Calibration & Editing

George Moellenbrock

Tenth Summer Synthesis Imaging Workshop
University of New Mexico, June 13-20, 2006

Synopsis

- Why calibration and editing?
- Formalism: Signals -> Visibility, Idealistic -> Realistic
- Solving the Measurement Equation
- Practical Calibration
- Scalar Calibration Example
- Evaluation of Calibration
- Full Polarization and the Matrix Formalism
- A Dictionary of Calibration Components
- New Calibration Challenges
- Editing and RFI
- Summary

Why Calibration and Editing?

- Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, stability, etc.)
- Need to accommodate engineering (e.g., frequency conversion, digital electronics, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal
- RFI

Determining *instrumental properties* (calibration)
is as important as
determining *radio source properties*

From Idealistic to Realistic

- Formally, we wish to user our interferometer to obtain the visibility function, which we intend to invert to obtain an image of the sky:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl\, dm$$

- In practice, we correlate (multiply & average) the electric field (voltage) samples, x_i & x_j, received at pairs of telescopes (i,j)

$$V_{ij} = \left\langle x_i(t) \cdot x_j^*(t) \right\rangle_{\Delta t} = V\left(u_{ij}, v_{ij}\right)$$

  – Averaging duration is set by the expected timescales for variation of the correlation result (typically 10s or less for the VLA)
  – x_i & x_j, are delay- and Doppler-compensated in the correlator to select a point on the sky in the frame of the final image (the phase center), and keep it stationary during the average and from sample to sample
- Single radio telescopes are devices for collecting the signal $x_i(t)$ and providing it to the correlator.

What signal is really collected?

- The net signal delivered by antenna *i*, $x_i(t)$, is a combination of the desired signal, $s_i(t,l,m)$, corrupted by a factor $J_i(t,l,m)$ and integrated over the sky, and diluted by noise, $n_i(t)$:

$$x_i(t) = \int_{sky} J_i(t,l,m) s_i(t,l,m)\, dldm + n_i(t)$$
$$= s_i'(t) + n_i(t)$$

- $J_i(t,l,m)$ is the product of a host of effects which we must *calibrate*
- In some cases, effects implicit in the $J_i(t,l,m)$ term corrupt the signal irreversibly and the resulting data must be *edited*
- $J_i(t,l,m)$ is a complex number
- $J_i(t,l,m)$ is *antenna-based*
- Usually, $|n_i| >> |s_i|$

Correlation of realistic signals

- The correlation of two realistic signals from different antennas:

$$\left\langle x_i \cdot x_j^* \right\rangle_{\Delta t} = \left\langle (s_i' + n_i) \cdot (s_j' + n_j)^* \right\rangle_{\Delta t}$$
$$= \left\langle s_i' \cdot s_j'^* \right\rangle + \left\langle s_i' \cdot n_j^* \right\rangle + \left\langle n_i \cdot s_j'^* \right\rangle + \left\langle n_i \cdot n_j^* \right\rangle$$

  - Noise doesn't correlate—even if $|n_i| >> |s_i|$, the correlation process isolates desired signals:

$$= \left\langle s_i' \cdot s_j'^* \right\rangle_{\Delta t}$$
$$= \left\langle \int_{sky} J_i s_i\, dl' dm' \cdot \int_{sky} J_j^* s_j^*\, dldm \right\rangle_{\Delta t}$$

  - In integral, only $s_i(t,l,m)$, from the same directions correlate (i.e., when $l=l'$, $m=m'$), so order of integration and signal product can be reversed:

$$= \left\langle \int_{sky} J_i J_j^* s_i s_j^*\, dldm \right\rangle_{\Delta t}$$

Correction of realistic signals (cont)

- We can recast $s_i(t,l,m)$ & $s_j(t,l,m)$ in terms of the single signal, $s(t,l,m)$, which departed the distant radio sources, and propogated toward each of our telescopes:

$$V_{ij} = \left\langle \int_{sky} J_i J_j^* s^2(t,l,m) e^{-i2\pi(u_{ij}l+v_{ij}m)} dldm \right\rangle_{\Delta t}$$

- On the timescale of the averaging, the only meaningful average is of the *squared* signal itself (direction-dependent), which is just the image of the source:

$$= \int_{sky} J_i J_j^* \left\langle s^2(t,l,m) \right\rangle_{\Delta t} e^{-i2\pi(u_{ij}l+v_{ij}m)} dldm$$

$$= \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij}l+v_{ij}m)} dldm$$

- If all *J=1,* we of course recover the ideal expression:

$$= \int_{sky} I(l,m) e^{-i2\pi(u_{ij}l+v_{ij}m)} dldm$$

Aside: Auto-correlations and Single Dishes

- The *auto*-correlation of a signal from a *single* antenna:

$$\left\langle x_i \cdot x_i^* \right\rangle = \left\langle (s_i' + n_i) \cdot (s_i' + n_i)^* \right\rangle$$
$$= \left\langle s_i' \cdot s_i'^* \right\rangle + \left\langle n_i \cdot n_i^* \right\rangle$$
$$= \left\langle \int_{sky} |J_i|^2 |s_i|^2 \, dldm \right\rangle + \left\langle |n_i|^2 \right\rangle$$
$$= \left\langle \int_{sky} |J_i|^2 I(l,m) dldm \right\rangle + \left\langle |n_i|^2 \right\rangle$$

  - This is an integrated power measurement plus noise
  - Desired signal *not* isolated from noise
  - Noise usually dominates
- Single dish radio astronomy calibration strategies dominated by switching schemes to isolate desired signal

9

The Scalar Measurement Equation

$$V_{ij}^{obs} = \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

- First, isolate non-direction-dependent effects, and factor them from the integral:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} \left(J_i^{sky} J_j^{sky*}\right) I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

- Next, we recognize that it is often possible to assume $J^{sky}=1$, and we have a relationship between ideal and observed Visibilities:

$$= \left(J_i^{vis} J_j^{vis*}\right) \int_{sky} I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

$$V_{ij}^{obs} = \left(J_i^{vis} J_j^{vis*}\right) V_{ij}^{ideal}$$

10

Solving the Measurement Equation

- The J's can be factored into a series of components representing physical elements along the signal path:

$$V_{ij}^{obs} = \left(J_i^4 J_j^{4*}\right)\left(J_i^3 J_j^{3*}\right)\left(J_i^2 J_j^{2*}\right)\left(J_i^1 J_j^{1*}\right) V_{ij}^{ideal}$$

- Depending upon availability of estimates for various J terms, we can re-arrange the equation and solve for any single term, if we know V^{ideal}:

$$\left[\left(J_i^3 J_j^{3*}\right)^{-1}\left(J_i^4 J_j^{4*}\right)^{-1} V_{ij}^{obs}\right] = \left(J_i^2 J_j^{2*}\right)\left[\left(J_i^1 J_j^{1*}\right) V_{ij}^{ideal}\right]$$

