

Polarization in Interferometry


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Polarization in interferometry

- Physics of Polarization
- Interferometer Response to Polarization
- Polarization Calibration & Observational Strategies
- Polarization Data & Image Analysis
- Astrophysics of Polarization
- Examples

- References:
 - Synth Im. II lecture 6, also parts of 1, 3, 5, 32
 - “Tools of Radio Astronomy” Rohlfs & Wilson




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WARNING!

- Polarimetry is an exercise in bookkeeping!
 - many places to make sign errors!
 - many places with complex conjugation (or not)
 - possible different conventions (e.g. signs)
 - different conventions for notation!
 - lots of matrix multiplications
- And be assured...
 - I've mixed notations (by stealing slides ☺)
 - I've made sign errors ☹ (I call it “choice of convention” ☺)
 - I've probably made math errors ☹
 - I've probably made it too confusing by going into detail ☹
 - But ... persevere (and read up on it later) ☺


DON'T PANIC !




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Polarization Fundamentals

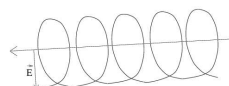


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Physics of polarization


- Maxwell's Equations + Wave Equation
 - $\mathbf{E} \cdot \mathbf{B} = 0$ (perpendicular) ; $E_z = B_z = 0$ (transverse)
- Electric Vector – 2 orthogonal independent waves:
 - $E_x = E_1 \cos(kz - \omega t + \delta_1)$ $k = 2\pi / \lambda$
 - $E_y = E_2 \cos(kz - \omega t + \delta_2)$ $\omega = 2\pi \nu$
 - describes helical path on surface of a cylinder...



– parameters $E_1, E_2, \delta = \delta_1 - \delta_2$ define ellipse

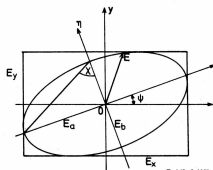


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The Polarization Ellipse


- Axes of ellipse E_a, E_b
 - $S_0 = E_1^2 + E_2^2 = E_a^2 + E_b^2$ Poynting flux
 - δ phase difference $\tau = kz - \omega t$
 - $E_\xi = E_a \cos(\tau + \delta) = E_x \cos \Psi + E_y \sin \Psi$
 - $E_\eta = E_b \sin(\tau + \delta) = -E_x \sin \Psi + E_y \cos \Psi$



Rohlfs & Wilson

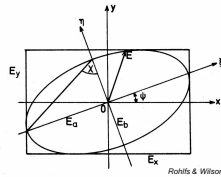


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The polarization ellipse continued...

- Ellipticity and Orientation
 - $E_1 / E_2 = \tan \alpha$ $\tan 2\psi = -\tan 2\alpha \cos \delta$
 - $E_a / E_b = \tan \chi$ $\sin 2\chi = \sin 2\alpha \sin \delta$
 - handedness ($\sin \delta > 0$ or $\tan \chi > 0 \rightarrow$ right-handed)



Rohls & Wilson

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Polarization ellipse – special cases

- Linear polarization
 - $\delta = \delta_1 - \delta_2 = m\pi$ $m = 0, \pm 1, \pm 2, \dots$
 - ellipse becomes straight line
 - electric vector position angle $\psi = \alpha$
- Circular polarization
 - $\delta = \frac{1}{2} (1 + m) \pi$ $m = 0, 1, \pm 2, \dots$
 - equation of circle $E_x^2 + E_y^2 = E^2$
 - orthogonal linear components:
 - $E_x = E \cos \tau$
 - $E_y = \pm E \sin (\tau - \pi/2)$
 - note quarter-wave delay between E_x and E_y !



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Orthogonal representation

- A monochromatic wave can be expressed as the superposition of two *orthogonal linearly polarized waves*
- A arbitrary elliptically polarized wave can also equally well be described as the superposition of two orthogonal *circularly polarized waves*!
- We are free to choose the orthogonal basis for the representation of the polarization
- NOTE: Monochromatic waves MUST be (fully) polarized – IT'S THE LAW!



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Linear and Circular representations

- Orthogonal Linear representation:
 - $E_\xi = E_a \cos (\tau + \delta) = E_x \cos \psi + E_y \sin \psi$
 - $E_\eta = E_b \sin (\tau + \delta) = -E_x \sin \psi + E_y \cos \psi$
- Orthogonal Circular representation:
 - $E_\xi = E_a \cos (\tau + \delta) = (E_r + E_l) \cos (\tau + \delta)$
 - $E_\eta = E_b \sin (\tau + \delta) = (E_r - E_l) \cos (\tau + \delta - \pi/2)$
 - $E_r = \frac{1}{2} (E_a + E_b)$
 - $E_l = \frac{1}{2} (E_a - E_b)$

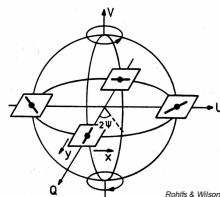
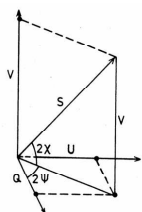


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The Poincare Sphere

- Treat 2ψ and 2χ as longitude and latitude on sphere of radius S_0



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Stokes parameters

- Spherical coordinates: radius I, axes Q, U, V
 - $S_0 = I = E_a^2 + E_b^2$
 - $S_1 = Q = S_0 \cos 2\chi \cos 2\psi$
 - $S_2 = U = S_0 \cos 2\chi \sin 2\psi$
 - $S_3 = V = S_0 \sin 2\chi$
- Only 3 independent parameters:
 - $S_0^2 = S_1^2 + S_2^2 + S_3^2$
 - $I^2 = Q^2 + U^2 + V^2$
- Stokes parameters I, Q, U, V
 - form complete description of wave polarization
 - NOTE: above true for monochromatic wave!



