Fundamentals of Radio Interferometry

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Outline

- **Antennas – Our Connection to the Universe**
- **The Monochromatic, Stationary Interferometer**
- **The Relation between Brightness and Visibility**
- **Coordinate Systems**
- **Making Images**
- **The Consequences of Finite Bandwidth**
- **Adding Time Delay and Motion**
- **Heterodyning**
- **The Consequences of Finite Time Averaging**

Telescopes – our eyes (ears?) on the Universe

- Nearly all we know of our universe is through observations of electromagnetic radiation.
- The purpose of an astronomical telescope is to determine the characteristics of this emission:
  - Angular distribution
  - Frequency distribution
  - Polarization characteristics
  - Temporal characteristics
- Telescopes are sophisticated, but imperfect devices, and proper use requires an understanding of their capabilities and limitations.

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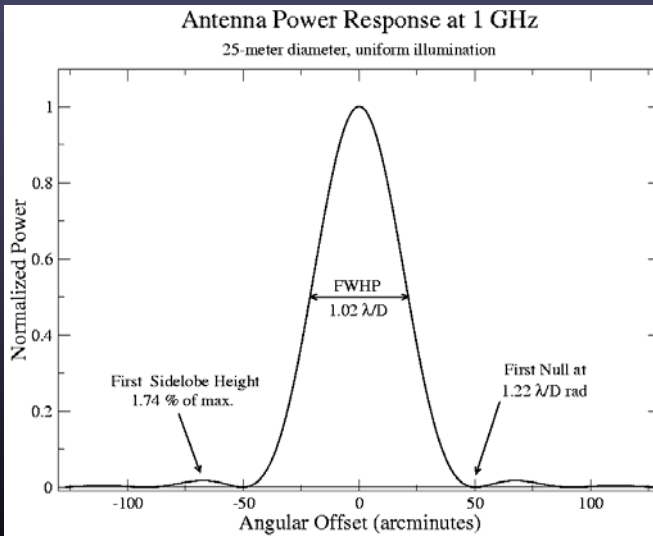
Antennas – the Single Dish

- The simplest radio telescope (other than elemental devices such as a dipole or horn) is a parabolic reflector – a 'single dish'.
- The detailed characteristics of single dishes are covered in the next lecture. Here, I comment only on four important characteristics:
  1. A directional gain.
  2. An angular resolution given by: $\theta \sim \lambda/D$.
  3. The presence of 'sidelobes' – finite response at large angles.
  4. The angular response contains no sharp edges.
- A basic understanding of the origin of these characteristics will aid in understanding the functioning of an interferometer.

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The Standard Parabolic Antenna Response

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The power response of a uniformly illuminated circular parabolic antenna of 25-meter diameter, at a frequency of 1 GHz.
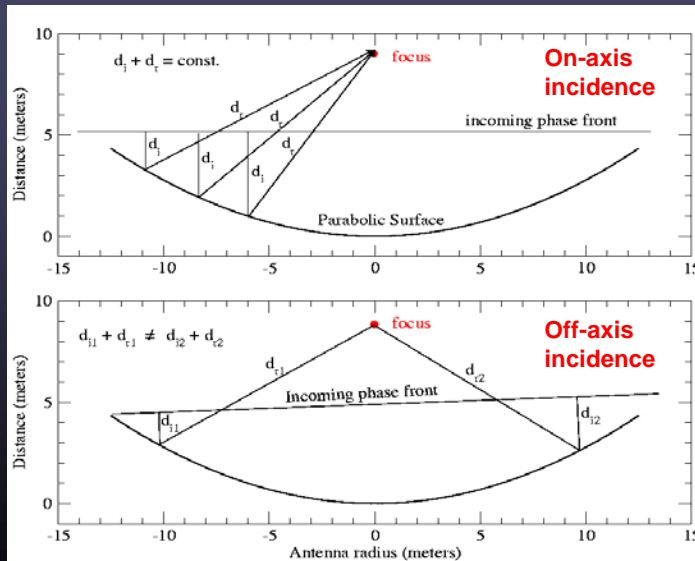
Antenna Power Response at 1 GHz
25-meter diameter, uniform illumination

FWHP
1.02 λ/D

First Sidelobe Height
1.74 % of max.

First Null at
1.22 λ/D rad

Normalized Power

Angular Offset (arcminutes)

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Beam Pattern Origin

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- An antenna's response is a result of coherent phase summation of the electric field at the focus.

- First null will occur at the angle where one extra wavelength of path is added across the full width of the aperture:

θ ~ λ/D

$d_i + d_r = const.$

focus

On-axis incidence

incoming phase front

d_r

d_i

Parabolic Surface

Distance (meters)

$d_{i1} + d_{r1} \neq d_{i2} + d_{r2}$

focus

Off-axis incidence

d_{r1}

d_{r2}

Incoming phase front

d_{i1}

d_{i2}

Distance (meters)

Antenna radius (meters)

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Getting Better Resolution

- The 25-meter aperture of a VLA antenna provides insufficient resolution for modern astronomy.
  - 30 arcminutes at 1.4 GHz, when we want 1 arcsecond or better!
- The trivial solution of building a bigger telescope is not practical. 1 arcsecond resolution at λ = 20 cm requires a 40 kilometer aperture.
  - The world's largest fully steerable antenna (operated by the NRAO at Green Bank, WV) has an aperture of only 100 meters $\Rightarrow$ 4 times better resolution than a VLA antenna.
- As building a 40-kilometer wide antenna is not feasible, we must consider a means of synthesizing the equivalent aperture, through combinations of elements.
- This method, termed 'aperture synthesis', was developed in the 1950s in England and Australia. Martin Ryle (University of Cambridge) earned a Nobel Prize for his contributions.

