Calibration & Editing

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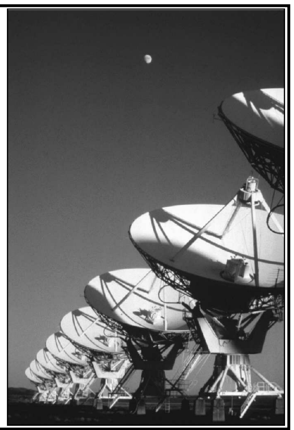


Synopsis

- Why calibration and editing?
- Formalism: Visibilities, signals, matrices
- Solving the Measurement Equation
- Practical Calibration Planning
- Spectral Line Example / Calibration Evaluation
- A Dictionary of Calibration Components
- Editing and RFI
- Summary

Why Calibration and Editing?

- Synthesis radio telescopes, though well-designed, are not perfect (e.g., surface accuracy, receiver noise, polarization purity, stability, etc.)
- Need to accommodate engineering (e.g., frequency conversion, digital electronics, etc.)
- Hardware or control software occasionally fails or behaves unpredictably
- Scheduling/observation errors sometimes occur (e.g., wrong source positions)
- Atmospheric conditions not ideal (not limited to "bad" weather)
- RFI

Determining *instrumental properties* (calibration)
is as important as
determining *radio source properties*

From Idealistic to Realistic

- Formally, we wish to obtain the visibility function, which we intend to invert to obtain an image of the sky:

$$V(u,v) = \int_{sky} I(l,m) e^{-i2\pi(ul+vm)} dl\, dm$$

- In practice, we correlate (multiply & average) the electric field (voltage) samples, x_i & x_j, received at pairs of telescopes (i,j)

$$V_{ij} = \left\langle K_i(t) x_i(t) \cdot K_j^*(t) x_j^*(t) \right\rangle_{\Delta t} = V(u_{ij}, v_{ij})$$

  - K_i is geometric compensation (delays, fringe rotation) which sets the position on the sky of the phase center
  - Averaging duration is set by the expected timescales for variation of the correlation result (typically 10s or less for the VLA)
- Single radio telescopes are devices for collecting the signal $x_i(t)$ and providing it to the correlator.

What signal is really collected?

- The net signal delivered by antenna i, $x_i(t)$, is a combination of the desired signal, $s_i(t,l,m)$, corrupted by a factor $J_i(t,l,m)$ and integrated over the sky, and noise, $n_i(t)$:

$$x_i(t) = \int_{sky} J_i(t,l,m) s_i(t,l,m)\, dldm + n_i(t)$$

$$= s_i'(t) + n_i(t)$$

- $J_i(t,l,m)$ is the product of a host of effects which we must *calibrate*
- In some cases, effects implicit in the $J_i(t,l,m)$ term corrupt the signal irreversibly and the resulting data must be *edited*
- $J_i(t,l,m)$ is a complex number
- $J_i(t,l,m)$ is *antenna-based*
- Usually, $|n_i| >> |s_i|$

Correlation of realistic signals

- The correlation of two realistic signals from different antennas:

$$\left\langle K_i x_i \cdot K_j^* x_j^* \right\rangle_{\Delta t} = \left\langle K_i (s_i' + n_i) \cdot K_j^* (s_j' + n_j)^* \right\rangle_{\Delta t}$$

$$= \left\langle K_i s_i' \cdot K_j^* s_j'^* \right\rangle + \left\langle K_i s_i' \cdot K_j^* n_j^* \right\rangle + \left\langle K_i n_i \cdot K_j^* s_j'^* \right\rangle + \left\langle K_i n_i \cdot K_j^* n_j^* \right\rangle$$

  - Noise doesn't correlate—even if $|n_i| >> |s_i|$, the correlation process isolates desired signals:

$$= \left\langle K_i s_i' \cdot K_j^* s_j'^* \right\rangle_{\Delta t}$$

$$= \left\langle \int_{sky} K_i J_i s_i\, dl'dm' \cdot \int_{sky} K_j^* J_j^* s_j^*\, dldm \right\rangle_{\Delta t}$$

  - In integral, only $s_i(t,l,m)$, from the same directions correlate (i.e., when $l=l'$, $m=m'$), so order of integration and signal product can be reversed:

$$= \left\langle \int_{sky} J_i J_j^* K_i s_i K_j^* s_j^*\, dldm \right\rangle_{\Delta t}$$

Correlation of realistic signals (cont)

- Using the geometry of the situation, we can recast s_i & s_j in terms of the single signal, *s*, which arrived at each of the telescopes from the distant sky:

$$V_{ij} = \left\langle \int_{sky} J_i J_j^* s^2(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm \right\rangle_{\Delta t}$$

- On the timescale of the averaging, the only meaningful average is of the *squared* signal itself (direction-dependent), which is just the image of the source:

$$= \int_{sky} J_i J_j^* \left\langle s^2(l,m) \right\rangle_{\Delta t} e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

$$= \int_{sky} J_i J_j^* I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

- If all *J=1,* we of course recover the Fourier transform expression:

$$= \int_{sky} I(l,m) e^{-i2\pi(u_{ij}l + v_{ij}m)} dldm$$

Correlation of realistic signals (cont)

- The *auto*-correlation of a signal from a *single* antenna:

$$\left\langle K_i x_i \cdot K_i^* x_i^* \right\rangle = \left\langle (s_i' + n_i) \cdot (s_i' + n_i)^* \right\rangle \qquad \left(|K|^2 = 1 \right)$$

$$= \left\langle s_i' \cdot s_i'^* \right\rangle + \left\langle n_i \cdot n_i^* \right\rangle$$

$$= \left\langle \int_{sky} |J_i|^2 |s_i|^2 dldm \right\rangle + \left\langle |n_i|^2 \right\rangle$$

$$= \left\langle \int_{sky} |J_i|^2 I(l,m) dldm \right\rangle + \left\langle |n_i|^2 \right\rangle$$

  - This is an integrated power measurement plus noise
  - Desired signal *not* isolated from noise
  - Noise usually dominates

  – Single dish radio astronomy calibration strategies dominated by switching schemes to isolate desired signal

Full-Polarization Formalism (matrices!)

