Cross Correlators

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Outline

- The correlation function
- What is a correlator?
- Simple correlators
- Sampling and quantization
- Spectral line correlators
- Software correlators

This lecture is complementary to Chapter 4 of ASP 180

The VLBA Correlator

The Correlation Function

$$C_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle_T$$

- If $i = j$ it is an auto-correlation (AC). Otherwise it is a cross-correlation (CC).
- Useful for
  - Determining timescales (AC)
  - Motion detection (2-D CC)
  - Optical character recognition (2-D CC)
  - Pulsar timing / template matching (CC)

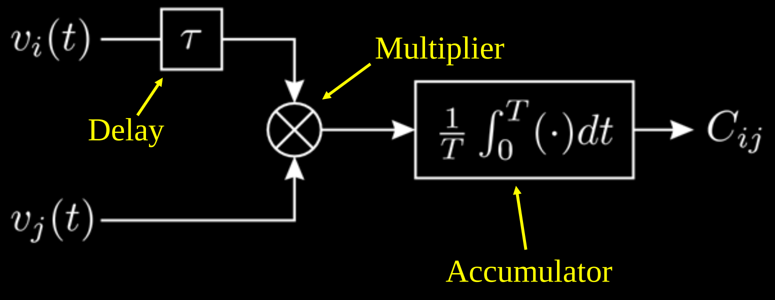
What is a Correlator?

In radio astronomy, a correlator is any device that combines sampled voltage time series from one or more antennas to produce sets of complex visibilities, V_{ij} .

- Visibilities are in general a function of
  - Frequency / polarization
  - Antenna pair
  - Time
- They are used for
  - Imaging
  - Spectroscopy / polarimetry
  - Astrometry

A Real (valued) Cross Correlator

$$C_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle_T$$



Visibilities

What astronomers really want is the complex visibility

$$V_{ij} = \left\langle E_i(t) E_j^*(t+\tau) \right\rangle$$

where the real part of $E_i(t)$ is the voltage measured by antenna i .

So what is the imaginary part of $E_i(t)$ *?*

It is the same as the real part but with each frequency component *phase* lagged by 90 degrees.

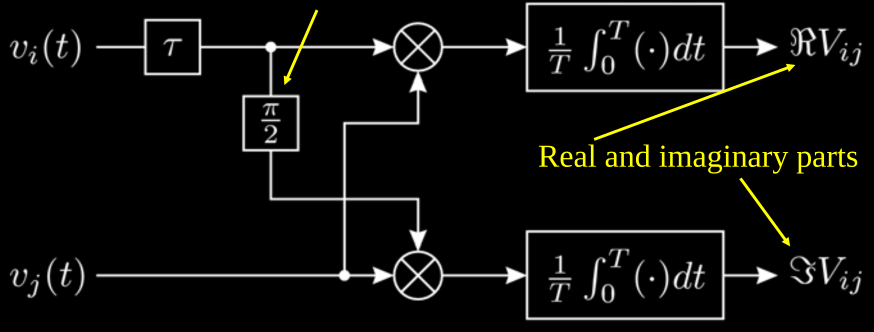
$$E_i(t) = v_i(t) + \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{v_i(t')}{t-t'} dt'$$

Hilbert transform

The Complex Correlator

$$V_{ij} = \langle v_i(t) v_j(t+\tau) \rangle + i \langle \mathcal{H}\left[v_i(t)\right] v_j(t+\tau) \rangle$$

