Calibration & Editing

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Synopsis

- Why calibration and editing?
- Idealistic formalism -> Realistic practice
- Editing and RFI
- Practical Calibration
- Baseline- and Antenna-based Calibration
- Scalar Calibration Example
- Full Polarization Generalization
- A Dictionary of Calibration Effects
- Calibration Heuristics
- New Calibration Challenges
- Summary

Why Calibration and Editing?

- Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, stability, etc.)
- Need to accommodate deliberate engineering (e.g., frequency conversion, digital electronics, filter bandpass, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- RFI

Determining *instrumental properties* (calibration)
is a prerequisite to
determining *radio source properties*

From Idealistic to Realistic

- Formally, we wish to use our interferometer to obtain the visibility function, which we intend to invert to obtain an image of the sky:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl\, dm$$

- In practice, we correlate (multiply & average) the electric field (voltage) samples, x_i & x_j, received at pairs of telescopes (*i,j*) and processed through the observing system:

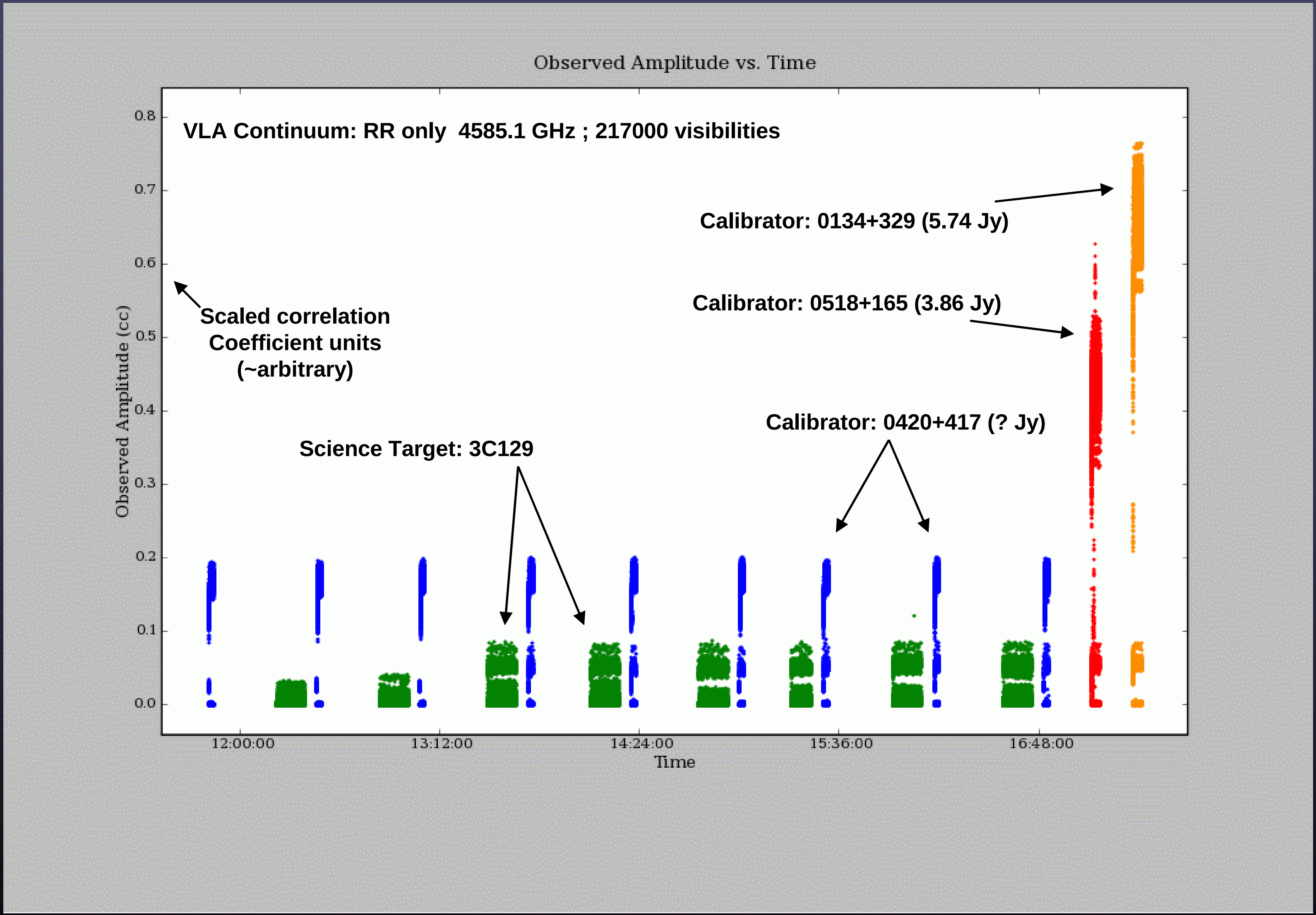
$$V_{ij}^{obs}(u_{ij}, v_{ij}) = \left\langle x_i(t) \cdot x_j^*(t) \right\rangle_{\Delta t} = J_{ij} V(u_{ij}, v_{ij})$$

  - Averaging duration is set by the expected timescales for variation of the correlation result (typically 10s or less for the VLA)
- J_{ij} is an *operator* characterizing the net effect of the observing process for baseline *(i,j),* which we must *calibrate*
- Sometimes J_{ij} corrupts the measurement irrevocably, resulting in data that must be *edited*

What Is Delivered by a Synthesis Array?

- An enormous list of complex numbers!
- E.g., the VLA (traditionally):
  - At each timestamp: 351 baselines (+ 27 auto-correlations)
  - For each baseline: 1 or 2 Spectral Windows (“IFs”)
  - For each spectral window: 1-512 channels
  - For each channel: 1, 2, or 4 complex correlations
    - RR or LL or (RR,LL), or (RR,RL,LR,LL)
  - With each correlation, a weight value
  - Meta-info: Coordinates, field, and frequency info
- $N = N_t \times N_{bl} \times N_{spw} \times N_{chan} \times N_{corr}$ visibilities
  - ~200000xN_{chan}xN_{corr} vis/hour at the VLA (up to ~few GB per observation)
- ALMA (~3-5X the baselines!), EVLA will generate an order of magnitude larger number each of spectral windows and channels (up to ~few 100 GB per observation!)

What does the raw data look like?



Data Examination and Editing

- After observation, initial data examination and editing very important
  - Will observations meet goals for calibration and science requirements?
- What to edit:
  - Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)
  - Any persistently 'dead' antennas (check operator's logs)
  - Periods of poor weather? (check operator's log)
  - Any antennas shadowing others? Edit such data.
  - Amplitude and phase should be continuously varying—edit outliers
  - Radio Frequency Interference (RFI)?
- Caution:
  - Be careful editing noise-dominated data (noise bias).
  - Be conservative: those antennas/timeranges which are bad on calibrators are probably bad on weak target sources—edit them
  - Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have significantly different amplitude response which may not be fatal—it may only need to be calibrated
  - Choose reference antenna wisely (ever-present, stable response)
- Increasing data volumes increasingly demand automated editing algorithms

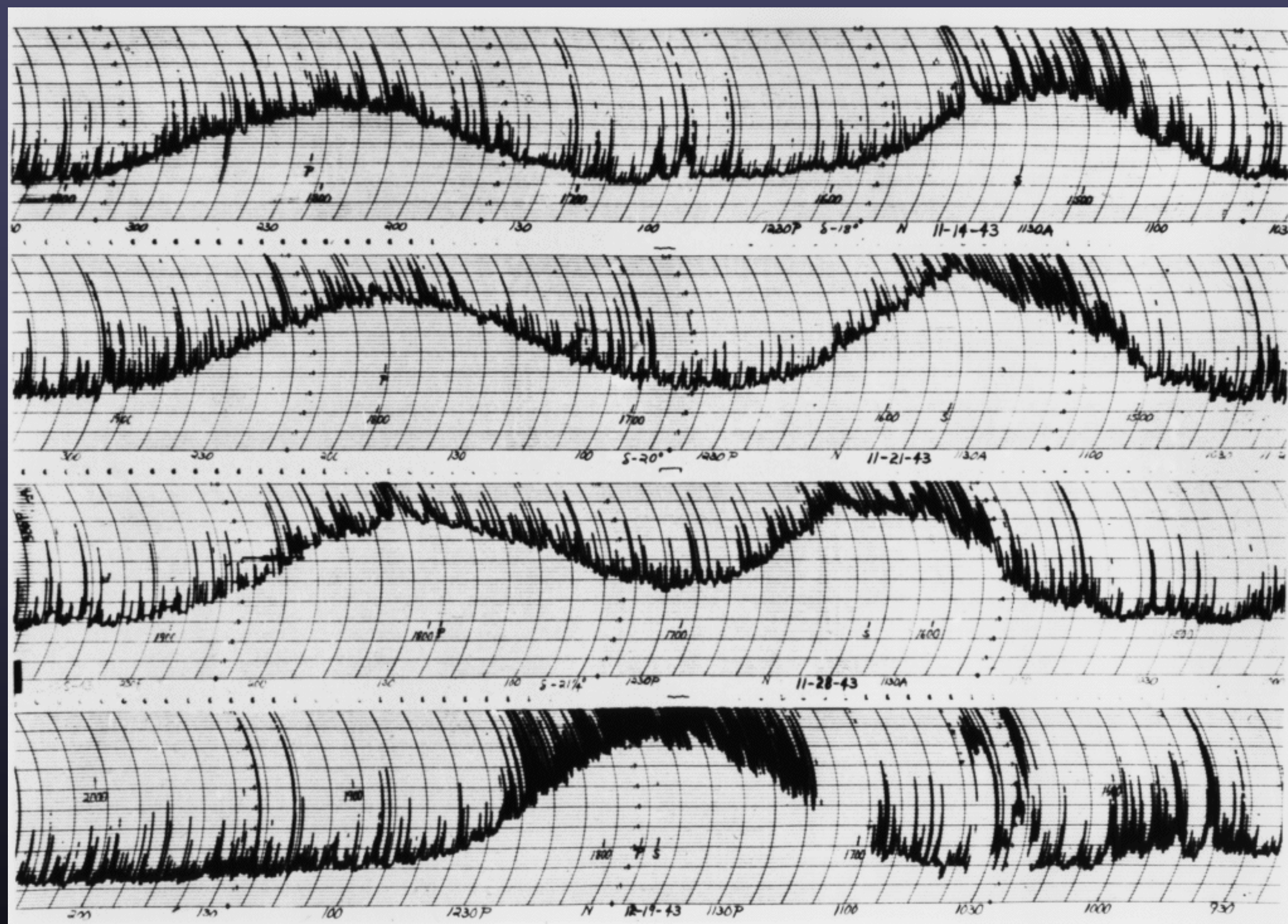
Radio Frequency Interference (RFI)

- RFI originates from man-made signals generated in the antenna electronics or by external sources (e.g., satellites, cell-phones, radio and TV stations, automobile ignitions, microwave ovens, computers and other electronic devices, etc.)
  – Adds to total noise power in all observations, thus decreasing the fraction of desired natural signal passed to the correlator, thereby reducing sensitivity and possibly driving electronics into non-linear regimes
  – Can correlate between antennas if of common origin and baseline short enough (insufficient decorrelation via geometry compensation), thereby obscuring natural emission in spectral line observations