- Re-solve for dominant effects using refined estimates of the more subtle terms (a la self-calibration)?
- After obtaining optimal estimates for all relevant J, data can be corrected:

$$V_{ij}^{corrected} = \left(J_i^1 J_j^{1*}\right)^{-1}\left(J_i^2 J_j^{2*}\right)^{-1}\left(J_i^3 J_j^{3*}\right)^{-1}\left(J_i^4 J_j^{4*}\right)^{-1} V_{ij}^{obs}$$

Solving the Measurement Equation

11

- Formally, solving for any antenna-based visibility calibration component is always the same non-linear fitting problem:

$$V_{ij}^{corrected \cdot obs} = \left(J_i^{solve} J_j^{solve*}\right) V_{ij}^{corrupted \cdot ideal}$$

  - Viability of the solution depends on isolation of different effects using *proper calibration observations*, and *appropriate solving strategies*
  - The relative importance of the different components enables deferring or even ignoring the more subtle effects

Antenna-based Calibration

12

- Success of synthesis telescopes relies on antenna-based calibration
  - N antenna-based factors, N(N-1)/2 visibility measurements
  - Fundamentally, only information that cannot be factored into antenna-based terms is believable as being of astronomical origin
- Closure: calibration-independent observables:
  - Closure phase (3 baselines):

$$\phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} = \left(\phi_{ij}^{real} + \theta_i - \theta_j\right) + \left(\phi_{jk}^{real} + \theta_j - \theta_k\right) + \left(\phi_{ki}^{real} + \theta_k - \theta_i\right)$$
$$= \phi_{ij}^{real} + \phi_{jk}^{real} + \phi_{ki}^{real}$$

  - Closure amplitude (4 baselines):

$$\left|\frac{V_{ij}^{obs} V_{kl}^{obs}}{V_{ik}^{obs} V_{jl}^{obs}}\right| = \left|\frac{J_i J_j V_{ij}^{real} J_k J_l V_{kl}^{real}}{J_i J_k V_{ik}^{real} J_j J_l V_{jl}^{real}}\right| = \left|\frac{V_{ij}^{real} V_{kl}^{real}}{V_{ik}^{real} V_{jl}^{real}}\right|$$

- Beware of non-closing errors!

13

Practical Calibration

- A priori calibrations (provided by the observatory)
  - Antenna positions, earth orientation and rate
  - Clocks
  - Antenna pointing, gain, voltage pattern
  - Calibrator coordinates, flux densities, polarization properties
- Absolute engineering calibration?
  - Very difficult, requires heroic efforts by observatory scientific and engineering staff
  - Concentrate on ensuring stability on adequate timescales
- Cross-calibration a better choice
  - Observe nearby ***point sources*** against which calibration can be solved, and transfer solutions to target observations
  - Choose appropriate calibrators; usually strong point sources because we can predict their visibilities
  - Choose appropriate timescales for calibration

14

"Absolute" Astronomical Calibrations

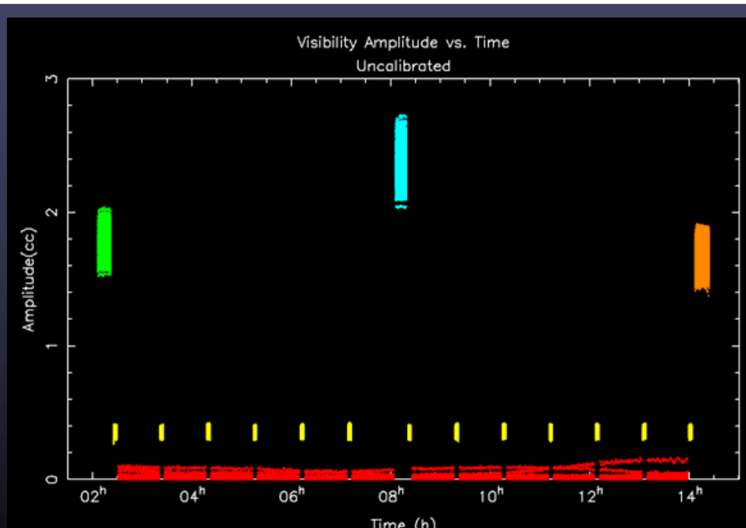
- Flux Density Calibration
  - Radio astronomy flux density scale set according to several "constant" radio sources
  - Use resolved models where appropriate
- Astrometry
  - Most calibrators come from astrometric catalogs; directional accuracy of target images tied to that of the calibrators
  - Beware of resolved and evolving structures (especially for VLBI)
- Linear Polarization Position Angle
  - Usual flux density calibrators also have significant stable linear polarization position angle

Simple Scalar Calibration Example

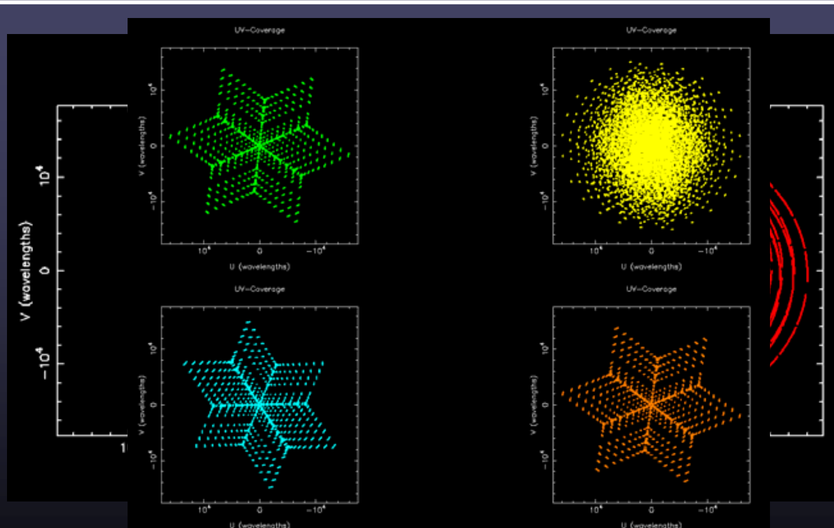
- Sources:
  - Science Target: NGC2403
  - Near-target calibrator: 0841+708 (8 degrees from target)
  - Flux Density calibrators: 3C48 (15.88 Jy), 3C147 (21.95 Jy), 3C286 (14.73 Jy)
- Signals:
  - RR correlation only (total intensity only)
  - 1419.79 MHz (HI), one 3.125 MHz channel
  - (continuum version of a spectral line observation)
- Array:
  - VLA C-configuration
- Simple multiplicative “Gain” calibration:

$$
\begin{aligned}
V_{ij}^{obs} &= G_{ij} V_{ij}^{true} \\
&= \left(G_i G_j^*\right) V_{ij}^{true}
\end{aligned}
$$

