# Noise and Interferometry

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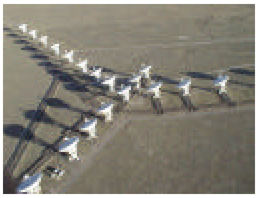


VLA 5e9 Hz

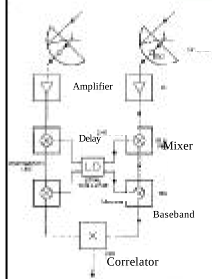
VLTI 5e14 Hz

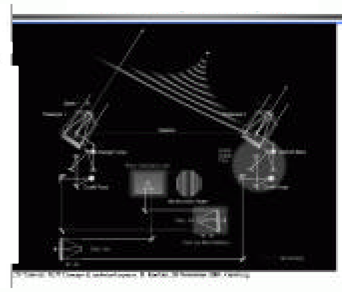


## Radio vs. Optical Interferometry

Radio: heterodyne + correlator

Optical: mirrors + direct detector (CCD)





## The Delay Lines

- The 'Paranal-Express'
  - Stroke: 60 m (120m in OPL)
  - Resolution: <5nm
  - Max. velocity: 0.5 m/s
  - Stability (jitter): <14nm rms
  - Power dissipation: <15W

Stability = 1e8



## References

'Noise and interferometry,' V. Radhakrishnan 1999, SIRA II

'Bolometers for infrared and millimeter waves,' P. Richards, J. 1994, Appl. Phys., 76, 1

'Letter on Brown and Twiss effect,' E. Purcell 1956, Nature, 178, 1449

'Thermal noise and correlations in photon detection,' Zmuidzinas 2000, Caltech

'The intensity interferometer,' Brown and Twiss 1974, (Taylor-Francis)

'Multiple telescope infrared interferometry,' Townes and Sutton 1981, (ESO conference, eds. Ulich and van de Stadt)

'Coherent vs. incoherent detection for interferometry,' de Graauw 1981, (ESO conference, eds Ulich and va de Stadt)

'Measurement of power spectra,' Blackman and Tuckey (Dover)

'Celestial masers,' Cook (Cambridge Univ. Press) chap 4

'Fundamentals of statistical physics,' Reif (McGraw-Hill) chap 9



**Photon statistics:** Bose-Einstein statistics for gas without number conservation (indistiguishable particles or wave function symmetric under particle exchange, spin 0; Reif Chap 9)

Thermal equilibrium => **Planck distribution function**

n_s = relative number of particles in state s =

number of photons in standing mode in box at temperature T =

number of photons/s/Hz/ster/pol in beam in free space (Richards 1994)

$$\langle n_s \rangle = \left(e^{h\nu_s/kT} - 1\right)^{-1}$$

**Photon noise**: fluctuations in # of photons arriving each second in free space beam

$$\langle \Delta n_s^2 \rangle \equiv \langle (n_s - \langle n_s \rangle)^2 \rangle = \langle n_s \rangle + \langle n_s \rangle^2$$

$\langle n_s \rangle$ = Poisson Stats = shot noise = counting stats = Maxwell – Boltzmann stats

$\langle n_s^2 \rangle$ = Wave noise = noise in a 'classical signal'



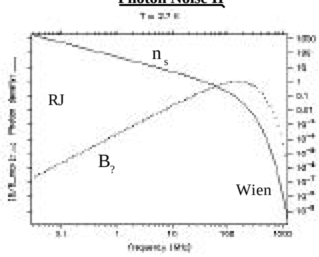
**Electron statistics:** Fermi-Dirac (indistiguishable particles, but number of particles in each state = 0 or 1, or antisymmetric wave function under particle exchange, spin ½)

$$\langle n_s \rangle = \left(e^{h\nu_s/kT} + 1\right)^{-1}$$

$$\langle \Delta n_s^2 \rangle \equiv \langle (n_s - \langle n_s \rangle)^2 \rangle = \langle n_s \rangle - \langle n_s \rangle^2$$

eg. maximum $\langle n_s \rangle = 1 \Rightarrow$ all states are filled $\therefore$ variance = 0

## Photon Noise II




$$\langle \Delta n_s^2 \rangle \equiv \langle (n_s - \langle n_s \rangle)^2 \rangle = \langle n_s \rangle + \langle n_s \rangle^2$$

