FleCSPH for Modeling Binary White Dwarf Mergers

M. Alexander Kaltenborn, Wesley Even, Oleg Korobkin, Hyun Lim, Julien Loiseau,
Chris Mauney, Irina Sagert

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Outline

Smoothed-Particle Hydrodynamics

- What is SPH?
- Applications

FleCSPH

- LANL SPH code
  - Functionality
- Numerical results
  - Conservation
  - scaling
- Current Research

Binary White Dwarfs

- Brief introduction to BWD systems

BWD Simulations

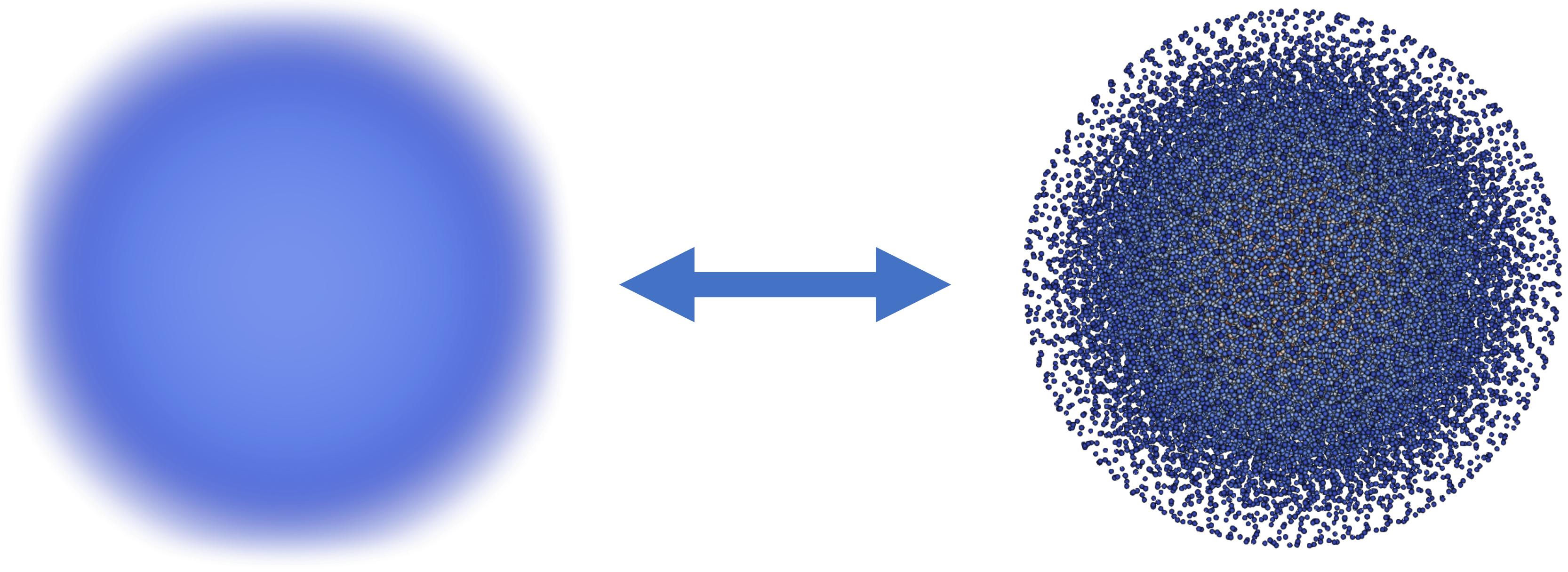
- Our results
- Comparison to the literature

Summary

- Concluding remarks

Smoothed-Particle Hydrodynamics

Smoothed-Particle Hydrodynamics (SPH)



- Lagrangian Method
  - fluid quantities carried by moving interpolation points (i.e., particles), which follow the fluid motion
- For any physical quantity, A:

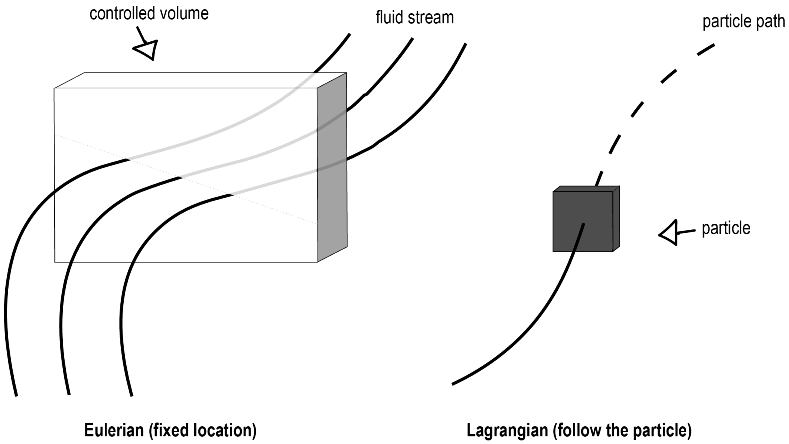
$$A(\vec{r})_a \simeq \sum_b \frac{m_b}{\rho_b} A(\vec{r}_b) \underline{W(|\vec{r} - \vec{r}_b|, \underline{h})}$$

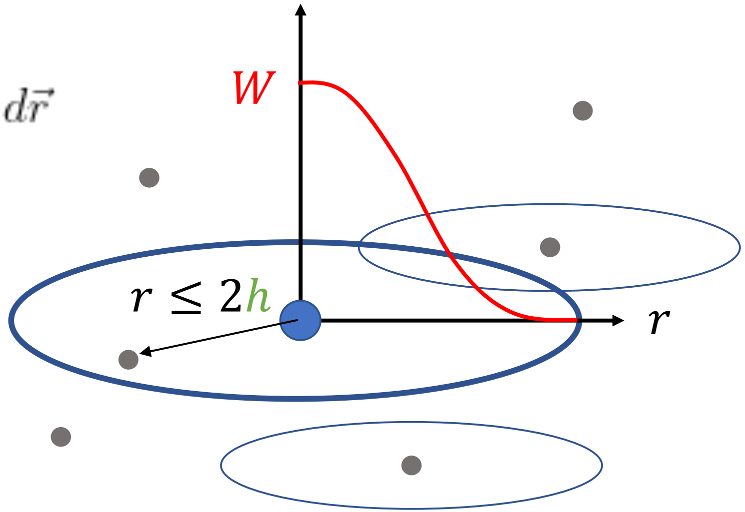
  - Derived from delta function:

$$A(\vec{r})_a = \int A(\vec{r})\delta(\vec{r} - \vec{r}_a)d\vec{r} \approx \int A(\vec{r}) W(|\vec{r} - \vec{r}_a|, h) d\vec{r}$$

  - Derivatives are determined by:

$$\partial_x A(\vec{r})_a = \sum_b \frac{m_b}{\rho_b} A(\vec{r})_b\, \partial_x W(|\vec{r}_a - \vec{r}_b|, h)$$





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SPH Equations

Euler equations

- Conservation of
  - Mass $$\frac{d\rho}{dt} = -\nabla \cdot (\rho \vec{v})$$
  - Energy $$\frac{du}{dt} = \left(\frac{P}{\rho^2}\right)\frac{d\rho}{dt}$$
  - Momentum $$\frac{d\vec{v}}{dt} = -\frac{1}{\rho}\nabla P$$
  - Entropy $$\frac{d(\rho s)}{dt} = -\nabla \cdot (\rho s \vec{v})$$

