VLBI Astrometry

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Harvard-Smithsonian CfA

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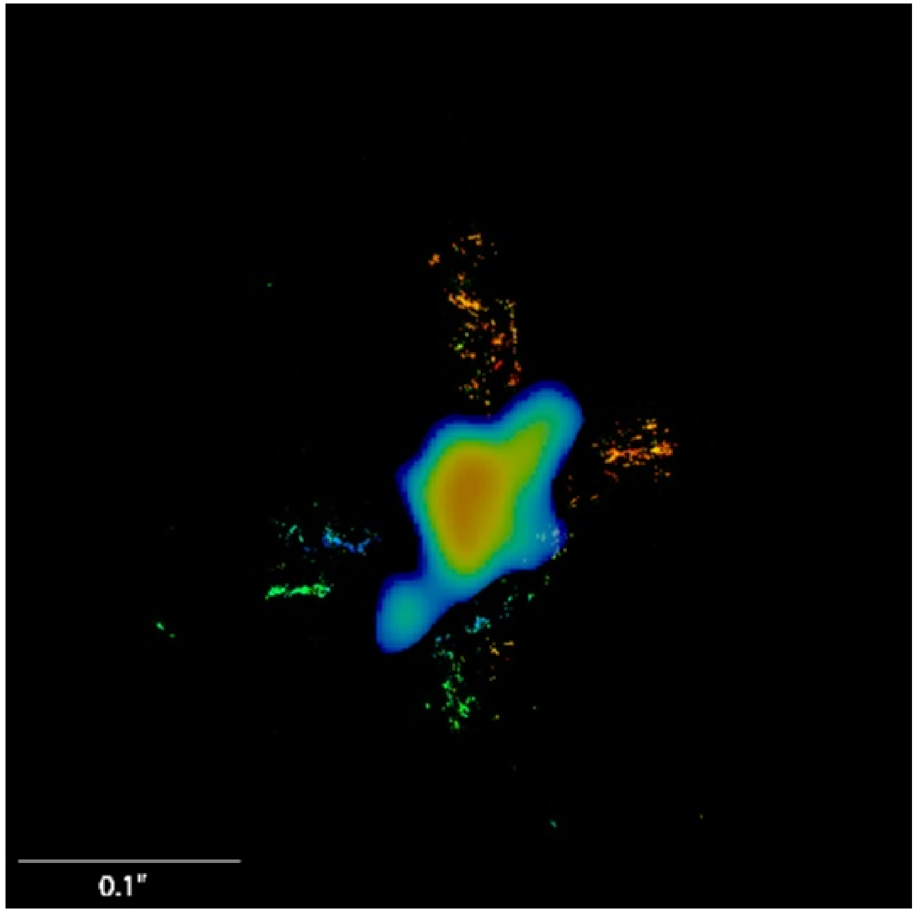
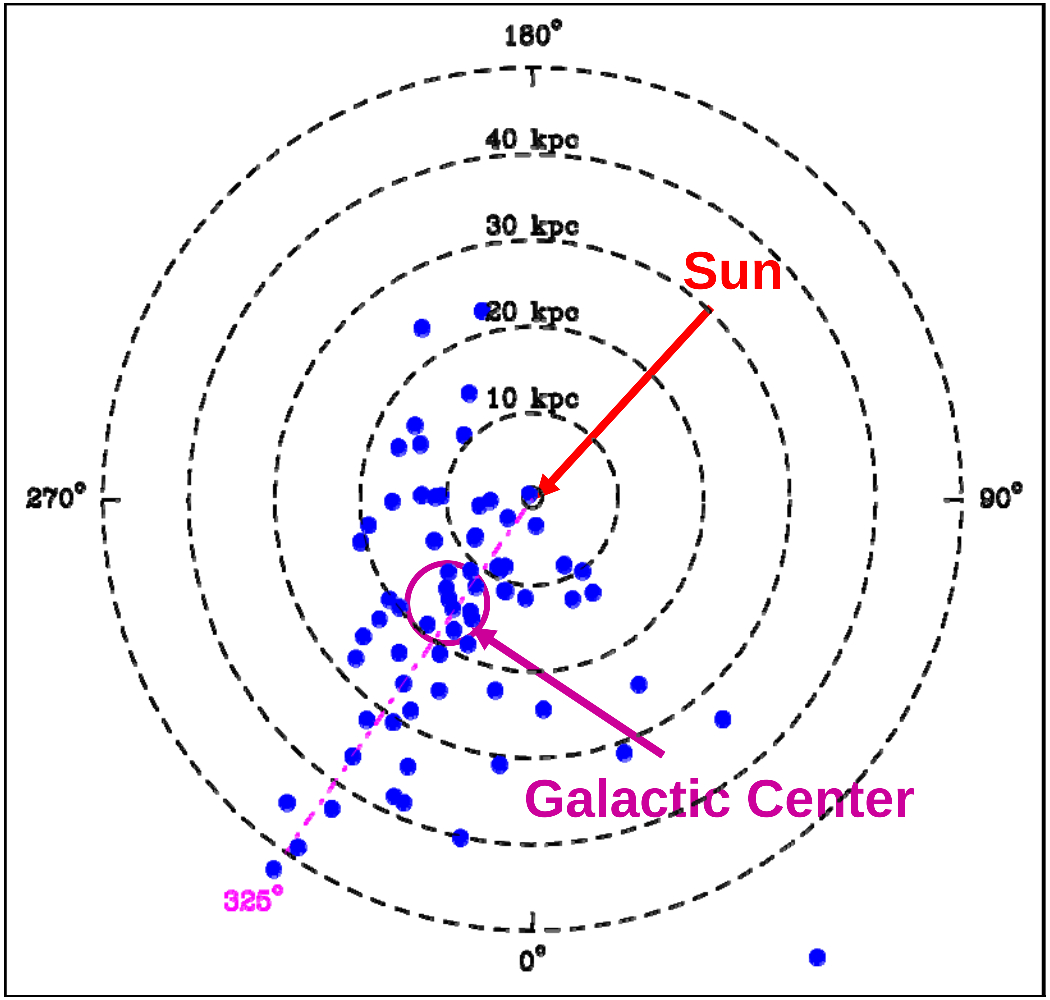


Calibrator

“From little things, big things grow…” Bruce Springsteen/Paul Kelly

Where is the Galactic Center?



Harlow Shapley (ca. 1920)

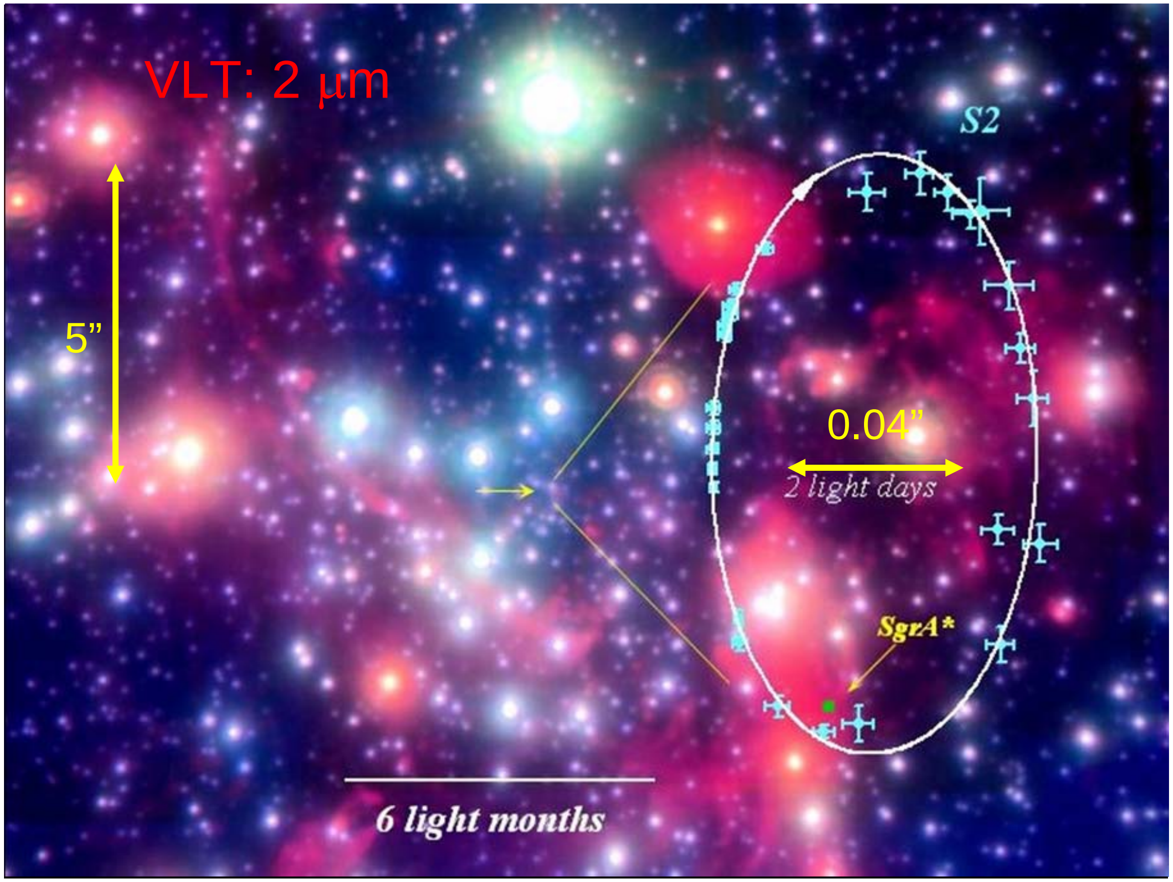
Globular clusters based on Cepheid distances

