Memo 3: Analysis of the Cassini Oct 2004 Experiments

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ABSTRACT

The Cassini spacecraft and four nearby calibrators was observed at 8 GHz on October 14, 16, 17, 19 and 20. From a comparison of the image position among the quasars and the days, a nominal position accuracy of ≈ 0.5 (mas) D(deg) $\mathrm{H}^{-0.5}$(hr), where D is the calibrator-spacecraft separation and H is the observing length, was obtained. At 8 GHz, calibrator-spacecraft separations more than about 4° are not recommended. Various subselection of data show little position degradation down to 3-min observations, or with six VLBA telescopes as long as the northern telesopes (BR, NL) are included. The use of multi-calibrator observations in order to remove the phase gradients can obtain position accuracies about a factor of three smaller (with caveats) when additional calibrators are used. The results are used in a Memo 4 to determine a conservative observation scheme for VLBA observation of spacecraft at 32 GHz and to search for suitable calibrators.

1. Phase Referencing the Cassini October 2004 Observations:

Five Cassini experiments were made in October 2004 in order to investigate the positional accuracy and variations among the days using the derived images. An analysis of the total delays in now in progress at JPL. Days 3 and 4 have three-hour observations, whereas days 1, 2 and 5 have one-hour observations. The calibrators are not very close to Cassini, none of them $< 2.5^\circ$. Their relative locations are shown in Fig. 1.

All source scans were one minute in length, and Cassini was observed every other scan. The main ICRF calibrator J0748 was observed once every 4 minutes and the other three calibrators were observed once every 12 minutes. One observation of the bandpass calibrator was made every hour. The quasars were observed with an 8 MHz bandwidth centered on the Cassini frequency of 8.4307 GHz. The Cassini signal covered a bandwidth of about 0.3 MHz. The calibrations are discussed in Memo 1 and the AIPS reduction script used to reduce the October 17 data set are given. The spacecraft was used as the phase reference source.

1.1. Position Accuracy

After calibration of the temporal phase using Cassini, images were then made of the four quasars for each observation day. Fig. 2 shows the derived position of each of the four calibrators, as measured by their location of the peak intensity, on the five observing days. Two of the calibrators, J0748 and J0802, have positition uncertainties less than 0.5 mas; hence their offset of about 5 mas north indicates that the delay model used for Cassini in the VLBA correlator was about 5 mas south of its true position. Analysis at JPL of the total delays can determine if this offset is accurate, or is produced by an inaccurate calculation in the correlator for the spacecraft position. It is possible that the Cassini position provided by JPL may be accuratly tied to Saturn to an accuracy of 0.2 mas, but that the ephemeris position of Saturn may be in error by 5 mas with respect to the ICRF frame.

There is clearly a relative displacement of the assumed positions among the four calibrators, particularly for J0743 which is more the 2 mas west of the other three calibrators that are within 1.2 mas of each other. The position uncertainties from the ICRF catalog are 0.25, 0.6, 2.5, 0.4 mas for J0748, J0750, J0743, J0802; respectively, in each coordinate; hence their separations are consistent with these errors. Although the ICRF grid is defined to < 0.1 mas, the accuracy of any one quasar is considerably less. This problem will be discussed in more detail in Memo 4.

The most important point from Fig. 2 is the scatter of the position of each calibrator among the days. On October 17, 19, and 20 the rms scatter is about 0.2 mas for the calibrators about 3° away from the spacecraft, and about 0.5 mas for the more distant calibrator. On October 14 and 16, the rms scatter is more than twice as large. This shows that the accuracy of the position measurments can vary by over a factor of two, and depends on the mean weather conditions over the array telescopes, and the average elevation of the observations. Combining this analysis with that of the September 2002 Jupiter deflection experiment, we find that the average position error S in mas is

$$S = 0.35 \text{ to } 0.75\ D\ H^{-0.5};\ H > 1 \tag{1}$$

where D is the target-calibration separation in degrees, and H is the observations length in hours. Comments on the obseravation length are given in the last section.