- Need dual-polarization basis (*p,q*) to fully sample the incoming EM wave front, where *p,q = R,L* (circular basis) or *p,q = X,Y* (linear basis):

$$\vec{I}_{circ} = \vec{S}_{circ} \vec{I}_{Stokes} \qquad \vec{I}_{lin} = \vec{S}_{lin} \vec{I}_{Stokes}$$

$$\begin{pmatrix} RR \\ RL \\ LR \\ LL \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & i & 0 \\ 0 & 1 & -i & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+V \\ Q+iU \\ Q-iU \\ I-V \end{pmatrix} \qquad \begin{pmatrix} XX \\ XY \\ YX \\ YY \end{pmatrix} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & i \\ 0 & 0 & 1 & -i \\ 1 & -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} I+Q \\ U+iV \\ U-iV \\ I-Q \end{pmatrix}$$

- Devices can be built to sample these basis states in the signal domain (Stokes Vector is defined in "power" domain)
- Some components of J_i involve mixing of basis states, so dual-polarization matrix description desirable or even required for proper calibration

Full-Polarization Formalism: Signal Domain

- Substitute:

$$s_i \rightarrow \vec{s}_i = \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i, \qquad J_i \rightarrow \overleftrightarrow{J}_i = \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}$$

- The *Jones matrix* thus corrupts a signal as follows:

$$\vec{s}_i' = \overleftrightarrow{J}_i \vec{s}_i \quad \text{(sky integral omitted)}$$

$$\begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i = \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}_i \begin{pmatrix} s^p \\ s^q \end{pmatrix}_i$$

$$= \begin{pmatrix} J^{p\rightarrow p} s^p + J^{q\rightarrow p} s^q \\ J^{p\rightarrow q} s^p + J^{q\rightarrow q} s^q \end{pmatrix}_i$$

Full-Polarization Formalism: Correlation

- Four correlations are possible from two polarizations. The *outer product* (a 'bookkeeping' product) represents correlation in the matrix formalism:

$$\left\langle \vec{s}_i' \otimes \vec{s}_j'^* \right\rangle = \left\langle \begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_i \otimes \begin{pmatrix} s'^p \\ s'^q \end{pmatrix}_j^* \right\rangle = \begin{pmatrix} \left\langle s_i'^p \cdot s_j'^{*p} \right\rangle \\ \left\langle s_i'^p \cdot s_j'^{*q} \right\rangle \\ \left\langle s_i'^q \cdot s_j'^{*p} \right\rangle \\ \left\langle s_i'^q \cdot s_j'^{*q} \right\rangle \end{pmatrix}$$

- A very useful property of outer products:

$$\left(\vec{s}_i' \otimes \vec{s}_j'^* \right) = \left(\overleftrightarrow{J}_i \vec{s}_i \right) \otimes \left(\overleftrightarrow{J}_j^* \vec{s}_j^* \right) = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^* \right) \left(\vec{s}_i \otimes \vec{s}_j^* \right)$$

Full-Polarization Formalism: Correlation (cont)

- The outer product for the Jones matrix:

$$\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^* = \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}_i \otimes \begin{pmatrix} J^{p\rightarrow p} & J^{q\rightarrow p} \\ J^{p\rightarrow q} & J^{q\rightarrow q} \end{pmatrix}_j^*$$

$$= \begin{pmatrix} J_i^{p\rightarrow p} J_j^{*p\rightarrow p} & J_i^{p\rightarrow p} J_j^{*q\rightarrow p} & J_i^{q\rightarrow p} J_j^{*p\rightarrow p} & J_i^{q\rightarrow p} J_j^{*q\rightarrow p} \\ J_i^{p\rightarrow p} J_j^{*p\rightarrow q} & J_i^{p\rightarrow p} J_j^{*q\rightarrow q} & J_i^{q\rightarrow p} J_j^{*p\rightarrow q} & J_i^{q\rightarrow p} J_j^{*q\rightarrow q} \\ J_i^{p\rightarrow q} J_j^{*p\rightarrow p} & J_i^{p\rightarrow q} J_j^{*q\rightarrow p} & J_i^{q\rightarrow q} J_j^{*p\rightarrow p} & J_i^{q\rightarrow q} J_j^{*q\rightarrow p} \\ J_i^{p\rightarrow q} J_j^{*p\rightarrow q} & J_i^{p\rightarrow q} J_j^{*q\rightarrow q} & J_i^{q\rightarrow q} J_j^{*p\rightarrow q} & J_i^{q\rightarrow q} J_j^{*q\rightarrow q} \end{pmatrix} = \overleftrightarrow{J}_{ij}$$

  – J_{ij} is a 4x4 *Mueller matrix*
  – Antenna and array design driven by minimizing off-diagonal terms!