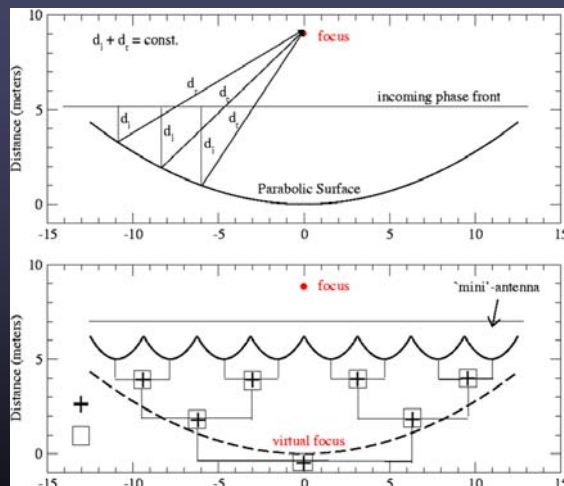R. Perley, Synthesis Imaging Summer School, 15-22 June 2004

Aperture Synthesis – Basic Concept

If the source emission is unchanging, there is no need to collect all of the incoming rays at one time.

One could imagine sequentially combining pairs of signals. If we break the aperture into N sub-apertures, there will be N(N−1)/2 pairs to combine.
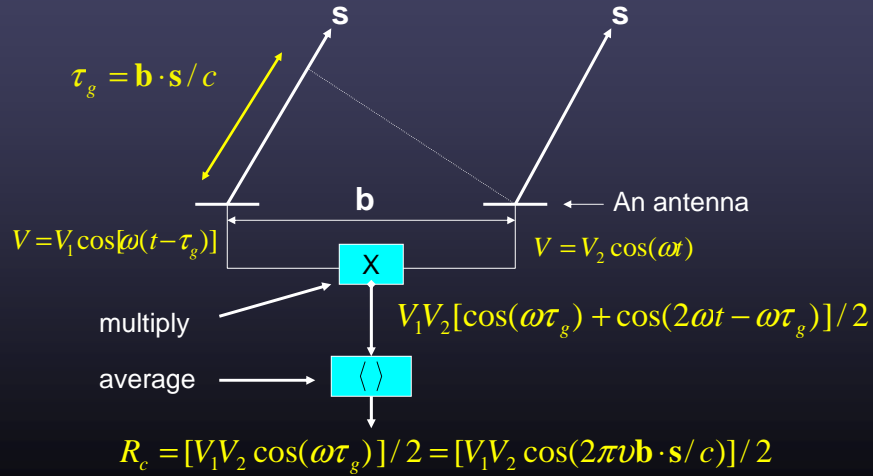
This approach is the basis of aperture synthesis.

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The Stationary, Monochromatic Interferometer

A small (but finite) frequency width, and no motion.
Consider radiation from a small solid angle $d\Omega$, from direction **s**.

s

s

$\tau_g = \mathbf{b} \cdot \mathbf{s} / c$

b

← An antenna

$V = V_1 \cos[\omega(t - \tau_g)]$

$V = V_2 \cos(\omega t)$

X

multiply

$V_1 V_2 [\cos(\omega \tau_g) + \cos(2\omega t - \omega \tau_g)] / 2$

average

$\langle\ \rangle$

$R_c = [V_1 V_2 \cos(\omega \tau_g)] / 2 = [V_1 V_2 \cos(2\pi \upsilon \mathbf{b} \cdot \mathbf{s} / c)] / 2$
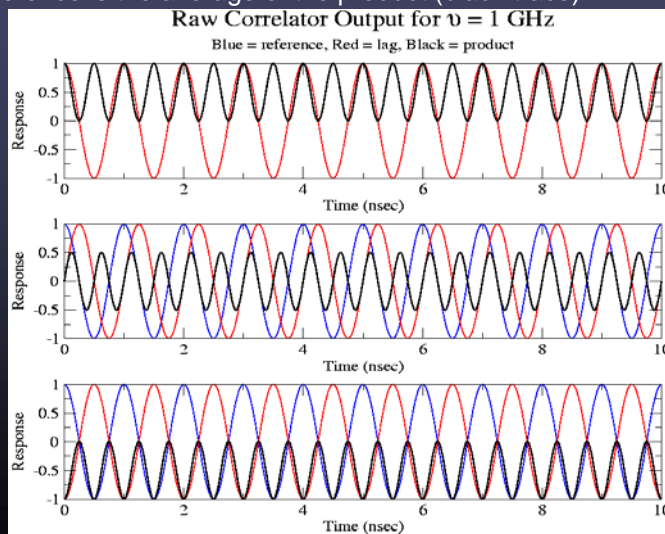

Examples of the Signal Multiplications

The two input signals are shown in red and blue.
The desired coherence is the average of the product (black trace)

In Phase:
$\tau_g = n\lambda/c$

Quadrature Phase:
$\tau_g = (2n+1)\lambda/4c$

Anti-Phase:
$\tau_g = (2n+1)\lambda/2c$

Raw Correlator Output for $\upsilon = 1$ GHz

Blue = reference, Red = lag, Black = product

Response

Time (nsec)

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- The averaged signal is independent of the time t, but is dependent on the lag, τ_g – a function of direction, and hence on the distribution of the brightness.
- In this expression, we use 'V' to denote the voltage of the signal. This depends upon the source intensity by:

$$V \propto E \propto \sqrt{I}$$

so the term V_1V_2 is proportional to source intensity, I_v.
(measured in Watts.m^{-2}.Hz^{-2}.ster^{-2}).

- The strength of the product is also dependent on the antenna areas and electronic gains – but these factors can be calibrated for.
- To determine the dependence of the response over an extended object, we integrate over solid angle.
- Provided there is no spatial coherence between emission from different directions, this integration gives a simple result.

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- The response from an extended source is obtained by integrating the response over the solid angle of the sky:

$$R_C = \iint I_v(\mathbf{s}) \cos(2\pi v \mathbf{b} \cdot \mathbf{s}/c)\, d\Omega$$

where I have ignored (for now) any frequency dependence, and presume no spatial coherence.