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Stokes parameters and polarization ellipse

- Spherical coordinates: radius I , axes Q , U , V
 - $S_0 = I = E_a^2 + E_b^2$
 - $S_1 = Q = S_0 \cos 2\chi \cos 2\psi$
 - $S_2 = U = S_0 \cos 2\chi \sin 2\psi$
 - $S_3 = V = S_0 \sin 2\chi$
- In terms of the polarization ellipse:
 - $S_0 = I = E_1^2 + E_2^2$
 - $S_1 = Q = E_1^2 - E_2^2$
 - $S_2 = U = 2 E_1 E_2 \cos \delta$
 - $S_3 = V = 2 E_1 E_2 \sin \delta$



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Stokes parameters special cases

- Linear Polarization
 - $S_0 = I = E^2 = S$
 - $S_1 = Q = I \cos 2\psi$
 - $S_2 = U = I \sin 2\psi$
 - $S_3 = V = 0$
 - Circular Polarization
 - $S_0 = I = S$
 - $S_1 = Q = 0$
 - $S_2 = U = 0$
 - $S_3 = V = S$ (RCP) or $-S$ (LCP)
- Note: cycle in 180°



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Quasi-monochromatic waves

- Monochromatic waves are fully polarized
- Observable waves (averaged over $\Delta\nu/\nu \ll 1$)
- Analytic signals for x and y components:
 - $E_x(t) = a_1(t) e^{i(\phi_1(t) - 2\pi\nu t)}$
 - $E_y(t) = a_2(t) e^{i(\phi_2(t) - 2\pi\nu t)}$
 - actual components are the real parts $\text{Re } E_x(t)$, $\text{Re } E_y(t)$
- Stokes parameters
 - $S_0 = I = \langle a_1^2 \rangle + \langle a_2^2 \rangle$
 - $S_1 = Q = \langle a_1^2 \rangle - \langle a_2^2 \rangle$
 - $S_2 = U = 2 \langle a_1 a_2 \cos \delta \rangle$
 - $S_3 = V = 2 \langle a_1 a_2 \sin \delta \rangle$



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Stokes parameters and intensity measurements

- If phase of E_y is retarded by ϵ relative to E_x , the electric vector in the orientation θ is:
 - $E(t, \theta, \epsilon) = E_x \cos \theta + E_y e^{i\epsilon} \sin \theta$
- Intensity measured for angle θ :
 - $I(\theta, \epsilon) = \langle E(t, \theta, \epsilon) E^*(t, \theta, \epsilon) \rangle$
- Can calculate Stokes parameters from 6 intensities:
 - $S_0 = I = I(0^\circ, 0) + I(90^\circ, 0)$
 - $S_1 = Q = I(0^\circ, 0) - I(90^\circ, 0)$
 - $S_2 = U = I(45^\circ, 0) - I(135^\circ, 0)$
 - $S_3 = V = I(45^\circ, \pi/2) - I(135^\circ, \pi/2)$
 - this can be done for single-dish (intensity) polarimetry!



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Partial polarization

- The observable electric field need not be fully polarized as it is the superposition of monochromatic waves
- On the Poincare sphere:
 - $S_0^2 \geq S_1^2 + S_2^2 + S_3^2$
 - $I^2 \geq Q^2 + U^2 + V^2$
- Degree of polarization p :
 - $p^2 S_0^2 = S_1^2 + S_2^2 + S_3^2$
 - $p^2 I^2 = Q^2 + U^2 + V^2$



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Summary – Fundamentals

- Monochromatic waves are polarized
- Expressible as 2 orthogonal independent transverse waves
 - elliptical cross-section \rightarrow polarization ellipse
 - 3 independent parameters
 - choice of basis, e.g. linear or circular
- Poincare sphere convenient representation
 - Stokes parameters I , Q , U , V
 - I intensity; Q, U linear polarization, V circular polarization
- Quasi-monochromatic "waves" in reality
 - can be partially polarized
 - still represented by Stokes parameters



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Antenna & Interferometer Polarization



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Interferometer response to polarization

- Stokes parameter recap:
 - intensity I
 - fractional polarization $(p\ I)^2 = Q^2 + U^2 + V^2$
 - linear polarization Q, U $(m\ I)^2 = Q^2 + U^2$
 - circular polarization V $(v\ I)^2 = V^2$
- Coordinate system dependence:
 - I independent
 - V depends on choice of "handedness"
 - $V > 0$ for RCP
 - Q, U depend on choice of "North" (plus handedness)
 - Q "points" North, U 45 toward East
 - EVPA $\Phi = \frac{1}{2} \tan^{-1} (U/Q)$ (North through East)



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Reflector antenna systems

- Reflections
 - turn RCP \leftrightarrow LCP
 - E-field allowed only in plane of surface
- Curvature of surfaces
 - introduce cross-polarization
 - effect increases with curvature (as t/D decreases)
- Symmetry
 - on-axis systems see linear cross-polarization
 - off-axis feeds introduce asymmetries & R/L squint
- Feedhorn & Polarizers
 - introduce further effects (e.g. "leakage")

