Wide-Field Imaging: I

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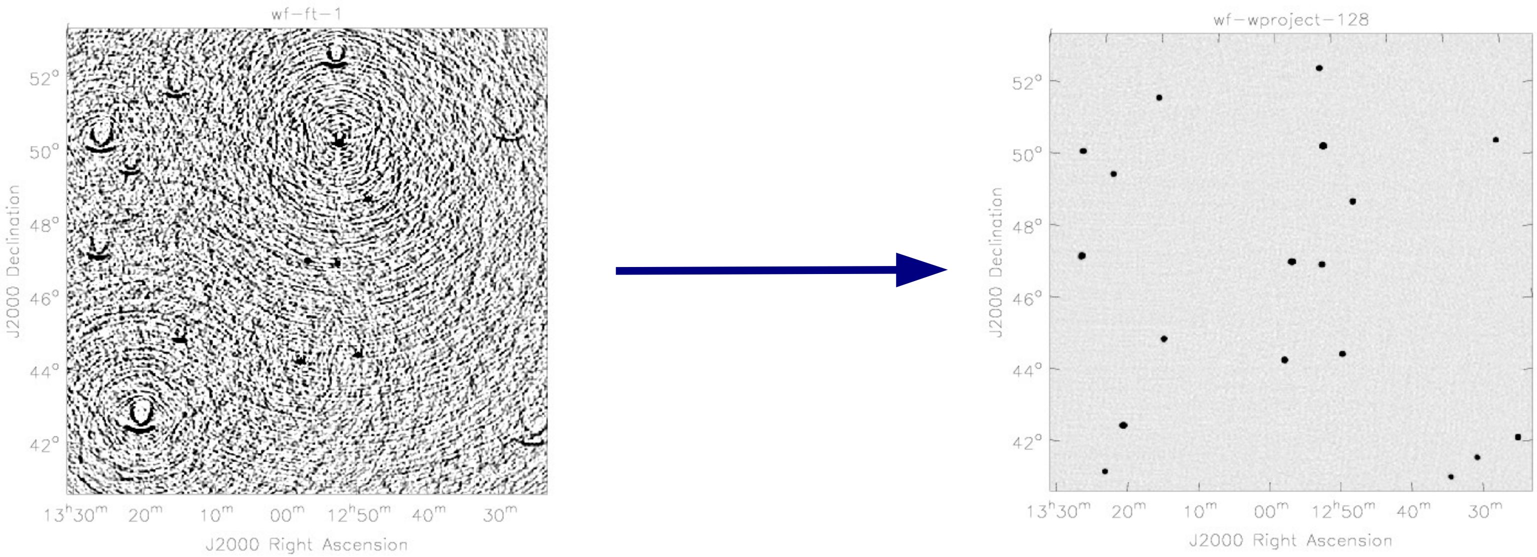
Wide-field imaging

- What do we mean by wide–field imaging
  - W-Term: 2D Fourier transform approximation breaks down
  - Full-beam imaging: Antenna Primary Beam (PB) effects cannot be ignored
  - Mosaicking: Imaging fields with emission larger than the antenna Field-of-View (FoV)

The W-Term

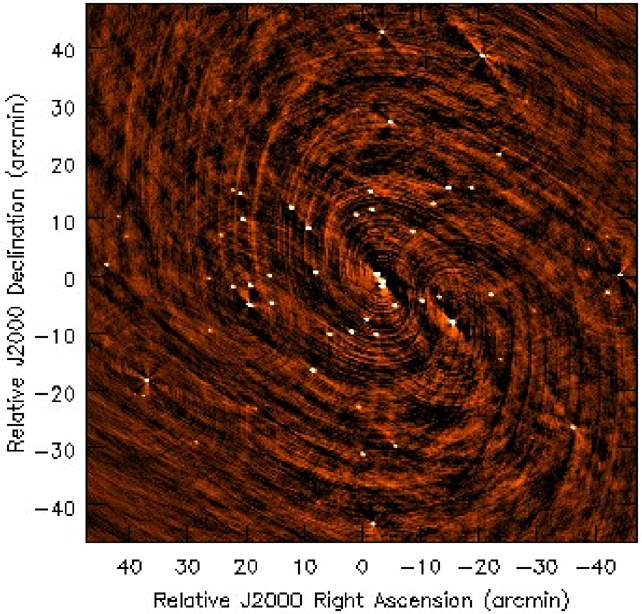
- 2D approximation of the measurement equation (ME) breaks down (“The W-term problem”).

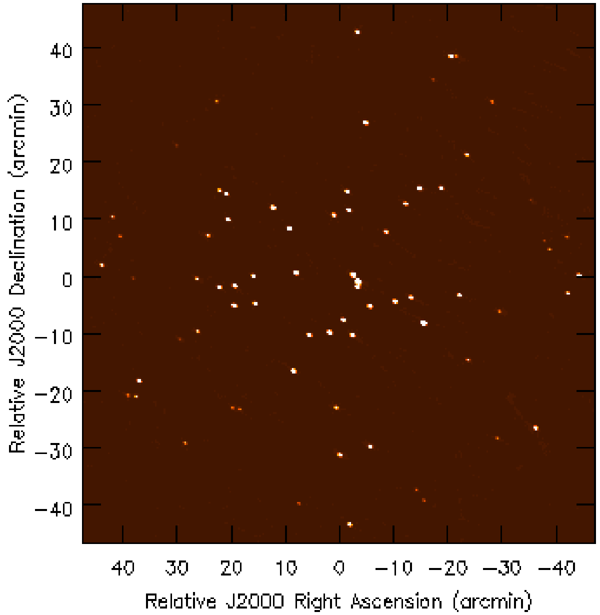


- Imaging dynamic range throughout the image is limited by deconvolution errors due to the sources away from the (phase) center.

Primary Beam Effects

- Antenna Primary Beam (PB) pattern cannot be approximated by unity (“Full Beam Imaging”).

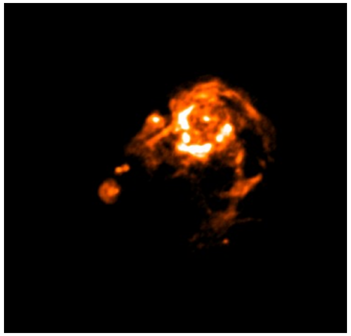


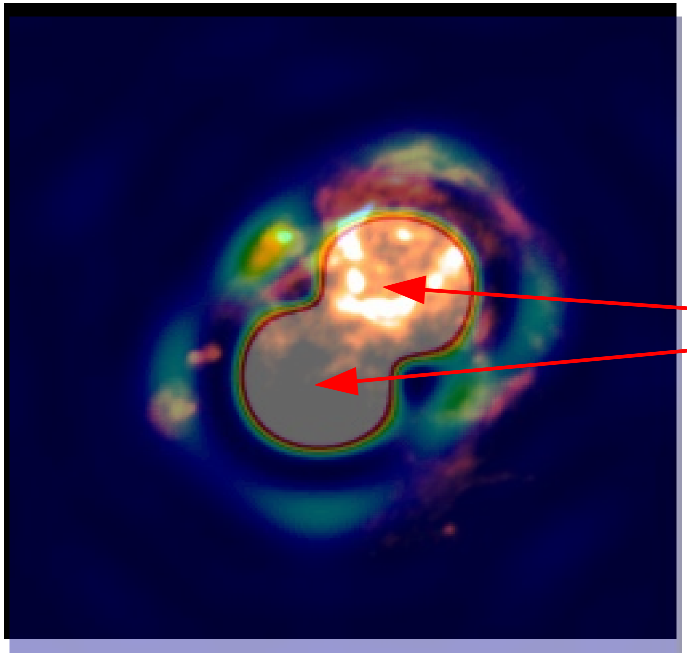


- Imaging dynamic range throughout the image is limited by the deconvolution errors due to the sources in the half-power points and the side lobes.

Mosaicking (see later lectures)

- Imaging emission wider than the Field-of-View (FoV) ("Mosaicking")
- Dominant sources of errors
  - Antenna Pointing
  - PB effects: rotation, multiple types of antenna in the array (ALMA)
  - Deconvolution errors for extended emission





Theory re-cap: Measurement Eq.

$$V^{Obs}(u_{ij}, v_{ij}; \nu) = \int I(l, m, \nu) \ e^{\iota [u_{ij} l + v_{ij} m]} dl \ \ dm$$

- van-Cittert Zernike Theorem: Coherence function is a 2D Fourier Transform of the Sky Brightness distribution

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- Full ME

$$V_{ij}^{Obs}(\nu) = J_{ij}(\nu, t) \int J_{ij}^{S}(s, \nu, t) \ I(s, \nu) \ e^{\iota s.b_{ij}} \ ds$$

Data

Corruptions

Sky

Geometry

$$s = (l, m, n) = (l, m, \sqrt{1 - l^2 - m^2} - 1): \ \textit{Directioncosins}$$

$$b_{ij} = (u_i - u_j, v_i - v_j, w_i - w_j) = (u_{ij}, v_{ij}, w_{ij}): \textit{The Baseline vector}$$

Theory re-cap: Measurement Eq.

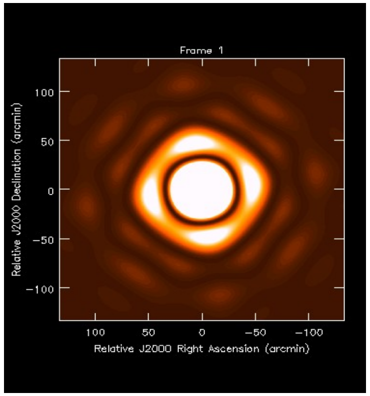
$$V_{ij}^{Obs}(\nu) = J_{ij}(\nu,t) \int J_{ij}^{S}(s,\nu,t)\ I(s,\nu)\ e^{\iota s.b_{ij}}\ ds$$

- $J_{ij} = J_i \otimes J_j^*$ Direction independent effects (e.g. Complex Gains *a la* SelfCal)

  Constant across the field of view

  $J_{ij}^S = J_i^S \otimes J_j^{S^*}$ Direction dependent effects (e.g. Antenna PB, ionosphere,...)

  - Time variable direction dependent gains due to PB rotation for Az-El antennas



- Geometry: W-Term $e^{\iota s.b_{ij}} = e^{\iota[ul + vm + w(\sqrt{1-l^2-m^2}-1)]}$

The W-Term: Theory

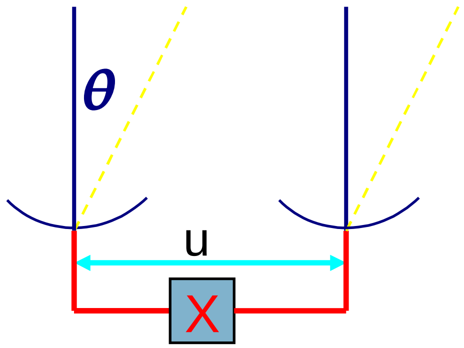
- Ignore J_{ij} (nominally calibrated data) and J^s_{ij} (ignore effects of PB)

$$V_{ij}^{Obs}(\nu)=\int I(s,\nu)\; e^{\iota\left[u_{ij}l+v_{ij}m+w_{ij}\left(\sqrt{1-l^2-m^2}-1\right)\right]}\; ds$$

- FoV is small: $l^2+m^2 \ll 1$
- Array is co-planar $w \ll \left(\sqrt{u^2_{max}+v^2_{max}}\right)$
  - vCZ: 2D Fourier transform works

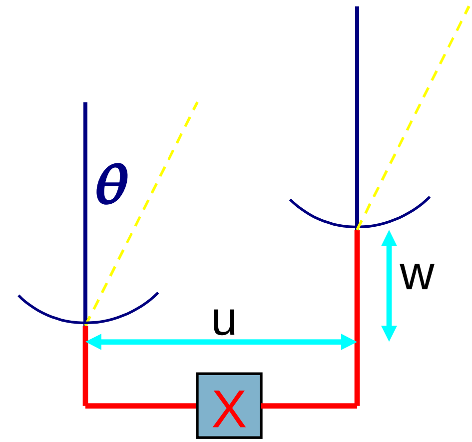
- When FoV **or** w_{ij} is "large", data and the image are not related by a simple 2D Fourier transform relationship.