Key point: the vector **s** is a function of direction, so the phase in the cosine is dependent on the angle of arrival.

This expression links what we want – the source brightness on the sky) ($I_v(\mathbf{s})$) – to something we can measure (R_C, the interferometer response).
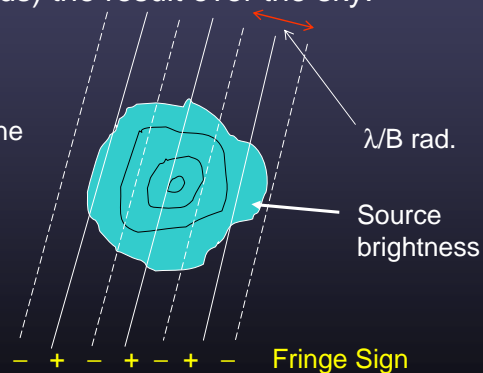
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A Schematic Illustration

The COS correlator can be thought of 'casting' a sinusoidal fringe pattern, of angular scale λ/B radians, onto the sky. The correlator multiplies the source brightness by this wave pattern, and integrates (adds) the result over the sky.

Orientation set by baseline geometry.
Fringe separation set by baseline length and wavelength.

λ/B rad.

Source brightness

− + − + − + −   Fringe Sign
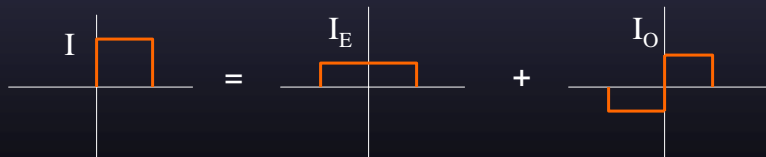
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Odd and Even Functions

- But the measured quantity, R_c, is insufficient – it is only sensitive to the 'even' part of the brightness, $I_E(\mathbf{s})$.
- Any real function, $I(x,y)$, can be expressed as the sum of two real functions which have specific symmetries:

An even part: $I_E(x,y) = (I(x,y) + I(-x,-y))/2 = I_E(-x,-y)$

An odd part: $I_O(x,y) = (I(x,y) – I(-x,-y))/2 = -I_O(-x,-y)$

I I_E I_O

=   +

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Recovering the 'Odd' Part: The SIN Correlator

The integration of the cosine response, R_c, over the source brightness is sensitive to only the even part of the brightness:

$$R_C = \iint I(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega = \iint I_E(\mathbf{s})\cos(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

since the integral of an odd function (I_O) with an even function (cos x) is zero.

To recover the 'odd' part of the intensity, I_O, we need an 'odd' coherence pattern. Let us replace the 'cos' with 'sin' in the integral:
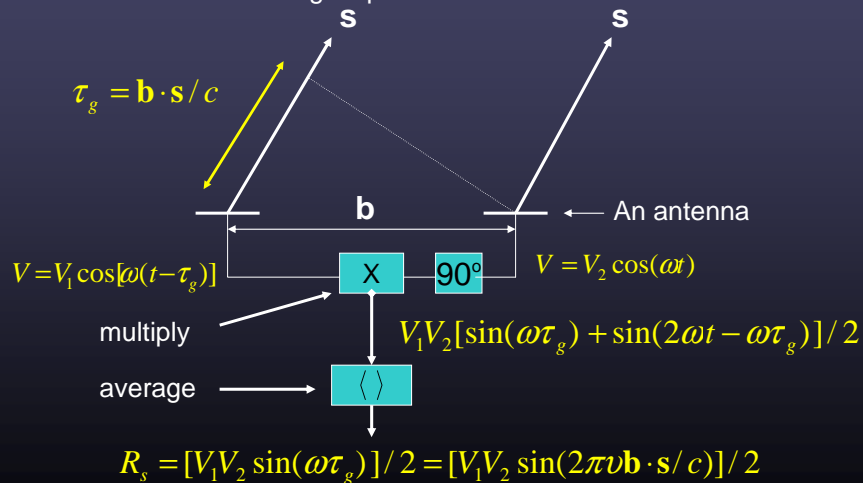
$$R_S = \iint I(\mathbf{s})\sin(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega = \iint I_O(\mathbf{s})\sin(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)d\Omega$$

since the integral of an even times an odd function is zero. To obtain this necessary component, we must make a 'sine' pattern.

Making a SIN Correlator

- We generate the 'sine' pattern by inserting a 90 degree phase shift in one of the signal paths.

s   **s**

$$\tau_g = \mathbf{b}\cdot\mathbf{s}/c$$

b   ← An antenna

$V = V_1\cos[\omega(t-\tau_g)]$ X   90° $V = V_2\cos(\omega t)$

multiply $V_1V_2[\sin(\omega\tau_g) + \sin(2\omega t - \omega\tau_g)]/2$

average $\langle\,\rangle$

$$R_s = [V_1V_2\sin(\omega\tau_g)]/2 = [V_1V_2\sin(2\pi\nu\mathbf{b}\cdot\mathbf{s}/c)]/2$$

Define the Complex Visibility

We now DEFINE a complex function, V, to be the complex sum of the two independent correlator outputs:

$$V = R_C - iR_S = Ae^{-i\phi}$$

where

$$A = \sqrt{R_C^2 + R_S^2}$$

$$\phi = \tan^{-1}\left(\frac{R_S}{R_C}\right)$$

This gives us a beautiful and useful relationship between the source brightness, and the response of an interferometer:
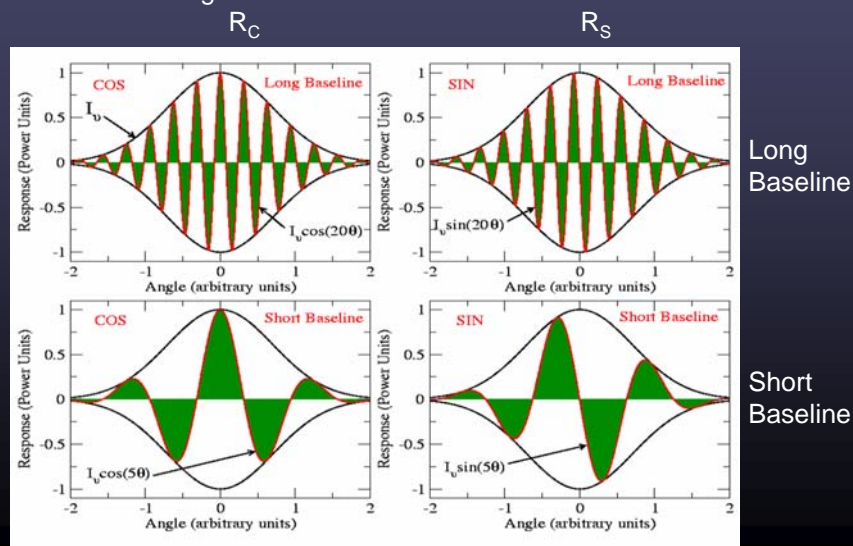
$$V(\mathbf{b}) = R_C - iR_S = \iint I_\nu(s)e^{-2\pi i\nu\mathbf{b}\cdot\mathbf{s}/c}\,d\Omega$$

Although it may not be obvious (yet), this expression can be inverted to recover $I(\mathbf{s})$ from $V(\mathbf{b})$.

Picturing the Visibility

- The intensity, I_ν, is in black, the 'fringes' in red. The visibility is the net dark green area.

R_C R_S

Long Baseline

Short Baseline

Comments on the Visibility

- The Visibility is a function of the source structure and the interferometer baseline.
- The Visibility is NOT a function of the absolute position of the antennas (provided the emission is time-invariant, and is located in the far field).
- The Visibility is Hermitian: $V(u,v) = V^*(-u,-v)$. This is a consequence of the intensity being a real quantity.
- There is a unique relation between any source brightness function, and the visibility function.
- Each observation of the source with a given baseline length provides one measure of the visibility.
- Sufficient knowledge of the visibility function (as derived from an interferometer) will provide us a reasonable estimate of the source brightness.

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Examples of Visibility Functions

- Top row: 1-dimensional even brightness distributions.
- Bottom row: The corresponding real, even, visibility functions.
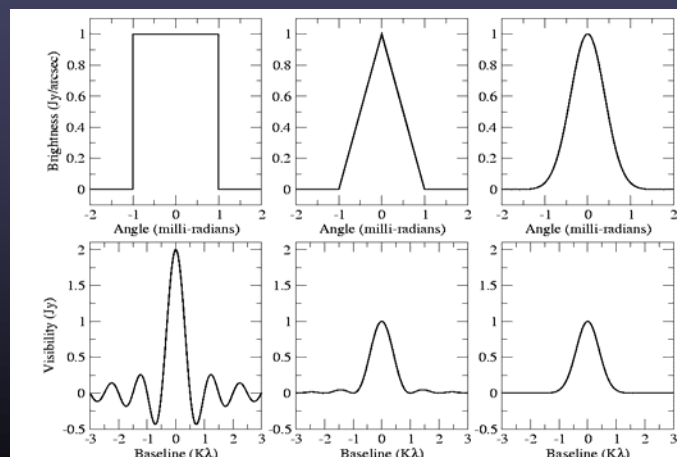
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Geometry – the perfect, and not-so-perfect

To give better understanding, we now specify the geometry.

Case A: A 2-dimensional measurement plane.

Let us imagine the measurements of $V_\nu(\mathbf{b})$ to be taken entirely on a plane. Then a considerable simplification occurs if we arrange the coordinate system so one axis is normal to this plane.

Let (u,v,w) be the coordinate axes, with w normal to the plane. All distances are measured in wavelengths. Then, the components of the unit direction vector, $\mathbf{s}$, are:

$$\mathbf{s} = (l, m, n) = \left(l, m, \sqrt{1 - l^2 - m^2} \right)$$

and for the solid angle $d\Omega = dldm \Big/ \sqrt{1 - l^2 - m^2}$

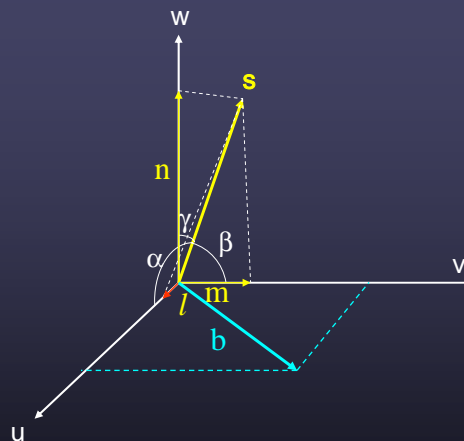R. Perley, Synthesis Imaging Summer School, 15-22 June 2004

Direction Cosines

The unit direction vector **s** is defined by its projections on the (u,v,w) axes. These components are called the **Direction Cosines**.

$l = \cos(\alpha)$

$m = \cos(\beta)$

$n = \cos(\gamma) = \sqrt{1 - l^2 - m^2}$

The baseline vector **b** is specified by its coordinates (u,v,w) (measured in wavelengths).

$$\mathbf{b} = (\lambda u, \lambda v, \lambda w) = (\lambda u, \lambda v, 0)$$

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The 2-d Fourier Transform Relation

Then, $\nu \mathbf{b.s}/c = ul + vm + wn = ul + vm$,   from which we find,

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2i\pi(ul+vm)} \, dl\,dm$$

which is a **2-dimensional Fourier transform** between the projected
brightness: $I_\nu / \cos(\gamma)$
and the spatial coherence function (visibility): $V_\nu(u,v)$.

And we can now rely on a century of effort by mathematicians on how
to invert this equation, and how much information we need to
obtain an image of sufficient quality. Formally,

$$I_\nu(l,m) = \cos(\gamma)\iint V_\nu(u,v)e^{i2\pi(ul+vm)}du\,dv$$

With enough measures of V, we can derive I.