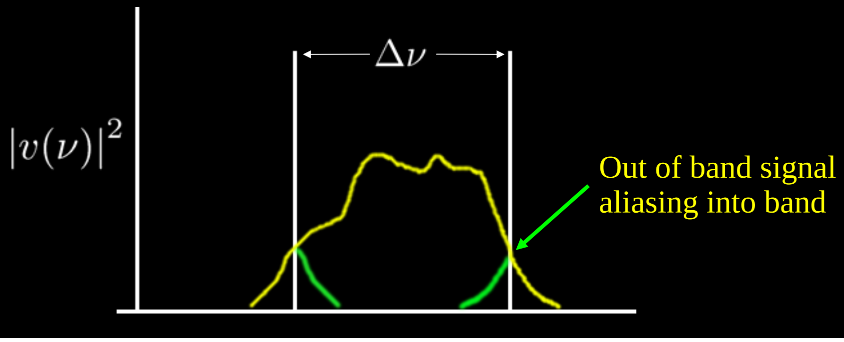
Hilbert transform



Real and imaginary parts

Nyquist-Shannon Sampling Theorem

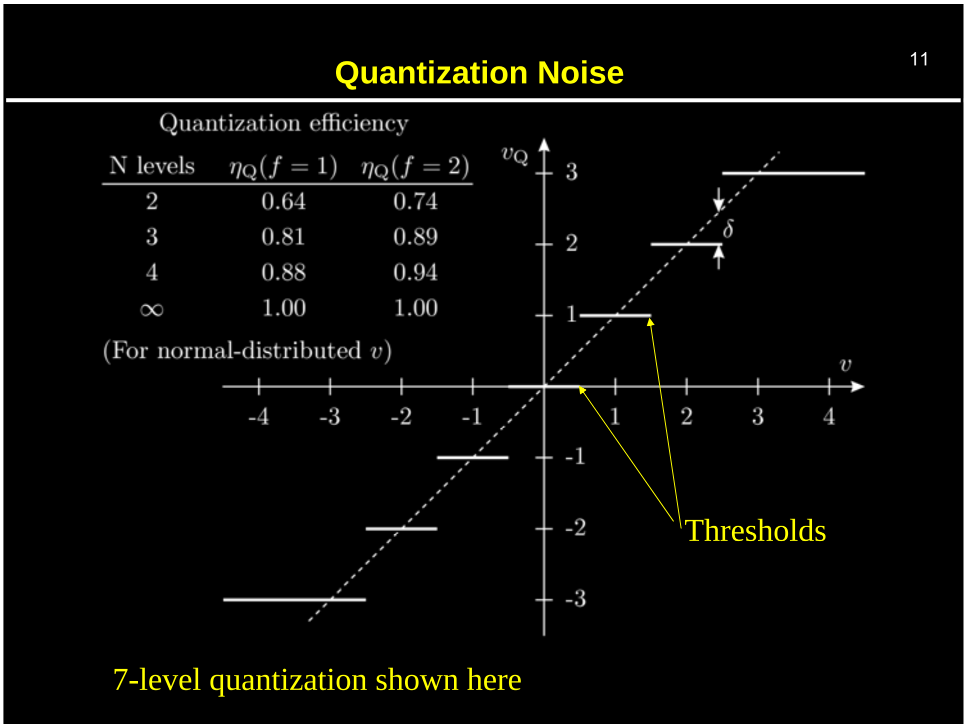
- If $v(t)$ is a real-valued time series sampled at "uniform" intervals, Δt, then a bandwidth $\Delta\nu = \frac{1}{2\Delta t}$ can be accurately reconstructed.
  - Uniform in which time system?
- $v(t)$ must be band limited.
  - Out of band signal is aliased into the band



Out of band signal aliasing into band

Quantization

- Sampling involves quantization of the signal
  - Quantization noise – non-Gaussian!
  - Strong signals become non-linear
  - Sampling theorem violated
    - Can no longer faithfully reconstruct original signal
- Quantization is often quite coarse
  - 3 levels at VLA
  - 2 or 4 at VLBA
  - Thresholds must be chosen carefully
- Unwanted noise lessens the impact of quantization at expense of sensitivity.
  - Usually $T_{sys} >> T_{source}$



