Sensitivity

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Outline

- What is Sensitivity?
- Antenna Performance Measures
- Interferometer Sensitivity
- Sensitivity of a Synthesis Image
- Other factors
- Summary

What is Sensitivity & Why Should You Care?

- Measure of weakest detectable emission
- Important throughout research program
  - Sound observing proposal
  - Sensible error analysis in journal
- Expressed in units involving Janskys
  - Unit for interferometer is Jansky (Jy)
  - Unit for synthesis image is Jy beam^{-1}
  - 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}
- Common current units: milliJy, microJy
- Common future units: nanoJy

Measures of Antenna Performance

System Temperature

- Point at blank sky. What is received power *P* ?
- Write *P* as equivalent temperature *T* of matched termination at receiver input
  - Rayleigh-Jeans limit to Planck law $P = k_B \times T \times \Delta\nu$
  - Boltzmann constant k_B
  - Observing bandwidth $\Delta\nu$
- Amplify *P* by g^2 where *g* is voltage gain
- **noise power $P_N = g^2 \times k_B \times T_{sys} \times \Delta\nu$**
  - T_{sys} includes cosmic background, sky, receiver, ground …
  - *Position and time dependence*

Measures of Antenna Performance

Source (Antenna) Temperature T_a

- Point at source of interest
- Received $P = P_n + P_a$

  $T_a \Rightarrow$ source power $P_a = g^2 \times k_B \times T_a \times \Delta\nu$

Measures of Antenna Performance

Gain

- Source power $P_a = g^2 \times k_B \times T_a \times \Delta\nu$
  - Let $T_a = K \times S$ for source flux density S, constant K
  - Then $P_a = g^2 \times k_B \times K \times S \times \Delta\nu$ (1)
- But source power also $P_a = ½ \times g^2 \times \eta_a \times A \times S \times \Delta\nu$ (2)
  - Antenna area A, efficiency η_a
  - Rx accepts 1/2 radiation from unpolarized source (Malus' Law)
- Equate (1), (2) and solve for K

  $$K = (\eta_a \times A) / (2 \times k_B) = T_a / S$$

  - K is antenna's gain or "sensitivity", unit Kelvin Jy^{-1}
- K is "flux collection ability" → **Big K's (Figure of merit).**

Measures of Antenna Performance

System Equivalent Flux Density

- Antenna temperature $T_a = K \times S$
  - Source power $P_a = g^2 \times k_B \times K \times S \times \Delta\nu$
- Express system temperature analogously
  - Let $T_{sys} = K \times SEFD$
  - *SEFD* is system equivalent flux density, unit Jy
  - System noise power $P_N = g^2 \times k_B \times K \times SEFD \times \Delta\nu$
- *SEFD* measures overall antenna performance

$$SEFD = T_{sys} / K$$

  - **Small SEFD is better**

Interferometer Sensitivity

Real Correlator - 1

- Simple correlator with single real output that is product of voltages from antennas *j,i*
  - $SEFD_i = T_{sysi} / K_i$ and $SEFD_j = T_{sysj} / K_j$
  - Each antenna collects bandwidth $\Delta\nu$
- Interferometer built from these antennas has
  - Accumulation time τ_{acc}, system efficiency η_s
  - Source, system noise powers imply sensitivity ΔS_{ij}
- Weak source limit
  - $S << SEFD_i$
  - $S << SEFD_j$

$$\Delta S_{ij} = \frac{1}{\eta_s} \times \sqrt{\frac{SEFD_i \times SEFD_j}{2 \times \Delta\nu \times \tau_{acc}}}$$

Interferometer Sensitivity

Real Correlator - 2

- For $SEFD_i = SEFD_j = SEFD$ drop subscripts

$$\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$$

  - Units Jy
- Interferometer system efficiency η_s
  - Accounts for electronics, digital losses
  - See instrument documentation

Interferometer Sensitivity

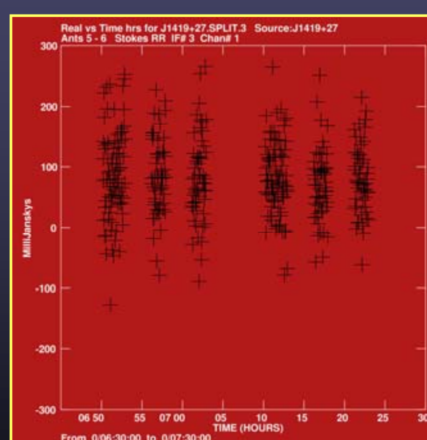
Abscissa spans 30 minutes.
Ordinate spans +/-300 milliJy.

