Cross Correlators

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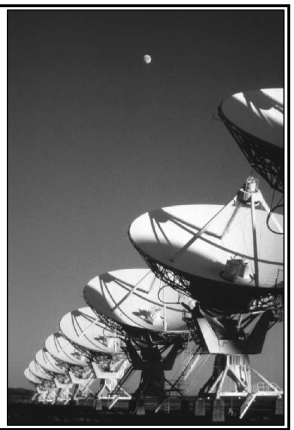

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Outline

- The correlation function
- What is a correlator?
- Simple correlators
- Sampling and quantization
- Spectral line correlators
- The EVLA correlator in detail

This lecture is complementary to Chapter 4 of ASP 180

The VLBA Correlator

The Correlation Function

$$C_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle_T$$

- If $i = j$ it is an auto-correlation (AC). Otherwise it is a cross-correlation (CC).
- Useful for
  - Determining timescales (AC)
  - Motion detection (2-D CC)
  - Optical character recognition (2-D CC)
  - Pulsar timing / template matching (CC)

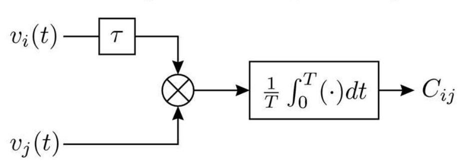
What is a Correlator?

A correlator is a hardware or software device that combines sampled voltage time series from one or more antennas to produce sets of complex visibilities, V_{ij}.

- Visibilities are in general a function of
  - Frequency
  - Antenna pair
  - Time
- They are used for
  - Imaging
  - Spectroscopy / polarimetry
  - Astrometry

A Real (valued) Cross Correlator

$$C_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle_T$$



Visibilities
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What astronomers really want is the complex visibility

$$V_{ij} = \left\langle E_i(t) E_j^*(t+\tau) \right\rangle$$

where the real part of $E_i(t)$ is the voltage measured by antenna i.

So what is the imaginary part of $E_i(t)$?

It is the same as the real part but with each frequency component *phase* lagged by 90 degrees.

$$E_i(t) = v_i(t) + \frac{i}{\pi} \int_{-\infty}^{\infty} \frac{v_i(t')}{t-t'} dt'$$

The Complex Correlator
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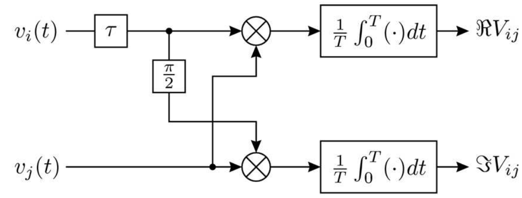
$$V_{ij} = \langle v_i(t) v_j(t+\tau) \rangle + i \langle \mathcal{H}[v_i(t)]\, v_j(t+\tau) \rangle$$



Time Series, Sampling, and Quantization
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- $\{v_i(t)\}$ are real-valued time series sampled at "uniform" intervals, Δt.
- The sampling theorem allows this to accurately reconstruct a bandwidth of $\Delta\nu = \frac{1}{2\Delta t}$.
- Sampling involves quantization of the signal
  - Quantization noise
  - Strong signals become non-linear
  - Sampling theorem violated!

Quantization Noise
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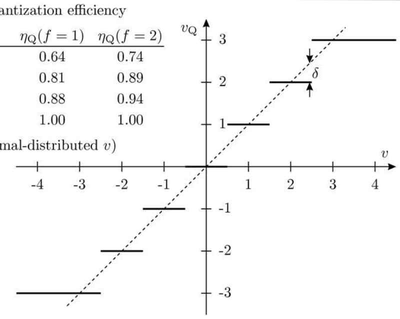
Quantization efficiency

N levels	$\eta_Q(f=1)$	$\eta_Q(f=2)$
2	0.64	0.74
3	0.81	0.89
4	0.88	0.94
∞	1.00	1.00

(For normal-distributed v)



Automatic Gain Control (AGC)
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- Normally prior to sampling the amplitude level of each time series is adjusted so that quantization noise is minimized.
- This occurs on timescales very long compared to a sample interval.
- The magnitude of the amplitude is stored so that the true amplitudes can be reconstructed after correlation.