Quantization Noise

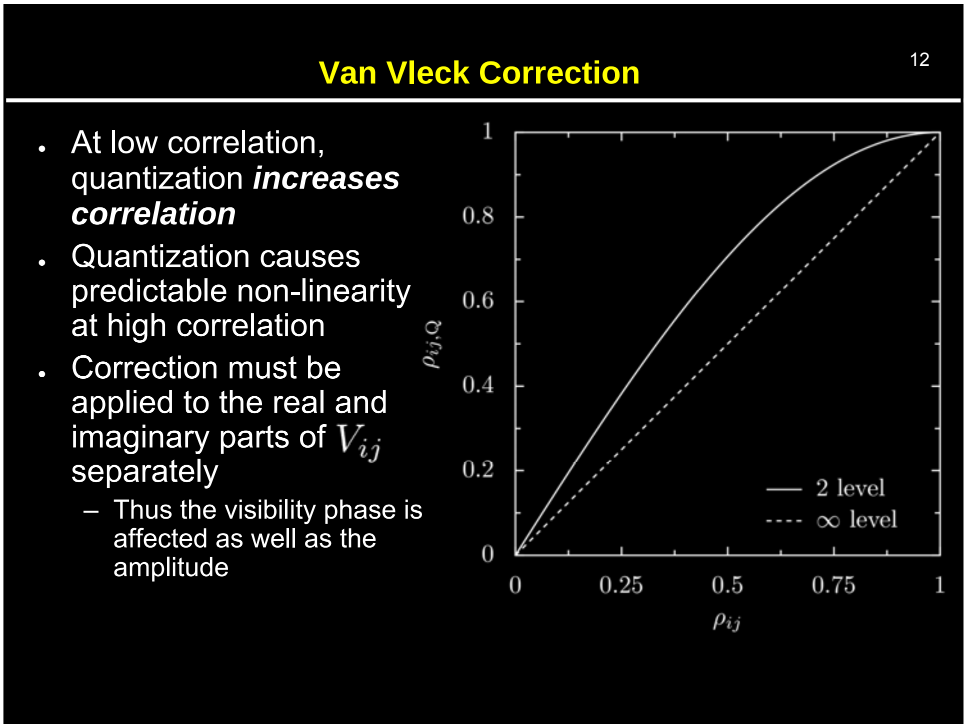
Quantization efficiency

N levels	$\eta_{\mathrm{Q}}(f=1)$	$\eta_{\mathrm{Q}}(f=2)$
2	0.64	0.74
3	0.81	0.89
4	0.88	0.94
∞	1.00	1.00

(For normal-distributed v)

Thresholds

7-level quantization shown here



Van Vleck Correction

- At low correlation, quantization ***increases correlation***
- Quantization causes predictable non-linearity at high correlation
- Correction must be applied to the real and imaginary parts of V_{ij} separately
  - Thus the visibility phase is affected as well as the amplitude

The Delay Model

- τ is the difference between the geometric delays of antenna j and antenna i. It can be + or - .
- The *delay center* moves across the sky with Earth rotation
  - τ is changing constantly
- Fringes at the delay center are *stopped.*
  - Long time integrations can be done
  - Wide bandwidths can be used
- Simple delay models incorporate:
  - Antenna locations
  - Source position
  - Earth orientation
- VLBI delay models must include much more!

Fractional Sample Delay Compensation

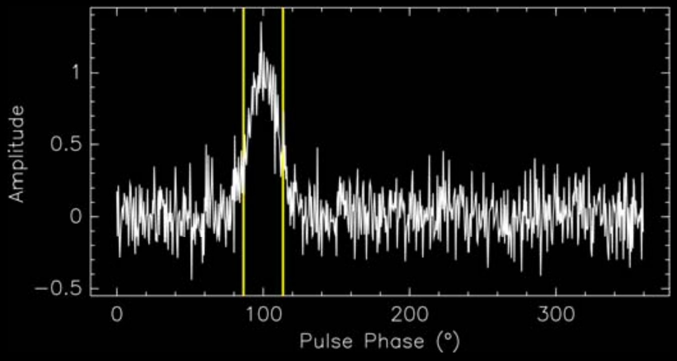
$$\tau = n\Delta t + \epsilon$$

- Delays must be corrected to better than Δt.
- Integer delay is usually done with digital delay lines.
- Fractional sample delay is trickier
- It is implemented differently at different correlators
  - Analog delay lines (DRAO array)
  - Add delay to the sampling clock (VLA)
  - Correct phases after multiplier (VLBA)

Note: this topic is covered extensively in ASP 180.

Pulsar Gating

- Pulsars emit regular pulses with small duty cycle
- Period in range 1 ms to 8 s; $\Delta t \ll P_{\mathrm{pulsar}} < T$
- Blanking during off-pulse improves sensitivity
- Propagation delay is frequency dependent



Spectral Line Correlators

- Chop up bandwidth for
  - Calibration
    - Bandpass calibration
    - Fringe fitting
  - Spectroscopy
  - Wide-field imaging
- Conceptual version
  - Build analog filter bank
  - Attach a complex correlator to each filter
- But…
  - Every channel is an edge channel
  - Bandwidth is wasted

Practical Spectral Line Correlators

- Want to use a single filter & sampler
  - Easier to calibrate
  - Practical, up to a point
- The FX architecture
  - F : Replace filterbank with digital Fourier transform
  - X : Use a complex-correlator for each frequency channel
  - Then integrate
- The XF architecture
  - X : Measure correlation function at many lags
  - Integrate
  - F : Fourier transform
- Other architectures or combinations of the above are possible