- Least predictable, least controllable threat to a radio astronomy observation

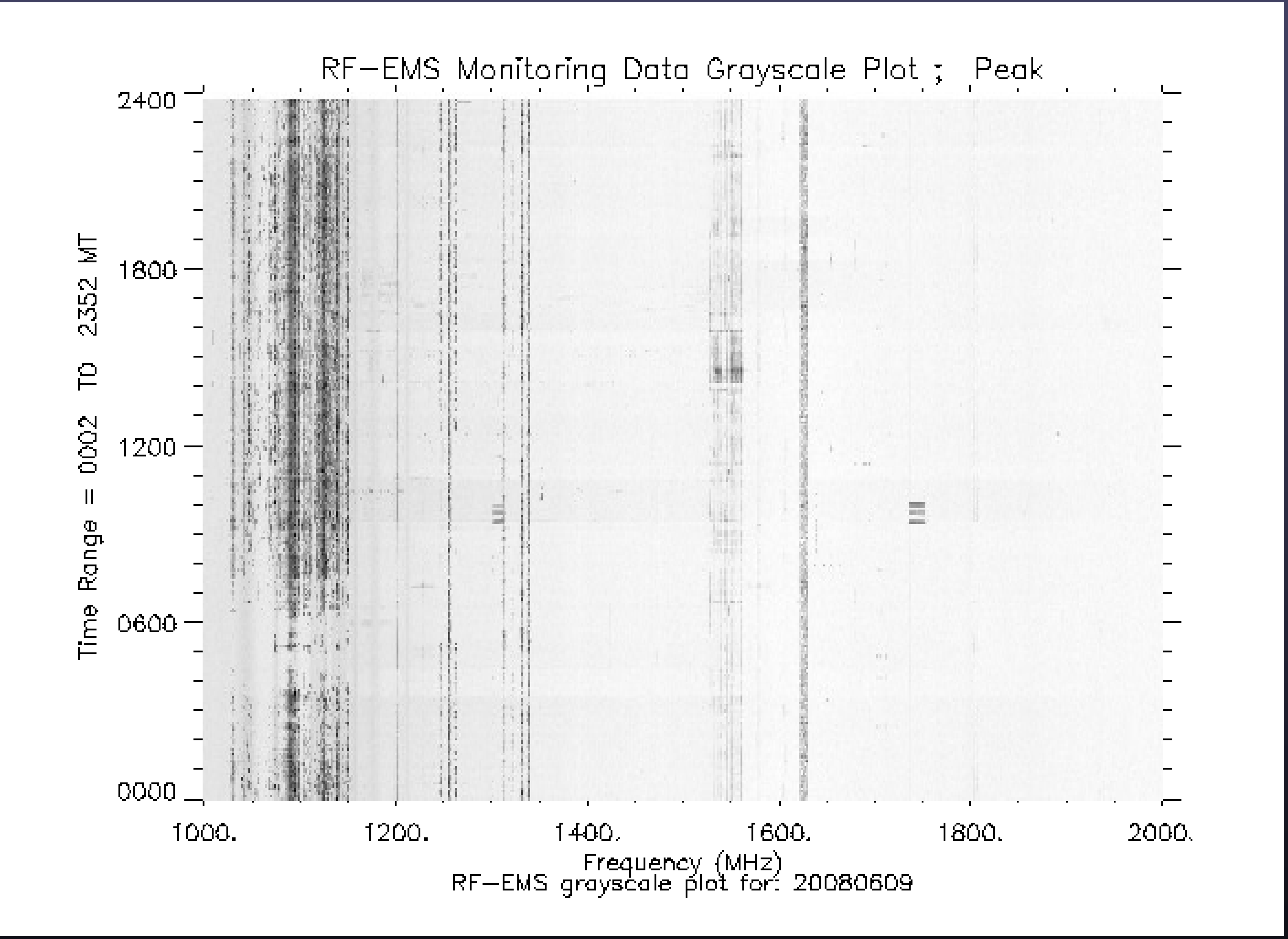
Radio Frequency Interference

- Has always been a problem (Reber, 1944, in total power)!



Radio Frequency Interference (cont)

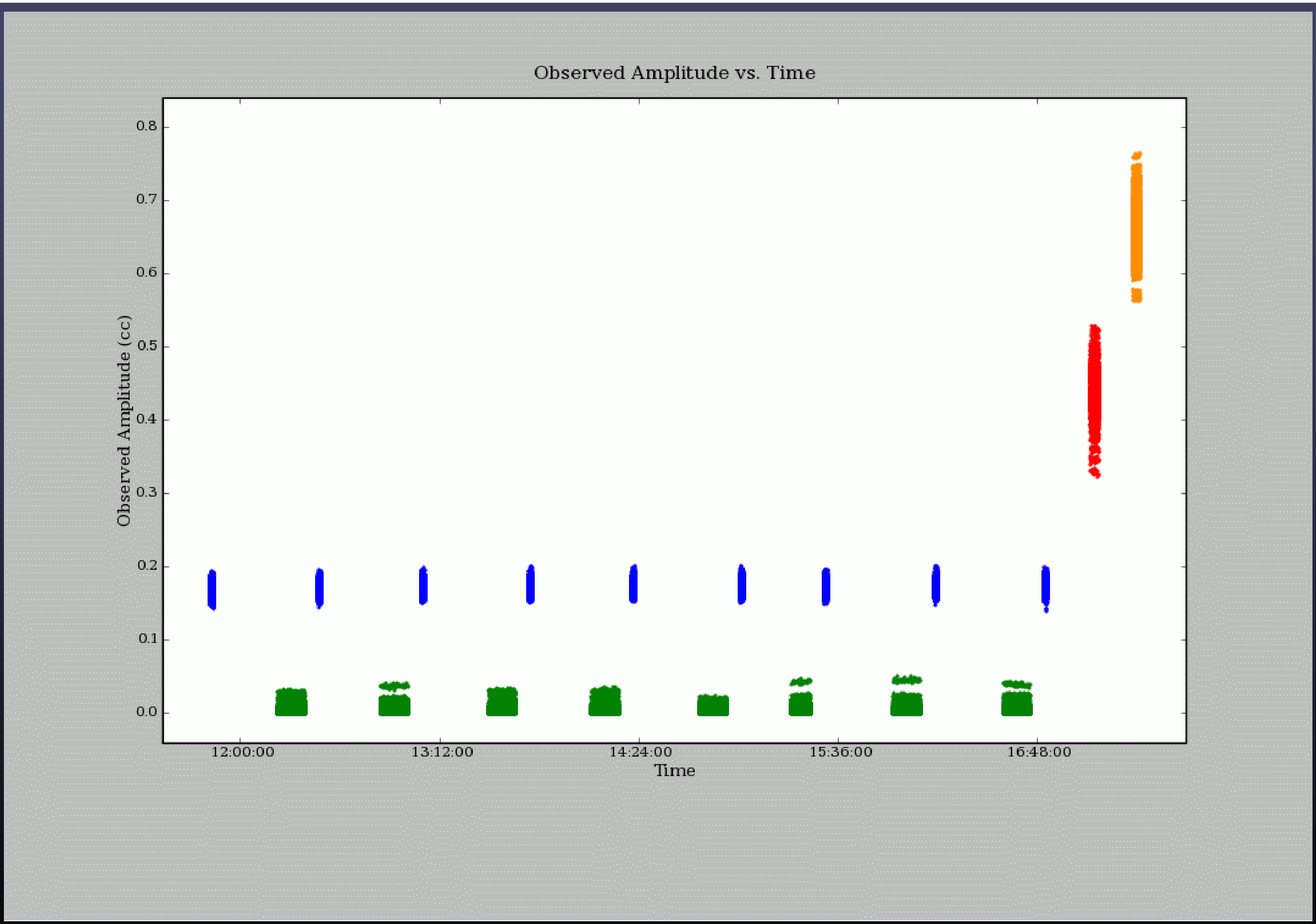
- Growth of telecom industry threatening radioastronomy!



Radio Frequency Interference (cont)

- RFI Mitigation
  - Careful electronics design in antennas, including filters, shielding
  - High-dynamic range digital sampling
  - Observatories world-wide lobbying for spectrum management
  - Choose interference-free frequencies: but try to find 50 MHz (1 GHz) of clean spectrum in the VLA (EVLA) 1.6 GHz band!
  - Observe continuum experiments in spectral-line modes so affected channels can be edited
- Various off-line mitigation techniques under study
  - E.g., correlated RFI power appears at celestial pole in image domain…

Editing Example



Practical Calibration Considerations

- A priori “calibrations” (provided by the observatory)
  - Antenna positions, earth orientation and rate
  - Clocks
  - Antenna pointing, gain, voltage pattern
  - Calibrator coordinates, flux densities, polarization properties
  - Tsys, nominal sensitivity
- Absolute *engineering* calibration?
  - Very difficult, requires heroic efforts by observatory scientific and engineering staff
  - Concentrate instead on ensuring instrumental *stability* on adequate timescales
- ***Cross-calibration*** a better choice
  - Observe nearby ***point sources*** against which calibration (J_{ij}) can be solved, and transfer solutions to target observations
  - Choose appropriate calibrators; usually strong point sources because we can easily predict their visibilities
  - Choose appropriate timescales for calibration

"Absolute" Astronomical Calibrations

- Flux Density Calibration
  - Radio astronomy flux density scale set according to several "constant" radio sources
  - Use resolved models where appropriate
  - VLA nominal scale: 10 Jy source: correlation coeff ~ 1.0
- Astrometry
  - Most calibrators come from astrometric catalogs; directional accuracy of target images tied to that of the calibrators
  - Beware of resolved and evolving structures (especially for VLBI)
- Linear Polarization Position Angle
  - Usual flux density calibrators also have significant stable linear polarization position angle for registration

- Relative calibration solutions (and dynamic range) insensitive to errors in these "scaling" parameters

Baseline-based Cross-Calibration

$$V_{ij}^{obs} = J_{ij} V_{ij}^{true}$$

- Simplest, most-obvious calibration approach: measure complex response of *each baseline* on a standard source, and scale science target visibilities accordingly
  - "Baseline-based" Calibration
- Only option for single baseline "arrays" (e.g., ATF)
- Calibration precision same as calibrator visibility sensitivity (on timescale of calibration solution).
- Calibration accuracy very sensitive to departures of calibrator from known structure
  - Un-modeled calibrator structure transferred (in inverse) to science target!