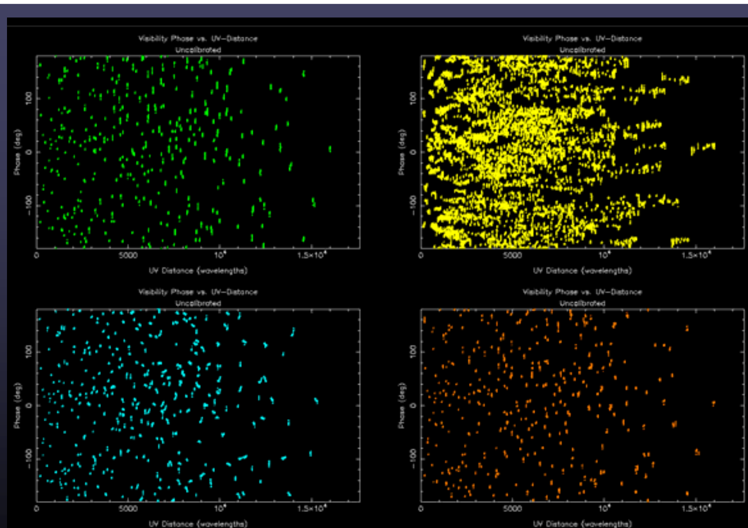
Observing Sequence



UV-coverages

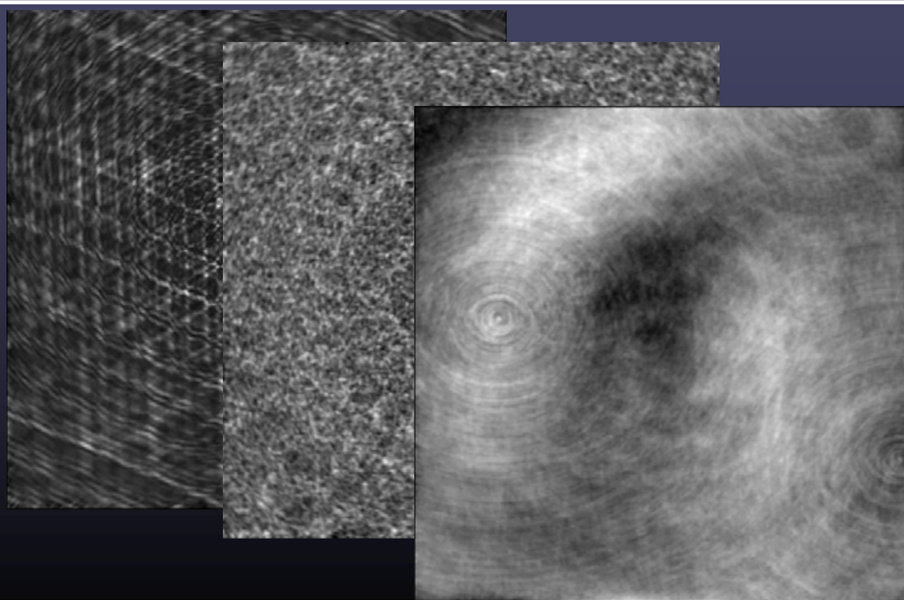


Views of the Uncalibrated data



Uncalibrated Images

19



Baseline-based Calibration

20

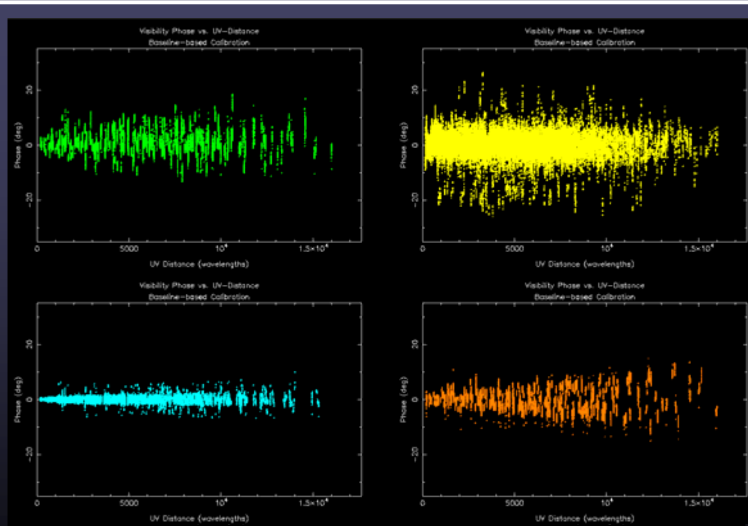
- Similar patterns among calibrators suggest a common calibration error
- Simplest thing to do is to pick the strongest calibrator, assume it is a point source, and determine a single set of per-baseline calibration factors (351 complex gains), and apply to all sources:

$$V_{ij}^{obs} = G_{ij} V_{ij}^{true}$$

  – Since we know this calibrator's flux density, the calibration solution will be properly scaled if we use this information in V^{ideal}
- How well does it work?

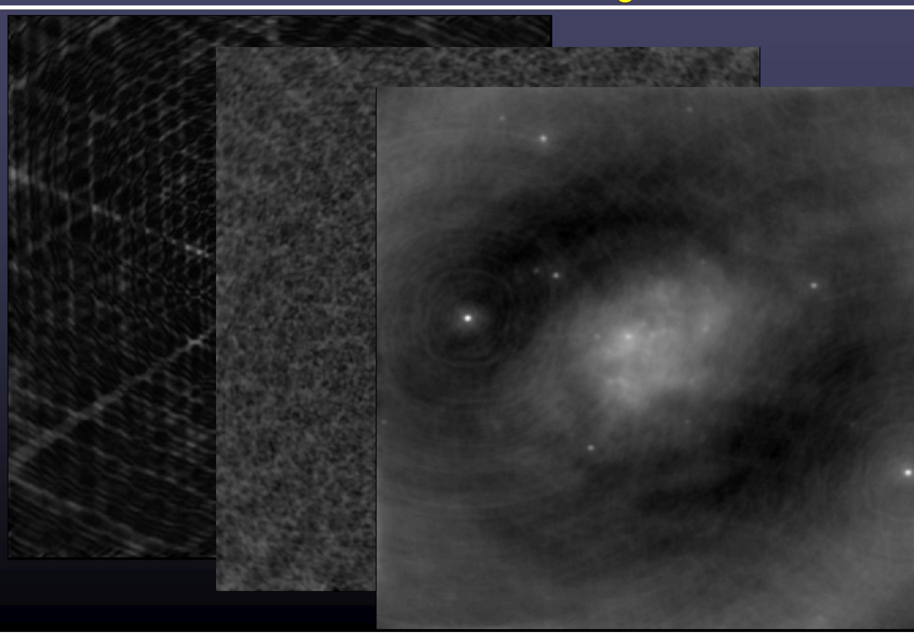
Baseline-based Calibration

21



Baseline-based Calibration Image Result?