1.2. Image Quality

The images, after cleaning, for all four sources, after phase calibration using Cassini, are shown in Fig. 3. The cross shows the nominal position of each of the source as determined by averaging the position for the five days, with extra weight given to Oct 17 and 19 with the longer observations. These positions should be accurate to 0.2 mas or less. *Because of the reciprocity between target and calibrator phases, these are the image structures that would be obtained for Cassini (mirror-reflected) if the indicated quasar were used as the reference source.*

Although the four calibrators are close to point sources, the images are far from point-like. The peak flux density at 8 GHz ranges between 50% and 80% of the true source peak flux density. The structures tend to be doublish with unequal components separated by as much as 2 mas. The images on October 14 show the most distortion, whereas the images on October 17, 19, 20 are less distorted and the brightness peak lies within 0.5 mas of the nominal (average among the days) position. The structure comparison associated with sources on the opposite side of Cassini are consistent with the effects of a phase gradient in the sky. For example, the structure distortion of J0748 and J0802 are antisymmetric with respect to their nominal positions. The distortions and offsets between J0748 and J0743 are similar since they are reasonably close in the sky.

Given a phase-distorted image, what is the best estimate of the true position, using the image alone? The emission centroid, rather than the peak, is generally a somewhat better approximation of the true (really nominal) position, than the location of the peak intensity. However, there are exceptions on October 14 for J0750 and J0743 when the centroid position is even further from the true position than that of the peak intensity. The algorithm in Eq. 1 was derived mainly from the scatter of the quasar positions.

1.3. Data Sub-Selection

We also made images for the October 17 data for the entire session of three hours, for 20 minutes and for 1 minute of data. Remarkably, there is little change in the source distortion or position, as seen in Fig. 4. This is not unexpected since the u-v coverage and the systematic phase errors do not change much over periods less than a few hours. Thus, with the very fast cycling between spacecraft and quasar, in principle three minutes of data (quasar-spacecraft-quasar, each for one minute) is sufficient to determine their relative positions! Of course redundancy is important in determining whether a very short span of data is unaffected with anomolous errors, and observations of additional calibrators also add confidence to the results. In addition, the one-minute and 20-minute periods were chosen in the middle of the session, whereas the 3-hour data covered more low elevation data which is of lower quality. These results suggest that 30 minutes of observations should be sufficient to determine an accurate spacecraft position, especially if the observation period is chosen at the highest source elevations. It may be possible even with the present slow ftp lines to send three to five minutes of telescope data to the correlator in order to obtain a preliminary spacecraft position within a few hours after the experiment.

Experimentation with a subset of telescopes has been analyzed in Memo 4. Because the longer baselines tend to have large phase errors (mainly caused by the lower elevation of the source as seen by these telescopes), the astrometric accuracy by including the longest baselines is not significantly increased, and the image quality may, in fact, be decreased. As shown in Memo 4, at 32 GHz where the calibrator source separation is more than 3°, more reliable spacecraft positions can be obtained by omitting MK, SC and HN from consideration. However, in order to have a psr which does not have large sidelobes and reasonable north-south resolution, the inclusion of north-south baselines is important. Hence, a subarray of the VLBA should include BR and NL. Perhaps, a minimal subarray would be BR, NL, OV, KP, PT (LA), FD. More investigations are needed for observations of spacecraft below $\delta 10^\circ$ to gain more confidence in using only six elements of the VLBA.

2. Correction of Phase Errors using Several Calibrators:

It is clear that the interactions of the phase errors (mainly the model errors of the correlator tropospheric model) produce the complicated structures changes and shifts of the position of a source from day to day. These cause of these phase residuals are discussed in Memo 1 and were designated as d_r, and represent the phase errors which have an angular dependence in the sky. For the October 17 observations these phase errors are shown in Fig. 5. For HN and MK there is a significant departure of the source phases with time in a relatively systematic manner; the rms noise component is relatively small. At the beginning of the observation period, the elevation of the sources at HN and MK were 65° and 35°, respectively. At the end of the observation period, the elevations at HN and MK were 35° and 65°, respectively. Hence, the phase offset is closely tied to source elevation for the antenna. Only for NL did the elevation drop below 45° during any part of the experiment. A similar residual phase behavior was also observed on all days for MK except for October 14 which had significant phase errors for most of the telescopes.

A closer look at the residual phases for MK is shown in Fig. 6. The phase residuals for the four sources are, in fact, consistent with that caused by a phase gradient in the sky. The phase gradient for MK can be fit approximately to a change in the atmospheric zenith path length of 20 cm. Clearly, the large phase gradients are associated with relatively low elevations when small errors in the assumed zenith path delay produce large errors in phase. Since any atmospheric phase error varies as sec(z) where z is the observations zenith angle, the phase difference among source will vary as d(sec(z))/dz = tan(z)sec(z). For zenith angle greater than about 50°, this error term increases dramatically. Thus, the systematic phase offsets for HN, MK and SC are more a reflection of the low elevation of these observations rather than a larger error of the tropospheric model compared with the other telescopes.