Interferometers with 2-d Geometry

- Which interferometers can use this special geometry?
  - a) Those whose baselines, over time, lie on a plane (any plane).
    - All E-W interferometers are in this group. For these, the w-coordinate points to the NCP.
      - WSRT (Westerbork Synthesis Radio Telescope)
      - ATCA (Australia Telescope Compact Array)
      - Cambridge 5km telescope (almost).
  - b) Any coplanar array, at a single instance of time.
    - VLA or GMRT in snapshot (single short observation) mode.
- What's the 'downside' of this geometry?
  - Full resolution is obtained only for observations that are in the w-direction. Observations at other directions lose resolution.
    - E-W interferometers have no N-S resolution for observations at the celestial equator!!!
    - A VLA snapshot of a source will have no 'vertical' resolution for objects on the horizon.

Case B: A 3-dimensional measurement volume:

- What if the interferometer does not measure the coherence function within a plane, but rather does it through a volume? In this case, we adopt a slightly different coordinate system. First we write out the full expression:

$$V_\nu(u,v,w) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2i\pi(ul+vm+wn)} dldm$$

(Note that this is not a 3-D Fourier Transform).

- Then, orient the coordinate system so that the w-axis points to the center of the region of interest, (u points east and v north) and make use of the small angle approximation:

$$n = \cos\gamma = \sqrt{1-\sin^2\gamma} = \sqrt{1-\theta^2} \cong 1-\theta^2/2$$

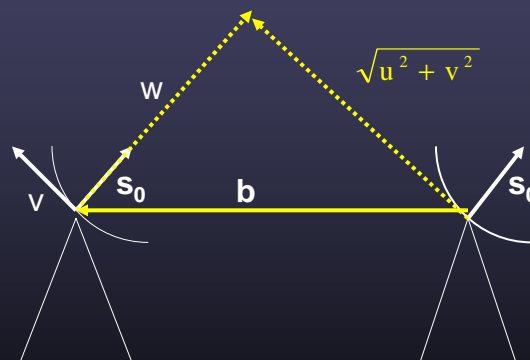where θ is the polar angle from the center of the image. The w-component is the 'delay distance' of the baseline.

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w points to the source, **u** towards the east, and **v** towards the NCP. The direction cosines l and m then increase to the east and north, respectively.

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$$V_\nu(u,v,w) = e^{-2i\pi w} \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2i\pi(ul+vm-w\theta^2/2)} dldm$$

The quadratic term in the phase can be neglected if it is much less than unity:

$$w\theta^2 << 1$$

Or, in other words, if the maximum angle from the center is:

$$\theta_{max} < \sqrt{\frac{1}{w}} \le \sqrt{\frac{\lambda}{B}} \sim \sqrt{\theta_{syn}} \qquad \text{(angles in radians!)}$$

then the relation between the Intensity and the Visibility again becomes a 2-dimensional Fourier transform:

$$V_\nu'(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2i\pi(ul+vm)} dldm$$

where the modified visibility is defined as:

$$V_\nu' = V_\nu e^{2\pi i w}$$

and is, in fact, the visibility we would have measured, had we been able to put the baseline on the w = 0 plane.

- This coordinate system, coupled with the small-angle approximation, allows us to use two-dimensional transforms for any interferometer array.
- Remember that this is an approximation! The visibilities really are measured in a volume, and we can't make that go away.
- How do we make images when the small-angle approximation breaks down?

  That's a longer story, for another day. (Short answer: we know how to do this, and it takes a lot more computing).

We have shown that under certain (and attainable) assumptions about electronic linearity and narrow bandwidth, a complex interferometer measures the visibility, or complex coherence:

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2i\pi(ul+vm)} \, dl \, dn$$

(u,v) are the projected baseline coordinates, measured in wavelengths, on a plane oriented facing the phase center, and

(*l*,m) are the sines of the angles between the phase center and the emission, in the EW and NS directions, respectively.

This is a Fourier transform relation, and it can be in general be solved, to give:

$$I_\nu(l,m) = \cos(\gamma) \iint V(u,v) e^{+2i\pi(ul+vm)} \, du \, dv$$

This relationship presumes knowledge of V(u,v) for all values of u and v. In fact, we have a finite number, N, measures of the visibility, so to obtain an image, the integrals are replaced with a sum:

$$I_\nu(l,m) = \frac{1}{N} \sum_{n=1}^{N} V_n(u_n,v_n) \exp[2\pi i(u_n l + v_n m)] \Delta u \Delta v$$

If we have N_v visibilities, and N_m cells in the image, we have ~$N_v N_m$ calculations to perform – a number that can exceed 10^{12}!

- The sum on the last page is in general complex, while the sky brightness is real. What's wrong?
- In fact, each measured visibility represents two visibilities, since $V(-u,-v) = V^*(u,v)$.
- This is because interchanging two antennas leaves R_c unchanged, but changes the sign of R_s.
- Mathematically, as the sky is real, the visibility must be Hermitian.
- So we can modify the sum to read:

$$I_\nu(l,m) = \frac{1}{N}\sum_{n=1}^{N}A_n\cos[2\pi(u_n l + v_n m) + \phi_n]\Delta u\Delta v$$

- Each cosine represents a two-dimensional sinusoidal function in the image, with unit amplitude, and orientation given by: $\alpha = \tan^{-1}(u/v)$.
- The cosinusoidal sea on the image plane is multiplied by the visibility amplitude A_n, and a shifted by the visibility phase ϕ_n.
- Each individual measurement adds a (shifted and amplified) cosinusoid to the image.
- The basic (raw, or dirty) map is the result of this summation process.
- The actual process, including the use of FFTs, is covered in the 'imaging' lecture.

