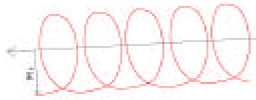


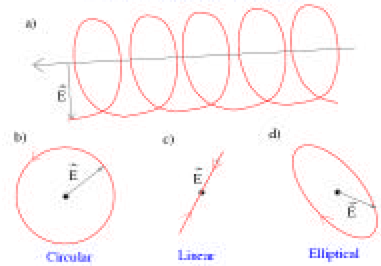
# Polarization in Interferometry

Greg Taylor

- What is polarized light?
- What fun science can be done with polarimetry?
- How do interferometers measure polarization?
- How do you calibrate and image in full polarization?



## Polarization of Light






## What is Polarized Light?

- Light is oscillating electric and magnetic fields
- Polarization is labeled by the shape of the trace of the tip of the **E** vector
- Each polarization has an orthogonal state
- Incoherent light can contain many polarization states

Stokes Parameters describe partially polarized light

$I = RR + LL$
$Q = RL + LR$ For circular feeds
$U = i(LR - RL)$
$V = LL - RR$

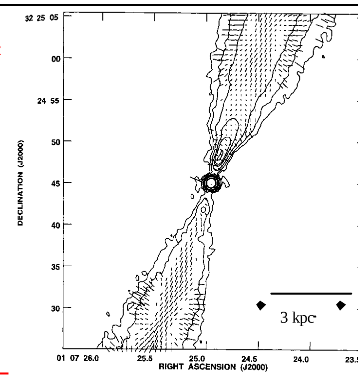
Alternate representation:

- pol. angle (EVPA) $\phi = 0.5 \operatorname{atan}(U/Q)$
- polarized intensity $p = \operatorname{sqrt}(Q^2 + U^2)$
- fractional linear $m = p / I$
- fractional circular $v = |V| / I$



3C 31
VLA @ 8.4 GHz

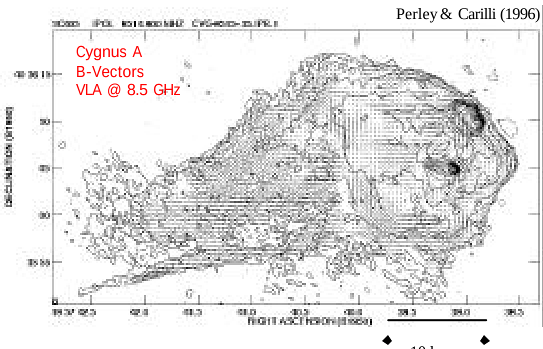
E-Vectors
Laing (1996)



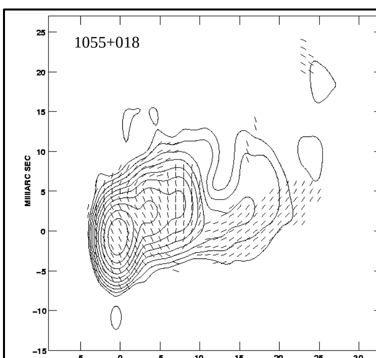



Perley & Carilli (1996)