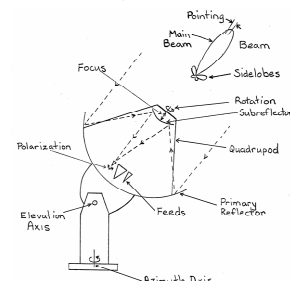


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Optics – Cassegrain radio telescope

- Paraboloid illuminated by feedhorn:

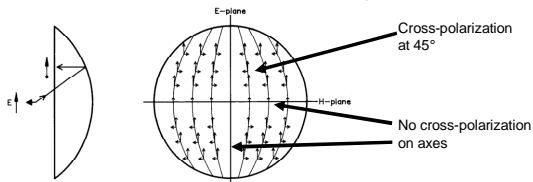


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Optics – telescope response

- Reflections
 - turn RCP \leftrightarrow LCP
 - E-field (currents) allowed only in plane of surface
- "Field distribution" on aperture for E and B planes:

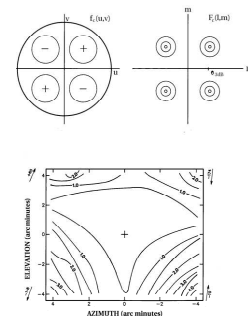


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Polarization field pattern

- Cross-polarization
 - 4-lobed pattern
- Off-axis feed system
 - perpendicular elliptical linear pol. beams
 - R and L beams diverge (beam squint)
- See also:
 - "Antennas" lecture by P. Napier



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Feeds – Linear or Circular?

- The VLA uses a circular feedhorn design
 - plus (quarter-wave) polarizer to convert circular polarization from feed into linear polarization in rectangular waveguide
 - correlations will be between R and L from each antenna
 - RR RL LR RL form complete set of correlations
- Linear feeds are also used
 - e.g. ATCA, ALMA (and possibly EVLA at 1.4 GHz)
 - no need for (lossy) polarizer!
 - correlations will be between X and Y from each antenna
 - XX XY YX YY form complete set of correlations
- Optical aberrations are the same in these two cases
 - but different response to electronic (e.g. gain) effects

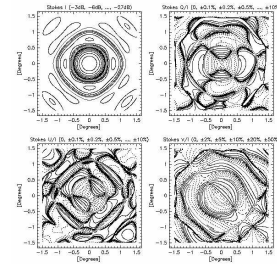


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Example – simulated VLA patterns

- EVLA Memo 58 “Using Grasp8 to Study the VLA Beam” W. Brisken

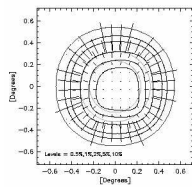


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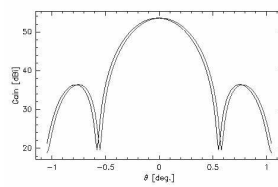


Example – simulated VLA patterns

- EVLA Memo 58 “Using Grasp8 to Study the VLA Beam” W. Brisken



Linear Polarization



Circular Polarization cuts in R & L

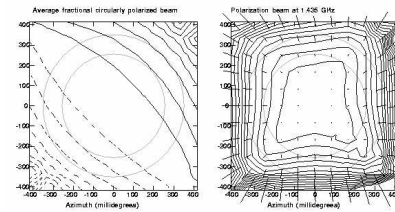


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Example – measured VLA patterns

- AIPS Memo 86 “Widefield Polarization Correction of VLA Snapshot Images at 1.4 GHz” W. Cotton (1994)



Circular Polarization

Linear Polarization

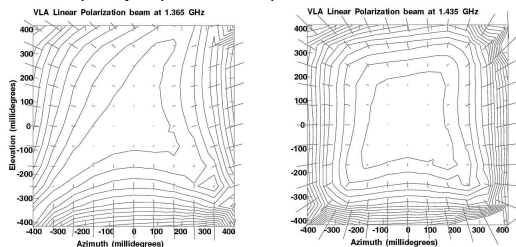


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Example – measured VLA patterns

- frequency dependence of polarization beam :

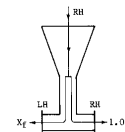


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Beyond optics – waveguides & receivers

- Response of polarizers
 - convert R & L to X & Y in waveguide
 - purity and orthogonality errors
- Other elements in signal path:
 - Sub-reflector & Feedhorn
 - symmetry & orientation
 - Ortho-mode transducers (OMT)
 - split orthogonal modes into waveguide
 - Polarizers
 - retard one mode by quarter-wave to convert LP → CP
 - frequency dependent!
 - Amplifiers
 - separate chains for R and L signals



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Back to the Measurement Equation

- Polarization effects in the signal chain appear as error terms in the Measurement Equation

– e.g. "Calibration" lecture, G. Moellenbrock:

- F = ionospheric Faraday rotation
- T = tropospheric effects
- P = parallactic angle
- E = antenna voltage pattern
- D = polarization leakage
- G = electronic gain
- B = bandpass response

$$\vec{J}_i = \vec{B}_i \vec{G}_i \vec{D}_i \vec{E}_i \vec{P}_i \vec{T}_i \vec{F}_i$$