The W-Term: Geometric interpretation



$$l = \sin(\theta)$$

$$n = \cos(\theta)$$



- Phase of the visibilities for direction $\boldsymbol{\theta}$
  - For the interferometer in a plane: $\phi = 2\pi u l$
  - For the interferometer not in a plane: $\phi = 2\pi \left[u l + w(n-1) \right]$
- 2D approximation valid only when: (1) w is small compared to u, or (2) $\theta \approx 0$

The W-Term: Optics interpretation

- Physically we measure...

$$V^o_{12} = \langle \boldsymbol{E}'_1(u, v, w \neq 0)\, \boldsymbol{E}_2^*(0{,}0{,}0)\rangle$$

- ...and interpret as if it were

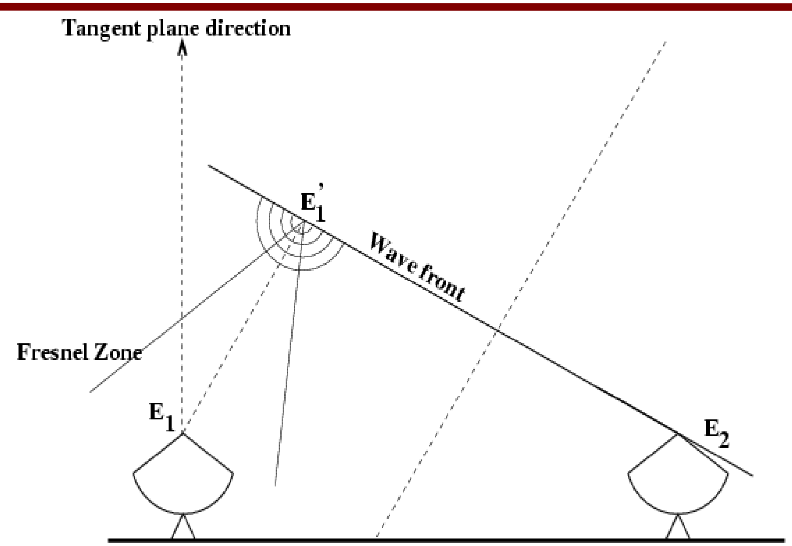
$$V_{12} = \langle \boldsymbol{E}_1(u, v, w = 0)\, \boldsymbol{E}_2^*(0{,}0{,}0)\rangle$$

- $\boldsymbol{E}_1$ is equal to $\boldsymbol{E}'_1$ propagated using Fresnel diffraction theory

$$V^o_{12} = V_{12}(u, v, w=0) * G(u, v, w), where$$

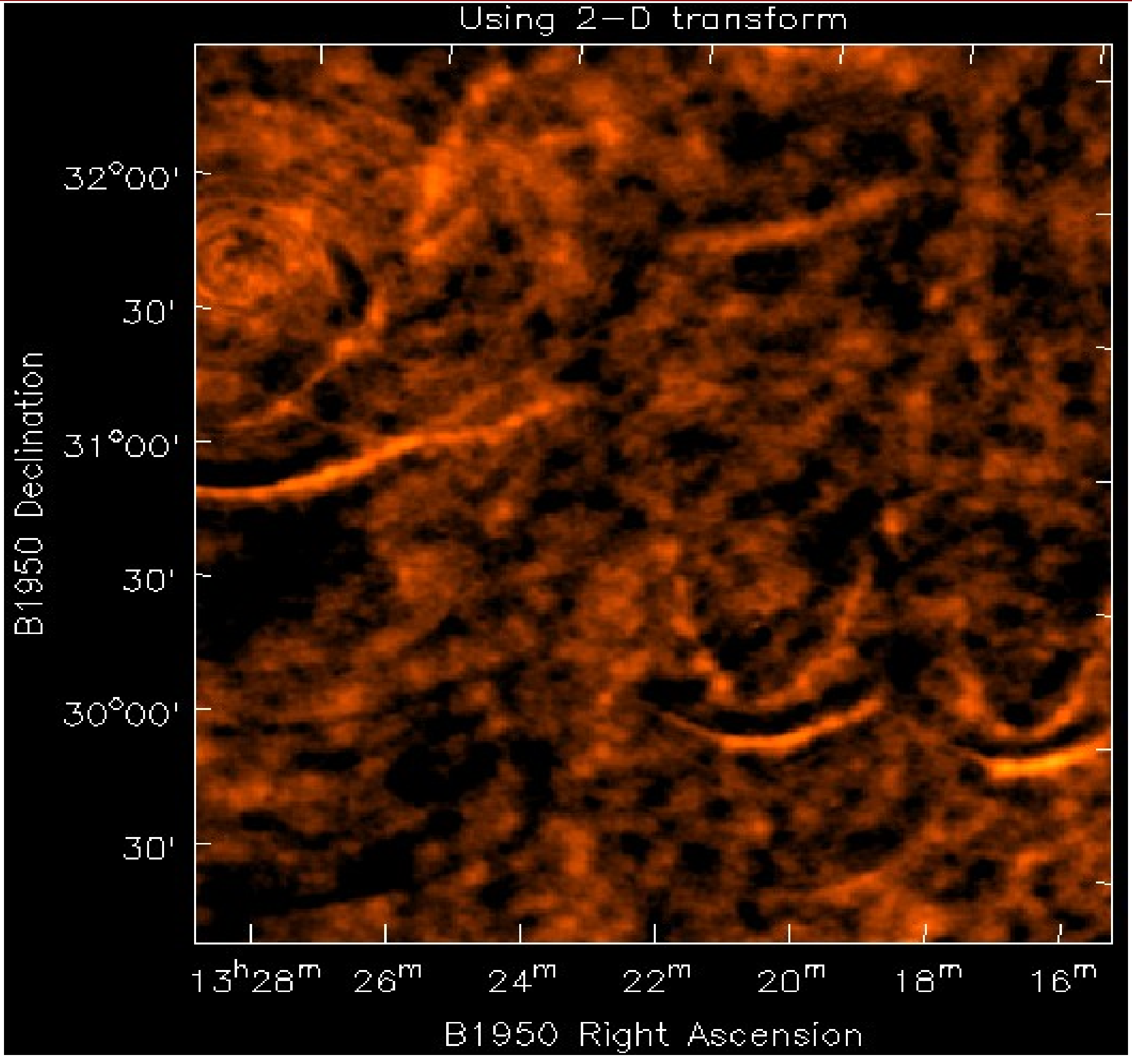
$$G(u, v, w) = Fresnel\ Propagator = FT\left[e^{2\pi\iota w\sqrt{1-l^2-m^2}}\right]$$

$$V^o_{12} = \int I(l, m)\, e^{2\pi\iota\left[u_{12} l + v_{12} m\right]}\, e^{2\pi\iota w_{12}\sqrt{1-l^2-m^2}-1}\, dl\, dm$$



- A single interferometer is sensitive to *multiple* Fourier component
- Concept of redundant baselines is more restrictive than is usually thought!

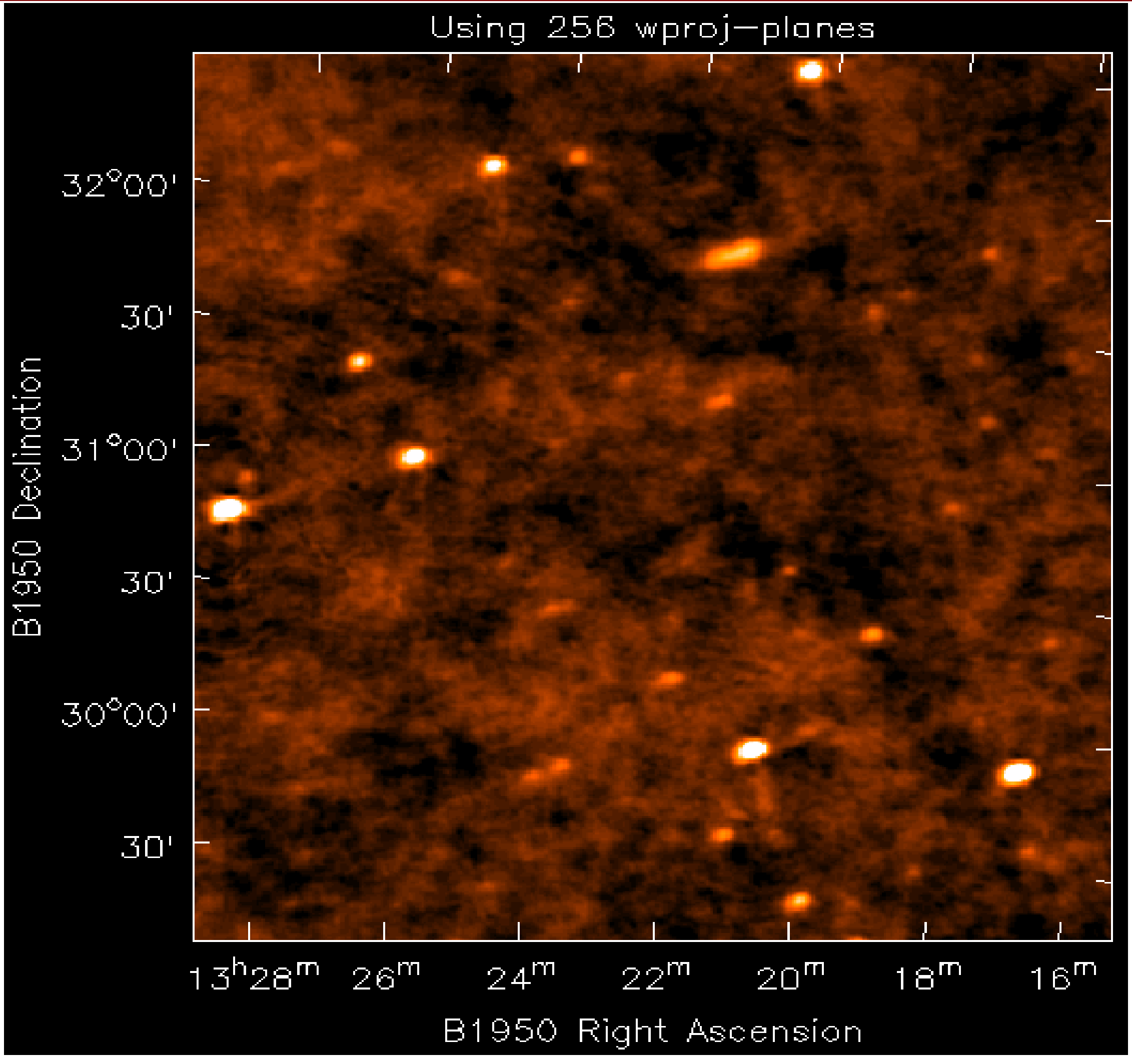
Example: No W-Term Correction



- W-term is a phase error
- Sources move in the image in a systematic way

- Hermitian but a "dispersive" effect

Example: After W-Term Correction



Solutions: 3D Imaging

- Do a 3D inversion of the ME to make a 3D “Image”

$$F(l,m,n)=\int V(u,v,w)\; e^{\iota\left[u_{ij}l+v_{ij}m+w_{ij}n\right]}\; du\, dv\, dw$$

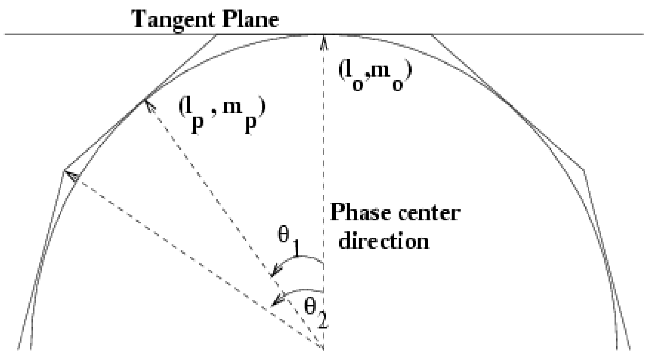
- Relate F(l,m,n) to the physical image as

$$I(l,m)=\frac{F(l,m,n)}{\sqrt{1-l^2-m^2}}\,\delta(l^2+m^2+n^2-1)$$

- Interpretation
  - Physical emission I(l,m) exists along the surface of a unit sphere inside the 3D-Image F(l,m,n)
- Resulting algorithm is not efficient
- Not used very often (read “never used” :-))

Solutions: Faceted Imaging

- Interpret *I(l,m)* as emission on the surface of Celestial Sphere of unit radius: $l^2+m^2+n^2=1$
  - Approximate the celestial sphere by a set of tangent planes – a.k.a. “facets”
  - Use 2D imaging on each facet
  - Re-project the facet-images to a single 2D plane



- Number of facets required

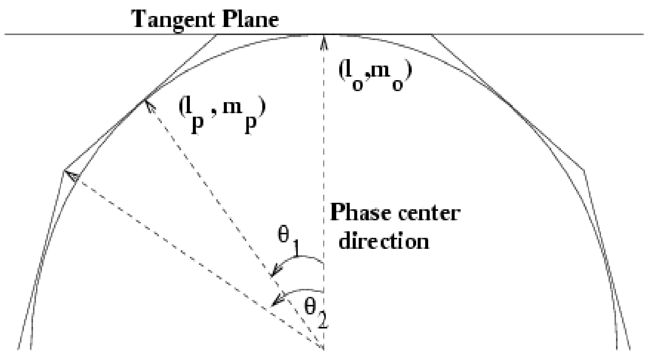
$$N_{Poly} = \theta_f^2 \frac{B_{max}}{\lambda}$$

$$= B_{max} \frac{\lambda}{D^2} \qquad D \equiv Antenna\ diameter\ ;\ \theta_f = Full\ Antenna\ Beam$$

Solutions: Faceted Imaging

- Re-phase the entire data towards the direction of each facet center (tangent point)
- Since *l^2+m^2 is small* (by construction), make a 2D image for each facet
  - *What happens when emission extends over multiple facets?*
- Re-project the facet images to a single plane

$$\begin{pmatrix} l \\ m \end{pmatrix} \approx \boldsymbol{R}_2 \begin{pmatrix} l' \\ m' \end{pmatrix} + \begin{pmatrix} l_p \\ m_p \end{pmatrix}$$



  - Intuitively easy to understand
  - Emission extending over multiple facets is an issue
  - Multiple images
  - Available in CASA and AIPS

Solutions: UV plane equivalent

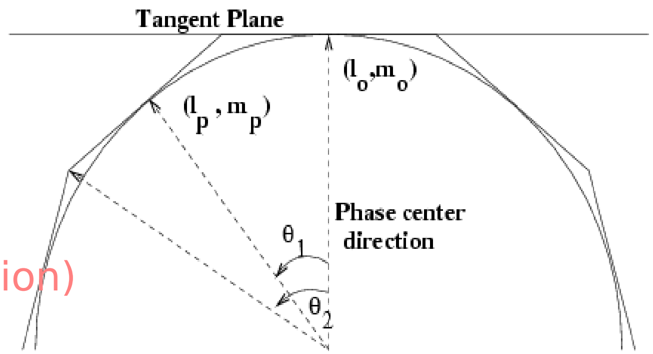
- Since the facet-images are related to the single-plane image by a linear co-ordinate transformation, there must exist a equivalent operation in the visibility plane.

