Astronomy 12 — Spring 1999 (S.T. Myers)

Homework Set 6

Due Thursday March 25, 1999 (5pm)

1. White dwarfs are made mostly of degenerate Carbon. Consider a white dwarf of radius 7000 km, and an effective temperature of 30,000 K. What is the luminosity of this object (in W and in $L_\odot$)?.

If this white dwarf has a mass of 1 $M_\odot$, calculate the total internal thermal energy (assume thermal energy of kT per atom). Use this to deduce a *cooling time*, the time over which it will radiate away its current thermal energy at its current luminosity.

We expect a white dwarf to be formed with an initial temperature equal to that of a (massive) stellar core at the end of triple-alpha, $T \sim 10^9$ K. Assuming the same radius as above, calculate the luminosity, thermal energy and thus the cooling time.

How do these cooling times compare to the age of our galaxy, 10^{10} years? Set up an equation giving the cooling time as a function of temperature, and use this to find the temperature for which the cooling time of the white dwarf is approximately the age of the galaxy. This is the coolest white dwarf that should be around today.

2. Consider a neutron star with mass $1.4M_\odot$ and radius of 10 km. Estimate the total gravitational potential (binding) energy of the neutron star.

Gamma ray bursts are events detected by satellites where a "burst" of high energy gamma rays are seen over a timescale of less than 1 second. One possibility is that these are caused by the collision of two neutron stars. Assume that the result of this collision is a single neutron star with twice the mass of the original, and calculate the new radius of this object by noting that the mass-radius relationship for a degenerate object

$$R \propto M^{-1/3}$$

should hold for neutron stars as well as white dwarfs, as they are supported by degeneracy pressure.

Calculate the binding energy of the merged object, and compare it to the sum of the binding energies of two initial neutron stars. How much energy will be liberated in the collision, assuming only the change in binding energy? Will the new object be a neutron star, or a black hole?

3. Consider a pulsar of mass $1.4M_\odot$ and radius of 10 km, with a rotation period of 0.033 seconds (like the Crab nebula pulsar). The strong magnetic field will cause ionized gas near the neutron star to rotate along with the pulsar. At what distance from the center of the neutron star will the co-rotation velocity equal the speed of light? Note that beyond this radius, known as the *light cylinder*, matter cannot be induced to co-rotate with the pulsar, and thus defines a limit to the region of strong influence of the neutron star.

As we disussed in class, high-luminosity stars expel material in stellar winds. The system Centaurus X-3 has an O6 primary (mass of $10.5M_\odot$) and a neutron star secondary (mass of $1.4M_\odot$, radius 10km, rotation period 4.84 sec) with a binary period of 2.087 days. Compute the orbital radius of the binary, and the radius of the light-cylinder of the neutron star. If the wind is isotropic (uniform and spherically symmetric), then we can assume that the part of the spherical wind that hits the light cylinder (a disk of that radius) falls onto the neutron star and releases its energy. Draw a sketch to illustrate the geomtery.

Using a typical O-star mass loss rate of $10^{-8}M_\odot$ per year, estimate the accretion luminosity of the wind material (in Watts and in $L_\odot$) that we might expect to be emitted in X-rays.