Projected onto Galactic Plane

Galactic Center Stellar Orbits

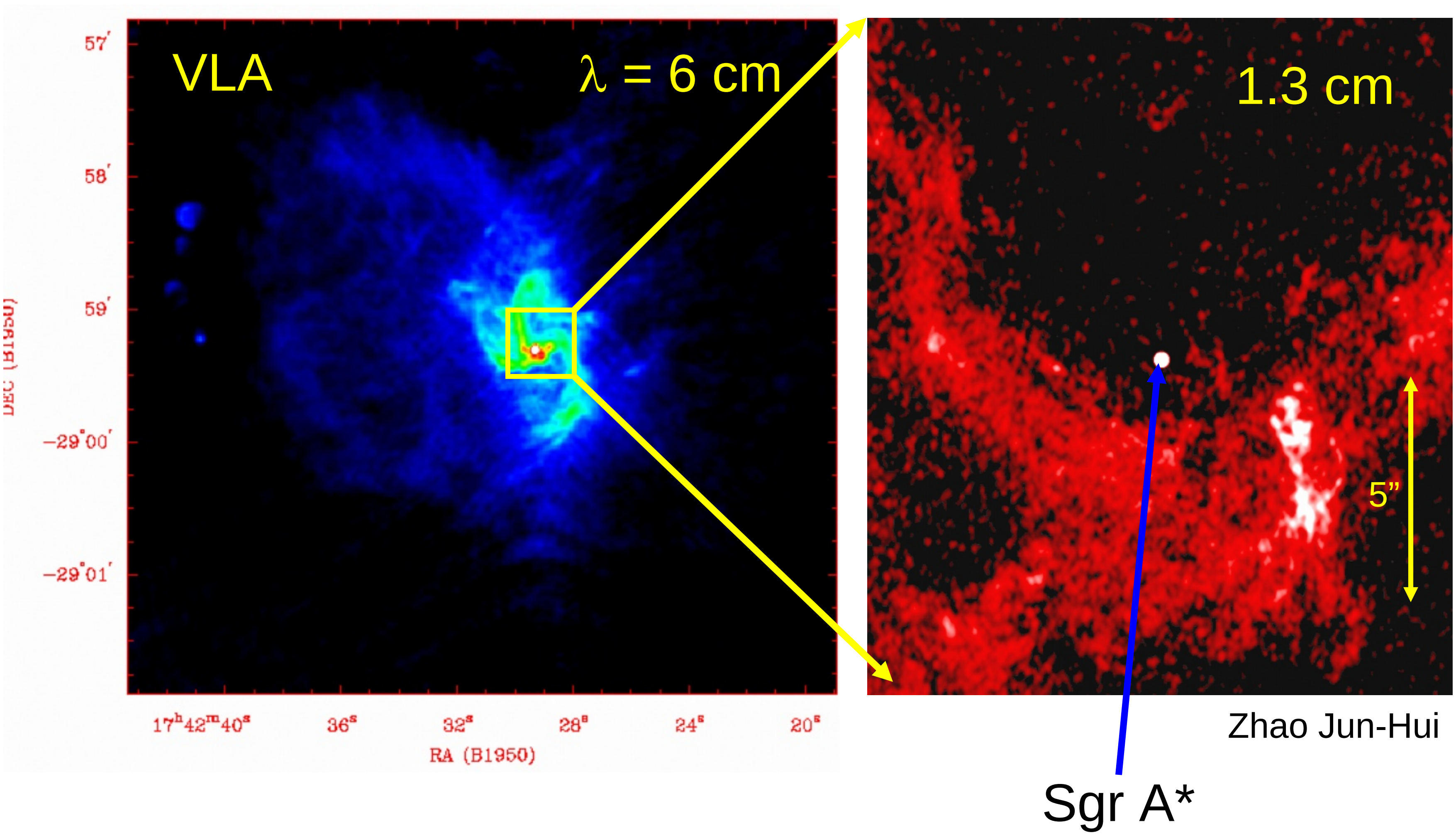
- $M = 4 \times 10^6\ M_{sun}$
- $R < 50$ AU
- Den. $> 10^{17}\ M_{sun}/pc^3$

Genzel et al / Ghez et al

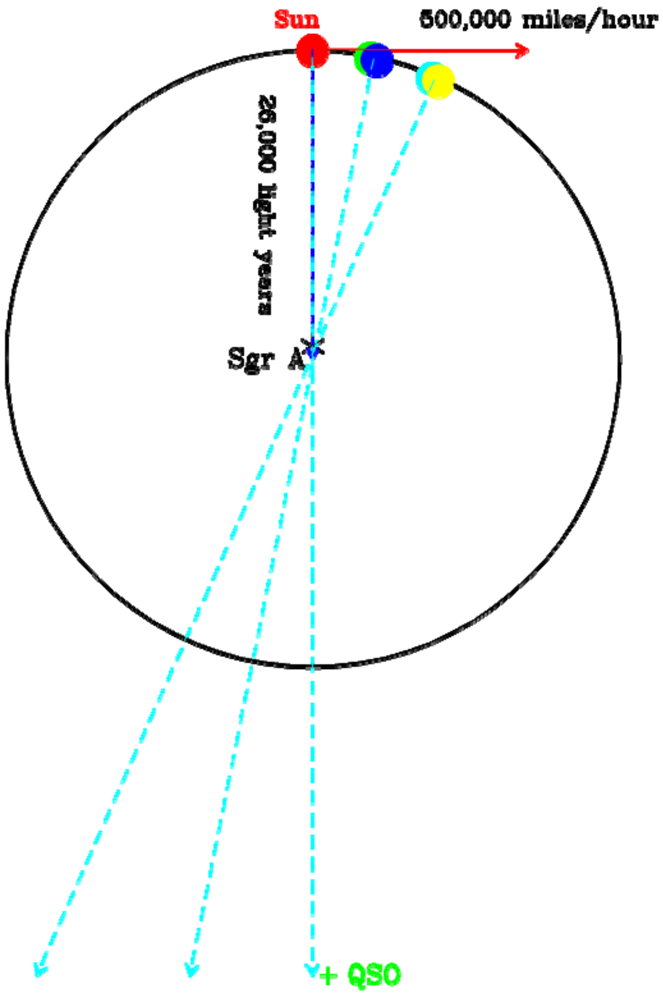


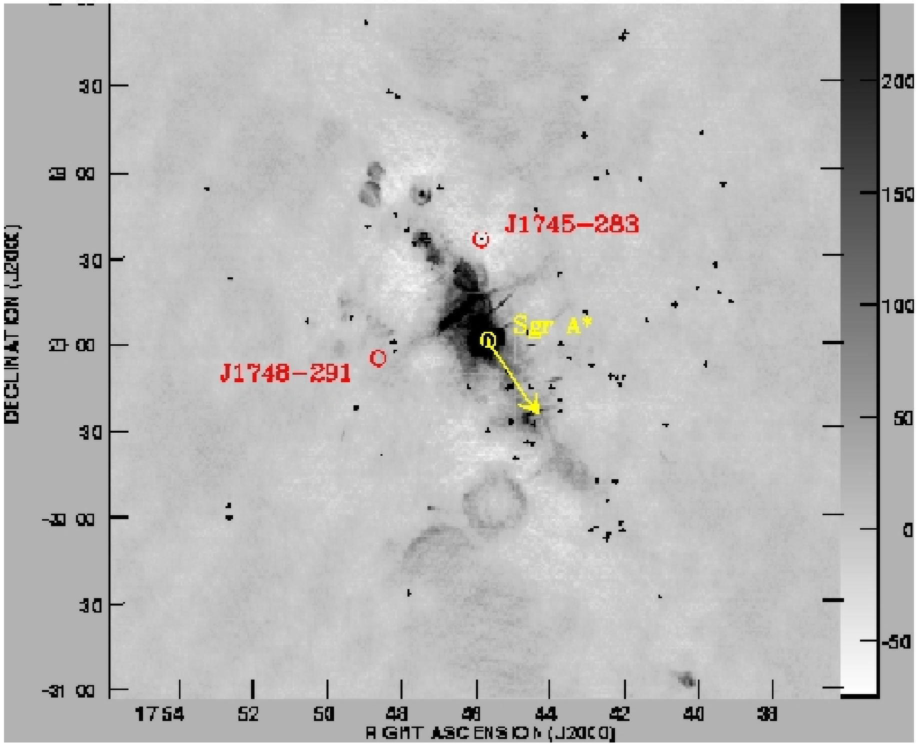
What can radio observations tell us?...