Wien (optical): $n_s < 1 \Rightarrow rms \propto \sqrt{n_s}$ (counting stats) = 'quantum regime'

RJ (radio): $n_s > 1 \Rightarrow rms \propto n_s$ (wave noise) = 'classical regime'



## Photon noise III: Black bodies, spatial modes, and temperatures (Richards 94)

Number of free space modes per polarization in solid angle $\Omega$, area A:

$$N = A\Omega/\lambda^2$$

e.g. for a diffraction limited beam N = 1 ('single mode reciever')

Power per mode: $P_\nu = h\nu\,(e^{h\nu/kT} - 1)^{-1}$ W

BB Brightness per unit area, unit solid angle:

$$B_\nu = N \times P_\nu = \frac{2h\nu^3}{c^2}(e^{h\nu/kT} - 1)^{-1} \ \mathrm{W\ m^{-2}\ ster^{-1}}$$

Resolved source: $T_A = T_B^{src}$ (2nd law of Thermodyn.)

Unresolved source: $T_A = T_B^{\prime}(\Omega = \Omega_{tel} > \Omega_{src}) < T_B^{src}$

Rayleigh-Jean limit: $h\nu/kT << 1$

$$kT_A = \frac{\lambda^2}{2}\frac{S_\nu}{\Omega_{tel}} = \frac{1}{2}S_\nu A$$



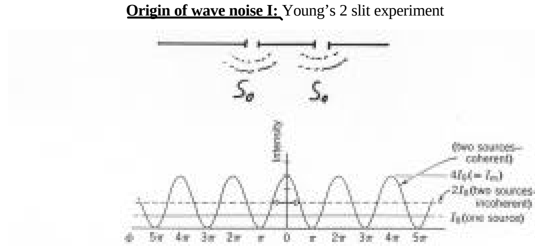
## Disclaimer on Wave noise

**Richards 1994:** 'The first term in equation 11 can be obtained more directly. For Poisson statistics the mean square fluctuation in the number of photons arriving in 1s is just equal to the number of photons arriving. This term has been verified experimentally in many experiments. The second term, by contrast, has not been measured unambiguously.'

**Zmuidzinas 2000:** 'Richards has recently discussed the second term of Eq. 4.1, raising questions about the theoretical and experimental justification for this term. However, as discussed in section III, the second term is needed in order to recover the Dickey radiometer equation for single mode detectors in the high background limit; we therefore disagree that there is no empirical justification for this term.'



## Origin of wave noise I: Young's 2 slit experiment



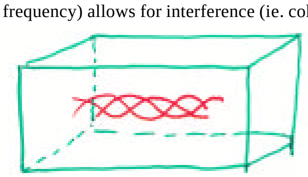

Single Source: $I \propto E^2$ = '1 photon'

Two incoherent sources: $I \propto 2\langle E^2 \rangle$ = '2 photons'

Two coherent sources: $I \propto \langle 2E \rangle^2$ = '0 to 4 photons'



## Origin of wave noise II: 'Bunching of Bosons' in phase space (time and frequency) allows for interference (ie. coherence).



Bosons can, and will, occupy the exact same phase space if allowed, such that interference (destructive or constructive) will occur. Restricting phase space (ie. narrowing the bandwidth and sampling time) leads to interference within the beam. This naturally leads to fluctuations that are proportional to intensity (= wave noise).



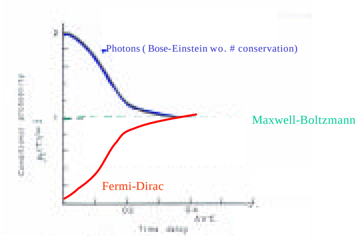
## Origin of wave noise III

"Think then, of a stream of wave packets each about c/Δν long, in a random sequence. There is a certain probability that two such trains accidentally overlap. When this occurs they interfere and one may find four photons, or none, or something in between as a result. It is proper to speak of interference in this situation because the conditions of the experiment are just such as will ensure that these photons are in the same quantum state. **To such interference one may ascribe the 'abnormal' density fluctuations in any assemblage of bosons.**

Were we to carry out a similar experiment with a beam of electrons we should find a suppression of the normal fluctuations instead of an enhancement. The accidental overlapping wave trains are precisely the configurations excluded by the Pauli principle." Purcell 1959