Antenna-based Cross Calibration

- Measured visibilities are formed from a product of antenna-based signals. Can we take advantage of this fact?
- The net signal delivered by antenna *i*, $x_i(t)$, is a combination of the desired signal, $s_i(t,l,m)$, corrupted by a factor $J_i(t,l,m)$ and integrated over the sky, and diluted by noise, $n_i(t)$:

$$x_i(t) = \int_{sky} J_i(t,l,m) s_i(t,l,m)\, dldm + n_i(t)$$
$$= s_i'(t) + n_i(t)$$

- $J_i(t,l,m)$ is the product of a series of effects encountered by the incoming signal
- $J_i(t,l,m)$ is an *antenna-based* complex number
- Usually, $|n_i| >> |s_i|$

Correlation of Realistic Signals - I

- The correlation of two realistic signals from different antennas:

$$\left\langle x_i \cdot x_j^* \right\rangle_{\Delta t} = \left\langle \left(s_i' + n_i\right) \cdot \left(s_j' + n_j\right)^* \right\rangle_{\Delta t}$$
$$= \left\langle s_i' \cdot s_j'^* \right\rangle + \left\langle s_i' \cdot n_j^* \right\rangle + \left\langle n_i \cdot s_j'^* \right\rangle + \left\langle n_i \cdot n_j^* \right\rangle$$

  - Noise signal doesn't correlate—even if $|n_i|$>> $|s_i|$, the correlation process isolates desired signals:

$$= \left\langle s_i' \cdot s_j'^* \right\rangle_{\Delta t}$$
$$= \left\langle \int_{sky} J_i s_i \, dl' dm' \cdot \int_{sky} J_j^* s_j^* \, dl dm \right\rangle_{\Delta t}$$

  - In integral, only $s_i(t,l,m)$, from the same directions correlate (i.e., when $l=l'$, $m=m'$), so order of integration and signal product can be reversed:

$$= \left\langle \int_{sky} J_i J_j^* s_i s_j^* \, dl dm \right\rangle_{\Delta t}$$

Correlation of Realistic Signals - II

- The s_i & s_j differ *only* by the relative arrival phase of signals from different parts of the sky, yielding the Fourier phase term (to a good approximation):

$$V_{ij} = \left\langle \int_{sky} J_i J_j^* s^2(t,l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm \right\rangle_{\Delta t}$$

- On the timescale of the averaging, the only meaningful average is of the *squared* signal itself (direction-dependent), which is just the image of the source:

$$= \int_{sky} J_i J_j^* \left\langle s^2(t,l,m) \right\rangle_{\Delta t} e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

$$= \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

- If all *J=1*, we of course recover the ideal expression:

$$= \int_{sky} I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

Aside: Auto-correlations and Single Dishes

- The *auto*-correlation of a signal from a *single* antenna:

$$\langle x_i \cdot x_i^* \rangle = \langle (s_i' + n_i) \cdot (s_i' + n_i)^* \rangle$$

$$= \langle s_i' \cdot s_i'^* \rangle + \langle n_i \cdot n_i^* \rangle$$

$$= \left\langle \int_{sky} |J_i|^2 |s_i|^2 \, dl dm \right\rangle + \langle |n_i|^2 \rangle$$

$$= \left\langle \int_{sky} |J_i|^2 I(l,m) \, dl dm \right\rangle + \langle |n_i|^2 \rangle$$

  - This is an integrated power measurement plus noise
  - Desired signal *not* isolated from noise
  - Noise usually dominates
- Single dish radio astronomy calibration strategies dominated by switching schemes to isolate desired signal from the noise

The Scalar Measurement Equation

$$V_{ij}^{obs} = \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

- First, isolate non-direction-dependent effects, and factor them from the integral:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} \left(J_i^{sky} J_j^{sky*}\right) I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

- Next, we recognize that over small fields of view, it is possible to assume $J^{sky}=1$, and we have a relationship between ideal and observed Visibilities:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

$$V_{ij}^{obs} = \left(J_i^{vis} J_j^{vis*}\right) V_{ij}^{true} = J_i J_j^* V_{ij}^{true}$$

- Standard calibration of most existing arrays reduces to solving this last equation for the J_i

Solving for the J_i

- We can write: $$\frac{V_{ij}^{obs}}{V_{ij}^{true}} - \left(J_i J_j^*\right) = 0$$

- …and define chi-squared: $$\chi^2 = \sum_{\substack{i,j \\ i \neq j}} \left| \frac{V_{ij}^{obs}}{V_{ij}^{true}} - \left(J_i J_j^*\right) \right|^2 w_{ij}$$

- …and minimize chi-squared w.r.t. each *Ji*, yielding (iteration):

$$J_i = \sum_{\substack{j \\ j \neq i}} \left(\frac{V_{ij}^{obs}}{V_{ij}^{true}} J_j w_{ij} \right) \Bigg/ \sum_{\substack{j \\ j \neq i}} \left(\left| J_j \right|^2 w_{ij} \right) \qquad \left(\frac{\partial \chi^2}{\partial J_i^*} = 0 \right)$$

- …which we recognize as a weighted average of J_i, itself:

$$J_i = \sum_{\substack{j \\ j \neq i}} \left(J_i' w_{ij}' \right) \Bigg/ \sum_{\substack{j \\ j \neq i}} w_{ij}'$$

Solving for J_i (cont)

- For a uniform array (same sensitivity on all baselines, ~same calibration magnitude on all antennas), it can be shown that the error in the calibration solution is:

$$\sigma_{J_i} \approx \sigma_{V^{obs}}(\Delta t) \Big/ V^{true} \langle J \rangle \sqrt{N_{ant} - 1}$$

- SNR improves with calibrator strength and square-root of N_{ant} (c.f. baseline-based calibration).
- Other properties of the antenna-based solution:
  - Minimal degrees of freedom (N_{ant} factors, $N_{ant}(N_{ant}-1)/2$ measurements)
  - Constraints arise from both antenna-basedness and consistency with a variety of (baseline-based) visibility measurements in which each antenna participates
  - Net calibration for a baseline involves a phase difference, so absolute directional information is lost
  - Closure…

Antenna-based Calibration and Closure

- Success of synthesis telescopes relies on antenna-based calibration
  - Fundamentally, any information that can be factored into antenna-based terms, could *be* antenna-based effects, and not source visibility
  - For $N_{ant} > 3$, source visibility cannot be entirely obliterated by any antenna-based calibration
- Observables independent of antenna-based calibration:
  - Closure phase (3 baselines):

$$\phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} = \left(\phi_{ij}^{true} + \theta_i - \theta_j\right) + \left(\phi_{jk}^{true} + \theta_j - \theta_k\right) + \left(\phi_{ki}^{true} + \theta_k - \theta_i\right)$$

$$= \phi_{ij}^{true} + \phi_{jk}^{true} + \phi_{ki}^{true}$$

  - Closure amplitude (4 baselines):

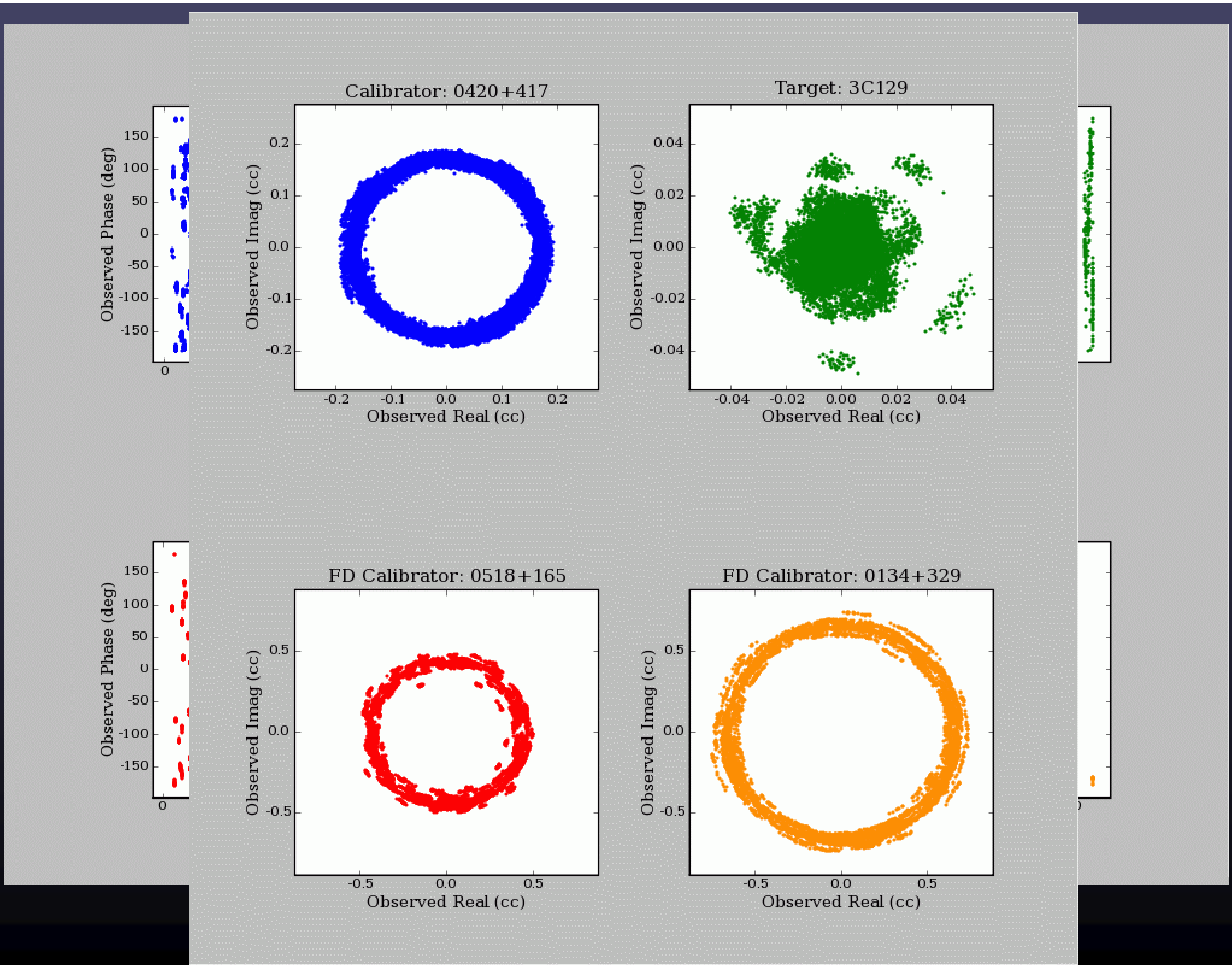
$$\left|\frac{V_{ij}^{obs} V_{kl}^{obs}}{V_{ik}^{obs} V_{jl}^{obs}}\right| = \left|\frac{J_i J_j V_{ij}^{true} J_k J_l V_{kl}^{true}}{J_i J_k V_{ik}^{true} J_j J_l V_{jl}^{true}}\right| = \left|\frac{V_{ij}^{true} V_{kl}^{true}}{V_{ik}^{true} V_{jl}^{true}}\right|$$

- Baseline-based calibration formally violates closure!