This type of phase error is often noticed for MK and it is believed that the model troposphere used in the correlator for MK may have a systematic bias. In four of the five observation sessions in October, the phase error for MK was similar (Oct 14 had anomolous phase errors for several telescopes). A correction of the MK tropospheric model delay of 20 cm corrected about 75% of the phase error seen in Fig. 6. The MK anomoly should be investigated in more detail, using several calibrators (which can remove the phase error), or by special observations between MK and Kokee, a telescope at a lower elevation in the Hawaiian Islands.

This phase gradient over each telescope can be corrected using the AIPS task ATMCA. This routine assumes that the relative position of the calibrators are accurate (more on this later), and interprets the residual phases among the calibrators as those associated with a phase gradient over each antenna in the vicinity of the sources. In this procedure, the gradient is not limited to the elevation direction. The occasional lobe ambiguities (J0750 at the beginning) can be easily interpreted. After application of ATMCA, the revised residual phases for the antennas, similar to Fig. 5 are shown in Fig. 7. The phases residuals for HN and MK are significantly reduced and the image coherence of all of the calibrators is now > 90%. The scatter among the source positions for each of the days is shown in Fig. 8. The rms scatter of the source positions is now much smaller than before the use of ATMCA, with an rms of < 0.2 mas, except for J0802, and this is disturbing.

The source J0802 shows an position change of 0.6 mas toward the north-west, even when the Oct 14 observation with the poorest phase behavior is removed. This cause of this apparent motion was investigated. Although the source has some extended structure, removing the structure dependence for ATMCA (and that of J0748) made virtually no difference. Slight adjustments of the assumed position of the sources also made no difference; that is, the position of J0802 still showed the drift, while the others sources remained stationary. Any structure changes in J0802, based on good quality self-cal images for each day, show no significant changes to the level 0.1 mas. If only J0748 and J0802 are used in ATMCA (they are on opposite sides of the Cassini), the apparent motion of J0802 is decreased by a factor or two. The conclusion is that this source, the most southern of the group with the lowest elevation during the observations, is affected by phase curvature in the region of sky near the sources. The apparent motion during the period is just fortuitous.

In order for ATMCA to be effective, the apriori positions of all of the secondary calibrators are assumed to be accurately known. For the Cassini observations, we assumed the average position from all of the days, with extra weighting to those on October 17 and 19. What happens if there are errors in the relative position of the calibrators? Can ATMCA still be used, perhaps iteratively, to also determine better source positions as well as phase gradients in the sky? This data set with significant calibrator position uncertainties offers a chance to answer this question.

The answer is **no** for short observations less than a few hours. We processed the observations of October 14 using ATMCA and the positions derived for the calibrator observations on this day alone, and they were quite far from the nominal positions. We found that it is difficult to separate the true phase gradients in the sky from those produced by source position errors. In other words, there is a relatively strong correlation between tropospheric phase gradients and position offsets, which cannot be separated with a short observation period. Although some source position errors derived on October 14 were more than 1.0 mas, the source positions derived after use of ATMCA and reimaging change by less than 0.2 mas, and half the time in the opposite direction. Similar results were obtained with the same processing for the October 16 data set with better phase stability. For this reason, we will suggest below that if multi-calibrators are used for spacecraft tracking, the relative calibration positions must be accurately determined first.

3. Analysis of the Total Delays:

In Memo 1 a comparison in the derivation of the spacecraft position—by using the position of the peak intensity on the image, or by analyzing the total delay difference between the spacecraft and a quasar—was given. *Both methods require an image in which the approximate position of the spacecraft can be determined.* In order to obtain a result which is independent of the VLBA correlator, the residual spacecraft phases, converted to delay, is added to the VLBA correlator models (plus other delay components found during the reductions), to determine to observed total delay. Memo 2 describes these two analysis types, using the residual image and using the total delay, for the Mer-B observations in Jan 2004.

A similar analysis should be done with the Cassini, October 2004 data. Total spacecraft and quasar delays were determined and these were sent to JPL for analysis. As of Jan 10, 2005 an analysis of the total delay data had not yet been made. From private communications from the navigation group, there was general agreement that the offset of the Cassini spacecraft from the JPL orbit was similar to that obtained from the images.