$$\boldsymbol{I}(\boldsymbol{C}\,l) \rightarrow |det(\boldsymbol{C})|^{-1}\,\boldsymbol{V}(\boldsymbol{C}^{-1^T}u)$$

where ***C*** is the image domain co-ordinate transform

l and *u* are the image and visibility plane co-ordinates

- Projection error: $\epsilon = \sin(\theta_1)(1-\cos(\theta_2)) \approx \frac{1}{2}\theta_1\theta_2^2$
  - Error same as in image plane faceting!
  - Produces a single image (no edge effects)
  - Global deconvolution possible (extended emission)
  - Use of advanced algorithms for extended emission possible
  - Region definition as in the usual case

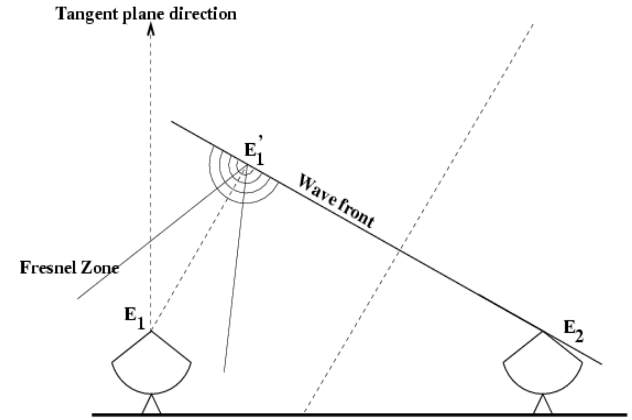


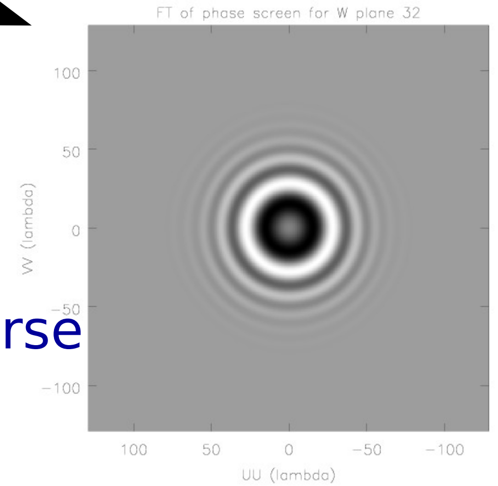
Available in CASA and possibly in AIPS

Solutions: W-Projection

- Optics interpretation
  - $V^o_{12} = \int I(l,m) e^{2\pi\iota[u_{12}l + v_{12}m]} e^{2\pi\iota w_{12}\sqrt{1-l^2-m^2}-1} \, dl\, dm$
  - $$\boxed{\begin{gathered} V^o(u,v,w) = V(u,v,w=0) * G(u,v,w), where \\ G(u,v,w) = FT\left[e^{2\pi\iota w\sqrt{1-l^2-m^2}}\right] \end{gathered}}$$
  - Size of the Fresnel Zone:

$$\frac{r_f}{\lambda} \approx \sqrt{w}$$

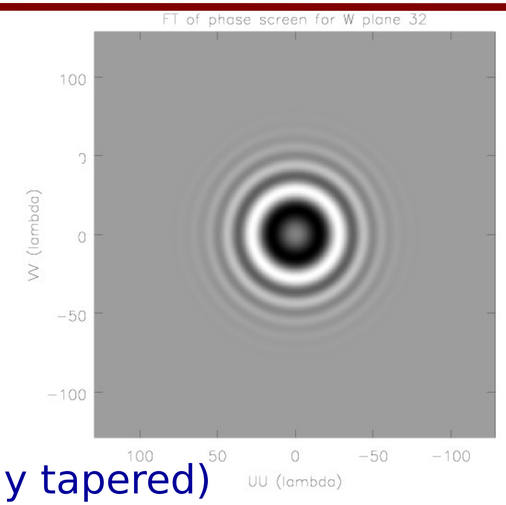




- If $G^T G$ is unitary (or even approximately so)

  G^T and G can be used in the forward and inverse transforms to produce distortion free images

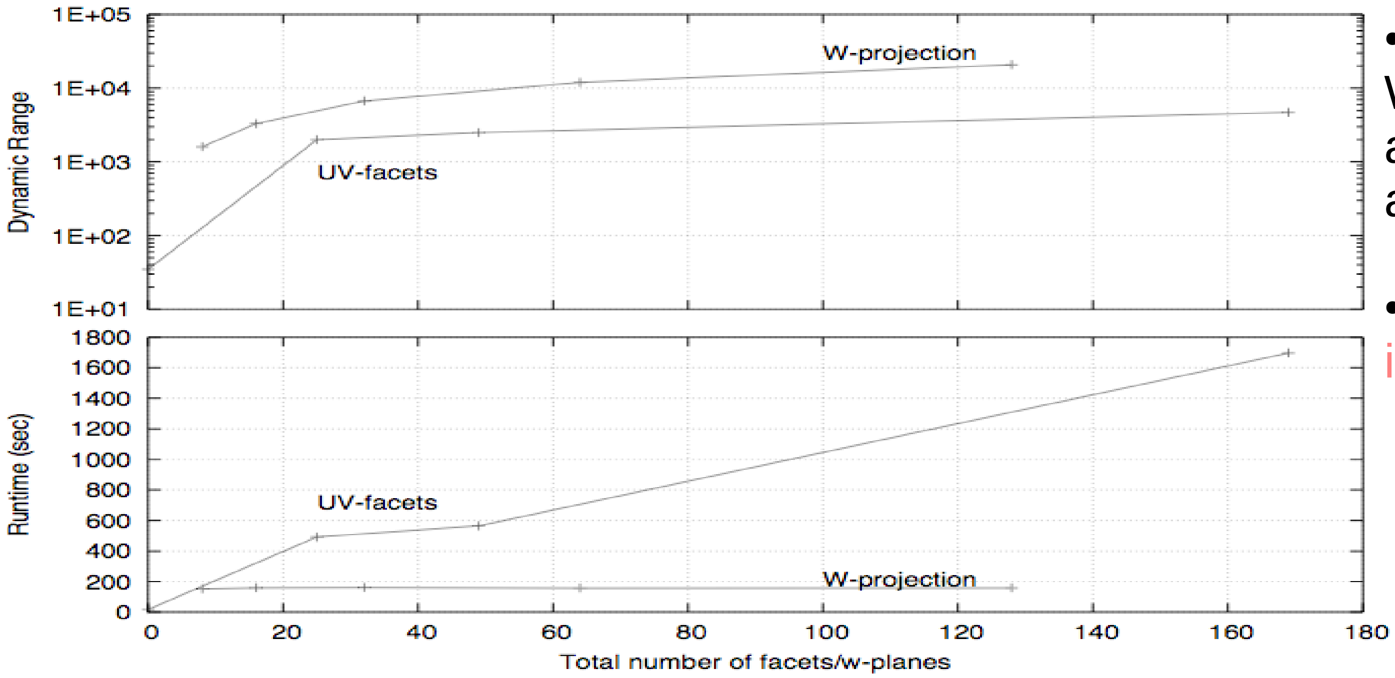
The W-Projection Algorithm



- Optics interpretation
  - $V^o(u,v,w) = V(u,v,w=0) * G(u,v,w)$
- Algorithm
  - Model prediction (major cycle) [Residual computation]
    - Perform a 2D FFT of the model image (appropriately tapered) (this is $V(u,v,w=0)$)
    - Evaluate the above convolution equation as part of Gridding to get $V^o(u,v,w)$
  - Compute the Dirty Image (minor cycle) [Deconvolution]
    - Use $G^T(u,v,w)$ on each $V^o(u,v,w)$ during gridding to evaluate $V(u,v)$
    - Perform a 2D FFT^{-1} of $V(u,v)$

W-Projection: Performance

- Scaling laws:
  - Facet imaging: $(N^2_{Facets} \; N^2_{GCF})N_{vis}$
  - W-Projection: $(N^2_{WPlanes} + N^2_{GCF})N_{vis}$
  - Ratio: $\approx N^2_{GCF}$ *for large number of facets/WPlanes*

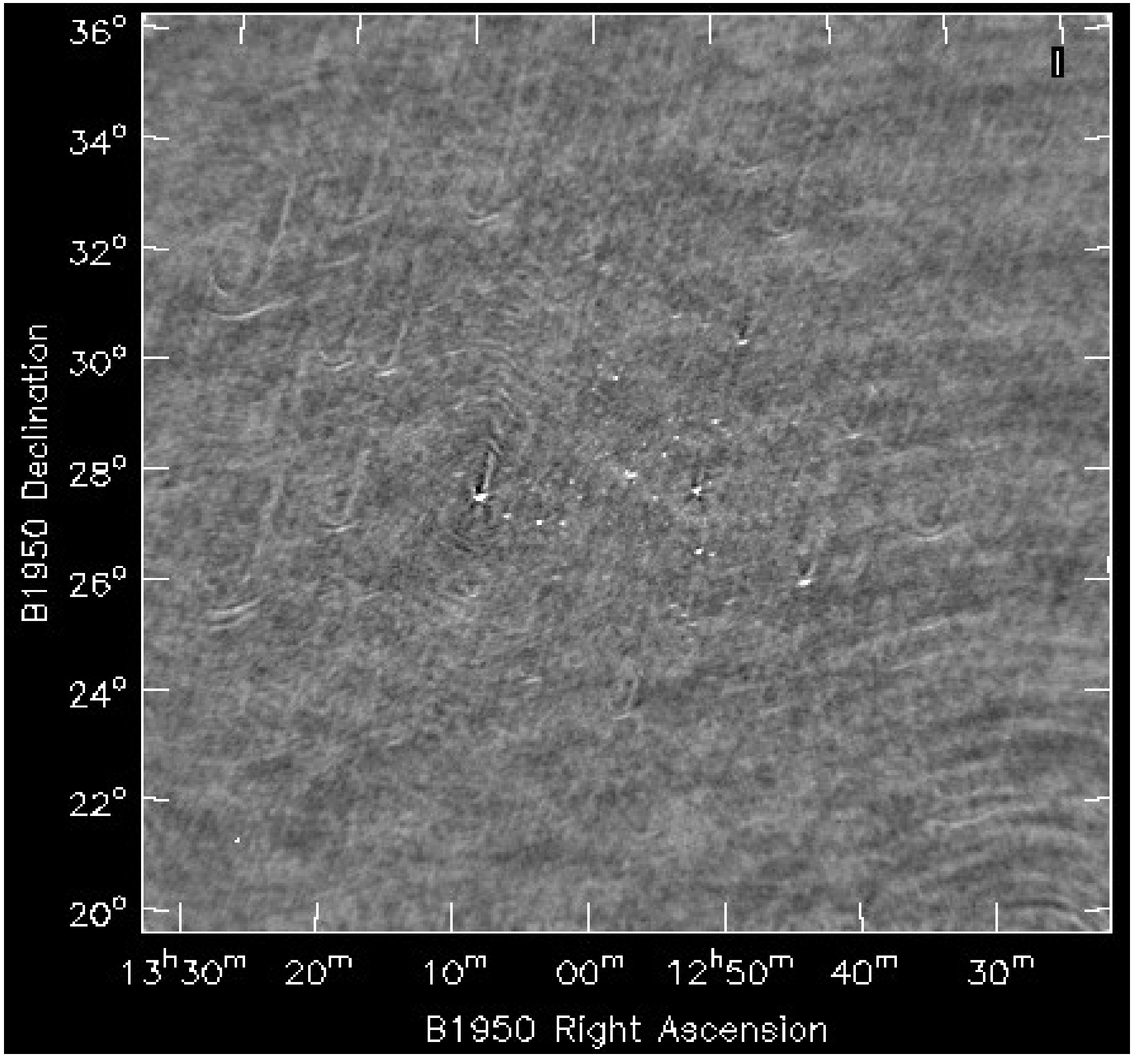


- In practice WProjection algorithm is about 10x faster
- Size of G(u,v,w) increases with W

