3D Hydrodynamics of Convection and Waves in Stars

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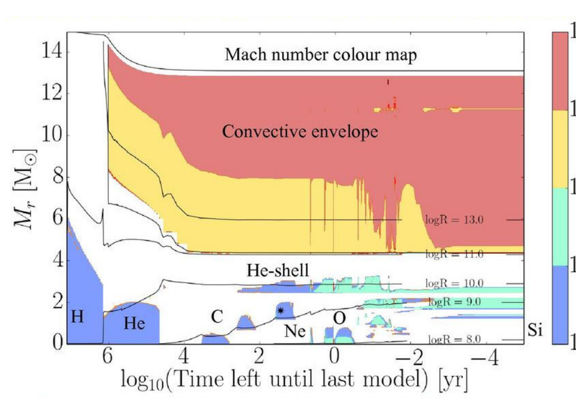
38th Annual New Mexico Symposium
February 17, 2023

Stellar models

- one-dimensional, spherically symmetric
- sequence of hydrostatic profiles
- evolution through nuclear burning much longer timescales than dynamic and thermal evolution
- parametrized treatment of convection, convective boundary mixing, all kinds of instabilities, ...
- rotation only possible for certain classes of profiles

Challenges of 3D Hydrodynamics

- Mach number: $M = \frac{u}{c}$
- stellar interiors are usually at low Mach numbers
- speed of sound $c = \sqrt{\gamma \frac{p}{\rho}} \propto \sqrt{\frac{T}{\mu}}$
- common schemes for the full Euler equations show problems for $M \to 0$
- solution dominated by numerical dissipation at low Mach numbers



credit Raphael Hirschi

Possible solutions

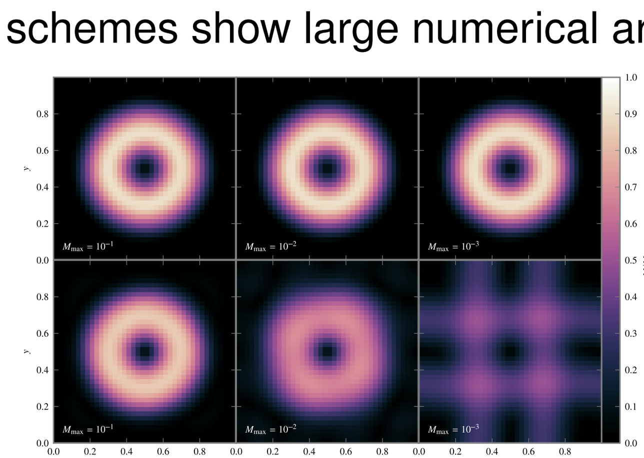
- increase spatial resolution (not feasible in many situations)
- reduce numerical viscosity for $M \to 0$
- change the PDEs to remove problematic terms (e.g., anelastic approximation)

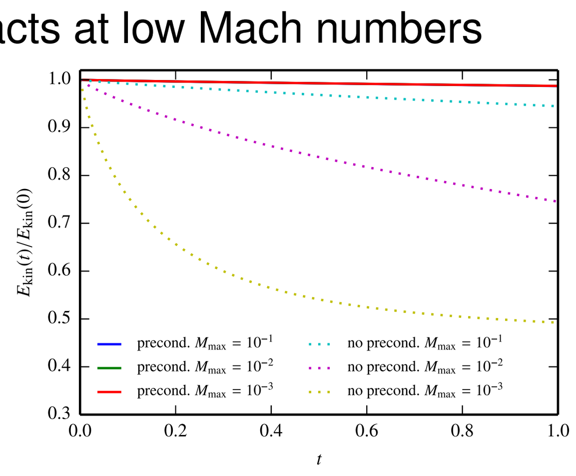
Low Mach number fluxes

Many standard schemes show large numerical artifacts at low Mach numbers

preconditioned Roe solver (Miczek et al., 2015)