22



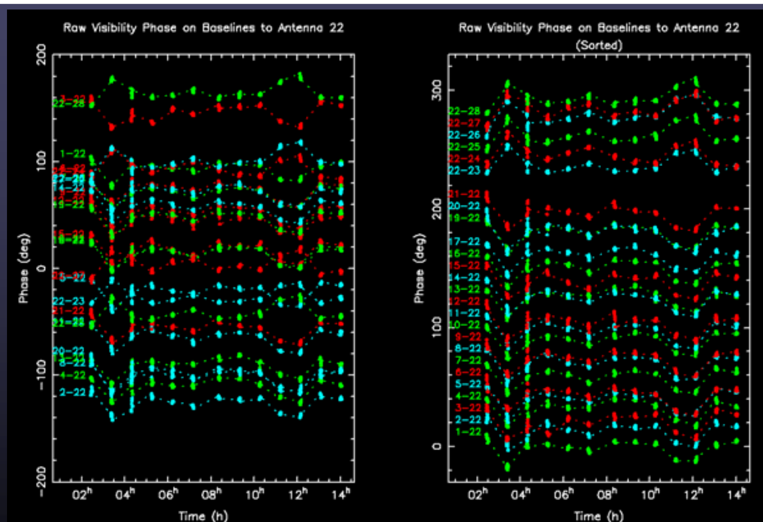
Rationale for Antenna-based Calibration

23

- A stable system seems to have allowed baseline-based calibration to work ok, but…
  - Transfer to other sources not perfect (gain variability?)
  - We've violated closure! (What if calibrator not a point source? Target source data's measured visibility function irrevocably changed by any errors in this assumption)
- Can we really leverage antenna-based calibration?
  - Per solution, only 27 degrees of freedom instead of 351
- How can we tell if effects are really antenna-based?
  - Similar time-variability on all baselines to individual antennas!?

Rationale for Antenna-based Calibration

24



The Antenna-based Calibration Solution

- Solve for 27 antenna-based gain factors on 600s timescale (1 solution per scan on near-target calibrator, 2 solutions per scan on flux-density calibrators):

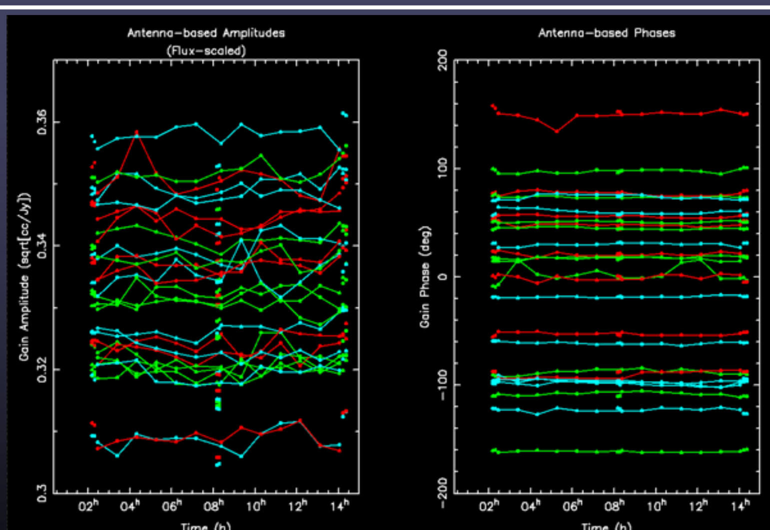
$$V_{ij}^{obs} = \left(G_i G_j^*\right) V_{ij}^{ideal}$$

  - 13 scans X 27 antennas = 351 gains for near-target calibrator; same # of d.o.f. as single baseline-based calibration, and we get time-dependent calibration, too!

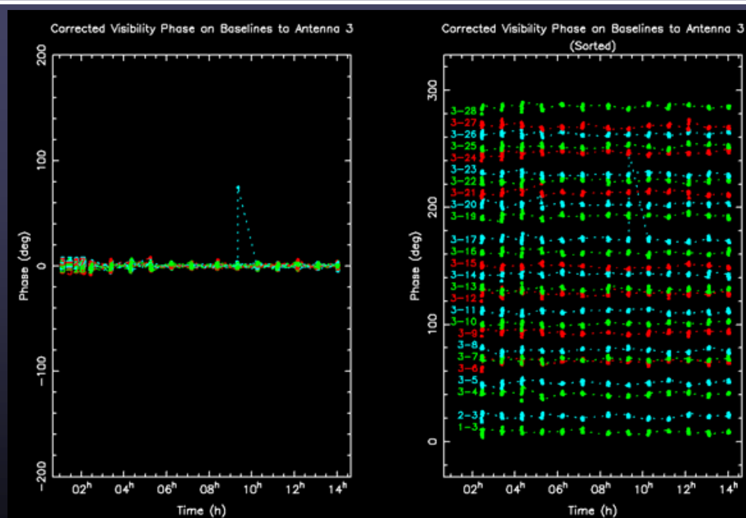
- Bootstrap flux density scale by scaling gain amplitudes of near-target calibrator (assumed 1 Jy above) according to gain amplitudes of flux density calibrators:

$$\left|G'_{i(n-t)}\right| = \left|G_{i(n-t)}\right| \left(\frac{\left\langle \left|G_{i(f-d)}\right| \right\rangle_i}{\left\langle \left|G_{i(n-t)}\right| \right\rangle_i} \right)$$

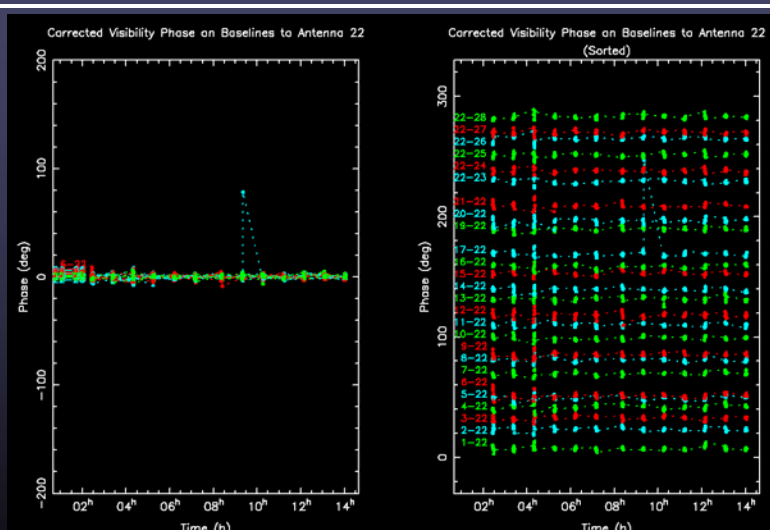
The Antenna-based Calibration Solution



Did Antenna-based Calibration Work?