W-Projection: Performance

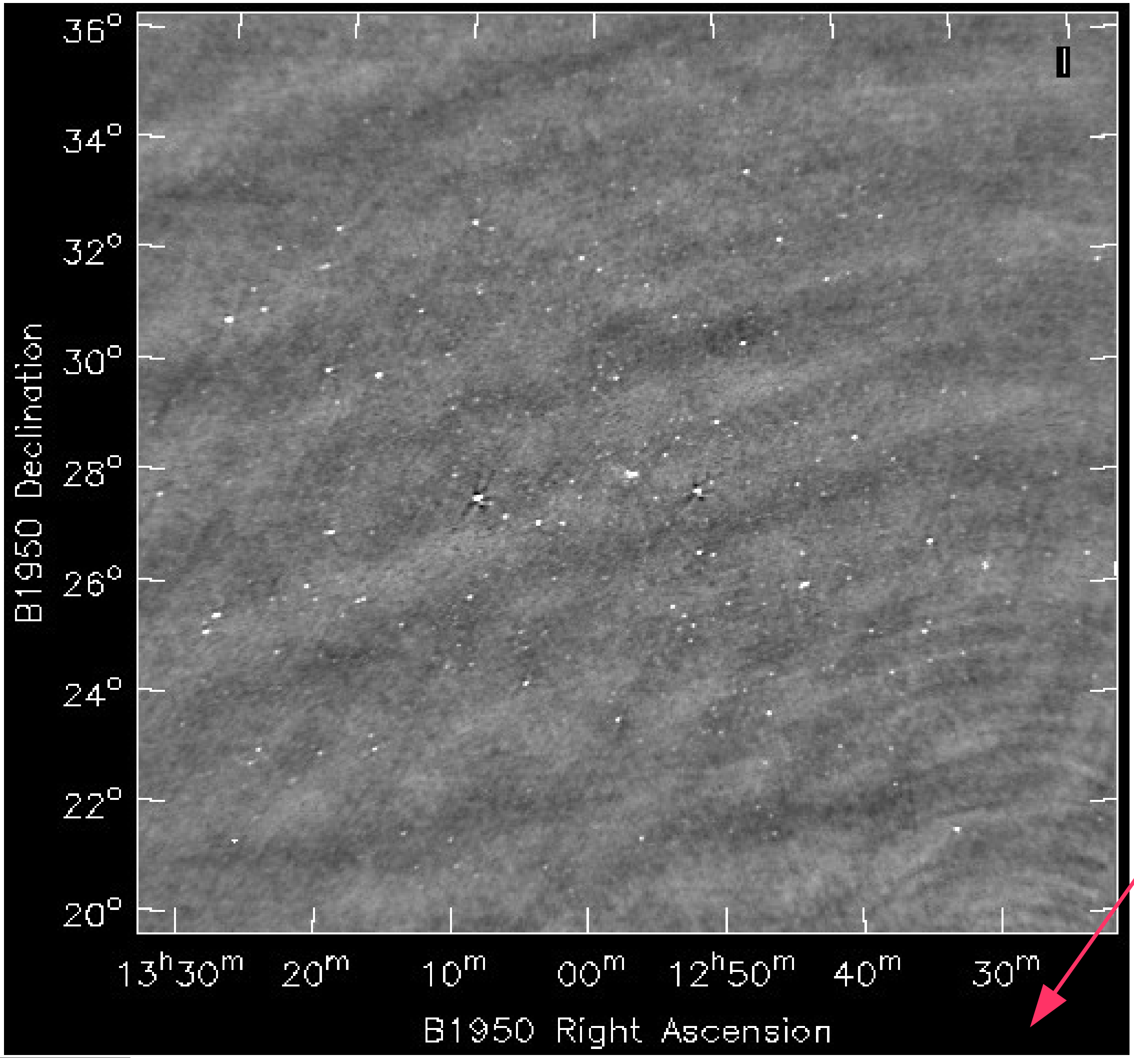
- WProjection about 10x faster than facet based imaging
- Algorithm complexity is significantly lower
  - This is more important than is realized!
- Users practically see no difference between “wide field” imaging and normal imaging
- Fits in the general mathematical framework of advanced imaging techniques
  - Works naturally with any minor cycle algorithm (Hogobm-, Clark-, CS-, MS-, Asp-Clean, etc.)
  - Naturally integrates with full-beam imaging/calibration
  - Naturally includes multi-field imaging
- May not be useful for full-sky imaging telescopes with long baselines
  - But can be combined with faceted imaging (implemented in CASA)

Examples: 74MHz, before correction



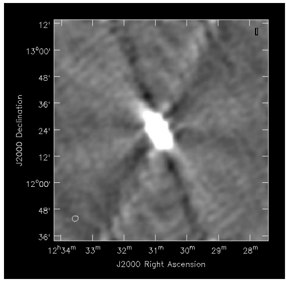
Courtesy:
K. Golap

Examples: WProjection imaging



Courtesy:
K. Golap

Sub-image of
an “outlier” field



A small digression

- Linear optimization view of deconvolution

$$\boldsymbol{V^o = A\, I^o + N}$$

$$\boldsymbol{V^M = A\, I^M}$$

- ***N*** is Gaussian Random Variable – in the data domain
- χ^2 Is the optimal estimator. Deconvolution is then equivalent to

minimize: $\chi^2 = |\boldsymbol{V^o} - A I^M|^2$ *where* $I^M = \sum_k P_k$; P_k is the Pixel Model

$$\boxed{\frac{\partial \chi^2}{\partial\, \text{Pixel Model}} \equiv \text{Dirty Image}}$$

$$\boxed{I_i^M = I_{i-1}^M + \alpha\, \Delta\, \chi^2}$$

- Various algorithms differ in (1) parametrization of P_k, (2) types of constraints, and (3) how the constraints are applied

A small digression: Projection methods

- ME entirely in the visibility domain: $V_{ij}^{Obs} = E_{ij}\left[V^o\right]$

  where E_{ij} represents a direction dependent (DD) effect

- Construct a K_{ij} *which models the desired DD effect*

- If $K^T_{ij} E_{ij} \sim \mathbf{1}$ (Unitary Operator), compute update direction (Dirty Image) as

$$FT\left[K_{ij}^T V_{ij}^{Res}\right] \longrightarrow I^{Dirty} = I^o * PSF$$

Deconvolution: Minor Cycle

- Accurate residual computation (Chisq) as

$$V_{ij}^{Res} = K_{ij} FT^{-1}\left[I^M\right] - V_{ij}^{Obs}$$

Deconvolution: Major Cycle

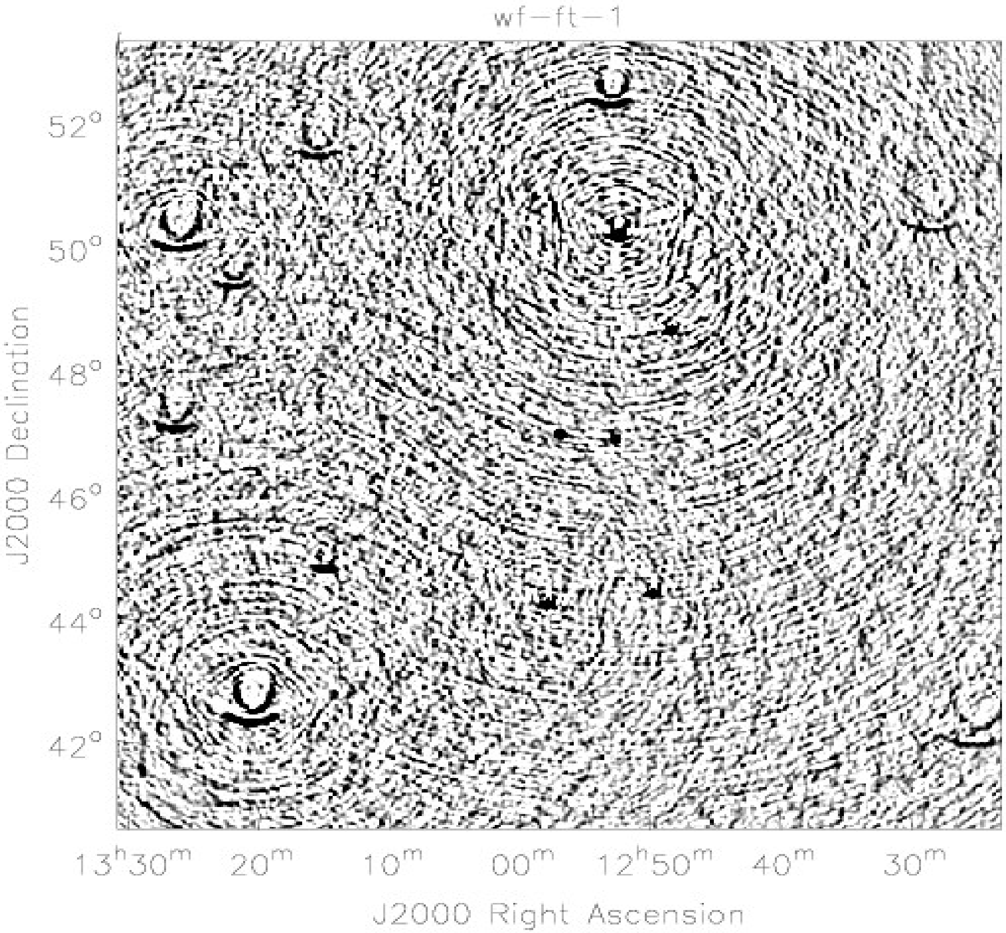
- Iterations will converge – if the operator is approximately unitary

Image domain vs. data domain

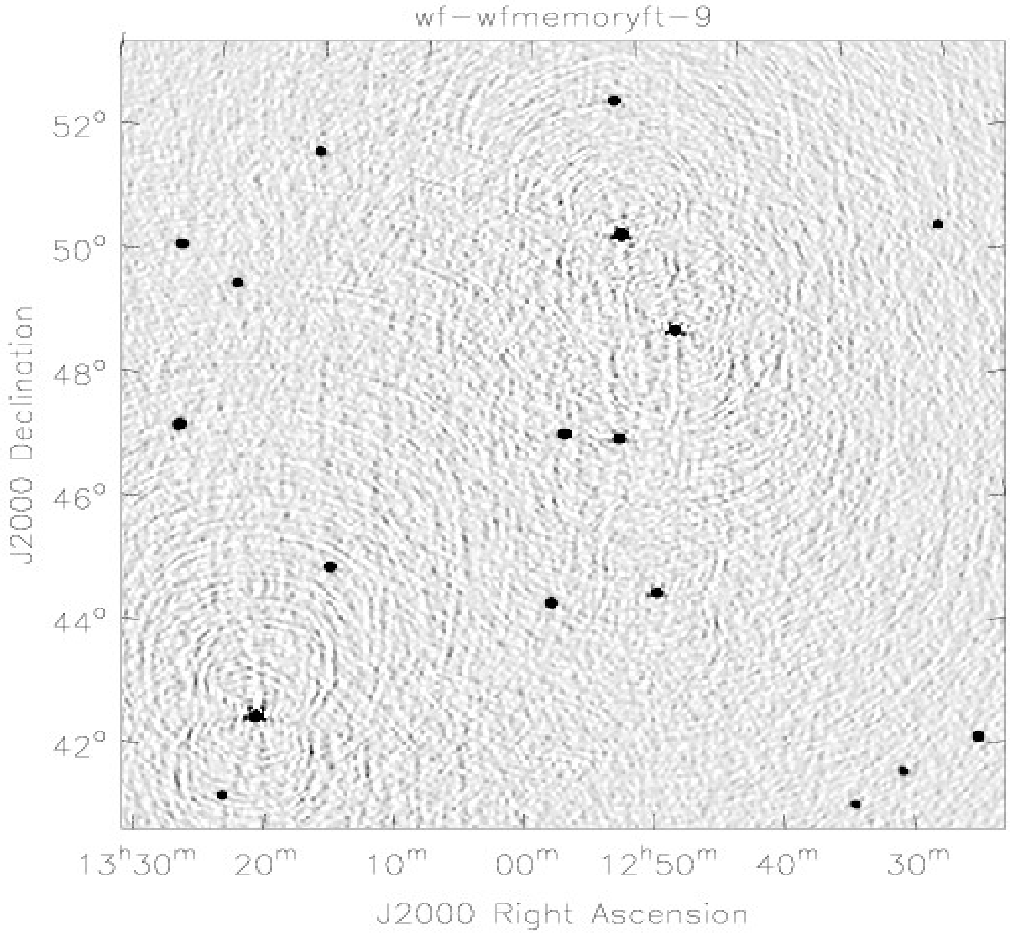
- Projection methods utilize the available data optimally
  - Non-linear operations (e.g. deconvolution) are done on images constructed using the entire data
  - Errors are corrected using parametrized models
  - Global corrections rather than local correction

- Since DD effects are typically also time varying, image domain based correction are non-optimal
  - Necessarily require data partitioning
  - Signal-to-noise available to solvers corresponds to a fraction of the total data
  - Require many more DoF: At least one per direction of interest
  - Local corrections ==> Could lead to “Closure errors”

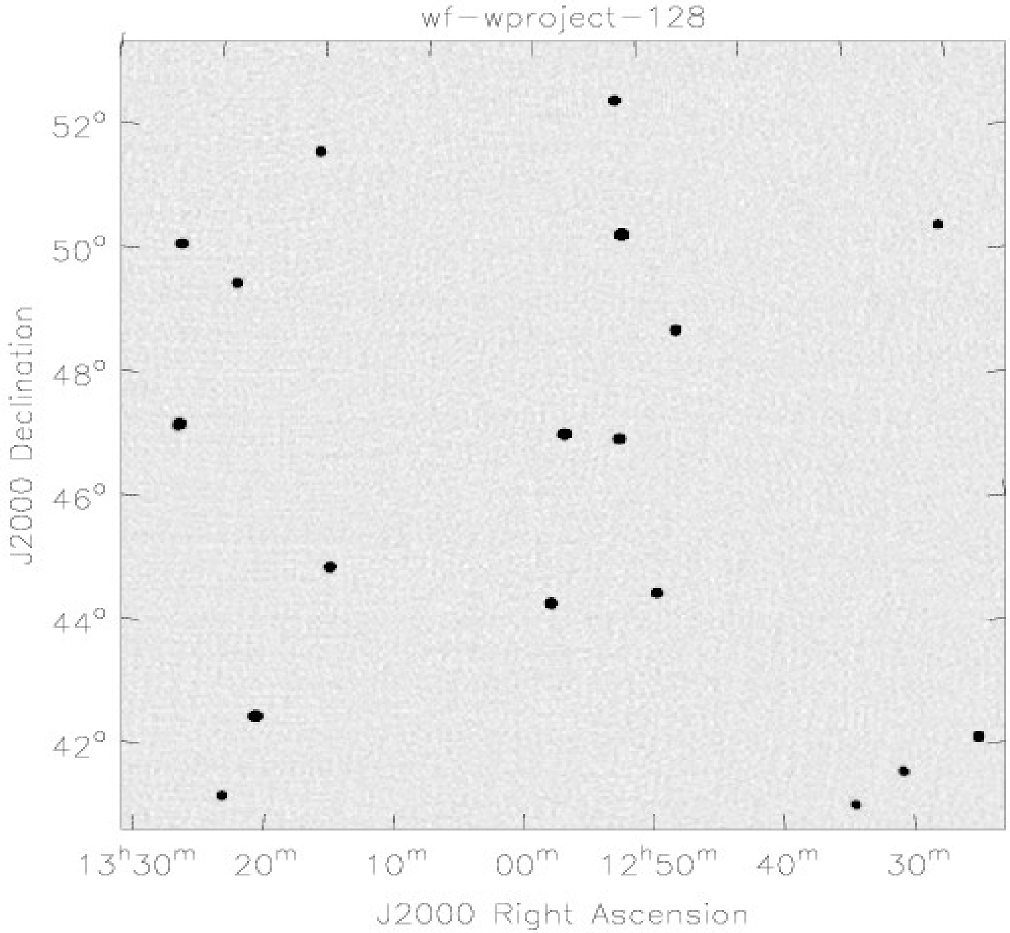
Examples: 2D Imaging



Examples: Facet Imaging



Examples: WProjection Imaging

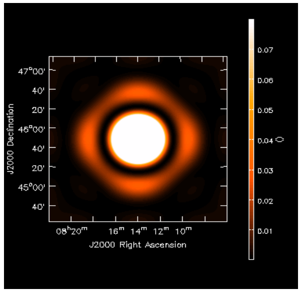


Full beam imaging

- Now ignore J_{ij} (i.e., using calibrated data). Ignore also the w-term for the moment:

$$V_{ij}^{Obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\; I(s,\nu)\; e^{\iota\left(u_{ij}l + v_{ij}m\right)}\; ds$$