standard Roe solver



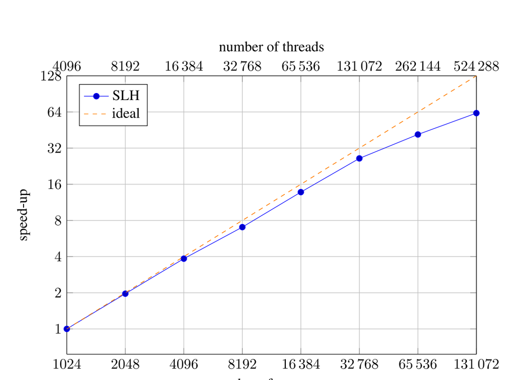


Implicit time stepping

- time step increases by $\sim \frac{1}{M}$, but much larger effort per step
- typically use ESDIRK scheme (Explicit first stage, Singly Diagonally Implicit Runge–Kutta)
- a nonlinear system of eqs. needs to be solved at each (sub)step
- solved using Newton–Raphson method
- good initial guess: previous step
- Jacobian matrix solved using iterative methods (BiCGSTAB, GMRES, Multigrid)

example: memory requirements (512^3 example)

- size: $5 N_x N_y N_z$ equations (6.7×10^8)
- Jacobian: $(5 N_x N_y N_z)^2$ matrix ($4.5 \times 10^{17} \widehat{=} 3.1$ EiB)
- nonzero entries: $325 N_x N_y N_z$ ($4.3 \times 10^{10} \widehat{=} 325$ GiB)
- sparseness: 9.7×10^{-6}%



Seven-League Hydro (SLH) code

- solves the compressible Euler equations in 1-, 2-, 3-D
- explicit and implicit time integration
- schemes suited for all Mach numbers (e.g., AUSM$^+$-up)
- works for low and high Mach numbers on the same grid
- hybrid (MPI, OpenMP) parallelization (works up to 458 752 cores)
- several solvers for the linear system: BiCGSTAB, GMRES, Multigrid, (direct)
- arbitrary curvilinear meshes using a rectangular computational mesh
- gravity solver (monopole, Multigrid)
- radiation in the diffusion limit
- general equation of state
- general nuclear reaction network

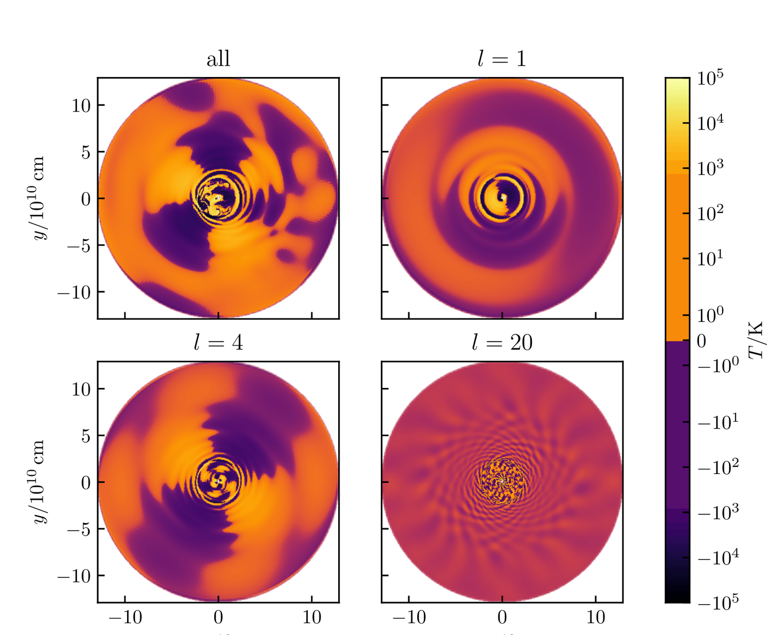
Anelastic simulations

- modified equation set
- removes sound waves
- tracks deviations from background $(\bar{\cdot})$

