Astronomy 12 Spring 1999 (S.T. Myers)

Equations and Constants - Exam 2

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Constants

$c = 2.998 \times 10^5$ km/s

$G = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

$h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$

$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

$k = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$

$m_p = 1.67 \times 10^{-27}\,\mathrm{kg}$

$m_e = 9.11 \times 10^{-31}\,\mathrm{kg}$

$M_\odot = 2 \times 10^{30}$ kg

$R_\odot = 6.96 \times 10^5$ km

$L_\odot = 3.83 \times 10^{26}$ W

$T_\odot = 5770$ K

$M_\oplus = 5.98 \times 10^{24}$ kg

$R_\oplus = 6378$ km

$g_\oplus = 9.807$ m/s^2

Solar Constant $= 1370$ W/m^2

$1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$

$1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m} = 206265\,\mathrm{AU}$

$1\,\mathrm{radian} = 206265''$

1 year $= 3.156 \times 10^7$ seconds $= 365.26$ days

$1\,\mathrm{Jy} = 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$

1 Å$= 10^{-10}$ m

χ (H I) $= 13.62$ eV

χ (H^-) $= 0.75$ eV

Mass ^{1}H $= 1.007825$ amu

Mass ^{4}He $= 4.00268$ amu

$1\mathrm{amu} = 1.66054 \times 10^{-27}\mathrm{kg}$

$\sigma_T = 6.65 \times 10^{-25}$ cm^2

$e = 1.602 \times 10^{-19}$ C

$k_q = (4\pi\epsilon_0)^{-1} = 8.99 \times 10^9$ Nm^2/C

Equations

Orbital Mechanics

$$M = m_1 + m_2 \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$GM\,P^2 = 4\pi^2\,a^3 \qquad L = \mu v_\theta r = \mu v_p r_p = \mu v_a r_a$$

$$E = \frac{1}{2}\mu v^2 - \frac{GM\mu}{r} = -\frac{GM\mu}{2a} \qquad F = -\frac{G m_1 m_2}{r^2}$$

$$v_{circ}^2 = \frac{G\,M}{r} \qquad v_{esc}^2 = \frac{2\,G\,M}{r}$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1$$

$$m_1\vec{r}_1 = -m_2\vec{r}_2 \qquad m_1\vec{v}_1 = -m_2\vec{v}_2$$

Electromagnetism, Light, and Atoms

$$E = h\nu = \frac{hc}{\lambda} \qquad F = \frac{L}{4\pi r^2}$$

$$F_{surf} = \sigma\,T^4 \qquad \lambda_{max}\,T = 0.29\ \mathrm{cm\,K}$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\,\frac{1}{e^{h\nu/kT} - 1} \qquad B_\nu \approx \frac{2h\nu^3}{c^2}\,e^{-h\nu/kT} \quad (h\nu \gg kT)$$

$$B_\nu \approx \frac{2kT}{\lambda^2} \quad (h\nu \ll kT) \qquad U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \qquad F = q\vec{v} \times \vec{B}$$

$$\lambda_e = \frac{h}{p} = \frac{h}{m_e v} \qquad \mu v r_n = n\hbar \qquad \hbar = \frac{h}{2\pi}$$

$$r_n = a_0\,n^2\,Z^{-1} \qquad E_n = -\frac{E_0 Z^2}{n^2}$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2} = 0.53\,\mathring{A} \qquad E_0 = \frac{e^2}{8\pi\epsilon_0 a_0} = 13.62\ \mathrm{eV}$$

$$\frac{1}{\lambda_{mn}} = \frac{1}{\lambda_0}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad \lambda_0 = 911.8\,Z^{-2}\,\mathring{A}$$

$$\frac{\Delta\nu}{\nu} \approx \frac{v_r}{c} \qquad \frac{\Delta\lambda}{\lambda} \approx \frac{v_r}{c}$$

$$u_{rad} = \frac{4\sigma}{c}\,T^4 \qquad P_{rad} = \frac{4\sigma}{3c}\,T^4 = \frac{1}{3}\,u_{rad}$$

Intensity, Flux & Radiative Transfer:

$$F = \int_0^\infty d\nu\, F_\nu = \int_0^\infty d\lambda\, F_\lambda \qquad I = \int_0^\infty d\nu\, I_\nu = \int_0^\infty d\lambda\, I_\lambda$$

$$F_\lambda\, d\lambda = \int_0^{4\pi} d\Omega\, \cos(\theta)\, I_\lambda\, d\lambda \qquad F_{obs} = \int_0^\infty d\lambda\, \Phi_f(\lambda)\, F_\lambda$$

$$d\Omega = \sin(\theta)\, d\theta\, d\phi \qquad E_\lambda\, d\lambda = I_\lambda\, d\lambda\, dt\, dA\, \cos(\theta)\, d\Omega$$

$$F = I \cdot \Omega \qquad \Omega \approx \pi\theta_R^2$$

$$dI_\nu = -\kappa_\nu \rho I_\nu dx \qquad dI_\nu = j_\nu \rho dx$$

$$d\tau_\nu = -\kappa_\nu \rho dx \qquad S_\nu = j_\nu/\kappa_\nu$$

$$z = x\, \cos(\theta) \qquad \tau = \tau_0 \sec(\theta)$$

$$-dI_\nu/d\tau_\nu = I_\nu - S_\nu \qquad I_\nu(\tau) = I_0\, e^{-\tau} + S_\nu(1 - e^{-\tau})$$