Signal Correlation and Matrices (cont)

- And finally, for fun, the correlation of corrupted signals:

$$\overleftrightarrow{J}_i\vec{s}_i \otimes \overleftrightarrow{J}_j^*\vec{s}_j^* = \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^*\right)\left(\vec{s}_i \otimes \vec{s}_j^*\right)$$

$$= \begin{pmatrix} J_i^{p\to p}J_j^{*p\to p} & J_i^{p\to p}J_j^{*q\to p} & J_i^{q\to p}J_j^{*p\to p} & J_i^{q\to p}J_j^{*q\to p} \\ J_i^{p\to p}J_j^{*p\to q} & J_i^{p\to p}J_j^{*q\to q} & J_i^{q\to p}J_j^{*p\to q} & J_i^{q\to p}J_j^{*q\to q} \\ J_i^{p\to q}J_j^{*p\to p} & J_i^{p\to q}J_j^{*q\to p} & J_i^{q\to q}J_j^{*p\to p} & J_i^{q\to q}J_j^{*q\to p} \\ J_i^{p\to q}J_j^{*p\to q} & J_i^{p\to q}J_j^{*q\to q} & J_i^{q\to q}J_j^{*p\to q} & J_i^{q\to q}J_j^{*q\to q} \end{pmatrix} \begin{pmatrix} s_i^p \cdot s_j^{*p} \\ s_i^p \cdot s_j^{*q} \\ s_i^q \cdot s_j^{*p} \\ s_i^q \cdot s_j^{*q} \end{pmatrix}$$

$$= \begin{pmatrix} J_i^{p\to p}J_j^{*p\to p}s_i^p \cdot s_j^{*p} + J_i^{p\to p}J_j^{*q\to p}s_i^p \cdot s_j^{*q} + J_i^{q\to p}J_j^{*p\to p}s_i^q \cdot s_j^{*p} + J_i^{q\to p}J_j^{*q\to p}s_i^q \cdot s_j^{*q} \\ J_i^{p\to p}J_j^{*p\to q}s_i^p \cdot s_j^{*p} + J_i^{p\to p}J_j^{*q\to q}s_i^p \cdot s_j^{*q} + J_i^{q\to p}J_j^{*p\to q}s_i^q \cdot s_j^{*p} + J_i^{q\to p}J_j^{*q\to q}s_i^q \cdot s_j^{*q} \\ J_i^{p\to q}J_j^{*p\to p}s_i^p \cdot s_j^{*p} + J_i^{p\to q}J_j^{*q\to p}s_i^p \cdot s_j^{*q} + J_i^{q\to q}J_j^{*p\to p}s_i^q \cdot s_j^{*p} + J_i^{q\to q}J_j^{*q\to p}s_i^q \cdot s_j^{*q} \\ J_i^{p\to q}J_j^{*p\to q}s_i^p \cdot s_j^{*p} + J_i^{p\to q}J_j^{*q\to q}s_i^p \cdot s_j^{*q} + J_i^{q\to q}J_j^{*p\to q}s_i^q \cdot s_j^{*p} + J_i^{q\to q}J_j^{*q\to q}s_i^q \cdot s_j^{*q} \end{pmatrix}$$

- UGLY, but we rarely need to worry about detail at this level---just let this occur "inside" the matrix formalism, and work with the notation

The Measurement Equation

- We can now write down the calibration situation in a general way---the Measurement Equation:

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^*\right)\overleftrightarrow{S}\vec{I}(l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

- ...and consider how to solve it!

The Measurement Equation - Simplified

$$\vec{V}_{ij}^{obs} = \int_{sky} \left(\overleftrightarrow{J}_i \otimes \overleftrightarrow{J}_j^*\right)\overleftrightarrow{S}\vec{I}(l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

- First, isolate non-direction-dependent effects, and factor them from the integral:

$$= \left(\overleftrightarrow{J}_i^{vis} \otimes \overleftrightarrow{J}_j^{vis*}\right)\int_{sky} \left(\overleftrightarrow{J}_i^{sky} \otimes \overleftrightarrow{J}_j^{sky*}\right)\overleftrightarrow{S}\vec{I}(l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

- Next, we recognize that it is often possible to assume $J^{sky}=1$, and we have a relationship between ideal and observed Visibilities:

$$= \left(\overleftrightarrow{J}_i^{vis} \otimes \overleftrightarrow{J}_j^{vis*}\right)\int_{sky} \overleftrightarrow{S}\vec{I}(l,m)e^{-i2\pi(u_{ij}l+v_{ij}m)}dldm$$

$$\vec{V}_{ij}^{obs} = \left(\overleftrightarrow{J}_i^{vis} \otimes \overleftrightarrow{J}_j^{vis*}\right)\vec{V}_{ij}^{ideal}$$

Solving the Measurement Equation

- The J terms can be factored into a series of components representing physical elements along the signal path:

$$\vec{V}_{ij}^{obs} = \left(\overleftrightarrow{J}_i^1 \otimes \overleftrightarrow{J}_j^{1*}\right)\left(\overleftrightarrow{J}_i^2 \otimes \overleftrightarrow{J}_j^{2*}\right)\left(\overleftrightarrow{J}_i^3 \otimes \overleftrightarrow{J}_j^{3*}\right)\left(\overleftrightarrow{J}_i^{\cdots} \otimes \overleftrightarrow{J}_j^{\cdots *}\right)\vec{V}_{ij}^{ideal}$$

- Depending upon availability of estimates for various J terms, we can re-arrange the equation and solve for any single term, if we know V^{ideal}:

$$\left[\left(\overleftrightarrow{J}_i^2 \otimes \overleftrightarrow{J}_j^{2*}\right)^{-1}\left(\overleftrightarrow{J}_i^1 \otimes \overleftrightarrow{J}_j^{1*}\right)^{-1}\vec{V}_{ij}^{obs}\right] = \left(\overleftrightarrow{J}_i^{solve} \otimes \overleftrightarrow{J}_j^{solve*}\right)\left[\left(\overleftrightarrow{J}_i^4 \otimes \overleftrightarrow{J}_j^{4*}\right)\left(\overleftrightarrow{J}_i^{\cdots} \otimes \overleftrightarrow{J}_j^{\cdots *}\right)\vec{V}_{ij}^{ideal}\right]$$