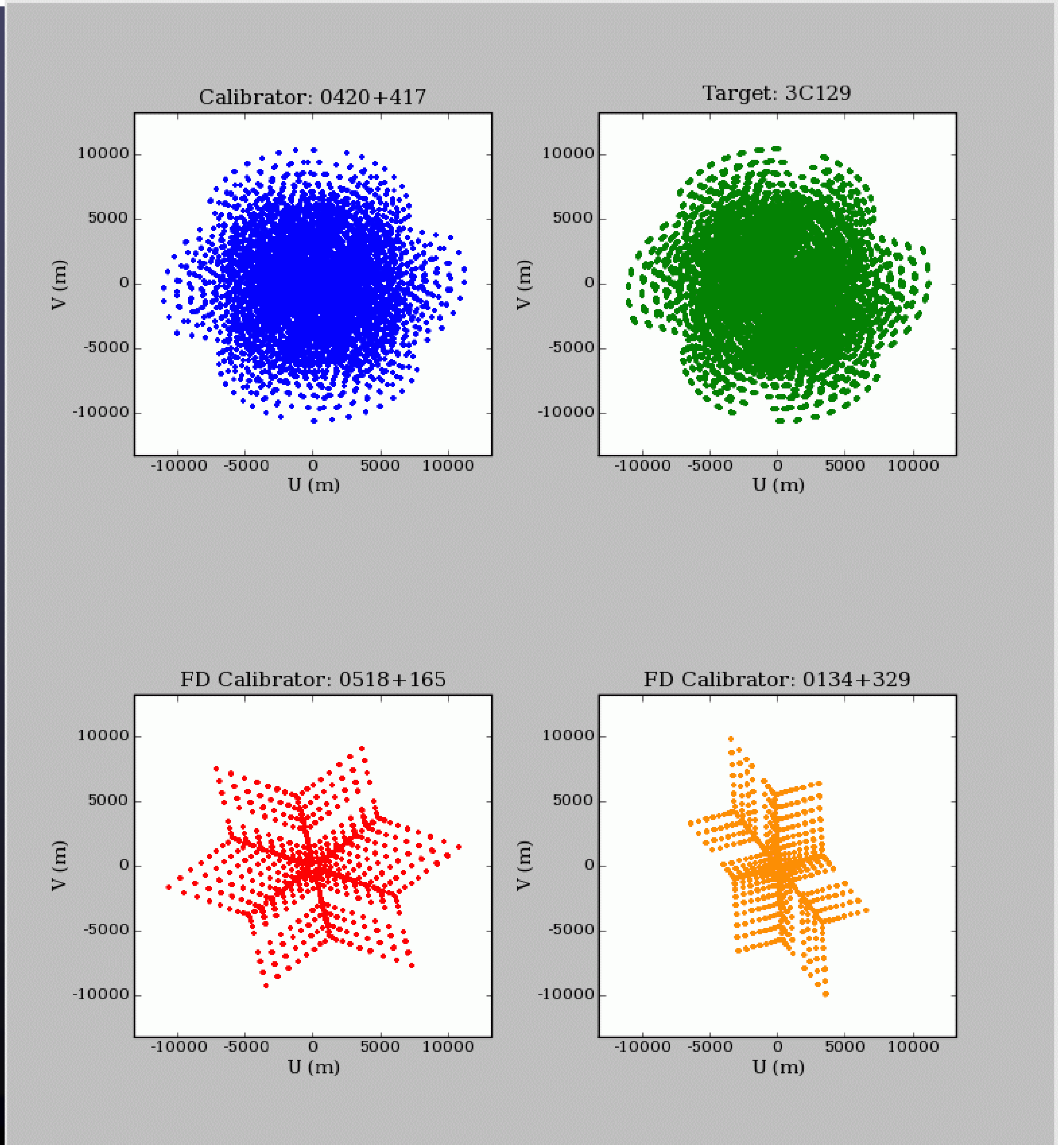
Simple Scalar Calibration Example

- Sources:
  - Science Target: 3C129
  - Near-target calibrator: 0420+417 (5.5 deg from target; unknown flux density, assumed 1 Jy)
  - Flux Density calibrators: 0134+329 (3C48: 5.74 Jy), 0518+165 (3C138: 3.86 Jy), both resolved (use standard model images)
- Signals:
  - RR correlation only (total intensity only)
  - 4585.1 MHz, 50 MHz bandwidth (single channel)
  - (scalar version of a continuum polarimetry observation)
- Array:
  - VLA B-configuration (July 1994)

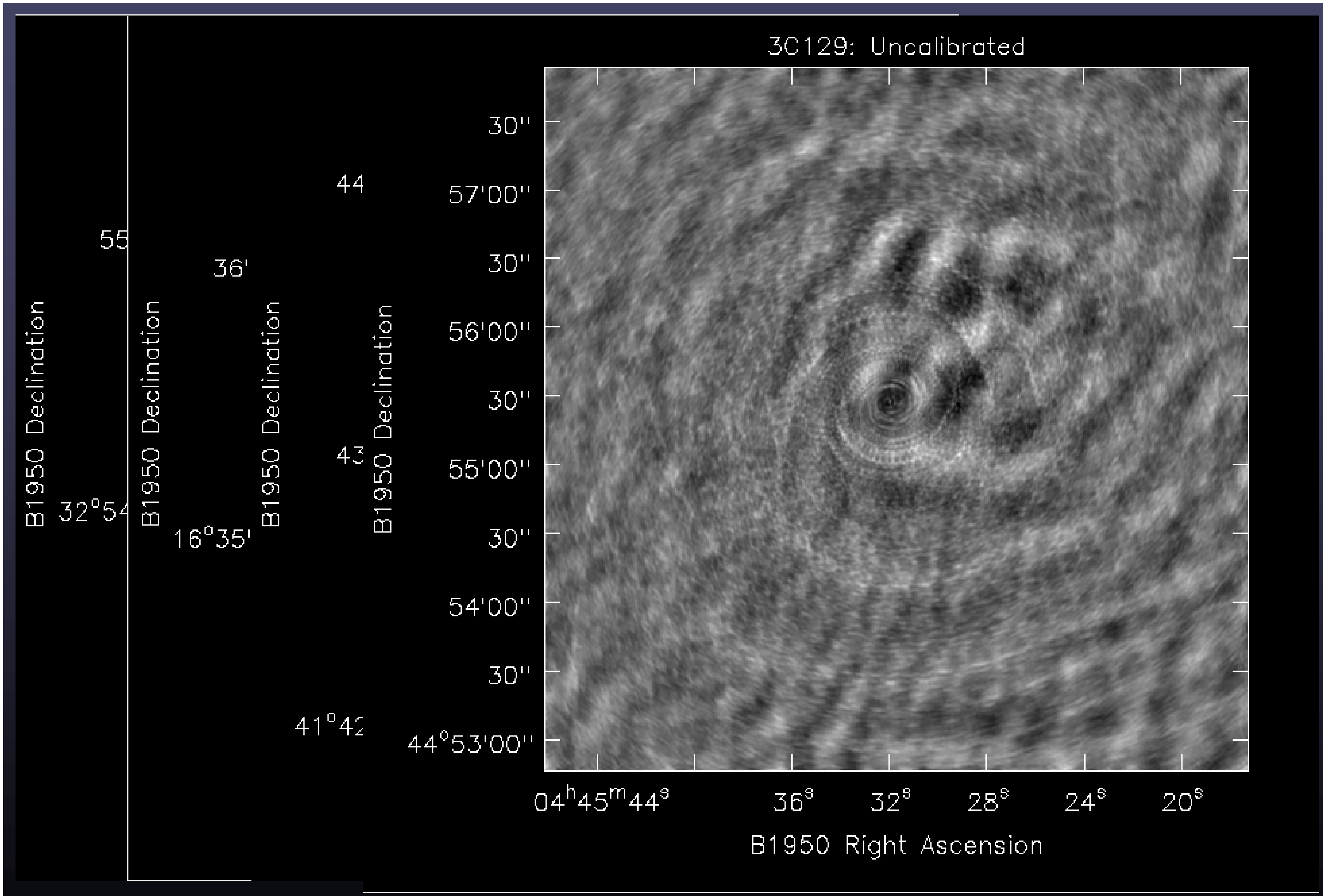
Views of the Uncalibrated Data



UV-Coverages



Uncalibrated Images



The Calibration Process

- Solve for antenna-based gain factors for each scan on all calibrators:

$$V_{ij}^{obs} = \left(J_i J_j^*\right) V_{ij}^{true}$$

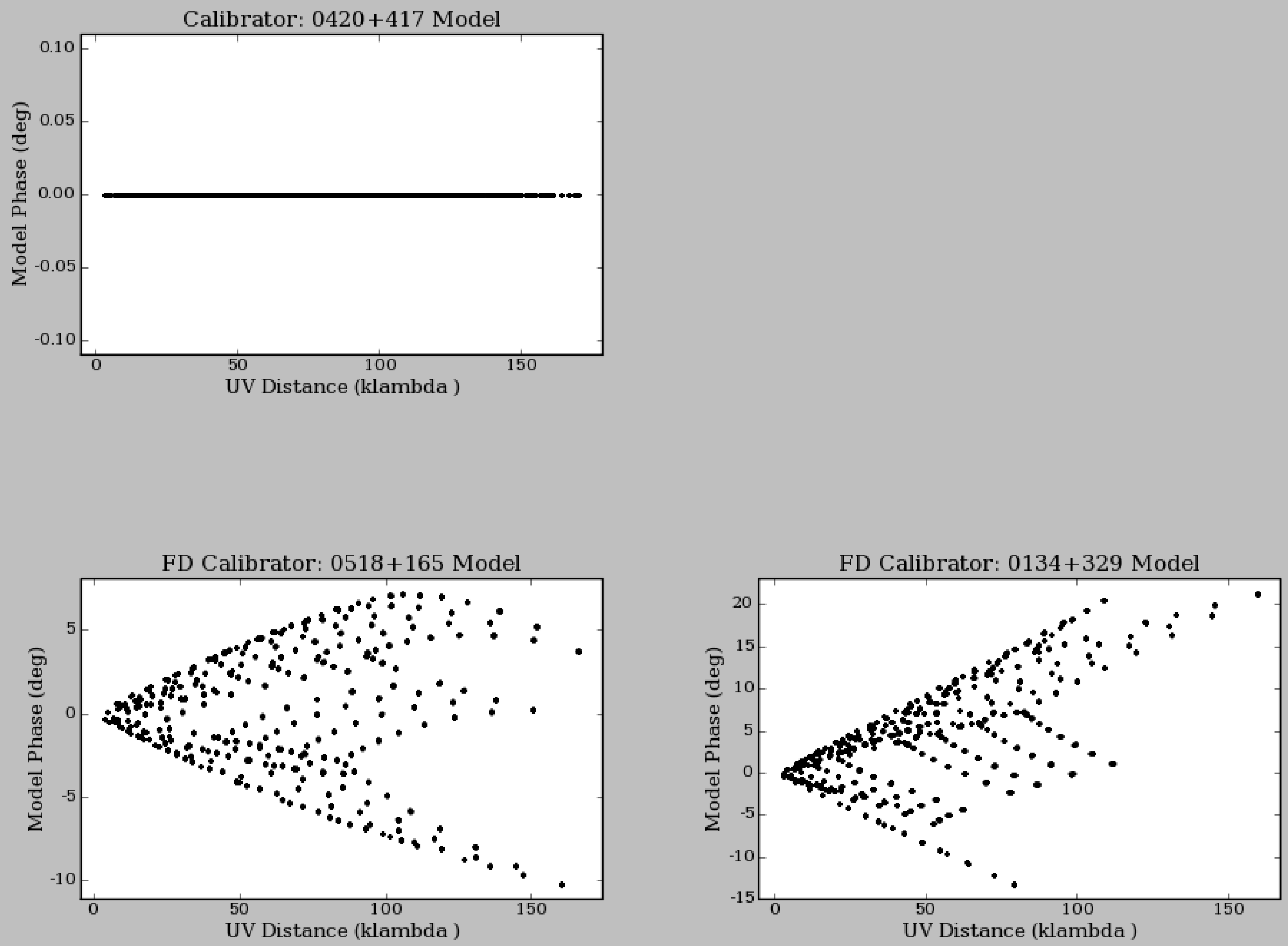
- Bootstrap flux density scale by enforcing constant mean power response:

$$\left|J'_{i(nt)}\right| = \left|J_{i(nt)}\right| \left(\frac{\left\langle \left|J_{i(fd)}\right|^2 \right\rangle_i}{\left\langle \left|J_{i(nt)}\right|^2 \right\rangle_i} \right)^{1/2}$$