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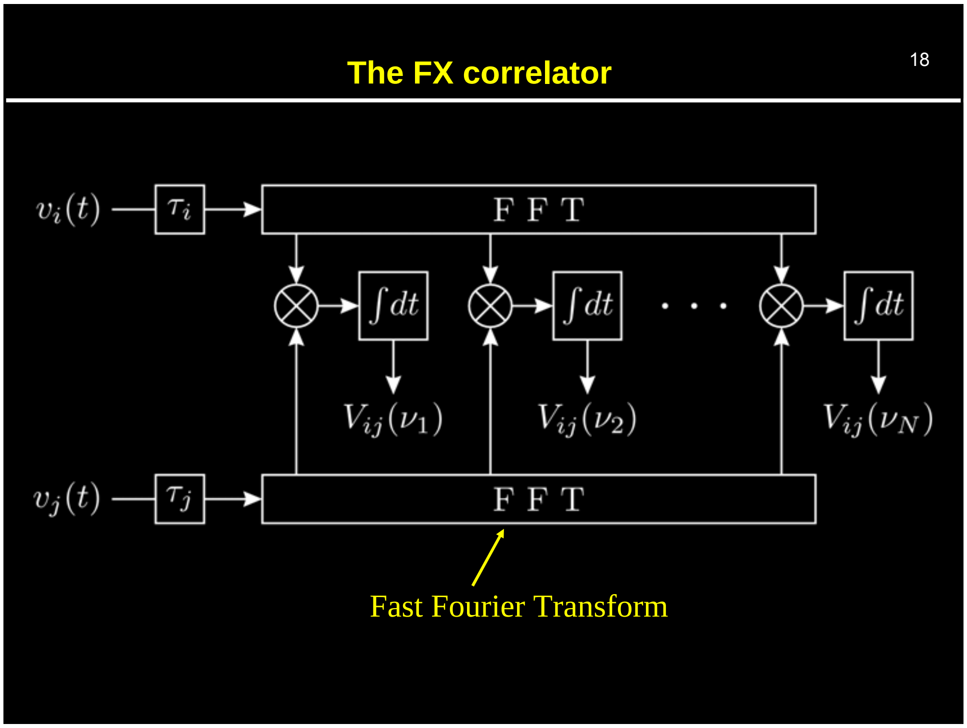


The FX correlator

Fast Fourier Transform

FX Correlators

- Spectrum is available *before* integration
  - Can apply fractional sample delay per channel
  - Can apply pulsar gate per channel
- Most of the digital parts run *N* times slower than the sample rate

FX Spectral Response

- FX Correlators derive spectra from truncated time series

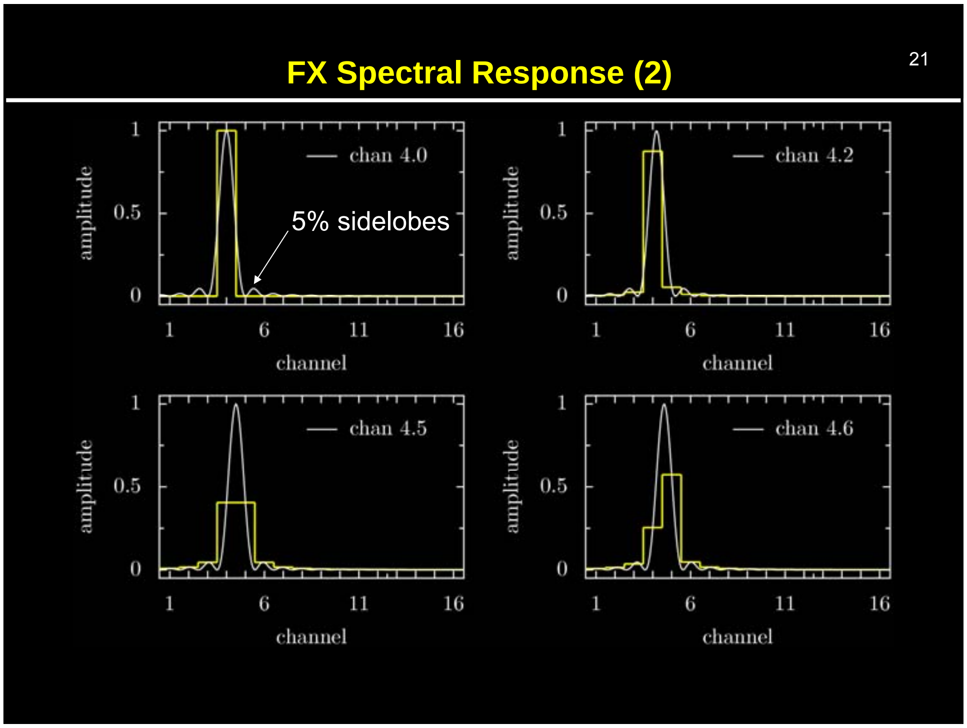
$$\begin{aligned} v(\nu) &= \mathcal{F}\left[v(t)\cdot\sqcap\left(\frac{t}{N\Delta t}\right)\right] \\ &= \mathcal{F}\left[v(t)\right]\star\mathcal{F}\left[\sqcap\left(\frac{t}{N\Delta t}\right)\right] \\ &\propto \mathcal{F}\left[v(t)\right]\star\operatorname{sinc}\left(N\Delta t\nu\right) \end{aligned}$$

