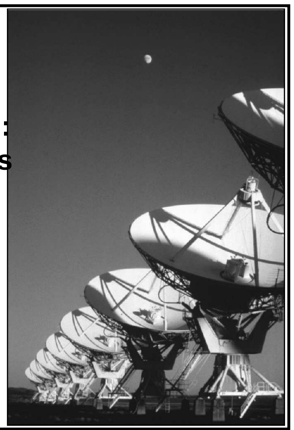

Wide Field Imaging I: Non-Coplanar Arrays

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Introduction

- From the first lecture, we have a general relation (the measurement equation) between the complex visibility V(u,v,w), and the sky intensity $I(l,m)$:

$$V(u,v,w) = \iint I(l,m)\exp\{-2\pi i[ul + vm + w(n-1)]\}\, dldm/n$$

where $n = \sqrt{1-l^2-m^2} = \cos\theta$

- This equation is valid for:
  - spatially incoherent radiation from the far field,
  - phase-tracking interferometer
  - narrow bandwidth
- Under certain conditions, a 2-d geometry can be applied, in which case the M.E. becomes a 2-d Fourier transform, and can be easily inverted to solve for $I(l,m)$

Heading toward 3-d

- For the VLA, the 'certain condition' is that the field of view be small.

$$\theta_{\max} < \sqrt{\lambda / B}$$

- For the VLA, at λ = 20 cm, in its A-configuration, this angle is about 10 arcmin.
- The problem worsens for lower frequencies, and smaller antennas.

$$N_{2D} = \left[\frac{\theta_{\rm FWHM}}{\theta_{\max}}\right]^2 = \left[\frac{\lambda}{D}\sqrt{\frac{B}{\lambda}}\right]^2 = \frac{\lambda B}{D^2}$$

- So – how do we handle this problem?

The '3-D' Formalism

- The general relationship is not a Fourier transform. It thus doesn't have an immediate inversion.
- But, we can consider the 3-D Fourier transform of V(u,v,w), giving a 3-D 'image volume' F(*l,m,n*), and try relate this to the desired intensity, I(*l,m*).
- The mathematical details are straightforward, but tedious, and are given in detail on pp 384-385 in the White Book.

The 3-D Image Volume

- We find that:

$$F(l,m,n) = \iiint \mathrm{V_0(u,v,w)}\exp[2\pi i(\mathrm{u}l + \mathrm{v}m + \mathrm{w}n)]d\mathrm{u}d\mathrm{v}d\mathrm{w}$$

where

$$\mathrm{V_0(u,v,w)} = \exp(-2\pi i\,\mathrm{w})\,\mathrm{V(u,v,w)}$$

Is related to the desired intensity, I(*l,m*), by:

$$F(l,m,n) = \frac{I(l,m)}{\sqrt{1-l^2-m^2}}\,\delta\left(\sqrt{1-l^2-m^2}-1\right)$$

This relation looks daunting, but in fact has a lovely geometric interpretation.

Interpretation

- The modified visibility V_0(u,v,w) is simply the observed visibility with no 'fringe tracking'.
- It's what we would measure if the fringes were held fixed, and the sky moves through them.
- The bottom equation states that the image volume is everywhere empty (F(*l,m,n*)=0), except on a spherical surface of unit radius where, $l^2 + m^2 + n^2 = 1$
- The desired intensity, I(*l,m*)/n, is the value of F(*l,m,n*) on this unit surface
- Note: The image volume is not a physical space. It is a mathematical construct.

Benefits of a 3-D Fourier Relation

- The identification of a 3-D Fourier relation means that all the relationships and theorems mentioned for 2-d imaging in earlier lectures carry over directly.
- These include:
  - Effects of finite sampling of V(u,v,w).
  - Effects of maximum and minimum baselines.
  - The 'dirty beam' (now a 'beam ball'), sidelobes, etc.
  - Deconvolution, 'clean beams', self-calibration.
- All these are, in principle, carried over unchanged, with the addition of a third dimension.
- But the real world makes this straightforward approach unattractive.

Coordinates

- Where on the unit sphere are sources found?

