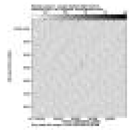
SENSITIVITY



Outline

What is Sensitivity & Why Should You Care?
What Are Measures of Antenna Performance?
What is the Sensitivity of an Interferometer?
What is the Sensitivity of a Synthesis Image?
Summary

What is Sensitivity & Why Should You Care?

- Measure of weakest detectable radio emission
- Important throughout research program
  - Technically sound observing proposal
  - Sensible error analysis in publication
- Expressed in units involving Janskys
  - Unit for interferometer is Jansky (Jy)
  - Unit for synthesis image is Jy beam^{-1}
- 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}
- Common to use milliJy or microJy

Measures of Antenna Performance

Source and System Temperatures

- What is received power P?
- Write P as equivalent temperature of matched termination at receiver input
  - Rayleigh-Jeans limit to Planck law $P = k_B \times T \times \Delta n$
  - Boltzmann constant k_B
  - Observing bandwidth Δn
- Amplify P by g^2 where g is voltage gain
- Separate powers from source, system noise
  - Source antenna temperature T_a => source power $P_a = g^2 \times k_B \times T_a \times \Delta n$
  - System temperature T_{sys} => noise power $P_N = g^2 \times k_B \times T_{sys} \times \Delta n$

Measures of Antenna Performance

Gain

- Source power $P_a = g^2 \times k_B \times T_a \times \Delta n$
  - Let $T_a = K \times S$ for source flux density S, constant K
  - Then $P_a = g^2 \times k_B \times K \times S \times \Delta n$ (1)
- But source power also $P_a = \frac{1}{2} \times g^2 \times h_a \times A \times S \times \Delta n$ (2)
  - Antenna area A, efficiency h_a
  - Receiver accepts 1/2 radiation from unpolarized source
- Equate (1), (2) and solve for K

$$K = (h_a \times A)/(2 \times k_B) = T_a / S$$

  - K is antenna's gain or "sensitivity", unit degree Jy^{-1}
- K measures antenna performance but no T_{sys}

Measures of Antenna Performance

System Equivalent Flux Density

- Antenna temperature $T_a = K \times S$
  - Source power $P_a = g^2 \times k_B \times K \times S \times \Delta n$
- Express system temperature analogously
  - Let $T_{sys} = K \times SEFD$
  - $SEFD$ is system equivalent flux density, unit Jy
  - System noise power $P_N = g^2 \times k_B \times K \times SEFD \times \Delta n$
- $SEFD$ measures overall antenna performance

$$SEFD = T_{sys} / K$$

  - Depends on T_{sys} and $K = (h_a \times A)/(2 \times k_B)$
  - Examples in Table 9-1

Interferometer Sensitivity

Real Correlator - 1

- Simple correlator with single real output that is product of voltages from antennas j,i
  - $SEFD_i = T_{sysi} / K_i$ and $SEFD_j = T_{sysj} / K_j$
  - Each antenna collects bandwidth Δn
- Interferometer built from these antennas has
  - Accumulation time t_{acc}, system efficiency h_s
  - Source, system noise powers imply sensitivity ΔS_{ij}
- Weak source limit
  - $S << SEFD_i$
  - $S << SEFD_j$

$$\Delta S_{ij} = \frac{1}{h_s} \times \sqrt{\frac{SEFD_i \times SEFD_j}{2 \times \Delta n \times t_{acc}}}$$

Interferometer Sensitivity

Real Correlator - 2

- For $SEFD_i = SEFD_j = SEFD$ drop subscripts

$$\Delta S = \frac{1}{h_s} \times \frac{SEFD}{\sqrt{2 \times \Delta n \times t_{acc}}}$$

  - Units Jy
- Interferometer system efficiency h_s
  - Accounts for electronics, digital losses
  - Eg: VLA continuum
    - *Digitize in 3 levels, collect data 96.2% of time*
    - *Effective* $h_s = 0.81 \times \sqrt{0.962} = 0.79$

Interferometer Sensitivity

Complex Correlator

- Delivers two channels
  - Real S_R, sensitivity ΔS
  - Imaginary S_I, sensitivity ΔS
- Eg: VLBA continuum
  - Figure 9-1 at 8.4 GHz
  - Observed scatter $S_R(t)$, $S_I(t)$
  - Predicted ΔS = 69 milliJy