Cygnus A
B-Vectors
VLA @ 8.5 GHz



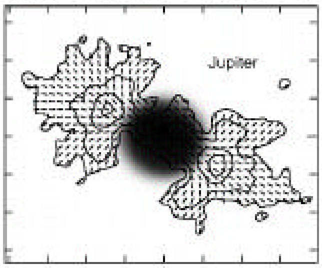



Attridge et al 1999

VLBA @ 5 GHz

>SNTHS IMAGN SUMMR SCHUL



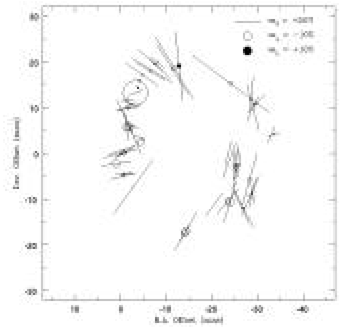

June 20-27, 2000
Socorro, NM, USA



R Aquirrii

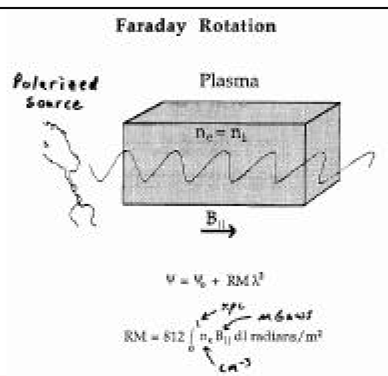
Stellar SiO Masers
Boboltz et al 1998

VLBA @ 43 GHz



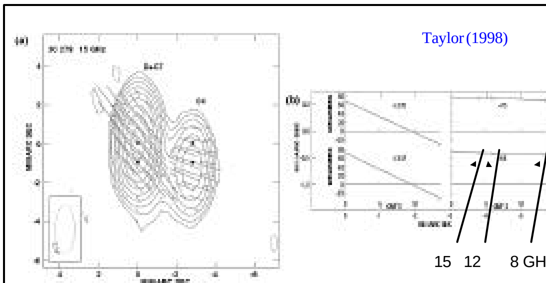
(b) Epoch 2 (29 Dec. 1995, $\phi = 0.78$)



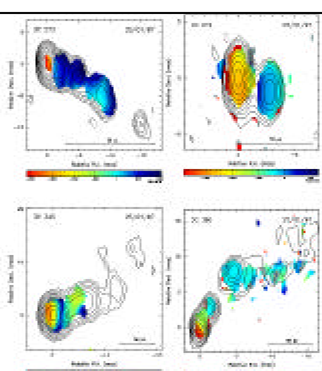




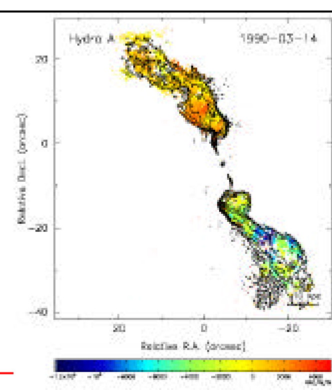

Taylor (1998)

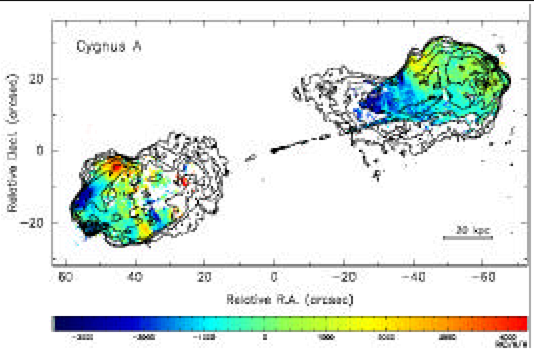











See Review of "Cluster Magnetic Fields" by Carilli & Taylor 2002 (ARA&A)



## Antenna Response

- Jones' matrix:

$$J_i = G_i D_i P_i$$

- $G_i$, is the "gain" given by

$$G_i = \begin{pmatrix} g_{ip} & 0 \\ 0 & g_{iq} \end{pmatrix},$$

where $g_{ip}$ and $g_{iq}$ are complex gain factors for the two orthogonally polarized signals.

- $D_i$, models imperfections in the feed polarization response.

$$D_i = \begin{pmatrix} 1 & d_{ip} \\ -d_{iq} & 1 \end{pmatrix},$$

where $d_{ip}$ and $d_{iq}$ are complex "leakage" terms.



## Antenna Response continued

- $P_i$, includes effects of parallactic angle, $\chi$

Antennas with equatorial mounts have $\chi = 0$.