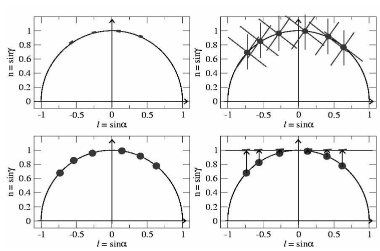
$$l = \cos\delta \sin\Delta\alpha$$
$$m = \sin\delta\cos\delta_0 - \cos\delta\sin\delta_0\cos\Delta\alpha$$
$$n = \sin\delta\sin\delta_0 + \cos\delta\cos\delta\ \cos\Delta\alpha$$

where δ_0 = the reference declination, and
$\Delta\alpha$ = the offset from the reference right ascension.

However, where the sources appear on a 2-d plane is a different matter.

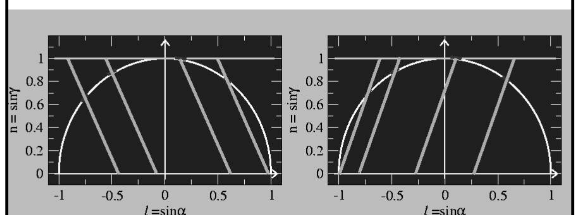
Illustrative Examples

Upper Left: True Image. Upper right: Dirty Image.
Lower Left: After deconvolution. Lower right: After projection



Snapshots in 3D Imaging

- A snapshot VLA observations, seen in '3D', creates 'line beams' (orange lines) , which uniquely project the sources (red bars) to the image plane (blue).
- Except for the tangent point, the apparent locations of the sources move in time.



Apparent Source Movement

- As seen from the sky, the plane containing the VLA rotates through the day.
- This causes the 'line-beams' associated with the snapshot images to rotate.
- The apparent source position in a 2-D image thus rotates, following a conic section. The loci of the path is:

$$l' = l + \left(\sqrt{1-l^2-m^2} - 1\right)\tan Z \sin\chi$$
$$m' = m - \left(\sqrt{1-l^2-m^2} - 1\right)\tan Z \cos\chi$$

where Z = the zenith distance, and χ = parallactic angle.

Wandering Sources

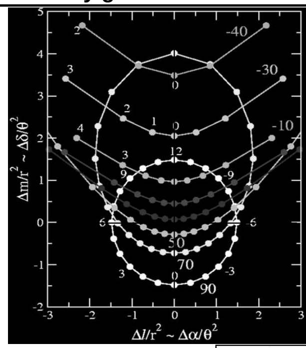
- The apparent source motion is a function of zenith distance and parallactic angle, given by:

$$\tan\chi = \frac{\cos\phi\sin H}{\sin\phi\cos\delta - \cos\phi\sin\delta\cos H}$$
$$\cos Z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos H$$

where
H = hour angle
δ = declination
ϕ = antenna latitude

And around they go …

- On the 2-d (tangent) image plane, source positions follow conic sections.
- The plots show the loci for declinations 90, 70, 50, 30, 10, -10, -30, and -40.
- Each dot represents the location at integer HA.
- The path is a circle at declination 90.
- The only observation with no error is at HA=0, δ=34.



How bad is it?

- In practical terms …
- The offset is $(\cos\theta - 1)\tan Z \sim (\theta^2 \tan Z)/2$
- At the antenna beam half-power, $\theta \sim \lambda/2D$
- So the position error, ε, measured in synthesized beamwidths, (λ/B) at this distance can be written as

$$\varepsilon = \frac{\lambda B}{8D^2}\tan Z$$

- For the VLA's A-configuration, this offset error (in beamwidths) can be written:

$$\varepsilon \sim 5\lambda_m \tan Z$$

- This is very significant at meter wavelengths!

So, What can we do?

- There are a number of ways to deal with this problem.

1. Compute the entire 3-d image volume.
   - The most straightforward approach.
   - But this approach is hugely wasteful in computing resources!
   - The minimum number of 'vertical planes' needed is: $B\theta^2/\lambda$
   - The number of volume pixels to be calculated is: $4B^3\theta^2/\lambda^3$
   - But the number of pixels actually needed is: $4B^2/\lambda^2$
   - So the fraction of effort which is wasted is $1 - \lambda/(B\theta^2)$.
   - And this about 90% at 20cm wavelength in A-configuration, for a full primary beam image.

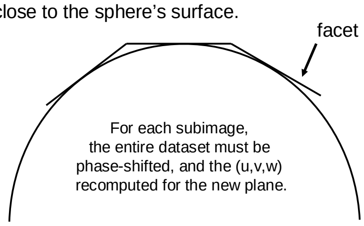
Deep Cubes!