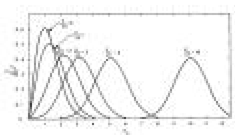
$$\Delta S = \frac{1}{h_s} \times \frac{SEFD}{\sqrt{2 \times \Delta n \times t_{acc}}}$$

  - Resembles observed scatter

Interferometer Sensitivity

Measured Amplitude

- Measured visibility amplitude $S_m = \sqrt{S_R^2 + S_I^2}$
  - Standard deviation (sd) of S_R or S_I is ΔS
- True visibility amplitude S
- Probability $\Pr(S_m / \Delta S)$
  - Figure 9-2



  - Behavior with true $S / \Delta S$
    - *High: Gaussian, sd* ΔS
    - *Zero: Rayleigh, sd* $\Delta S \sqrt{2-(p/2)}$
    - *Low: Rice.* S_m *gives biased estimate of S. Use unbias method.*

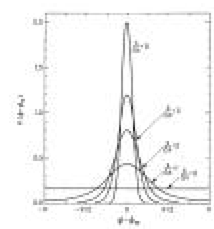
Interferometer Sensitivity

Measured Phase

- Measured visibility phase

$$f_m = \arctan(S_I / S_R)$$

- True visibility phase f
- Probability $\Pr(f - f_m)$
  - Figure 9-2
  - Behavior with true $S/\Delta S$
    - *High: Gaussian*
    - *Zero: Uniform*



- Seek weak detection in phase, not amplitude

Image Sensitivity

Single Polarization

- Simplest weighting case where visibility samples
  - Have same interferometer sensitivities $\Delta S = \frac{1}{h_s} \times \frac{SEFD}{\sqrt{2 \times \Delta n \times t_{acc}}}$
  - Have same signal-to-noise ratios w
  - Combined with natural weight ($W=1$), no taper ($T=1$)
- Image sensitivity is sd of mean of L samples, each with sd ΔS, ie, $\Delta I_m = \Delta S / \sqrt{L}$
  - No. of interferometers $\frac{1}{2} \times N \times (N-1)$ $\quad L = \frac{1}{2} \times N \times (N-1) \times (t_{int} / t_{acc})$
  - No. of accumulation times t_{int} / t_{acc}
  - So $\Delta I_m = \frac{1}{h_s} \times \frac{SEFD}{\sqrt{N \times (N-1) \times \Delta n \times t_{int}}}$

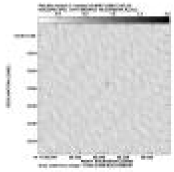
Image Sensitivity

Dual Polarizations - 1

- Single-polarization image sensitivity ΔI_m
- Dual-polarization data => image Stokes I,Q,U,V
  - Gaussian noise in each image
  - Mean zero, $\Delta I = \Delta Q = \Delta U = \Delta V = \Delta I_m / \sqrt{2}$
- Polarized flux density $P = \sqrt{Q^2 + U^2}$
  - Rayleigh noise, sd $\Delta Q \times \sqrt{2-(p/2)} = \Delta U \times \sqrt{2-(p/2)}$
  - Cf. visibility amplitude, Figure 9-2
- Polarization position angle $c = \frac{1}{2} \times \arctan(U/Q)$
  - Uniform noise between $\pm p/2$
  - Cf. visibility phase, Figure 9-2, $\pm p$

Image Sensitivity

Dual Polarizations – 2



- Eg: VLBA continuum
  - Figure 9-3 at 8.4 GHz
  - Observed
    - *T: Stokes I, simplest weighting*
    - *B: Gaussian noise* ΔI *= 90 microJy beam*$^{-1}$
  - Predicted

$$\Delta I = \Delta I_m / \sqrt{2} = \Delta S / \sqrt{2 \times L}$$

$$L = \frac{1}{2} \times N \times (N-1) \times (t_{int} / t_{acc})$$

- *Previous eg* ΔS
- *Plus here L = 77,200*
- *So* ΔI *= 88 microJy beam*$^{-1}$

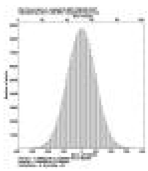
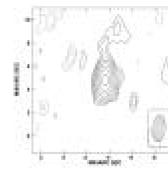