Antenna i

Baseline ij (outer product)

$$\begin{aligned} \vec{J}_i \otimes \vec{J}_j^* &= (\vec{B}_i \vec{G}_i \vec{D}_i \vec{E}_i \vec{P}_i \vec{T}_i \vec{F}_i \otimes \vec{B}_j^* \vec{G}_j^* \vec{D}_j^* \vec{E}_j^* \vec{P}_j^* \vec{T}_j^* \vec{F}_j^*) \\ &= (\vec{B}_i \otimes \vec{B}_j^*) (\vec{G}_i \otimes \vec{G}_j^*) (\vec{D}_i \otimes \vec{D}_j^*) (\vec{E}_i \otimes \vec{E}_j^*) (\vec{P}_i \otimes \vec{P}_j^*) (\vec{T}_i \otimes \vec{T}_j^*) (\vec{F}_i \otimes \vec{F}_j^*) \\ &= \vec{B}_{ij} \vec{G}_{ij} \vec{D}_{ij} \vec{E}_{ij} \vec{P}_{ij} \vec{T}_{ij} \vec{F}_{ij} \end{aligned}$$

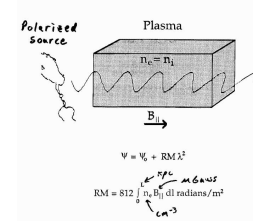


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Ionospheric Faraday Rotation, F

- Birefringency due to magnetic field in ionospheric plasma



– also present in radio sources!



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Ionospheric Faraday Rotation, F

- The ionosphere is *birefringent*; one hand of circular polarization is delayed w.r.t. the other, introducing a phase shift:

$$\Delta\phi = 0.15 \lambda^2 \int B_l n_e ds \text{ deg} \quad (\lambda \text{ in cm, } n_e \text{ in } 10^{14} \text{ cm}^{-2}, B_l \text{ in G})$$

- rotates the linear polarization position angle:

$$\vec{F}^{RL} = \begin{pmatrix} e^{i\Delta\phi} & 0 \\ 0 & e^{-i\Delta\phi} \end{pmatrix}; \quad \vec{F}^{XY} = \begin{pmatrix} \cos \Delta\phi & -\sin \Delta\phi \\ \sin \Delta\phi & \cos \Delta\phi \end{pmatrix}$$

- more important at longer wavelengths:

$$TEC = \int n_e ds \sim 10^{14} \text{ cm}^{-2}; \quad B_l \sim 1\text{G}; \quad \lambda = 20\text{cm} \rightarrow \Delta\phi \sim 60^\circ$$

- ionosphere most active at solar maximum and sunrise/sunset
- watch for direction dependence (in-beam)
- see "Low Frequency Interferometry" (C. Brogan)



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Parallactic Angle, P

- Orientation of sky in telescope's field of view

- Constant for equatorial telescopes

- Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan \left(\frac{\cos(l) \sin(h(t))}{\sin(l) \cos(\delta) - \cos(l) \sin(\delta) \cos(h(t))} \right)$$

l = latitude, $h(t)$ = hour angle, δ = declination

- Rotates the position angle of linearly polarized radiation (c.f. F)

$$\vec{P}^{RL} = \begin{pmatrix} e^{i\chi} & 0 \\ 0 & e^{-i\chi} \end{pmatrix}; \quad \vec{P}^{XY} = \begin{pmatrix} \cos \chi & -\sin \chi \\ \sin \chi & \cos \chi \end{pmatrix}$$

- defined per antenna (often same over array)
- P modulation can be used to aid in calibration



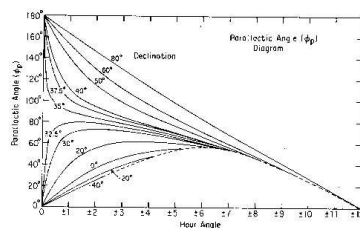
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Parallactic Angle, P

- Parallactic angle versus hour angle at VLA:

- fastest swing for source passing through zenith



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Antenna voltage pattern, E

- Direction-dependent gain and polarization

- includes primary beam

- Fourier transform of cross-correlation of antenna voltage patterns

- includes polarization asymmetry (squint)

$$E^{pq} = \begin{pmatrix} e^p(l, m) & 0 \\ 0 & e^q(l, m) \end{pmatrix}$$

- can include off-axis cross-polarization (leakage)

- convenient to reserve D for on-axis leakage
- will then have off-diagonal terms

- important in wide-field imaging and mosaicing
- when sources fill the beam (e.g. low frequency)



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Polarization Leakage, D

- Polarizer is not ideal, so orthogonal polarizations not perfectly isolated
 - Well-designed systems have $d < 1\text{-}5\%$
 - A geometric property of the antenna, feed & polarizer design
 - frequency dependent (e.g. quarter-wave at center ν)
 - direction dependent (in beam) due to antenna
 - For R,L systems
 - parallel hands affected as $d \cdot Q + d \cdot U$, so only important at high dynamic range (because $Q,U \sim d$, typically)
 - cross-hands affected as $d \cdot I$ so almost always important

$$\tilde{D}^{pq} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix} \quad \text{Leakage of } q \text{ into } p \text{ (e.g. L into R)}$$



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Coherency vector and correlations

- Coherency vector:

$$\mathbf{e} = \langle \tilde{s}_i \otimes \tilde{s}_j^* \rangle = \left\langle \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i \otimes \begin{pmatrix} s^p \\ s^q \end{pmatrix}_j^* \right\rangle = \begin{pmatrix} \langle s_i^p \cdot s_j^{*p} \rangle \\ \langle s_i^p \cdot s_j^{*q} \rangle \\ \langle s_i^q \cdot s_j^{*p} \rangle \\ \langle s_i^q \cdot s_j^{*q} \rangle \end{pmatrix}$$