(Slide added based on discussions)

The Correlation Coefficient
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- The correlation coefficient,ρ_{ij} measures the likeness of two time series in an amplitude independent manner:

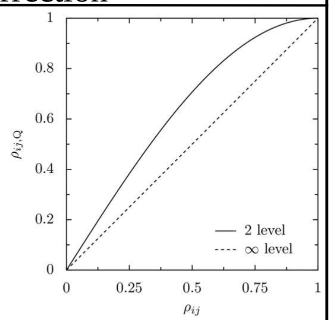
$$\rho_{ij} = \frac{|V_{ij}|}{\sqrt{V_{ii} V_{jj}}}$$

- Normally the correlation coefficient is much less than 1
- Because of AGC, the correlator actually measures the correlation coefficient. The visibility amplitude is restored by dividing by the AGC gain.

(Slide added based on discussions)

Van Vleck Correction

- At low correlation, quantization ***increases correlation***
- Quantization causes predictable non-linearity at high correlation V_{ij}
- Correction must be applied to the real and imaginary parts of separately
  - Thus the visibility phase is affected as well as the amplitude



The Delay Model

- τ is the difference between the geometric delays of antenna j and antenna i. It can be + or - .
- The *delay center* moves across the sky
  - τ is changing constantly
- Fringes at the delay center are stopped.
  - Long time integrations can be done
  - Wide bandwidths can be used
- Simple delay models incorporate:
  - Antenna locations
  - Source position
  - Earth orientation
- VLBI delay models must include much more!

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Fractional Sample Delay Compensation

$$\tau = n\Delta t + \epsilon$$

- Delays must be corrected to better than Δt.
- Integer delay is usually done with digital delay lines.
- Fractional sample delay is trickier
- It is implemented differently at different correlators
  - Analog delay lines (DRAO array)
  - Add delay to the sampling clock (VLA)
  - Correct phases after multiplier (VLBA)

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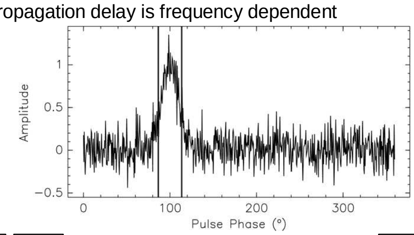
Pulsar Gating

- Pulsars emit regular pulses with small duty cycle
- Period in range 1 ms to 8 s; $\Delta t \ll P_{\text{pulsar}} < T$
- Blanking during off-pulse improves sensitivity
- Propagation delay is frequency dependent



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Spectral Line Correlators

- Chop up bandwidth for
  - Calibration
    - Bandpass calibration
    - Fringe fitting
  - Spectroscopy
  - Wide-field imaging
- Conceptual version
  - Build analog filter bank
  - Attach a complex correlator to each filter

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Practical Spectral Line Correlators

- Use a single filter / sampler
  - Easier to calibrate
  - Practical, up to a point
- The FX architecture
  - F : Replace filterbank with digital Fourier transform
  - X : Use a complex-correlator for each frequency channel
  - Then integrate
- The XF architecture
  - X : Measure correlation function at many lags
  - Integrate
  - F : Fourier transform
- Other architectures possible

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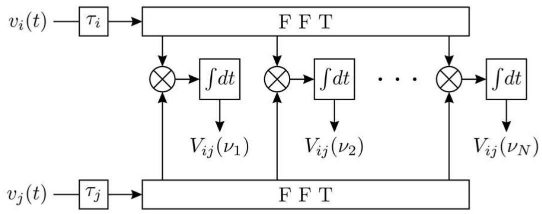
The FX correlator



FX Correlators

- Spectrum is available **before integration**
  - Can apply fractional sample delay per channel
  - Can apply pulsar gate per channel
- Most of the digital parts run *N* times slower than the sample rate

FX Spectral Response

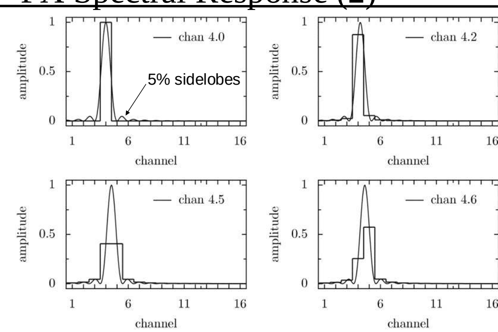
- FX Correlators derive spectra from truncated time series

$$\begin{aligned} v(\nu) &= \mathcal{F}\left[v(t)\cdot\sqcap\left(\frac{t}{N\Delta t}\right)\right] \\ &= \mathcal{F}[v(t)]\star\mathcal{F}\left[\sqcap\left(\frac{t}{N\Delta t}\right)\right] \\ &\propto \mathcal{F}[v(t)]\star\operatorname{sinc}(N\Delta t\nu) \end{aligned}$$

