Array Design

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presented at
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Outline

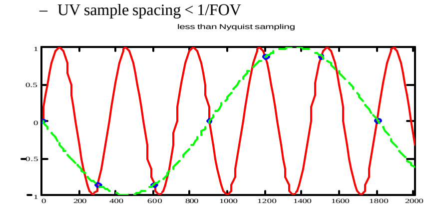
- Statement of the problem
- UV metrics
- Imaging metrics
- Beam metrics
  - PSF and synthesized beam
  - Sidelobe statistics
  - Optimization
- ALMA configurations

Problem

- Given a collection of antennas, what is the the best array configuration?
- Considerations
  - Best science output
  - Geography
  - Flexibility
  - Costs
- Need methods for evaluating configurations

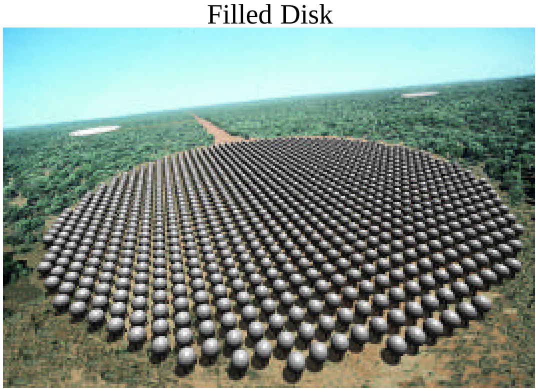
UV metrics

- Brightness = FT of complex visibility
- Resolution $\delta\varphi > \lambda/\text{max\_baseline}$
- Nyquist theorem
  - UV sample spacing < 1/FOV



- Simple DFT would seem to require complete sampling of the UV-plane

Filled Disk



First Arrays

- Large telescopes on tracks
  - => linear, “Tee”, and star arrays
  - Often with regular spacing
- Science was mostly small fields
- Ryle strived for complete sampling

Westerbork

Minimum Redundancy

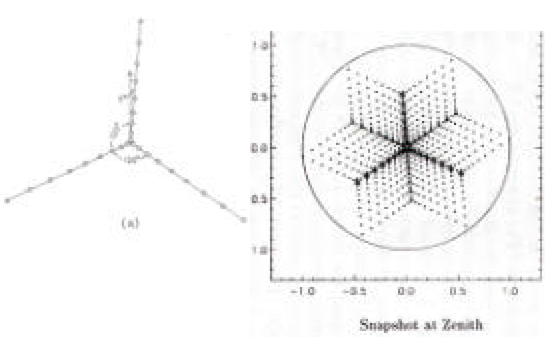
- Possible in 1-D
  - Earth rotation gives good 2-D coverage for polar sources
- Not solved for 2-D snapshots

Early Configuration Design

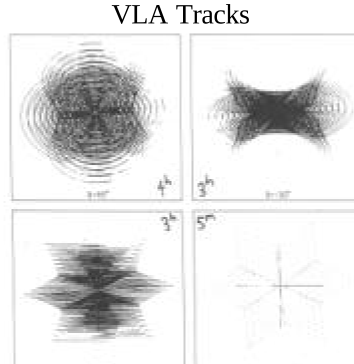
- Layout antennas with some idea in mind
- Look at UV coverage
- Iterate
- Seemed OK for small numbers of antennas