For alt-az mounted antennas, Parallactic angle has an effect on the measured signals which depends on the feed polarization type:

$$P_i^+ = \begin{pmatrix} \cos(\chi) & -\sin(\chi) \\ \sin(\chi) & \cos(\chi) \end{pmatrix}$$ for linear or

$$P_i^\odot = \begin{pmatrix} e^{-j\chi} & 0 \\ 0 & e^{j\chi} \end{pmatrix}$$ for circular feeds

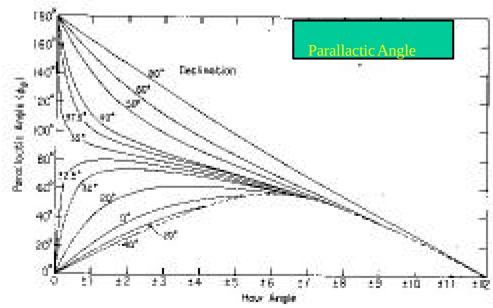
where $j = \sqrt{-1}$.

The parallactic angle is given by

$$\chi = \tan^{-1}\left(\frac{\cos(\lambda)\sin(h)}{\sin(\lambda)\cos(\delta) - \cos(\lambda)\sin(\delta)\cos(h)}\right)$$

where $\delta$ is the source declination, $\lambda$ is the latitude of the antenna, and $h$ is the source hour angle.








## Interferometer Response

- Cross correlate all 4 feed combinations to get $v = (pp, qp, pq, qq)$.
- Interferometer $(i, k)$ response is given by "Mueller" matrix from the outer product of the Jones matrices:

$$v = (J_i \otimes J_k^*)\, S\, s$$

$s$ = true Stokes visibility vector $(I, Q, U, V)$ and $^*$ indicates the complex conjugate.

$S$ = coordinate transformation matrix from the Stokes system to that of the correlations:

$$S^+ = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & j \\ 0 & 0 & 1 & -j \\ 1 & -1 & 0 & 0 \end{pmatrix}$$ for linear or

$$S^\odot = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & j & 0 \\ 0 & 1 & -j & 0 \\ 1 & 0 & 0 & -1 \end{pmatrix}$$ for circular.



## Linearized Interferometer Response

- For crossed linearly polarized feeds

$$v_{pp} = \tfrac{1}{2} g_{ip} g_{kp}^* (I + Q\cos 2\chi + U\sin 2\chi),$$
$$v_{pq} = \tfrac{1}{2} g_{ip} g_{kq}^* ((d_{ip} - d_{kq}^*) I - Q\sin 2\chi + U\cos 2\chi + jV),$$
$$v_{qp} = \tfrac{1}{2} g_{iq} g_{kp}^* ((d_{kp}^* - d_{iq}) I - Q\sin 2\chi + U\cos 2\chi - jV),$$
$$v_{qq} = \tfrac{1}{2} g_{iq} g_{kq}^* (I - Q\cos 2\chi - U\sin 2\chi),$$

- for circularly polarized feeds:

$$v_{pp} = \tfrac{1}{2} g_{ip} g_{kp}^* (I + V),$$
$$v_{pq} = \tfrac{1}{2} g_{ip} g_{kq}^* ((d_{ip} - d_{kq}^*) I + e^{-2j\chi}(Q + jU)),$$
$$v_{qp} = \tfrac{1}{2} g_{iq} g_{kp}^* ((d_{kp}^* - d_{iq}) I + e^{2j\chi}(Q - jU)),$$
$$v_{qq} = \tfrac{1}{2} g_{iq} g_{kq}^* (I - V).$$

- Linearized equations may limit dynamic range

## Calibration

- Corrupting effects:
  - Atmosphere
  - Instrumental gain variations
  - Instrumental imperfections
- Details depend on feed polarization type
- Astronomical and other measurements needed to calibrate
- Astronomical calibration sources
  - Preferably unresolved
  - Synchrotron emission usually has:
    1) weak circular polarization (<0.1%)
    2) significant linear polarization (1–10%)
  - Physically small means time variable



