Cross Correlators

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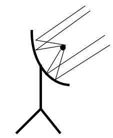
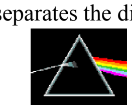
NRAO/Socorro

What is a Correlator?

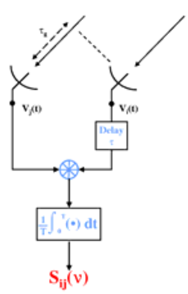
- In an optical telescope…
  - a lens or a mirror collects the light & brings it to a focus



  - a spectrograph separates the different frequencies



- In an interferometer, the correlator performs both these tasks, by correlating the signals from each telescope (antenna) pair:



The basic observables are the complex visibilities:

amplitude & phase

as functions of

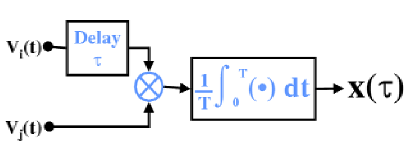
baseline, time, and frequency.

The correlator takes in the signals from the individual telescopes, and writes out these visibilities.

Correlator Basics

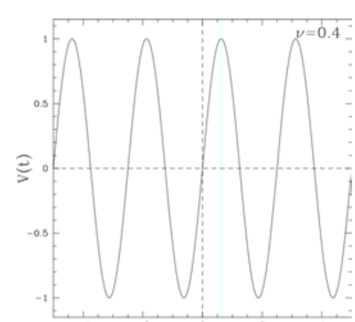
The cross-correlation of two real signals $v_i(t)$ and $v_j(t)$ is

$$x_{ij}(\tau) \equiv \langle v_i(t)\, v_j(t+\tau) \rangle$$

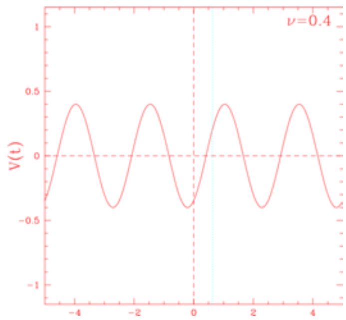


A simple (real) correlator.

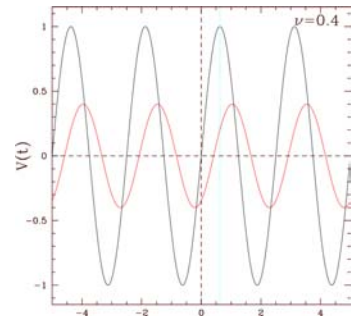
Antenna 1:



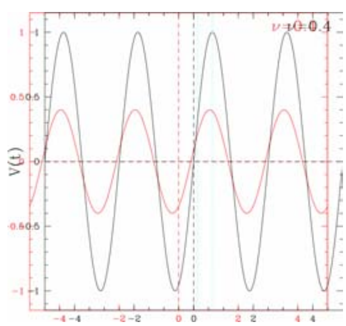
Antenna 2:



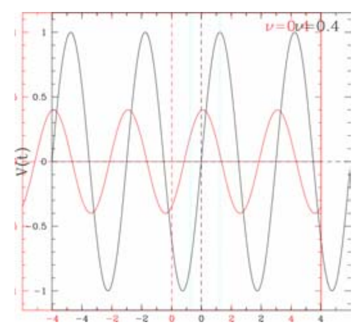
τ=0:



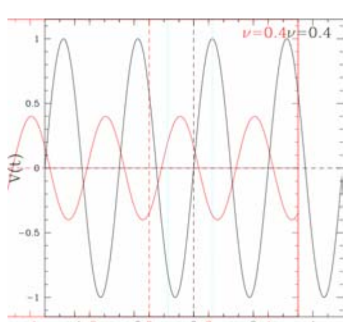
τ=0.5:



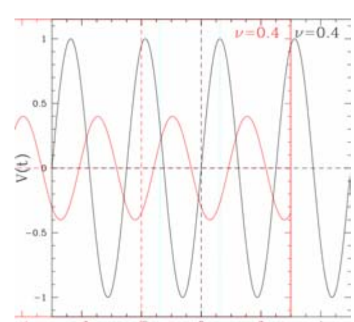
τ=1:



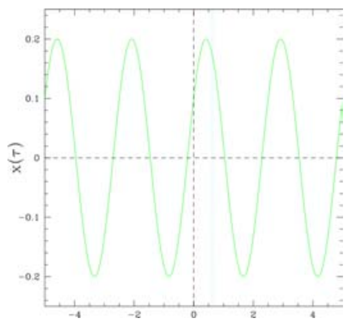
τ=1.5:



τ=2:



→Correlation:



Correlation of a Single Frequency

For a monochromatic signal:

$$v_i(t) = \sin 2\pi\nu_0 t$$
$$v_j(t) = \sin(2\pi\nu_0 t + \phi)$$

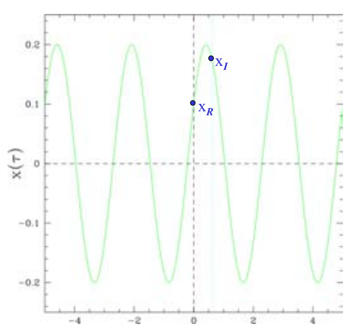
and the correlation function is

$$x_{ij}(\tau) = \langle \sin 2\pi\nu_0 t)\, \sin(2\pi\nu_0(t+\tau)+\phi)\rangle$$
$$= x_R \cos 2\pi\nu_0(\tau-\tau_0) + x_I \sin 2\pi\nu_0(\tau-\tau_0)$$

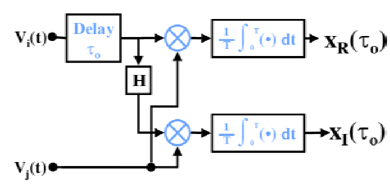
So we need only measure $R_{ij} = x_R + ix_I$, with

- $x_R = x_{ij}(\tau_0)$
- $x_I = x_{ij}(\tau_0 + \Delta\tau)$, with $\Delta\tau = 1/(4\nu_0)$ $(\Delta\phi = 90°)$.

→Correlation:



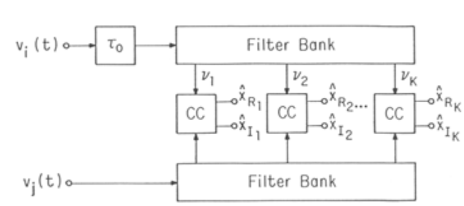
At a given frequency, all we can know about the signal is contained in two numbers: the real and the imaginary part, or the amplitude and the phase.



A complex correlator.

Broad-band Continuum Correlators

1. The simple approach:
   - use a filterbank to split the signal up into quasi-monochromatic signals ν_k
   - hook each of these up to a different complex correlator, with the appropriate (different) delay: $\Delta\tau_k = 1/(4\nu_k)$
   - add up all the outputs
2. The clever approach:
   instead of sticking in a delay, put in a filter that shifts the phase for *all* frequencies by π/2



$$\hat{R}_{ij}(\tau_0) = \sum_{k=1}^{K} \hat{x}_{R_k} + i\hat{x}_{I_k}$$

Figure 4–4. A wide-band complex correlator synthesized from narrow-band complex correlators, or a spectroscopic correlator. Each box labeled "CC" is as indicated in Figure 4–3.

Spectral Line Correlators

1. The simple approach:
   - use a filterbank to split the signal up into quasi-monochromatic signals ν_k
   - hook each of these up to a different complex correlator, with the appropriate (different) delay: $\Delta\tau_k = 1/(4\nu_k)$
   - record all the outputs: $R_{ij}(\nu, t)$

Fourier Transforms: a motivational exercise

The frequency spectrum is the Fourier transform of the cross-correlation (lag) function.

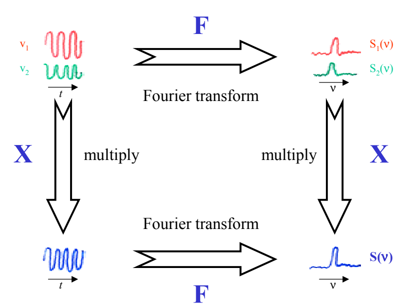
- Short lags (small delays) ⟺ high frequencies
- Long lags (large delays) ⟺ low frequencies

…so measuring a range of lags corresponds to measuring a range of frequencies!

Spectral Line Correlators (cont'd)

2. Clever approach #1: the FX correlator
   - F: replace the filterbank with a Fourier transform
   - X: use the simple (complex) correlator above to measure the cross-correlation at each frequency
   - average over time, & record the results
3. Clever approach #2: the XF correlator
   - X: measure the correlation function at a bunch of different lags (delays)
   - average over time
   - F: Fourier transform the resulting time (lag) series to obtain spectra
   - record the results

FX vs. XF



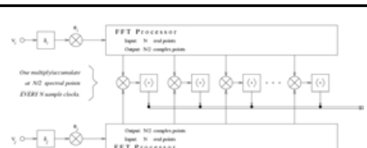


Fig. 4-6: FX correlator baseline processing.