VLA

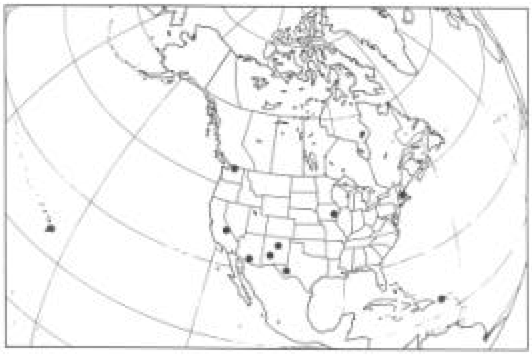
VLA Zenith Snapshot



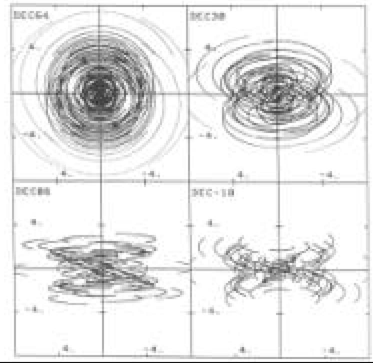
VLA Tracks



VLBA



VLBA UV Coverage

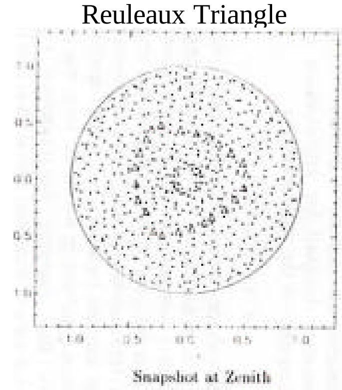


More Recent UV Designs

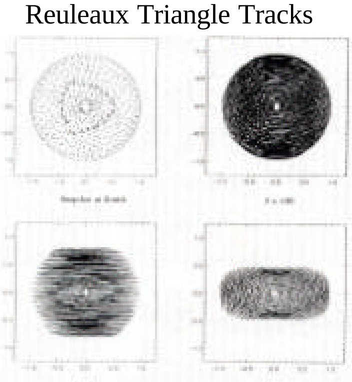
- Optimize UV coverage
- Good snapshot coverage

- Uniform UV coverage
  - Reuleaux triangles (SMA)
- Complete coverage
  - MMA circular configuration
- Match image visibility distribution
  - Gaussian UV coverage

Reuleaux Triangle



Reuleaux Triangle Tracks



Imaging Metrics

- Use the imaging tools we have to evaluate configurations

Image Evaluation

- Set of test pictures
- Generate model visibilities
- Process images
- Compare images to pictures
  - Dynamic range
  - Fidelity
- Repeat for different configurations
- Modify configuration?

- Iterate

General Results

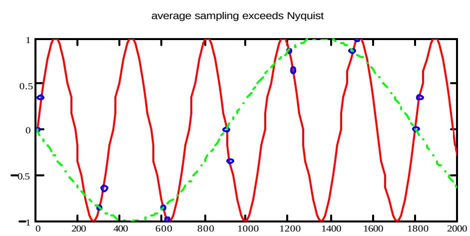
- More UV coverage gives better images
- Guassian UV coverage gives better images, especially for wide fields

Why are the images so good?

- Small fields greatly relaxes the Nyquist sampling
- Many algorithms are essentially model fitting
- Complete Nyquist sampling is not necessary

Generalized Nyquist Theorem

- Uniform complete sampling not required



- Just need the average number of samples to exceed Nyquist

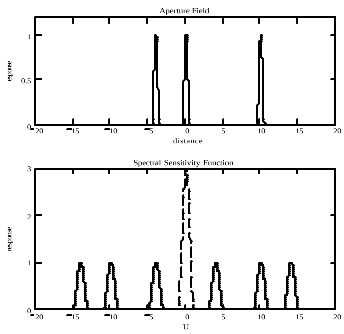
Imaging Approach to Array Design

- Selection of test pictures prejudices the design
- Multiple definitions of dynamic range and fidelity
- Time consuming
- Difficult to evolve to better configuration

- But **imaging is the bottom line** for what we expect to get out of an array
- Part of the ALMA configuration design process

- Can we quantify the effect of the missing UV data?