- Some observations:
  - J_{ij}^{S} is direction dependent: complex gain potentially different for different direction in the sky

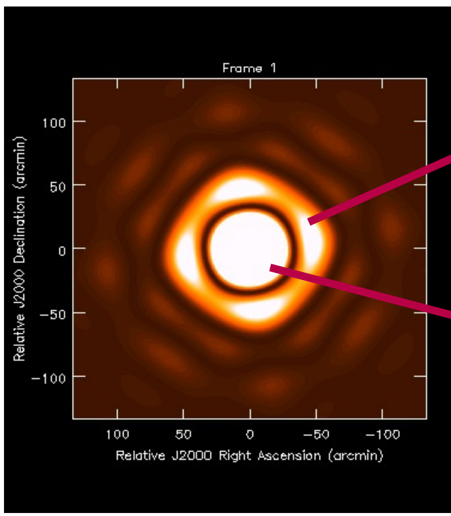


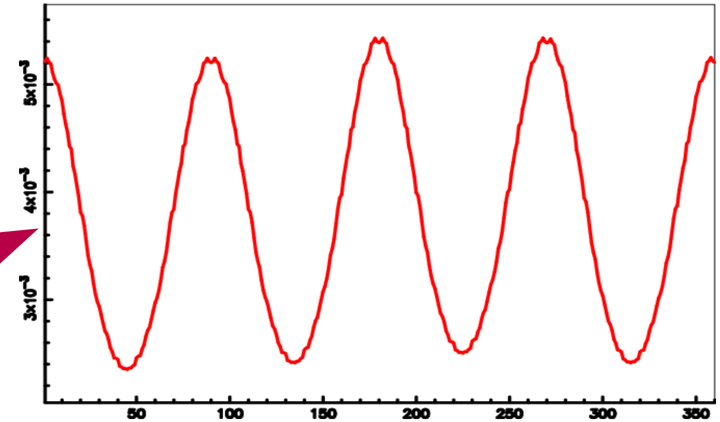
  - $J_{ij}^{S} = J_{i}^{S} \otimes J_{j}^{S*}$: This is true for most instrumental, atmospheric /ionospheric corruptions (all effects that obey "closure relationship")
  - When $J_{i}^{S} = J_{j}^{S}$ and stationary in time (e.g. PB of ideal, identical antennas), its effects can be corrected in the image domain

$$\frac{I^{Obs}}{J^{s}(s,\nu)} = I(s,\nu)$$

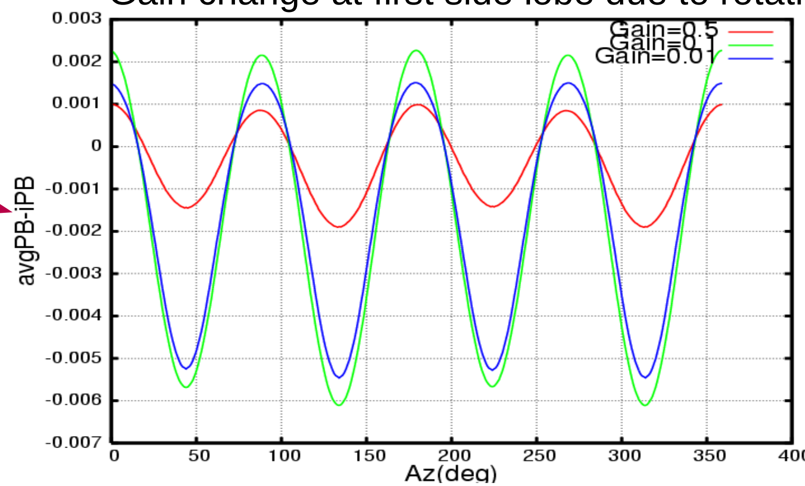
More observations...

- J^s_i are in general complex (complex Primary Beams!)
- In real life, $J^S_i \neq J^S_j$
- In real life, J^s_i vary with time...





Gain change at first side lobe due to rotation

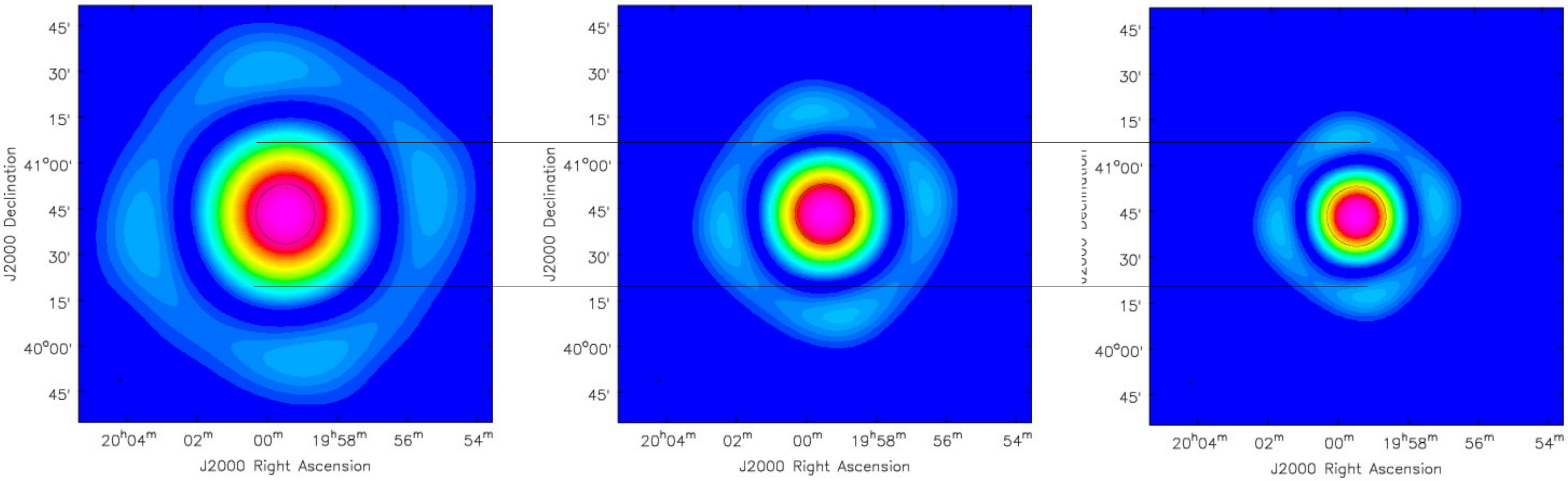


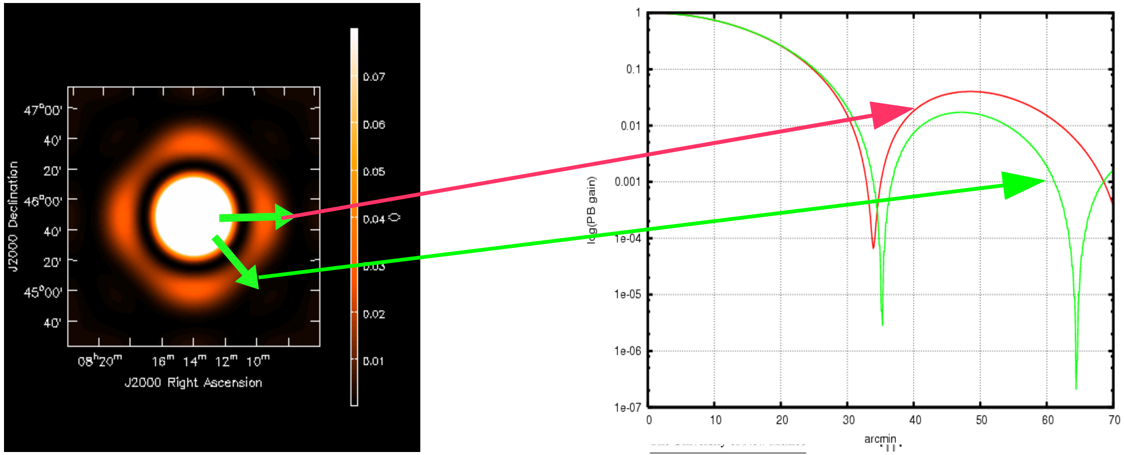
Source of time variability

- Pointing errors
- Geometric distortions

Frequency dependence

- ... J^s_i vary with frequency...
  - To the first order, PBs scale with frequency