SPH discretization

- Automatically satisfied

$$\boxed{\frac{d\rho_a}{dt} = \sum_b m_b \vec{v}_{ab} \cdot \nabla_a W_{ab}}$$

$$\frac{du_a}{dt} = \frac{P_a}{\rho_a^2}\sum_b m_b \vec{v}_{ab} \cdot \nabla_a W_{ab}$$

$$\frac{d\vec{v}_a}{dt} = -\sum_b m_b \left(\frac{P_a}{\rho_a^2} + \frac{P_b}{\rho_b^2}\right)\nabla_a W_{ab}$$

$$\boxed{\frac{ds_a}{dt} = 0}$$

adiabatic flows

FleCSPH

FleCSPH

https://github.com/laristra/flecsph

- FleCSPH (Loiseau et al., Software X (2020)): SPH code build with the LANL FleCSI numerical framework as part of the LANL Ristra Project. FleCSPH is a general-purpose SPH code, but has been applied primarily to astrophysical problems. Some core capabilities:
  1. Implementation of different SPH Kernels
  2. Astrophysical Equations of State (EoSs), both analytic and tabulated
     1. Ideal gas, polytropic, piecewise polytropic, cold white dwarf (ZTWD), ZTWD+ideal gas
     2. Finite-temperature nuclear matter (StellarCollapse), Helmholtz
  3. Material EoSs: Liquid, Mie-Grüneisen, Osborne, Tillotson
  4. Artificial viscosity: constant and with shock trigger (Cullen and Dehnen (2010))
  5. External potentials for boundary conditions and relaxation
  6. Newtonian Gravity via N-body calculations or Fast-Multipole Method
  7. Fixed general-relativistic background metric for static and rotating stars

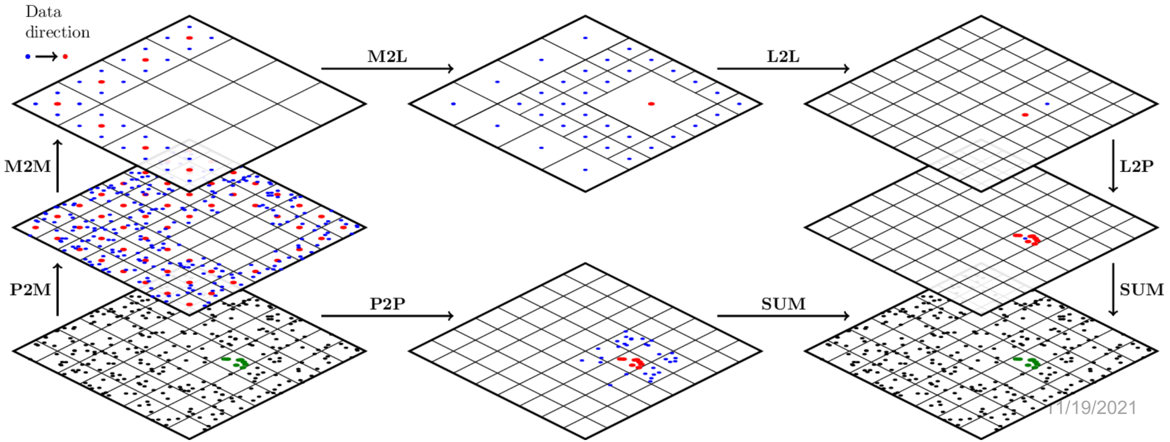
Fast Multipole Method (FMM)

Approximation of long-range forces (gravitation/electromagnetic)

- Based on Taylor series

$$\vec{f}(\vec{r}) = \vec{f}(\vec{r_m}) + ||\frac{\partial \vec{f}}{\partial \vec{r}}|| \cdot (\vec{r} - \vec{r_m}) + \frac{1}{2}(\vec{r} - \vec{r_m})^{\mathsf{T}} \cdot ||\frac{\partial \vec{f}}{\partial \vec{r} \partial \vec{r}}|| \cdot (\vec{r} - \vec{r_m})$$

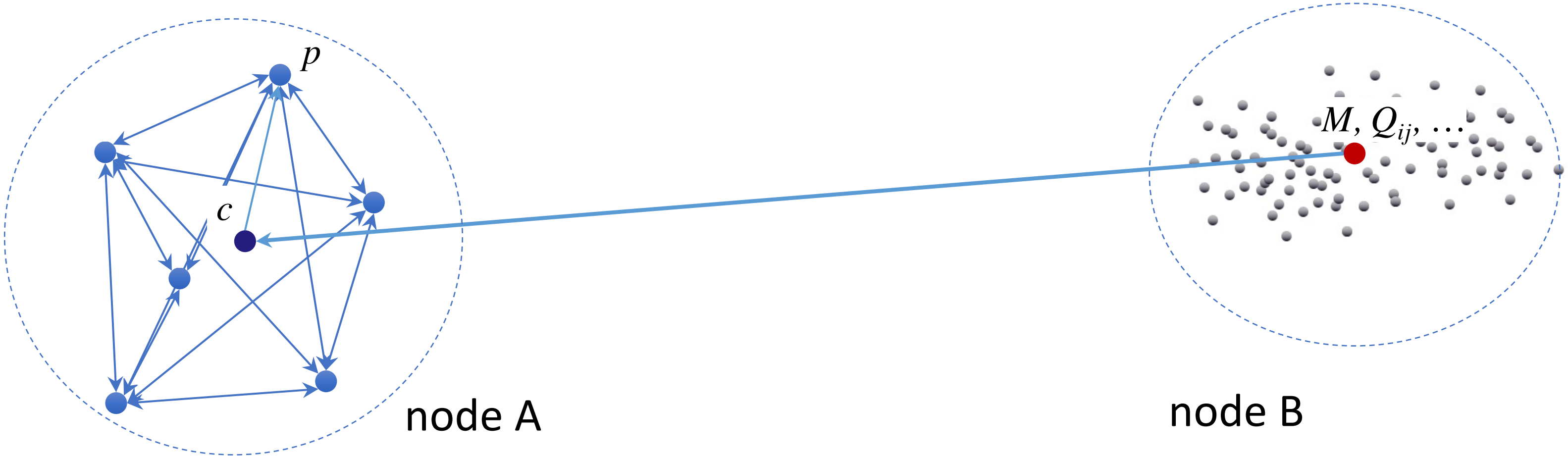
- Multiples steps:
  - P2M, M2M, M2L, L2L, …
- Using the whole tree
  - Global communications



Newtonian gravity with FMM

Local particles interact individually – O(N²) cost:

Distant particles interact through tree nodes – O(N log N) cost:



$$\vec{a}_{ab} = -G \sum_b m_b \frac{\vec{r}_b - \vec{r}_a}{|\vec{r}_b - \vec{r}_a|^3}$$

$$\vec{a}_c = -G \sum_p M \frac{\vec{r}_c - \vec{r}_p}{|\vec{r}_c - \vec{r}_p|^3} + (\dots)$$

$$\vec{a}_p = \vec{a}_c(\vec{r}_c) + \left\| \frac{\partial \vec{a}_c}{\partial \vec{r}_c} \right\| \cdot (\vec{r} - \vec{r}_c) + \frac{1}{2}(\vec{r} - \vec{r}_c)^T \cdot \left\| \frac{\partial \vec{a}_c}{\partial \vec{r}_c \partial \vec{r}_c} \right\| \cdot (\vec{r} - \vec{r}_c) + (\dots)$$