Fourier transform

- Results in convolved visibility spectrum

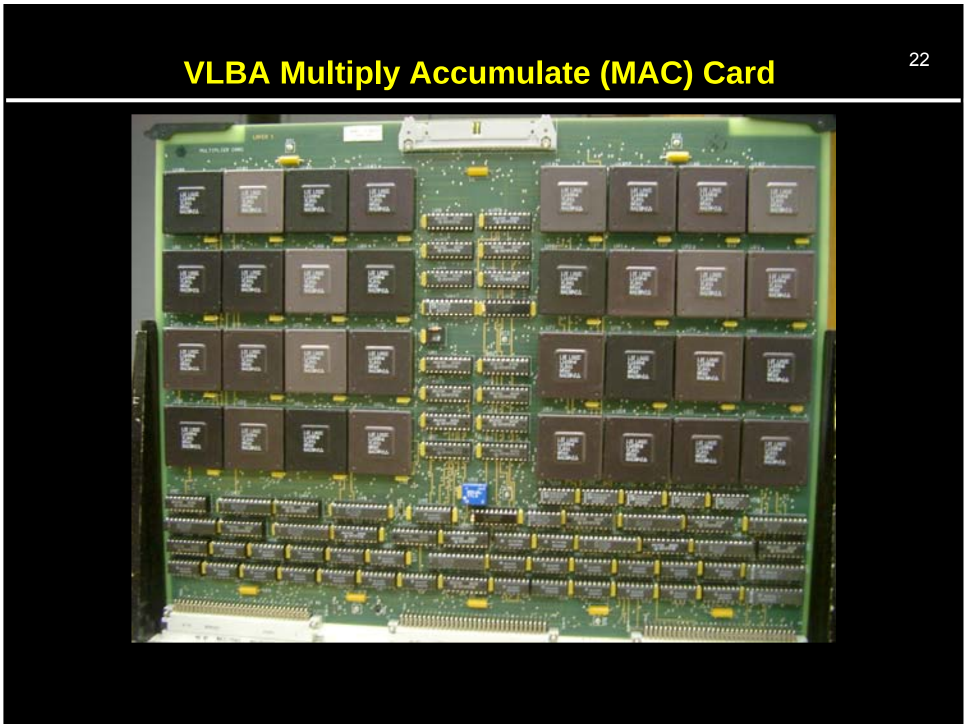
Convolution

$$\begin{aligned} V_{ij}(\nu) &= \left\langle\left(\mathcal{F}\left[v_i(t)\right]\star\operatorname{sinc}\left(N\Delta t\nu\right)\right)\left(\mathcal{F}\left[v_j(t)\right]\star\operatorname{sinc}\left(N\Delta t\,\nu\right)\right)^*\right\rangle \\ &= \left\langle\mathcal{F}\left[v_i(t)\right]\mathcal{F}\left[v_j(t)\right]^*\right\rangle\star\operatorname{sinc}^2\left(N\Delta t\,\nu\right) \end{aligned}$$



FX Spectral Response (2)

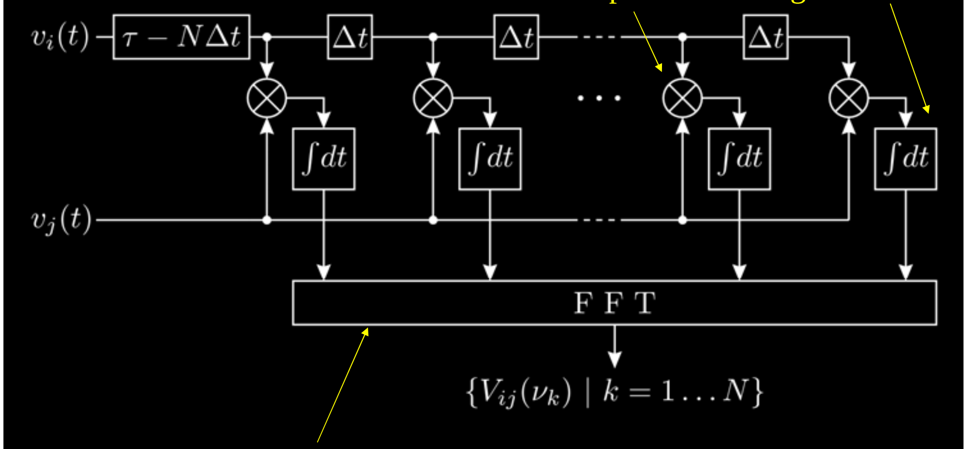
5% sidelobes



VLBA Multiply Accumulate (MAC) Card

The XF Correlator (real version)