- Optical
  - Optical transfer function
    - Autocorrelation of field
- Radio
  - Spectral sensitivity function
    - Transfer function or UV sampling



Beam Metrics

- Optical
  - Optical transfer function
    - Autocorrelation of field
  - Point Spread Function
    - FT of optical transfer function
- Radio
  - Spectral response function
    - Transfer function or UV samples
  - Synthesized beam x Primary Beam
    - FT of spectral response function

- The PSF is the optical image generated by a point source. It is a measure of the instrumental or array artifacts that the imaging algorithms must deal with.
- A good PSF => good images

PSF and Radio Astronomy

- Voltage pattern of array beam

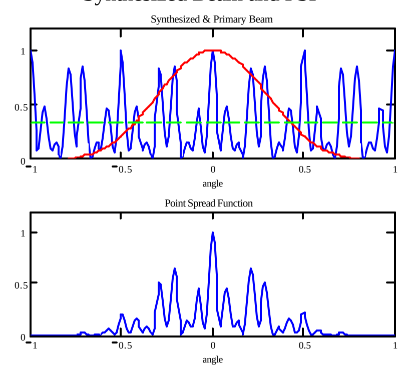
$$U_{PSF}(\vec{p}) = \frac{1}{N}\sum_{n=1}^{N} U_n(\vec{p})\exp\left(-ik\,\vec{p}\cdot\vec{r}_n\right).$$

- Point Spread Function = synthesized beam plus single antenna response

$$PSF(\vec{p}) = \left|U_{PSF}(\vec{p})\right|^2 = \frac{1}{N^2}\sum_{m=1}^{N}\sum_{n=1}^{N} B_{mn}(\vec{p})\exp\left(-ik\,\vec{p}\cdot(\vec{r}_m - \vec{r}_n)\right)$$

- Same as the power from the array used as a transmitter

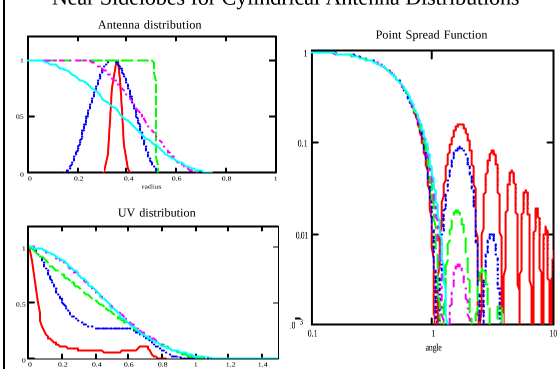
Synthesized Beam and PSF



Sidelobes

- Near sidelobes from large scale distribution
- Far sidelobes from small scale distribution and gaps
- Visibility data can be weighted to improve the sidelobes at the expense of S/N
- At mm and sub-mm, sensitivity is a dominant design criteria and data weighting should be avoided

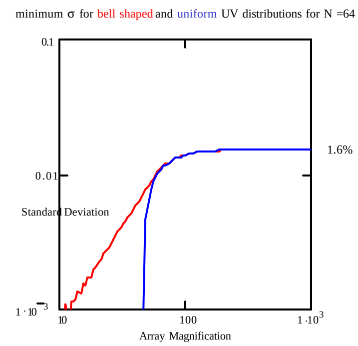
Near Sidelobes for Cylindrical Antenna Distributions



Far Sidelobes

- Caused by gaps in UV coverage
- Average of PSF
  - Including single dish measurements
    - PSF > 0
    - Average = 1/(N-1)
  - Interferometric data only
    - PSF > -1/(N-1)
    - Average = 0
- Standard deviation
  - σ > 1/N for Magnification >> N
  - N = number of antennas
  - Magnification = (primary beam width)/(synthesized beam width)

Standard Deviation vs. Magnification



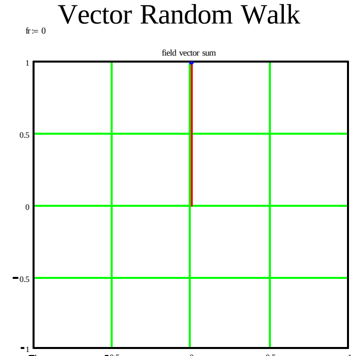
Sidelobe Calculation

$$PSF(\vec{p}) = \frac{1}{N^2} B(\vec{p}) \left| \sum_{n=1}^{N} \exp\left(-ik\,\vec{p}\cdot\vec{r}_n\right) \right|^2$$