- After obtaining estimates for all relevant J, data can be corrected:

$$\vec{V}_{ij}^{corrected} = \left(\overleftrightarrow{J}_i^{\cdots} \otimes \overleftrightarrow{J}_j^{\cdots *}\right)^{-1}\left(\overleftrightarrow{J}_i^3 \otimes \overleftrightarrow{J}_j^{3*}\right)^{-1}\left(\overleftrightarrow{J}_i^2 \otimes \overleftrightarrow{J}_j^{2*}\right)^{-1}\left(\overleftrightarrow{J}_i^1 \otimes \overleftrightarrow{J}_j^{1*}\right)^{-1}\vec{V}_{ij}^{obs}$$

Solving the Measurement Equation

- Formally, solving for any calibration component is always the same non-linear fitting problem:

$$\vec{V}_{ij}^{corrected \cdot obs} = \left(\overleftrightarrow{J}_i^{solve} \otimes \overleftrightarrow{J}_j^{solve*}\right)\vec{V}_{ij}^{corrupted \cdot ideal}$$

  - Algebraic particulars are stored safely and conveniently inside the matrix formalism (out of sight, out of mind!)
  - Viability of the solution depends on the underlying algebra (hardwired in calibration applications) and relies on *proper calibration observations*
  - The relative importance of the different components enables deferring or even ignoring the more subtle effects

Antenna-based Calibration

- Success of synthesis telescopes relies on antenna-based calibration
  - N antenna-based factors, N(N-1) visibility measurements
  - Fundamentally, only information that cannot be factored into antenna-based terms is believable as being of astronomical origin
- Closure: calibration-independent observables (diagonal components):
  - Closure phase (3 baselines):

$$\phi_{ij}^{obs} + \phi_{jk}^{obs} + \phi_{ki}^{obs} = \phi_{ij}^{real} + (\theta_i - \theta_j) + \phi_{jk}^{real} + (\theta_j - \theta_k) + \phi_{ki}^{real} + (\theta_k - \theta_i)$$

$$= \phi_{ij}^{real} + \phi_{jk}^{real} + \phi_{ki}^{real}$$

  - Closure amplitude (4 baselines):

$$\left|\frac{V_{ij}^{obs}V_{kl}^{obs}}{V_{ik}^{obs}V_{jl}^{obs}}\right| = \left|\frac{J_iJ_jV_{ij}^{real}J_kJ_lV_{kl}^{real}}{J_iJ_kV_{ik}^{real}J_jJ_lV_{jl}^{real}}\right|$$

$$= \left|\frac{V_{ij}^{real}V_{kl}^{real}}{V_{ik}^{real}V_{jl}^{real}}\right|$$

  – Tim Cornwell's lecture "Self-calibration" (Wednesday)
- Beware of non-closing errors!

Planning for Good Calibration

- A priori calibrations (provided by the observatory)
  - Antenna positions, earth orientation and rate
  - Clocks
  - Antenna pointing, gain, voltage pattern
  - Calibrator coordinates, flux densities, polarization properties
- Absolute calibration?
  - Very difficult, requires heroic efforts by visiting observers and observatory scientific and engineering staff
- Cross-calibration a better choice
  - Observe nearby point sources against which calibration components can be solved, and transfer solutions to target observations
  - Choose appropriate calibrators for different components; usually strong point sources because we can predict their visibilities
  - Choose appropriate timescales for each component
- Simple (common) example, Gain and Bandpass:

$$\vec{V}_{ij}^{obs} = \left(\vec{B}_i \otimes \vec{B}_j^*\right)\left(\vec{G}_i \otimes \vec{G}_j^*\right)\vec{V}_{ij}^{ideal}$$
$$= \vec{B}_{ij}\vec{G}_{ij}\vec{V}_{ij}^{ideal}$$

"Electronic" Gain, *G*

- Catch-all for most amplitude and phase effects introduced by antenna electronics (amplifiers, mixers, quantizers, digitizers)
  - Most commonly treated calibration component
  - Dominates other effects for standard VLA observations
  - Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
  - Often also includes ionospheric and tropospheric effects which are typically difficult to separate unto themselves
  - Excludes frequency dependent effects (see *B*)

$$G^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

Bandpass Response, *B*

- *G*-like component describing frequency-dependence of antenna electronics, etc.
  - Filters used to select frequency passband not square
  - Optical and electronic reflections introduce ripples across band
  - Often assumed time-independent, but not necessarily so
  - Typically (but not necessarily) normalized

$$B^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

Spectral-Line Calibration Example

- Observation: total intensity spectral line imaging of weak target
  - A weak target source (**2**)
  - A nice near-by point-like *G, T* calibrator (**3**), observed alternately, but too weak for good *B* calibration (flux density unknown)
  - One observation of strong flux density calibrator (**5**)
  - One observation of a strong source for B calibration **(4)**
- Schedule (each digit is a fixed duration):

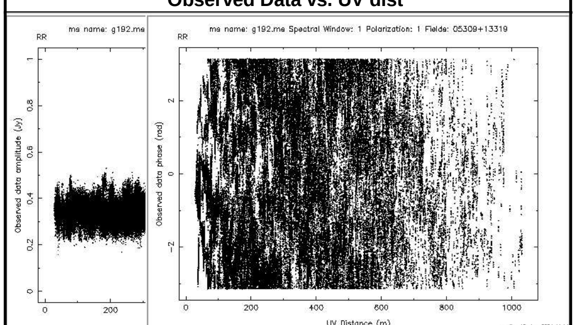
3-222-3-222-3-222-3-222-3-222-3-4444-3-222-3-222-3-222-555

Spectral-Line Calibration Example (cont)

