Astronomy 12 Spring 1999 (S.T. Myers)

Midterm Examination 1 Tuesday 9 Feburary 1999

Constants

$c = 2.998 \times 10^5$ km/s

$\mathrm{G} = 6.67 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$

$\mathrm{h} = 6.626 \times 10^{-34}\,\mathrm{J\,s}$

$\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$

$\mathrm{k} = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$

$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$

$\mathrm{m_p} = 1.67 \times 10^{-27}\,\mathrm{kg}$

$\mathrm{m_e} = 9.11 \times 10^{-31}\,\mathrm{kg}$

$M_\odot = 2 \times 10^{30}$ kg

$R_\odot = 6.96 \times 10^5$ km

$L_\odot = 3.83 \times 10^{26}$ W

$T_\odot = 5770$ K

$M_{bol,\odot} = +4.75$

$M_{V,\odot} = +4.83$

$M_\oplus = 5.98 \times 10^{24}$ kg

$R_\oplus = 6378$ km

$1\,\mathrm{AU} = 1.496 \times 10^8\,\mathrm{km}$

$1\,\mathrm{pc} = 3.086 \times 10^{16}\,\mathrm{m} = 206265\,\mathrm{AU}$

$1\,\mathrm{radian} = 206265''$

1 year $= 3.156 \times 10^7$ seconds $= 365.26$ days

$1\,\mathrm{Jy} = 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$

$1\ \text{Å} = 10^{-10}$ m

Equations

Orbital Mechanics

$$M = m_1 + m_2 \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$G M\, P^2 = 4\pi^2\, a^3 \qquad L = \mu v_\theta r = \mu v_p r_p = \mu v_a r_a$$

$$E = \frac{1}{2}\mu v^2 - \frac{GM\mu}{r} = -\frac{GM\mu}{2a} \qquad F = -\frac{G m_1 m_2}{r^2}$$

$$v_{circ}^2 = \frac{G\,M}{r} \qquad v_{esc}^2 = \frac{2\,G\,M}{r}$$

$$\vec{r} = \vec{r}_2 - \vec{r}_1 \qquad \vec{v} = \vec{v}_2 - \vec{v}_1$$

$$m_1 \vec{r}_1 = -m_2 \vec{r}_2 \qquad m_1 \vec{v}_1 = -m_2 \vec{v}_2$$

Electromagnetism, Light, and Atoms

$$E = h\nu = \frac{hc}{\lambda} \qquad F = \frac{L}{4\pi r^2}$$

$$F_{surf} = \sigma\, T^4 \qquad \lambda_{max}\, T = 0.29 \mathrm{cm\,K}$$

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\, \frac{1}{e^{h\nu/kT} - 1} \qquad B_\lambda(T) = \frac{2hc^2}{\lambda^5}\, \frac{1}{e^{hc/\lambda kT} - 1}$$

$$B_\nu(T) \approx \frac{2kT}{\lambda^2} \quad (h\nu \ll kT) \qquad B_\nu(T) \approx \frac{2h\nu^3}{c^2}\, e^{-h\nu/kT} \quad (h\nu \gg kT)$$

$$\frac{\Delta\nu}{\nu} \approx \frac{v_r}{c} \qquad \frac{\Delta\lambda}{\lambda} \approx \frac{v_r}{c}$$

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} \qquad U = \frac{q_1 q_2}{4\pi\epsilon_0 r}$$

$$\lambda_e = \frac{h}{m_e v} \qquad \mu v r_n = n\hbar \qquad \hbar = \frac{h}{2\pi}$$

$$r_n = a_0\, n^2\, Z^{-1} \qquad E_n = -\frac{E_0 Z^2}{n^2}$$

$$a_0 = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2} = 0.53\, \mathring{A} \qquad E_0 = \frac{e^2}{8\pi\epsilon_0 a_0} = 13.62 \text{ eV}$$

$$\frac{1}{\lambda_{mn}} = \frac{1}{\lambda_0}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \qquad \lambda_0 = 911.8\, Z^{-2}\, \mathring{A}$$

Intensity and Flux

$$F = \int_0^\infty d\nu\, F_\nu = \int_0^\infty d\lambda\, F_\lambda$$

$$F_\lambda\, d\lambda = \int_0^{4\pi} d\Omega\, \cos(\theta)\, I_\lambda\, d\lambda$$

$$d\Omega = \sin(\theta)\, d\theta\, d\phi$$

$$F = I \cdot \Omega$$

$$I = \int_0^\infty d\nu\, I_\nu = \int_0^\infty d\lambda\, I_\lambda$$

$$F_{obs} = \int_0^\infty d\lambda\, \Phi_f(\lambda)\, F_\lambda$$

$$E_\lambda\, d\lambda = I_\lambda\, d\lambda\, dt\, dA\, \cos(\theta)\, d\Omega$$

$$\Omega \approx \pi\theta_R^2$$

Stellar Distances, Magnitudes, and Properties

$$D \approx d\,\theta$$

$$\frac{f_b}{f_a} = 10^{\frac{m_a - m_b}{2.5}}$$

$$m_b - m_a = 2.5\log(f_a/f_b)$$

$$L \propto M^{-3.5} \quad \text{[MS]}$$

$$d_{pc} = 1/p''$$

$$\frac{L_b}{L_a} = 10^{\frac{M_a - M_b}{2.5}}$$

$$m - M = 5\log d_{pc} - 5$$

$$t_{MS} \propto \frac{M}{L}$$

Binary Stars

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} = \frac{v_1}{v_2}$$

$$M\, \sin^3 i = \frac{P v_{rad}^3}{2\pi G}$$

$$R_B = \frac{v}{2}\,(t_2 - t_1)$$

$$\frac{I_B}{I_A} = \frac{F_{max} - F_{pri}}{F_{max} - F_{sec}}$$

$$M = \frac{4\pi^2\, a^3}{G\, P^2} = \frac{P\, v^3}{2\pi G}$$

$$f(m, M, i) = \frac{m_2^3}{M^2}\, \sin^3 i = \frac{P v_{1r}^3}{2\pi G}$$

$$R_A = \frac{v}{2}\,(t_3 - t_1)$$

$$\frac{T_B}{T_A} = \left(\frac{I_B}{I_A}\right)^{1/4}$$