$2N$ multipliers and integrators



Real to complex FFT; often done in software

XF Spectral Response

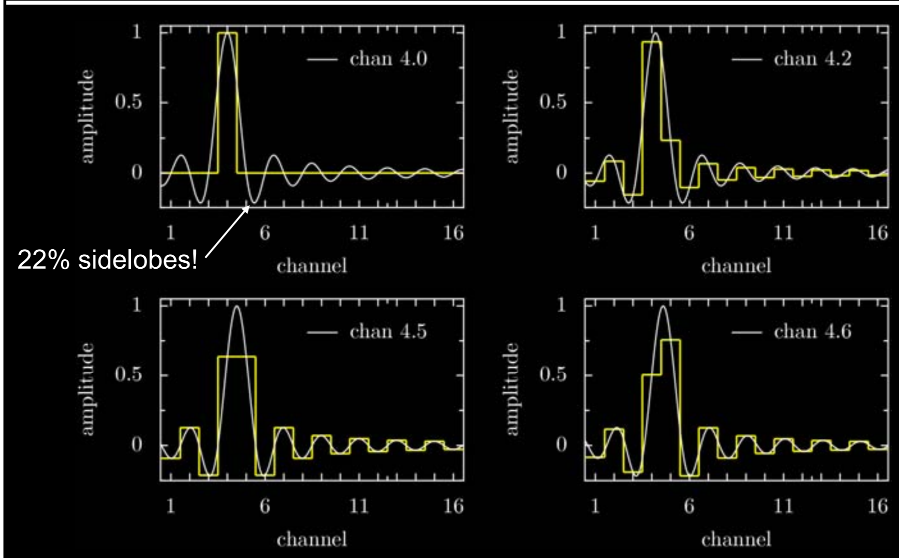
- XF correlators measure lags over a finite delay range

$$V_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle \cdot \sqcap\left(\frac{\tau}{N\Delta t}\right)$$

- Results in convolved visibility spectrum

$$\begin{aligned} V_{ij}(\nu) &= \mathcal{F}\left[\langle v_i(t) v_j(t+\tau) \rangle \cdot \sqcap\left(\frac{\tau}{N\Delta t}\right)\right] \\ &= \mathcal{F}\left[\langle v_i(t) v_j(t+\tau) \rangle\right] \star \mathrm{sinc}(N\Delta t\, \nu) \end{aligned}$$

XF Spectral Response (2)



22% sidelobes!

Hanning Smoothing

- Multiply lag spectrum by Hanning taper function

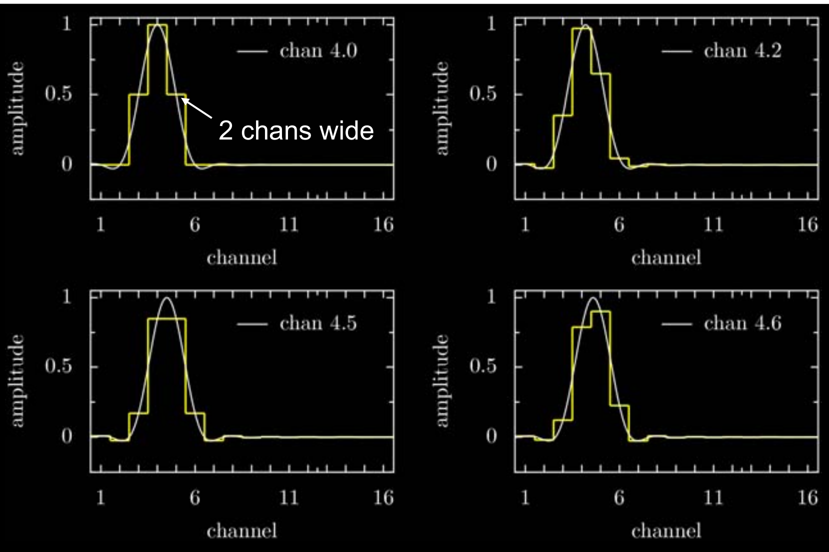
$$H(\tau) = \tfrac{1}{2}\left(1 + \cos\tfrac{\pi\tau}{N\Delta t}\right)$$

- This is equivalent to convolution of the spectrum by

$$H(\nu) = \tfrac{1}{2}\delta(\nu) + \tfrac{1}{4}\delta\left(\nu - \tfrac{1}{2N\Delta t}\right) + \tfrac{1}{4}\delta\left(\nu + \tfrac{1}{2N\Delta t}\right)$$