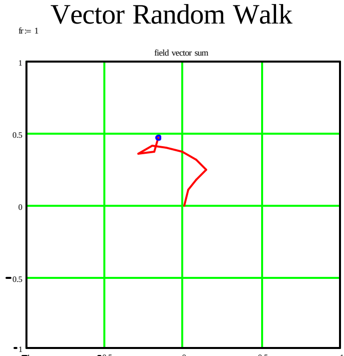
- Sidelobes are sum of N unit vectors of different orientations

Vector rotation $= \frac{2p}{l}\,\vec{p} \bullet \vec{r}$

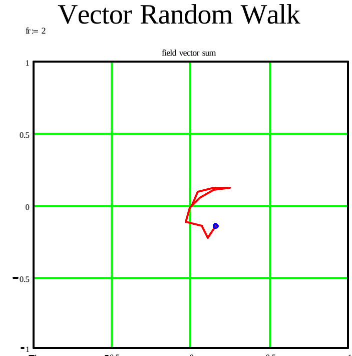
Vector Random Walk



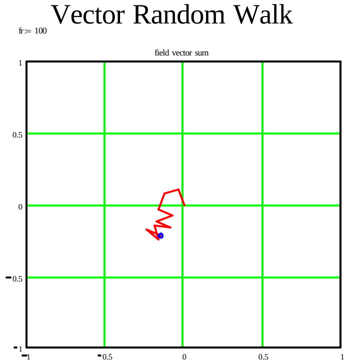
Vector Random Walk



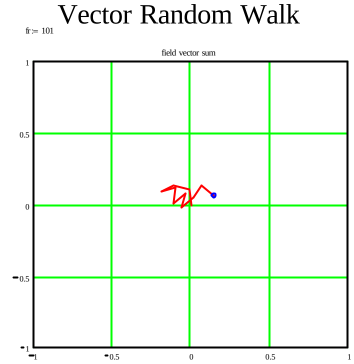
Vector Random Walk



Vector Random Walk



Vector Random Walk



Statistical Distribution of Sidelobes

- Should have a Rayleigh distribution

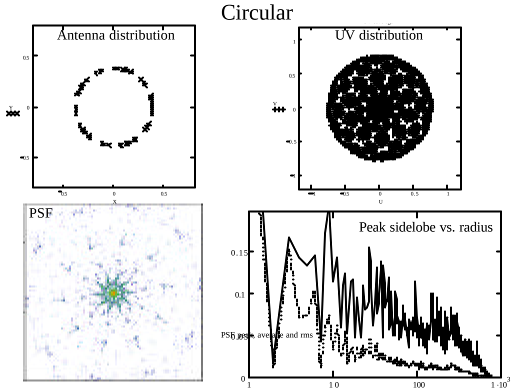
$$g(s) = N \exp(-Ns)$$

- Expected maximum sidelobe

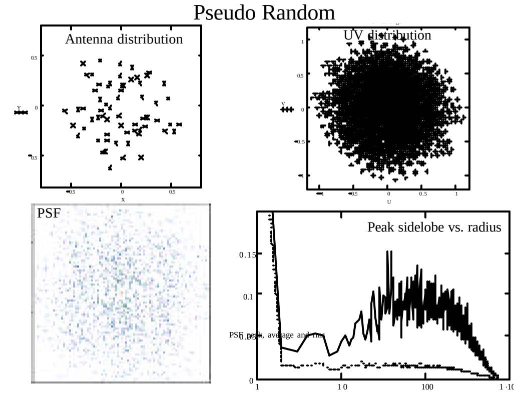
$$s_{\max} \approx \frac{2}{N} \ln(mag)$$

- Test against three configurations
  - UV coverage optimized circular array
  - Random Gaussian array
  - Systematic Gaussian array