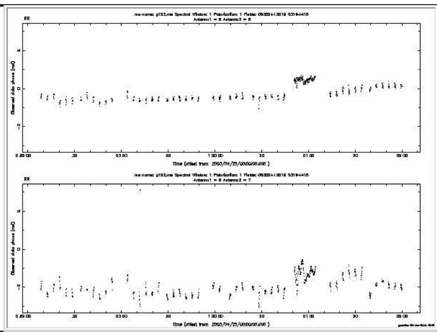
target source = **2**
G calibrator = **3**
B calibrator = **4**
Flux calibrator = **5**

- Calibration sequence:
  - On **4**, solve for *G*: $\left(\vec{V}_{ij(4)}^{obs}\right) = \vec{G}_{ij(4)}\left(\vec{V}_{ij(4)}^{ideal}\right)$
  - On **4**, solve for *B*, using *G*: $\left(\vec{V}_{ij(4)}^{obs}\right) = \vec{B}_{ij(4)}\left(\vec{G}_{ij(4)}\vec{V}_{ij(4)}^{ideal}\right)$
  - On **3,5**, solve for *G*, using *B*: $\left(\vec{B}_{ij(4)}^{-1}\vec{V}_{ij(3,5)}^{obs}\right) = \vec{G}_{ij(3,5)}\left(\vec{V}_{ij(3,5)}^{ideal}\right)$
  - Scale **3**'s *G*s according to **5**'s *G*s: $\left|\vec{G}'_{i(3)}\right| = \left|\vec{G}_{i(3)}\right|\left(\left\langle\left|\vec{G}_{i(5)}\right|\right\rangle \Big/ \left\langle\left|\vec{G}_{i(3)}\right|\right\rangle\right)$
  - Transfer *B, G* to **2**: $\vec{V}_{ij(2)}^{corrected} = \left(\vec{G}'^{-1}_{ij(3)}\vec{B}_{ij(4)}^{-1}\vec{V}_{ij(2)}^{obs}\right)$

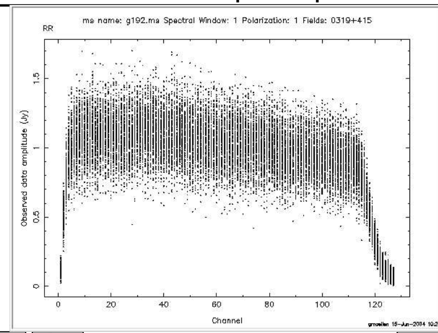
Observed Data vs. UV dist



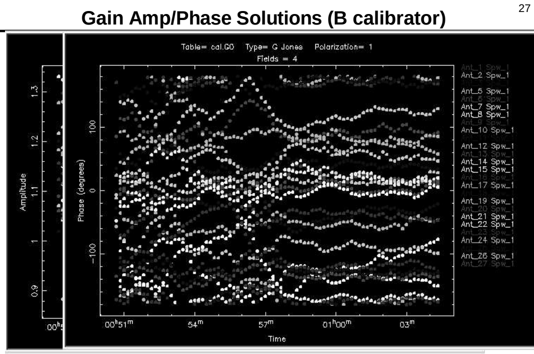
Observed Data – Phase vs. Time



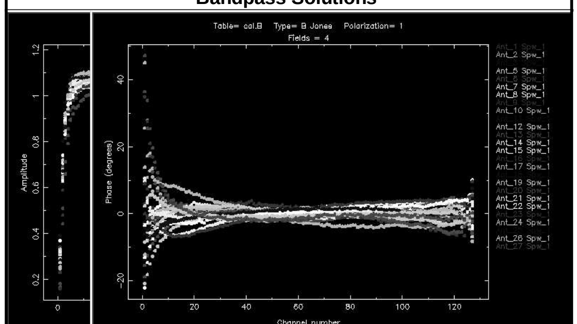
Observed Data – Amplitude Spectrum



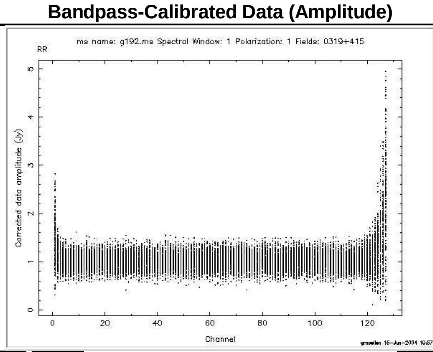
Gain Amp/Phase Solutions (B calibrator)



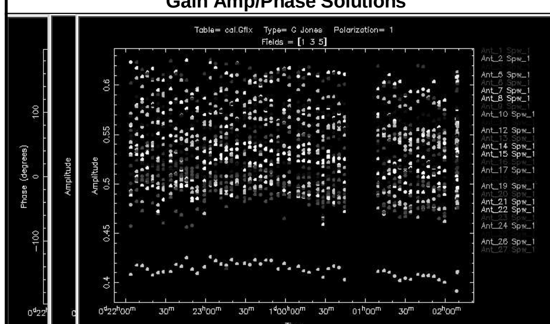
Bandpass Solutions



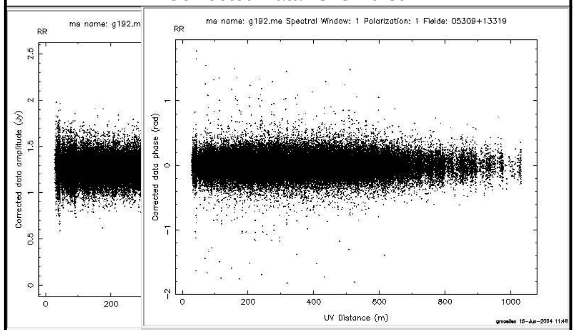
Bandpass-Calibrated Data (Amplitude)