$$\nabla \cdot \bar{\rho} \mathbf{v} = 0$$

$$\frac{\partial \mathbf{v}}{\partial t} = -(\mathbf{v} \cdot \nabla)\mathbf{v} - \nabla P - C g \hat{\mathbf{r}} + 2(\mathbf{v} \times \hat{\mathbf{z}} \Omega) + \bar{\nu} \left(\nabla^2 \mathbf{v} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{v}) \right)$$

$$\frac{\partial T}{\partial t} = -(\mathbf{v} \cdot \nabla) T + (\gamma - 1) T h_\rho v_r - v_r \left(\frac{\partial \bar{T}}{\partial r} - (\gamma - 1) \bar{T} h_\rho \right) + \frac{\bar{Q}}{c_v \bar{\rho}} + \frac{1}{c_v \bar{\rho}} \nabla \cdot (c_p \overline{\kappa \rho} \nabla T) + \frac{1}{c_v \bar{\rho}} \nabla \cdot (c_p \kappa_r \bar{\rho} \nabla \bar{T})$$



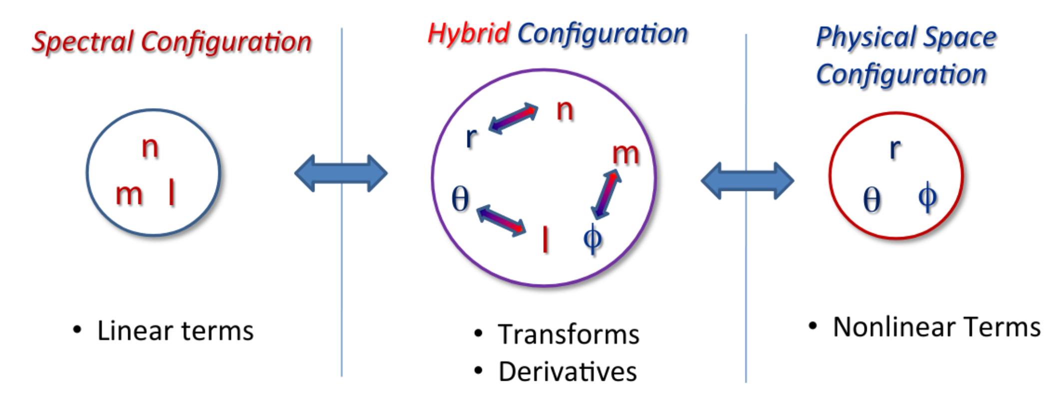
spherical harmonic decomposition

References

Edelmann, P. V. F., Ratnasingam, R. P., Pedersen, M. G., et al. 2019, ApJ, 876, 4, [1903.09392]
Horst, L., Edelmann, P. V. F., Andrássy, R., et al. 2020, A&A, 641, A18, [2006.03011]
Matsui, H., Heien, E., Aubert, J., et al. 2016, Geochemistry, Geophysics, Geosystems, 17, 1586
Miczek, F., Röpke, F. K., & Edelmann, P. V. F. 2015, A&A, 576, A50, [1409.8289]
Rogers, T. M. & McElwaine, J. N. 2017, ApJ, 848, L1, [1709.04920]
Varghese, A., Ratnasingam, R. P., Vanon, R., Edelmann, P. V. F., & Rogers, T. M. 2023, ApJ, 942, 53, [2211.06432]

Rayleigh code

- 3D pseudo-spectral MHD code
- original developer: Nick Featherstone (SWRI Boulder)
- openly developed on GitHub with a team of 6 core developers
- GPLv3 licensed github.com/geodynamics/Rayleigh
- 2D domain decomposition (more efficient parallelization)
- efficient scaling up to 10^4 cores (Matsui et al., 2016)
- custom reference states (e.g., from MESA stellar evolution code)
- support for active and passive scalars to track species

