Astronomy 12 Spring 1999 (S.T. Myers)

Equations and Constants - Final Exam

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Constants

$c = 2.998 \times 10^5$ km/s

$\mathrm{G} = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

$\mathrm{h} = 6.626 \times 10^{-34}\,\mathrm{J\,s}$

$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

$\mathrm{k} = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$

$\mathrm{m_p} = 1.67 \times 10^{-27}\,\mathrm{kg}$

$\mathrm{m_e} = 9.11 \times 10^{-31}\,\mathrm{kg}$

$M_\odot = 2 \times 10^{30}$ kg

$R_\odot = 6.96 \times 10^5$ km

$L_\odot = 3.83 \times 10^{26}$ W

$T_\odot = 5770$ K

$M_\oplus = 5.98 \times 10^{24}$ kg

$R_\oplus = 6378$ km

$g_\oplus = 9.807$ m/s^2

Solar Constant = 1370 W/m^2

$1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$

$1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m} = 206265\,\mathrm{AU}$

$1\,\mathrm{radian} = 206265''$

1 year $= 3.156 \times 10^7$ seconds $= 365.26$ days

$1\,\mathrm{Jy} = 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$

1 Å$= 10^{-10}$ m

χ (H I) = 13.62 eV

χ (H$^-$) = 0.75 eV

Mass ^{1}H = 1.007825 amu

Mass ^{4}He = 4.00268 amu

1amu $= 1.66054 \times 10^{-27}$kg

$\sigma_T = 6.65 \times 10^{-25}$ cm^2

$e = 1.602 \times 10^{-19}$ C

$k_q = (4\pi\epsilon_0)^{-1} = 8.99 \times 10^9$ Nm^2/C

Equations

Orbital Mechanics

$$M = m_1 + m_2$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$G M\, P^2 = 4\pi^2\, a^3$$

$$L = \mu v_\theta r = \mu v_p r_p = \mu v_a r_a$$

$$E = \frac{1}{2}\mu v^2 - \frac{G M \mu}{r} = -\frac{G M \mu}{2a}$$

$$F = -\frac{G m_1 m_2}{r^2}$$

$$v_{circ}^2 = \frac{G\, M}{r}$$

$$v_{esc}^2 = \frac{2\, G\, M}{r}$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{v} = \vec{v}_2 - \vec{v}_1$$

$$m_1 \vec{r}_1 = -m_2 \vec{r}_2$$

$$m_1 \vec{v}_1 = -m_2 \vec{v}_2$$

Electromagnetism, Light, and Atoms

$$E = h\nu = \frac{hc}{\lambda}$$

$$F = \frac{L}{4\pi r^2}$$

$$F_{surf} = \sigma\, T^4$$

$$\lambda_{max}\, T = 0.29 \text{ cm K}$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$$

$$B_\nu \approx \frac{2h\nu^3}{c^2}\, e^{-h\nu/kT} \quad (h\nu \gg kT)$$

$$B_\nu \approx \frac{2kT}{\lambda^2} \quad (h\nu \ll kT)$$

$$U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$F = q\vec{v} \times \vec{B}$$

$$\lambda_e = \frac{h}{p} = \frac{h}{m_e v}$$

$$\mu v r_n = n\hbar \qquad \hbar = \frac{h}{2\pi}$$

$$r_n = a_0\, n^2\, Z^{-1}$$

$$E_n = -\frac{E_0 Z^2}{n^2}$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2} = 0.53\, \mathring{A}$$

$$E_0 = \frac{e^2}{8\pi\epsilon_0 a_0} = 13.62 \text{ eV}$$

$$\frac{1}{\lambda_{mn}} = \frac{1}{\lambda_0}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\lambda_0 = 911.8\, Z^{-2}\, \mathring{A}$$

$$\frac{\Delta\nu}{\nu} \approx \frac{v_r}{c}$$

$$\frac{\Delta\lambda}{\lambda} \approx \frac{v_r}{c}$$

$$u_{rad} = \frac{4\sigma}{c}\, T^4 = a\, T^4$$

$$P_{rad} = \frac{4\sigma}{3c}\, T^4 = \frac{1}{3}\, u_{rad}$$

Intensity, Flux & Radiative Transfer:

$$F = \int_0^\infty d\nu\, F_\nu = \int_0^\infty d\lambda\, F_\lambda$$

$$I = \int_0^\infty d\nu\, I_\nu = \int_0^\infty d\lambda\, I_\lambda$$

$$F_\lambda\, d\lambda = \int_0^{4\pi} d\Omega\, \cos(\theta)\, I_\lambda\, d\lambda$$

$$F_{obs} = \int_0^\infty d\lambda\, \Phi_f(\lambda)\, F_\lambda$$

$$d\Omega = \sin(\theta)\, d\theta\, d\phi$$

$$E_\lambda\, d\lambda = I_\lambda\, d\lambda\, dt\, dA\, \cos(\theta)\, d\Omega$$