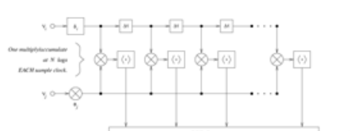


Fig. 4-1: Lag (XF) correlator baseline processing.

Details, Details

Why digital?

- precise & repeatable
- lots of duplication
- accurate & stable delay lines

…but there are some complications as well…

Digitization

1. Sampling: v(t) ⇒ v(t_k), with t_k=(0,1,2,…)Δt

– For signal v(t) limited to 0≤ν≤Δν, this is lossless if done at the Nyquist rate:

Δt≤1/(2Δν)

– *n.b.* wider bandwidth ⇒ finer time samples!

– limits accuracy of delays/lags

2. Quantization: v(t) ⇒ v(t) + δt

– quantization noise

– quantized signal is *not* band-limited ⇒ oversampling helps

Quantization & Quantization Losses

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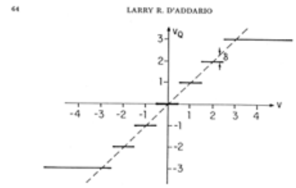


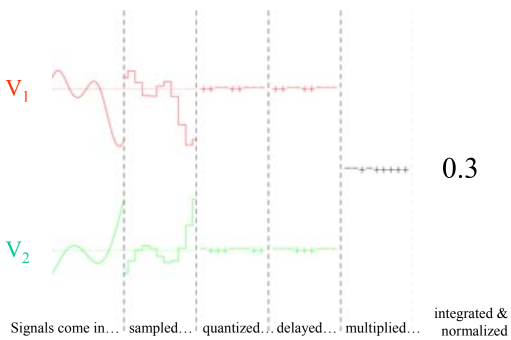
Figure 4-6. An example of a quantizer transfer function (solid lines); this quantizer has seven levels. The dashed line is the line defined by $v_q = v$, and the difference between it and the transfer function is the quantization noise, δ.

Table 4-1.
Signal-to-Noise Ratio vs. Quantization and Sampling Rate

Quantization	Sampling Rate	S/N (digital) / S/N (continuous)
2-level (1 bit)	2Δν	.64
	4Δν	.74
3-level	2Δν	.81*
	4Δν	.89
4-level	2Δν	.88
	4Δν	.94
∞-level (continuous)	2Δν	1.00
	4Δν	1.00

*VLA Case.
All cases assume rectangular bandpasses of width Δν, signal levels adjusted to maximize the signal-to-noise ratio, and small correlation coefficients.

Michael's Miniature Correlator



Cross-Correlating a Digital Signal

We measure the cross-correlation of the digitized (rather than the original) signals.

- digitized CC is monotonic function of original CC
- 1-bit (2-level) quantization:

$$x_{ij}(\tau) = \sigma_i \sigma_j \sin \frac{\pi \rho_{ij}(\tau)}{2}$$

– σ_i is average signal power level – *NOT* kept for 2-level quantization!

–roughly linear for correlation coefficient $x_{ij}(\tau) \ll 1$

- For high correlation coefficients, requires non-linear correction: the Van Vleck correction

Van Vleck Correction

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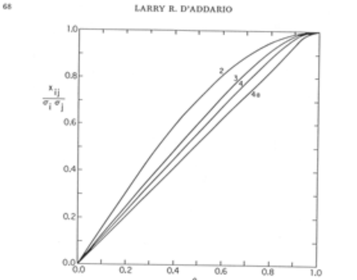


Figure 4-7. Quantization correction functions for various quantizations. In each case the signal powers are set for maximum signal-to-noise ratio. The curves are labeled according to the number of quantization levels; 4a uses a simplified multiplier (see Cooper, 1970).

Spectral Response; Gibbs Ringing

- **XF correlator:** limited number of lags N

⇒ 'uniform' coverage to max. lag $N\Delta t$

⇒ Fourier transform gives spectral response

$$\frac{\sin{(N \Delta \tau)\, \nu}}{(N \Delta \tau)\, \nu}$$

- 22% sidelobes!