## Calibration of Circular Feeds

- Parallel correlations sensitive to Stokes $I$ & $V$

$$v_{pp} = \frac{1}{2} g_p g_p^* (I + V),$$
$$v_{qq} = \frac{1}{2} g_q g_q^* (I - V),$$

- Assume $V = 0$ for calibrator
- Can seperate and solve for gains for $p$ and $q$
- Instrumental ($d$) and source polarization ($Q, U$) sum of two vectors:

$$v_{pq} = \frac{1}{2} g_p g_q^* [(d_p - d_q^*) I + e^{-2i\psi}(Q + jU)],$$
$$v_{qp} = \frac{1}{2} g_q g_p^* [(d_q^* - d_p) I + e^{2i\psi}(Q - jU)]$$

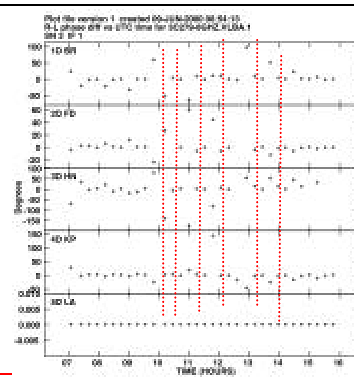
- Calibrator observations of a range of PA gives clean separation
- Independent gain calibration for $p$ and $q$ allows arbitrary phase offset – refer all phases to same "reference" antenna
- $p - q$ phase difference is that of the reference antenna
- Need observations of calibrator of known polarization angle aka Electric Vector Position Angle (EVPA)



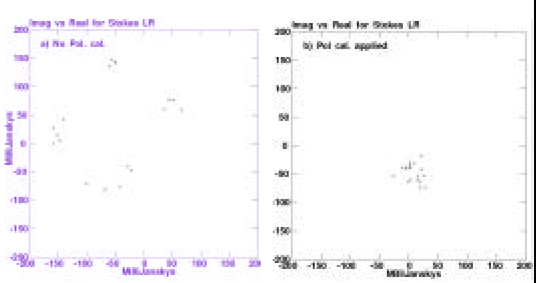
## Calibration procedure (Circular Feeds)

- From frequent observations of a calibrator:
  1) determine $p, q$ gains ($g$) assuming $V = 0$
  2) determine source ($Q, U$) and instrumental ($d$) polarization
- From observations of a calibrator with known polarization angle:
  3) determine $p - q$ phase difference of reference antenna
- Things that can go wrong:
  1) time variable $p - q$ phase difference
  2) time variable $d$ terms










## Calibration of Linear Feeds

- Parallel correlations sensitive to $I, Q$, & $U$

$$v_{pp} = \frac{1}{2} g_p g_p^* (I + Q \cos 2\chi + U \sin 2\chi),$$
$$v_{qq} = \frac{1}{2} g_q g_q^* (I - Q \cos 2\chi - U \sin 2\chi),$$

- Calibrator $Q$ and $U$ usually cannot be ignored (few %)
- Phase unaffected by polarization of a point source at the phase center
- Cannot seperate $p, q$ gains and calibrator polarization
- $p - q$ phase offset not known
- May be unknown orientation error of $p$ and $q$
- Need obs of source with known polarization

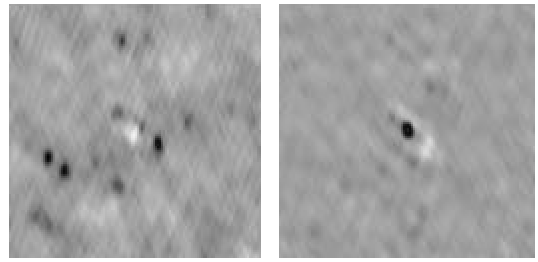
$$v_{pq} = \frac{1}{2} g_p g_q^* [(d_p - d_q^*) I - Q \sin 2\chi + U \cos 2\chi + jV],$$
$$v_{qp} = \frac{1}{2} g_q g_p^* [(d_q^* - d_p) I - Q \sin 2\chi + U \cos 2\chi - jV],$$