## Origin of wave noise IV

**Photon arrival time:** normalized probability of detecting a second photoelectron after interval t in a plane wave of linearly polarized light with Gaussian spectral profile of width $\Delta\nu$ (Mandel 1963). Exactly the same factor 2 as in Young's slits!




$$\text{Relevant timescale} \approx 1/\Delta n$$

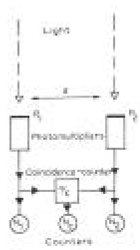


## Origin of wave noise V

'If we were to split a beam of electrons by a nonpolarizing mirror, allowing the beams to fall on separate electron multipliers, the outputs of the latter would show a negative cross-correlation. A split beam of classical particles would, of course, show zero cross correlation. As usual in fluctuation phenomena, the behavior of fermions and bosons deviate in opposite directions from that of classical particles. The Brown-Twiss effect is thus, from a particle point of view, a characteristic quantum effect.'' Purcell 1959



**Intensity Interferometry:** rectifying signal with square-law detector ('photon counter') destroys phase information. Cross correlation of inten sities still results in a finite correlation, proportional to the square of the E-field correlation coefficient as measured by a 'normal' interferometer. Exact same phenomenon as increased co rrelation for t < 1/ $\Delta\nu$ in lag-space above, ie. correlation of the wave noise itself = 'Brown and Twiss effect'



$$\overline{N}_c = \overline{N}_1 \overline{N}_2 2t\left[1 + \frac{1}{2}g^2\right]$$ $\gamma$ = correlation coefficient

Advantage: timescale = $1/\Delta\nu$ (not $1/\nu$)

=> insensitive to poor optics, 'seeing'

Disadvantage: No visibility phase information

lower SNR

Works best for high n_s: hot stars in near IR



**Signal-to-Noise I:** allow for A, t, $\Delta\nu$

$$\text{Variance per measurement} = \langle \Delta n_s^2 \rangle = \langle n_s \rangle + \langle n_s \rangle^2$$

$$n_s = A \cdot \frac{S_?}{h?} \text{ photons/sec/Hz @ Rx and (\# measurements of } n_s) = ?\ ? \cdot t$$

$$n_s < 1 \text{ (counting stats): rms} \propto \langle n_{s,src} \rangle^{1/2}$$

$$SNR = \left(\frac{n_{s,src}}{\sqrt{n_{s,src}}}\right) (??t)^{1/2} = \left(A\frac{S_?}{h?}??t\right)^{1/2}$$

$$\therefore \text{SNR always increases as } A^{1/2}$$

$$n_s > 1 \text{ (wave noise): rms} \propto \langle n_s \rangle$$

using: $kT = S_? A$ and $T_{tot} = T_A + T_{Rx} + T_{bg}$

recover the Dickey radiometry equation:

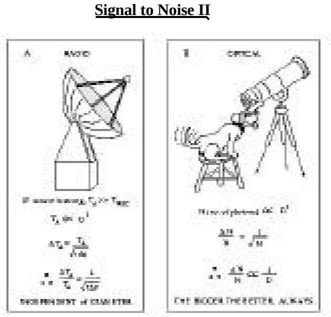
$$SNR = \frac{n_{s,src}}{n_{s,tot}} (?\ ?t)^{1/2} = \frac{T_A}{T_A + T_{Rx} + T_{bg}} (?\ ?t)^{1/2}$$

Weak source: $T_A << T_{Rx} + T_{bg} \Rightarrow SNR \propto A\,(?\ ?t)^{1/2}$

Strong (point) source: $T_A >> T_{Rx} + T_{bg} \Rightarrow SNR \propto (?\ ?t)^{1/2} \neq \text{func}(A)$



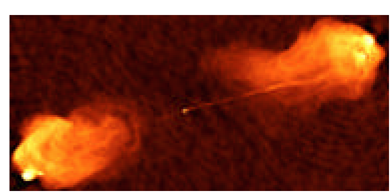
## Signal to Noise II



For strong **point** source



## Photon occupation number I: bright radio source



Cygnus A: $S_{1.4GHz} = 1400$ Jy, $z = 0.057$

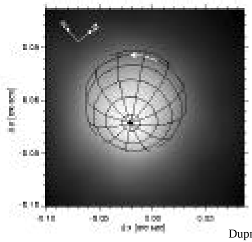
$$\text{VLA(25m)}: T_A = 140 \text{ K} \Rightarrow \frac{h?}{kT} = 0.0005$$

$$\Rightarrow n_s = (e^{\frac{h?}{kT}} - 1)^{-1} = 2000 \text{ Hz}^{-1} \text{ sec}^{-1}$$