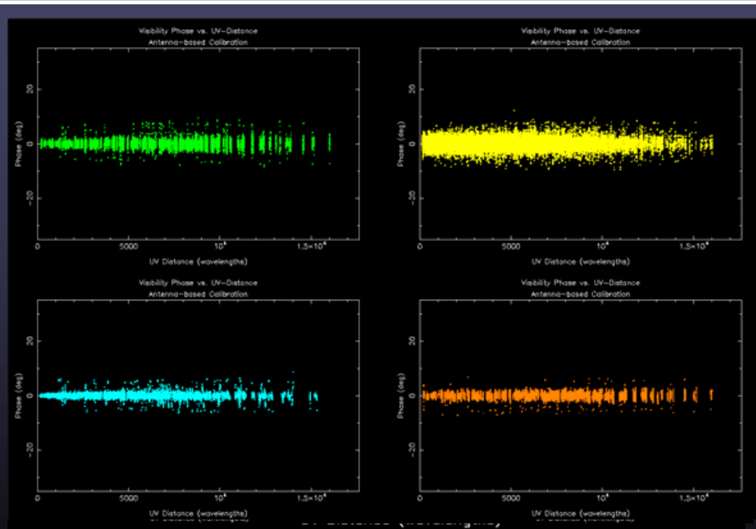


Did Antenna-based Calibration Work?



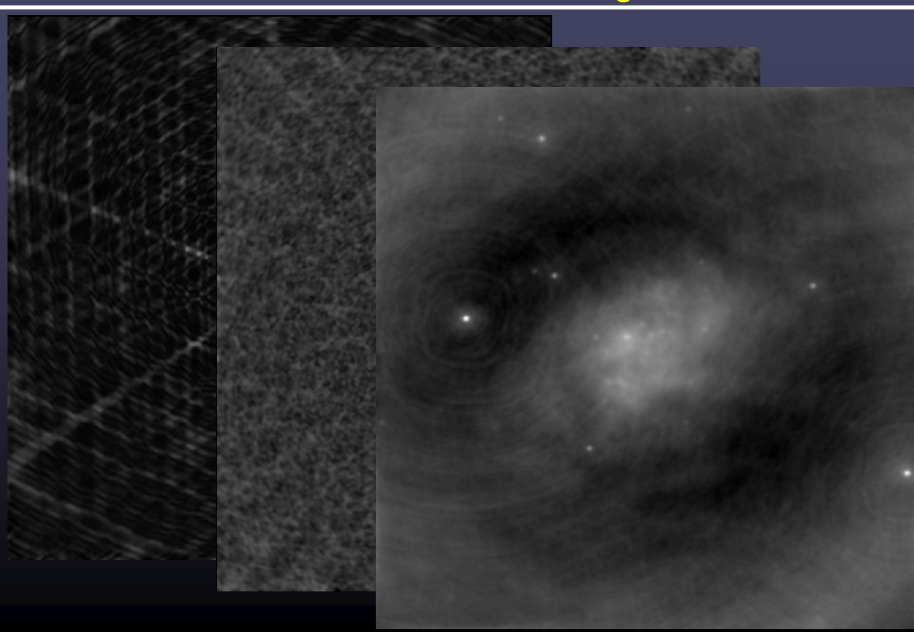
29

Antenna-based Calibration



30

Antenna-based Calibration Image Result?



Evaluating Calibration Performance

- Are solutions continuous?
  - Noise-like solutions are just that—noise
  - Discontinuities indicate instrumental glitches
  - Any additional editing required?
- Are calibrator data fully described by antenna-based effects?
  - Phase and amplitude *closure errors* are the baseline-based residuals
  - Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration components
    - Michael Rupen's lecture: "Self-Calibration" (Wednesday)
- Any evidence of unsampled variation? Is interpolation of solutions appropriate?
  - Reduce calibration timescale, if SNR permits

Summary of Scalar Example

- Simple Baseline-based calibration may be ok for a stable system and well-known calibrators (quick and dirty first look)
- Dominant calibration effects are ***antenna-based***
  - Minimizes degrees of freedom
  - Preserves closure for possible self-calibration
  - Permits higher dynamic range honestly!
- Point-like calibrators effective
- Flux density bootstrapping

33

Full-Polarization Formalism (Matrices!)

- Need dual-polarization basis (*p,q*) to fully sample the incoming EM wave front, where *p,q* = *R,L* (circular basis) or *p,q* = *X,Y* (linear basis):

$$\vec{I}_{circ} = \overleftrightarrow{S}_{circ}\vec{I}_{Stokes}$$

$$\begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

$$\vec{I}_{lin} = \overleftrightarrow{S}_{lin}\vec{I}_{Stokes}$$

$$\begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

- Devices can be built to sample these linear or circular basis states in the signal domain (Stokes Vector is defined in "power" domain)
- Some components of J_i involve mixing of basis states, so dual-polarization matrix description desirable or even required for proper calibration

34

Full-Polarization Formalism: Signal Domain

- Substitute:

$$s_i \rightarrow \vec{s}_i = \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i, \qquad J_i \rightarrow \overleftrightarrow{J}_i = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}$$

- The *Jones matrix* thus corrupts a signal as follows:

$$\vec{s}\,'_i = \overleftrightarrow{J}_i\vec{s}_i \qquad \text{(sky integral omitted)}$$

$$\begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_i \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i$$

$$= \begin{pmatrix} J^{p\to p}s^p + J^{q\to p}s^q \\ J^{p\to q}s^p + J^{q\to q}s^q \end{pmatrix}_i$$

Full-Polarization Formalism: Correlation

35

- Four correlations are possible from two polarizations. The *outer product* (a ‘bookkeeping’ product) represents correlation in the matrix formalism:

$$\vec{V}_{ij}^{obs} = \left\langle \vec{s}'_i \otimes \vec{s}'^{*}_j \right\rangle = \left\langle \begin{pmatrix} s'^{p} \\ s'^{q} \end{pmatrix}_i \otimes \begin{pmatrix} s'^{p} \\ s'^{q} \end{pmatrix}^{*}_j \right\rangle = \begin{pmatrix} \left\langle s'^{p}_i \cdot s'^{*p}_j \right\rangle \\ \left\langle s'^{p}_i \cdot s'^{*q}_j \right\rangle \\ \left\langle s'^{q}_i \cdot s'^{*p}_j \right\rangle \\ \left\langle s'^{q}_i \cdot s'^{*q}_j \right\rangle \end{pmatrix}$$

- A very useful property of outer products:

$$\vec{V}_{ij}^{obs} = \left(\vec{s}'_i \otimes \vec{s}'^{*}_j\right) = \left(\overleftrightarrow{J}_i \vec{s}_i\right) \otimes \left(\overleftrightarrow{J}^{*}_j \vec{s}^{*}_j\right) = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}^{*}_j\right)\left(\vec{s}_i \otimes \vec{s}^{*}_j\right) = \overleftrightarrow{J}_{ij} \vec{V}_{ij}^{true}$$

Full-Polarization Formalism: Correlation (cont)

36

- The outer product for the Jones matrix:

$$\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}^{*}_j = \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}_i \otimes \begin{pmatrix} J^{p\to p} & J^{q\to p} \\ J^{p\to q} & J^{q\to q} \end{pmatrix}^{*}_j$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} & J_i^{p\to p} J_j^{*q\to p} & J_i^{q\to p} J_j^{*p\to p} & J_i^{q\to p} J_j^{*q\to p} \\ J_i^{p\to p} J_j^{*p\to q} & J_i^{p\to p} J_j^{*q\to q} & J_i^{q\to p} J_j^{*p\to q} & J_i^{q\to p} J_j^{*q\to q} \\ J_i^{p\to q} J_j^{*p\to p} & J_i^{p\to q} J_j^{*q\to p} & J_i^{q\to q} J_j^{*p\to p} & J_i^{q\to q} J_j^{*q\to p} \\ J_i^{p\to q} J_j^{*p\to q} & J_i^{p\to q} J_j^{*q\to q} & J_i^{q\to q} J_j^{*p\to q} & J_i^{q\to q} J_j^{*q\to q} \end{pmatrix} = \overleftrightarrow{J}_{ij}$$

  - J_{ij} is a 4x4 *Mueller matrix*
  - Antenna and array design driven by minimizing off-diagonal terms!