- Calibrator $Q$ and $U$ affect real part of cross pol. correlations
- Calibrator $V$ affects imaginary part of cross pol. correlations but unaffected by PA

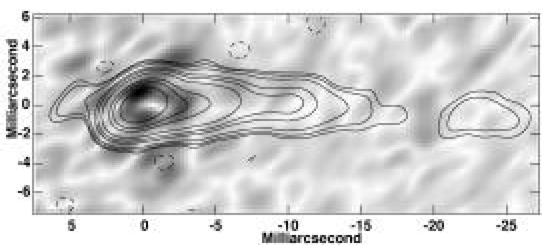
## Calibration procedure (Linear Feeds)

- From frequent observations of a calibrator solve jointly for:
  1) Gains ($g$)
  2) Calibrator polarization ($Q, U$)
  3) Instrumental polarization ($d$)
- Iterative scheme may work (e.g., ATCA)
- From obs. of a calibrator of known EVPA
  4) Determine feed orientations
  5) Determine $p - q$ phase difference
- Things that can go wrong:
  1) time variable $p - q$ phase difference
  2) time variable $d$ terms

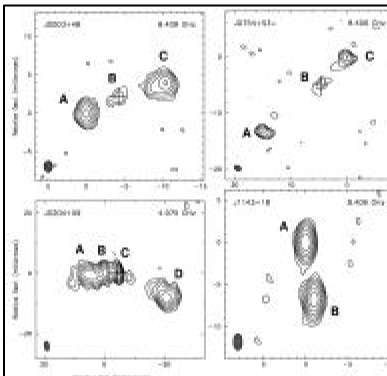








Compact Symmetric Objects (CSOs)

VLBA @ 8.4 GHz

Peck & Taylor (2001)

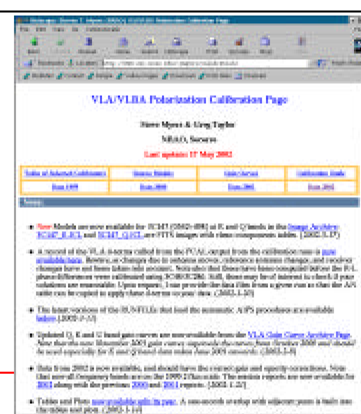


## VLBA 22 GHz Observations of 3C120

José-Luis Gómez IAA (Spain)
Alan P. Marscher BU (USA)
Antonio Alberdi IAA (Spain)
Svetlana Marchenko-Jorstad BU (USA)
Cristina García-Miró IAA (Spain)

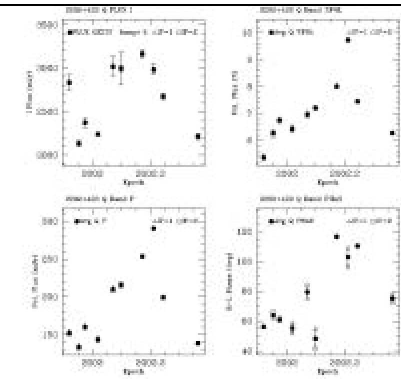


Practical VLBI polarization angle calibration



1) Find a Calibrator

2) Estimate
$\phi$_VLA

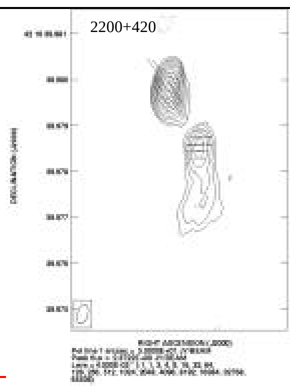




3) Sum Q and U over VLBA image.