Where is the Galactic Center?



Sgr A* Proper Motion





Sgr A* Proper Motion

IR Stellar Orbits:

$M_{IR} \sim 4 \times 10^6\ M_{sun}$

$R < 50$ AU

Radio Observations:

Sgr A* motionless ➔

$M > 10\%$ of M_{IR}

Observed size:

$R < 0.5$ AU

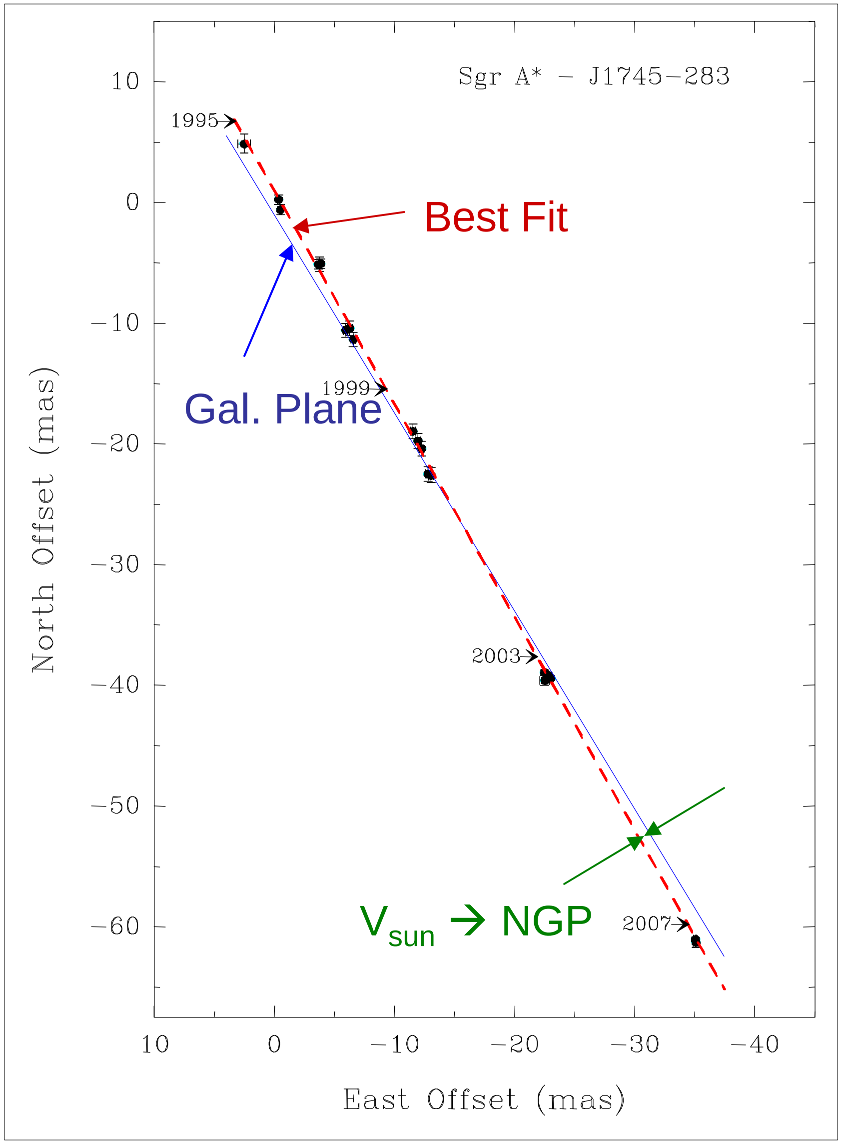
IR + Radio data combined:

Dark mass = luminous source

Density $> 10^{22}\ M_{sun}/pc^3$

Overwhelming evidence for a Super-Massive Black Hole

How do we make such measurements?



Micro-arcsec Astrometry with the VLBA

Comparable to GAIA & SIM

Fringe spacing:

$\theta_f \sim \lambda/D$ ~ 1 cm / 8000 km = 250 μas

Centroid Precision:

0.5 θ_f / SNR ~ 10 μas

Systematics:

path length errors ~ 2 cm (~2 λ)

shift position by ~ $2\theta_f$

Relative positions (to QSOs):

$\Delta\Theta$ ~ 1 deg (0.02 rad)

cancel systematics: $\Delta\Theta * 2\theta_f$ ~ 10 μas

Signal to Noise Limitations

$$\sigma_S = \frac{b\ \mathrm{SEFD}}{\sqrt{2B\tau\ \ N(N-1)/2}} \quad \approx 0.2\ \ \mathrm{mJy}$$

$$\begin{aligned} b &= 1.2 \\ 2B &= 512 \times 10^6\ \ \mathrm{Hz} \\ \tau &= 3,600\ \ \mathrm{sec} \\ N &= 10\ \ \mathrm{antennas} \\ \mathrm{SEFD} &= 1500\ \ \mathrm{Jy} \end{aligned}$$

$$\sigma_\theta = 0.5\ \mathrm{FWHM/SNR} \quad \approx 0.05\ (S/2\ \mathrm{mJy})^{-1}\ \mathrm{mas}$$

$$\begin{aligned} \mathrm{FWHM} &\approx 1\ \mathrm{mas} \\ \mathrm{SNR} &\approx 5S(\mathrm{mJy}) \qquad (= S\,/\,\sigma_S\,) \end{aligned}$$

Systematic Limitations

$$\sigma_\theta = \mathrm{FWHM}\ (c\ \Delta\tau/\lambda)\ \Delta\theta \quad \approx 0.05\ \mathrm{mas}$$

$$c\ \Delta\tau \approx c\ \Delta\tau_0\ \sec ZA \tan ZA$$

(will explain formula later)

$$\begin{aligned}
\mathrm{FWHM} &\approx 1\ \mathrm{mas} \\
c\ \Delta\tau_0 &\approx 1\ \ \mathrm{cm} \\
ZA &\approx 60\ \ \mathrm{deg} \\
\lambda &\approx 1.3\ \ \mathrm{cm} \\
\Delta\theta &\approx 1\ \ \mathrm{deg}
\end{aligned}$$

Signal to Noise vs. Systematic Limitations

σ_θ (noise) $\approx 0.05\ (S/2\ \mathrm{mJy})^{-1}$ mas

σ_θ (systematics) $\approx 0.05\ (c\Delta\tau/4\ \mathrm{cm})$ mas

Typically, σ_θ (noise) $< \sigma_\theta$ (systematics)

for $S > 2$ mJy

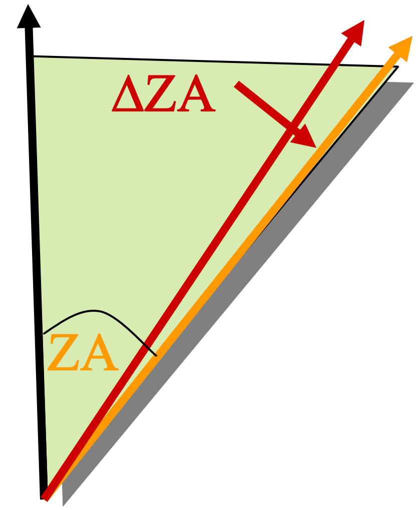
If $S > 2$ mJy, use more observing time to calibrate.