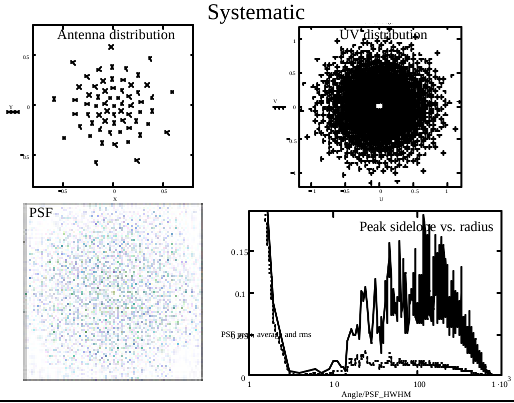
Circular



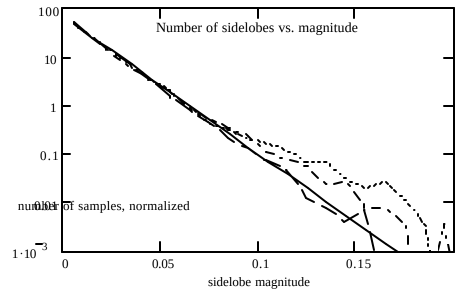
Pseudo Random



Systematic



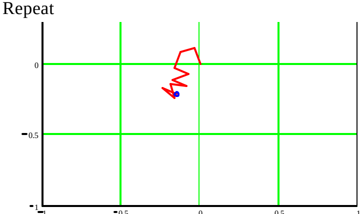
Distribution of Sidelobes



Histograms of the PSF sidelobe amplitudes for the initial circular array (dotted), pseudo-random (dashed) and systematic (dot-dash). The solid line is the theoretical distribution for pseudo-random configurations.

Optimization

- Minimize peak sidelobe
  - Find peak sidelobe
  - Adjust a single antenna to reduce this peak
  - Check that no other peak now exceeds new peak
  - Repeat

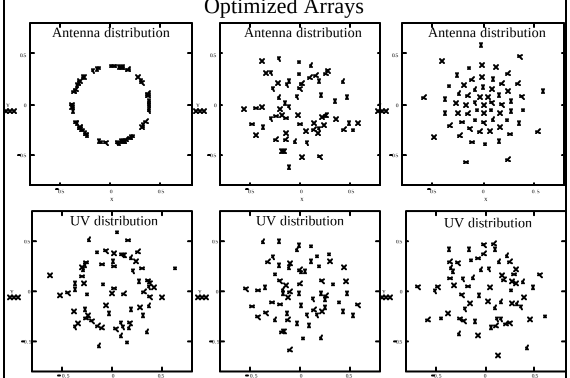


Sequential Optimization

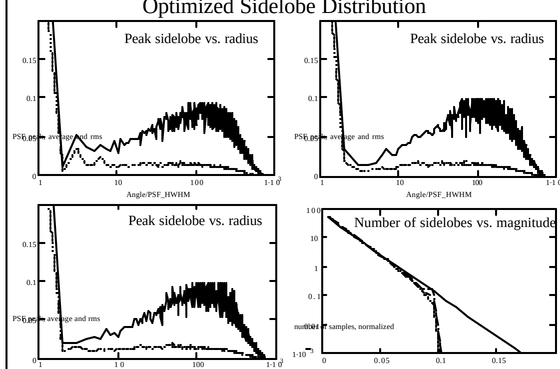
- Rotation of the antenna unit vectors is $\frac{2p}{l}\vec{p}\bullet\vec{r}$
-
- Near sidelobes require large antenna shifts
- Farther sidelobes require small shifts that don’t effect the near sidelobes
- You can start by minimizing the near sidelobes and then progress outward without degrading nearer sidelobes
- Expected maximum sidelobe after optimization is