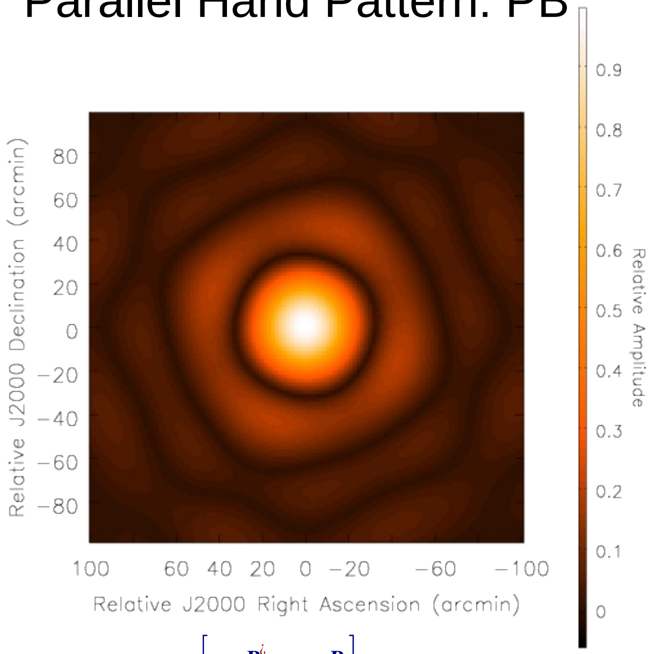




Polarization dependence

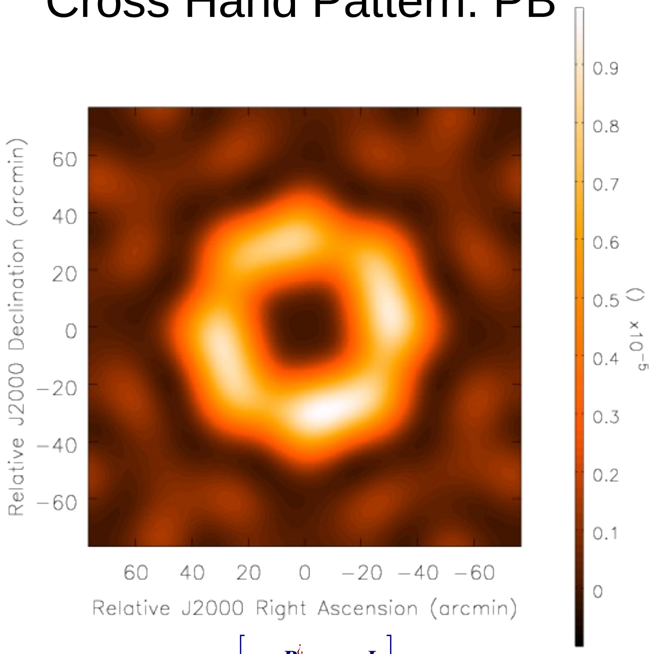
- ... $J^s{}_i$ vary with polarization

Parallel Hand Pattern: PB^{RR}



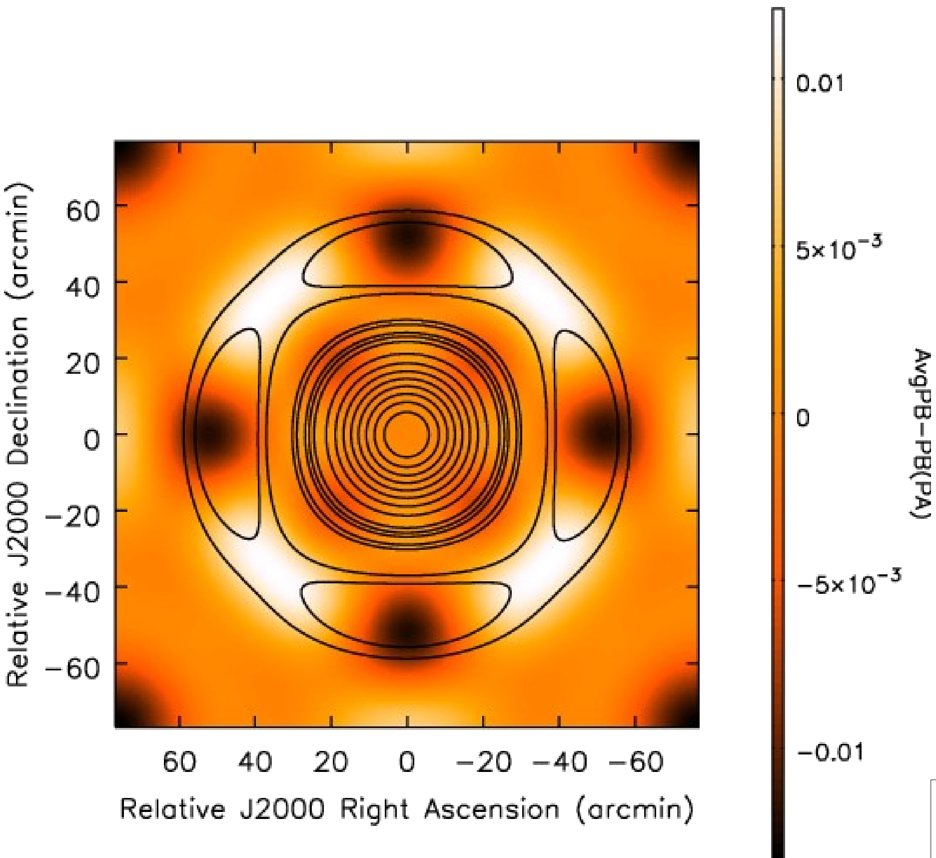
$$FT\left[E^{R*} * E^{R}\right]$$

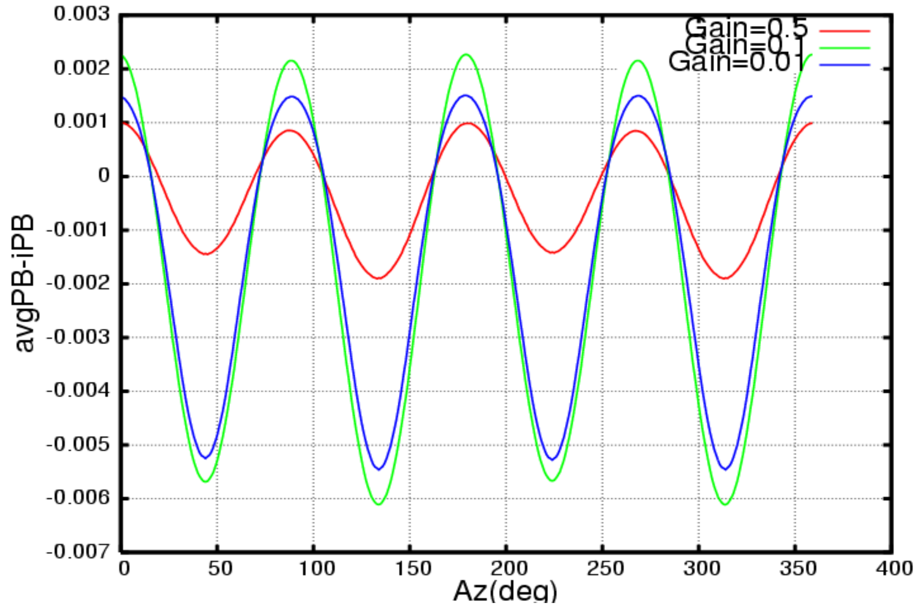
Cross Hand Pattern: PB^{RL}



$$FT\left[E^{R*} * E^{L}\right]$$

Error propagation

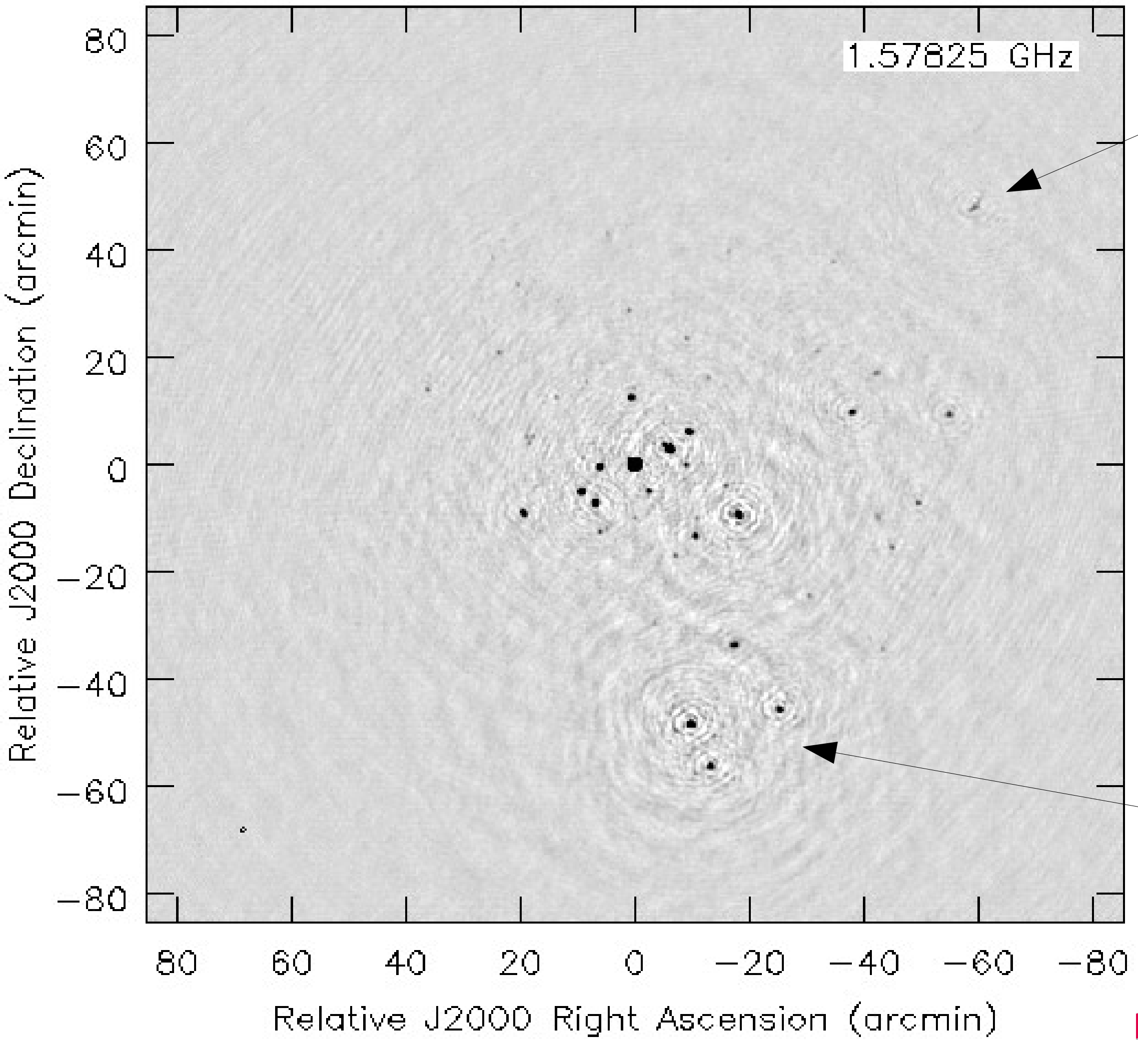




Azimuthal cuts at 50%, 10% and 1% of the Stokes-I error pattern AvgPB - PB(t_o)

$$\Delta I^R = \sum_\psi PSF(\psi) * \left[\delta PB(\psi) I^o\right]$$

Examples: Stokes-I and -V imaging

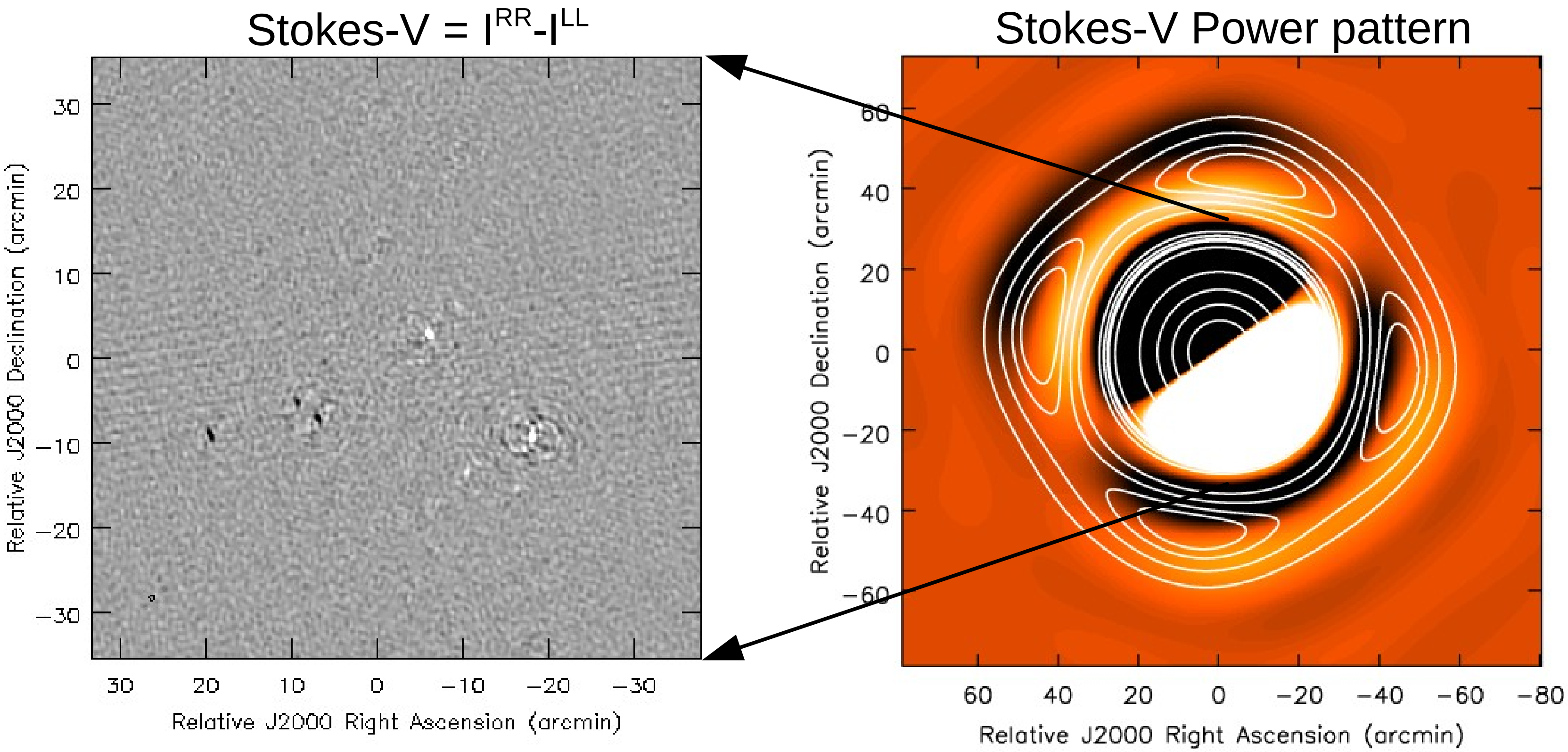


W-Term errors!

- 3C147, EVLA, L-Band
- High DR "700,000:1"

Errors due PB side-lobes?

Examples: Stokes-V



Theory: Full-beam imaging

- Re-cap – the simplified ME

$$V_{ij}^{Obs}(\nu) = \int J_{ij}^{S}(s,\nu,t)\ \ I(s,\nu)\ \ e^{\iota\left(u_{ij}l + v_{ij}m\right)}\ \ ds \qquad J_{ij}^{S} = J_{i}^{S} \otimes J_{j}^{S*}$$

- Re-write it as

$$V_{ij}^{Obs} = E_{ij} * \left[V\right] = E_{i}^{*} * E_{j} * \left[V^{o}\right]$$

E_i: *Antenna Aperture Illumination Patter*

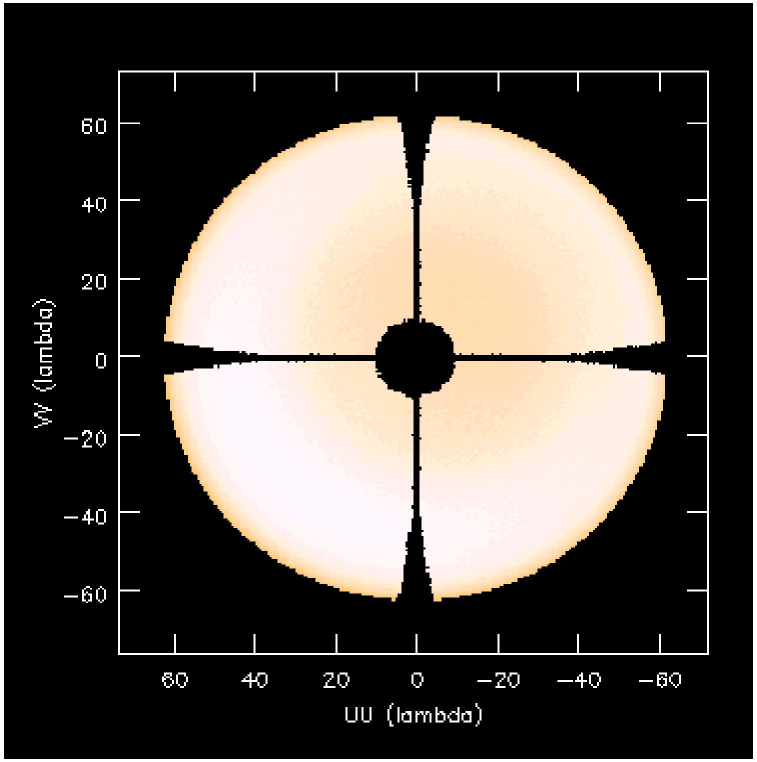
$E_i = FT\left[J_i^s\right]$

V^{obs} is equal to true visibilities convolved with the auto-correlation of antenna Aperture Illumination pattern.