Atmospheric & Ionospheric Errors

Frequency (maser)		Un-modeled Zenith Path Length	
		Atmosphere	Ionosphere
43 GHz	(SiO)	5 cm	0.5 cm
22	(H_2O)	5	2
12	(CH_3OH)	5	6
6.7	(CH_3OH)	5	20
1.6	(OH)	5	300

Relative Atmospheric Delay Errors

$$\tau_A \approx \tau_0 \ \sec ZA$$



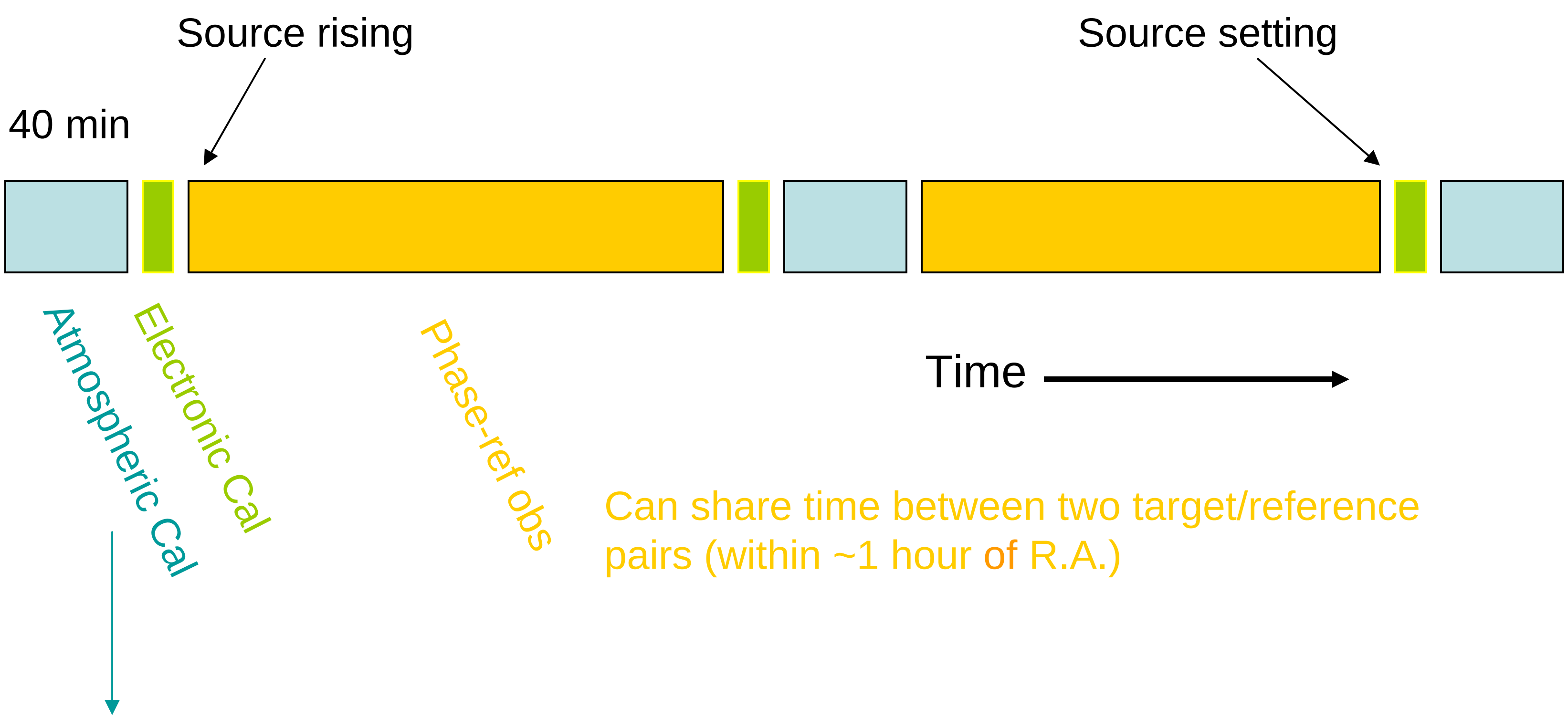
Difference between target and reference sources:

$$\Delta\tau_A = \left(\frac{\partial \tau_A}{\partial ZA}\right) \Delta ZA$$

Note: $\sec(ZA) \ \tan(ZA) \approx 3.5 \quad \text{for } ZA = 60°$

$\sec(ZA) \ \tan(ZA) \approx 8.0 \quad \text{for } ZA = 70°$

Typical Observing Sequence



Can share time between two target/reference pairs (within ~1 hour of R.A.)

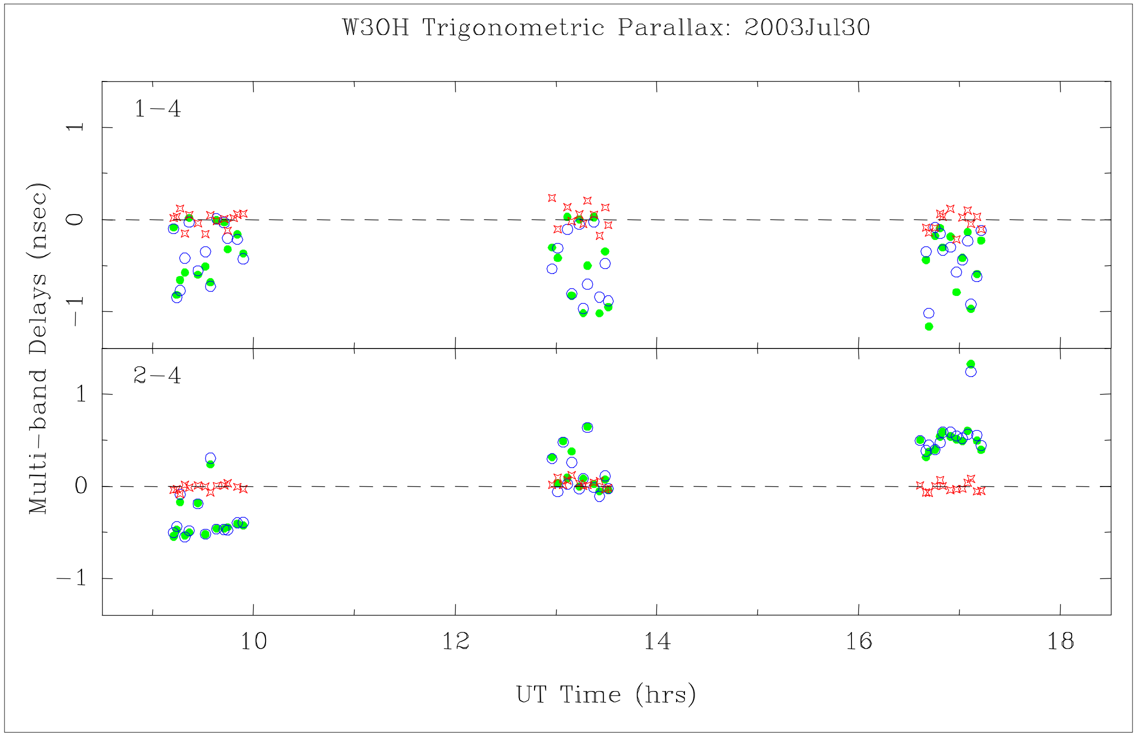
QSO-1(ZA=10^0), QSO-2(ZA=50^0), … QSO-15(ZA=30^0)

Atmospheric Delay Calibration

- Measure zenith delay (τ_0) above each antenna
- Spread observing bands to cover 500 MHz

  σ_τ ~ (1/BW) * (1/SNR)
- Observe QSOs over range of elevations
- Fit to atmospheric model:

  τ_0 ~ 0.5 cm accuracy



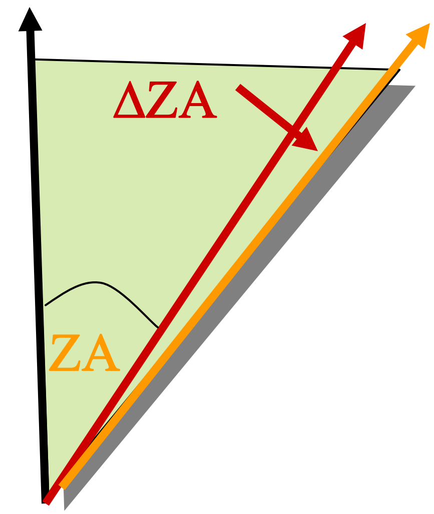
Data **Model** **Residuals**

Position Errors

Effects of position error of phase reference source:

- 1st order correction:
  position shift of Target

- 2nd order corrections:
  small shift of Target
  distorts image



Target → Reference $\Delta\Theta$ = 1 degree

Reference pos. err σ_θ = 0.1 arcsec

Position Errors

Effects of position error of phase reference source:

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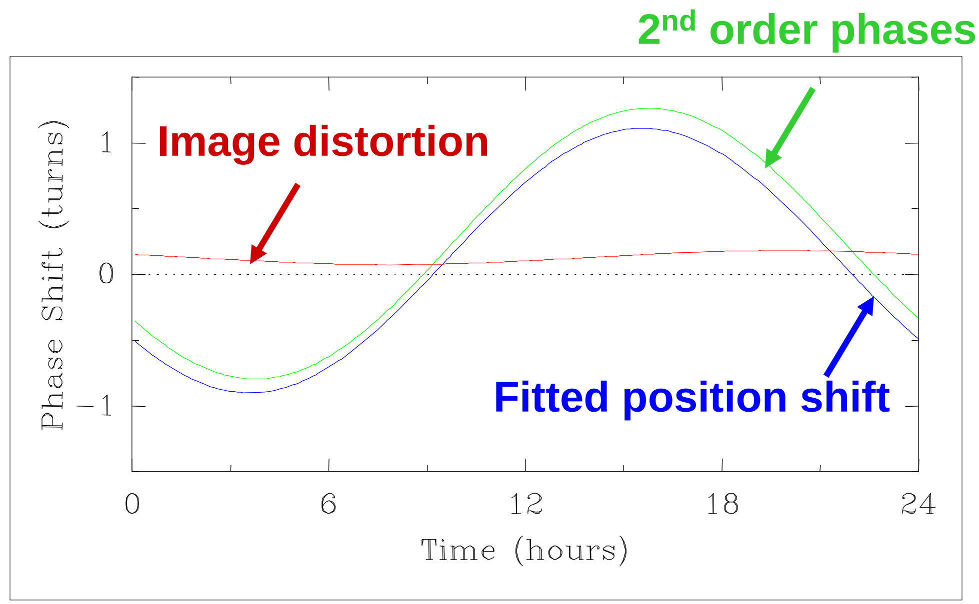