$$s_{\max, opt} \approx \frac{1}{N}\left[1+2\ln(mag)-\ln(N)\right]$$

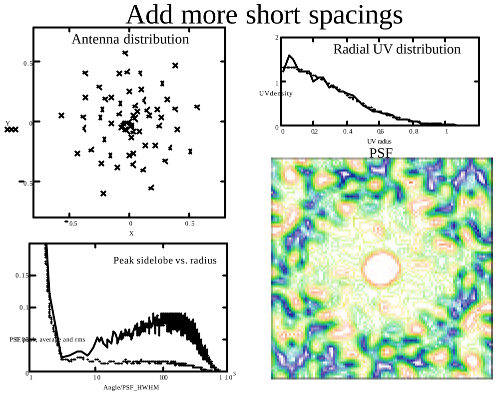
Optimized Arrays



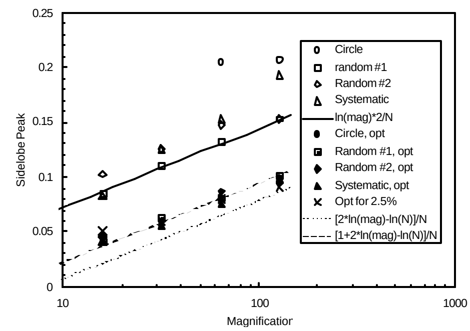
Optimized Sidelobe Distribution



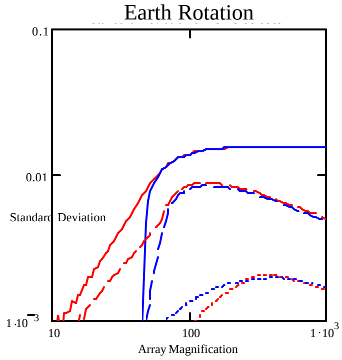
Add more short spacings



Results for several design studies



Earth Rotation



Lower limit to the standard deviation for bell shaped (red curves) and uniform UV coverage (blue curves) for 64 antenna arrays for snapshot (solid), 6 min (dashed) and 1 hr tracks (dotted).

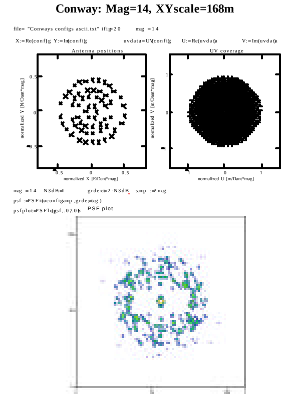
PSF Metric

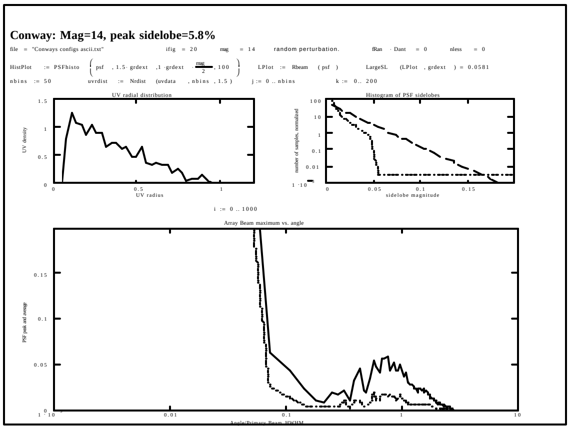
- Simple to calculate
- Gives quantifiable results
- Can be compared to idealized expectations
- Directly related to imaging artifacts
- Easy to evolve into better configurations
  - Can easily incorporate physical constraints
- Major part of ALMA configuration design