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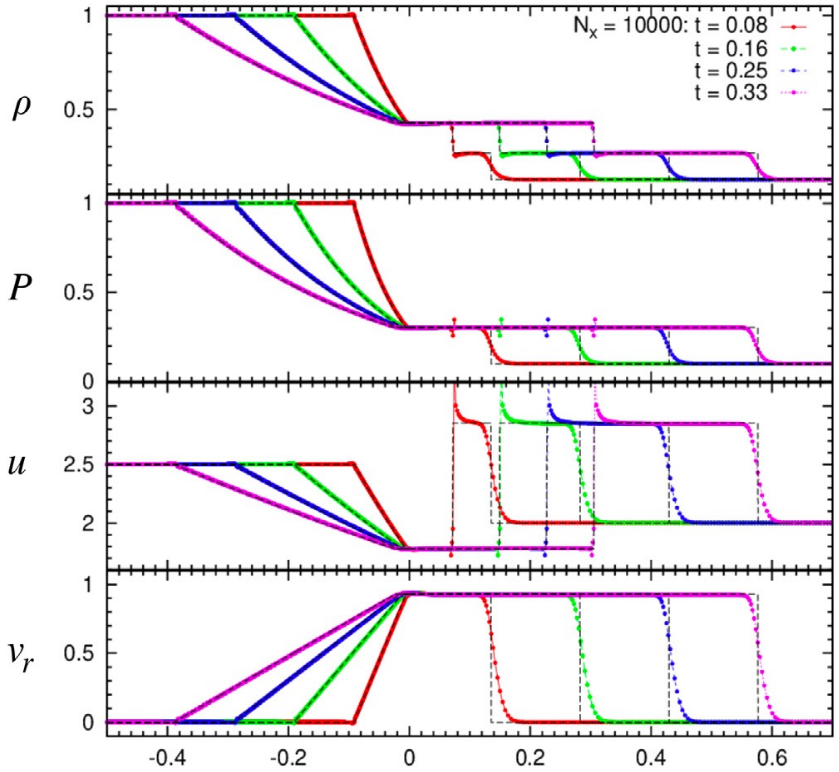
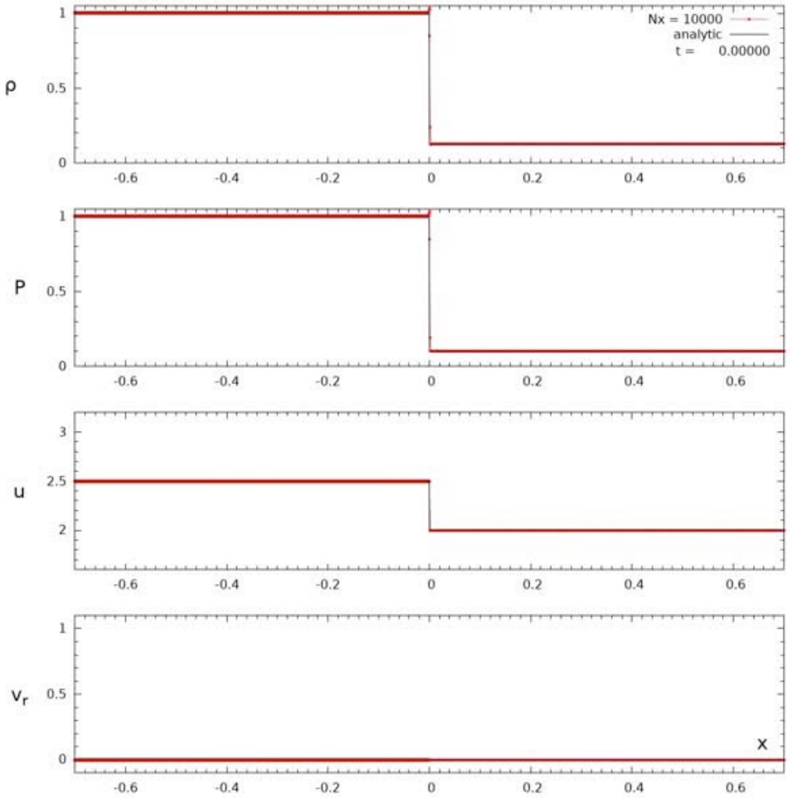
FleCSPH Test Case: Sodtube

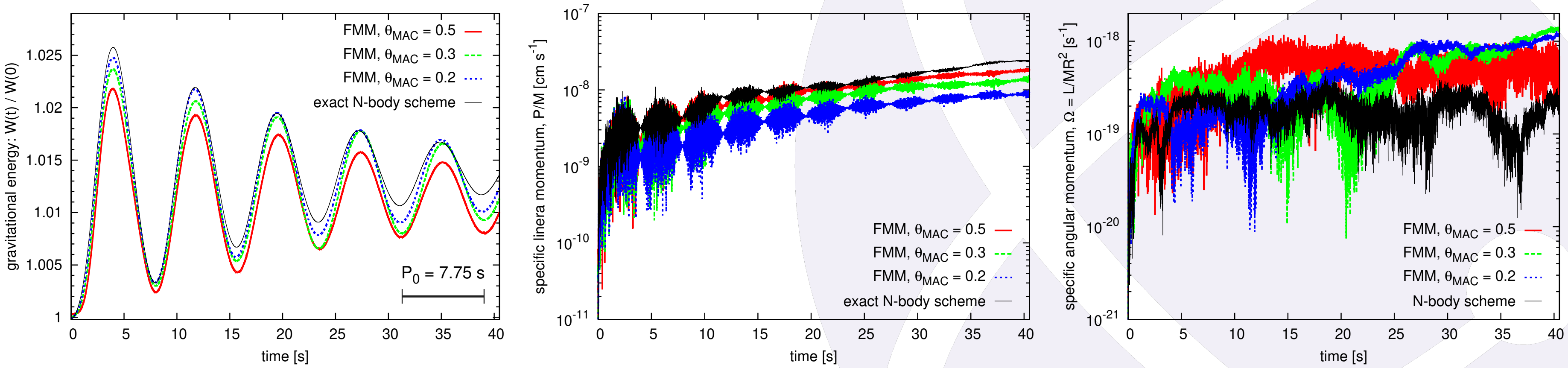


One-dimensional Sod shock tube with 10,000 particles. Panels, from top to bottom: density, pressure, specific internal energy, and velocity. Each panel contains four different times. (Loiseau et al., Software X, 2020)



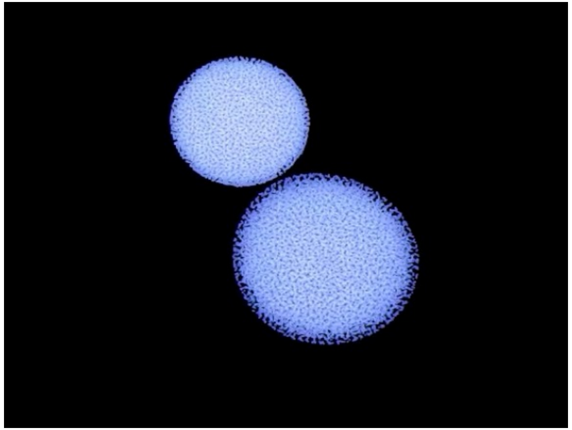
FleCSPH Test Case: Gravity Stellar Oscillations

- Truncation error in the initial configuration triggers small oscillations of the star
- Oscillation damped by the viscosity during the evolution
- Checks consistency and conservation properties for the coupled hydrodynamics and gravity



Oscillations of a star near equilibrium (14,993 particles). Left panel: gravitational energy evolution for the exact N-body gravity and the FMM approximation with three different MAC values: tan(θMAC) = 0.2, 0.3 and 0.5. Center and Right: evolution of specific linear and angular momenta, respectively.