37

Signal Correlation and Matrices (cont)

- And finally, for fun, the correlation of corrupted signals:

$$\overleftrightarrow{J}_i \vec{s}_i \otimes \overleftrightarrow{J}_j^* \vec{s}_j^* = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^*\right)\left(\vec{s}_i \otimes \vec{s}_j^*\right)$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} & J_i^{p\to p} J_j^{*q\to p} & J_i^{q\to p} J_j^{*p\to p} & J_i^{q\to p} J_j^{*q\to p} \\ J_i^{p\to p} J_j^{*p\to q} & J_i^{p\to p} J_j^{*q\to q} & J_i^{q\to p} J_j^{*p\to q} & J_i^{q\to p} J_j^{*q\to q} \\ J_i^{p\to q} J_j^{*p\to p} & J_i^{p\to q} J_j^{*q\to p} & J_i^{q\to q} J_j^{*p\to p} & J_i^{q\to q} J_j^{*q\to p} \\ J_i^{p\to q} J_j^{*p\to q} & J_i^{p\to q} J_j^{*q\to q} & J_i^{q\to q} J_j^{*p\to q} & J_i^{q\to q} J_j^{*q\to q} \end{pmatrix} \begin{pmatrix} s_i^p \cdot s_j^{*p} \\ s_i^p \cdot s_j^{*q} \\ s_i^q \cdot s_j^{*p} \\ s_i^q \cdot s_j^{*q} \end{pmatrix}$$

$$= \begin{pmatrix} J_i^{p\to p} J_j^{*p\to p} s_i^p \cdot s_j^{*p} + J_i^{p\to p} J_j^{*q\to p} s_i^p \cdot s_j^{*q} + J_i^{q\to p} J_j^{*p\to p} s_i^q \cdot s_j^{*p} + J_i^{q\to p} J_j^{*q\to p} s_i^q \cdot s_j^{*q} \\ J_i^{p\to p} J_j^{*p\to q} s_i^p \cdot s_j^{*p} + J_i^{p\to p} J_j^{*q\to q} s_i^p \cdot s_j^{*q} + J_i^{q\to p} J_j^{*p\to q} s_i^q \cdot s_j^{*p} + J_i^{q\to p} J_j^{*q\to q} s_i^q \cdot s_j^{*q} \\ J_i^{p\to q} J_j^{*p\to p} s_i^p \cdot s_j^{*p} + J_i^{p\to q} J_j^{*q\to p} s_i^p \cdot s_j^{*q} + J_i^{q\to q} J_j^{*p\to p} s_i^q \cdot s_j^{*p} + J_i^{q\to q} J_j^{*q\to p} s_i^q \cdot s_j^{*q} \\ J_i^{p\to q} J_j^{*p\to q} s_i^p \cdot s_j^{*p} + J_i^{p\to q} J_j^{*q\to q} s_i^p \cdot s_j^{*q} + J_i^{q\to q} J_j^{*p\to q} s_i^q \cdot s_j^{*p} + J_i^{q\to q} J_j^{*q\to q} s_i^q \cdot s_j^{*q} \end{pmatrix}$$

- UGLY, but we rarely need to worry about detail at this level---just let this occur “inside” the matrix formalism, and work with the notation

38

The Matrix Measurement Equation

- We can now write down the Measurement Equation in matrix notation:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^*\right) \overleftrightarrow{S}\vec{I}(l,m)\, e^{-i2\pi\left(u_{ij}l + v_{ij}m\right)}\, dl\, dm$$

- …and consider how the J_i are products of many effects.

A Dictionary of Calibration Components

- J_i contains many components:
  - F = ionospheric Faraday rotation
  - T = tropospheric effects
  - P = parallactic angle
  - E = antenna voltage pattern
  - D = polarization leakage
  - G = electronic gain
  - B = bandpass response
  - K = geometric compensation
  - M, A = baseline-based corrections

$$\breve{J}_i = \breve{K}_i \breve{B}_i \breve{G}_i \breve{D}_i \breve{E}_i \breve{P}_i \breve{T}_i \breve{F}_i$$

- Order of terms follows signal path (right to left)
- Each term has matrix form of J_i with terms embodying its particular algebra (on- vs. off-diagonal terms, etc.)
- Direction-dependent terms must stay inside FT integral
- The full matrix equation (especially after correlation!) is daunting, but usually only need to consider the terms individually or in pairs, and rarely in open form (matrix formulation = shorthand)

Ionospheric Faraday Rotation, *F*

- The ionosphere is *birefringent*; one hand of circular polarization is delayed w.r.t. the other, introducing a dispersive phase shift:

$$\Delta\phi \approx 0.15\ \lambda^2 \int B_{\|} n_e ds\ \ \mathrm{deg}$$

$$\lambda \text{ in cm}, \quad n_e ds \text{ in } 10^{14}\,\mathrm{cm}^{-2}, \quad B_{\|} \text{ in G}$$

  - Rotates the linear polarization position angle
  - More important at longer wavelengths (λ^2)

$$TEC = \int n_e ds\ \sim 10^{14}\,\mathrm{cm}^{-2}; \quad B_{\|} \sim 1\mathrm{G}; \quad \lambda = 20\mathrm{cm}\ \rightarrow \Delta\phi \sim 60^{\circ}$$

  - More important at solar maximum and at sunrise/sunset, when ionosphere is most active and variable
  - Beware of direction-dependence within field-of-view!
- – Tracy Clark's lecture: "Low Frequency Interferometry" (Monday)

$$\breve{F}^{RL} = \begin{pmatrix} e^{i\Delta\phi} & 0 \\ 0 & e^{-i\Delta\phi} \end{pmatrix}; \ \breve{F}^{XY} = \begin{pmatrix} \cos\Delta\phi & -\sin\Delta\phi \\ \sin\Delta\phi & \cos\Delta\phi \end{pmatrix}$$

Tropospheric Effects, *T*

41

- The troposphere causes polarization-independent amplitude and phase effects due to emission/opacity and refraction, respectively
  - Typically 2-3m excess path length at zenith compared to vacuum
  - Higher noise contribution, less signal transmission: Lower SNR
  - Most important at ν > 15 GHz where water vapor absorbs/emits
  - More important nearer horizon where tropospheric path length greater
  - Clouds, weather = variability in phase and opacity; may vary across array
  - Water vapor radiometry? Phase transfer from low to high frequencies?