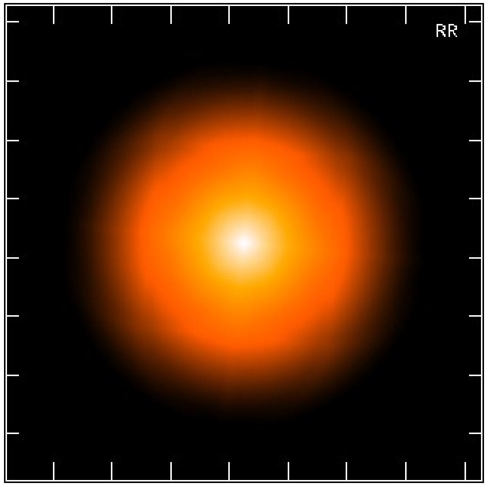
- If there exists a function K_{ij} such that $K^{T}_{ij} * E_{ij}$ ~ *Delta Function*
  - Gridding: $V_{ij}^{G} = K_{ij}^{T} * V_{ij}^{Obs} \quad = \quad K_{ij}^{T} * E_{ij} * \left[V^{o}\right] \quad \approx \quad \left[V^{o}\right]_{ij}$
  - Imaging: $FFT\left[V^{o}\right] \rightarrow I^{d}$
  - Prediction: $V_{ij}^{M} = K_{ij} * FFT\left[I^{M}\right]$

Solution: A-Projection algorithm

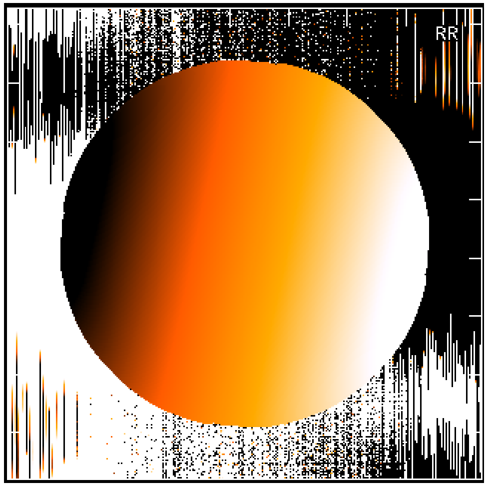
$\Re(K_i^R)$: *Antenna Aperture Illumination Patter*



$|K_{ij}|$

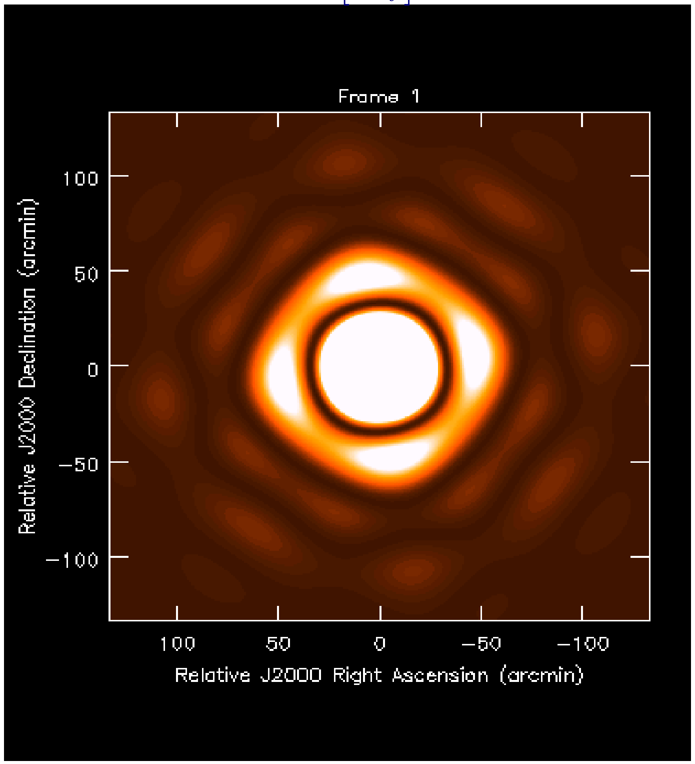


$phase(K_{ij})$

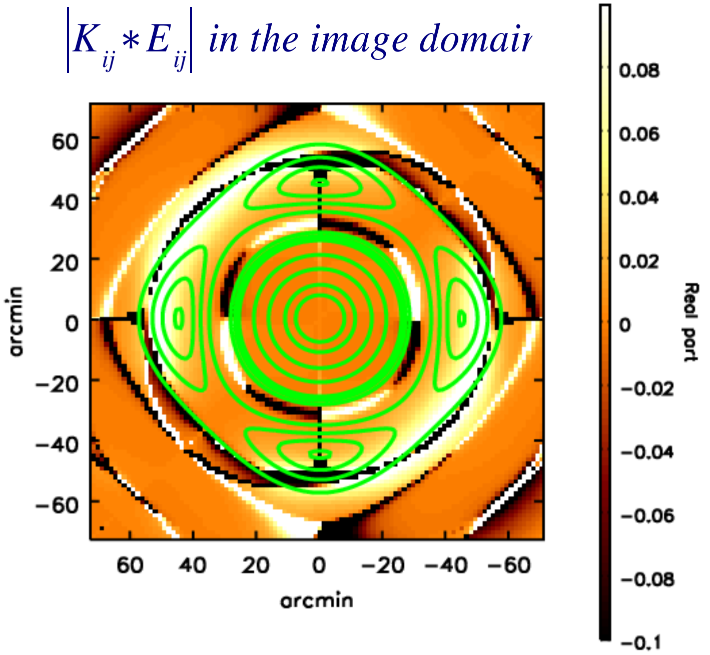


Solution: A-Projection algorithm

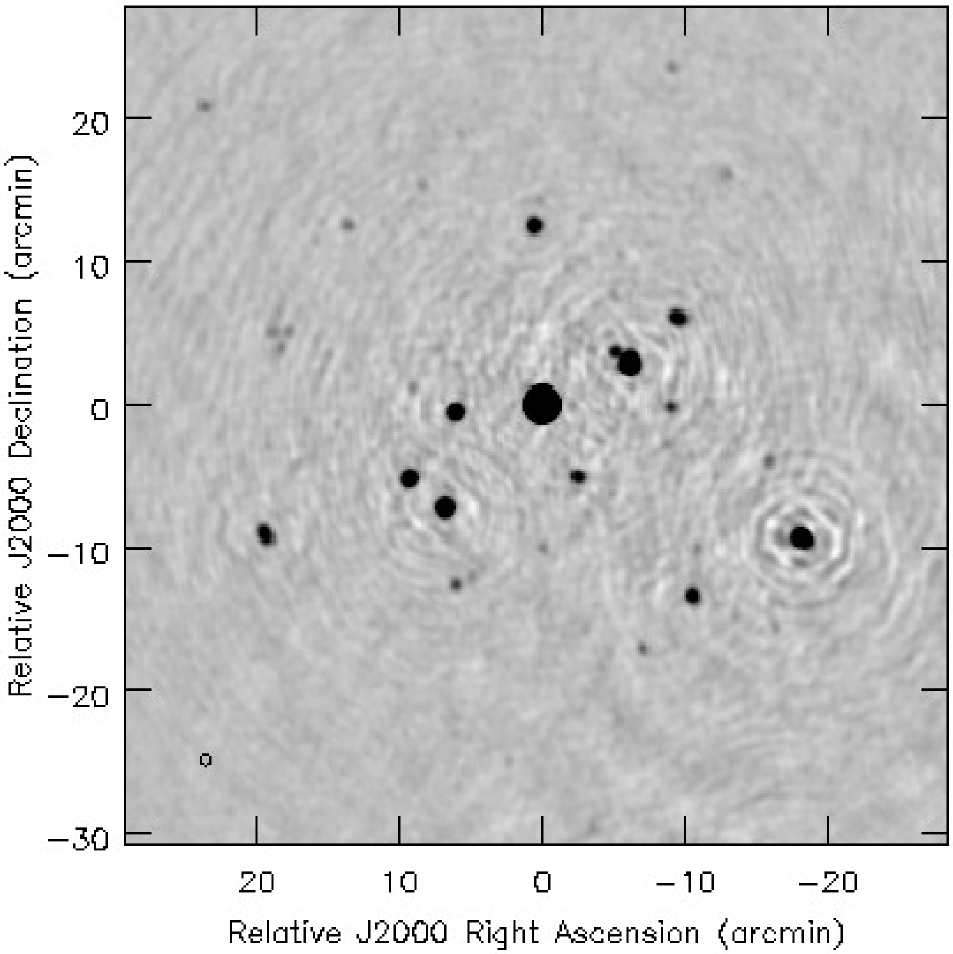
$FT\left[K_{ij}\right]$



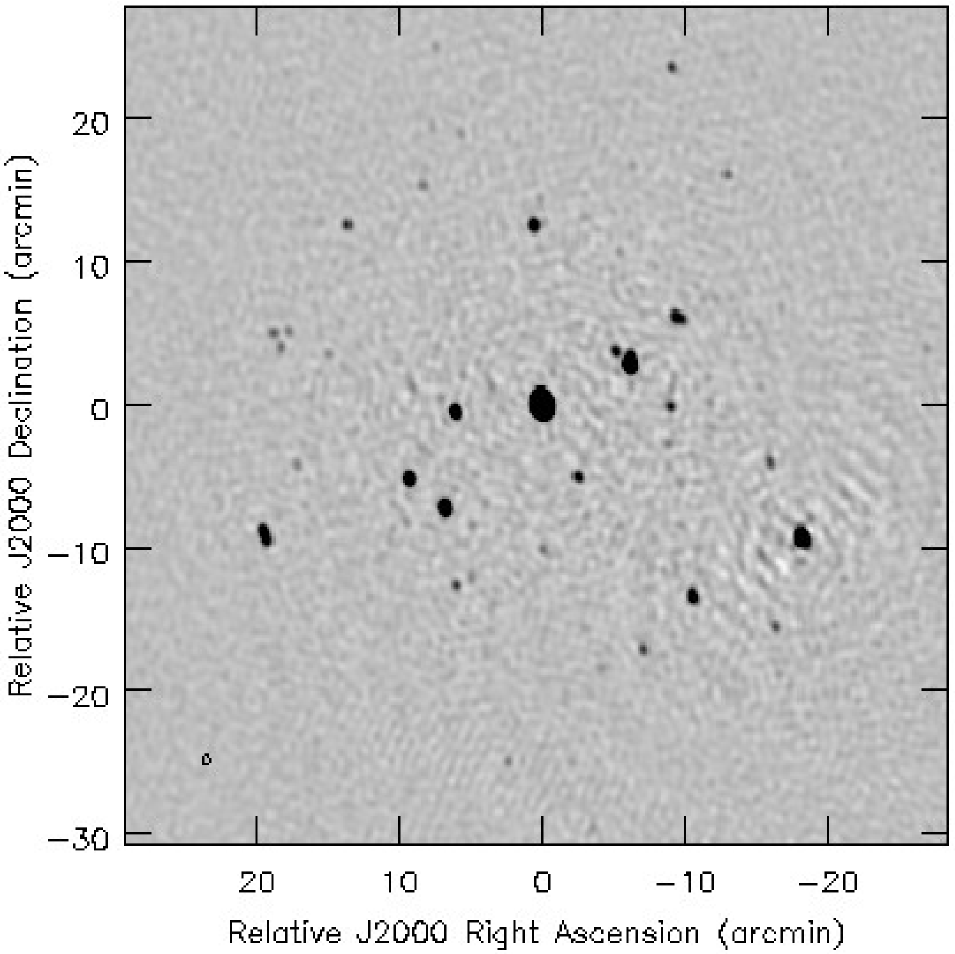
$\left|K_{ij} * E_{ij}\right|$ *in the image domain*