- e.g. for circularly polarized feeds:

$$\mathbf{e}_{circ} = \langle \tilde{s}_i \otimes \tilde{s}_j^* \rangle = \left\langle \begin{pmatrix} s^R \\ s^L \end{pmatrix}_i \otimes \begin{pmatrix} s^R \\ s^L \end{pmatrix}_j^* \right\rangle = \begin{pmatrix} \langle s_i^R \cdot s_j^{*R} \rangle \\ \langle s_i^R \cdot s_j^{*L} \rangle \\ \langle s_i^L \cdot s_j^{*R} \rangle \\ \langle s_i^L \cdot s_j^{*L} \rangle \end{pmatrix}$$



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Coherency vector and Stokes vector

- Example: circular polarization (e.g. VLA)

$$\mathbf{e}_{circ} = \tilde{S}_{circ} \mathbf{e}^S = \begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix}$$

- Example: linear polarization (e.g. ATCA)

$$\mathbf{e}_{lin} = \tilde{S}_{lin} \mathbf{e}^S = \begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$



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Visibilities and Stokes parameters

- Convolution of sky with measurement effects:

$$\vec{V}_{ij}^{obs} = \int_{sky} (\tilde{J}_i \otimes \tilde{J}_j^*) \tilde{S} \tilde{I}(l, m) e^{-i2\pi(u_j l + v_j m)} dldm$$

Instrumental effects, including "beam" $E(l, m)$ → coordinate transformation to Stokes parameters → (I, Q, U, V)

- e.g. with (polarized) beam E :

$$\vec{V}_{ij}^{obs} = \int_{sky} (\tilde{E}_i \otimes \tilde{E}_j^*) \tilde{S} \tilde{I}(l, m) e^{-i2\pi(u_j l + v_j m)} dldm$$

$$\vec{V}_{ij}^{obs} = \int_{uv} (\tilde{E}_i \otimes \tilde{E}_j^*) \tilde{S} \tilde{I}(u, v) e^{i2\pi(u_j l + v_j m)} dudv$$

- imaging involves inverse transforming these



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Example: RL basis

- Combining E , etc. (no D), expanding P, S :

$$\begin{aligned} V_{ij}^{RR} &= \int_{sky} E_{ij}^{RR}(l, m) [I(l, m) + V(l, m)] e^{i(\chi_i - \chi_j)} e^{-i2\pi(u_j l + v_j m)} dldm \\ V_{ij}^{RL} &= \int_{sky} E_{ij}^{RL}(l, m) [Q(l, m) + iU(l, m)] e^{i(\chi_i - \chi_j)} e^{-i2\pi(u_j l + v_j m)} dldm \\ V_{ij}^{LR} &= \int_{sky} E_{ij}^{LR}(l, m) [Q(l, m) - iU(l, m)] e^{-i(\chi_i - \chi_j)} e^{-i2\pi(u_j l + v_j m)} dldm \\ V_{ij}^{LL} &= \int_{sky} E_{ij}^{LL}(l, m) [I(l, m) - V(l, m)] e^{-i(\chi_i - \chi_j)} e^{-i2\pi(u_j l + v_j m)} dldm \end{aligned}$$

2χ for co-located array 0 for co-located array



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Example: RL basis imaging

- Parentetical Note:

- can make a pseudo- I image by gridding $RR+LL$ on the Fourier half-plane and inverting to a real image
- can make a pseudo- V image by gridding $RR-LL$ on the Fourier half-plane and inverting to real image
- can make a pseudo- $(Q+iU)$ image by gridding RL to the full Fourier plane (with LR as the conjugate) and inverting to a complex image
- does not require having full polarization RR, RL, LR, LL for every visibility
- More on imaging (& deconvolution) tomorrow!



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Leakage revisited...

- Primary on-axis effect is "leakage" of one polarization into the measurement of the other (e.g. $R \leftrightarrow L$)
 - but, direction dependence due to polarization beam!
- Customary to factor out on-axis leakage into D and put direction dependence in "beam"
 - example: expand RL basis with on-axis leakage

$$\hat{V}_{ij}^{RR} = V_{ij}^{RR} + d_i^R V_{ij}^{LR} + d_j^{*R} V_{ij}^{RL} + d_i^R d_j^{*R} V_{ij}^{LL}$$

$$\hat{V}_{ij}^{RL} = V_{ij}^{RL} + d_i^R V_{ij}^{LL} + d_j^{*L} V_{ij}^{RR} + d_i^R d_j^{*L} V_{ij}^{LR}$$

– similarly for XY basis



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Example: RL basis leakage

- In full detail:

$$V_{ij}^{RR} = \int_{\text{sky}} E_{ij}^{RR}(l, m) [(I + V) e^{i(x_i - x_j)} + d_i^R e^{-i(x_i + x_j)} (Q - iU) + d_j^{*R} e^{i(x_i + x_j)} (Q + iU)] (l, m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

2nd order: D²→I into I

$$V_{ij}^{RL} = \int_{\text{sky}} E_{ij}^{RL}(l, m) [(Q + iU) e^{i(x_i + x_j)} + d_i^R (I - V) e^{-i(x_i - x_j)} + d_j^{*L} (I + V) e^{i(x_i - x_j)} + d_i^R d_j^{*L} (Q - iU) e^{-i(x_i + x_j)}] (l, m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