A simple example

The rectangle below represents a piece of sky. The solid red lines are the maxima of the sinusoids, the dashed lines their minima. Two visibilities are shown, each with phase zero.
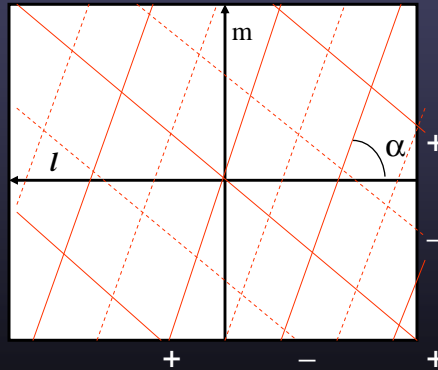
1-d Example: Point-Source

For a 1 Jy point source, all visibility amplitudes are 1 Jy, and all phases are zero. The lower panel shows the response when visibilities from 21 equally-spaced baselines are added.

The individual visibilities are shown in the top panel. Their (incremental) sums are shown in the lower panel.
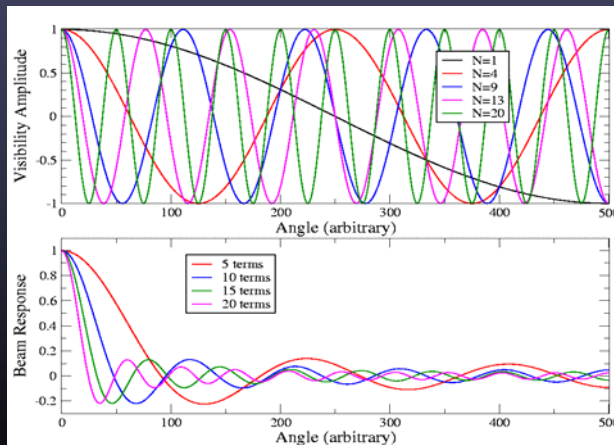
Example 2: Square Source

- For a centered square object, the visibility amplitudes decline with increasing baseline, and the phases are all zero or 180.
- Again, 21 baselines are included.
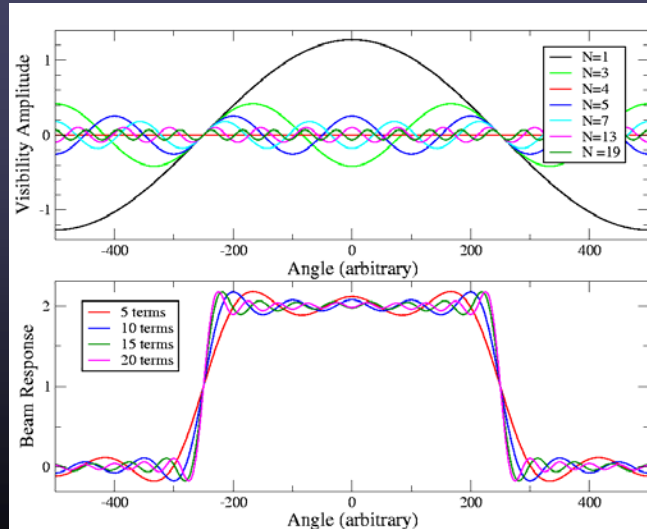
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The Effect of Bandwidth.

Real interferometers must accept a range of frequencies (amongst other things, there is no power in an infinitesimal bandwidth)! So we now consider the response of our interferometer over frequency.

To do this, we first define the frequency response functions, G(v), as the amplitude and phase variation of the signals paths over frequency.

Then integrate:

$$V = \frac{1}{\Delta v} \int_{v-\Delta v/2}^{v+\Delta v/2} I_v(\mathbf{s}) G_1(v) G_2^*(v) e^{2\pi i v \tau_g} \, dv$$

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The Effect of Bandwidth.

If the source intensity does not vary over frequency width, we get

$$V = \iint I_\nu(\mathbf{s}) \, \mathrm{sinc} \, (\tau_g \, \Delta\nu) e^{-2i\pi\nu_0\tau_g} \, d\Omega$$

where I have assumed the $G(\nu)$ are square, real, and of width $\Delta\nu$. The frequency ν_0 is the mean frequency within the bandwidth.

The fringe attenuation function, sinc(x), is defined as:

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

$$\approx 1 - \frac{(\pi x)^2}{6} \qquad \text{for } x \ll 1$$

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The Bandwidth/FOV limit

This shows that the source emission is attenuated by the function sinc(x), known as the 'fringe-washing' function. Noting that $\tau_g \sim$ (B/c) $\sin(\theta) \sim B\theta/\lambda\nu \sim (\theta/\theta_{res})/\nu$, we see that the attenuation is small when

$$\frac{\Delta\nu}{\nu} \frac{\theta}{\theta_{res}} \ll 1$$

The ratio $\Delta\nu/\nu$ is the fractional bandwidth. The ratio θ/θ_{res} is the source offset in units of the fringe separation, λ/B.
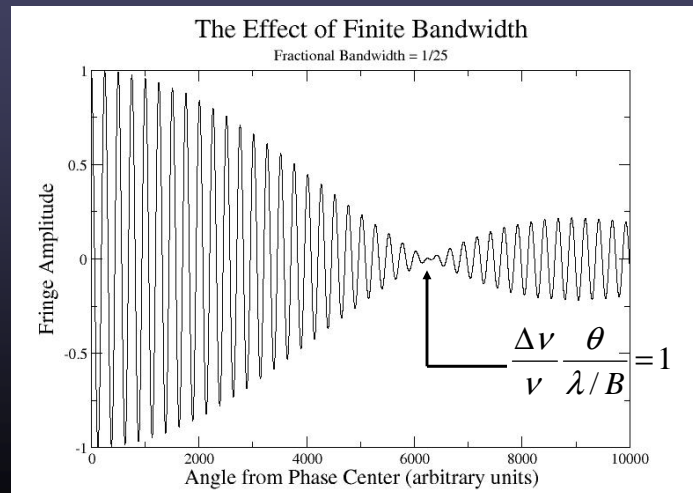
In words, this says that the attenuation is small if the fractional bandwidth times the angular offset in resolution units is less than unity. Significant attenuation of the measured visibility is to be expected if the source offset is comparable to the interferometer resolution divided by the fractional bandwidth.

