Astronomy 12 — Spring 1999 (S.T. Myers)

Homework Set 1

Due Thursday January 28, 1999 (5pm)

Revised 27Jan99, due date extended to 01Feb99.

Make sure that your work is neat, legible, and complete. If we can't read it, it can't get credit. It is important to show all your work, and to tell where you got any numbers that you used in the problem (given in the question, from some page of the text, or from the tables in the Appendix, etc). These homework sets are as much about teaching you how to go about using your astronomical knowledge to solve the problems as they are about getting the right answer. There will be partial credit, but if we cannot follow your work, we will not give you credit. You may work together on homework assignments, but you should all contribute and not just copy each others work. Use your good judgement and follow the honor code.

To ease you into problem solving, I am giving you some hints in this assignment. These problems include explanatory text to place them in an astronomical context, and not all information is directly relevant to the problem, and you may have to look up necessary information — this is not meant as a trick, but to give you a feel for actual scientific problem solving as it occurs in research. Whenever possible, use drawings to diagram a problem as a visual aid (as I do on the blackboard in lecture).

1. The goal of this problem is to demonstrate the relation between the *intensity* and surface flux from a blackbody. Note that in this case, we are computing a radiated intensity and flux, and so the relevant solid angle is π steradians.

In lecture, we discussed the blackbody specific intensity function

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\,\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

which has units of W m^{-2} Hz^{-1} sr^{-1}. The Sun has an effective brightness temperature of $T = 5770$ K. What is the wavelength (in Å) at which the blackbody emission peaks? (Hint: use Wien's Law.)

Compute the frequency corresponding to the wavelength of the blackbody maximum. At this wavelength (frequency) what is the value of the intensity B_ν of the blackbody emission? (Be sure to use the flux per unit frequency B_ν, and not the flux per unit wavelength B_λ.) Convert this into a flux density $F_\nu = \pi\, I_\nu$ and then estimate the total flux $F \sim \nu\, F_\nu$ (the approximate flux per log-frequency interval or in the band from $\nu \pm \nu/2$, where ν is the frequency of the blackbody maximum).

The surface flux from a blackbody like the Sun should be given exactly by the Stefan-Boltzmann equation

$$F_{surf} = \sigma\, T^4$$

where $\sigma = 5.67 \times 10^{-8}$ Wm^{-2}K^{-4} is the Stefan-Boltzmann constant. Compare this exact result to the estimate we got above. We often plot νF_ν when we want to graph flux density from an intensity.

2. Sometimes, instead of computing radiated intensities (where the solid angle is effectively π steradians), we need to compute the *received* flux, where the solid angle is determined by the apparent angular size of the source of emission. Because the flux densities encountered in astronomy are so small, in radio astronomy we define the unit of flux density to be the Jansky (Jy), where

$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}.$$

If we observe an astronomical object, like a distant star, we need to calibrate our measurement. The temperatures of the solar system planets depend upon their distance from the Sun (as we found in ASTR011), with Jupiter having a temperature of approximately 122 K (see Appendix A3 of the text). At its closest approach to Earth, Jupiter is 4.2 AU away; Jupiter has a radius of 71400 km. What is the apparent angular radius (in arcseconds) and apparent solid angle (in sr) of the disk of Jupiter at this distance as seen from the Earth? (Hint: make sure to convert between radians and arcseconds correctly, and use the small angle approximation for angular diameter vs. distance.)

3. Suppose you are observing at a wavelength of $\lambda = 1$ cm (frequency $\nu = 30$ GHz. For Jupiter at T=122 K, calculate the value of the blackbody Boltzmann factor $h\nu/kT$. You should find that this is much less than unity, and thus you can use the Rayleigh-Jeans approximation:

$$B_\nu(T) \approx \frac{2kT}{\lambda^2} \qquad \frac{h\nu}{kT} \ll 1 \qquad \lambda = \frac{c}{\nu}.$$

Use this to calculate the blackbody intensity B_ν of Jupiter at 30 GHz, in units of Jy/sr (note the solid angle dependence).

Use the solid angle Ω of the disk of Jupiter at closest approach found in the previous problem to calculate the observed flux density (in Jy)

$$F_\nu = I_\nu \, \Omega$$

of Jupiter at this frequency. Then, multiply by the frequency ν to estimate the received flux

$$F_{rec} \sim \nu \, F_\nu$$

as we did for the emitted surface flux in problem 1. The power received by our telescope is then given by multiplying by the physical area A of the telescope

$$P_{rec} = F_{rec} \, A$$

for which we will assume has a cross-section given by a circular disk of diameter 10 meters (radius 5 meters) — this is actually a rather small radio telescope. The largest, Arecibo, is over 300 meters in diameter! Compare the power P_{rec} collected from Jupiter to power levels we are used to, like 1 Watt or even 1 mW (0.001W) like in most computer circuits. This is why radio astronomy is so difficult — the power levels are tiny!

We can now compare a measurement of a radio source to the brightness of Jupiter to get its actual flux density!

4. The star Sirius (α Canis Majoris) is the brightest star (other than the Sun) in the sky. It has an observed parallax of $p = 0\overset{''}{.}377$. What is its distance from the Sun in parsecs?

Sirius is actually is a binary star, with a faint companion. The apparent bolometric magnitude of the bright primary Sirius-A is $m_{bol,A} = -1.55$. What is its absolute bolometric magnitude M_{bol}? Assuming the Sun has an absolute bolometric magnitude of +4.75, calculate the luminosity of Sirius (in solar units $L_\odot$).

The secondary Sirius-B has an apparent bolometric magnitude $m_{bol,B} = +5.69$. Calculate the absolute bolometric magnitude and luminosity of Sirius-B.

5. The orbit of the Sirius system is nearly edge-on to our line of sight, and has an observed orbital period of 49.94 years. The relative orbit of the two stars is seen to have an apparent semimajor axis a of $7.''62$. Compute the orbital semimajor axis a in AU, and thus use Kepler's 3rd Law to find the total mass M of the system.

After careful astrometric monitoring of the system, the individual orbits of A and B about the barycenter of the system are measured, with a ratio of semimajor axes of $a_A/a_B = 0.466$. What is the ratio of the masses $\mathrm{M}_A/\mathrm{M}_B$? Use the total mass found above to find the individual component masses M_A and M_B (in solar masses $\mathrm{M}_\odot$).

By definition, our Sun has a mass 1 $\mathrm{M}_\odot$ and luminosity 1 $\mathrm{L}_\odot$, and thus the *mass-to-light ratio* for our Sun is $\mathrm{M/L} = 1\ \mathrm{M}_\odot/\mathrm{L}_\odot$. Use the above calculations to determine M/L for Sirius-A and Sirius-B. If all stars burned fuel at the same rate, you might expect that the M/L were the same for all stars. Speculate on what you just found means, both in terms of using the luminosity to infer the mass, and the lifetime of the star, if it burns fuel at the rate given by L and has a fuel resevoir proportional to M.