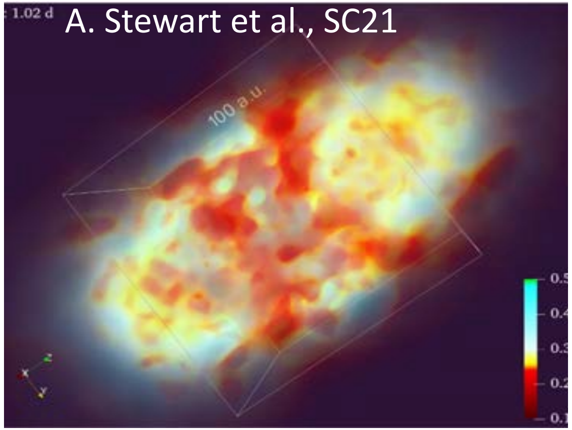
FleCSPH Current Research



Visualization: Pascal Grosset

Compact Mergers

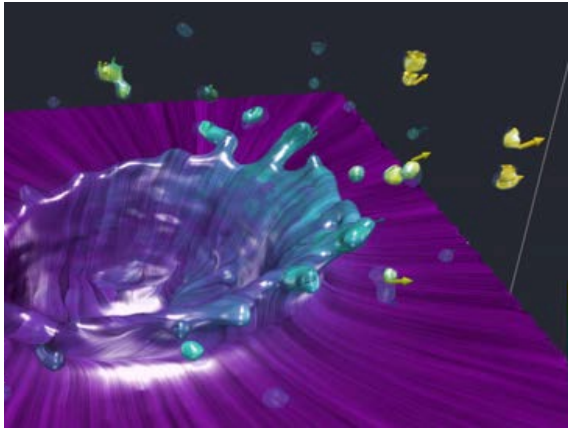
- Binary white dwarfs
- Binary neutron stars
- Neutron star and white dwarf systems



Visualization: Oleg Korobkin

Kilanovae

- Rapid neutron capture nucleosynthesis after neutron star merger
- Optical and infrared spectra



Visualization: Roxana Bujack

Astrophysical Solids

- Asteroid impacts
- Dynamics of the solid neutron star crust in single rotating neutron stars and binary neutron star mergers

Binary White Dwarfs

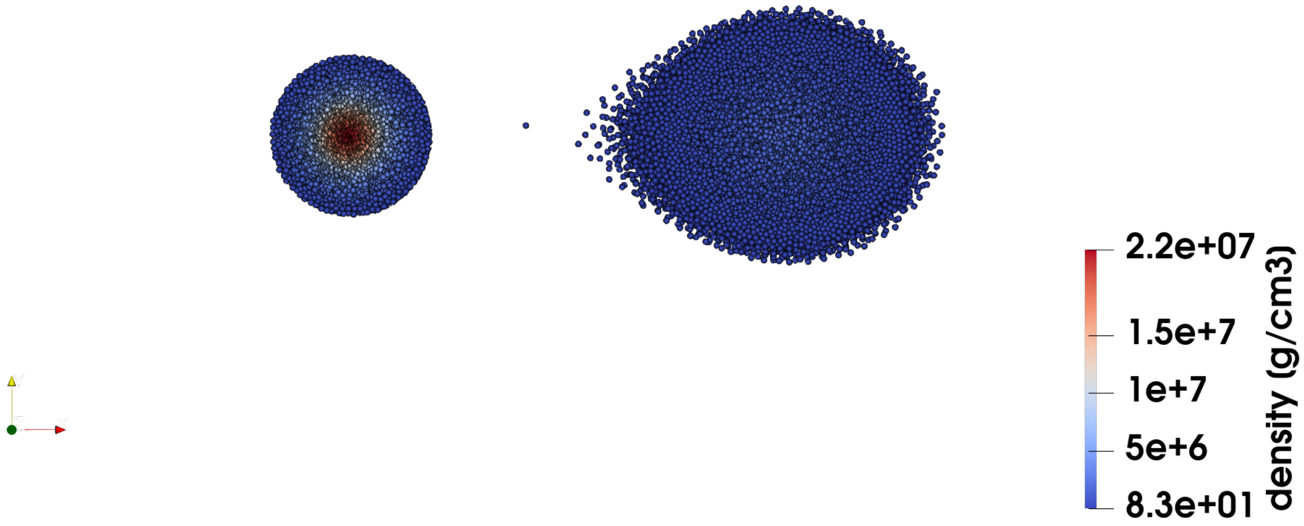
Why simulate Binary White Dwarfs

- Type Ia supernovae (SNeIa) are commonly accepted to be the observed transient produced after a thermonuclear detonation inside a white dwarf star
- SNeIa typically accretion from an evolved main-sequence star onto a white dwarf
  - SNeIa rate from these systems is incompatible with observations
- Extremely luminous SNeIa speculated to be derived from double-degenerate white dwarf mergers
- Tidal dissipation and gravitational radiation drive the binary to merger
- The less massive white dwarf begins to accrete material onto the other
  - Fundamentally unstable, requiring simulations to capture dynamics

BWD Simulations

BWD Merger Simulation

- Co-rotating system, composed of 298,341 particles
  - M_1= 1.0 M_s and M_2 = 0.5 M_s, q = 0.5
  - ZTWD+thermal EoS
  - Initial oribital period of 74.5s