$$F = I \cdot \Omega$$

$$\Omega \approx \pi\theta_R^2$$

$$dI_\nu = -\kappa_\nu \rho I_\nu dx$$

$$dI_\nu = j_\nu \rho dx$$

$$d\tau_\nu = -\kappa_\nu \rho dx$$

$$S_\nu = j_\nu / \kappa_\nu$$

$$z = x\, \cos(\theta)$$

$$\tau = \tau_0 \sec(\theta)$$

$$-dI_\nu / d\tau_\nu = I_\nu - S_\nu$$

$$I_\nu(\tau) = I_0\, e^{-\tau} + S_\nu(1 - e^{-\tau})$$

$$\tau \sim \kappa\, n\, \ell$$

$$\tau \sim \sigma\, \rho\, \ell$$

Stellar Distances, Magnitudes, and Properties

$$f = \frac{L}{4\pi d^2}$$

$$L = 4\pi R^2 \cdot \sigma T^4$$

$$D \approx d\, \theta$$

$$d_{pc} = 1/p''$$

$$\frac{f_b}{f_a} = 10^{\frac{m_a - m_b}{2.5}}$$

$$\frac{L_b}{L_a} = 10^{\frac{M_a - M_b}{2.5}}$$

$$m_b - m_a = 2.5 \log(f_a / f_b)$$

$$m - M = 5 \log d_{pc} - 5$$

$$L \propto M^{3.5} \quad [\mathrm{MS}]$$

$$t_{MS} \propto \frac{M}{L}$$

Binary Stars

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} = \frac{v_1}{v_2}$$

$$M = \frac{4\pi^2\, a^3}{G\, P^2} = \frac{P\, v^3}{2\pi G}$$

$$M\, \sin^3 i = \frac{P v_{rad}^3}{2\pi G}$$

$$f(m, M, i) = \frac{m_2^3}{M^2}\, \sin^3 i = \frac{P v_{1r}^3}{2\pi G}$$

$$R_B = \frac{v}{2}\,(t_2 - t_1)$$

$$R_A = \frac{v}{2}\,(t_3 - t_1)$$

$$\frac{I_B}{I_A} = \frac{F_{max} - F_{pri}}{F_{max} - F_{sec}}$$

$$\frac{T_B}{T_A} = \left(\frac{I_B}{I_A}\right)^{1/4}$$

Stellar Structure:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} \qquad \frac{dM}{dr} = 4\pi r^2\rho$$

$$\frac{dL}{dr} = 4\pi r^2\rho\epsilon(r) \qquad P = \frac{\rho kT}{\mu m_H} + \frac{4\sigma}{3c}T^4 \propto \rho^\gamma$$

$$\frac{dT}{dr}(\mathrm{rad}) = -\frac{3\bar{\kappa}\rho}{16\sigma T^3}\frac{L}{4\pi r^2} \qquad \frac{dT}{dr}(\mathrm{conv}) = \left(\frac{\gamma-1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

$$U_g \sim -\frac{GM^2}{R} \qquad U_{th} \sim \frac{M}{\mu m_H}kT$$

$$P_{deg} \propto \rho^{5/3} \qquad R_{deg} \propto M^{-1/3}$$

$$M_{wd} < 1.4\ \mathrm{M}_\odot \qquad M_{ns} \lesssim 3\ \mathrm{M}_\odot$$

Atmospheres, Photospheres, and Thermodynamics

$$p_{gas} = nkT = \frac{\rho kT}{\mu m_H} \qquad u_{gas} = \frac{1}{2}\rho v_{rms}^2 = \frac{3}{2}nkT$$

$$p \approx p_0\, e^{-mgh/kT} \qquad \ell \approx 1/n\sigma$$

$$\frac{N_B}{N_A} = \frac{g_B}{g_A}e^{(E_A-E_B)/kT} \qquad \frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{N_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

$$g_n = 2\,n^2 \qquad Z = g_1 + \sum_{j=2}^{\infty} g_j\, e^{(E_1-E_j)/kT}$$

$$\frac{N_{HII}}{N_{HI}} = \frac{2.41\times10^{21}}{N_e}T^{3/2}\,e^{-157882/T} \qquad \frac{N_{HII}}{N_{HI}} = \frac{3.33\times10^{-2}}{P_e}T^{5/2}\,e^{-157882/T}$$

$$\frac{N_{II}}{N} = \frac{N_{II}/N_I}{1+N_{II}/N_I} \qquad \frac{N_2}{N} = \frac{N_2/N_I}{1+N_{II}/N_I}$$

Dust, Interstellar Medium and Star Formation:

$$m_v - M_v = 5\log d_{pc} - 5 + A_v \qquad \tau_v = 0.4\,A_v\,\ln 10 \approx A_v$$

$$B-V(\mathrm{obs}) = B-V(\mathrm{true}) + CE \qquad CE \approx A_v/3$$

$$\frac{p}{k} = n_1\,T_1 = n_2\,T_2 \qquad \sigma_\lambda = Q_\lambda\,\pi a^2 \qquad Q_\lambda \sim \frac{a}{\lambda}$$

Sound Waves and Pulsating Stars

$$v_s^2 = \frac{dp}{d\rho} \approx \frac{\gamma p}{\rho} \qquad P_{ceph} \approx \sqrt{\frac{3\pi}{2\gamma G\rho}} \qquad \left(\gamma \approx \frac{5}{3}\right)$$