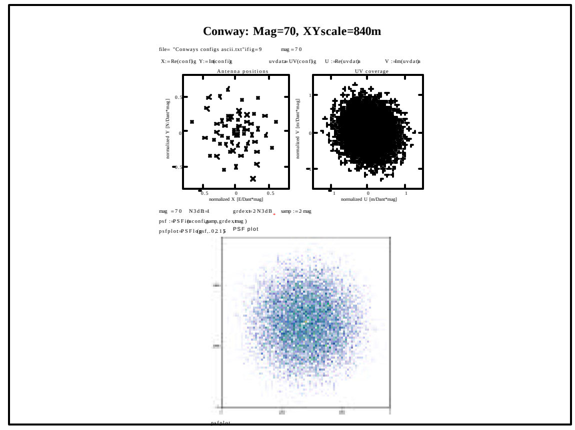
ALMA configurations

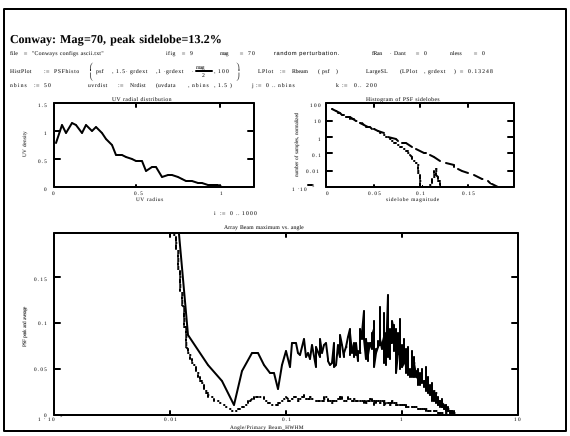
- Geography and boundaries are significant constraints
- Design approach
  - Large scale Gaussian UV distribution
  - Logarithmic spiral for zooming
  - Finally sidelobe optimization
- Multiple configurations
  - Start in compact, close packed
  - Transition to self similar logarithmic spiral
  - Largest configuration circle or star
- Final design in progress

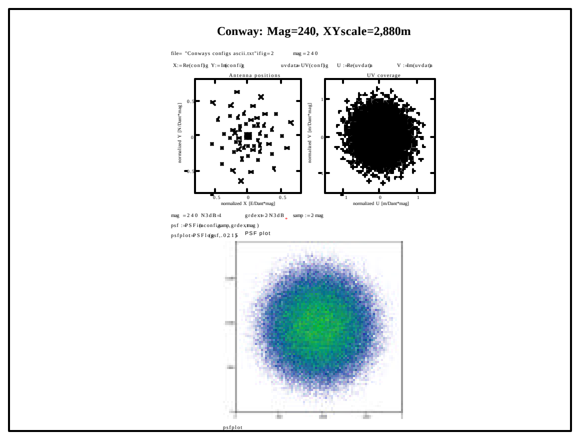
Conway: Mag=14, XYscale=168m

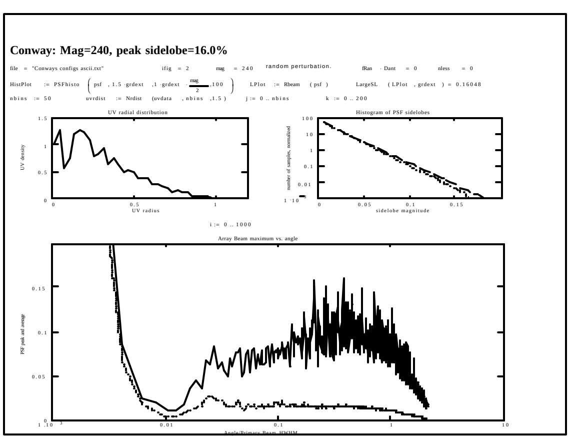



Conway: Mag=14, peak sidelobe=5.8%
UV radial distribution
UV density
UV radius
Histogram of PSF sidelobes
number of samples, normalized
sidelobe magnitude
Array Beam maximum vs. angle
PSF peak and average
Angle/Primary Beam_HWHM


Conway: Mag=70, XYscale=840m
Antenna positions
UV coverage
PSF plot
psfplot


Conway: Mag=70, peak sidelobe=13.2%
UV radial distribution
UV density
UV radius
Histogram of PSF sidelobes
number of samples, normalized
sidelobe magnitude
Array Beam maximum vs. angle
PSF peak and average
Angle/Primary Beam_HWHM


Conway: Mag=240, XYscale=2,880m
Antenna positions
UV coverage
PSF plot
psfplot


Conway: Mag=240, peak sidelobe=16.0%
UV radial distribution
UV density
UV radius
Histogram of PSF sidelobes
number of samples, normalized
sidelobe magnitude
Array Beam maximum vs. angle
PSF peak and average
Angle/Primary Beam_HWHM