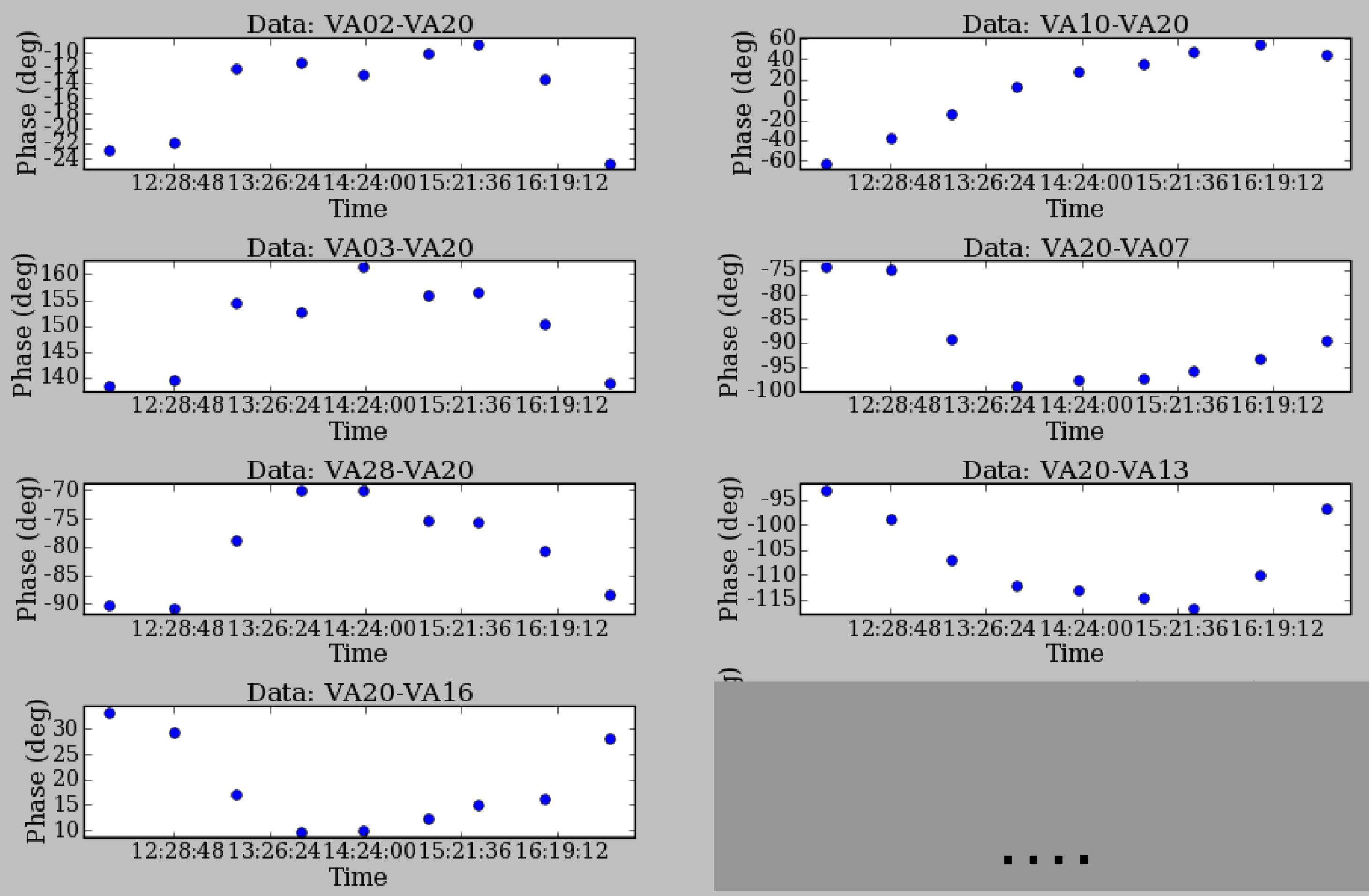
- Correct data (interpolate, as needed):

