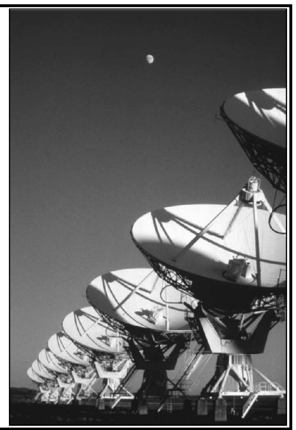
Sensitivity

Joan Wrobel



Ninth Synthesis Imaging Summer School
Socorro, June 15-22, 2004

Outline

- What is Sensitivity & Why Should You Care?
- What Are Measures of Antenna Performance?
- What is the Sensitivity of an Interferometer?
- What is the Sensitivity of a Synthesis Image?
- Summary



What is Sensitivity & Why Should You Care?

- Measure of weakest detectable emission
- Important throughout research program
  - Sound observing proposal
  - Sensible error analysis in journal
- Expressed in units involving Janskys
  - Unit for interferometer is Jansky (Jy)
  - Unit for synthesis image is Jy beam^{-1}
  - 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}
- Common current units: milliJy, microJy
- Common future units: nanoJy

Measures of Antenna Performance

Source & System Temperatures

- What is received power P ?
- Write P as equivalent temperature T of matched termination at receiver input
  - Rayleigh-Jeans limit to Planck law $P = k_B \times T \times \Delta\nu$
  - Boltzmann constant k_B
  - Observing bandwidth $\Delta\nu$
- Amplify P by g^2 where g is voltage gain
- Separate powers from source, system noise
  - Source antenna temperature $T_a \Rightarrow$ source power $P_a = g^2 \times k_B \times T_a \times \Delta\nu$
  - System temperature $T_{sys} \Rightarrow$ noise power $P_N = g^2 \times k_B \times T_{sys} \times \Delta\nu$

Measures of Antenna Performance

Gain

- Source power $P_a = g^2 \times k_B \times T_a \times \Delta\nu$
  - Let $T_a = K \times S$ for source flux density S, constant K
  - Then $P_a = g^2 \times k_B \times K \times S \times \Delta\nu$ (1)
- But source power also $P_a = ½ \times g^2 \times \eta_a \times A \times S \times \Delta\nu$ (2)
  - Antenna area A, efficiency η_a
  - Rx accepts 1/2 radiation from unpolarized source
- Equate (1), (2) and solve for K

$$K = (\eta_a \times A) / (2 \times k_B) = T_a / S$$

  - K is antenna's gain or "sensitivity", unit degree Jy^{-1}
- K measures antenna performance but no T_{sys}

Measures of Antenna Performance

System Equivalent Flux Density

- Antenna temperature $T_a = K \times S$
  - Source power $P_a = g^2 \times k_B \times K \times S \times \Delta\nu$
- Express system temperature analogously
  - Let $T_{sys} = K \times SEFD$
  - *SEFD* is system equivalent flux density, unit Jy
  - System noise power $P_N = g^2 \times k_B \times K \times SEFD \times \Delta\nu$
- *SEFD* measures overall antenna performance

$$SEFD = T_{sys} / K$$

  - Depends on T_{sys} and $K = (\eta_a \times A) / (2 \times k_B)$
  - Examples in Table 9-1

Interferometer Sensitivity

Real Correlator - 1

- Simple correlator with single real output that is product of voltages from antennas *j,i*
  - $SEFD_i = T_{sysi} / K_i$ and $SEFD_j = T_{sysj} / K_j$
  - Each antenna collects bandwidth $\Delta\nu$

- Interferometer built from these antennas has
  - Accumulation time τ_{acc}, system efficiency η_s
  - Source, system noise powers imply sensitivity ΔS_{ij}
- Weak source limit
  - $S << SEFD_i$
  - $S << SEFD_j$

$$\Delta S_{ij} = \frac{1}{\eta_s} \times \sqrt{\frac{SEFD_i \times SEFD_j}{2 \times \Delta\nu \times \tau_{acc}}}$$

Interferometer Sensitivity

Real Correlator - 2

- For $SEFD_i = SEFD_j = SEFD$ drop subscripts

$$\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$$

  - Units Jy

- Interferometer system efficiency η_s
  - Accounts for electronics, digital losses
  - E.g.: VLA continuum
    - *Digitize in 3 levels, collect data 96.2% of time*
    - *Effective* $\eta_s = 0.81 \times \sqrt{0.962} = 0.79$

Interferometer Sensitivity

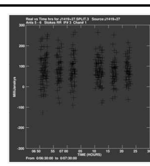
Abscissa spans 30 minutes.
Ordinate spans +/-300 milliJy.