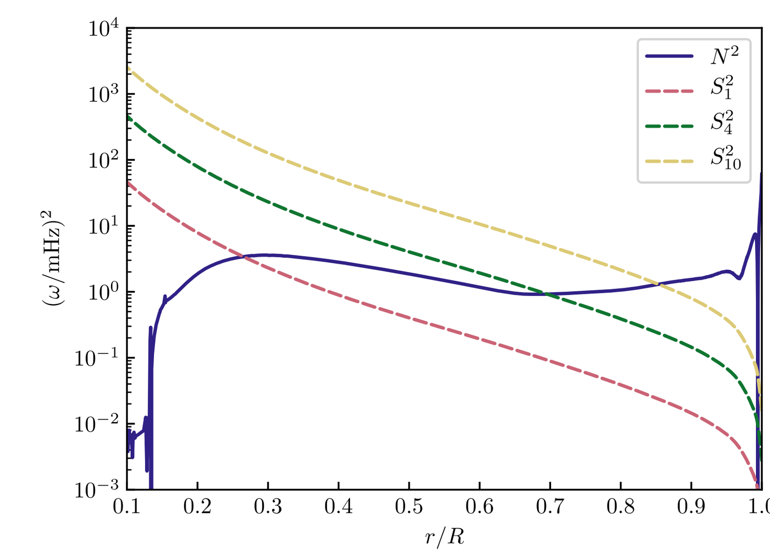


credit: Featherstone (2015)

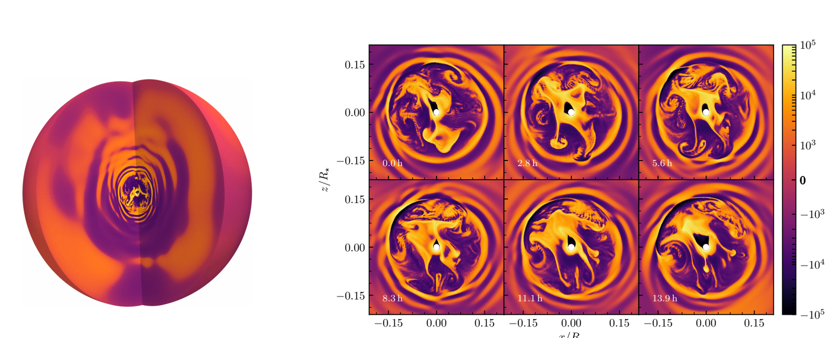
COMPUTATIONAL INFRASTRUCTURE for GEODYNAMICS

Initial Model (Edelmann et al., 2019)

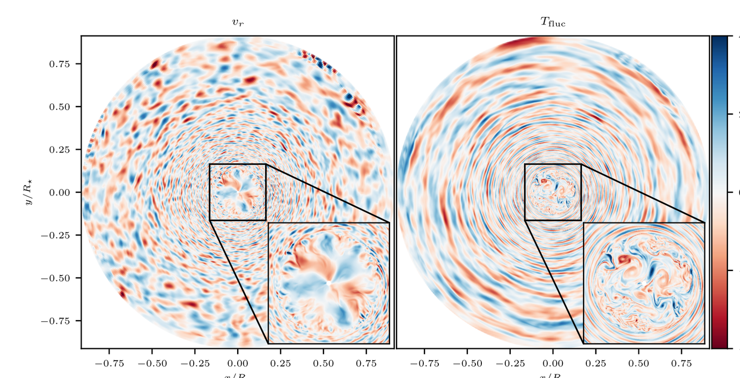
- MESA model, 3 $M_\odot$ (B-type)
- Z=0.02, ZAMS
- full sphere
- 1% to 90% of stellar radius
- no rotation (so far)
- increased viscosity and radiative diffusivity
- increased luminosity
- resolution: 1500 (r) $\times$ 128 (ϑ) $\times$ 256 (φ)