$$V_{ij}^{corrected} = \left(J'^{-1}_i J'^{*-1}_j\right) V_{ij}^{obs}$$

A priori Models Required for Calibrators

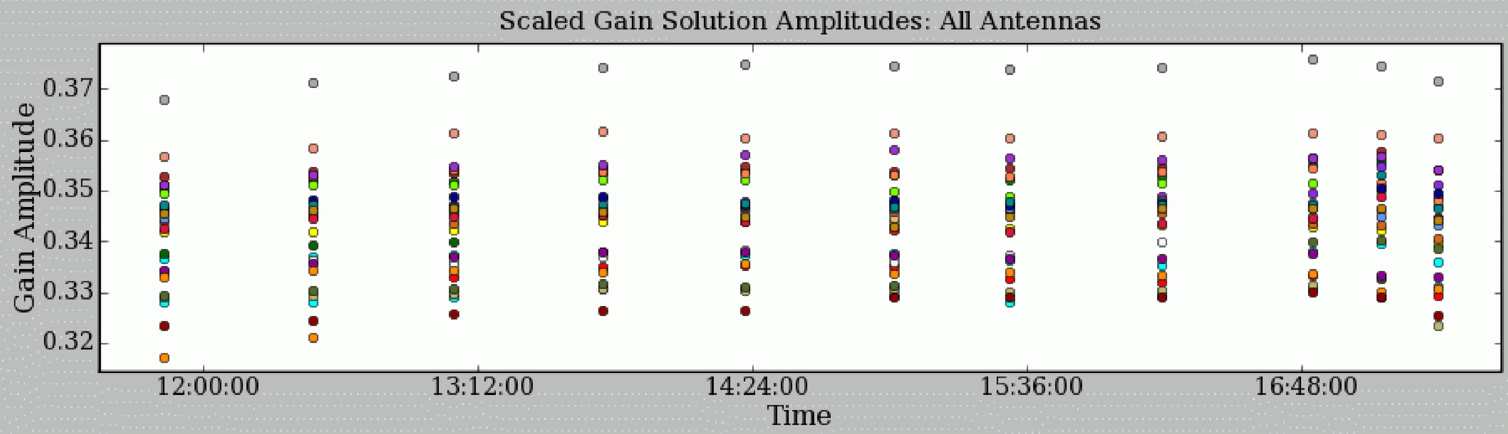


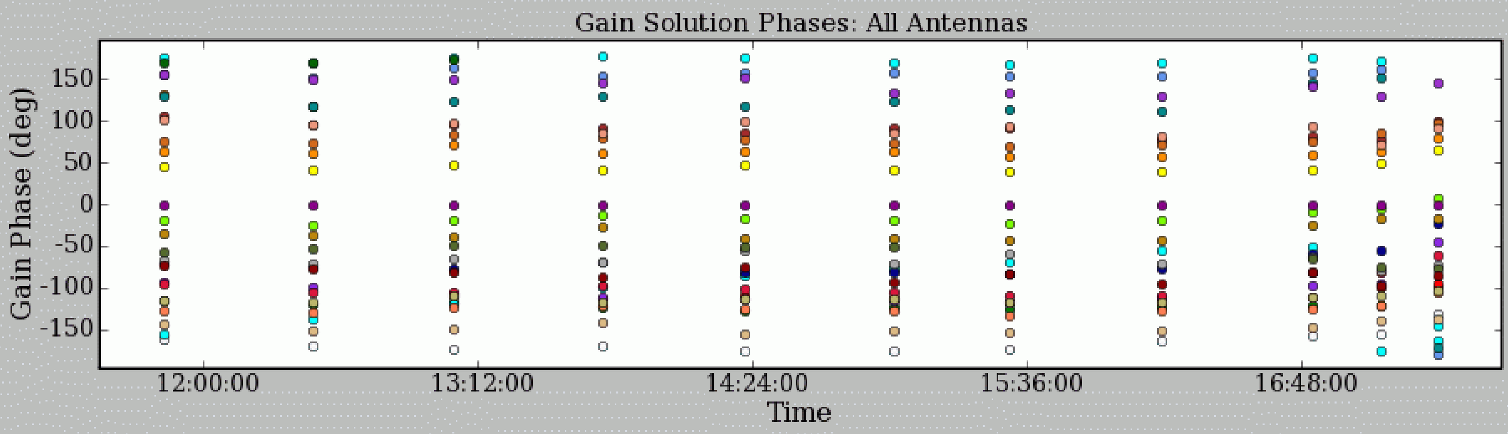
Rationale for Antenna-based Calibration



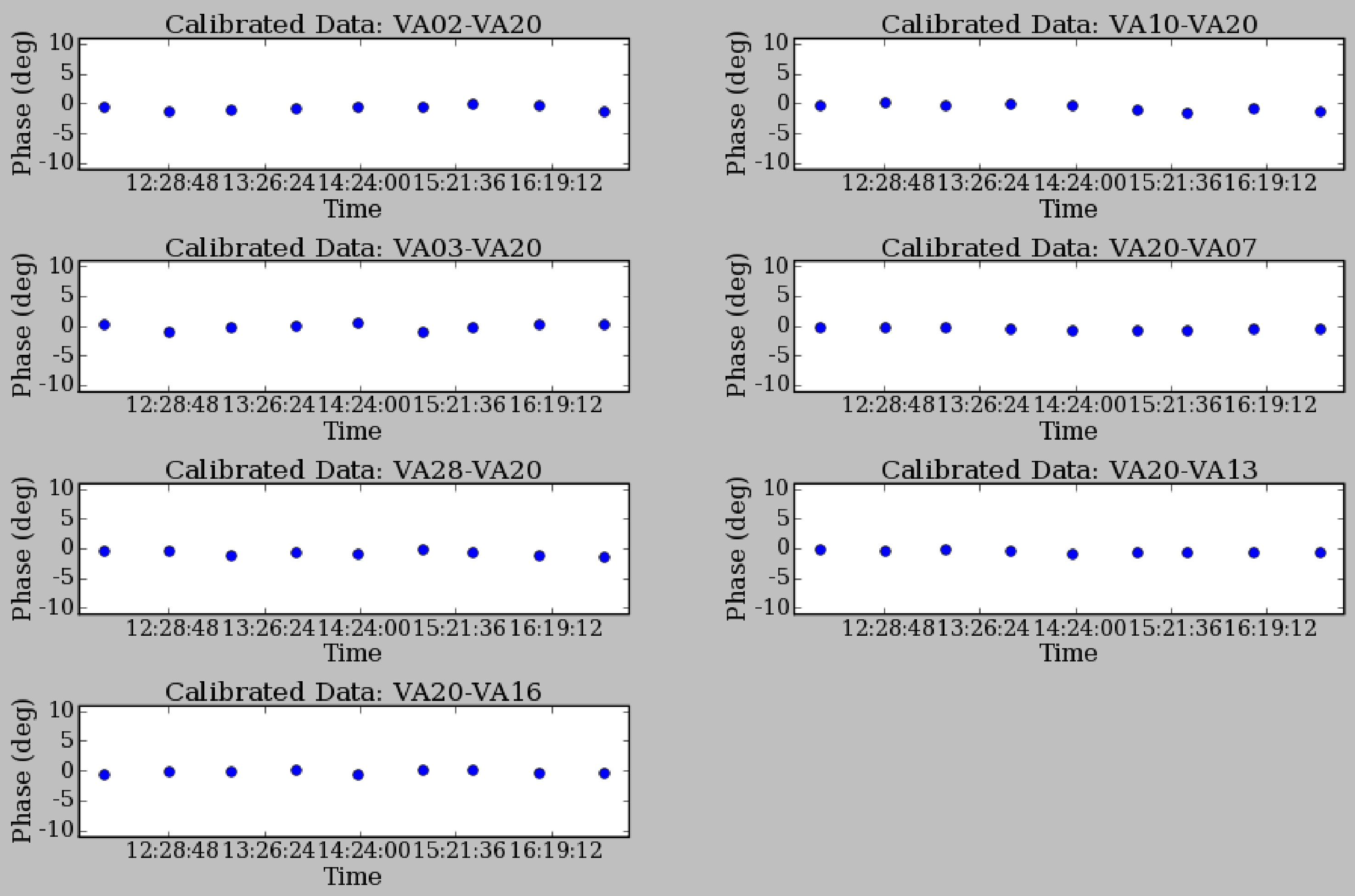
Baselines to antenna VA20 (0420+417 only, scan averages)

The Antenna-based Calibration Solution



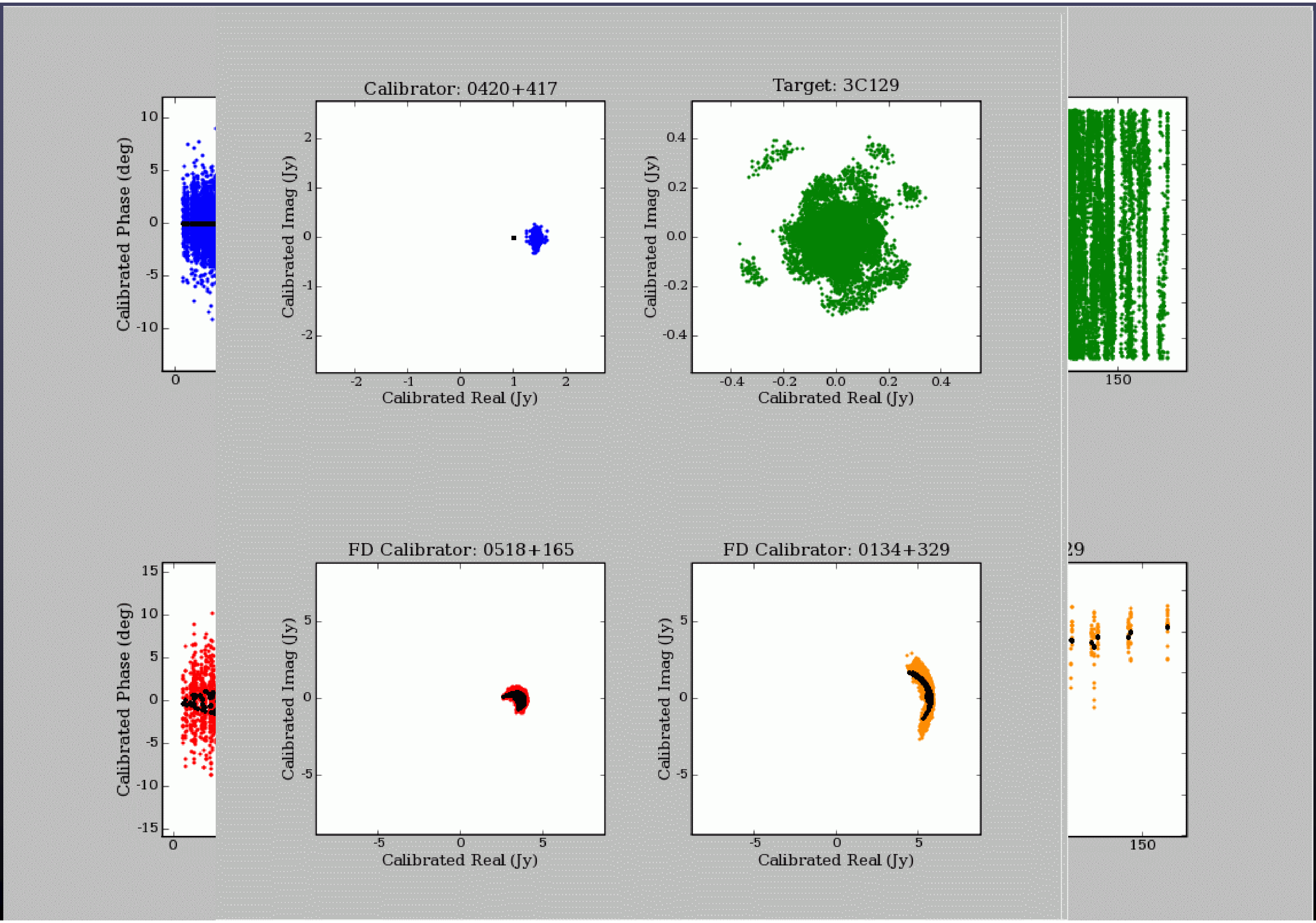


Did Antenna-based Calibration Work?

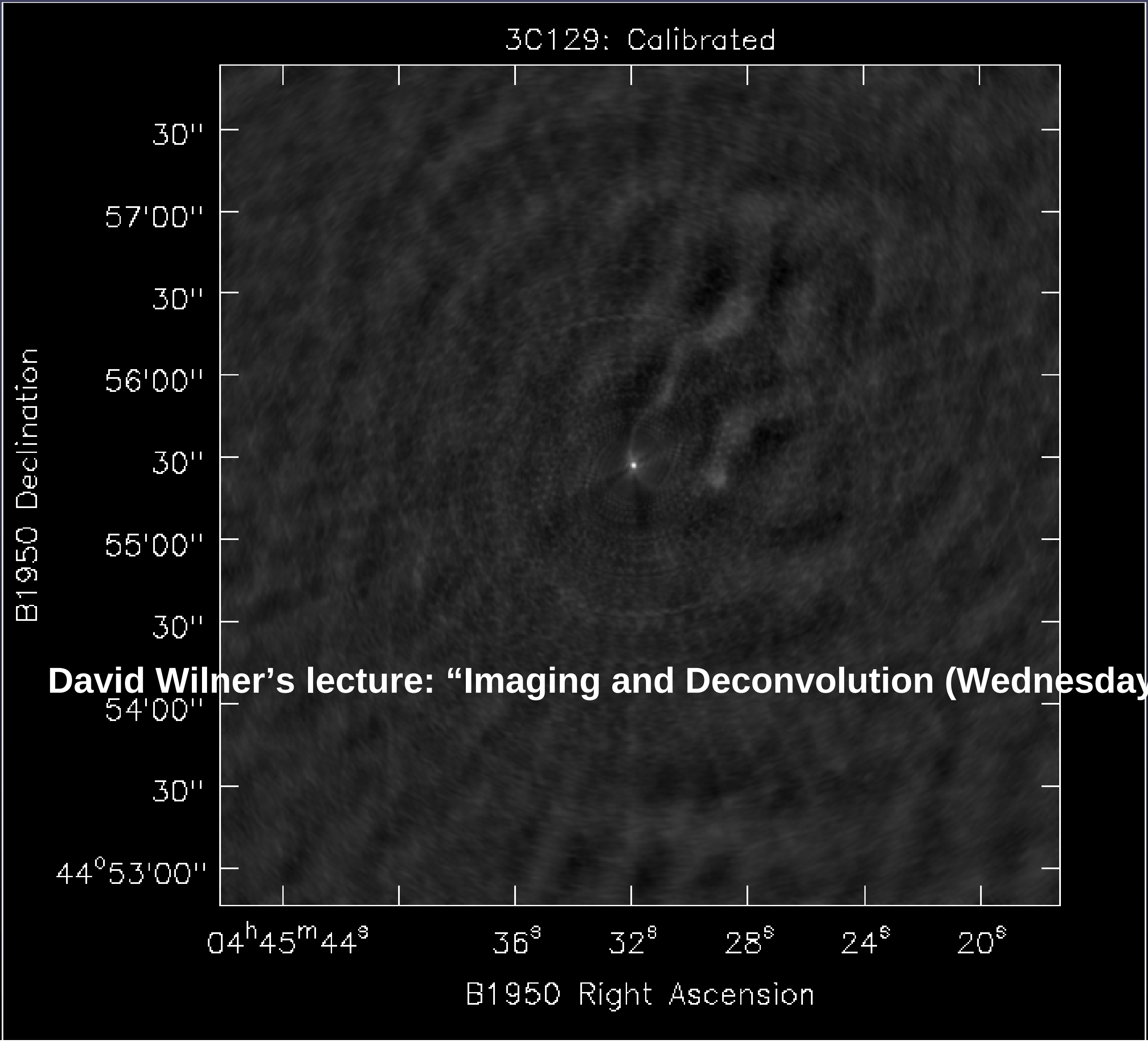


Baselines to antenna VA20 (0420+417 only)

Antenna-based Calibration Visibility Result



Antenna-based Calibration Image Result



David Wilner's lecture: "Imaging and Deconvolution (Wednesday)

Evaluating Calibration Performance

- Are solutions continuous?
  - Noise-like solutions are just that—noise
  - Discontinuities indicate instrumental glitches
  - Any additional editing required?
- Are calibrator data fully described by antenna-based effects?
  - Phase and amplitude *closure errors* are the baseline-based residuals
  - Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration components
    - Mark Claussen's lecture: "Advanced Calibration" (Wednesday)
- Any evidence of unsampled variation? Is interpolation of solutions appropriate?
  - Reduce calibration timescale, if SNR permits
- Ed Fomalont's lecture: "Error Recognition" (Wednesday)

Summary of Scalar Example

- Dominant calibration effects are ***antenna-based***
  - Minimizes degrees of freedom
  - More precise
  - Preserves closure
  - Permits higher dynamic range *safely*!