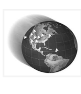
Complex Correlator

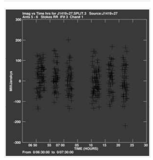
- Delivers two channels
  - Real S_R, sensitivity ΔS
  - Imaginary S_I, sensitivity ΔS
- Eg: VLBA continuum
  - Figure 9-1 at 8.4 GHz
  - Observed scatter $S_R(t)$, $S_I(t)$
  - Predicted ΔS = 69 milliJy

$$\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$$

  - Resembles observed scatter



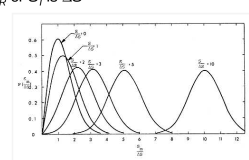




Interferometer Sensitivity

Measured Amplitude

- Measured visibility amplitude $S_m = \sqrt{S_R^2 + S_I^2}$
  - Standard deviation (s.d.) of S_R or S_I is ΔS
- True visibility amplitude S
- Probability $\Pr(S_m/\Delta S)$
  - Figure 9-2



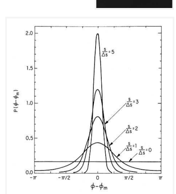
  - Behavior with true $S/\Delta S$
    - *High: Gaussian, s.d.* ΔS
    - *Zero: Rayleigh, s.d.* $\Delta S \times \sqrt{2-(\pi/2)}$
    - *Low: Rice.* S_m *gives biased estimate of S. Use unbias method.*

Interferometer Sensitivity

Measured Phase

- Measured visibility phase

  $\phi_m = \arctan(S_I/S_R)$
- True visibility phase ϕ
- Probability $\Pr(\phi - \phi_m)$
  - Figure 9-2



  - Behavior with true $S/\Delta S$
    - *High: Gaussian*
    - *Zero: Uniform*
- Seek weak detection in phase, not in amplitude

Image Sensitivity

Single Polarization

- Simplest weighting case where visibility samples
  - Have same interferometer sensitivities $\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$
  - Have same signal-to-noise ratios *w*
  - Combined with natural weight (*W=1),* no taper (*T=1)*
- Image sensitivity is s.d. of mean of *L* samples, each with s.d. ΔS, i.e., $\Delta I_m = \Delta S / \sqrt{L}$
  - ***N*** antennas, # of interferometers $\frac{1}{2} \times N \times (N-1)$
  - # of accumulation times t_{int}/τ_{acc}
  - $L = \frac{1}{2} \times N \times (N-1) \times (t_{int}/\tau_{acc})$
  - So $\Delta I_m = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{N \times (N-1) \times \Delta\nu \times t_{int}}}$

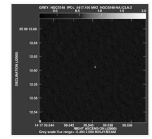
Image Sensitivity

Dual Polarizations - 1

- Single-polarization image sensitivity ΔI_m
- Dual-polarization data $\Rightarrow$ image Stokes *I,Q,U,V*
  - Gaussian noise in each image
  - Mean zero, s.d. $\Delta I = \Delta Q = \Delta U = \Delta V = \Delta I_m/\surd 2$
- Linearly polarized flux density $P = \sqrt{Q^2 + U^2}$
  - Rayleigh noise, s.d. $\Delta Q \times \sqrt{2-(\pi/2)} = \Delta U \times \sqrt{2-(\pi/2)}$
  - Cf. visibility amplitude, Figure 9-2
- Polarization position angle $\chi = ½ \times \arctan(U/Q)$
  - Uniform noise between $\pm\pi/2$
  - Cf. visibility phase, Figure 9-2, $\pm\pi/2$

Image Sensitivity

Dual Polarizations – 2



- Eg: VLBA continuum
  - Figure 9-3 at 8.4 GHz
  - Observed
    - *Stokes I, simplest weighting*
    - *Gaussian noise ΔI = 90 microJy beam^{-1}*
  - Predicted