  – Crystal Brogan's lecture: "Millimeter Interferometry and ALMA" (Thursday)

$$\overset{\tau}{T}^{pq} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Parallactic Angle, *P*

42

- Orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos(l)\sin(h(t))}{\sin(l)\cos(\delta) - \cos(l)\sin(\delta)\cos(h(t))}\right)$$

$$l = \text{latitude},\ h(t) = \text{hour angle},\ \delta = \text{declination}$$

  - Rotates the position angle of linearly polarized radiation (c.f. *F*)
  - Analytically known, and its variation provides leverage for determining polarization-dependent effects

  – Rick Perley's lecture: "Polarization in Interferometry" (today!)

$$\overset{\tau}{P}^{RL} = \begin{pmatrix} e^{i\chi} & 0 \\ 0 & e^{-i\chi} \end{pmatrix};\ \overset{\tau}{P}^{XY} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}$$

Antenna Voltage Pattern, *E*

- Antennas of all designs have direction-dependent gain
  - Important when region of interest on sky comparable to or larger than λ/D
  - Important at lower frequencies where radio source surface density is greater and wide-field imaging techniques required
  - Beam squint: E^p and E^q not parallel, yielding spurious polarization
  - For convenience, direction dependence of polarization leakage (*D*) may be included in *E* (off-diagonal terms then non-zero)
- – Rick Perley's lecture: "Wide Field Imaging I" (Thursday)
- – Debra Shepherd's lecture: "Wide Field Imaging II" (Thursday)

$$E^{pq} = \begin{pmatrix} e^p(l,m) & 0 \\ 0 & e^q(l,m) \end{pmatrix}$$

Polarization Leakage, *D*

- Antenna & polarizer are not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed feeds have *d* ~ a few percent or less
  - A geometric property of the optical design, so frequency-dependent
  - For *R,L* systems, total-intensity imaging affected as ~*dQ, dU,* so only important at high dynamic range (*Q,U,d each ~few %*, typically)
  - For *R,L* systems, linear polarization imaging affected as ~*dI,* so almost always important
- – Rick Perley's lecture: "Polarization in Interferometry" (today!)

$$\vec{D}^{pq} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

"Electronic" Gain, *G*

- Catch-all for most amplitude and phase effects introduced by antenna electronics (amplifiers, mixers, quantizers, digitizers)
  - Most commonly treated calibration component
  - Dominates other effects for standard VLA observations
  - Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
  - Often also includes ionospheric and tropospheric effects which are typically difficult to separate unto themselves
  - Excludes frequency dependent effects (see *B*)

$$G^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

Bandpass Response, *B*

- *G*-like component describing frequency-dependence of antenna electronics, etc.
  - Filters used to select frequency passband not square
  - Optical and electronic reflections introduce ripples across band
  - Often assumed time-independent, but not necessarily so
  - Typically (but not necessarily) normalized
  - – Claire Chandler's lecture: "Spectral Line Observing I" (Wednesday)
  - – Lynn Matthew's lecture: "Spectral Line Observing II" (Wednesday)

$$B^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

Geometric Compensation, *K*

- Must get geometry right for Synthesis Fourier Transform relation to work in real time; residual errors here require "Fringe-fitting"
    - Antenna positions (geodesy)
    - Source directions (time-dependent in topocenter!) (astrometry)
    - Clocks
    - Electronic pathlengths
    - Importance scales with frequency and baseline length
  - – Ylva Pihlstrom's lecture: "Very Long Baseline Interferometry" (Thursday)

$$K^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

Non-closing Effects: *M, A*

- Correlator-based errors which do not decompose into antenna-based components
  - – Digital correlators designed to limit such effects to well-understood and ***uniform*** scaling laws (absorbed in *G*)
  - – Simple noise
  - – Additional errors can result from averaging in time and frequency over variation in antenna-based effects and visibilities (practical instruments are finite!)
  - – Correlated "noise" (e.g., RFI)
  - – Virtually indistinguishable from source structure effects
  - – Geodetic observers consider determination of radio source structure—a baseline-based effect—as a required *calibration* if antenna positions are to be determined accurately
  - – Diagonal 4x4 matrices, M_{ij} multiplies, A_{ij} adds

Calibrator Rules of Thumb

- *T, G, K:*
  - Strong and point-like sources, as near to target source as possible
  - Observe often enough to track phase and amplitude variations: calibration intervals of up to 10s of minutes at low frequencies (beware of ionosphere!), as short as 1 minute or less at high frequencies
  - Observe at least one calibrator of known flux density at least once
- *B:*
  - Strong enough for good narrow-bandwidth sensitivity (often, *T, G* calibrator is ok), point-like if visibility might change across band
  - Observe often enough to track variations (e.g., waveguide reflections change with temperature and are thus a function of time-of-day)
- *D:*
  - Best calibrator for full calibration is strong and unpolarized
  - If polarized, observe over a broad range of parallactic angle to disentangle *D*s and source polarization (often, *T, G* calibrator is ok)
- *F:*
  - Requires strongly polarized source observed often enough to track variation

The Full Matrix Measurement Equation

- The net *J* can be written:

$$
\begin{aligned}
\vec{J}_i \otimes \vec{J}_j^* &= \vec{M}_{ij}\left(\vec{K}_i\vec{B}_i\vec{G}_i\vec{D}_i\vec{E}_i\vec{P}_i\vec{T}_i\vec{F}_i \otimes \vec{K}_j^*\vec{B}_j^*\vec{G}_j^*\vec{D}_j^*\vec{E}_j^*\vec{P}_j^*\vec{T}_j^*\vec{F}_j^*\right)\\
&= \overleftrightarrow{M}_{ij}\left(\overleftrightarrow{K}_i \otimes \overleftrightarrow{K}_j^*\right)\left(\overleftrightarrow{B}_i \otimes \overleftrightarrow{B}_j^*\right)\left(\overleftrightarrow{G}_i \otimes \overleftrightarrow{G}_j^*\right)\left(\overleftrightarrow{D}_i \otimes \overleftrightarrow{D}_j^*\right)\\
&\qquad\left(\overleftrightarrow{E}_i \otimes \overleftrightarrow{E}_j^*\right)\left(\overleftrightarrow{P}_i \otimes \overleftrightarrow{P}_j^*\right)\left(\overleftrightarrow{T}_i \otimes \overleftrightarrow{T}_j^*\right)\left(\overleftrightarrow{F}_i \otimes \overleftrightarrow{F}_j^*\right)\\
&= \overleftrightarrow{M}_{ij}\overleftrightarrow{K}_{ij}\overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\overleftrightarrow{D}_{ij}\overleftrightarrow{E}_{ij}\overleftrightarrow{P}_{ij}\overleftrightarrow{T}_{ij}\overleftrightarrow{F}_{ij}
\end{aligned}
$$