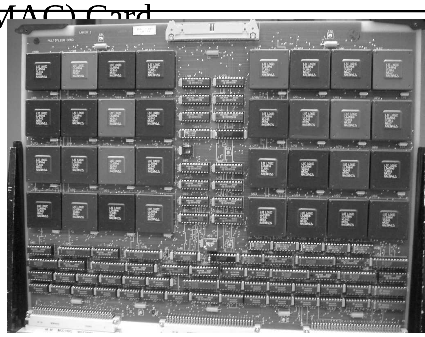
- Results in convolved visibility spectrum

$$\begin{aligned} V_{ij}(\nu) &= \left\langle(\mathcal{F}[v_i(t)]\star\operatorname{sinc}(N\Delta t\nu))\,(\mathcal{F}[v_j(t)]\star\operatorname{sinc}(N\Delta t\,\nu))^*\right\rangle \\ &= \left\langle\mathcal{F}[v_i(t)]\,\mathcal{F}[v_j(t)]^*\right\rangle\star\operatorname{sinc}^2(N\Delta t\,\nu) \end{aligned}$$

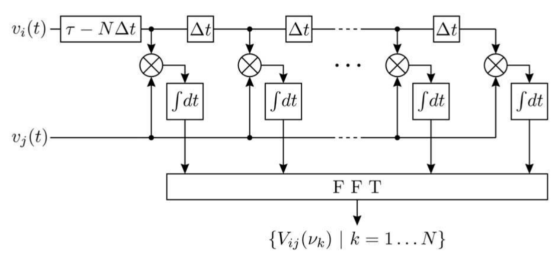
FX Spectral Response (2)



VLBA Multiply Accumulate (MAC) Card



The XF Correlator (real version)



XF Spectral Response
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- XF correlators measure lags over a finite delay range

$$V_{ij}(\tau) = \langle v_i(t) v_j(t+\tau) \rangle \cdot \sqcap \left(\frac{t}{N\Delta t} \right)$$

- Results in convolved visibility spectrum

$$V_{ij}(\nu) = \mathcal{F}\left[\langle v_i(t) v_j(t+\tau) \rangle \cdot \sqcap \left(\frac{t}{N\Delta t} \right) \right]$$
$$= \mathcal{F}\left[\langle v_i(t) v_j(t+\tau) \rangle \right] \star \mathrm{sinc}\left(N\Delta t\, \nu \right)$$

XF Spectral Response (2)
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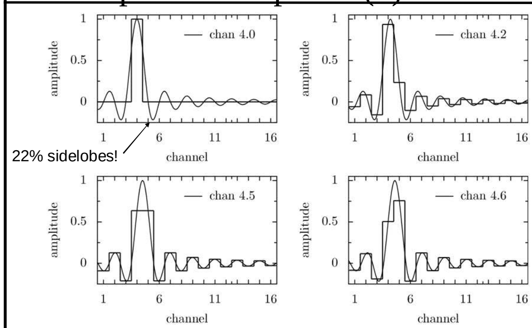

Hanning Smoothing
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- Multiply lag spectrum by Hanning taper function

$$H(\tau) = \frac{1}{2}\left(1 + \cos\frac{\pi\tau}{N\Delta t}\right)$$

- This is equivalent to convolution of the spectrum by

$$H(\nu) = \delta(\nu) - \frac{1}{2}\delta\left(\nu - \frac{1}{2N\Delta t}\right) - \frac{1}{2}\delta\left(\nu + \frac{1}{2N\Delta t}\right)$$

- Note that sensitivity and spectral resolution are reduced.

Hanning Smoothing (2)
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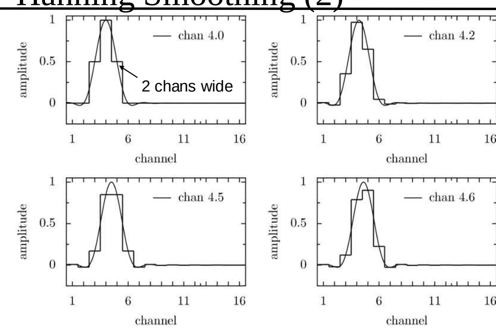