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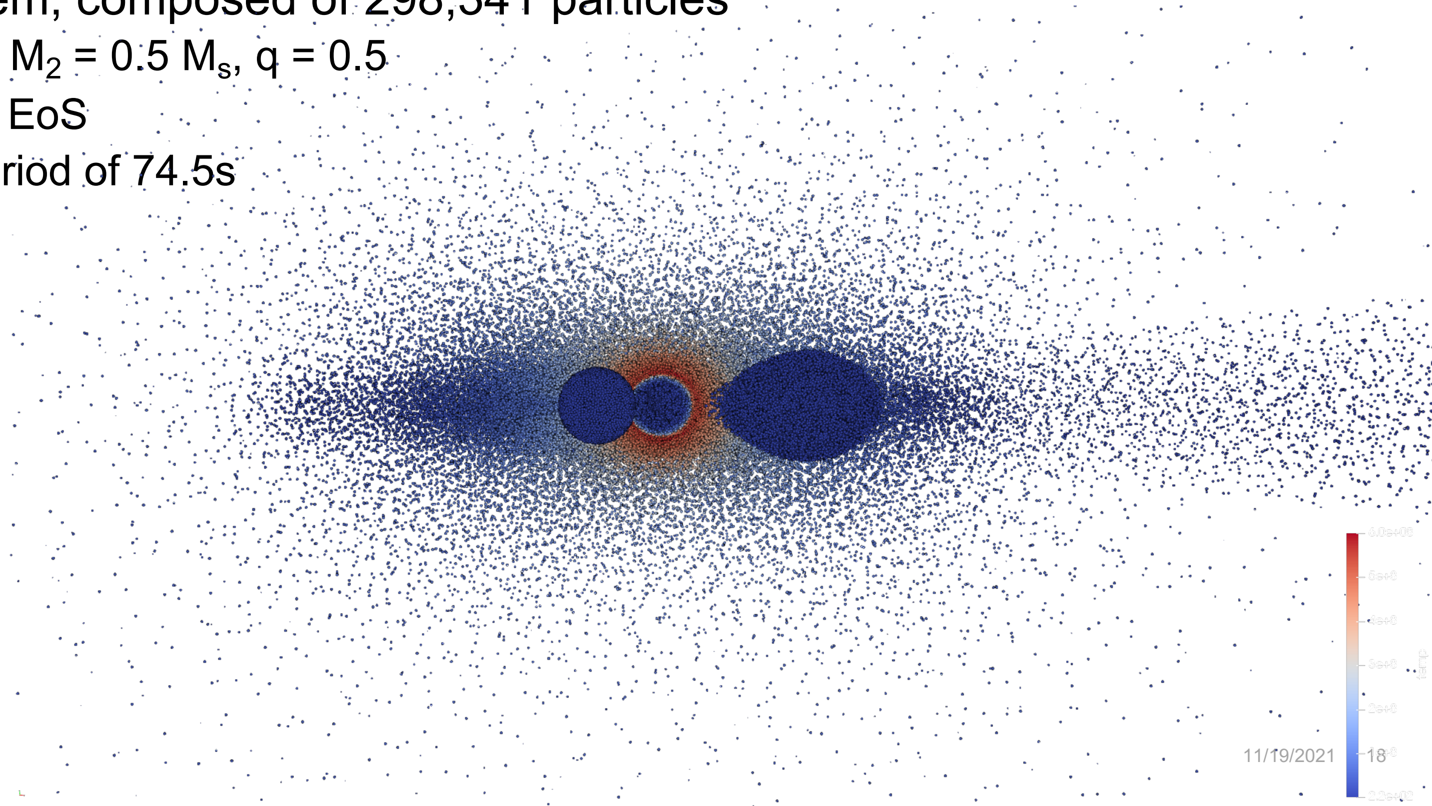
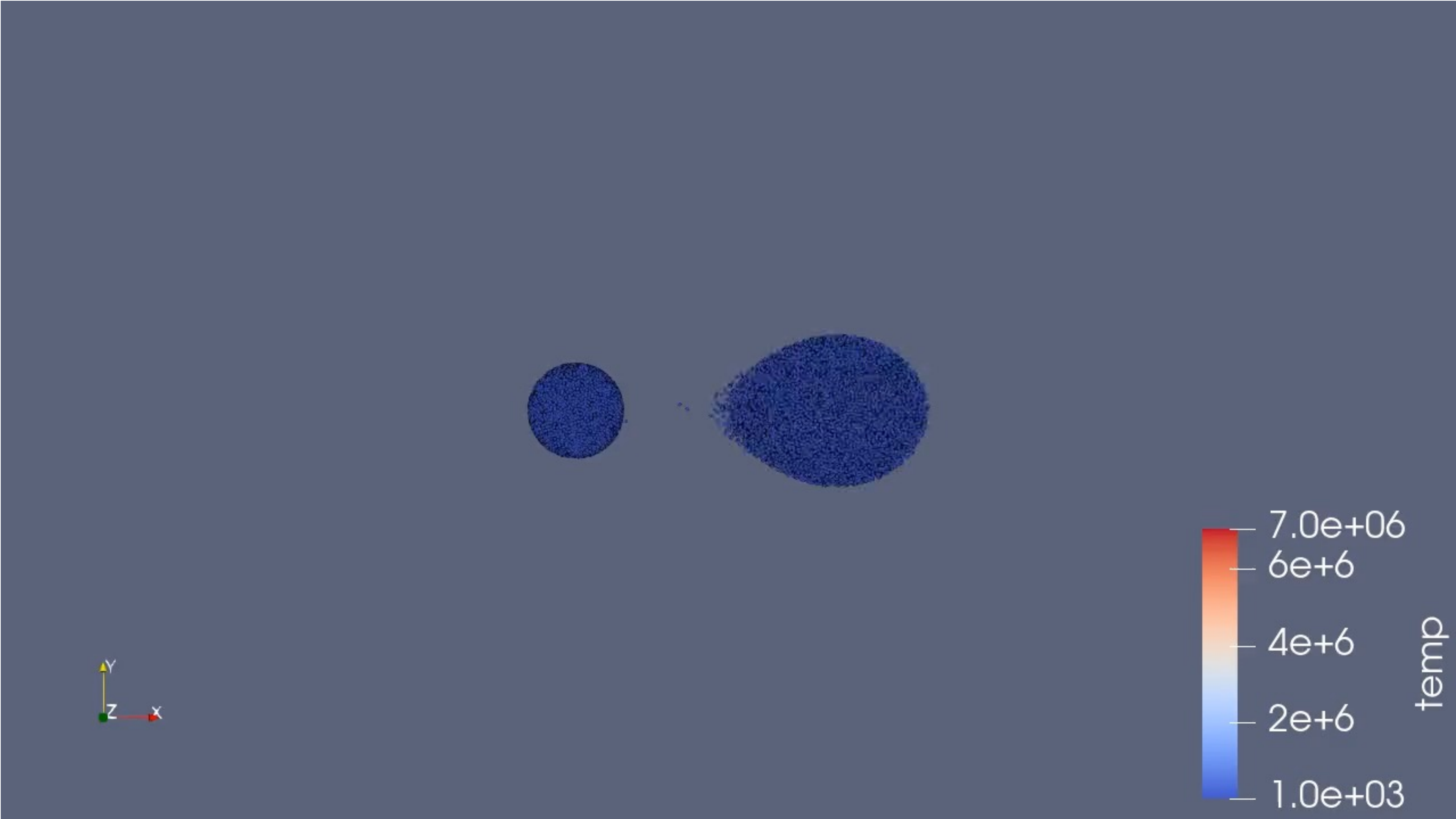
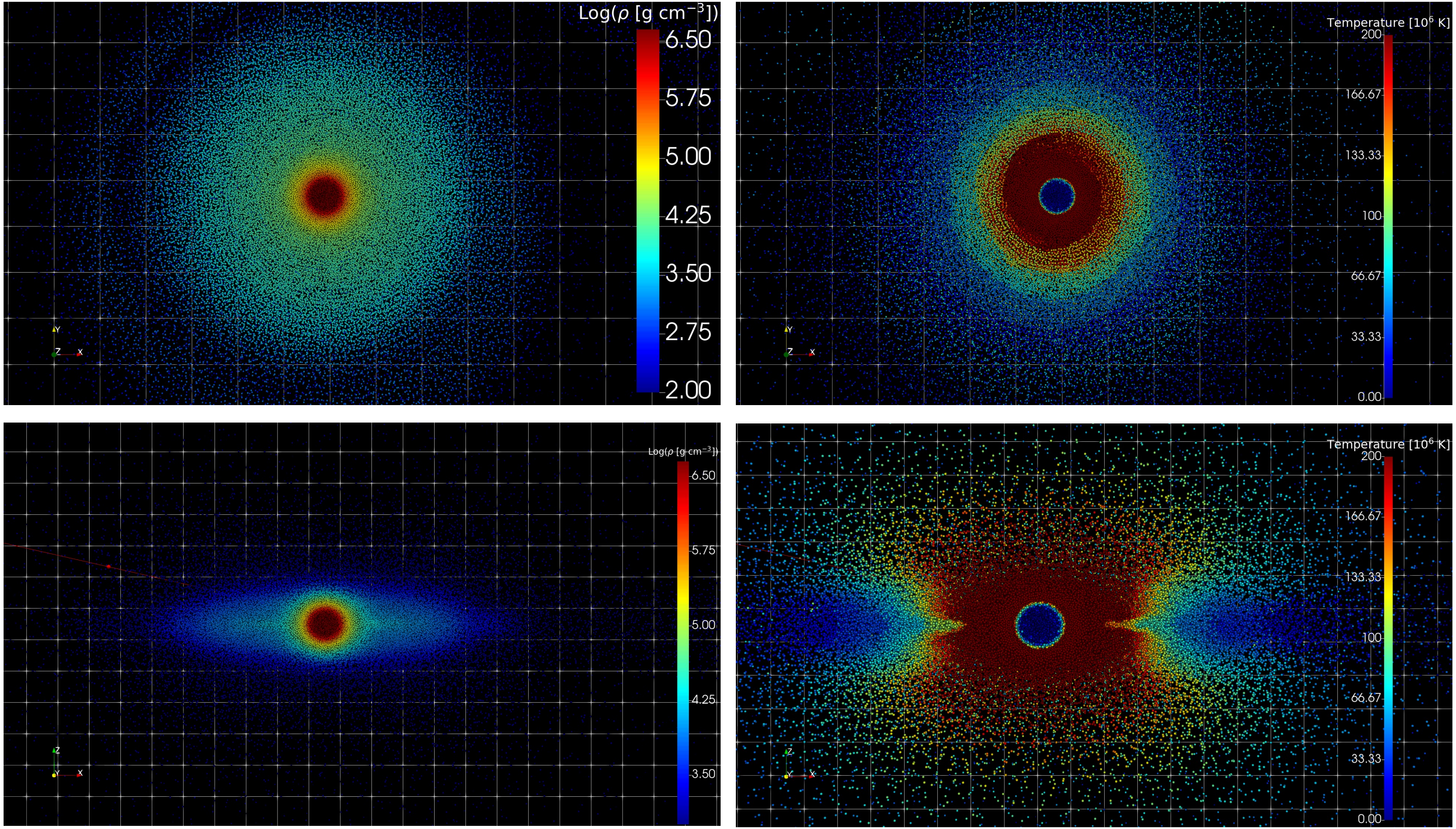