- The total general *Measurement Equation* has the form:

$$
\vec{V}_{ij} = \vec{M}_{ij}\vec{K}_{ij}\vec{B}_{ij}\vec{G}_{ij}\int_{sky}\vec{D}_{ij}\vec{E}_{ij}\vec{P}_{ij}\vec{T}_{ij}\vec{F}_{ij}\,\vec{S}\vec{I}(l,m)\,e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)}\,dl\,dm + \vec{A}_{ij}
$$

  - *S* maps the Stokes vector, *I*, to the polarization basis of the instrument, all calibration terms cast in this basis

New Calibration Challenges

51

- Bandpass Calibration
  - Parameterized solutions (narrow-bandwidth, high resolution regime)
  - Spectrum of calibrators (wide absolute bandwidth regime)
- Phase vs. Frequency (self-) calibration
  - Troposphere and Ionosphere introduce time-variable phase effects which are easily parameterized in frequency (non-dispersive and dispersive delays)
- Frequency-dependent Instrumental Polarization
  - Contribution of geometric optics is wavelength-dependent (standing waves)
- Frequency-dependent Primary Beam
- Increased sensitivity: Can implied dynamic range be reached by conventional means?

Why not just solve for generic J_i matrix?

52

- It has been proposed (Hamaker 2000, 2006) that we can self-calibrate the generic J_i matrix, apply “post-calibration” constraints to ensure consistency, and recover full polarization measurements of the sky
- Important for low-frequency arrays where isolated calibrators are unavailable (such arrays see the whole sky)
- May have a role for EVLA & ALMA, where conventional calibration observations are not available
- Currently under study…

53

Data Examination and Editing

- After observation, initial data examination and editing very important
  - Will observations meet goals for calibration and science requirements?
  - Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)
  - Any persistently 'dead' antennas ($J_i=0$ during otherwise normal observing)? (check operator's logs)
  - Periods of poor weather? (check operator's log)
  - Any antennas shadowing others? Edit such data.
  - Amplitude and phase should be continuously varying—edit outliers
  - Be conservative: those antennas/timeranges which are bad on calibrators are probably bad on weak target sources—edit them
  - Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have significantly different amplitude response which may not be fatal—it may only need to be calibrated
  - Radio Frequency Interference (RFI)?
  - Choose reference antenna wisely (ever-present, stable response)
  - Increasing data volumes demand automated editing algorithms

54

Radio Frequency Interference

- RFI originates from man-made signals generated in the antenna electronics or by external sources (e.g., satellites, cell-phones, radio and TV stations, automobile ignitions, microwave ovens, computers and other electronic devices, etc.)
  - Adds to total noise power in all observations, thus decreasing sensitivity to desired natural signal, possibly pushing electronics into non-linear regimes
  - As a contribution to the n_i term, can correlate between antennas if of common origin and baseline short enough (insufficient decorrelation via K_i)

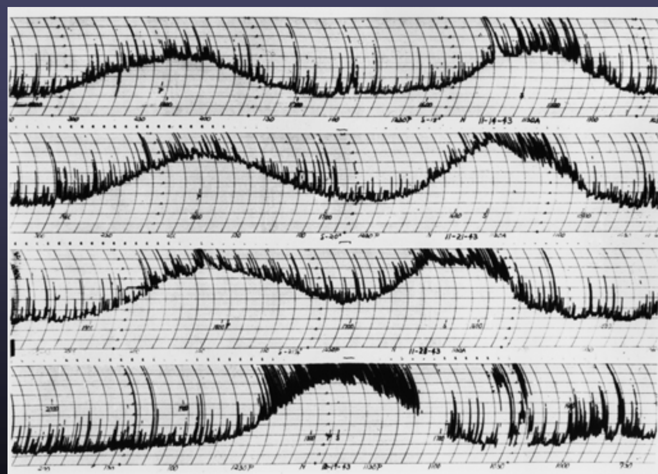
$$\begin{aligned}
\left\langle K_i x_i \cdot K_j^* x_j^* \right\rangle_{\Delta t} &= \left\langle K_i \left(s_i' + n_i + n_{RFI}\right) \cdot K_j^* \left(s_j' + n_j + n_{RFI}\right)^* \right\rangle_{\Delta t} \\
&= \left\langle K_i s_i' \cdot K_j^* s_j'^* \right\rangle + \left\langle K_i n_{RFI} \cdot K_j^* n_{RFI}^* \right\rangle \\
&= \left\langle K_i s_i' \cdot K_j^* s_j'^* \right\rangle + \left\langle K_i K_j^* \left| n_{RFI} \right|^2 \right\rangle
\end{aligned}$$

  - When RFI is correlated, it obscures natural emission in spectral line observations

55

Radio Frequency Interference

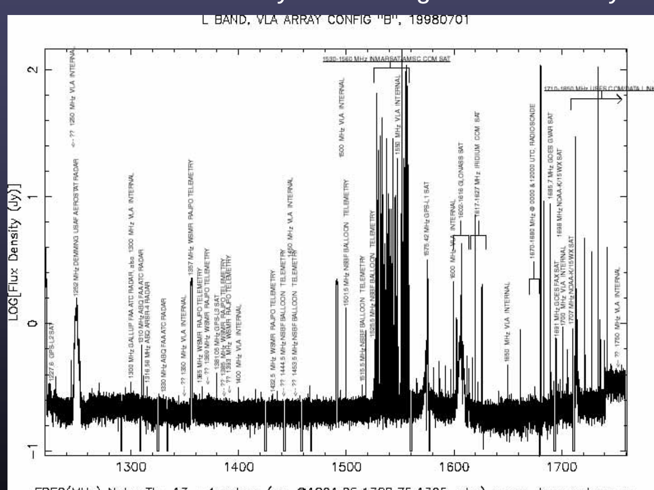
- Has always been a problem (Reber, 1944, in total power)!



56

Radio Frequency Interference (cont)

- Growth of telecom industry threatening radioastronomy!



57

Radio Frequency Interference (cont)

- RFI Mitigation
  - Careful electronics design in antennas, including filters, shielding
  - High-dynamic range digital sampling
  - Observatories world-wide lobbying for spectrum management
  - Choose interference-free frequencies: try to find 50 MHz (1 GHz) of clean spectrum in the VLA (EVLA) 1.6 GHz band!
  - Observe continuum experiments in spectral-line modes so affected channels can be edited
- Various off-line mitigation techniques under study
  - E.g., correlated RFI power appears at celestial pole in image domain…

58

Summary

- Determining calibration is as important as determining source structure—can't have one without the other
- Calibration dominated by antenna-based effects, permits separation of calibration from astronomical information (closure)
- Calibration formalism algebra-rich, but can be described piecemeal in comprehendible segments, according to well-defined effects
- Calibration determination is a single standard fitting problem
- Calibration an iterative process, improving various components in turn
- Point sources are the best calibrators
- Observe calibrators according requirements of calibration components
- Data examination and editing an important part of calibration
- Beware of RFI! (Please, no cell phones at the VLA site tour!)