Relativity & Quantum Mechanics

$$E = \gamma m c^2 \approx m c^2 + \frac{1}{2}mv^2 \quad (v \ll c) \qquad \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$$\phi = \frac{GM}{R} \qquad \lambda = \frac{h}{p}$$

$$\lambda_{obs} = \left(1 - \frac{2\phi}{c^2}\right)\lambda_{em} \qquad R_{sw} = \frac{2GM}{c^2}$$

$$\Delta x \cdot \Delta p \geq \hbar \qquad \Delta E \cdot \Delta t \geq \hbar$$

Galaxies, Quasars, and Clusters of Galaxies:

$$v_c^2 = \frac{GM(<R)}{R} \qquad t_{ff} = \sqrt{\frac{\pi^2 R^3}{8GM}}$$

$$E = K + U \qquad \langle U \rangle = -2\langle K \rangle$$

$$L \propto v_{max}^4 \qquad L \propto \sigma_v^4$$

$$M_{vir} \approx \frac{5\sigma_r^2 R}{G} \qquad T_{vir} \approx \frac{m\sigma_r^2}{k}$$

$$\sigma_r^2 = \frac{1}{3}\sigma_v^2 \qquad L_{Ed} = \frac{4\pi Gc}{\bar{\kappa}} M$$

$$\epsilon_{ff} \approx 1.42 \times 10^{-40} \left(\frac{n_e}{1\mathrm{m}^{-3}}\right)^2 \left(\frac{T}{1\mathrm{K}}\right)^{1/2} \mathrm{W\ m}^{-3}$$

Cosmology:

$$v = Hd + v_{pec}$$

$$v \approx cz \quad (v \ll c)$$

$$R_0 = R(1+z) \qquad (R_0 \equiv 1)$$

$$\rho_0 = \rho R^3$$

$$\dot{R}^2 = \frac{8\pi G}{3}\rho R^2 - kc^2 + \frac{1}{3}\Lambda c^2 R^2$$

$$\dot{R} = \frac{dR}{dt}$$

$$\ddot{R} = -\frac{4\pi G}{3}\rho R \quad (\Lambda = 0)$$

$$\ddot{R} = \frac{d^2 R}{dt^2}$$

$$H = \frac{\dot{R}}{R}$$

$$q = -\frac{R\ddot{R}}{\dot{R}^2}$$

$$\rho_{cr} = \frac{3H^2}{8\pi G}$$

$$\Omega = \frac{\rho}{\rho_{cr}}$$

$$T_0 = T R$$

$$u_{rad} = a T^4 = a T_0^4 R^{-4}$$

$$H_0 dt = R^{1/2} dR \quad (u_{rad} \ll \rho c^2)$$

$$H_0 dt = R\, dR \quad (u_{rad} \gg \rho c^2)$$

$$ds^2 = c^2 dt^2 - \frac{R^2 dr^2}{1 - kr^2}$$

$$d_H = R(t) \int_0^t \frac{c\, dt'}{R(t')}$$

$$\chi = \int_{t_0}^{t_1} \frac{c\, dt}{R(t)}$$

$$\chi(z) = \frac{2c}{H_0}\left[1 - (1+z)^{-1/2}\right] \quad (k = 0)$$

$$f = \frac{L}{4\pi d_L^2}$$

$$\theta = \frac{D}{d_A}$$

$$d_L = (1+z)\chi$$

$$d_A = \frac{\chi}{(1+z)}$$

Important Energy Thresholds

Ionized Hydrogen	13.6 eV
mass of electron	511 KeV/c^2
mass of proton	938 MeV/c^2
mass of W,Z	90 GeV/c^2

Solar Magnitudes

	M_{bol}	M_V	$U-B$	$B-V$
Sun	$+4.76$	$+4.83$	$+0.16$	$+0.64$

Characteristic Absolute Visual Magnitudes

O and B stars	$+0$	–	-6
Classical Cepheid	-0.5	–	-6
Type II Cepheid	$+0$	–	-3
RR Lyrae Variable	$+0.5$	–	$+1$
Globular Cluster	-5	–	-9
Spiral Galaxy	-15	–	-21
Elliptical Galaxy	-9	–	-23
Irregular Galaxy	-13	–	-18
Type Ia Supernova		$\approx$	-20
Type II Supernova		$\approx$	-18

Common Elements and Isotopes:

(Key: $^{mass}_{num}X$) $\quad$ $^{1}_{1}$H $\quad$ $^{4}_{2}$He $\quad$ $^{12}_{6}$C $\quad$ $^{14}_{7}$N

$^{16}_{8}$O $\quad$ $^{20}_{10}$Ne $\quad$ $^{32}_{16}$S $\quad$ $^{56}_{26}$Fe

Solar Composition:

$X = \rho_{\mathrm{H}}/\rho \approx 0.75$ $\qquad$ $Y = \rho_{\mathrm{He}}/\rho \approx 0.23$ $\qquad$ $Z \approx 0.02$