- Point-like calibrators effective
- Flux density bootstrapping

Full-Polarization Formalism (Matrices!)

- Need dual-polarization basis (*p,q*) to fully sample the incoming EM wave front, where *p,q* = *R,L* (circular basis) or *p,q* = *X,Y* (linear basis):

$$\vec{I}_{circ} = \overleftrightarrow{S}_{circ}\vec{I}_{Stokes}$$

$$\begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

$$\vec{I}_{lin} = \overleftrightarrow{S}_{lin}\vec{I}_{Stokes}$$

$$\begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

- Devices can be built to sample these linear or circular basis states in the signal domain (Stokes Vector is defined in “power” domain)
- Some components of J_i involve mixing of basis states, so dual-polarization matrix description desirable or even required for proper calibration

Full-Polarization Formalism: Signal Domain

- Substitute:

$$s_i \rightarrow \vec{s}_i = \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i, \quad J_i \rightarrow \overleftrightarrow{J}_i = \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}$$

- The *Jones matrix* thus corrupts the vector wavefront signal as follows:

$$\vec{s}_i' = \overleftrightarrow{J}_i \vec{s}_i \qquad \text{(sky integral omitted)}$$

$$\begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i = \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}_i \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i$$

$$= \begin{pmatrix} J^{p\rightarrow p} s^p + J^{q\rightarrow p} s^q \\ J^{p\rightarrow q} s^p + J^{q\rightarrow q} s^q \end{pmatrix}_i$$

Full-Polarization Formalism: Correlation - I

- Four correlations are possible from two polarizations. The *outer product* (a ‘bookkeeping’ product) represents correlation in the matrix formalism:

$$\vec{V}_{ij}^{obs} = \left\langle \vec{s}_i' \otimes \vec{s}_j'^{*} \right\rangle = \left\langle \begin{pmatrix} s'^{p} \\ s'^{q} \end{pmatrix}_i \otimes \begin{pmatrix} s'^{p} \\ s'^{q} \end{pmatrix}_j^{*} \right\rangle = \begin{pmatrix} \left\langle s_i'^{p} \cdot s_j'^{*p} \right\rangle \\ \left\langle s_i'^{p} \cdot s_j'^{*q} \right\rangle \\ \left\langle s_i'^{q} \cdot s_j'^{*p} \right\rangle \\ \left\langle s_i'^{q} \cdot s_j'^{*q} \right\rangle \end{pmatrix}$$

- A very useful property of outer products:

$$\vec{V}_{ij}^{obs} = \left(\vec{s}_i' \otimes \vec{s}_j'^{*}\right) = \left(\overleftrightarrow{J}_i \vec{s}_i\right) \otimes \left(\overleftrightarrow{J}_j^{*} \vec{s}_j^{*}\right) = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^{*}\right)\left(\vec{s}_i \otimes \vec{s}_j^{*}\right) = \overleftrightarrow{J}_{ij} \vec{V}_{ij}^{true}$$

Full-Polarization Formalism: Correlation - II

- The outer product for the Jones matrix:

$$\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^* = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_i \otimes \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_j^*$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} & J_i^{p\to p} J_j^{*q\to p} & J_i^{q\to p} J_j^{*p\to p} & J_i^{q\to p} J_j^{*q\to p} \\ J_i^{p\to p} J_j^{*p\to q} & J_i^{p\to p} J_j^{*q\to q} & J_i^{q\to p} J_j^{*p\to q} & J_i^{q\to p} J_j^{*q\to q} \\ J_i^{p\to q} J_j^{*p\to p} & J_i^{p\to q} J_j^{*q\to p} & J_i^{q\to q} J_j^{*p\to p} & J_i^{q\to q} J_j^{*q\to p} \\ J_i^{p\to q} J_j^{*p\to q} & J_i^{p\to q} J_j^{*q\to q} & J_i^{q\to q} J_j^{*p\to q} & J_i^{q\to q} J_j^{*q\to q} \end{pmatrix} = \overleftrightarrow{J}_{ij}$$

  – J_{ij} is a 4x4 *Mueller matrix*
  – Antenna and array design driven by minimizing off-diagonal terms!

Full-Polarization Formalism: Correlation - III

- And finally, for fun, the correlation of corrupted signals:

$$\overleftrightarrow{J}_i \vec{s}_i \otimes \overleftrightarrow{J}_j^* \vec{s}_j^* = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^*\right)\left(\vec{s}_i \otimes \vec{s}_j^*\right)$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} & J_i^{p\to p} J_j^{*q\to p} & J_i^{q\to p} J_j^{*p\to p} & J_i^{q\to p} J_j^{*q\to p} \\ J_i^{p\to p} J_j^{*p\to q} & J_i^{p\to p} J_j^{*q\to q} & J_i^{q\to p} J_j^{*p\to q} & J_i^{q\to p} J_j^{*q\to q} \\ J_i^{p\to q} J_j^{*p\to p} & J_i^{p\to q} J_j^{*q\to p} & J_i^{q\to q} J_j^{*p\to p} & J_i^{q\to q} J_j^{*q\to p} \\ J_i^{p\to q} J_j^{*p\to q} & J_i^{p\to q} J_j^{*q\to q} & J_i^{q\to q} J_j^{*p\to q} & J_i^{q\to q} J_j^{*q\to q} \end{pmatrix} \begin{pmatrix} s_i^p \cdot s_j^{*p} \\ s_i^p \cdot s_j^{*q} \\ s_i^q \cdot s_j^{*p} \\ s_i^q \cdot s_j^{*q} \end{pmatrix}$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} s_i^p \cdot s_j^{*p} + J_i^{p\to p} J_j^{*q\to p} s_i^p \cdot s_j^{*q} + J_i^{q\to p} J_j^{*p\to p} s_i^q \cdot s_j^{*p} + J_i^{q\to p} J_j^{*q\to p} s_i^q \cdot s_j^{*q} \\ J_i^{p\to p} J_j^{*p\to q} s_i^p \cdot s_j^{*p} + J_i^{p\to p} J_j^{*q\to q} s_i^p \cdot s_j^{*q} + J_i^{q\to p} J_j^{*p\to q} s_i^q \cdot s_j^{*p} + J_i^{q\to p} J_j^{*q\to q} s_i^q \cdot s_j^{*q} \\ J_i^{p\to q} J_j^{*p\to p} s_i^p \cdot s_j^{*p} + J_i^{p\to q} J_j^{*q\to p} s_i^p \cdot s_j^{*q} + J_i^{q\to q} J_j^{*p\to p} s_i^q \cdot s_j^{*p} + J_i^{q\to q} J_j^{*q\to p} s_i^q \cdot s_j^{*q} \\ J_i^{p\to q} J_j^{*p\to q} s_i^p \cdot s_j^{*p} + J_i^{p\to q} J_j^{*q\to q} s_i^p \cdot s_j^{*q} + J_i^{q\to q} J_j^{*p\to q} s_i^q \cdot s_j^{*p} + J_i^{q\to q} J_j^{*q\to q} s_i^q \cdot s_j^{*q} \end{pmatrix}$$

- UGLY, but we rarely, if ever, need to worry about detail at this level---just let this occur “inside” the matrix formalism, and work with the notation

The Matrix Measurement Equation

- We can now write down the Measurement Equation in matrix notation:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^{*} \right) \overleftrightarrow{S} \vec{I}(l,m) e^{-i2\pi\left(u_{ij}l + v_{ij}m\right)} dldm$$

- …and consider how the J_i are products of many effects.

A Dictionary of Calibration Components

- J_i contains many components:
  - F = ionospheric effects
  - T = tropospheric effects
  - P = parallactic angle
  - X = linear polarization position angle
  - E = antenna voltage pattern
  - D = polarization leakage
  - G = electronic gain
  - B = bandpass response
  - K = geometric compensation
  - M, A = baseline-based corrections

$$\overleftrightarrow{J}_i = \overleftrightarrow{K}_i \overleftrightarrow{B}_i \overleftrightarrow{G}_i \overleftrightarrow{D}_i \overleftrightarrow{E}_i \overleftrightarrow{X}_i \overleftrightarrow{P}_i \overleftrightarrow{T}_i \overleftrightarrow{F}_i$$

- Order of terms follows signal path (right to left)
- Each term has matrix form of J_i with terms embodying its particular algebra (on- vs. off-diagonal terms, etc.)
- Direction-dependent terms must stay inside FT integral
- Full calibration is traditionally a bootstrapping process wherein relevant terms are considered in decreasing order of dominance, relying on approximate orthogonality

Ionospheric Effects, *F*

- The ionosphere introduces a dispersive phase shift:

$$\Delta\phi \approx 0.15\ \lambda^2 \int B_{\|} n_e ds\ \ \mathrm{deg}$$

$$\lambda \text{ in cm}, \quad n_e ds \text{ in } 10^{14}\,\mathrm{cm}^{-2}, \quad B_{\|} \text{ in G}$$

$$TEC = \int n_e ds\ \sim 10^{14}\,\mathrm{cm}^{-2}; \quad B_{\|} \sim 1\mathrm{G}; \quad \lambda = 20\mathrm{cm} \rightarrow \Delta\phi \sim 60^{\circ}$$

  - More important at longer wavelengths (λ^2)
  - More important at solar maximum and at sunrise/sunset, when ionosphere is most active and variable
  - Beware of direction-dependence within field-of-view!
  - The ionosphere is *birefringent*; one hand of circular polarization is delayed w.r.t. the other, thus rotating the linear polarization position angle

  – Tracy Clark’s lecture: “Low Frequency Interferometry” (Monday)

$$\overleftrightarrow{F}^{RL} = e^{i\Delta\phi}\begin{pmatrix} e^{i\varepsilon} & 0 \\ 0 & e^{-i\varepsilon} \end{pmatrix}; \ \overleftrightarrow{F}^{XY} = e^{i\Delta\phi}\begin{pmatrix} \cos\varepsilon & -\sin\varepsilon \\ \sin\varepsilon & \cos\varepsilon \end{pmatrix}$$

Tropospheric Effects, *T*

- The troposphere causes polarization-independent amplitude and phase effects due to emission/opacity and refraction, respectively
    - Typically 2-3m excess path length at zenith compared to vacuum
    - Higher noise contribution, less signal transmission: Lower SNR
    - Most important at ν > 20 GHz where water vapor and oxygen absorb/emit
    - More important nearer horizon where tropospheric path length greater
    - Clouds, weather = variability in phase and opacity; may vary across array
    - Water vapor radiometry? Phase transfer from low to high frequencies?
    - Zenith-angle-dependent parameterizations?
  - Crystal Brogan's lecture: "Millimeter Interferometry and ALMA" (Monday)

$$\overleftrightarrow{T}^{pq} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Parallactic Angle, *P*

- Visibility phase variation due to changing orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos(l)\sin(h(t))}{\sin(l)\cos(\delta) - \cos(l)\sin(\delta)\cos(h(t))}\right)$$

$$l = \text{latitude},\ \ h(t) = \text{hour angle},\ \ \delta = \text{declination}$$

