Range Imaging" (Monday)

Decoupling Calibration Effects

Multiplicative gain (G) term will soak up many different effects; known priors should be compensated for separately, especially when direction-dependent differences (e.g., between calibrator and target) will limit the accuracy of calibration transfer:

Zenith angle-dependent atmospheric opacity, refraction (T,F)

Zenith angle-dependent gain curve (E)

Antenna position errors (K)

Early calibration solves (e.g., G) are always subject to more subtle, uncorrected effects

E.g., instrumental polarization (D), which introduces gain calibration errors and causes apparent closure errors in *parallel-hand* correlations

When possible, iterate and alternate solves to decouple effects…

The Full Matrix Measurement Equation

The total general *Measurement Equation* has the form:

$$\vec{V}_{ij} = M_{ij} B_{ij} G_{ij} D_{ij} \int_{sky} E_{ij} P_{ij} T_{ij} F_{ij} S \vec{I}_{\nu}(l,m)\, e^{-i2\pi(u_{ij}l+v_{ij}m)}\, dl\, dm + \vec{A}_{ij}$$

- *S* maps the Stokes vector, *I*, to the polarization basis of the instrument, all calibration terms cast in this basis

Suppressing the direction-dependence (on-axis calibration):

$$\vec{V}_{ij} = M_{ij} B_{ij} G_{ij} D_{ij} P_{ij} T_{ij} \vec{V}_{ij}^{IDEAL} + \vec{A}_{ij}$$

Generally, only a subset of terms (up to 3 or 4) are considered, though highest-dynamic range observations may require more

Solve for terms in decreasing order of dominance

(Non-trivial direction-dependent solutions involve convolutional treatment of the visibilities, and is coupled to the imaging and deconvolution process)

Solving the Measurement Equation

Formally, solving for any antenna-based visibility calibration component is always the same general non-linear fitting problem:

$$V_{ij}^{corrected\ obs} = \left(J_i J_j^*\right) V_{ij}^{corrupted\ mod}$$

Observed and Model visibilities are corrected/corrupted by available prior calibration solutions

Resulting solution used as prior in subsequent solves, as necessary

Each solution is relative to priors and assumed source model

Iterate sequences, as needed ☾ generalized self-calibration

Viability of the overall calibration depends on isolation of different effects using *proper calibration observations*, and *appropriate solving strategies*

Heuristic mnemonics….

Calibration Heuristics – Spectral Line

Total Intensity Spectral Line (B,G):

– $Vobs = B\ G\ Vtrue$

1. Preliminary Gain solve on B-calibrator:
   – $Vobs = GB\ Vmod$
2. Bandpass Solve (using *GB*) on B-calibrator (then discard *GB*):
   – $Vobs = B\ (GB\ Vmod)$
3. Gain solve (using inverted *B*) on calibrators:
   – $(B'\ Vobs) = G\ Vmod$
4. Flux Density scaling:
   – G ☾ Gf (enforce gain consistency)
5. Correct with inverted solutions:
   – $Vcor = Gf'\ B'\ Vobs$
6. Image!

Heuristic notation! Rigorous math notation (antenna-basedness, subscripts, etc.) omitted.

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Calibration Heuristics – Polarimetry

Polarimetry (B,G,D,X,P):

– $Vobs = B\ G\ D\ X\ P\ Vtrue$

1. Preliminary Gain solve on B-calibrator:
   – $Vobs = GB\ Vmod$
2. Bandpass (B) Solve (using *GB*) on B-calibrator (then discard *GB*):
   – $Vobs = B\ (GB\ Vmod)$
3. Gain (G) solve (using P, inv *B*) on calibrators:
   – $(B'\ Vobs) = G\ (PVmod)$
4. Instrumental Polarization (D) solve (using P, inverse of G,B) on instrumental polarization calibrator:
5. $(G'\ B'\ Vobs) = D\ (P\ Vmod)$

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Calibration Heuristics – Polarimetry

5. Polarization position angle solve (using D,P, inverted G,B) on position angle calibrator:
   - *(G' B' Vobs)* = *X (D P Vmod)*
6. Flux Density scaling:
   - *G* ☾ *Gf* (enforce gain consistency)
7. Correct with inverted solutions:
   - *Vcor = P' X' D' Gf' B' Vobs*
8. Image!

New Calibration Challenges

Bandpass Calibration

- Parameterized solutions (narrow-bandwidth, high resolution regime)
- Spectrum of calibrators (wide absolute bandwidth regime)

'Delay-aware' gain (self-) calibration

- Troposphere and Ionosphere introduce time-variable phase effects which are easily parameterized in frequency and should be (c.f. sampling the calibration in frequency)

Frequency-dependent Instrumental Polarization

- Contribution of geometric optics is wavelength-dependent (standing waves)

Frequency-dependent voltage pattern

Wide-field voltage pattern accuracy (sidelobes)

Direction-dependent components

- E.g., Instrumental Polarization (polarized beam)
- Couples to the imaging process

Increased sensitivity: Can implied dynamic range be reached by conventional calibration and imaging techniques?

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Summary

Determining calibration is as important as determining source structure —can't have one without the other

Data examination and editing an important part of calibration

Beware of RFI! (Please, no cell phones at the VLA site tour!)

Calibration dominated by antenna-based effects, permits efficient separation of calibration from astronomical information (satisfies closure)

Full calibration formalism algebra-rich, but is *modular*

Calibration determination is a single standard fitting problem

Calibration an iterative process, improving various components in turn, as needed

Point sources are the best calibrators

Observe calibrators according requirements of calibration components

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