Target → Reference $\Delta\Theta$ = 1 degree

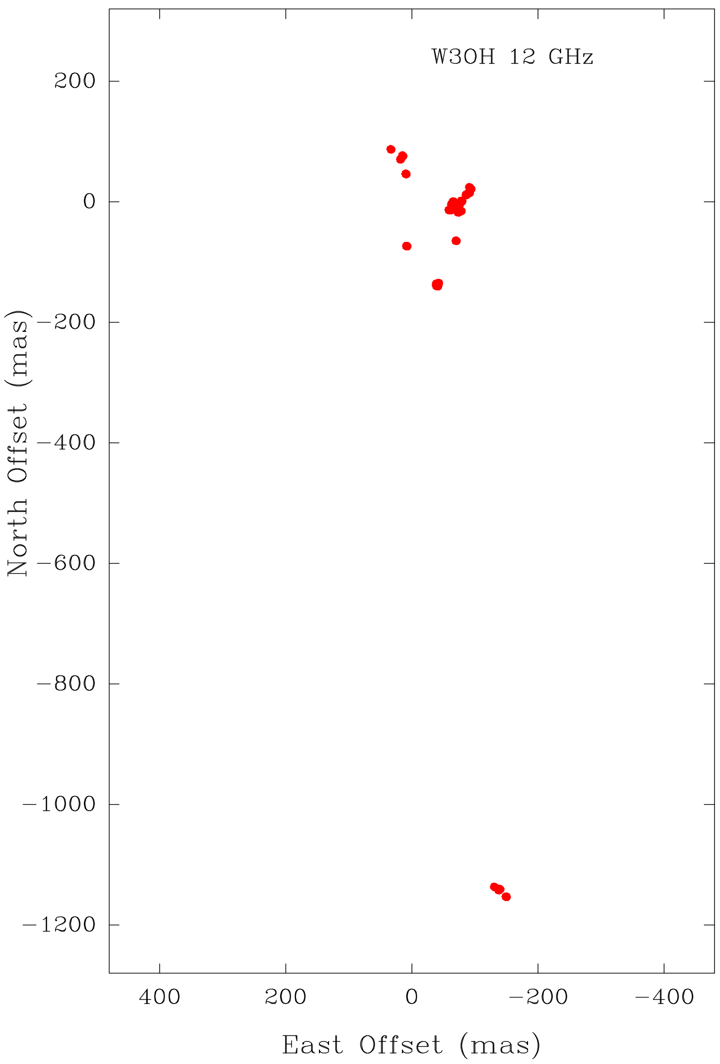
Reference pos. err σ_θ = 0.1 arcsec

Maser as Reference Source

Spread over ~ 1”

Don’t know which spot to use
before getting data

What to do…

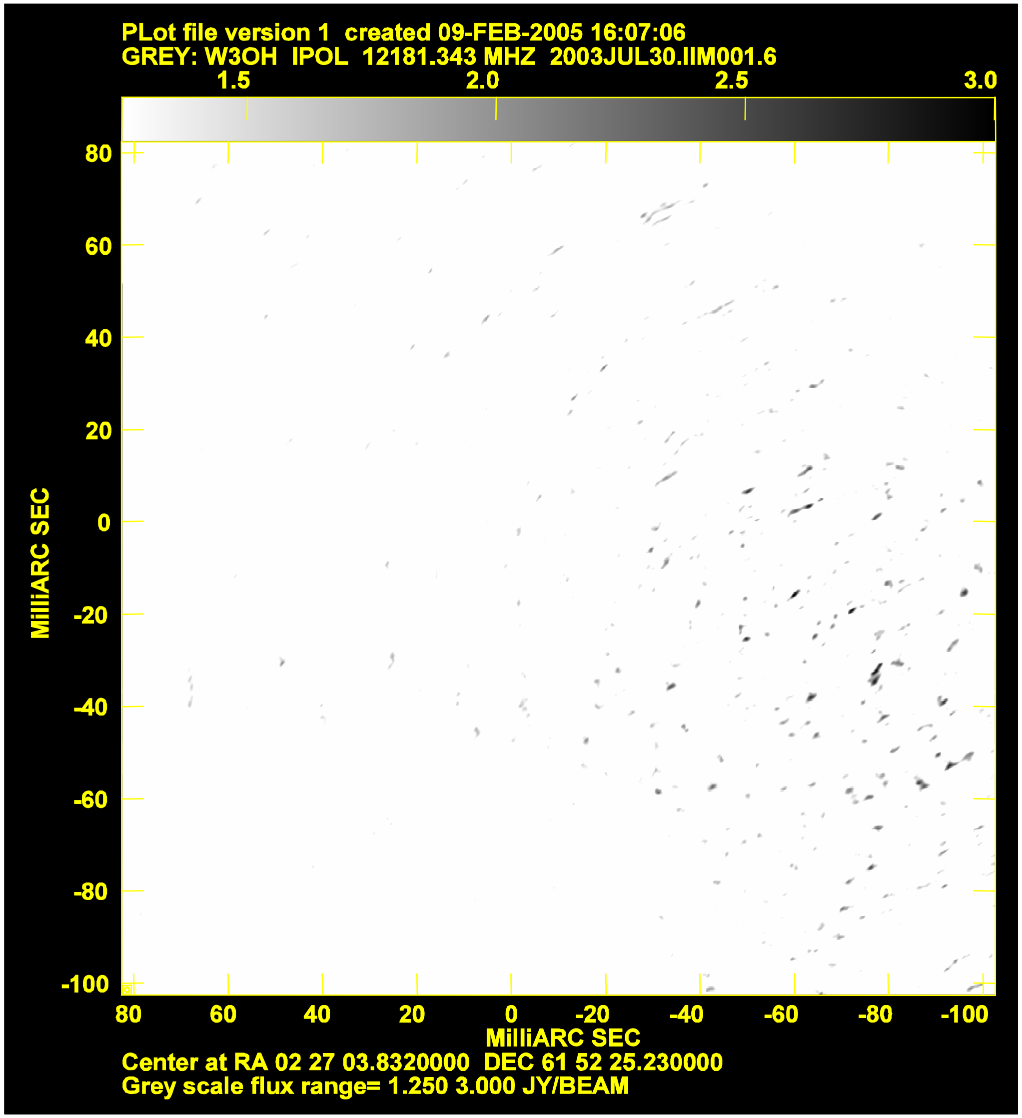


Reference Source Position

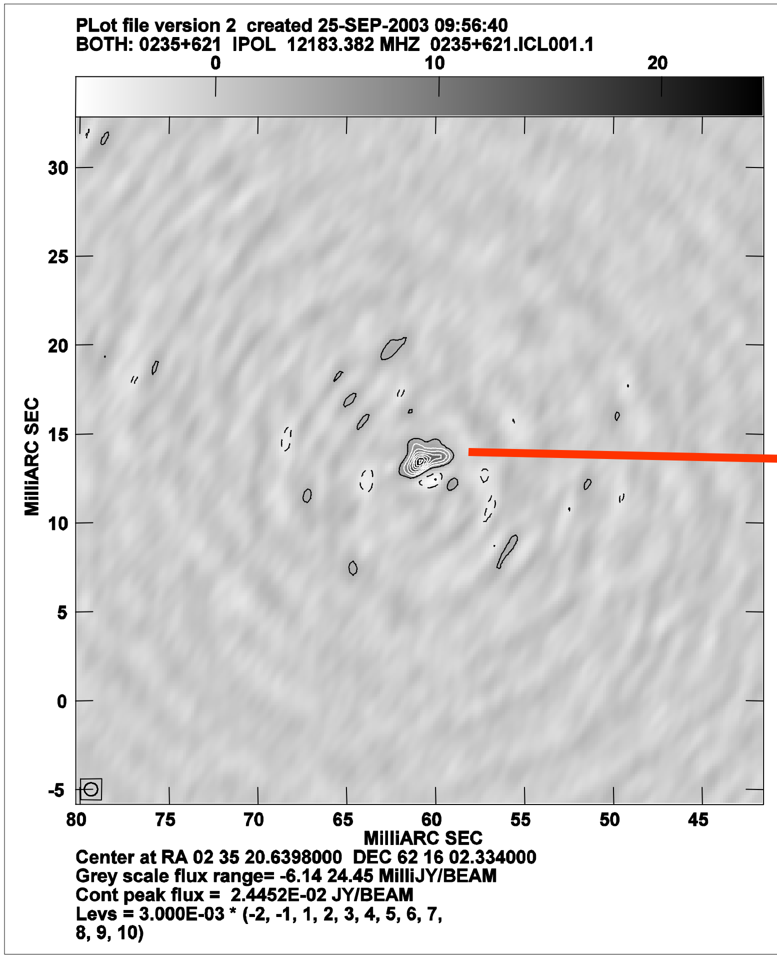
Measure with VLBI
- (need ICRF quasar within about 3 degrees on sky)