  - Rotates the position angle of linearly polarized radiation
  - Analytically known, and its variation provides leverage for determining polarization-dependent effects
  - Position angle calibration can be viewed as an offset in χ

– Steve Myers' lecture: "Polarization in Interferometry" (today!)

$$\overleftrightarrow{P}^{RL} = \begin{pmatrix} e^{i\chi} & 0 \\ 0 & e^{-i\chi} \end{pmatrix};\ \overleftrightarrow{P}^{XY} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}$$

Linear Polarization Position Angle, *X*

- Configuration of optics and electronics causes a linear polarization position angle offset
- Same algebraic form as *P*
- Calibrated by registration with a source of known polarization position angle
- For linear feeds, this is the orientation of the dipoles in the frame of the telescope

  – Steve Myers' lecture: "Polarization in Interferometry" (today!)

$$\overset{\leftrightarrow}{X}^{RL} = \begin{pmatrix} e^{i\Delta\chi} & 0 \\ 0 & e^{-i\Delta\chi} \end{pmatrix}; \;\; \overset{\leftrightarrow}{X}^{XY} = \begin{pmatrix} \cos\Delta\chi & -\sin\Delta\chi \\ \sin\Delta\chi & \cos\Delta\chi \end{pmatrix}$$

Antenna Voltage Pattern, *E*

- Antennas of all designs have direction-dependent gain
    - Important when region of interest on sky comparable to or larger than λ/D
    - Important at lower frequencies where radio source surface density is greater and wide-field imaging techniques required
    - Beam squint: E^p and E^q offset, yielding spurious polarization
    - For convenience, direction dependence of polarization leakage (*D*) may be included in *E* (off-diagonal terms then non-zero)
  - – Rick Perley's lecture: "Wide Field Imaging I" (Thursday)
  - – Debra Shepherd's lecture: "Wide Field Imaging II" (Thursday)

$$E^{pq} = \begin{pmatrix} e^p(l,m) & 0 \\ 0 & e^q(l,m) \end{pmatrix}$$

Polarization Leakage, *D*

- Antenna & polarizer are not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed feeds have *d* ~ a few percent or less
  - A geometric property of the optical design, so frequency-dependent
  - For *R,L* systems, total-intensity imaging affected as *~dQ, dU,* so only important at high dynamic range (*Q,U,d each ~few %*, typically)
  - For *R,L* systems, linear polarization imaging affected as *~dI,* so almost always important
- Best calibrator: Strong, point-like, observed over large range of parallactic angle (to separate source polarization from *D*)

  – Steve Myers' lecture: "Polarization in Interferometry" (today!)

$$\overset{\leftrightarrow}{D}^{pq} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

“Electronic” Gain, *G*

- Catch-all for most amplitude and phase effects introduced by antenna electronics and other generic effects
    - Most commonly treated calibration component
    - Dominates other effects for standard VLA observations
    - Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
    - Often also includes ionospheric and tropospheric effects which are typically difficult to separate unto themselves
    - Excludes frequency dependent effects (see *B*)
- Best calibrator: strong, point-like, near science target; observed often enough to track expected variations
    - Also observe a flux density standard

$$G^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

Bandpass Response, *B*

- *G*-like component describing frequency-dependence of antenna electronics, etc.
    - Filters used to select frequency passband not square
    - Optical and electronic reflections introduce ripples across band
    - Often assumed time-independent, but not necessarily so
    - Typically (but not necessarily) normalized
- Best calibrator: strong, point-like; observed long enough to get sufficient per-channel SNR, and often enough to track variations
    - – Ylva Pihlstrom's lecture: "Spectral Line Observing" (Wednesday)

$$B^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

Geometric Compensation, *K*

- Must get geometry right for Synthesis Fourier Transform relation to work in real time; residual errors here require "Fringe-fitting"
  - Antenna positions (geodesy)
  - Source directions (time-dependent in topocenter!) (astrometry)
  - Clocks
  - Electronic pathlengths
  - Longer baselines generally have larger relative geometry errors, especially if clocks are independent (VLBI)
  - Importance scales with frequency
- *K* is a clock- & geometry-parameterized version of *G* (see chapter 5, section 2.1, equation 5-3 & chapters 22, 23)
  - – Shep Doeleman's lecture: "Very Long Baseline Interferometry" (Thursday)

$$K^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

Non-closing Effects: *M, A*

- Baseline-based errors which do not decompose into antenna-based components
  - Digital correlators designed to limit such effects to well-understood and ***uniform*** (not dependent on baseline) scaling laws (absorbed in *G*)
  - Simple noise (additive)
  - Additional errors can result from averaging in time and frequency over variation in antenna-based effects and visibilities (practical instruments are finite!)
  - Correlated “noise” (e.g., RFI)
  - Difficult to distinguish from source structure (visibility) effects
  - Geodetic observers consider determination of radio source structure—a baseline-based effect—as a required *calibration* if antenna positions are to be determined accurately
  - Diagonal 4x4 matrices, M_{ij} multiplies, A_{ij} adds

The Full Matrix Measurement Equation

- The total general *Measurement Equation* has the form:

$$\vec{V}_{ij} = \overleftrightarrow{M}_{ij}\overleftrightarrow{K}_{ij}\overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\int_{sky}\overleftrightarrow{D}_{ij}\overleftrightarrow{E}_{ij}\overleftrightarrow{P}_{ij}\overleftrightarrow{T}_{ij}\overleftrightarrow{F}_{ij}\ \overleftrightarrow{S}\vec{I}(l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dl\,dm + \vec{A}_{ij}$$

  - *S* maps the Stokes vector, *I*, to the polarization basis of the instrument, all calibration terms cast in this basis

- Suppressing the direction-dependence:

$$\vec{V}_{ij}^{obs} = \overleftrightarrow{M}_{ij}\overleftrightarrow{K}_{ij}\overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\overleftrightarrow{D}_{ij}\overleftrightarrow{X}_{ij}\overleftrightarrow{P}_{ij}\overleftrightarrow{T}_{ij}\overleftrightarrow{F}_{ij}\vec{V}_{ij}^{true} + \vec{A}_{ij}$$

- Generally, only a subset of terms (up to 3 or 4) are considered, though highest-dynamic range observations may require more
- Solve for terms in decreasing order of dominance

Solving the Measurement Equation

- Formally, solving for any antenna-based visibility calibration component is always the same non-linear fitting problem:

$$V_{ij}^{corrected \cdot obs} = \left(J_i^{solve} J_j^{solve*} \right) V_{ij}^{corrupted \cdot true}$$

  - Viability of the solution depends on isolation of different effects using *proper calibration observations*, and *appropriate solving strategies*

Calibration Heuristics – Spectral Line

- Spectral Line (B,G): $\vec{V}_{ij}^{obs} = \overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\vec{V}_{ij}^{true}$

1. Preliminary G solve on B-calibrator:

$$\vec{V}^{obs} = \underline{\overleftrightarrow{G}}\vec{V}^{true}$$

2. B Solve on B-calibrator:

$$\vec{V}^{obs} = \underline{\overleftrightarrow{B}}\left(\overleftrightarrow{G}\vec{V}^{true}\right)$$

3. G solve (using B) on G-calibrator:

$$\left(\overleftrightarrow{B}^{-1}\vec{V}^{obs}\right) = \underline{\overleftrightarrow{G}}\vec{V}^{true}$$

4. Flux Density scaling:

$$\left|\overleftrightarrow{G}'\right| = \left|\overleftrightarrow{G}\right|\left(\left\langle\left|\overleftrightarrow{G}_{fd}\right|^2\right\rangle \Big/ \left\langle\left|\overleftrightarrow{G}\right|^2\right\rangle\right)^{1/2}$$

5. Correct:

$$\vec{V}^{corrected} = \left(\overleftrightarrow{G}'^{-1}\overleftrightarrow{B}^{-1}\vec{V}^{obs}\right)$$

6. Image!

Calibration Heuristics – Continuum Polarimetry

- Continuum Polarimetry (G,D,X,P): $\vec{V}_{ij}^{obs} = \overleftrightarrow{G}_{ij}\overleftrightarrow{D}_{ij}\overleftrightarrow{X}_{ij}\overleftrightarrow{P}_{ij}\vec{V}_{ij}^{true}$

1. Preliminary G solve on GD-calibrator (using P):

$$\vec{V}^{obs} = \underline{\overleftrightarrow{G}}\left(\overleftrightarrow{P}\vec{V}^{true}\right)$$

2. D solve on GD-calibrator (using P, G):

$$\left(\overleftrightarrow{G}^{-1}\vec{V}^{obs}\right) = \underline{\overleftrightarrow{D}}\left(\overleftrightarrow{P}\vec{V}^{true}\right)$$

3. Polarization Position Angle Solve (using P,G,D):

$$\left(\overleftrightarrow{D}^{-1}\overleftrightarrow{G}^{-1}\vec{V}^{obs}\right) = \underline{\overleftrightarrow{X}}\left(\overleftrightarrow{P}\vec{V}^{true}\right)$$

4. Flux Density scaling:

$$\left|\overleftrightarrow{G}'\right| = \left|\overleftrightarrow{G}\right|\left(\left\langle\left|\overleftrightarrow{G}_{fd}\right|^2\right\rangle \Big/ \left\langle\left|\overleftrightarrow{G}\right|^2\right\rangle\right)^{1/2}$$

5. Correct:

$$\vec{V}^{corrected} = \left(\overleftrightarrow{P}^{-1}\overleftrightarrow{X}^{-1}\overleftrightarrow{D}^{-1}\overleftrightarrow{G}'^{-1}\vec{V}^{obs}\right)$$

6. Image!

New Calibration Challenges

- Bandpass Calibration
  - Parameterized solutions (narrow-bandwidth, high resolution regime)
  - Spectrum of calibrators (wide absolute bandwidth regime)
- Phase vs. Frequency (self-) calibration
  - Troposphere and Ionosphere introduce time-variable phase effects which are easily parameterized in frequency and should be (c.f. sampling the calibration in frequency)
- Frequency-dependent Instrumental Polarization
  - Contribution of geometric optics is wavelength-dependent (standing waves)
- Frequency-dependent Voltage Pattern
- Increased sensitivity: Can implied dynamic range be reached by conventional calibration and imaging techniques?

Why not just solve for generic J_i matrix?

- It has been proposed (Hamaker 2000, 2006) that we can self-calibrate the generic J_i matrix, apply “post-calibration” constraints to ensure consistency of the astronomical absolute calibrations, and recover full polarization measurements of the sky
- Important for low-frequency arrays where isolated calibrators are unavailable (such arrays see the whole sky)
- May have a role for EVLA & ALMA

- Currently under study…

Summary

- Determining calibration is as important as determining source structure—can't have one without the other
- Data examination and editing an important part of calibration
- Beware of RFI! (Please, no cell phones at the VLA site tour!)
- Calibration dominated by antenna-based effects, permits efficient separation of calibration from astronomical information (closure)
- Full calibration formalism algebra-rich, but is *modular*
- Calibration determination is a single standard fitting problem
- Calibration an iterative process, improving various components in turn, as needed
- Point sources are the best calibrators
- Observe calibrators according requirements of calibration components