Wave excitation

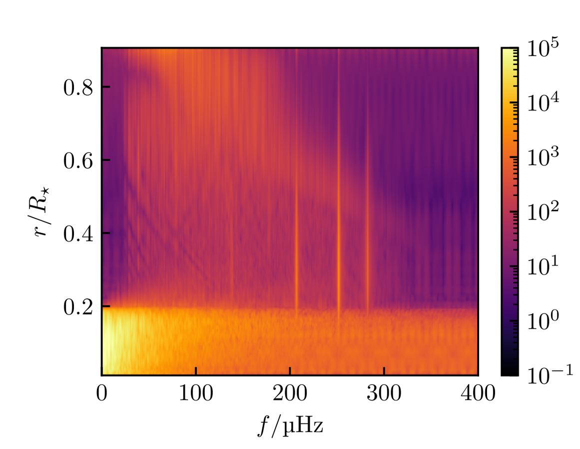


anelastic 3D (Edelmann et al., 2019)

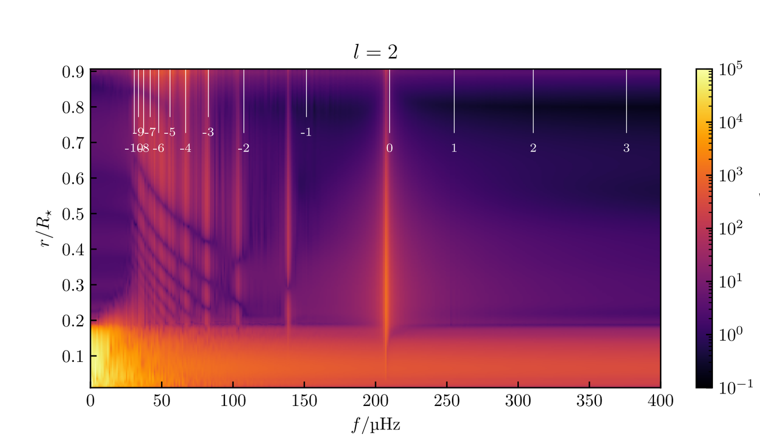


fully compressible 2D (Horst et al., 2020)

Spectra

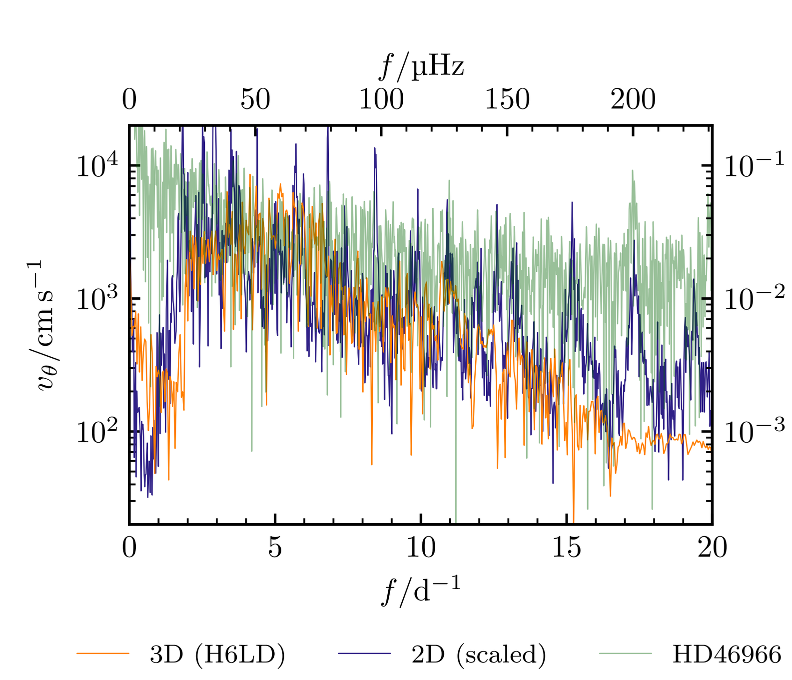


full radial velocity spectrum (all ℓ)