4. Robustness of the Position Determinations

Based on all of the VLBA observations of spacecraft, the robustness of the position determinations can be estimated. These results are based on 8.4 GHz observations. Extrapolation to the anticipated 32 GHz observations are given in Memo 4.

4.1. Extreme Spacecraft Characteristics

Inaccurate predicted orbital information has been indirectly tested on several spacecraft with inaccurate orbits. Because of the image forming properties on the VLBA, spacecraft offsets of 500 mas from nominal can be reliably obtained before the spacecraft image is smeared because of the sampling in time and and frequency. With the presents signal strengths of the spacecraft, a search area as large as $30''$ can be made if needed.

The signal strength of the spacecraft monitored by the VLBA have been very strong. The approximate signal to noise (SNR) in 2-sec integration for Cassini is 50:1 in a 1-MHz bandwidth. For a typical quasar of 0.5 Jy over a 16 MHz bandwidth, the SNR is 10:1 in 2-sec. In order to phase-reference successfully, the SNR of the quasar or the spacecraft should be $> 5 : 1$ over one minute to follow the instrumental phase; the SNR of the other source only need be $> 2 : 1$ over one minute, so that a 30-min observation would give a reasonably good position accuracy. Thus, there is a margin of a factor of 50 for the spacecraft signal to be used as the phase reference source. As will be discussed in Memo 4, in order to find a quasar at 32 GHz within one or two degrees from the spacecraft, the VLBA sensitivity will have to be increased.

We have monitored one spacecraft (MGS) in which the spacecraft tracking signal changed about 1.5 MHz between two different states. The image position and total delays of the spacecraft before and after the change of telemetry state showed no discontiuity to a level of 0.1 mas. These signals were, however, relatively broad-band, covering about 0.5 MHz each. We have had no VLBA observations using an unmodulated carrier, although interferometry theory suggests that the correlation properties (hence the imaging and delay reconstruction) will be the same for an incoherent source and a pure tone. There may, however, be some difference in the stochastic noise associated with the correlation properties of a pure tone that may have some effect on the error analysis.

4.2. Southern Declinations

The position accuracy and reliability of VLBA observations of spacecraft will degrade in the south. Based on the VLBA experience with quasars, position errors will increase for $\delta = -25^\circ$ by a factor 1.5 east/west because of larger troposphere errors and by a factor of 3.0 north/south because of the lower resolution in this direction. The loss in position error is less important than the possibility of extremely poor images because of the interaction of the relatively poor PSR and the larger tropospehring noise. As will be discussed in Memo 4, the importance of finding small spacecraft and quasar separations for phase referencing increases for spacecraft with $\delta < -10^\circ$.

The addition of a telescope in Australia or South Africa would also improve the position determination of spacecraft in the southern sky. These observations, to be effective, must be at least three hours in extended since only the longer baselines will be most effective in the position determination and good (u-v) coverage and troposophere averaging will be necessary.

4.3. Missing Telescopes and Short Observations

The results in §1.3 show that the entire VLBA is not critical for the determination of accurate spacecraft positions. The longest baselines often have the largest phase errors and produce somewhat more distortion in the images. The longest baselines can be removed, but inclusion of sufficient north-south resolution must be available, hence BR and NL must be included, and HN as well if the spacecraft is not too far south.

Observations as short as a few minutes may also provide good positions if the spacecraft signals are even only 1% of the present Cassini signal strength. But, there is no redundancy to determine on the accuracy of the data. However, this small volume of data could be transported by ftp to the correlator in a short period of time, and could be used as a quick, approximate determination of the spacecraft position within a few hours of the experiment. Observations of 30 minutes, split into two 15-min segments and separated by at least one hour may be the most efficient observing method in determining accurate positions with minimal data.

The main criterion in the determination of accurate positions is whether the *image* of the spacecraft or calibrator is sufficiently well-defined in order to determine an unambiguous position source. Even in the analysis of the total delays, the image quality and its offset is critical. Although short observation sessions with only six VLBA telescopes and one calibrator will usually produce a position accuracy which is nearly as good as a longer observation, there is little redundance and the ability to recover from poor weather conditions or low elevation observations.