XF Correlators : Recirculation
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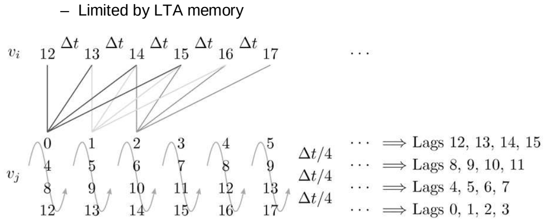
- Example: 4 lag correlator with recirculation factor of 4
  - 4 correlator cycles (red) per sample interval (Δt)
  - 4 lags calculated per cycle (blue for second sample interval)
  - Forms 16 lags total
  - Limited by LTA memory



VLA MAC Card
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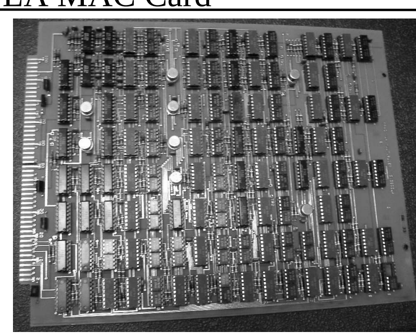

The EVLA WIDAR Correlator

- XF architecture duplicated 64 times, or “FXF”
  - Four 2GHz basebands per polarization
  - Digital filterbank makes 16 subbands per baseband
  - 16,384 channels/baseline at full sensitivity
  - 4 million channels with less bandwidth!
- Initially will support 32 stations with plans for 48
- 2 stations at 25% bandwidth or 4 stations at 6.25% bandwidth can replace 1 station input
- Correlator efficiency is about 95%
  - Compare to 81% for VLA
- VLBI ready

WIDAR Correlator (2)

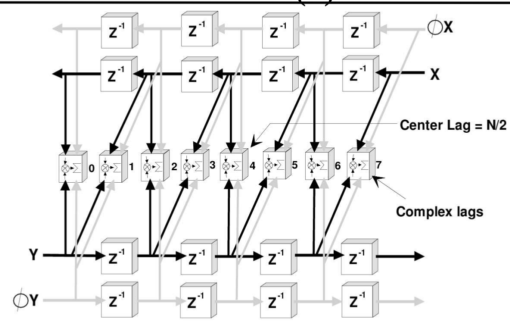

Figure from WIDAR memo 014, Brent Carlson

WIDAR Correlator (3)

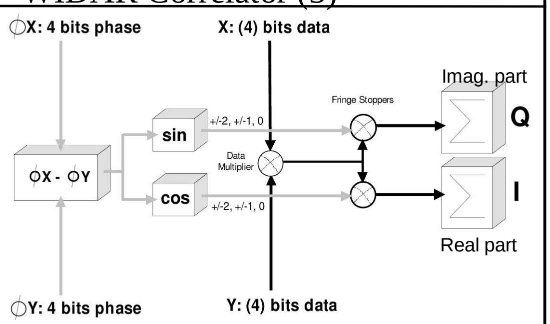

Figure from WIDAR memo 014, Brent Carlson

WIDAR Correlator Modes

Bandwidth MHz	Single Pol. Prod. No. Channels	Single Pol. Prod. Freq. Separ. kHz	Two Pol.Prod. No. Channels per pol	Two Pol.Prod. Freq. Separ. kHz	Four Pol.Prod. No. Channels per pol	Four Pol.Prod. Freq. Separ. kHz
8192	16,384	500	8,192	1000	4,096	2000
4096	16,384	250	8,192	500	4,096	1000
2048	32,768	62.5	16,384	31.25	8,192	250
1024	65,536	15.625	32,768	31.25	16,384	62.5
512	131,072	3.906	65,536	7.813	32,768	15.625
256	262,144	0.977	131,072	1.953	65,536	3.906
128	262,144	0.488	131,072	0.977	65,536	1.953
64	262,144	0.244	131,072	0.488	65,536	0.977
32	262,144	0.122	131,072	0.244	65,536	0.488
16	262,144	0.061	131,072	0.122	65,536	0.244
8	262,144	0.031	131,072	0.061	65,536	0.122
4	262,144	0.015	131,072	0.031	65,536	0.061
2	262,144	0.008	131,072	0.015	65,536	0.031
1	262,144	3.8 Hz	131,072	7.6 Hz	65,536	0.015
0.5	262,144	1.9 Hz	131,072	3.8 Hz	65,536	7.6 Hz
0.25	262,144	0.95 Hz	131,072	1.9 Hz	65,536	3.8 Hz
0.125	262,144	0.48 Hz	131,072	0.95 Hz	65,536	1.9 Hz
0.0625	262,144	0.24 Hz	131,072	0.48 Hz	65,536	0.95 Hz
0.03125	262,144	0.12 Hz	131,072	0.24 Hz	65,536	0.48 Hz