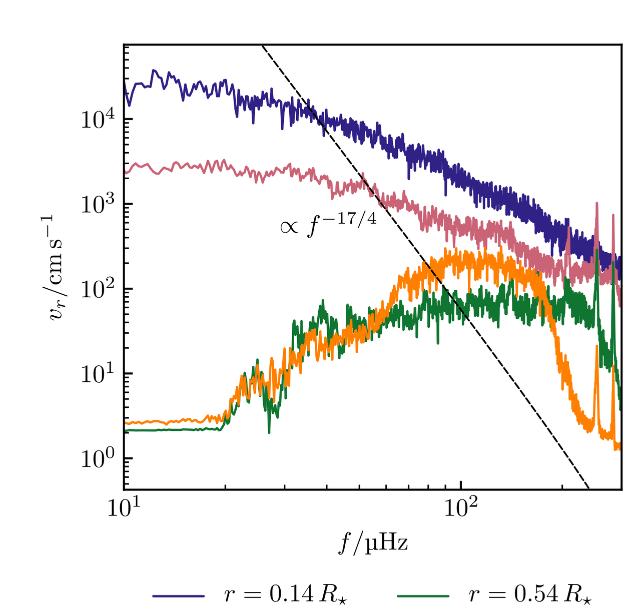


velocity spectrum of $\ell = 2$

Comparison to asteroseismological observations



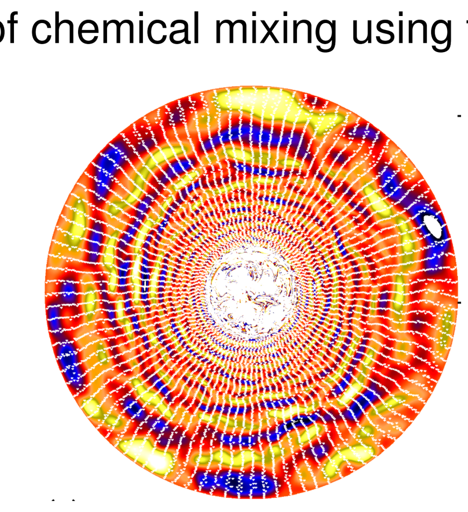
spectra of tangential velocity/brightness variations



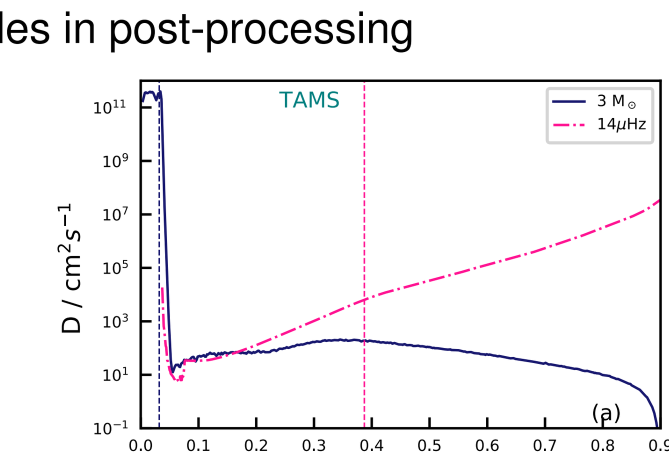
radial velocity spectrum

Chemical mixing by internal gravity waves

Tracing of chemical mixing using tracer particles in post-processing



(Rogers & McElwaine, 2017)



(Varghese et al., 2023)

Acknowledgements: PVFE was supported by the U.S. Department of Energy through the Los Alamos National Laboratory (LANL). LANL is operated by Triad National Security, LLC, for the National Nuclear Security Administration of the U.S. Department of Energy (Contract No. 89233218CNA000001).