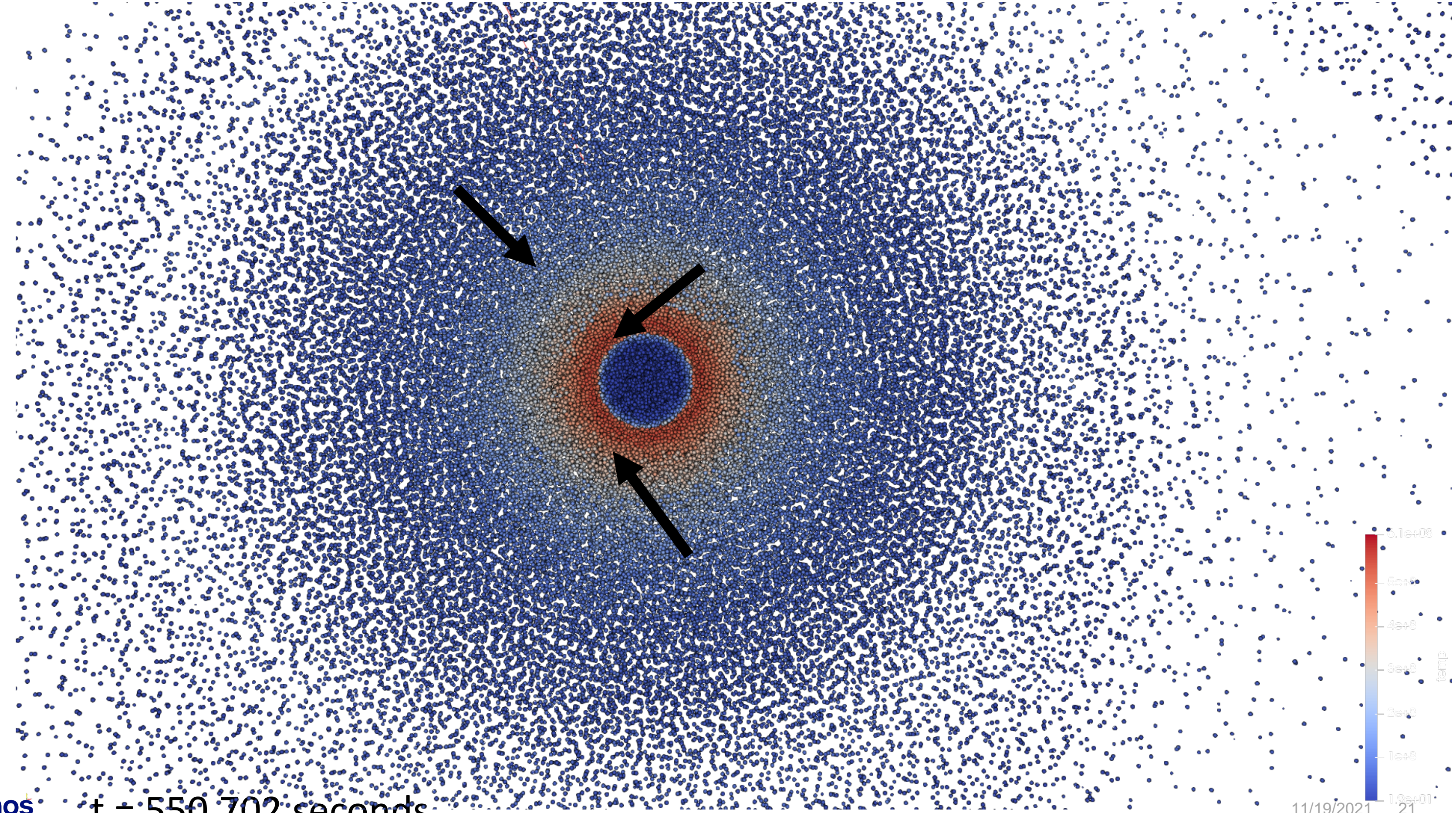
Y
Z
X
7.0e+06
6e+6
4e+6
2e+6
1.0e+03
temp