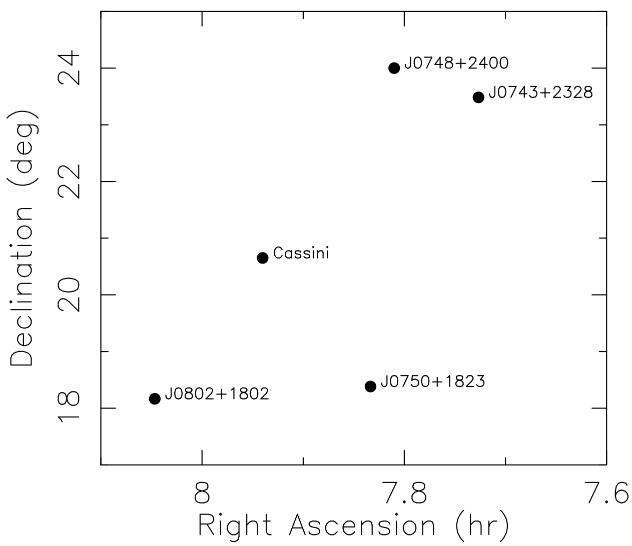

Fig. 1.— **Cassini-Calibrators Configuration:** The locations of the calibrators and the Cassini spacecraft in the sky during the October 2004 experiments. The spacecraft moved less than 0.1° during the observation period.

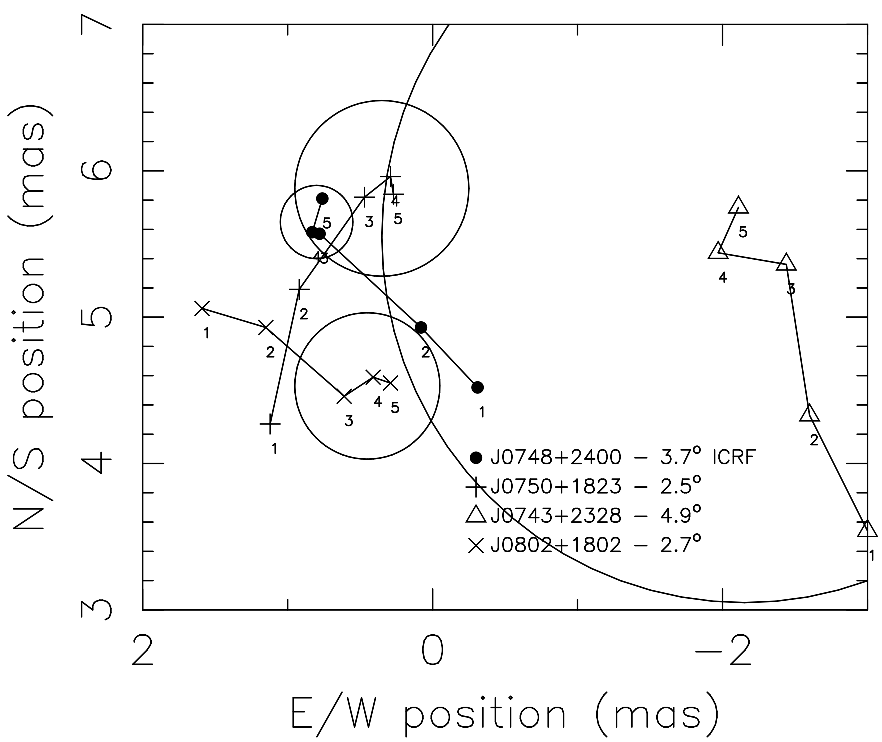

Fig. 2.— **The Locations of the Four Calibrators on the Five Observing Days:** The image location of each calibrator, designated by the symbols, are plotted for each of the observing days: 1=Oct 14, 2=Oct 16, 3=Oct 17, 4=Oct 19, 5=Oct 20. Cassini was used as the reference source. The circles, centered around the weighted average position over all the days, have with a diameter equal to the position uncertainty from the ICRF/VLBA catalogs.