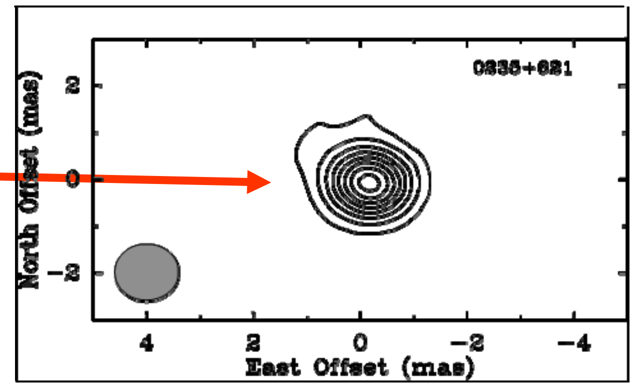
Measure with VLA
- (need largest configuration)

Use “raw” VLBI data
- Fringe rate map
- Invert raw data (Dirty Map)



Before and After Images





Calibration: Step 1

Fix phase errors in VLBA correlator model:

• Parallactic Angle (feed rotation effect)	CLCOR
• Atmospheric zenith delays (“geodetic” blocks)	DELZN/CLCOR
• Ionospheric zenith delays (global electron models)	TECOR
• Earth’s Orientation Parameter errors	CLCOR
• Source coordinate errors (if known)	CLCOR

Calibration: Step 2

Calibrate amplitudes (correlation coefficient → flux density):

• Correct for clipper bias	CLCAL
• Apply system temperatures/gain curves	APCAL/CLCAL

Calibration: Step 3

Align electronic phase shifts among bands:

- Determine band phases on strong source FRING
- Correct all data CLCAL

Fix spectral drift (Doppler shift from Earth's rotation)

- Apply bandpass corrections (if necessary) BPASS
- Fourier transform to delays,

  Apply phase-slope across delay function,

  Inverse Fourier transform back CVEL

Calibration: Step 4

Phase reference data to 1 source / band / spectral channel :

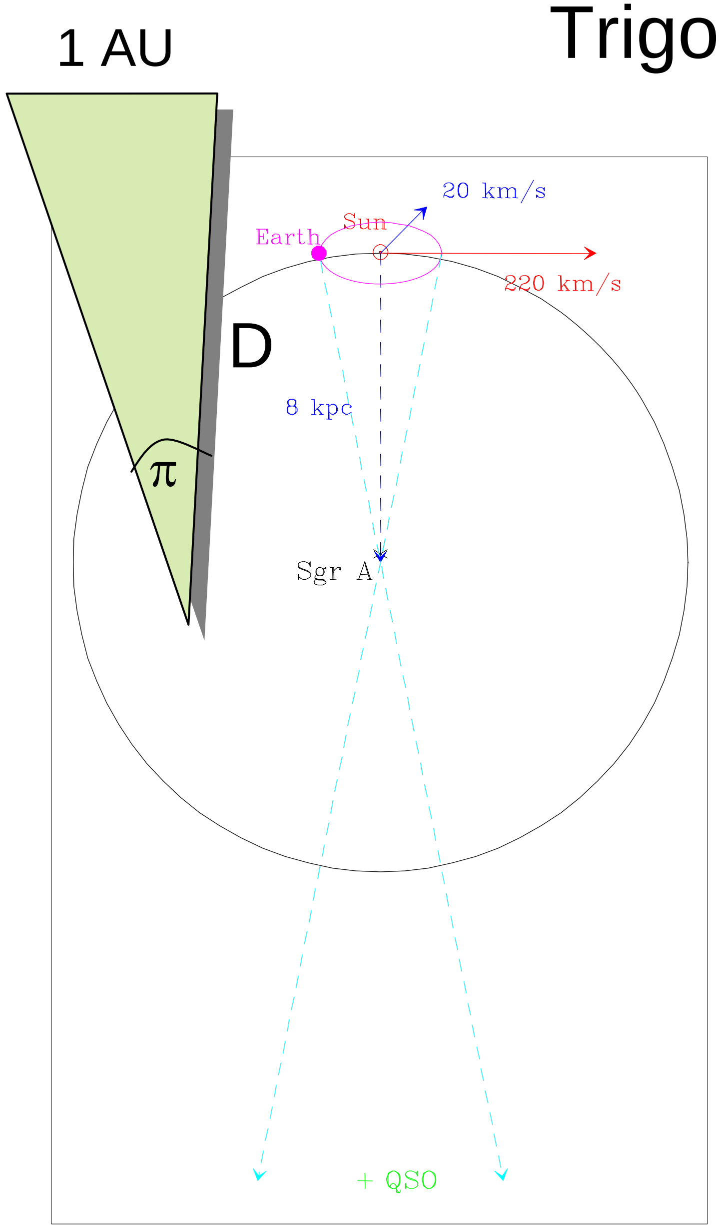
• Calculate phase reference	CALIB or FRING
• Apply phases to all data	CLCAL

Lunar Parallax

Hipparchus (189 BC)