Complex Correlator

- Delivers two channels
  - Real S_R, sensitivity ΔS
  - Imaginary S_I, sensitivity ΔS
- Eg: VLBA continuum
  - Figure 9-1 at 8.4 GHz
  - Observed scatter $S_R(t)$, $S_I(t)$
  - Predicted ΔS = 69 milliJy

$$\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$$

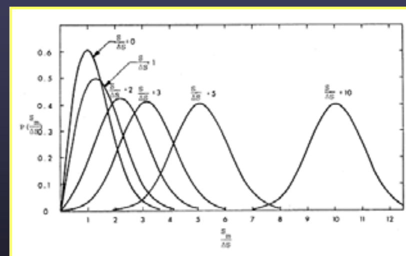
  - Resembles observed scatter



Interferometer Sensitivity

Measured Amplitude

- Measured visibility amplitude $S_m = \sqrt{S_R^2 + S_I^2}$
  - Standard deviation (s.d.) of S_R or S_I is ΔS
- True visibility amplitude S
- Probability $\Pr(S_m/\Delta S)$
  - Figure 9-2



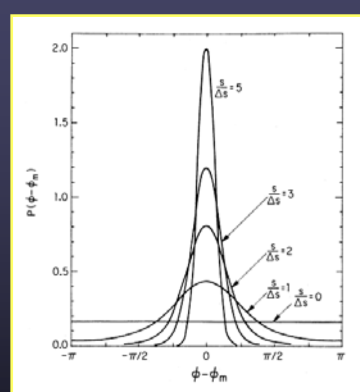
  - Behavior with true $S/\Delta S$
    - *High: Gaussian*
    - *Zero: Rayleigh*
    - *Low: Rice. S_m gives biased estimate of S.*

Interferometer Sensitivity

Measured Phase

- Measured visibility phase

  $\phi_m = \arctan(S_I/S_R)$
- True visibility phase ϕ
- Probability $\Pr(\phi - \phi_m)$
  - Figure 9-2



  - Behavior with true $S/\Delta S$
    - *High: Gaussian*
    - *Zero: Uniform*
- Seek weak detection in phase, not in amplitude

Image Sensitivity

Single Polarization

- Simplest weighting case where visibility samples
  - Have same interferometer sensitivities $\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$
  - Have same signal-to-noise ratios *w*
  - Combined with natural weight (*W=1),* no taper (*T=1)*
- Image sensitivity is s.d. of mean of *L* samples, each with s.d. ΔS, i.e., $\Delta I_m = \Delta S/\sqrt{L}$
  - ***N*** antennas, # of interferometers $\frac{1}{2} \times N \times (N-1)$
  - # of accumulation times $t_{\text{int}}/\tau_{acc}$
  - $L = \frac{1}{2} \times N \times (N-1) \times (t_{\text{int}}/\tau_{acc})$
  - So $\Delta I_m = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{N \times (N-1) \times \Delta\nu \times t_{\text{int}}}}$

Image Sensitivity

NGC5548 FOV 150 milliarcsec = 80 pc
Wrobel 2000, ApJ, 531, 716

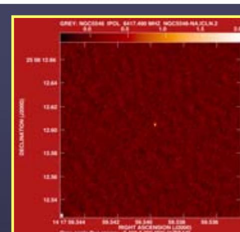
Dual Polarizations – 2

- Eg: VLBA continuum
  - Figure 9-3 at 8.4 GHz

  - Observed
    - *Stokes I, simplest weighting*
    - *Gaussian noise ΔI = 90 microJy beam^{-1}*
  - Predicted

    $\Delta I = \Delta I_m/\sqrt{2} = \Delta S/\sqrt{2 \times L}$

    $L = \frac{1}{2} \times N \times (N-1) \times (t_{\text{int}}/\tau_{acc})$
    - *Previous e.g. ΔS*
    - *Plus here L = 77,200*
    - *So s.d. ΔI = 88 microJy beam^{-1}*



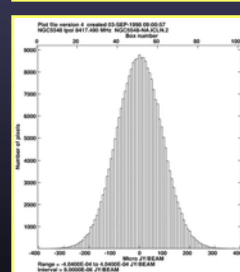


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Image Sensitivity

Dual Polarizations - 1

- Single-polarization image sensitivity ΔI_m
- Dual-polarization data $\Rightarrow$ image Stokes I,Q,U,V
  - Gaussian noise in each image, 2 samples
  - Mean zero, s.d. $\Delta I = \Delta Q = \Delta U = \Delta V = \Delta I_m/\sqrt{2}$
- Linearly polarized flux density $P = \sqrt{Q^2 + U^2}$
  - Rayleigh noise, $P \geq 0$
  - Recall visibility amplitude
- Polarization position angle $\chi = \frac{1}{2} \times \arctan(U/Q)$
  - Recall visibility phase

See Fig. 9.4 in text

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Other Sensitivity Factors

- **Beam size and T_B**
  Small synthesized beams require high T_B

  VLBA "sees" non-thermal sources, Fig. 9.5
- **Very bright sources: High dynamic range Imaging (Monday)**
- **Confusion**

Sensitivity

Summary – 1

- One antenna
  - System temperature T_{sys}
  - Gain K
- Overall antenna performance is measured by system equivalent flux density *SEFD*

$$SEFD = T_{sys} / K$$

  - Units Jy

Sensitivity

Summary - 2

- Connect two antennas to form interferometer
  - Antennas have same *SEFD*, observing bandwidth $\Delta\nu$
  - Interferometer system efficiency η_s
  - Interferometer accumulation time τ_{acc}
- Sensitivity of interferometer

$$\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$$

  - Units Jy

Sensitivity

Summary - 3

- Connect *N* antennas to form array
  - Antennas have same *SEFD*, observing bandwidth $\Delta\nu$
  - Array has system efficiency η_s
  - Array integrates for time t_{int}
  - Form synthesis image of single polarization
- Sensitivity of synthesis image

$$\Delta I_m = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{N \times (N-1) \times \Delta\nu \times t_{\mathrm{int}}}}$$

  - Units Jy beam^{-1}