BWD Merger Simulation



t = 550.702 seconds

BWD Merger Simulation



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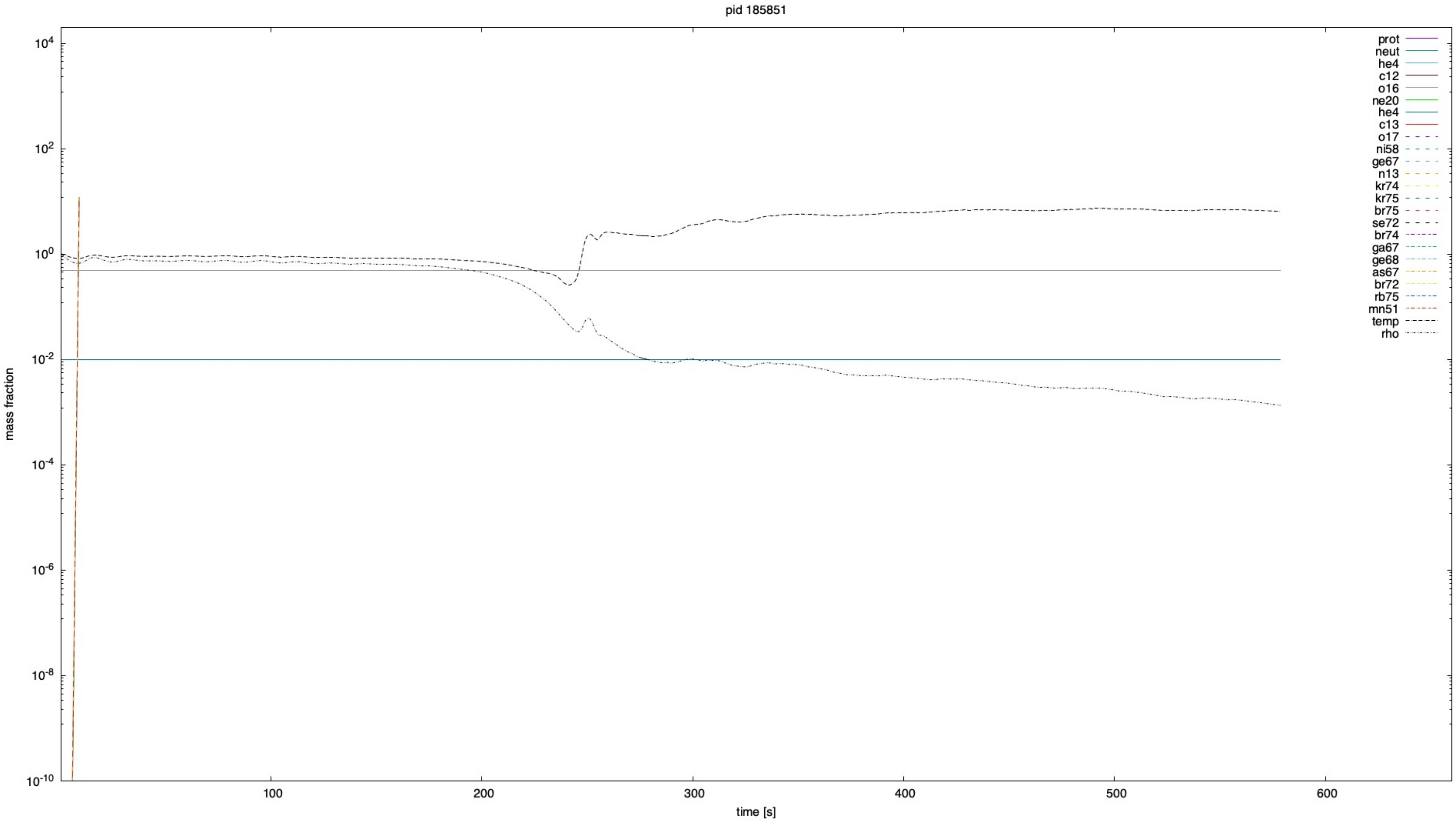

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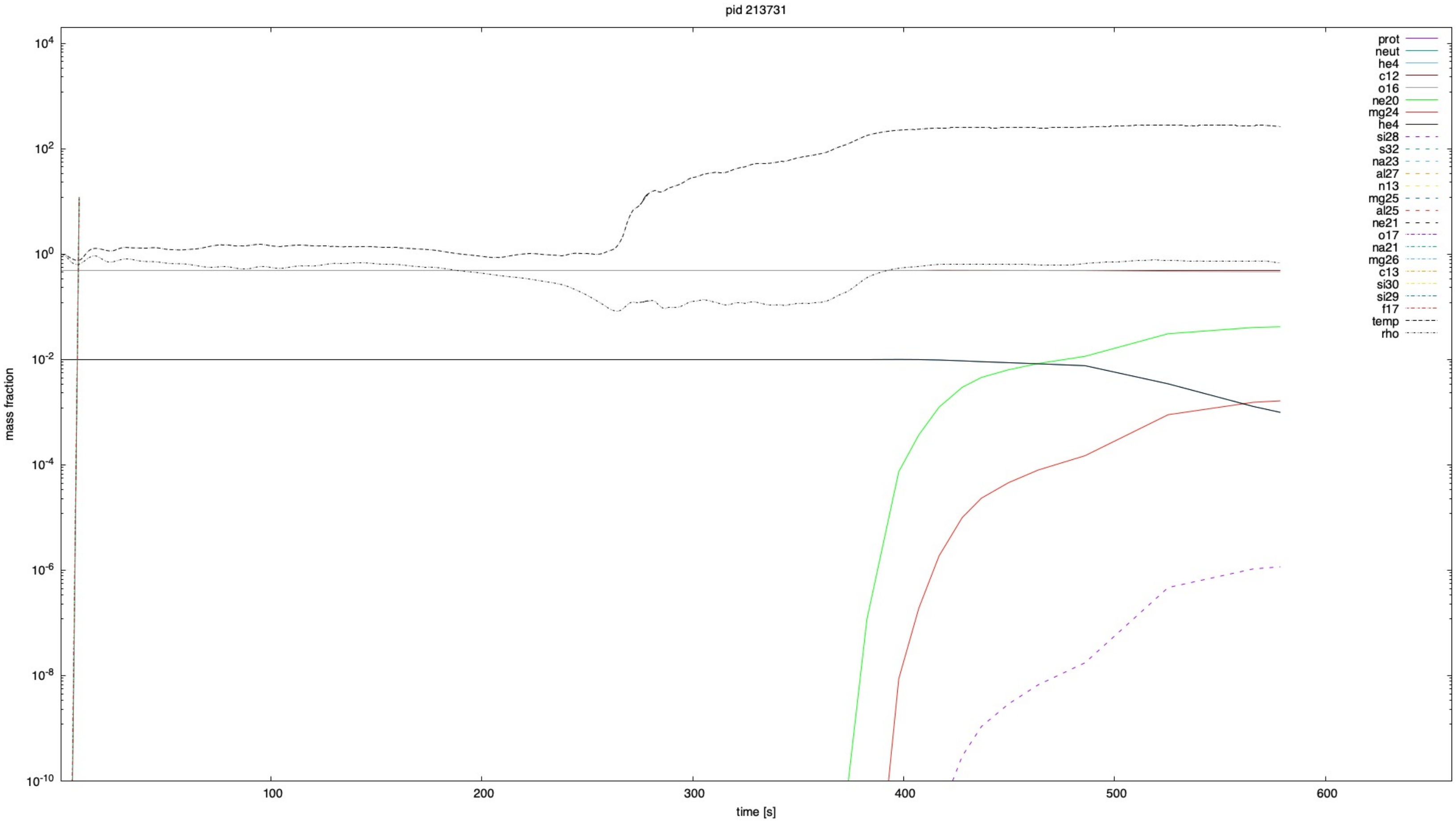