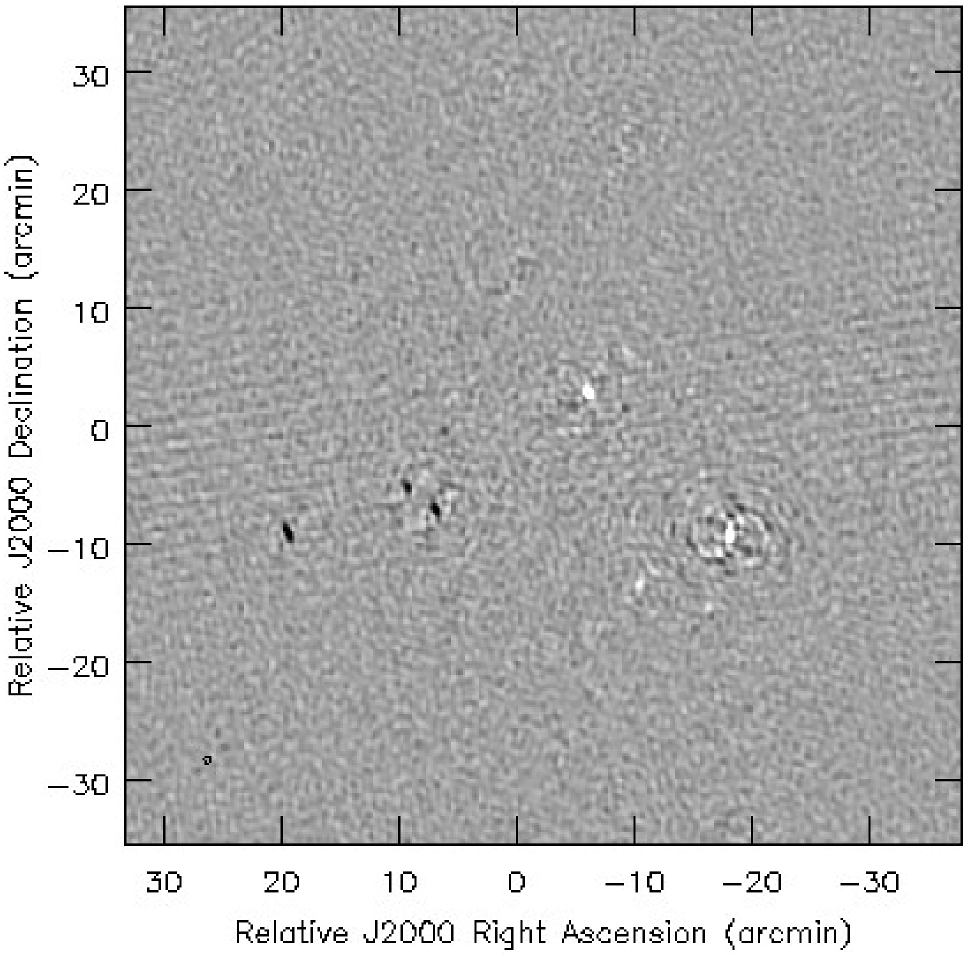
Stokes-I: Before correction



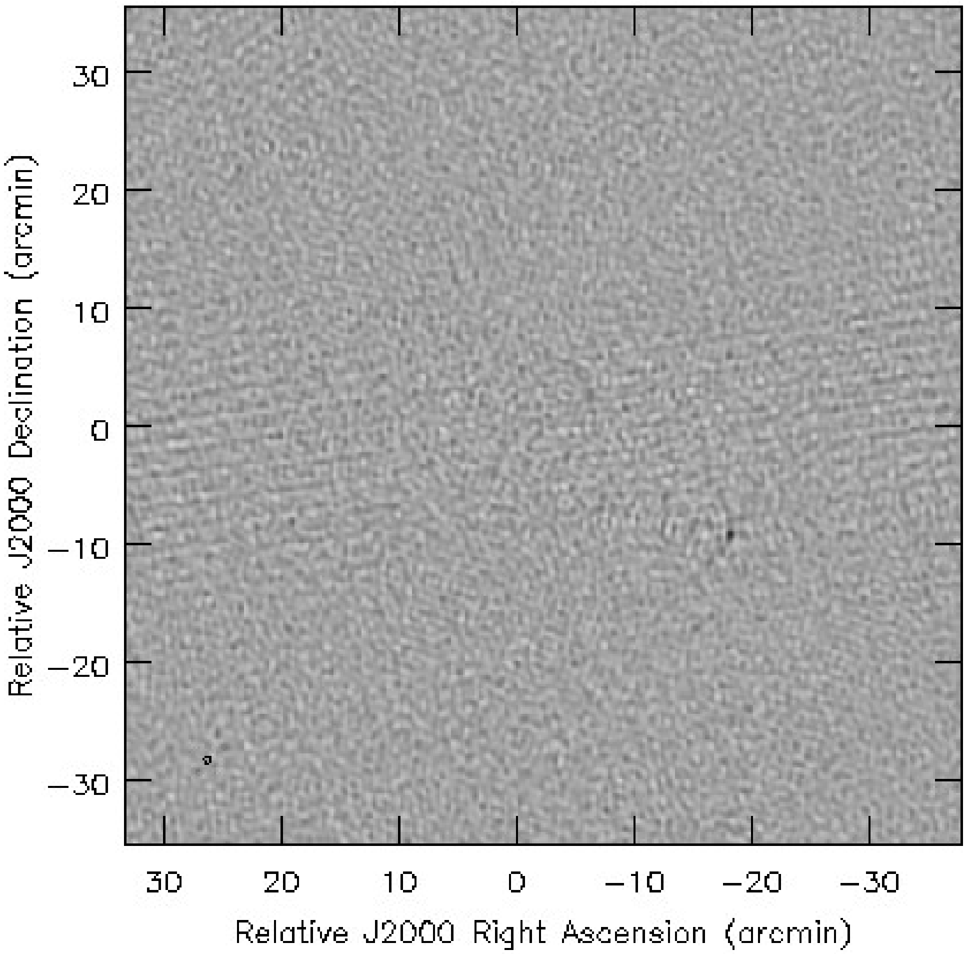
Stokes-I: After correction



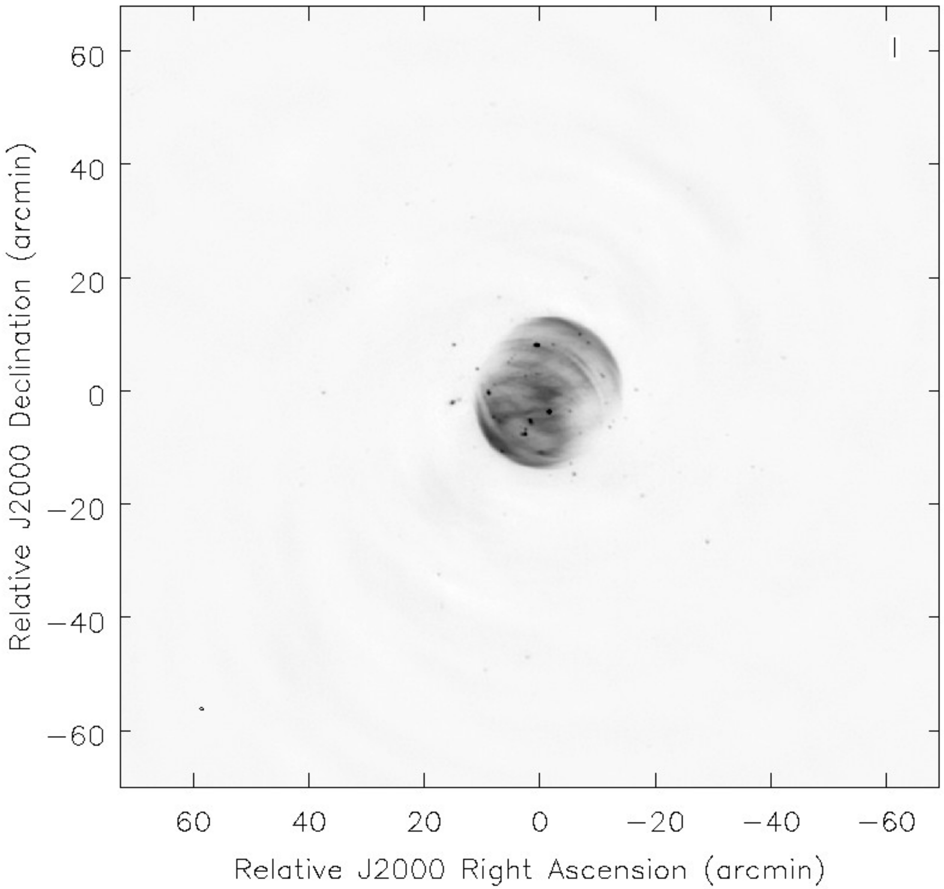
Stokes-V: Before correction

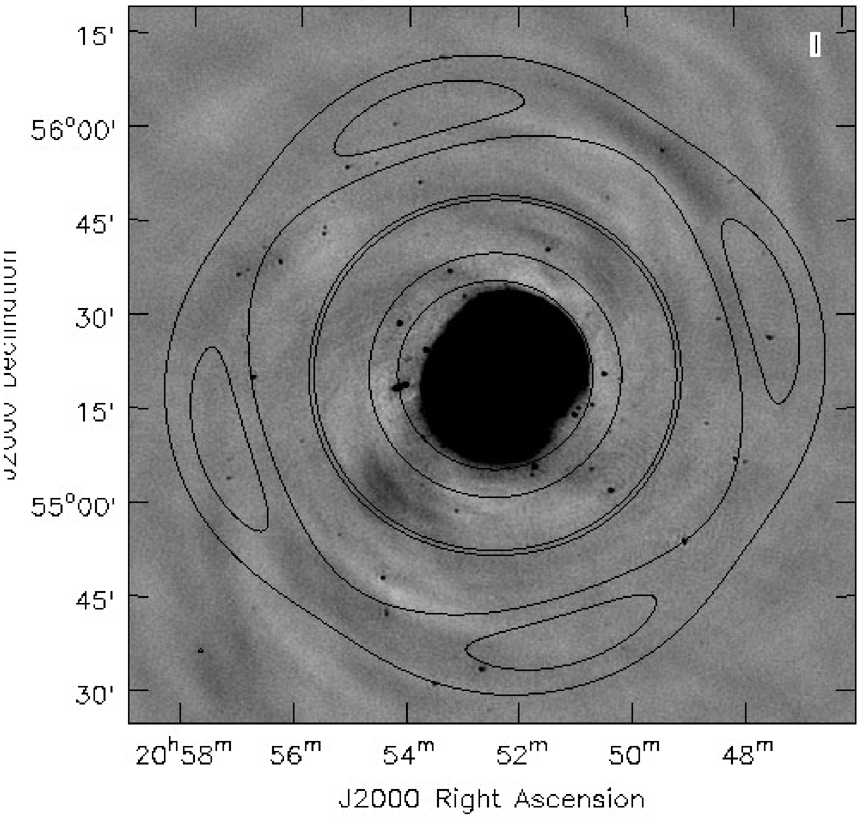


Stokes-V: After correction

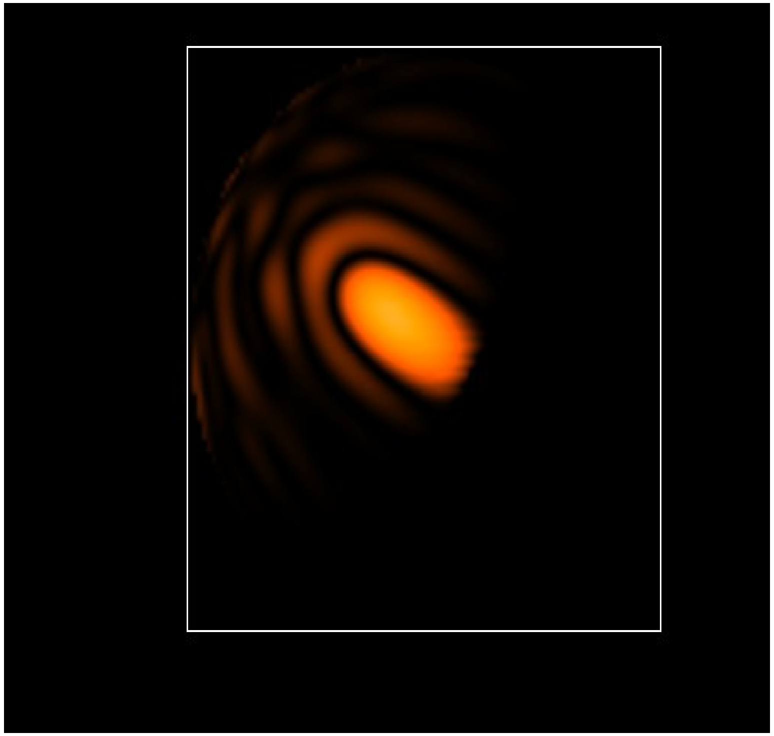


Examples: EVLA Imaging

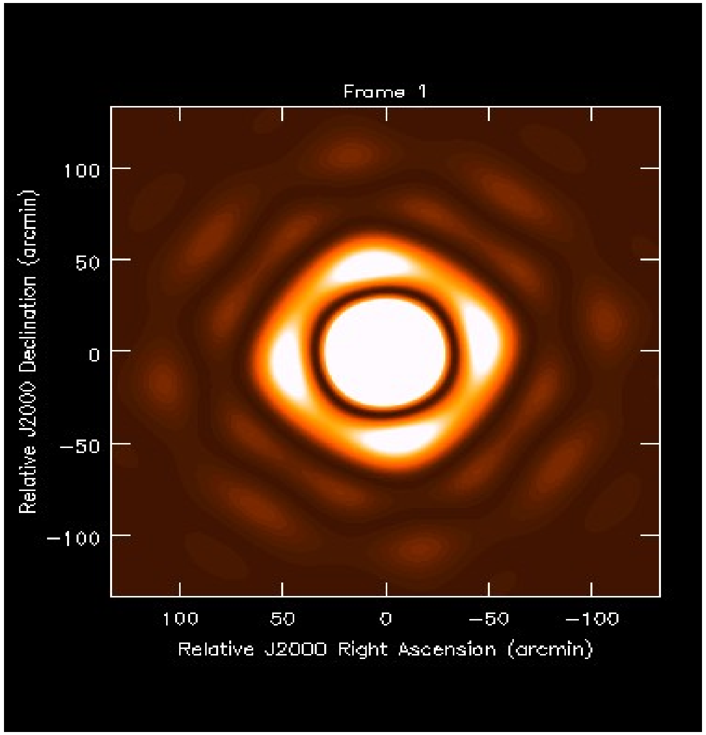




Examples: Time varying PBs



Simulations for LWA @50MHz
(Masaya Kuniyoshi (LWA/NRAO))



Model for EVLA PB at L-Band

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