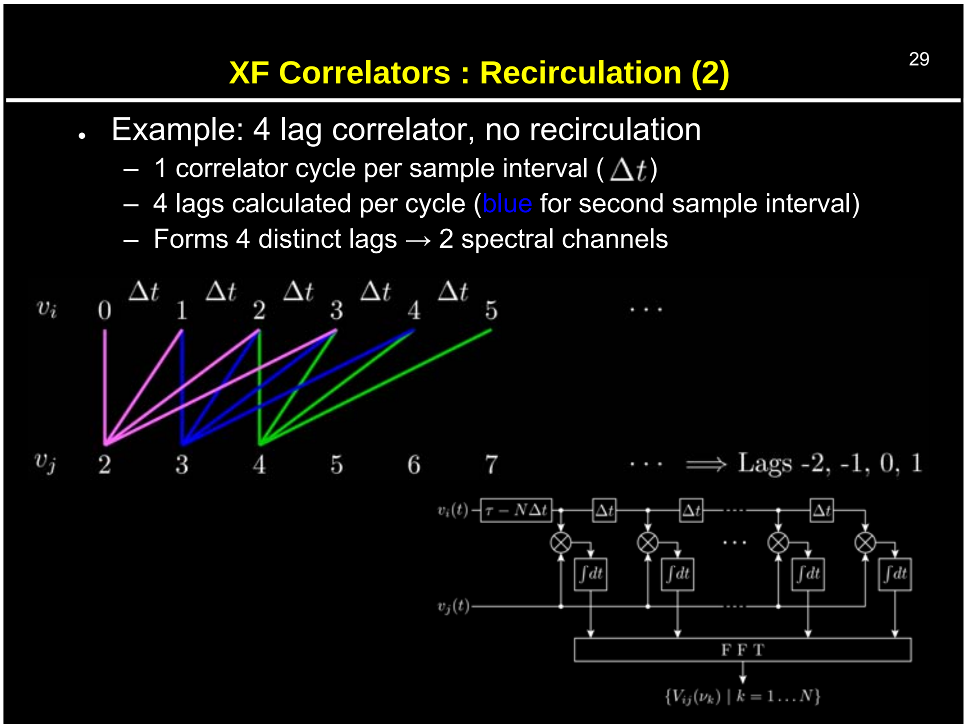
- Note that spectral resolution is reduced because the longest lags are down-weighted.

Hanning Smoothing (2)



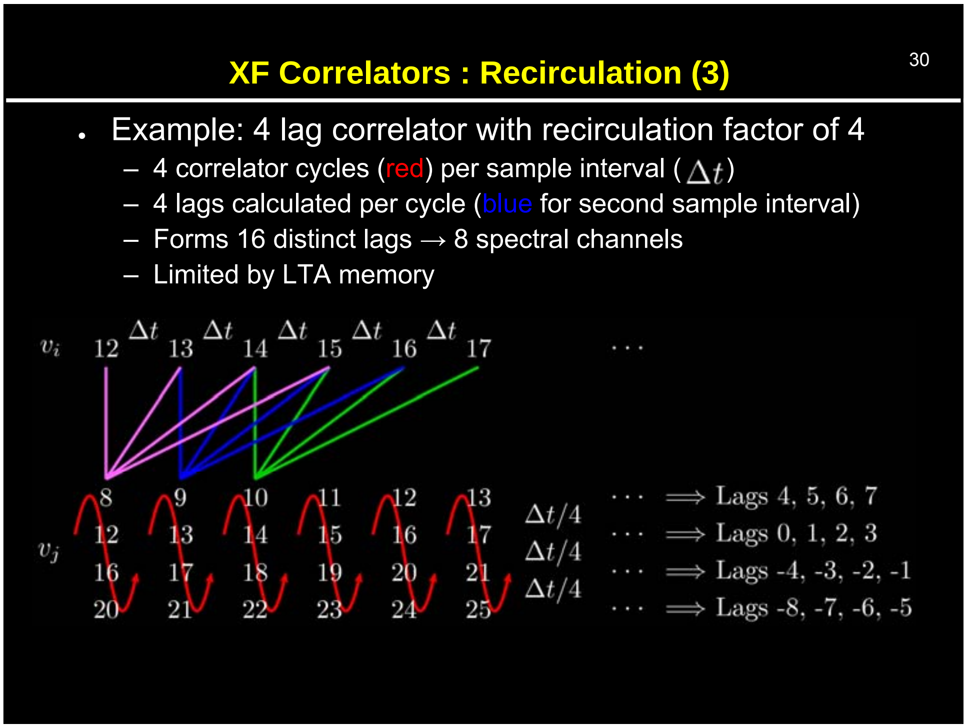
XF Correlators : Recirculation

- If the correlator runs at a fixed speed, then a slower input data rate can be processed with more lags in the same amount of time.
- A factor of two decrease in bandwidth can result in *four times* the spectral resolution.
  - x2 from reduced bandwidth
  - x2 from more lags