$\therefore$ wave noise dominated

[note : $T_{Rx} + T_{bg} \approx 40$ K $\therefore$ $T_{sys}$ is dominated by source]

## Photon occupation number II: optical source



Dupree et al. 1998

Betelgeuse

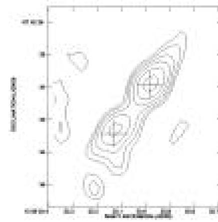
HST : $T_B = 3000K \Rightarrow h?/kT = 8$ at5e14 Hz

$\Rightarrow n_s = 0.0003\ Hz^{-1}\ sec^{-1}$

$\therefore$ 'counting noise' dominated



## Photon occupation number III: faint source



$1202-0725: S_{1.4GHz} = 0.2\,mJy,\ z = 4.7$

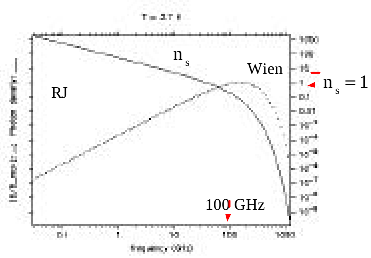
$T_A = 0.02\,mK \Rightarrow h?/kT = 3000$

$\therefore n_s << 1.$

Why do westill assumewavenoise dominates?

$T_{bg} \geq 2.7$ K ensuresn$_s > 1$ alwaysatcm?.






"Even the feeble microwave background ensures that the occupation number at most radio frequencies is already high. In other words, even though the particular contribution to the signal that we seek is very very weak, it is already in a classical sea of noise and if there are benefits to be derived from retaining the associated aspects, we would be foolish to pass them up." Radhakrishnan 1998



## Quantum noise I: Commutation relations

$?x = D$

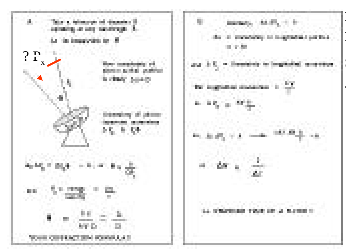
$?P_x = P_z?$

$P_z = \frac{h?}{c}$

$? = \frac{?}{D}$

$?P_x = \frac{h}{D}$

$?x \cdot ?P_x = h$



Bandlimited signal : ??

Power $= \Delta E \approx h\Delta?$

Sampling theory : $?t \approx \frac{1}{??}$

$?E \cdot ?t = h$



## Quantum noise II: Coherent Amplifiers

Photons:

$?E?t = h$

$?E = h?\Delta n_s$

$?t = \frac{?j}{?2p}$

$\Rightarrow ?j?n_s = 1\,rad\,Hz^{-1}\,sec^{-1}$

Coherent Amplifier: $?j < 1$ rad

$\Rightarrow ?n_s = 1 Hz^{-1} sec^{-1}$

Phase conservingamplifierhas minimumnoise of 1 photon/mode.

Rx temperature:

$?E = kT_{sys}$

$?t = \frac{?j}{?2p}$

$?E?t = h$

recover quantumlimited Rxnoise:

$T_{sys} = \frac{2ph?}{?j\ k}$

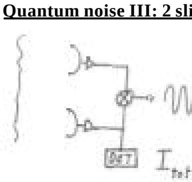
$T^Q_{sys}(1.4\,GHz) = 0.4\,K << T_{bg}$

$T^Q_{sys}(250\,GHz) = 75\,K >> T_{bg}$

Phase coherent amplifier automatically puts signal into RJ = 'classical' regime



## Quantum noise III: 2 slit paradox



Which slit does the photon enter? With a phase conserving amplifier it seems one could both detect the photon and 'build-up' the interference pattern (which we know can't be correct). But quantum noise dictates that the amplifier introduces 1 photon/mode noise, such that:

I_tot = 1 +/- 1

and we still cannot tell which slit the photon came through.

