Astronomy 12 — Spring 1999 (S.T. Myers)

Homework Set 8

Due Thursday April 15, 1999 (5pm)

1. The galaxy NGC 2639 is an Sa galaxy with a measured maximum rotational velocity of 324 km s^{-1} and an apparent magnitude of B = 12.22 mag (after making corrections for any extinction).

(a) The B-band Tully-Fisher relation for Sa spirals is

$$M_B = -9.95 \log v_{max} + 3.15$$

for maximum velocity in km/s. Use this relation to estimate the absolute B magnitude of NGC 2639.

(b) Determine the distance to NGC 2639 using its distance modulus.

(c) It is customary to measure a radius R_{25} to where a spiral galaxy's surface brightness falls to a level of 25 B-mag arcsec^{-2}. (Observationally, surface brightness is equivalent to intensity, and is measured in the peculiar units of magnitudes per arcsec^2, or the brightness from a star of that apparent magnitude in a square arcsecond.) Spiral galaxies appear to follow a standard relation

$$\log R_{25} = -0.249\, M_B - 4.00$$

for R_{25} in kpc. What is R_{25} for NGC 2639 in kiloparsecs?

(d) In the B band, the night sky has a brightness of $\mu_{sky} = 22$ B-mag arcsec^{-2}. If you want to measure a galaxy profile down to R_{25} ($\mu = 25$ B-mag arcsec^{-2}), how many times *fainter* than the sky background is this? This is why its hard to do photometry from the ground.

(e) To measure the mass of a galaxy, we can use Newton's Law (or Kepler's Law)

$$v^2 = \frac{GM}{R}$$

to give the mass inside some radius R given a rotational velocity v. Find the mass in $M_\odot$ within R_{25} for NGC 2639, assuming that the rotation curve is flat outside a few kpc.

(f) The Sun has an absolute visual magnitude of $M_V = 4.83$ and a color index of $B - V = 0.64$. Find M_B of the Sun, and use this to determine the luminosity of NGC 2639 in the B band, in $L_\odot$.

(g) Compute the mass-to-light ratio of NGC 2639 in the B band (in $M_\odot/L_\odot$) interior to R_{25}, using the numbers found above. Is this consistent with what we have learned about spiral galaxies?

2. The Virgo cluster of galaxies is the nearest substantial cluster. This cluster extends over nearly 10 degrees on the sky, and contains a number of bright galaxies. Our first task is to find the distance to Virgo. The following table gives a list of distance indicators and the results and uncertainties $d_i \pm \sigma_i$ for the Virgo cluster (adapted from Jacoby et al. 1992). More details on these specific indicators can be found in the online web version of this problem set.

	Method	Virgo Distance (Mpc)
1.	Cepheids	14.9 ± 1.2
2.	Novae	21.1 ± 3.9
3.	Planetary Nebula Luminosity Function	15.4 ± 1.1
4.	Globular Cluster Luminosity Function	18.8 ± 3.8
5.	Surface Brightness Fluctuations	15.9 ± 0.9
6.	Tully-Fisher relation (spirals)	15.8 ± 1.5
7.	Faber-Jackson/D-σ relation (ellipticals)	16.8 ± 2.4
8.	Type Ia Supernovae	19.4 ± 5.0

Derive an average distance by computing the *weighted mean*

$$d_{avg} = \frac{\sum_i (d_i/\sigma_i^2)}{\sum_i (1/\sigma_i^2)}$$

with the sums running over the eight methods $i = 1$ to 8. Note that this is derived by minimizing the chi-square

$$\chi^2 = \sum_i \left(\frac{d_i - D}{\sigma_i}\right)^2$$

and setting $d_{avg} = D$ at the minimum where $d\chi^2/dD = 0$.

The uncertainty in our estimated d_{avg} is given by finding where χ^2 increased by 1 above the minimum. This is given by

$$\sigma_d^2 = \frac{1}{\sum_i (1/\sigma_i^2)}$$

assuming the uncertainties really reflect Gaussian errors. What is the uncertainty in our average distance estimate to Virgo, in Mpc?

3. Spectra of galaxies in the Virgo cluster indicate an average recession velocity of 1136 km/s for the cluster. When combined with the estimated distance from Problem 2, what is the value of the Hubble constant H_0, assuming $v_r = H_0 \cdot r$, that is obtained?

Because Virgo represents a significant mass in our corner of the Universe, we must correct the velocity of Virgo for our motion *towards* the cluster, estimated to be around 168 km/s. What does this correction give us for the value of H_0?

Recent parallaxes from the Hipparcos satellite give some indication that our distance scale may be too short, and should be increased by 10%. If we make this correction to our average distance to Virgo, what is the new value of the Hubble constant that we derive?