- To give an idea of the scale of processing, the table below shows the number of 'vertical' planes needed to encompass the VLA's primary beam.
- For the A-configuration, each plane is at least 2048 x 2048.
- For the NMA, it's at least 16384 x 16384!
- And one cube would be needed for each spectral channel.

λ	NMA	A	B	C	D	E
400cm	2250	225	68	23	7	2
90cm	560	56	17	6	2	1
20cm	110	11	4	2	1	1
6cm	40	4	2	1	1	1
2cm	10	2	1	1	1	1
1.3cm	6	1	1	1	1	1

Polyhedron Imaging

- The wasted effort is in computing pixels we don't need.
- The polyhedron approach approximates the unit sphere with small flat planes, each of which stays close to the sphere's surface.



Polyhedron Approach, (cont.)

- How many facets are needed?
- If we want to minimize distortions, the plane mustn't depart from the unit sphere by more than the synthesized beam, λ/B. Simple analysis (see the book) shows the number of facets will be:

$$N_f \sim 2\lambda B/D^2$$

or twice the number needed for 3-D imaging.

- But the size of each image is much smaller, so the total number of cells computed is much smaller.
- The extra effort in phase computation and (u,v,w) rotation is more than made up by the reduction in the number of cells computed.
- This approach is the current standard.

Polyhedron Imaging

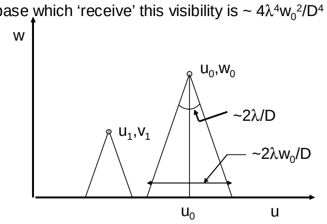
- Procedure is then:
  - Determine number of facets, and the size of each.
  - Generate each facet image, rotating the (u,v,w) and phase-shifting the phase center for each.
  - Jointly deconvolve the set. The Clark/Cotton/Schwab major/minor cycle system is well suited for this.
  - Project the finished images onto a 2-d surface.
- Added benefit of this approach:
  - As each facet is independently generated, one can imagine a separate antenna-based calibration for each.
  - Useful if calibration is a function of direction as well as time.
  - This is needed for meter-wavelength imaging.

W-Projection

- Although the polyhedron approach works well, it is expensive, and there are annoying boundary issues – where the facets overlap.
- The facet approach re-projects the dataset for each sub-image direction. Is it possible to project the data onto a single (u,v) plane, accounting for all the necessary phase shifts?
- Answer is YES! Tim Cornwell has developed a new algorithm, termed ‘w-projection’, to do this.
- Available only in AIPS++, this approach permits a single 2-d image/deconvolution, and eliminates the annoying edge effects which accompany re-projection.

W-Projection

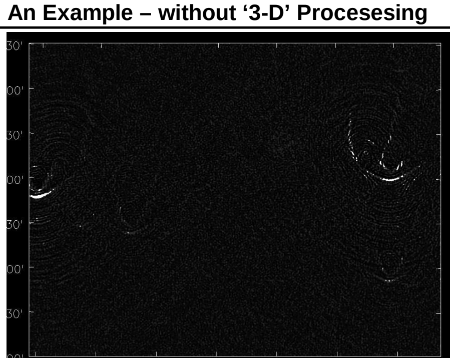
- Each visibility, at location (u,v,w) is mapped to the w=0 plane, with a phase shift proportional to the distance.
- Each visibility is mapped to ALL the points lying within a cone whose full angle is the same as the field of view of the desired map – $\sim 2\lambda/D$ for a full-field image.
- Area in the base of the cone is $\sim 4\lambda^2 w^2/D^2 < 4B^2/D^2$. Number of cells on the base which ‘receive’ this visibility is $\sim 4\lambda^4 w_0^2/D^4 < 4\lambda^2 B^2/D^4$.



W-Projection

- The phase shift for each visibility onto the w=0 plane is in fact a Fresnel diffraction function.
- Each 2-d cell receives a value for each observed visibility within an (upward/downwards) cone of full angle $\theta < \lambda/D$.
- In practice, the data are non-uniformly vertically gridded – speeds up the projection.
- There are a lot of computations, but they are done only once.
- Spatially-variant self-cal can be accommodated (but hasn’t yet).

An Example – without ‘3-D’ Procesesing



Example – with 3D processing