## Quantum noise IV: Einstein Coefficients

$$\text{Stimulated emission} = B_{ij},\ \text{Spontaneous Emission} = A_{ij} = \frac{8p?^3 h}{c^3} B_{ij}$$

$$\text{Stimulated Absorption: } B_{ij} = B_{ji}$$

$$\text{Radiative Transfer: } \frac{\partial I}{\partial x} = \frac{h?}{c??}\left[B_{ij} n_i - B_{ji} n_j\right] I + A_{ij} n_i \frac{h?}{4p??}$$

$$\frac{\text{Stimulated}}{\text{Spontaneous}} = \frac{\frac{h?}{c??} B_{ij} n_i I}{\frac{h?}{4p??} A_{ij} n_i} = \frac{c^2 I}{2h?^3}$$

$$I_? \to B_? = \frac{2k?^2}{c^2} T_B = \frac{2k}{?^2} T_B$$

$$\frac{\text{Stimulated}}{\text{Spontaneous}} = \frac{kT_B}{h?}$$

→ as expected for a quantum limited receiver

$$\left(\text{stimulated} = \frac{kT_B}{h?};\ \text{spontaneous} = 1\right).$$



## Quantum noise IVb: maser acts as quantum limited amplifier as dictated by the Einstein coefficients (Zmuindzinas 2000)

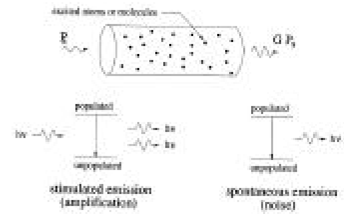



Fig. 2: An illustration of quantum noise in a maser amplifier. This (fictitious) maser amplifier consists of a tube filled with a gas of molecules or atoms, which are pumped in a way that causes some transition with frequency $\nu$ to be inverted. A signal arriving at the input with power $P_i$ is amplified by stimulated emission and emerges with power $GP_i$, where $G$ is the power gain of the amplifier. However, due to spontaneous emission, noise photons emerge from the amplifier output even when $P_i = 0$.



## Quantum noise V: Radio vs. Optical Interferometry

Quantum noise $= n_q = 1\ \text{Hz}^{-1}\ \text{sec}^{-1}$

Optical: $n_s \ll 1 \Rightarrow$ QN is disasterous ∴ better to use mirrors and director detectors (CCDs)

advantages: Rx noise is ≈ 0, large bandwidths

disadvantages: **adding elements means splitting signal ⇒ lower SNR**

complex, precise optics, delays

Radio: $n_s \gg 1 \Rightarrow$ QN is irrelevant ∴ might as well use phase conserving electronics

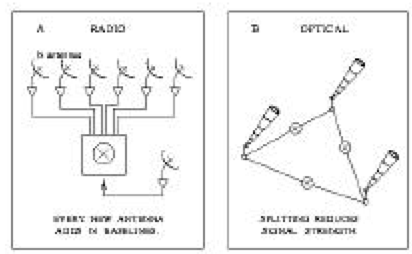
advantages: **adding antennas + full pol doesn't affect SNR**

easy IF electronics

disadvantages: high Rx noise



## Quantum noise Vb




without lowering SNR!



## Quantum limit VI: Heterodyne vs. direct detection interferometry

$$SNR_{Het} = n_s (? ?t)^{1/2}/1 \qquad SNR_{DD} = \left(\frac{n_s}{N} ? ?t\right)^{1/2}$$

$$\frac{SNR_{Het}}{SNR_{DD}} \approx (N n_s)^{1/2} \quad \text{with } N = \#\text{ elements}$$

Betelgeuse with HST at 5e14Hz:

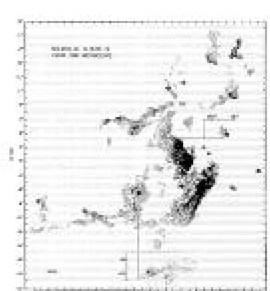
$$\frac{SNR_{Het}}{SNR_{DD}} = (N/3000)^{1/2} \Rightarrow \text{DD wins}$$

Cygnus A with VLA at 1.4 GHz:

$$\frac{SNR_{Het}}{SNR_{DD}} = (2000N)^{1/2} \Rightarrow \text{Heterodyne wins}$$



## Quantum limit VII: On the border



Orion at 345GHz in CO(3−2):

$T_A \approx 10K \Rightarrow h?/kT = 1.7 \Rightarrow n_s = 1/4$

$$\frac{SNR_{Het}}{SNR_{DD}} = (N/4)^{1/2} \Rightarrow \text{toss-up}$$