$$\tau \sim \kappa\, n\, \ell \qquad \tau \sim \sigma\, \rho\, \ell$$

Stellar Distances, Magnitudes, and Properties

$$D \approx d\,\theta \qquad d_{pc} = 1/p''$$

$$\frac{f_b}{f_a} = 10^{\frac{m_a - m_b}{2.5}} \qquad \frac{L_b}{L_a} = 10^{\frac{M_a - M_b}{2.5}}$$

$$m_b - m_a = 2.5 \log(f_a/f_b) \qquad m - M = 5 \log d_{pc} - 5$$

$$L \propto M^{-3.5} \ \text{[MS]} \qquad t_{MS} \propto \frac{M}{L}$$

Binary Stars

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} = \frac{v_1}{v_2} \qquad M = \frac{4\pi^2\, a^3}{G\, P^2} = \frac{P\, v^3}{2\pi G}$$

$$M\, \sin^3 i = \frac{P v_{rad}^3}{2\pi G} \qquad f(m, M, i) = \frac{m_2^3}{M^2}\, \sin^3 i = \frac{P v_{1r}^3}{2\pi G}$$

$$R_B = \frac{v}{2}\,(t_2 - t_1) \qquad R_A = \frac{v}{2}\,(t_3 - t_1)$$

$$\frac{I_B}{I_A} = \frac{F_{max} - F_{pri}}{F_{max} - F_{sec}} \qquad \frac{T_B}{T_A} = \left(\frac{I_B}{I_A}\right)^{1/4}$$

Stellar Structure:

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2} \qquad \frac{dM}{dr} = 4\pi r^2 \rho$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon(r) \qquad P = \frac{\rho k T}{\mu m_H} + \frac{4\sigma}{3c} T^4 \propto \rho^\gamma$$

$$\frac{dT}{dr}(\text{rad}) = -\frac{3\bar{\kappa}\rho}{16\sigma T^3}\frac{L}{4\pi r^2} \qquad \frac{dT}{dr}(\text{conv}) = \left(\frac{\gamma-1}{\gamma}\right)\frac{T}{P}\frac{dP}{dr}$$

$$U_g \sim -\frac{GM^2}{R} \qquad U_{th} \sim \frac{M}{\mu m_H} kT$$

$$P_{deg} \propto \rho^{5/3} \qquad R_{deg} \propto M^{-1/3}$$

$$M_{wd} < 1.4\ \mathrm{M}_\odot \qquad M_{ns} \lesssim 3\ \mathrm{M}_\odot$$

Atmospheres, Photospheres, and Thermodynamics

$$p_{gas} = nkT = \frac{\rho k T}{\mu m_H} \qquad u_{gas} = \frac{1}{2}\rho v_{rms}^2 = \frac{3}{2} nkT$$

$$p \approx p_0\, e^{-mgh/kT} \qquad \ell \approx 1/n\sigma$$

$$\frac{N_B}{N_A} = \frac{g_B}{g_A} e^{(E_A - E_B)/kT} \qquad \frac{N_{i+1}}{N_i} = \frac{2Z_{i+1}}{N_e Z_i}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2} e^{-\chi_i/kT}$$

$$g_n = 2n^2 \qquad Z = g_1 + \sum_{j=2}^{\infty} g_j\, e^{(E_1 - E_j)/kT}$$

$$\frac{N_{HII}}{N_{HI}} = \frac{2.41\times 10^{21}}{N_e} T^{3/2} e^{-157882/T} \qquad \frac{N_{HII}}{N_{HI}} = \frac{3.33\times 10^{-2}}{P_e} T^{5/2} e^{-157882/T}$$

$$\frac{N_{II}}{N} = \frac{N_{II}/N_I}{1 + N_{II}/N_I} \qquad \frac{N_2}{N} = \frac{N_2/N_I}{1 + N_{II}/N_I}$$

Dust, Interstellar Medium and Star Formation:

$$m_v - M_v = 5\log d_{pc} - 5 + A_v \qquad \tau_v = 0.4\, A_v\, \ln 10 \approx A_v$$

$$B - V(\text{obs}) = B - V(\text{true}) + CE \qquad CE \approx A_v/3$$

$$\frac{p}{k} = n_1 T_1 = n_2 T_2 \qquad \sigma_\lambda = Q_\lambda\, \pi a^2 \qquad Q_\lambda \sim \frac{a}{\lambda}$$

Relativity & Quantum Mechanics

$$E = \gamma\, m\, c^2 \approx m\, c^2 + \frac{1}{2} m v^2 \quad (v \ll c) \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\phi = \frac{GM}{R} \qquad \lambda = \frac{h}{p}$$

$$\lambda_{obs} = \left(1 - \frac{2\phi}{c^2}\right) \lambda_{em} \qquad R_{sw} = \frac{2GM}{c^2}$$

Solar Magnitudes

	M_{bol}	M_V	$U-B$	$B-V$
Sun	+4.76	+4.83	+0.16	+0.64

Common Elements and Isotopes:

(Key: ${}^{mass}_{num}X$)

${}^{1}_{1}\mathrm{H}$ ${}^{4}_{2}\mathrm{He}$ ${}^{12}_{6}\mathrm{C}$ ${}^{14}_{7}\mathrm{N}$

${}^{16}_{8}\mathrm{O}$ ${}^{20}_{10}\mathrm{Ne}$ ${}^{32}_{16}\mathrm{S}$ ${}^{56}_{26}\mathrm{Fe}$

Solar Composition:

$X = \rho_{\mathrm{H}}/\rho \approx 0.75$ $Y = \rho_{\mathrm{He}}/\rho \approx 0.23$ $Z \approx 0.02$