2nd order: D²→I into I

1st order: D→I into P

3rd order: D²→P into P



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Example: Linearized response

- Dropping terms in d^2 , dQ , dU , dV (and expanding G)

- For crossed linearly polarized feeds

$$\begin{aligned} v_{pp} &= \frac{1}{2} g_{ip} g_{kp}^* (I + Q \cos 2\chi + U \sin 2\chi), \\ v_{pq} &= \frac{1}{2} g_{ip} g_{kq}^* ((d_{ip} - d_{kq}) I - Q \sin 2\chi + U \cos 2\chi + jV), \\ v_{qp} &= \frac{1}{2} g_{iq} g_{kp}^* ((d_{kp} - d_{iq}) I - Q \sin 2\chi + U \cos 2\chi - jV), \\ v_{qq} &= \frac{1}{2} g_{iq} g_{kq}^* (I - Q \cos 2\chi - U \sin 2\chi), \end{aligned}$$

- for circularly polarized feeds:

$$\begin{aligned} v_{pp} &= \frac{1}{2} g_{ip} g_{kp}^* (I + V), \\ v_{pq} &= \frac{1}{2} g_{ip} g_{kq}^* ((d_{ip} - d_{kq}) I + e^{-2j\chi} (Q + jU)), \\ v_{qp} &= \frac{1}{2} g_{iq} g_{kp}^* ((d_{kp} - d_{iq}) I + e^{2j\chi} (Q - jU)), \\ v_{qq} &= \frac{1}{2} g_{iq} g_{kq}^* (I - V). \end{aligned}$$

– warning: using linear order can limit dynamic range!



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Summary – polarization interferometry

- Choice of basis: CP or LP feeds
- Follow the Measurement Equation
 - ionospheric Faraday rotation F at low frequency
 - parallactic angle P for coordinate transformation to Stokes
 - "leakage" D varies with v and over beam (mix with E)
- Leakage
 - use full (all orders) D solver when possible
 - linear approximation OK for low dynamic range



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Polarization Calibration & Observation

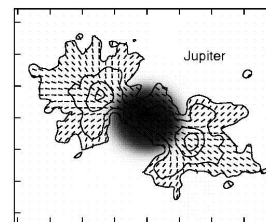


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So you want to make a polarization map...

>SNTHS IMAGN SUMMR SCHUL



June 20-27, 2000
Socorro, NM, USA



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Strategies for polarization observations

- Follow general calibration procedure (last lecture)
 - will need to determine leakage D (if not known)
 - often will determine G and D together (iteratively)
 - procedure depends on basis and available calibrators
- Observations of polarized sources
 - follow usual rules for sensitivity, uv coverage, etc.
 - remember polarization fraction is usually low! (few %)
 - if goal is to map E-vectors, remember to calculate noise in $\Phi = \frac{1}{2} \tan^{-1} U/Q$
 - watch for gain errors in V (for CP) or Q,U (for LP)
 - for wide-field high-dynamic range observations, will need to correct for polarized primary beam (during imaging)



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Strategies for leakage calibration

- Need a bright calibrator! Effects are low level...
 - determine gains G (mostly from parallel hands)
 - use cross-hands (mostly) to determine leakage
 - general ME D solver (e.g. aips++) uses all info
- Calibrator is unpolarized
 - leakage directly determined (ratio to I model), but only to an overall constant
 - need way to fix phase $p-q$ (ie. R-L phase difference), e.g. using another calibrator with known EVPA
- Calibrator of known polarization
 - leakage can be directly determined (for I,Q,U,V model)
 - unknown $p-q$ phase can be determined (from U/Q etc.)



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Other strategies

- Calibrator of unknown polarization
 - solve for model IQUV and D simultaneously or iteratively
 - need good parallactic angle coverage to modulate sky and instrumental signals
 - in instrument basis, sky signal modulated by $e^{i2\chi}$
- With a very bright strongly polarized calibrator
 - can solve for leakages and polarization per baseline
 - can solve for leakages using parallel hands!
- With no calibrator
 - hope it averages down over parallactic angle
 - transfer D from a similar observation
 - usually possible for several days, better than nothing!
 - need observations at same frequency



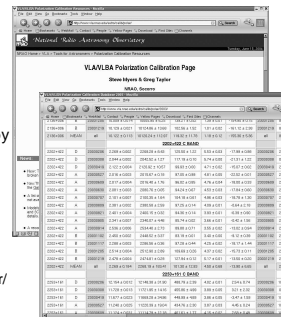
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Finding polarization calibrators

- Standard sources
 - planets (unpolarized if unresolved)
 - 3C286, 3C48, 3C147 (known IQU, stable)
 - sources monitored (e.g. by VLA)
 - other bright sources (bootstrap)

<http://www.vla.nrao.edu/astro/calib/polar/>

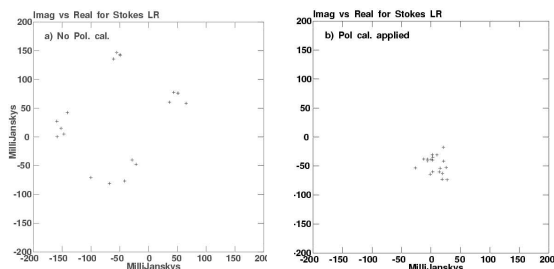


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Example: D-term calibration

- D-term calibration effect on RL visibilities :