Gain Amp/Phase Solutions

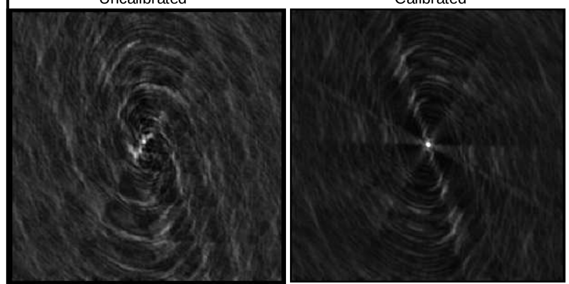


Corrected Data vs. UV dist



Effect of Calibration in the Image Plane

Uncalibrated | Calibrated



Evaluating Calibration Performance

- Are solutions continuous?
  - Noise-like solutions are just that—noise
  - Discontinuities indicate instrumental glitches
  - Any additional editing required?
- Are calibrator data fully described by antenna-based effects?
  - Phase and amplitude *closure errors* are the baseline-based residuals
  - Are calibrators sufficiently point-like? If not, self-calibrate: model calibrator visibilities (by imaging, deconvolving and transforming) and re-solve for calibration; iterate to isolate source structure from calibration components
    - – Tim Cornwell's lecture: "Self-Calibration" (Wednesday)
- Any evidence of unsampled variation? Is interpolation of solutions appropriate?
  - Self-calibration may be required, if possible

A Dictionary of Calibration Components

- J_i contains many components:
  - F = ionospheric Faraday rotation
  - T = tropospheric effects
  - P = parallactic angle
  - E = antenna voltage pattern
  - D = polarization leakage
  - G = electronic gain
  - B = bandpass response
  - K = geometric compensation

$$\vec{\vec{J}}_i = \vec{\vec{K}}_i \vec{\vec{B}}_i \vec{\vec{G}}_i \vec{\vec{D}}_i \vec{\vec{E}}_i \vec{\vec{P}}_i \vec{\vec{T}}_i \vec{\vec{F}}_i$$

- Order of terms follows signal path (right to left)
- Each term has matrix form of J_i with terms embodying its particular algebra (on- vs. off-diagonal terms, etc.)
- Direction-dependent terms involve FT in solution
- The full matrix equation (especially after correlation!) is daunting, but usually only need to consider the terms individually or in pairs, and rarely in open form (matrix formulation = shorthand)

Ionospheric Faraday Rotation, *F*

- The ionosphere is *birefringent*; one hand of circular polarization is delayed w.r.t. the other, introducing a phase shift:

$$\Delta\phi \approx 0.15\ \lambda^2 \int B_{\parallel} n_e ds \ \deg$$

$$\lambda \text{ in cm}, \quad n_e ds \text{ in } 10^{14}\text{cm}^{-2}, \quad B_{\parallel} \text{ in G}$$

  - Rotates the linear polarization position angle
  - More important at longer wavelengths (λ^2)

$$TEC = \int n_e ds \sim 10^{14}\text{cm}^{-2}; \quad B_{\parallel} \sim 1\text{G}; \quad \lambda = 20\text{cm} \rightarrow \Delta\phi \sim 60^{\circ}$$

  - More important at solar maximum and at sunrise/sunset, when ionosphere is most active and variable
  - Beware of direction-dependence within field-of-view
- – Crystal Brogan's lecture: "Low Frequency Interferometry" (Friday)

$$\vec{\vec{F}}^{RL} = \begin{pmatrix} e^{i\Delta\phi} & 0 \\ 0 & e^{-i\Delta\phi} \end{pmatrix}; \quad \vec{\vec{F}}^{XY} = \begin{pmatrix} \cos\Delta\phi & -\sin\Delta\phi \\ \sin\Delta\phi & \cos\Delta\phi \end{pmatrix}$$

Tropospheric Effects, *T*

- The troposphere causes polarization-independent amplitude and phase effects due to emission/opacity and refraction, respectively
  - Typically 2-3m excess path length at zenith compared to vacuum
  - Higher noise contribution, less signal transmission: Lower SNR
  - Most important at ν > 15 GHz where water vapor absorbs/emits
  - More important nearer horizon where tropospheric path length greater
  - Clouds, weather = variability in phase and opacity; may vary across array
  - Water vapor radiometry? Phase transfer from low to high frequencies?

– Debra Shepherd's lecture: "Millimeter Interferometry" (Friday)

$$\vec{\vec{T}}^{pq} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Parallactic Angle, *P*

- Orientation of sky in telescope's field of view
  - Constant for equatorial telescopes
  - Varies for alt-az-mounted telescopes:

$$\chi(t) = \arctan\left(\frac{\cos(l)\sin(h(t))}{\sin(l)\cos(\delta) - \cos(l)\sin(\delta)\cos(h(t))}\right)$$

$$l = \text{latitude},\ h(t) = \text{hour angle},\ \delta = \text{declination}$$

  - Rotates the position angle of linearly polarized radiation (c.f. *F*)
  - Analytically known, and its variation provides leverage for determining polarization-dependent effects

$$\vec{P}^{RL} = \begin{pmatrix} e^{i\chi} & 0 \\ 0 & e^{-i\chi} \end{pmatrix};\ \vec{P}^{XY} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}$$