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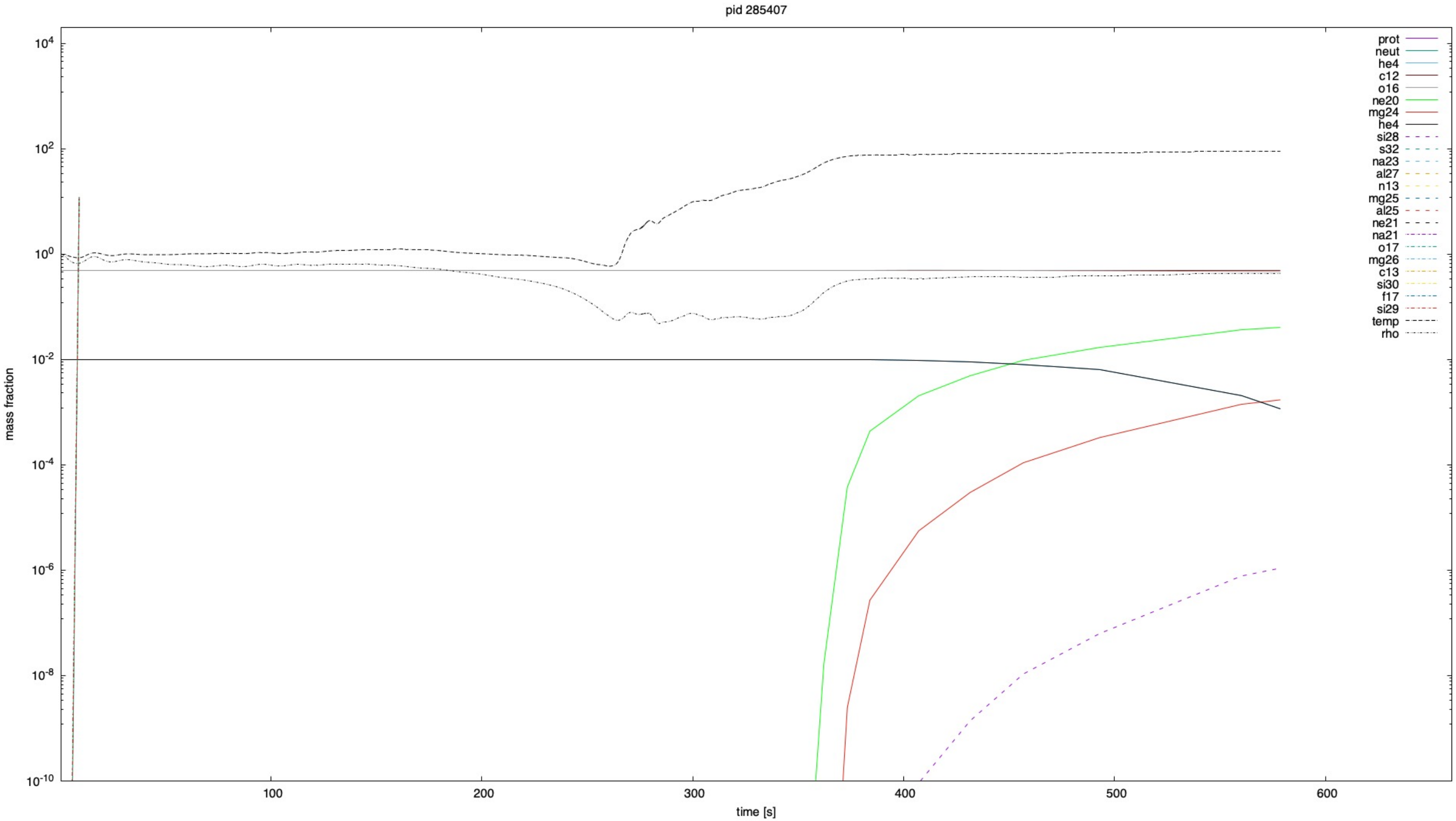
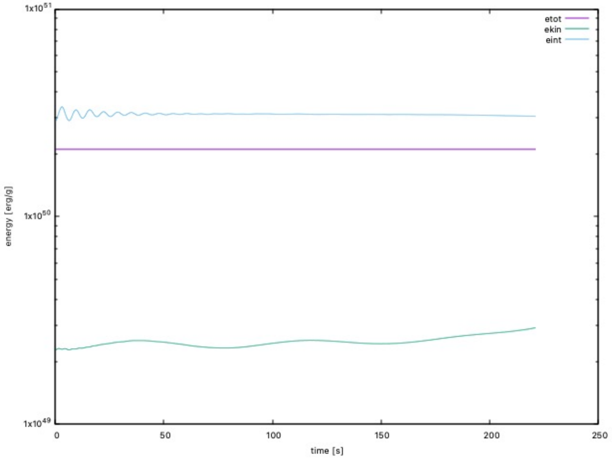
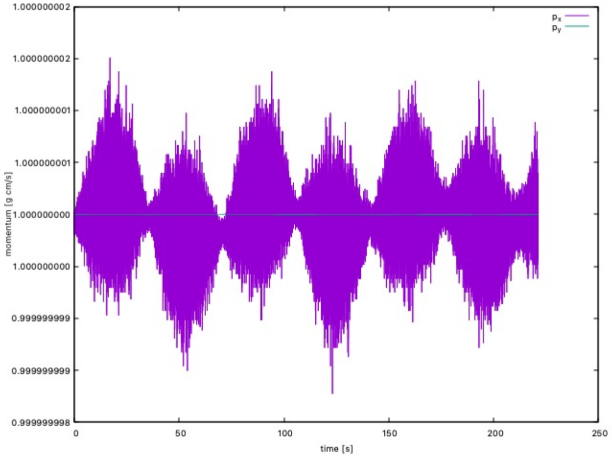


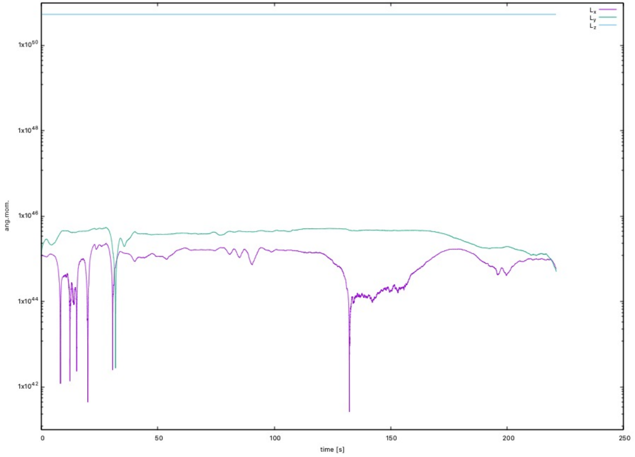
BWD Conservation



Binary system energy versus time



Binary system momentum (x,y) versus time



Binary system ang. momentum (x,y) versus time

The total, kinetic, and internal specific energy of the system over the first 215s

The momentum in the x- and y-directions of the system over the first 215s

The angular momentum in the x- and y-directions of the system over the first 215s

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Summary

Summary

- SPH is uniquely suited to handle astrophysical systems
- Compact merger modeling has been implemented into the LANL SPH code FleCSPH
- Initial conditions and setup remain important
- Future Applications of FleCSPH at LANL:
  - Compare WD-WD mergers with established results
  - Post-process nuclear burning results from mergers
  - Systematic study of WD-NS mergers and accretion disk formation