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Bandwidth Effect Example

Finite Bandwidth causes loss of coherence at large angles, because the amplitude of the interferometer fringes are reduced with increasing angle from the delay center.

The Effect of Finite Bandwidth
Fractional Bandwidth = 1/25

$$\frac{\Delta \nu}{\nu}\frac{\theta}{\lambda/B}=1$$

Fringe Amplitude (y-axis, values 1, 0.5, 0, -0.5, -1)

Angle from Phase Center (arbitrary units) (x-axis, 0, 2000, 4000, 6000, 8000, 10000)

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Avoiding Bandwidth Losses

- The trivial solution is to avoid observing large objects! (Not helpful).
- Although there are computational methods which allow recovery of the lost amplitude, the loss in SNR is unavoidable.
- The simple solution is to observe with a small bandwidth. But this causes loss of sensitivity.
- So, the best (but not cheapest!) solution is to observe with LOTS of narrow channels.
- Modern correlators will provide tens to hundreds of thousands of channels of appropriate width.
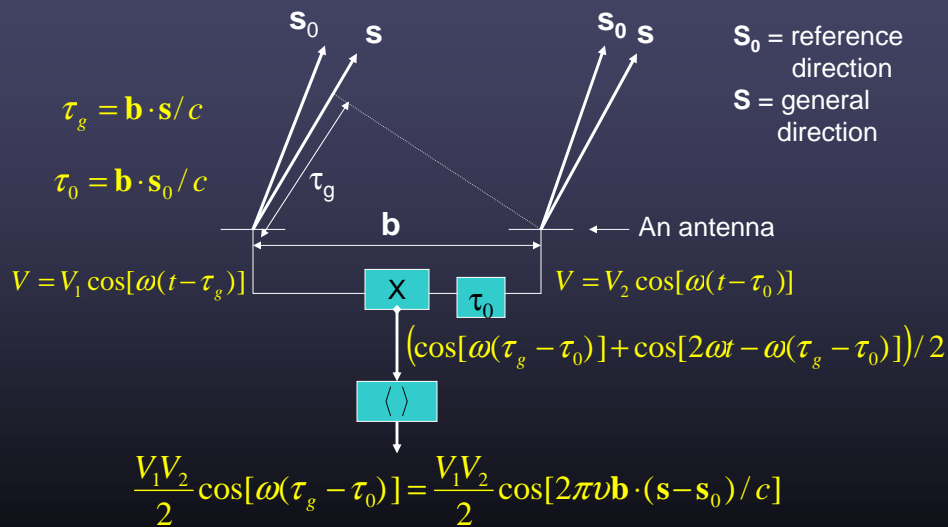
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- Another important consequence of observing with a finite bandwidth is that the sensitivity of the interferometer is not uniform over the sky.
- The current analysis, which applies to a finite bandwidth zero-delay interferometer, shows that only sources on a plane orthogonal to the interferometer baseline will be observed with full coherence.
- How can we recover the proper visibility for sources far from this plane?
- Add time delay to shift the maximum of the 'sinc' pattern to the direction of the source.

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The Stationary, Radio-Frequency Interferometer with inserted time delay 42

$\mathbf{s}_0$ $\mathbf{s}$ $\mathbf{s}_0$ $\mathbf{s}$ $\mathbf{S_0}$ = reference direction

$\tau_g = \mathbf{b} \cdot \mathbf{s}/c$ $\mathbf{S}$ = general direction

$\tau_0 = \mathbf{b} \cdot \mathbf{s}_0/c$ τ_g

$\mathbf{b}$ ← An antenna

$V = V_1 \cos[\omega(t - \tau_g)]$ X τ_0 $V = V_2 \cos[\omega(t - \tau_0)]$

$\left(\cos[\omega(\tau_g - \tau_0)] + \cos[2\omega t - \omega(\tau_g - \tau_0)] \right)/2$

$\langle \, \rangle$

$$\frac{V_1 V_2}{2} \cos[\omega(\tau_g - \tau_0)] = \frac{V_1 V_2}{2} \cos[2\pi \upsilon \mathbf{b} \cdot (\mathbf{s} - \mathbf{s}_0)/c]$$

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Coordinates

• It should be clear from inspection that the results of the last section are reproduced, with the 'fringes' and the bandwidth delay pattern, how centered about the direction defined by $\tau - \tau_g = 0$. The unattenuated field of view is as before:

$$\Delta\theta/\theta_{res} < \nu/\Delta\nu$$

• Remembering the coordinate system discussed earlier, where the w axis points to the reference center ($\mathbf{s}_0$), assuming the introduced delay is appropriate for this center, and that the bandwidth losses are negligible, we have:

$$\omega\tau_g = 2\pi\upsilon\mathbf{b}\cdot\mathbf{s}/c = 2\pi(ul + vm + wn)$$

$$\omega\tau_0 = 2\pi\upsilon\mathbf{b}\cdot\mathbf{s}_0/c = 2\pi w$$

$$n = \sqrt{1 - l^2 - m^2} = \cos\gamma$$

$$d\Omega = dl\,dm/n$$

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Extension to a Moving Source

• Inserting these, we obtain:

$$V_\nu(u,v) = \iint \frac{I_\nu(l,m)}{\sqrt{1-l^2-m^2}} e^{-2i\pi[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} dl\,dm$$

• The third term in the exponential is generally very small, and can be ignored in most cases, as discussed before.

• The extension to a moving source (or, more usually, to an interferometer located on a rotating object) is elementary – the delay term τ changes with time, so as to keep the peak of the fringe-washing function on the center of the region of interest.

• Also note that for a point object at the tracking center ($l = m = 0$), the phase is zero. This is because the added delay has exactly matched the phase lag of the radiation on the lagged antenna.