Antenna Voltage Pattern, *E*

- Antennas of all designs have direction-dependent gain
  - Important when region of interest on sky comparable to or larger than λ/D
  - Important at lower frequencies where radio source surface density is greater and wide-field imaging techniques required
  - Beam squint: E^p and E^q not parallel, yielding spurious polarization
  - For convenience, direction dependence of polarization leakage (*D*) may be included in *E* (off-diagonal terms then non-zero)
- – Rick Perley's lecture: "Wide Field Imaging I" (Friday)
- – Tim Cornwell's lecture: "Wide Field Imaging II" (Friday)

$$E^{pq} = \begin{pmatrix} e^p(l,m) & 0 \\ 0 & e^q(l,m) \end{pmatrix}$$

Polarization Leakage, *D*

- Polarizer is not ideal, so orthogonal polarizations not perfectly isolated
  - Well-designed feeds have *d* ~ a few percent or less
  - A geometric property of the feed design, so frequency dependent
  - For *R,L* systems, total-intensity imaging affected as ~*dQ, dU,* so only important at high dynamic range (*Q,U~d~few %*, typically)
  - For *R,L* systems, linear polarization imaging affected as ~*dI,* so almost always important
- – Steve Myers' lecture: "Polarization in Interferometry" (today!)

$$\vec{D}^{pq} = \begin{pmatrix} 1 & d^p \\ d^q & 1 \end{pmatrix}$$

"Electronic" Gain, *G*

- Catch-all for most amplitude and phase effects introduced by antenna electronics (amplifiers, mixers, quantizers, digitizers)
  - Most commonly treated calibration component
  - Dominates other effects for standard VLA observations
  - Includes scaling from engineering (correlation coefficient) to radio astronomy units (Jy), by scaling solution amplitudes according to observations of a flux density calibrator
  - Often also includes ionospheric and tropospheric effects which are typically difficult to separate unto themselves
  - Excludes frequency dependent effects (see *B*)

$$G^{pq} = \begin{pmatrix} g^p & 0 \\ 0 & g^q \end{pmatrix}$$

Bandpass Response, *B*

- *G*-like component describing frequency-dependence of antenna electronics, etc.
  - Filters used to select frequency passband not square
  - Optical and electronic reflections introduce ripples across band
  - Often assumed time-independent, but not necessarily so
  - Typically (but not necessarily) normalized

$$B^{pq} = \begin{pmatrix} b^p(\nu) & 0 \\ 0 & b^q(\nu) \end{pmatrix}$$

Geometric Compensation, *K*

- Must get geometry right for Synthesis Fourier Transform relation to work in real time; residual errors here require "Fringe-fitting"
  - Antenna positions (geodesy)
  - Source directions (time-dependent in topocenter!) (astrometry)
  - Clocks
  - Electronic pathlengths
  - Importance scales with frequency and baseline length
- – Craig Walker's lecture: "Very Long Baseline Interferometry" (Thursday)

$$K^{pq} = \begin{pmatrix} k^p & 0 \\ 0 & k^q \end{pmatrix}$$

Non-closing Effects: *M, A*

- Correlator-based errors which do not decompose into antenna-based components
  - Most digital correlators designed to limit such effects to well-understood and uniform scaling laws (absorbed in *G*)
  - Simple noise
  - Additional errors can result from averaging in time and frequency over variation in antenna-based effects and visibilities (practical instruments are finite!)
  - Correlated "noise" (e.g., RFI)
  - Virtually indistinguishable from source structure effects
  - Geodetic observers consider determination of radio source structure—a baseline-based effect—as a required *calibration* if antenna positions are to be determined accurately
  - Diagonal 4x4 matrices, M_{ij} multiplies, A_{ij} adds

The Whole M.E.

- The net *J* can be written:

$$\vec{J}_i \otimes \vec{J}_j^* = \overleftrightarrow{M}_{ij}\left(\overleftrightarrow{K}_i\overleftrightarrow{B}_i\overleftrightarrow{G}_i\overleftrightarrow{D}_i\overleftrightarrow{E}_i\overleftrightarrow{P}_i\overleftrightarrow{T}_i\overleftrightarrow{F}_i \otimes \overleftrightarrow{K}_j^*\overleftrightarrow{B}_j^*\overleftrightarrow{G}_j^*\overleftrightarrow{D}_j^*\overleftrightarrow{E}_j^*\overleftrightarrow{P}_j^*\overleftrightarrow{T}_j^*\overleftrightarrow{F}_j^*\right)$$
$$= \overleftrightarrow{M}_{ij}\left(\overleftrightarrow{K}_i \otimes \overleftrightarrow{K}_j^*\right)\left(\overleftrightarrow{B}_i \otimes \overleftrightarrow{B}_j^*\right)\left(\overleftrightarrow{G}_i \otimes \overleftrightarrow{G}_j^*\right)\left(\overleftrightarrow{D}_i \otimes \overleftrightarrow{D}_j^*\right)$$
$$\left(\overleftrightarrow{E}_i \otimes \overleftrightarrow{E}_j^*\right)\left(\overleftrightarrow{P}_i \otimes \overleftrightarrow{P}_j^*\right)\left(\overleftrightarrow{T}_i \otimes \overleftrightarrow{T}_j^*\right)\left(\overleftrightarrow{F}_i \otimes \overleftrightarrow{F}_j^*\right)$$
$$= \overleftrightarrow{M}_{ij}\overleftrightarrow{K}_{ij}\overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\overleftrightarrow{D}_{ij}\overleftrightarrow{E}_{ij}\overleftrightarrow{P}_{ij}\overleftrightarrow{T}_{ij}\overleftrightarrow{F}_{ij}$$

- The total *Measurement Equation* has the form:

$$\vec{V}_{ij} = \overleftrightarrow{M}_{ij}\overleftrightarrow{K}_{ij}\overleftrightarrow{B}_{ij}\overleftrightarrow{G}_{ij}\int \overleftrightarrow{D}_{ij}\overleftrightarrow{E}_{ij}\overleftrightarrow{P}_{ij}\overleftrightarrow{T}_{ij}\overleftrightarrow{F}_{ij}\ \overleftrightarrow{S}\vec{I}(l,m)\, e^{-i2\pi\left(u_{ij}l+v_{ij}m\right)}dl\ dm + \vec{A}_{ij}$$