Pete Lawrence's Digitalsky: http://www.digitalsky.org.uk

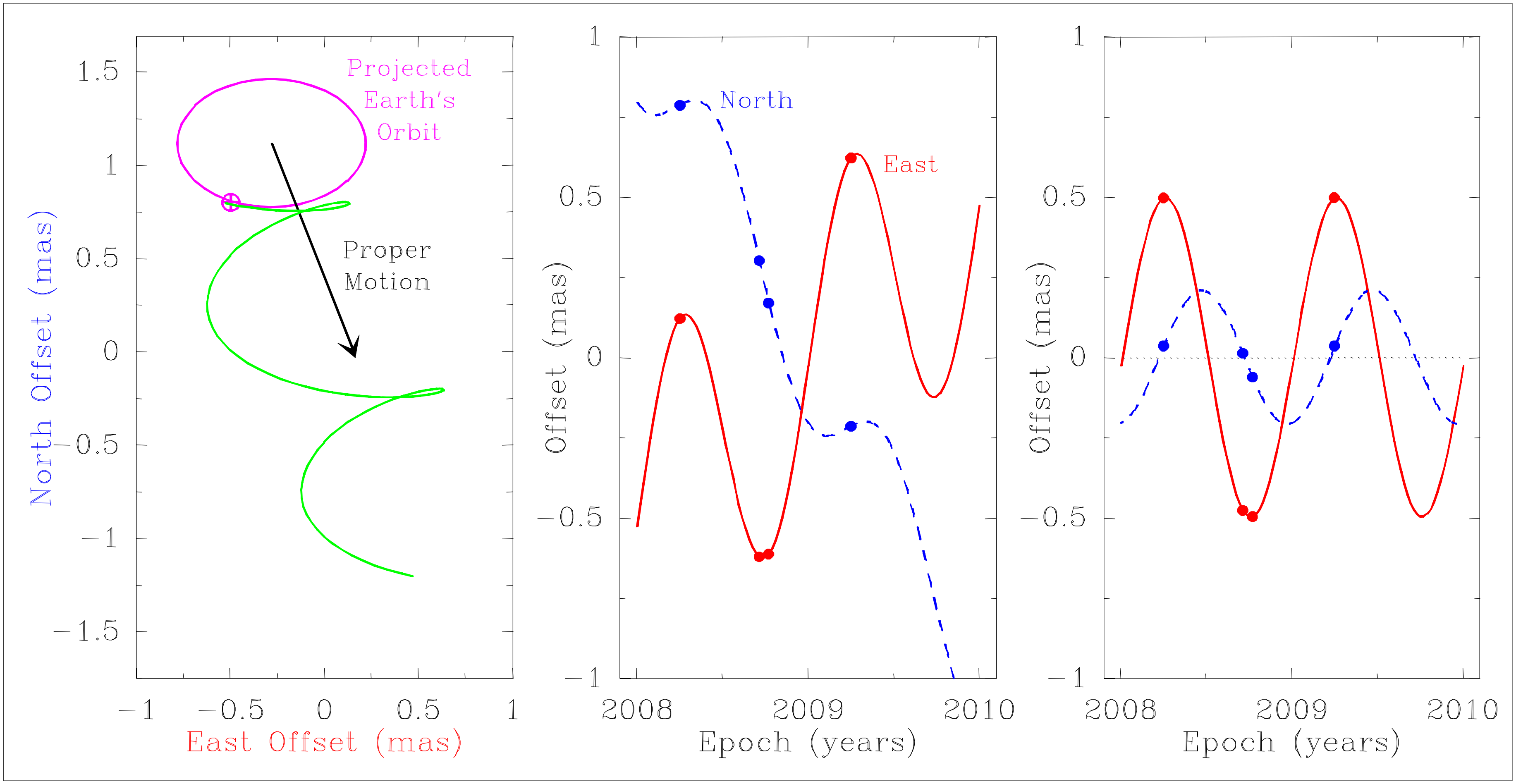
Trigonometric Parallax



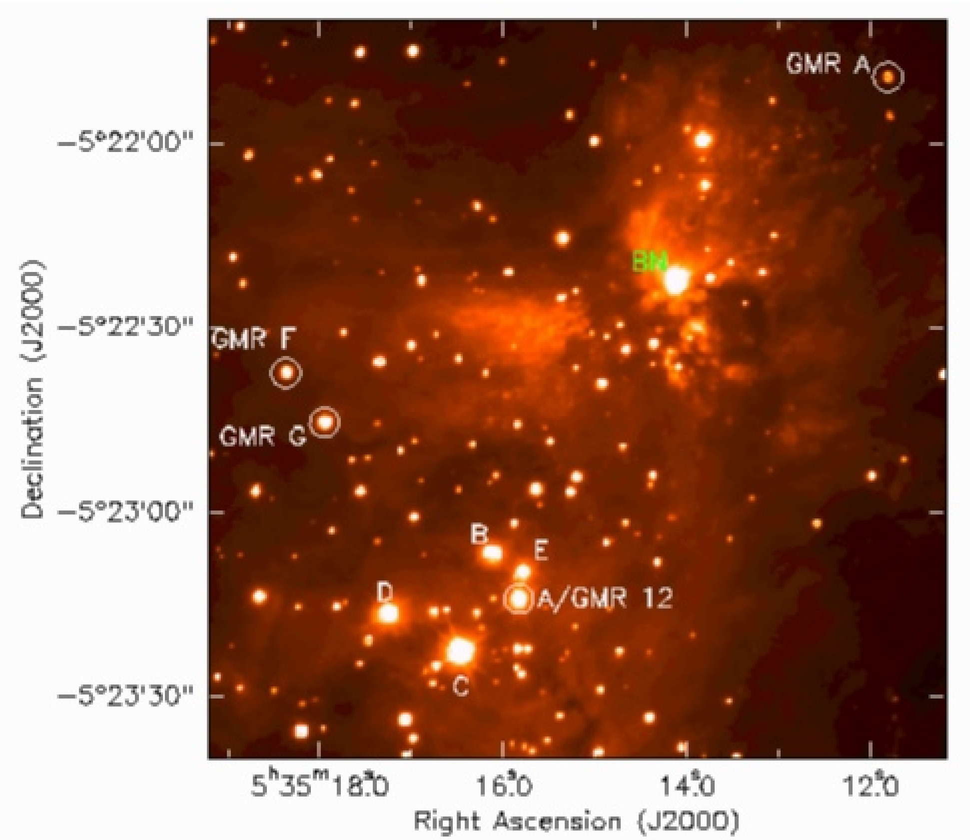
- Triangulation using Earth's orbit as one leg of triangle
- Bessel in 1830s measured first accurate stellar distance this way
- VLBA accuracy 10 micro-arcsec !

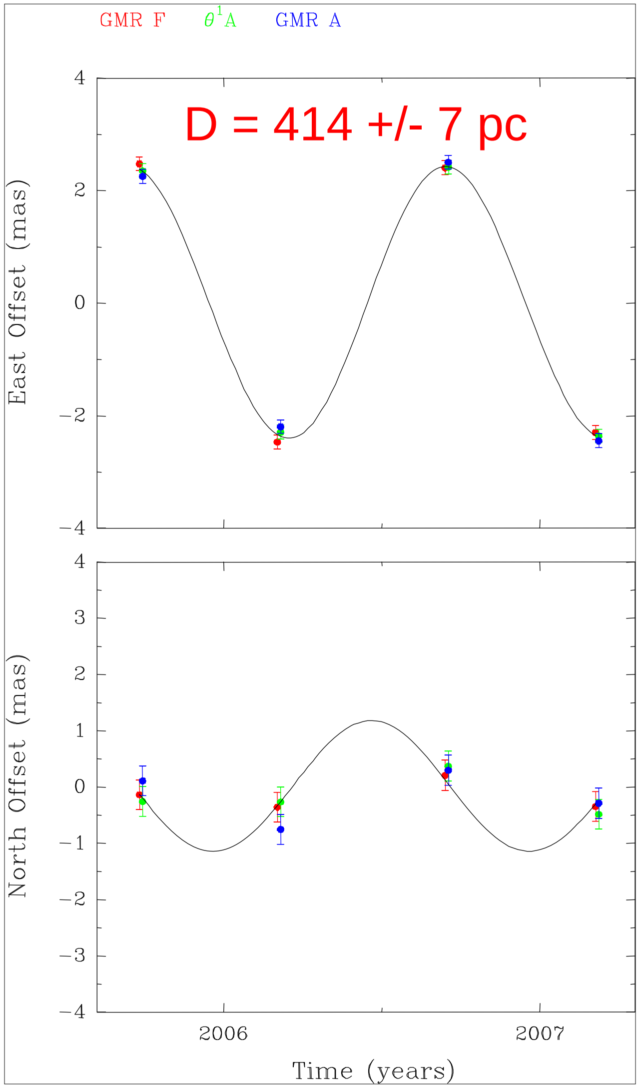
  10,000x better than Bessel
- With such "vision", could read this from the Moon!

Parallax 1.01



Orion Nebular Cluster Parallax





How to fit data & optimize observations?

Menten, Reid, Forbrich & Brunthaler (2007)

Least–Squares Fitting

Goal: Minimize $\chi^2 = \sum_{i=1}^{N} (d_i - m_i)^2$

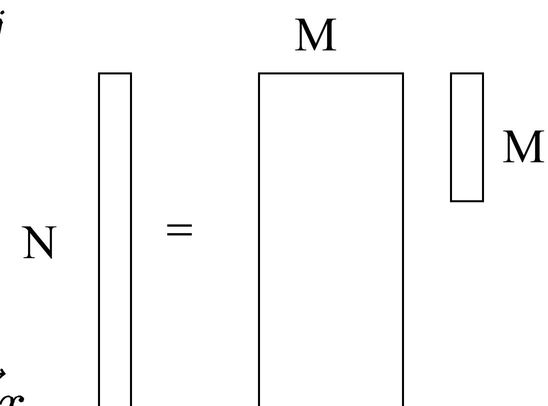
where d_i is i^{th} datum and m_i is its model value.

Taylor expand model about M parameters, x_j:

$$m_i = m_i\big|_0 + \sum_{j=1}^{M} \left(\frac{\partial m_i}{\partial x_j}\right)\Big|_0 \, \Delta x_j$$

$$r_i = d_i - m_i\big|_0 = \sum_{j=1}^{M} \left(\frac{\partial m_i}{\partial x_j}\right)\Big|_0 \, \Delta x_j$$

$$\vec{r} = P \; \vec{\Delta x}$$



$$P^T \vec{r} = P^T P \; \vec{\Delta x}$$

$$\left(P^T P\right)^{-1} P^T \vec{r} = \left(P^T P\right)^{-1} P^T P \; \vec{\Delta x}$$

Experiment Design

Least–squares solution: $\vec{\Delta x} = \left(P^T P\right)^{-1} P^T \vec{r}$

where $P = \text{Matrix}\left[\frac{\partial m_i}{\partial x_j}\right]$

"Design matrix" $\left(P^T P\right)^{-1}$

diagonal elements give parameter uncertainties (variances)

off-diagonals give parameter co-variances (correlations)