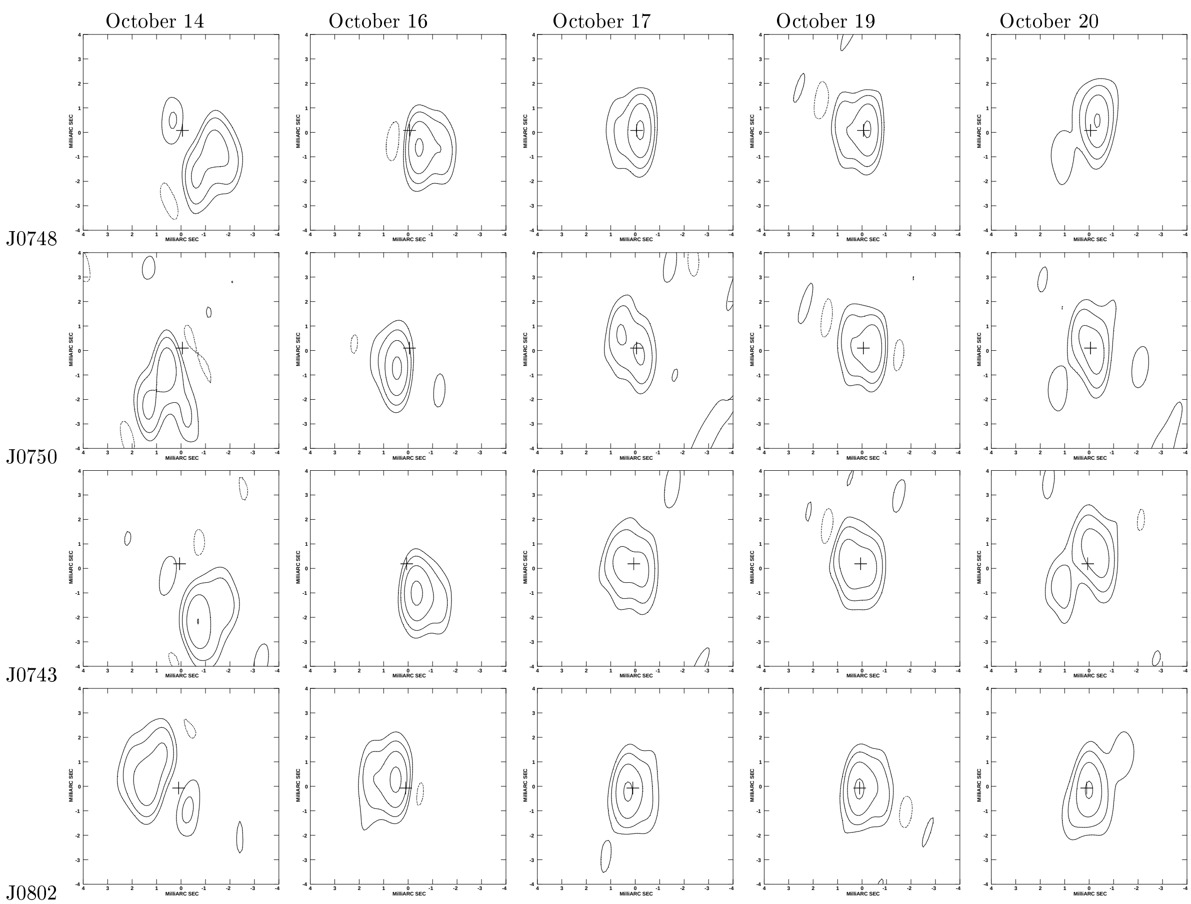

Fig. 3.— **The Images of the Calibrators:** The images of each of the four calibrators, after phase referencing to Cassini, are shown for each of the five days. The cross shows the weighted average position (Oct 17,19,20 were best) for each source, derived from Fig. 2. The contour levels are at -10%, 10%, 20%, 40%, 80% of the peak of the self-cal image of the sources.