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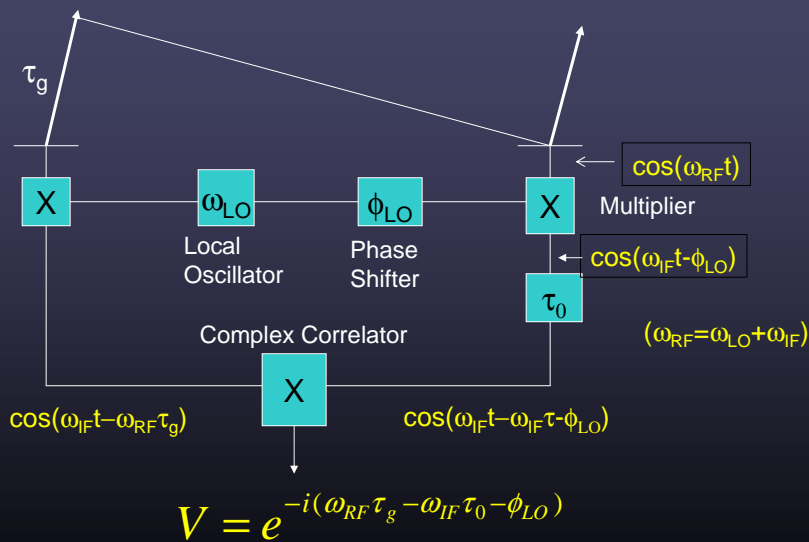
Consequence of IF Conversion

- This would be the end of the story (so far as the fundamentals are concerned) if all the internal electronics of an interferometer would work at the observing frequency (often called the 'radio frequency', or RF).

- Unfortunately, this cannot be done in general, as high frequency components are much more expensive, and generally perform more poorly, than low frequency components.

- Thus, nearly all radio interferometers use 'down-conversion' to translate the radio frequency information from the 'RF', to a lower frequency band, called the 'IF' in the jargon of our trade.

- For signals in the radio-frequency part of the spectrum, this can be done with almost no loss of information. But there is an important side-effect from this operation, which we now review.

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Downconversion

τ_g

$\cos(\omega_{RF}t)$

X · ω_{LO} · ϕ_{LO} · X · Multiplier

Local Oscillator · Phase Shifter

$\cos(\omega_{IF}t-\phi_{LO})$

τ_0

Complex Correlator

$(\omega_{RF}=\omega_{LO}+\omega_{IF})$

X

$\cos(\omega_{IF}t-\omega_{RF}\tau_g)$ · $\cos(\omega_{IF}t-\omega_{IF}\tau-\phi_{LO})$

$$V = e^{-i(\omega_{RF}\tau_g - \omega_{IF}\tau_0 - \phi_{LO})}$$

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Phase Addition

- This response will be identical to the delay-tracking, RF interferometer if the phase in the exponential is equal to $\omega_{RF}(\tau_g\text{-}\tau_0)$.
- This can be done by adjusting the LO phase such that

$$\phi_{LO} = \omega_{LO}\ \tau_0$$

- The reason this is necessary is that the delay, τ_0, has been added in the IF portion of the signal path, rather than at the frequency at which the delay actually occurs. Thus, the physical delay needed to maintain broad-band coherence is present, but because it is added at the 'wrong' frequency, an incorrect phase, equal to $\omega_{LO}\tau_0$, has been inserted, which must be corrected by addition of the 'missing' phase in the LO portion.
- When this is done, the response is identical to the ideal, delay-tracking RF interferometer.

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Time-Averaging Loss

- We have assumed everywhere that the values of the visibility are obtained 'instantaneously'. This is of course not reasonable, for we must average over a finite time interval.
- The time averaging, if continued too long, will cause a loss of measured coherence which is quite analogous to bandwidth smearing.
- The fringe-tracking interferometer keeps the phase constant for emission from the phase-tracking center. However, for any other position, the phase of a point of emission changes in time. The relation is:

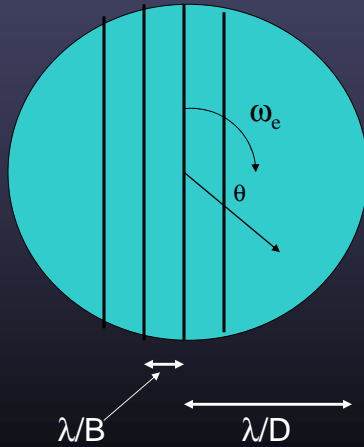$$\phi(t) = 2\pi\nu_F t = \frac{2\pi\omega_e B\theta}{\lambda}\,t$$

where θ is the source offset from the phase-tracking center.

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Time-Smearing Loss

- Simple derivation of fringe frequency:

  - Light blue area is antenna primary beam on the sky.
  - Fringes (black lines) rotate about the center at rate ω_e.
  - Time taken for a fringe to rotate by λ/B at angular distance θ is:
    $t = (\lambda/B)/\omega_e\theta > D/(\omega_e B)$
  - Fringe frequency is then
    $\nu_f = \omega_e B/D$

ω_e

θ

λ/B

λ/D

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Time-averaging Loss

- The net visibility obtained after an integration time, t, is found by integration:

$$V = \frac{V_0}{\delta t} \int_{-\delta t/2}^{\delta t/2} e^{i2\pi \upsilon_f t} dt = \mathrm{sinc}(\upsilon_f t)$$

- As with bandwidth loss, the condition for minimal time loss is that the integration time be much less than the inverse fringe frequency:

$$t << \frac{\lambda}{\omega_e B \theta} = \frac{D}{\omega_e B}$$

- For VLA in A-configuration, t << 10 seconds

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How to beat time smearing?

- The situation is the same as for bandwidth loss:
  - One can do processing to account for the lost signal, but the SNR cannot be recovered.
  - Only good solution is to reduce the integration time.
  - This makes for large databases, and more processing.