Parallax Design

"Design matrix" $\left(P^T P\right)^{-1}$

where $P = \text{Matrix}\left[\frac{\partial m_i}{\partial x_j}\right]$

Parallax model: $m_i = \Pi \cos(\omega \Delta t_i) + \alpha_0 + \mu_\alpha \Delta t_i$

where $x_1 = \Pi$, $x_2 = \alpha_0$, and $x_3 = \mu_\alpha$

$$P_{i1} = \frac{\partial m_i}{\partial \Pi} = \cos(\omega \Delta t_i)$$

$$P_{i2} = \frac{\partial m_i}{\partial \alpha_0} = 1$$

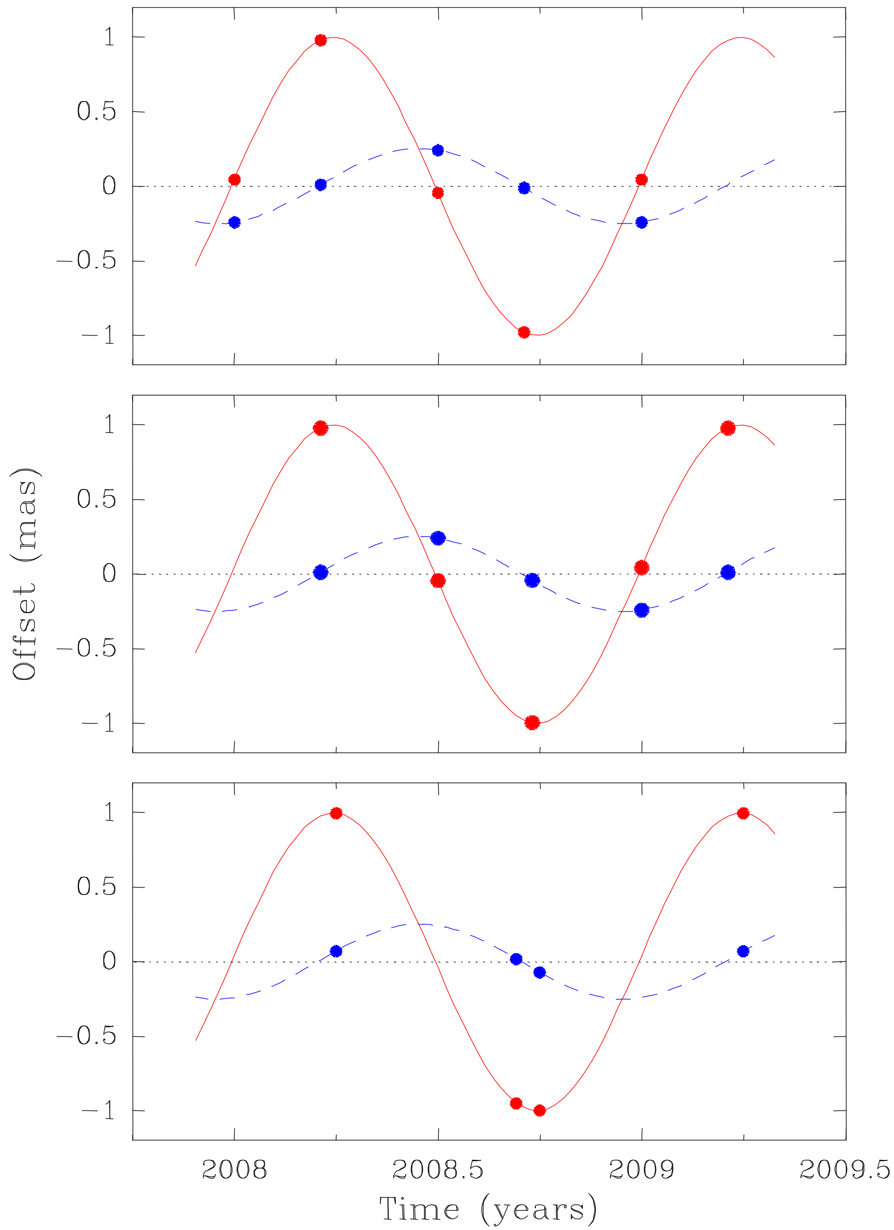
$$P_{i3} = \frac{\partial m_i}{\partial \mu_\alpha} = \Delta t_i$$

$$P \quad = \begin{bmatrix} \cos(\omega\Delta t_1) & 1 & \Delta t_1 \\ \cos(\omega\Delta t_2) & 1 & \Delta t_2 \\ \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots \\ \cos(\omega\Delta t_N) & 1 & \Delta t_N \end{bmatrix}$$

$$P^T P = \begin{bmatrix} \sum\cos^2(\omega\Delta t_i) & \sum\cos(\omega\Delta t_i) & \sum\cos(\omega\Delta t_i)\Delta t_i \\ \sum\cos(\omega\Delta t_i) & \sum 1 & \sum\Delta t_i \\ \sum\cos(\omega\Delta t_i)\Delta t_i & \sum\Delta t_i & \sum{\Delta t_i}^2 \end{bmatrix}$$

If data symmetric about $t = 0$, off-diagonal terms $\rightarrow 0$

$$\left(P^T P\right)^{-1} = \begin{bmatrix} 1/\sum\cos^2(\omega\Delta t_i) & 0 & 0 \\ 0 & 1/\sum 1 & 0 \\ 0 & 0 & 1/\sum{\Delta t_i}^2 \end{bmatrix}$$



$$\sigma_\pi = 1 \,/\, \mathrm{sqrt}(\ \Sigma \cos^2 \omega t\ )$$

$N = 5$

$$\sigma_\pi = 1 \,/\, \mathrm{sqrt}(\ 2\ ) = 0.7$$

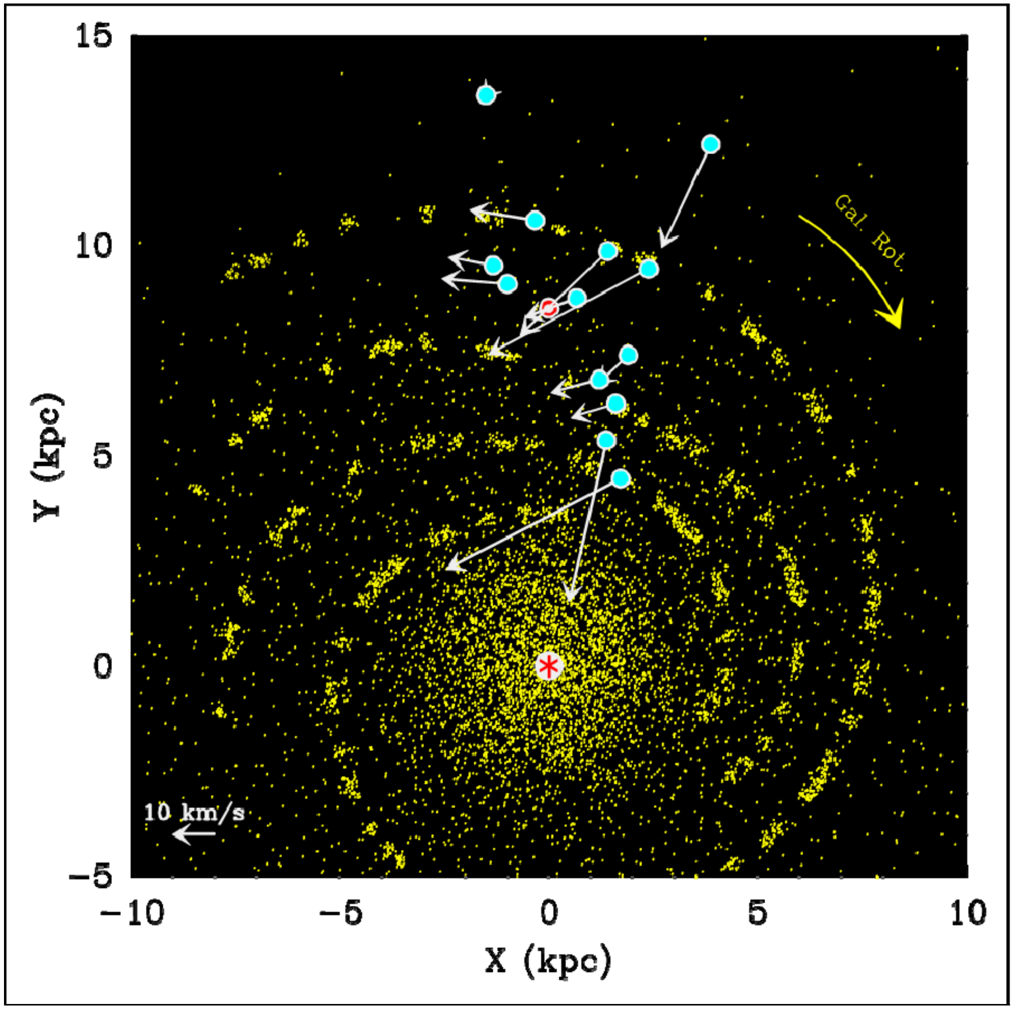
$N = 5$

$$\sigma_\pi = 1 \,/\, \mathrm{sqrt}(\ 3\ ) = 0.6$$

$N = 4$

$$\sigma_\pi = 1 \,/\, \mathrm{sqrt}(\ 4\ ) = 0.5$$

Peculiar Motions of Star Forming Regions



- Trace Spiral Arms
- In rotating frame see clear systematic motions:

  SFRs orbit galaxy slower than galaxy spins