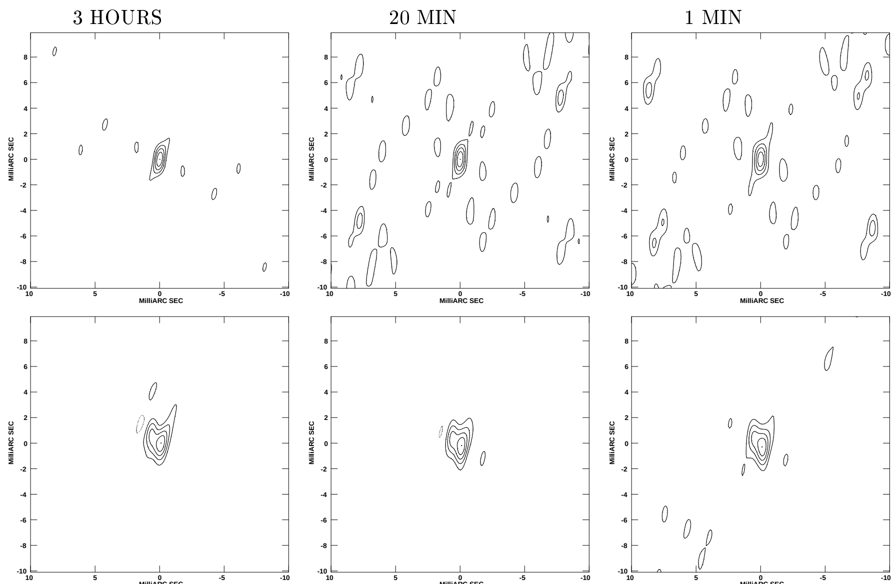

Fig. 4.— **Effect of Observation Length on Image Quality:** The upper panel shows the point-spread-function for the October 17 observations using the entire 3 hours of observations, 20 minutes near the mid observation point, and 1 minute near the mid observation point. The contour levels are 20% of the peak. The lower panel shows the cleaned image of J0748+2400 after calibration with Cassini, for the same integration periods. The contour levels are at 20% of the peak flux density. The source is unresolved and the distortions are caused by phase errors which persist over the observing day. See Fig. 5 which shows the representative phase errors.

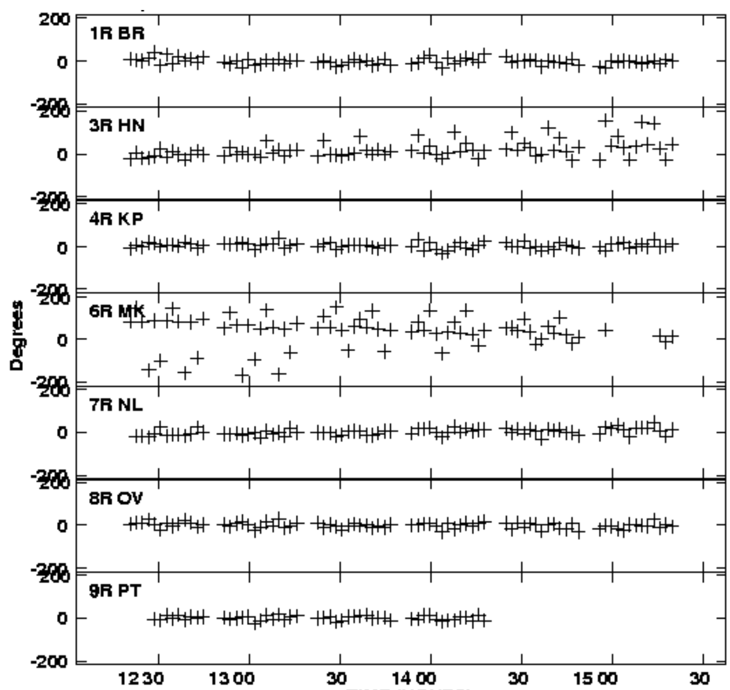

Fig. 5.— **The Residual Phases for the Calibrators:** The antenna-based phase residual, after phase referencing to the spacecraft and assuming the average source positions, are shown for the October 17 observations.

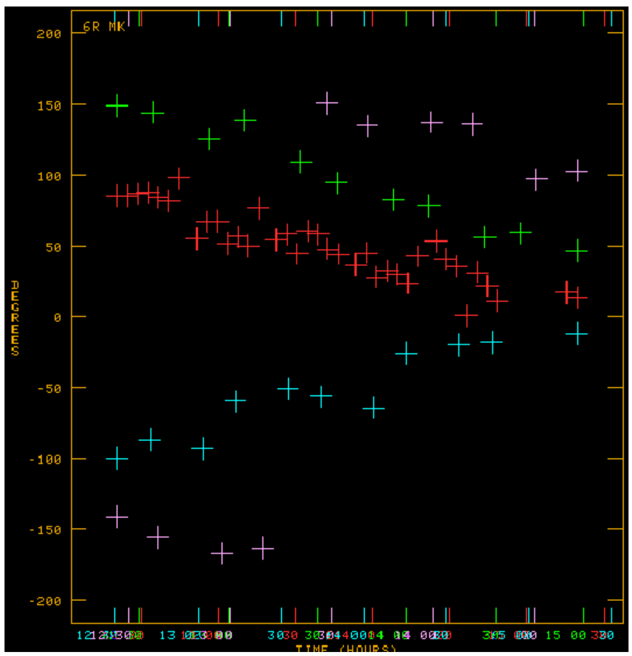

Fig. 6.— **The Residual Phases for MK:** A blow-up of the MK phases in Fig. 5. Red is J0748, light blue is J0802, purple is J0742, green is J0750. The sources J0748 and J0802 are on opposite sides of the calibrator, so it is expected that their residual phases would be asymmetric if produced by a phase gradient in the sky.