  - *S* maps the Stokes vector, *I*, to the polarization basis of the instrument, all calibration terms cast in this basis

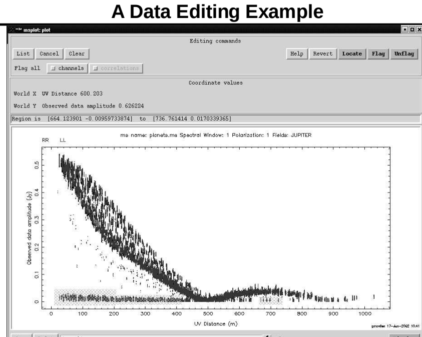
Calibrator Rules of Thumb

- *T, G, K:*
  - Strong and point-like sources, as near to target source as possible
  - Observe often enough to track phase and amplitude variations: calibration intervals of up to 10s of minutes at low frequencies (beware of ionosphere!), as short as 1 minute or less at high frequencies
  - Observe at least one calibrator of known flux density at least once
- *B:*
  - Strong enough for good sensitivity in each channel (often, *T, G* calibrator is ok), point-like if visibility might change across band
  - Observe often enough to track variations (e.g., waveguide reflections change with temperature and are thus a function of time-of-day)
- *D:*
  - Best calibrator for full calibration is strong and polarized
  - If polarized, observe over a broad range of parallactic angle to disentangle *D*s and source polarization (often, *T, G* calibrator is ok)
- *F:*
  - Choose strongly polarized source and observe often enough to track variation

Data Examination and Editing

- After observation, initial data examination and editing very important
  - Will observations meet goals for calibration and science requirements?
  - Some real-time flagging occurred during observation (antennas off-source, LO out-of-lock, etc.). Any such bad data left over? (check operator's logs)
  - Any persistently 'dead' antennas ($J_i=0$ during otherwise normal observing)? (check operator's logs)
  - Amplitude and phase should be continuously varying—edit outliers
  - Any antennas shadowing others? Edit such data.
  - Be conservative: those antennas/timeranges which are bad on calibrators are probably bad on weak target sources—edit them
  - Periods of poor weather? (check operator's log)
  - Distinguish between bad (hopeless) data and poorly-calibrated data. E.g., some antennas may have significantly different amplitude response which may not be fatal—it may only need to be calibrated
  - Radio Frequency Interference (RFI)?
  - Choose reference antenna wisely (ever-present, stable response)

A Data Editing Example



Radio Frequency Interference

- RFI originates from man-made signals generated in the antenna electronics or by external sources (e.g., satellites, cell-phones, radio and TV stations, automobile ignitions, microwave ovens, etc.)
  - Adds to total noise power in all observations, thus decreasing sensitivity to desired natural signal, possibly pushing electronics into non-linear regimes
  - As a contribution to the n_i term, can correlate between antennas if of common origin and baseline short enough (insufficient decorrelation via K_i)

$$\left\langle K_i x_i \cdot K_j^* x_j^* \right\rangle_{\Delta t} = \left\langle K_i\left(s_i' + n_i + n_{RFI}\right) \cdot K_j^*\left(s_j' + n_j + n_{RFI}\right)^* \right\rangle_{\Delta t}$$
$$= \left\langle K_i s_i' \cdot K_j^* s_j'^* \right\rangle + \left\langle K_i n_{RFI} \cdot K_j^* n_{RFI}^* \right\rangle$$
$$= \left\langle K_i s_i' \cdot K_j^* s_j'^* \right\rangle + \left\langle K_i K_j^* \left|n_{RFI}\right|^2 \right\rangle$$

  - When RFI is correlated, it obscures natural emission in spectral line observations

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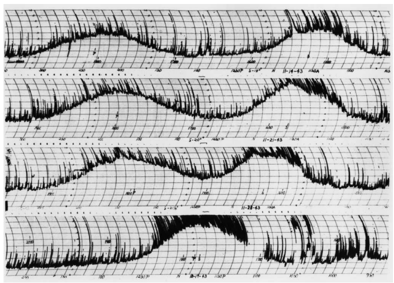
Radio Frequency Interference

- Has always been a problem (Reber, 1944, in total power)!



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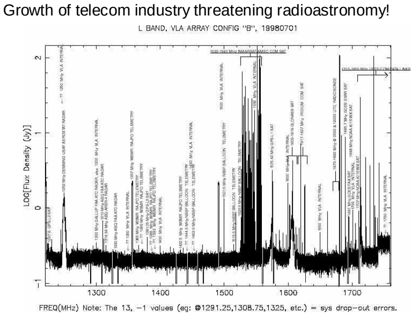
Radio Frequency Interference (cont)

- Growth of telecom industry threatening radioastronomy!



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Radio Frequency Interference (cont)

- RFI Mitigation
  - Careful electronics design in antennas, including notch filters
  - High-dynamic range digital sampling
  - Observatories world-wide lobbying for spectrum management
  - Choose interference-free frequencies: try to find 50 MHz (1 GHz) of clean spectrum in the VLA (EVLA) 1.6 GHz band!
  - Observe continuum experiments in spectral-line modes so affected channels can be edited
- Various off-line mitigation techniques under study
  - E.g., correlated RFI power appears at celestial pole in image domain…

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Summary

- Determining calibration is as important as determining source structure—can't have one without the other
- Calibration dominated by antenna-based effects, permits separation of calibration from astronomical information
- Calibration formalism algebra-rich, but can be described piecemeal in comprehendible segments, according to well-defined effects
- Calibration determination is a single standard fitting problem
- Calibration an iterative process, improving various components in turn
- Point sources are the best calibrators
- Observe calibrators according requirements of components
- Data examination and editing an important part of calibration

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