What is the *Hubble time*, $t_H = H_0^{-1}$, in years, assuming this H_0?

4. The Virgo cluster contains hundreds of galaxies moving around under the influence of its gravitational potential. The radial velocity dispersion of the Virgo cluster galaxies is $\sigma_r = 666$ km/s (is there a demonic plot here?). The virial theorem implies

$$M \approx \frac{5\sigma_r^2 R}{G}$$

where the factor 5 comes from assuming Virgo is a uniform density sphere. Using this relation, and assuming that the Virgo cluster is approximately a sphere of radius $R = 1.5$ Mpc, what is the total gravitational mass of the Virgo cluster, in $M_\odot$?

How long, in years, would it take a galaxy to cross the Virgo cluster? Assume the *rms* velocity σ_r. How does this compare to the Hubble time?

What is the Hubble velocity difference between opposite points on a diameter of the Virgo cluster? How does this compare to σ_r? How is this related to the timescales we just computed?

5. Like many large cluster such as Coma, the Virgo cluster contains a hot intergalactic medium (IGM), which is gas at an extreme temperature of 70 million K. This means that Virgo is emitting X-rays via the *thermal bremsstrahlung* (free-free) process that we discussed in class for the much cooler H II regions.

(a) The luminosity comes from electron-ion interactions, and can be shown to have an volume emissivity

$$\epsilon_{ff} = 1.42 \times 10^{-40}\ n_e^2\, T^{1/2}\ \mathrm{W/m^3}$$

where $L \sim \epsilon \cdot V$. If the X-ray luminosity of Virgo is $L_x = 1.5 \times 10^{36}$ W, what is the average electron number density n_e (in m^{-3}) in the cluster IGM?

(b) Again modeling Virgo as a uniform density sphere of radius 1.5 Mpc, what is the total mass (in $M_\odot$) of the IGM in the cluster, assuming that it is composed entirely of ionized hydrogen?

(c) What fraction of the gravitational mass is accounted for by the hot IGM?

(d) If the optical luminosity of the galaxies in the cluster give a total of $L_V = 1.2 \times 10^{12}\ L_\odot$, what is the mass-to-light ratio in the V-band for the Virgo cluster? If an "average" galaxy has a mass-to-light ratio of 10 $M_\odot/L_\odot$ in V-band, what is the approximate mass in galaxies in the cluster? How does this compare to the hot gas mass of the IGM?

(e) For the IGM gas, the average kinetic thermal energy per particle is

$$K = \frac{3}{2}kT$$

for both the electrons and protons. How long will it take to radiate this energy away in X-ray emission at the rate given above? How does this *cooling time* compare to the Hubble time? (Note: if the cooling time is less than the Hubble time, the cluster would cool off and stop emitting X-rays before it can grow in size. If the cooling time is longer it can stay hot indefinitely as it gets heated by infalling galaxies.)

6. How much energy can we get out of an accreting supermassive black hole? To answer this, we recycle the idea of the *Eddington luminosity* that we used to calculate the limiting luminosity of a big star:

$$L_{Ed} = \frac{4\pi G M c}{\kappa}$$

while for electron (Thomson) scattering

$$\kappa = \frac{\sigma_T}{m_H} = 0.04\ \mathrm{m^2 kg^{-1}}$$

which is just another way of saying if the luminosity is too high, the radiation pressure overcomes the gravitational attraction of the massive object (see Homework Set 4, Problem 3). Calculate the Eddington luminosity for a 10^8 $\mathrm{M}_\odot$ black hole.

The energy comes from the gravitational binding energy of the accreting matter. Derive an expression for the luminosity L of matter accreting at a rate $\dot{M}$ (in $\mathrm{M}_\odot$ per year) from infinity to a radius equal to $R_{acc} = qR_{sw}$, a multiple q of the Schwarschild radius for our black hole of mass M. Equating this to the Eddington luminosity for our black hole (scale to $M/10^8\ \mathrm{M}_\odot$), find an expression for the *Eddington accretion rate* $\dot{M}_{Ed}$ in terms of q and $M/10^8\ \mathrm{M}_\odot$. What is the Eddington accretion rate (in $\mathrm{M}_\odot$ per year) for our 10^8 $\mathrm{M}_\odot$ black hole, assuming the gas gets to $3\,R_{sw}$ ($q = 3$)?

The Eddington accretion rate is the maxiumum rate you can feed a black hole as radiation pressure keeps you from feeding it faster. Furthermore, observations show that active galactic nuclei (AGN) are emitting light at just this rate. Therefore, this is how fast you must be feeding stars into a supermassive black hole in a quasar or radio galaxy to keep it luminous!