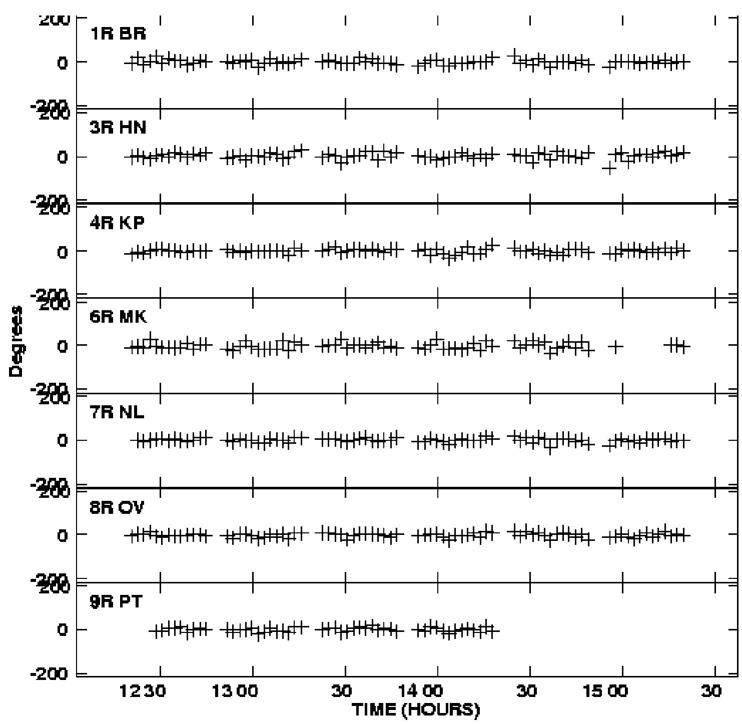

Fig. 7.— **The Residual Phases for the Calibrators after application of ATMCA:** The antenna-based phases are shown for the October 17 observations, using the best positions for the sources, and removing the phase gradient with ATMCA. Compare with Fig. 5.

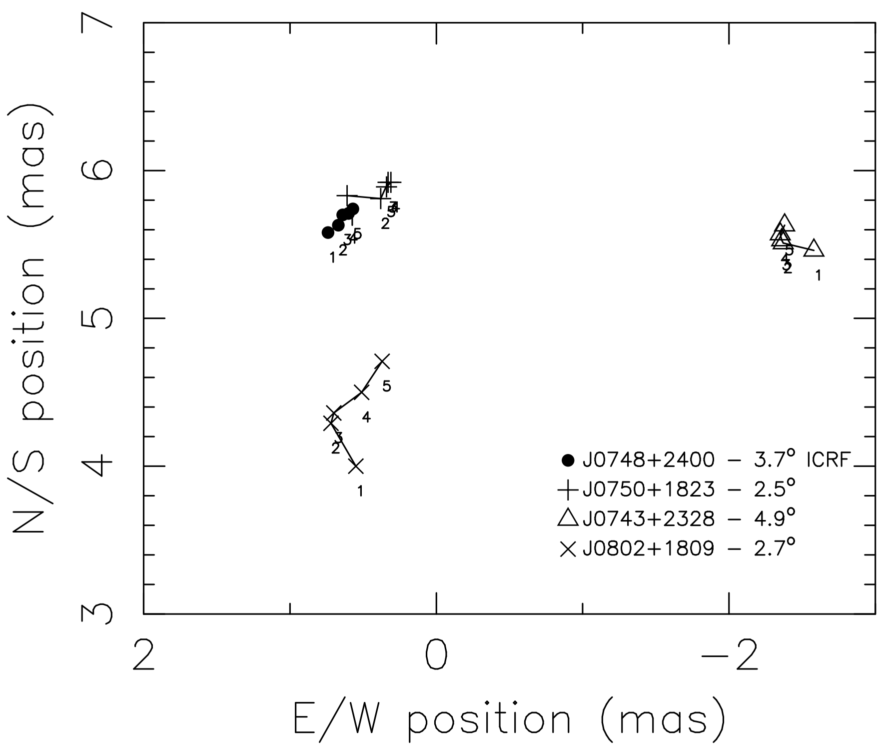

Fig. 8.— **The Locations of the Four Calibrators on the Five Observing Days after calibration with ATMCA:** The location of each calibrator, designated by the symbols, are plotted for each of the observing days: 1=Oct 14, 2=Oct 16, 3=Oct 17, 4=Oct 19, 5=Oct 20. Compare with Fig. 2.