XF Correlators : Recirculation (2)

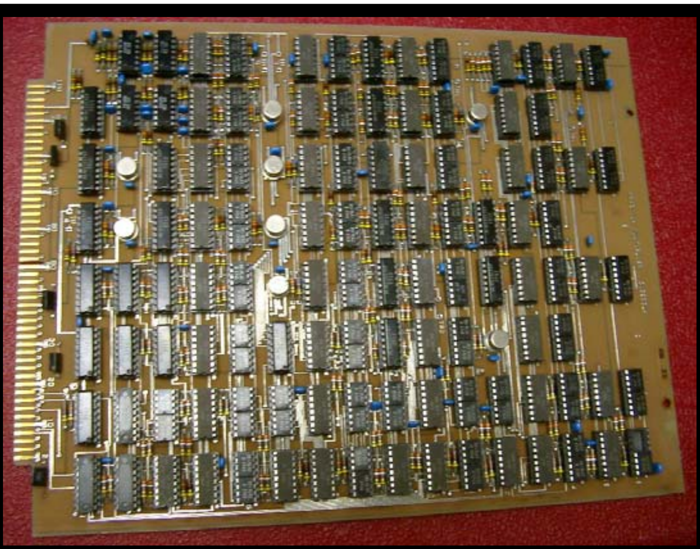
- Example: 4 lag correlator, no recirculation
  - 1 correlator cycle per sample interval (Δt)
  - 4 lags calculated per cycle (blue for second sample interval)
  - Forms 4 distinct lags → 2 spectral channels



XF Correlators : Recirculation (3)

- Example: 4 lag correlator with recirculation factor of 4
  - 4 correlator cycles (red) per sample interval (Δt)
  - 4 lags calculated per cycle (blue for second sample interval)
  - Forms 16 distinct lags → 8 spectral channels
  - Limited by LTA memory

VLA MAC Card



The EVLA WIDAR Correlator

- XF architecture duplicated 64 times, or “FXF”
  - Four 2 GHz basebands per polarization
  - Digital filter-bank makes 16 sub-bands per baseband
  - 16,384 channels/baseline at full sensitivity
  - 4 million channels with recirculation!
- Initially will support 32 stations; upgradable to 48
- 2 stations at 25% bandwidth or 4 stations at 6.25% bandwidth can replace 1 station input
- Correlator efficiency is about 95%
  - Compare to 81% for VLA
- VLBI ready
- Will add enormously to VLA capabilities!

Software Correlators

- Hardware correlator = special purpose computer
- Software correlator = general purpose computer running special purpose software
- Replace circuits with subroutines
- Typically FX correlators require least compute cycles and offer most flexibility

Software Correlators : Advantages

- **Accuracy** – In hardware extra precision means more wiring and circuitry and compromises are often made
- **Flexibility** – Spectral resolution, time resolution, number of inputs, ... not limited
- **Expandability** – A software correlator running on a computer cluster can be incrementally upgraded
- **Rapid development** – Changes and fixes don't require rewiring. Debugging is simpler.
- **Special modes** – Much easier to implement in software
- **Utilization** – All processor power is usable at all times
- **Cheaper** – In development

Software Correlators : Disadvantages

- Compared to equivalent hardware correlator:
  - Power hungry
  - Big
  - More expensive? (per processing power)

Software Correlators : Performance

- For a cluster of 3 GHz Pentium processors
  - VLA correlator ~ 150 CPUs
  - VLBA correlator ~ 250 CPUs
  - EVLA correlator ~ 200,000 CPUs!
- Other means of achieving high compute rates
  - Floating point accelerators, DSPs, FPGAs
  - The Cell processor
  - Graphics Processing units

Software Correlators : Niche Uses

- Baseband recorded data
  - Data rates limited by recording media
  - Media costs greater than processing costs!
- High spectral & time resolution
  - Masers
  - Spacecraft tracking
  - Very wide fields of view
- VLBI fringe checking

Generally good for VLBI!

Things To Remember

- Correlator = device to calculate the correlation function
  - Typically special purpose computers
  - Software correlators becoming practical
- Two major classes of spectral line correlators
  - XF (or lag) correlator (e.g. VLA)
  - FX correlator (e.g. VLBA)
- Geometric delays need to be compensated to high accuracy
- Correlated visibilities are imperfect due to
  - Quantization
  - Spectral response
  - Delay model errors