    $\Delta I = \Delta I_m/\surd 2 = \Delta S/\sqrt{2 \times L}$

    $L = ½ \times N \times (N-1) \times (t_{int}/\tau_{acc})$
    - *Previous e.g. ΔS*
    - *Plus here L = 77,200*
    - *So s.d. ΔI = 88 microJy beam^{-1}*

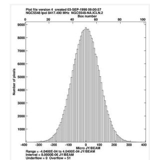


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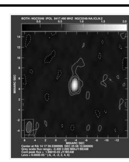
Image Sensitivity

Dual Polarizations – 3



- Eg: VLBA continuum
  - Figure 9-3 at 8.4 GHz
  - Observed
    - *I_{peak} = 2 milliJy beam^{-1}*
    - *Gaussian noise ΔI = 90 microJy beam^{-1}*
  - Position error from sensitivity?

    $$\Delta\theta = \frac{1}{2} \times \theta_{HPBW} \times \frac{\Delta I}{I_{peak}}$$
    - *Gaussian beam θ_{HPBW} = 1.5 milliarcsec*
    - *Then $\Delta\theta$ = 34 microarcsec*
    - *Other position errors dominate*

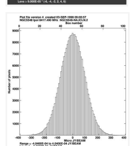


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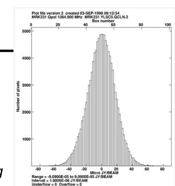
Image Sensitivity



Dual Polarizations – 4

- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - *Stokes Q, U images, simplest weighting*
    - *Gaussian $\Delta Q = \Delta U$ = 17 microJy beam^{-1}*
  - Predicted

    $\Delta Q = \Delta U = \Delta I_m/\surd 2 = \Delta S/\sqrt{2 \times L}$

    $$\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$$

    $L = ½ \times N \times (N-1) \times (t_{int}/\tau_{acc})$
  - So s.d. *$\Delta Q = \Delta U$ = 16 microJy beam^{-1}*

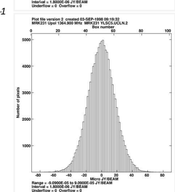


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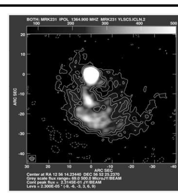
Image Sensitivity

Dual Polarizations – 5



- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - *Stokes I image*
    - *Simplest weighting*
    - *Gaussian noise $\Delta I > \Delta Q = \Delta U$*
  - Expect s.d. $\Delta I = \Delta Q = \Delta U = \Delta I_m/\surd 2$

    if each image limited by sensitivity
    - *Other factors can increase ΔI*
    - *Suspect dynamic range as I_{peak} = 10,000 ΔI*
    - *Lesson: Use sensitivity as tool to diagnose problems*

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Sensitivity

Summary – 1

- One antenna
  - System temperature T_{sys}
  - Gain *K*
- Overall antenna performance is measured by system equivalent flux density *SEFD*

  $$SEFD = T_{sys} / K$$
  - Units Jy

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Sensitivity

Summary - 2

- Connect two antennas to form interferometer
  - Antennas have same *SEFD,* observing bandwidth $\Delta\nu$
  - Interferometer system efficiency η_s
  - Interferometer accumulation time τ_{acc}
- Sensitivity of interferometer

$$\Delta S = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{2 \times \Delta\nu \times \tau_{acc}}}$$

  - Units Jy

Sensitivity

Summary - 3

- Connect *N* antennas to form array
  - Antennas have same *SEFD,* observing bandwidth $\Delta\nu$
  - Array has system efficiency η_s
  - Array integrates for time t_{int}
  - Form synthesis image of single polarization
- Sensitivity of synthesis image

$$\Delta I_m = \frac{1}{\eta_s} \times \frac{SEFD}{\sqrt{N \times (N-1) \times \Delta\nu \times t_{int}}}$$

  - Units Jy beam^{-1}

Westerbork's Most Sensitive Image

A Deep WSRT 1.4 GHz Radio Survey of the Spitzer Space Telescope FLSv Region, Morganti et al. 2004,

A&A, in press, astro-ph/0405418

ΔI = 8.5 microJy beam^{-1}