Image Sensitivity

Dual Polarizations – 3



- Eg: VLBA continuum
  - Figure 9-3 at 8.4 GHz
  - Observed
    - *T:* I_{peak} *= 2 milliJy beam*$^{-1}$
    - *B: Gaussian noise* ΔI *= 90 microJy beam*$^{-1}$
  - Position error from sensitivity?

$$\Delta q = \frac{1}{2} \times q_{HPBW} \times \frac{\Delta I}{I_{peak}}$$

- *Gaussian beam* q_{HBPW} *= 1.5 milliarcsec*
- *Then* Δq *= 34 microarcsec*
- *Other position errors dominate*

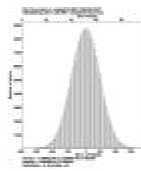
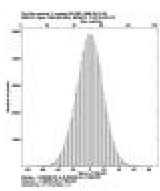


Image Sensitivity

Dual Polarizations – 4



- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - *Q, U images, simplest weighting*
    - *Gaussian* $\Delta Q = \Delta U$ *= 17 microJy beam*$^{-1}$
  - Predicted

$$\Delta Q = \Delta U = \Delta I_m / \sqrt{2} = \Delta S / \sqrt{2 \times L}$$

$$\Delta S = \frac{1}{h_s} \times \frac{SEFD}{\sqrt{2 \times \Delta n \times t_{acc}}}$$

$$L = \frac{1}{2} \times N \times (N-1) \times (t_{int} / t_{acc})$$

- $\Delta Q = \Delta U$ *= 16 microJy beam*$^{-1}$

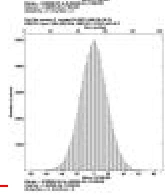
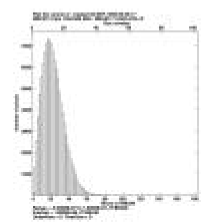


Image Sensitivity

Dual Polarizations – 5



- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - *Q, U images, simplest weighting*
    - $\Delta Q = \Delta U$ *= 17 microJy beam*$^{-1}$
    - *Form image of* $P = \sqrt{Q^2 + U^2}$
    - *Rayleigh noise in P*
    - *Sd 11 microJy beam*$^{-1}$
  - Predicted
    - *Sd* $\Delta Q \times \sqrt{2 - (p/2)} = \Delta U \times \sqrt{2 - (p/2)}$
    - *Sd 11 microJy beam*$^{-1}$

Image Sensitivity

Dual Polarizations – 6

- Eg: VLA continuum
  - Figure 9-4 at 1.4 GHz
  - Observed
    - *I, Q, U images, simplest weighting*
    - *Gaussian noise* $\Delta Q = \Delta U < \Delta I$
  - *I, Q, U* will have same sd if each is limited by sensitivity
    - *Recall* $\Delta I = \Delta Q = \Delta U = \Delta V = \Delta I_m / \sqrt{2}$
    - *Other factors can increase* ΔI
    - *Suspect dynamic range as* I_{peak} *= 10,000* ΔI
    - *Lesson: Use sensitivity as tool to diagnose problems*

Sensitivity

Summary – 1

- One antenna
  - System temperature T_{sys}
  - Gain *K*
- Overall antenna performance is measured by system equivalent flux density *SEFD*

$$SEFD = T_{sys} / K$$

  - Units Jy

Sensitivity

Summary - 2

- Connect two antennas to form interferometer
  - Antennas have same *SEFD*, observing bandwidth Δn
  - Interferometer system efficiency h_s
  - Interferometer accumulation time t_{acc}
- Sensitivity of interferometer

$$\Delta S = \frac{1}{h_s} \times \frac{SEFD}{\sqrt{2 \times \Delta n \times t_{acc}}}$$

  - Units Jy

Sensitivity

Summary - 3

- Connect *N* antennas to form array
  - Antennas have same *SEFD*, observing bandwidth Δn
  - Array has system efficiency h_s
  - Array integrates for time t_{int}
  - Form synthesis image of single polarization
- Sensitivity of synthesis image

$$\Delta I_m = \frac{1}{h_s} \times \frac{SEFD}{\sqrt{N \times (N-1) \times \Delta n \times t_{\text{int}}}}$$

  - Units Jy beam^{-1}