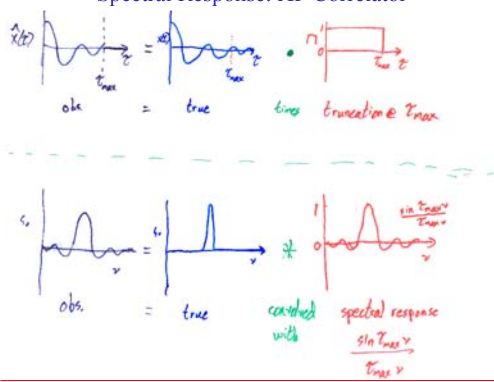
- Hanning smoothing

- **FX correlator:** as XF, but Fourier transform before multiplication ⇒ spectral response is

$$\left(\frac{\sin{(N \Delta \tau)\, \nu}}{(N \Delta \tau)\, \nu}\right)^2$$

- 5% sidelobes

Spectral Response: XF Correlator



sinc( ) *vs.* sinc²( )

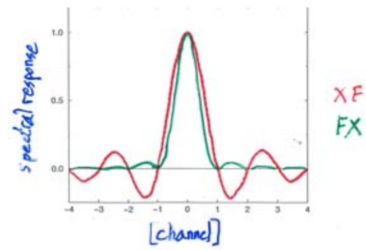


Figure 4-8. The sinc²(·) function, with sinc(·) [dashed] for comparison.

- *n.b.* radio frequency interference is spread across frequency by the spectral response
- **Gibbs phenomenon:** 'ringing' off the band edges

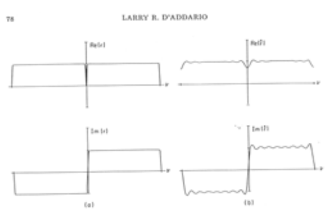


Figure 4-11. (a) The cross power spectrum resulting from a continuum source of unit flux in the reference direction: "true complex gain." Note the nonzero phase. (b) The computed cross power spectrum with 16 delays.

How to Obtain Finer Frequency Resolution

The size of a correlator (number of chips, speed, etc.) is generally set by the number of baselines $(\propto N_{ant}^2)$ and the maximum total bandwidth. [note also copper/connectivity costs…]

- **Subarrays**
  … trade antennas for channels
- **Bandwidth**
  -- cut $\Delta\nu$:
  $\Rightarrow$ same number of lags/spectral points across a smaller $\Delta\nu$: N_{chan} = *constant*
  $\Rightarrow$ narrower channels: $\nu \propto \Delta\nu$
  …limited by filters

-- recirculation:

- chips are generally running flat-out for max. $\Delta\nu$ (e.g. EVLA/WIDAR uses a 256 MHz clock with $\Delta\nu$ = 128 MHz/sub-band)
- For smaller $\Delta\nu$, chips are sitting idle most of the time: e.g., pass 32 MHz to a chip capable of doing 128 M multiplies per second

$\Rightarrow$ add some memory, and send two copies of the data with different delays

$\Rightarrow N_{chan} \propto 1/\Delta\nu$

$\Rightarrow \delta\nu \propto (\Delta\nu)^2$

…limited by memory & data output rates

VLA Correlator: Bandwidths and Numbers of Channels

Table 14: **Available bandwidths and number of spectral line channels in normal mode**

BW Code	Bandwidth MHz	Single IF Mode[1] No. Channels[4]	Single IF Mode[1] Freq. Separ. kHz	Two IF Mode[2] No. Channels[4] per IF	Two IF Mode[2] Freq. Separ. kHz	Four IF Mode[3] No. Channels[4] per IF	Four IF Mode[3] Freq. Separ. kHz
0	50	16	3125	8	6250	4	12500
1	25	32	781.25	16	1562.5	8	3125
2	12.5	64	195.313	32	390.625	16	781.25
3	6.25	128	48.828	64	97.656	32	195.313
4	3.125	256	12.207	128	24.414	64	48.828
5	1.5625	512	3.052	256	6.104	128	12.207
6	0.78125	512	1.526	256	3.052	128	6.104
8	0.1953125	256	0.763	128	1.526	64	3.052
9	0.1953125	512	0.381	256	0.763	128	1.526

Notes:
(1) Observing Modes 1A, 1B, 1C, 1D.
(2) Observing Modes 2AB, 2AC, 2AD, 2BC, 2BD, 2CD.
(3) Observing Modes 4, PA, PB. It is possible to use the output from one, two or four IFs in such a way as to obtain different combinations of number of spectral line channels and channel separation. The minimum and maximum number of channels is 4 and 512 respectively.
(4) These are the numbers of spectral line channels produced in the array processor. Any number of spectral line channels that is a power of 2, that is less than or equal to the number in the table and that is greater than or equal to 2 may be selected using the data selection options available within the OBSERVE and JObserve programs.