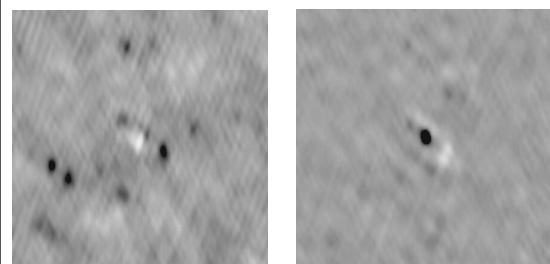


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Example: D-term calibration

- D-term calibration effect in image plane :
Bad D-term solution Good D-term solution



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Example: "standard" procedure for CP feeds

Calibration of Circular Feeds

- Parallel correlations sensitive to Stokes I & V

$$v_{ij} = \frac{1}{2} g_{ij} g_{ij}^* (I + V),$$

$$v_{ji} = \frac{1}{2} g_{ij} g_{ij}^* (I - V).$$

- Assume $V = 0$ for calibrator
- Can separate and solve for gains for p and q
- Instrumental (d) and source polarization (Q, U) sum of two vectors:

$$v_{ij} = \frac{1}{2} g_{ij} g_{ij}^* ((d_{ij} - d_{ij}^*) I + e^{2i\chi} (Q + jU)),$$

$$v_{ji} = \frac{1}{2} g_{ij} g_{ij}^* ((d_{ij} - d_{ij}^*) I + e^{2i\chi} (Q - jU))$$

- Calibrator observations of a range of PA gives clean separation
- Independent gain calibration for p and q allows arbitrary phase offset – refer all phases to same "reference" antenna
- $p - q$ phase difference is that of the reference antenna
- Need observations of calibrator of known polarization angle aka Electric Vector Position Angle (EVPA)

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Example: "standard" procedure for LP feeds

Calibration of Linear Feeds

- Parallel correlations sensitive to I, Q , & U

$$v_{ij} = \frac{1}{2} g_{ij} g_{ij}^* (I + Q \cos 2\chi + U \sin 2\chi),$$

$$v_{ji} = \frac{1}{2} g_{ij} g_{ij}^* (I - Q \cos 2\chi - U \sin 2\chi),$$

- Calibrator Q and U usually cannot be ignored (few %)
- Phase unaffected by polarization of a point source at the phase center
- Cannot separate p, q gains and calibrator polarization
- $p - q$ phase offset not known
- May be unknown orientation error of p and q
- Need obs of source with known polarization

$$v_{ij} = \frac{1}{2} g_{ij} g_{ij}^* ((d_{ij} - d_{ij}^*) I - Q \sin 2\chi + U \cos 2\chi + jV),$$

$$v_{ji} = \frac{1}{2} g_{ij} g_{ij}^* ((d_{ij} - d_{ij}^*) I - Q \sin 2\chi - U \cos 2\chi - jV),$$

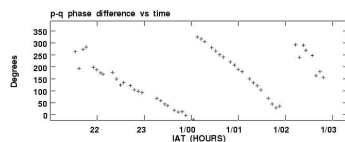
- Calibrator Q and U affect real part of cross pol. correlations
- Calibrator V affects imaginary part of cross pol. correlations but unaffected by PA

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Special Issues

- Low frequency – ionospheric Faraday rotation
 - important for 2 GHz and below (sometimes higher too)
 - λ^2 dependence (separate out using multi-frequency obs.)
 - depends on time of day and solar activity (& observatory location)
 - external calibration using zenith TEC (plus gradient?)
 - self-calibration possible (e.g. with snapshots)



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Special issues – continued...

- VLBI polarimetry
 - follows same principles
 - will have different parallactic angle at each station!
 - can have heterogeneous feed geometry (e.g. CP & LP)
 - harder to find sources with known polarization
 - calibrators resolved!
 - transfer EVPA from monitoring program



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Subtleties ...

- Antenna-based D solutions
 - closure quantities → undetermined parameters
 - different for parallel and cross-hands
 - e.g. can add d to R and d^* to L
 - need for reference antenna to align and transfer D solutions
- Parallel hands
 - are D solutions from cross-hands appropriate here?
 - what happens in full D solution (weighting issues?)

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Special Issues – observing circular polarization

- Observing circular polarization V is straightforward with LP feeds (from Re and Im of cross-hands)
- With CP feeds:
 - gain variations can masquerade as (time-variable) V signal
 - helps to switch signal paths through back-end electronics
 - R vs. L beam squint introduces spurious V signal
 - limited by pointing accuracy
 - requires careful calibration
 - relative R and L gains critical
 - average over calibrators (be careful of intrinsic V)
 - VLBI somewhat easier
 - different systematics at stations help to average out

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Special Issues – wide field polarimetry

- Actually an imaging & deconvolution issue
 - assume polarized beam $D \cdot E$ is known
- Deal with direction-dependent effects
 - beam squint (R,L) or beam ellipticity (X,Y)
 - primary beam
- Iterative scheme (e.g. CLEAN)
 - implemented in aips++
 - see lectures by Bhatnagar & Cornwell
- Described in EVLA Memo 62 “Full Primary Beam Stokes IQUV Imaging” T. Cornwell (2003) :