4) Compute
$\phi\_VLBA = 0.5 \, \mathrm{atan}\,(U/Q)$

5) Rotate VLBA data by
$(\phi\_VLA - \phi\_VLBA)$

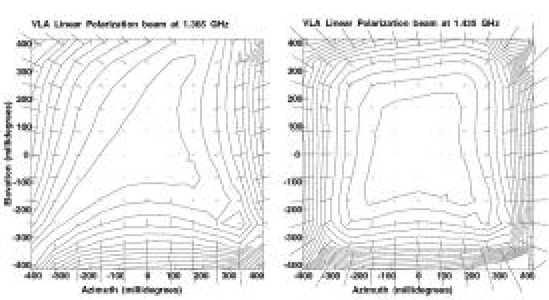




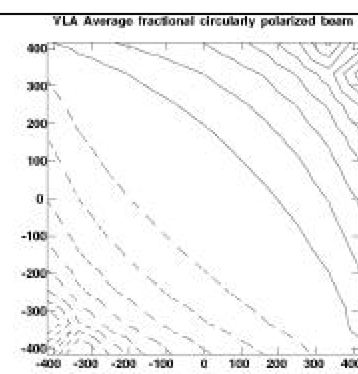

## Wide Field Polarimetry

- Instrumental poln. varies across antenna beam
- Instrumental poln. pattern rotates with PA
- "Snapshot" images can be corrected from / image and antenna poln. pattern
- General correction involves imaging/deconvolutio
- Long synthesis provides some reduction
- Pattern may be a strong function of frequency
- Off axis feeds (e.g., VLA) cause "Beam squint"
- Circularly polarized beams aren't concentric, so beware of strong off-axis instrumental circular poln.







## Ionospheric Faraday Rotation

- Causes rotation of the polarization angle, $\Delta\Phi$

$$\Delta\Phi = \frac{0.93 \times 10^{6} \int N\, B_{\|}\, ds}{(2\pi\nu)^2} \text{ radians} \qquad (1)$$

$N$ = electron density ($cm^{-3}$),
$B_{\|}$ = magnetic field $\|$ to propagation (Gauss),
$\nu$ = radio frequency (Hz),
$s$ = distance along the line of sight (cm).

- Effect decreases rapidly with frequency
- Earth's ionosphere may be a problem at $\nu < 2$ GHz
- Effect depends on time of day and solar activity
- Effect depends on line of sight through magnetic field



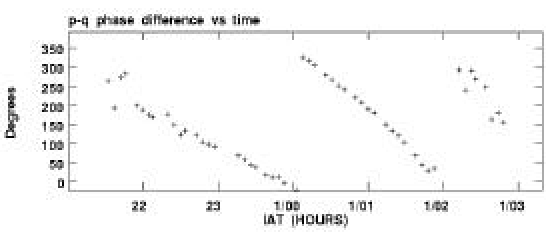
## External Calibration of IFR

1) Measure zenith total electron content
2) Assume ionospheric structure
3) Assume magnetic field structure (offset dipole)
4) Calculate time dependent RM correction for line of sight to calibrators and sources

## Self Calibration of IFR

1) Make snapshot reference polarization image
2) Divide data into time segments and determine RM or $p-q$ phase difference
3) Apply corrections to the data








## Imaging

- Similar to total intensity ($I$) imaging
- Apply calibration
- Convert to Stokes' representation
- Image/deconvolve $I, Q, U$ and $V$ independently
- $I$ positive but $Q, U$, and $V$ may not be
- Special case:
  - If some but not all RL, LR correlations present, can't make independent $Q, U$ image
  - Make $Q + jU$ image & use complex deconvolution
- Spatial frequency filtering effects
  - May miss structure on scales resolved on all baselines
  - Filtering effects can be different for $I, Q, U$ and $V$ (e.g., poln. angle variations either intrinsic or from RMs)
  - May lead to > 100% polarization