VLBI

- difficult to send the data to a central location in real time
- long baselines, unsynchronized clocks ⇒ relative phases and delays are poorly known
- So, record the data and correlate later
- Advantages of 2-level recording

Correlator Efficiency η_c

- quantization noise
- overhead
  - don't correlate all possible lags
  - blanking
- errors
  - incorrect quantization levels
  - incorrect delays

Choice of Architecture

- number of multiplies: FX wins as $\{N_{ant}, N_{chan}\}\Uparrow$
  multiplies per second ~ $N_{ant}^2 \, \Delta\nu \, N_{prod} \, N_{chan}$
- number of logic gates: XF multiplies are much easier than FX; which wins, depends on current technology
- shuffling the data about: "copper" favors XF over FX for big correlators
- bright ideas help: hybrid correlators, nifty correlator chips, etc.

New Mexico Correlators

	VLA	EVLA (WIDAR)	VLBA
Architecture	XF	filter-XF	FX
Quantization	3-level	16/256-level	2- or 4-level
N_{ant}	27	40	20
Max. Δν	0.2 GHz	16 GHz	0.256 GHz
N_{chan}	1-512	16,384-262,144	256-2048
Min. δν	381 Hz	0.12 Hz	61.0 Hz
dt_{min}	1.7 s	0.01 s	0.13 s
Power req't.	50 kW	135 kW	10-15 kW
Data rate	3.3×10^3 vis/sec	2.6×10^7 vis/sec	3.3×10^6 vis/sec

Current VLA

Bandwidth MHz	Single Pol. Prod. No. Channels	Single Pol. Prod. Freq. Separ. kHz	Two Pol.Prod. No. Channels per pol	Two Pol.Prod. Freq. Separ. kHz	Four Pol.Prod. No. Channels per pol	Four Pol.Prod. Freq. Separ. kHz
100	16	6250	8	12500	2	50000
50	16	3125	8	6250	4	12500
25	32	781.25	16	1562.5	8	3125
12.5	64	195.313	32	390.625	16	781.25
6.25	128	48.828	64	97.656	32	195.313
3.125	256	12.207	128	24.414	64	48.828
1.5625	512	3.052	256	6.104	128	12.207
0.78125	512	1.526	256	3.052	128	6.104
0.19531	512	0.381	256	0.763	128	1.526

EVLA/WIDAR

Bandwidth MHz	Single Pol. Prod. No. Channels	Single Pol. Prod. Freq. Separ. kHz	Two Pol.Prod. No. Channels per pol	Two Pol.Prod. Freq. Separ. kHz	Four Pol.Prod. No. Channels per pol	Four Pol.Prod. Freq. Separ. kHz
8192	16,384	500	8,192	1000	4,096	2000
4096	16,384	250	8,192	500	4,096	1000
2048	32,768	62.5	16,384	31.25	8,192	250
1024	65,536	15.625	32,768	31.25	16,384	62.5
512	131,072	3.906	65,536	7.813	32,768	15.625
256	262,144	0.977	131,072	1.953	65,536	3.906
128	262,144	0.488	131,072	0.977	65,536	1.953
64	262,144	0.244	131,072	0.488	65,536	0.977
32	262,144	0.122	131,072	0.244	65,536	0.488
16	262,144	0.061	131,072	0.122	65,536	0.244
8	262,144	0.031	131,072	0.061	65,536	0.122
4	262,144	0.015	131,072	0.031	65,536	0.061
2	262,144	0.008	131,072	0.015	65,536	0.031
1	262,144	3.8 Hz	131,072	7.6 Hz	65,536	0.015
0.5	262,144	1.9 Hz	131,072	3.8 Hz	65,536	7.6 Hz
0.25	262,144	0.95 Hz	131,072	1.9 Hz	65,536	3.8 Hz
0.125	262,144	0.48 Hz	131,072	0.95 Hz	65,536	1.9 Hz
0.0625	262,144	0.24 Hz	131,072	0.48 Hz	65,536	0.95 Hz
0.03125	262,144	0.12 Hz	131,072	0.24 Hz	65,536	0.48 Hz