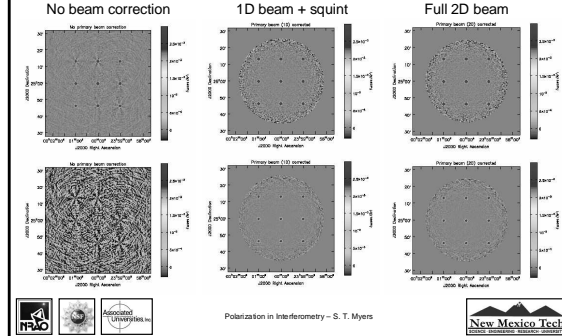


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Example: wide field polarimetry (Cornwell 2003)

- Simulated array of point sources

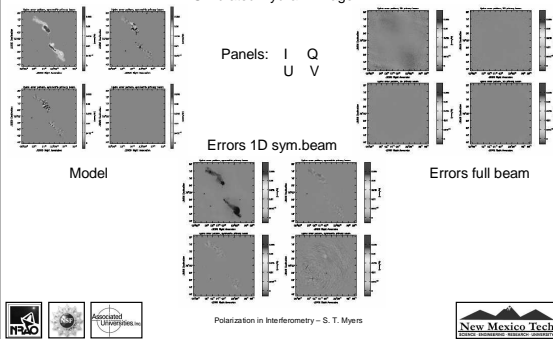


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Example: wide field polarimetry continued...

- Simulated Hydra A image



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Summary – Observing & Calibration

- Follow normal calibration procedure (previous lecture)
- Need bright calibrator for leakage D calibration
 - best calibrator has strong known polarization
 - unpolarized sources also useful
- Parallax angle coverage useful
 - necessary for unknown calibrator polarization
- Need to determine unknown $p-q$ phase
 - CP feeds need EVPA calibrator for R-L phase
 - if system stable, can transfer from other observations
- Special Issues
 - observing CP difficult with CP feeds
 - wide-field polarization imaging (needed for EVLA & ALMA)



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Polarization data analysis

- Making polarization images
 - follow general rules for imaging & deconvolution
 - image & deconvolve in I, Q, U, V (e.g. CLEAN, MEM)
 - note: Q, U, V will be positive and negative
 - in absence of CP, V image can be used as check
 - joint deconvolution (e.g. aips++, wide-field)
- Polarization vector plots
 - use “electric vector position angle” (EVPA) calibrator to set angle (e.g. R-L phase difference)
 - $\Phi = \frac{1}{2} \tan^{-1} U/Q$ for E vectors (B vectors \perp E)
 - plot E vectors with length given by p
- Faraday rotation: determine $\Delta\Phi$ vs. λ^2



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Polarization Astrophysics

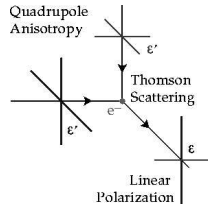


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Astrophysical mechanisms for polarization

- Magnetic fields
 - synchrotron radiation → LP (small amount of CP)
 - Zeeman effect → CP
 - Faraday rotation (of background polarization)
 - dust grains in magnetic field
 - maser emission
- Electron scattering
 - incident radiation with quadrupole
 - e.g. Cosmic Microwave Background
- and more...



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Astrophysical sources with polarization

- Magnetic objects
 - active galactic nuclei (AGN) (accretion disks, MHD jets, lobes)
 - protostars (disks, jets, masers)
 - clusters of galaxies IGM
 - galaxy ISM
 - compact objects (pulsars, magnetars)
 - planetary magnetospheres
 - the Sun and other (active) stars
 - the early Universe (primordial magnetic fields???)
- Other objects
 - Cosmic Microwave Background (thermal)
- Polarization levels
 - usually low (<1% to 5-10% typically)

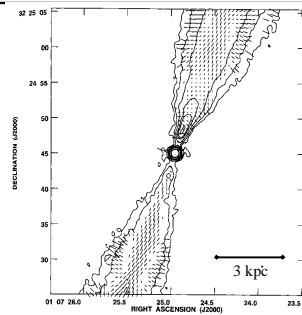


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Example: 3C31

- VLA @ 8.4 GHz
- E-vectors
- Laing (1996)

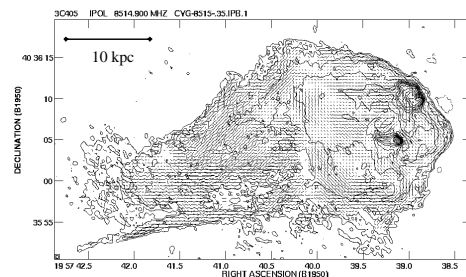


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Example: Cygnus A

- VLA @ 8.5 GHz B-vectors Perley & Carilli (1996)

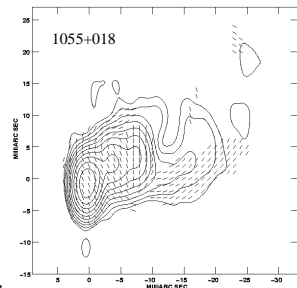


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Example: Blazar Jets

- VLBA @ 5 GHz Attridge et al. (1999)

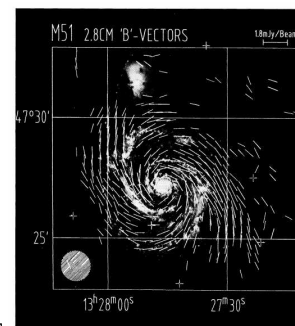


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Example: the ISM of M51

Neininger (1992)



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