Astronomy 11 — Fall 1998 (S.T. Myers)

Homework Set 8

Due Thursday December 10, 1998 (5pm)

1. The star EPS451 has a luminosity of $0.4\,\mathrm{L}_\odot$, a mass of $0.8\,\mathrm{M}_\odot$, and a radius of $0.87\,\mathrm{R}_\odot$.

Suppose you wanted to claim that the star's luminosity was generated by crashing comets in the surface of the star at the escape velocity from the surface. How much mass would need to be hitting the surface of EPS451 each second to generate the luminosity from the kinetic energy of the "impacts"?

We wish to estimate a lifetime under this scenario. The time it would take to accrete the current mass is a good starting point. Using this accretion rate, how long (in years) would it take before you have added a mass equal to the total mass of EPS451?

Does this seem like a reasonable scenario for the energy generation mechanism in stars like EPS451 and the Sun, and why or why not?

2. On the other hand, suppose you wanted to see if the stellar luminosity could be supplied from gravitational potential energy. Approximate the potential energy of a sphere as

$$U \sim -\frac{GM^2}{R}$$

where it is negative with respect to all the mass dispersed at infinity ($U = 0$). What is the total gravitational binding energy (in Joules) available in EPS451?

Approximately at what rate (in meters per year) and by what fraction (of the radius) would EPS451 have to contract each year to shine at constant luminosity equal to the current value?

For a lifetime, you can use the time it would take to either generate a total amount of energy equal to the current potential energy or the time it would take for the radius to change by a fraction of order unity ($dR/R \sim 1$). Compute these timescales (in years).

Does gravitational contraction appear to be a reasonable scenario for stellar energy generation, and why or why not?

3. Einstein's Special Theory of Relativity equates mass with energy, with

$$E = m\,c^2$$

being, in some sense, the conversion factor. Thus, the mass of an atomic nucleus in part contains the binding energy between the protons and neutrons inside it (due primarily to the strong nuclear force). The mass of a hydrogen nucleus (one proton) is 1.007825 amu and the mass of a helium-4 nucleus (two protons and two neutrons) is 4.00268 amu, where

$$1\ \mathrm{amu} = 1.67352 \times 10^{-27}\ \mathrm{kg}$$

is the *atomic mass unit*. What is the *mass defect*, or mass difference between four ^{1}H nuclei and one ^{4}He nucleus in amu? Also, express this mass difference as a fraction of the total mass of the original four protons.

The process of thermonuclear fusion (as it is carried out in the Sun) can be thought of as converting four hydrogen nuclei into one helium-4 nucleus, with the mass defect being turned into energy (heat). Calculate how much energy (in Joules) is generated in a single fusion reaction

$$4^1\mathrm{H} \rightarrow^4 \mathrm{He} + \mathrm{energy}$$

and then compute how many such fusions must be done each second to supply the observed luminosity of EPS451.

Assuming the composition of EPS451 is similar to the Sun (75% H, 25% He, by mass), what fraction of the the mass of hydrogen in EPS451 will have been turned into helium in 1 Gyr (10^9 years)? If 10% of the the total stellar mass were in the core of EPS451 and thus is available for fusion, how long (in Gyr) could EPS451 maintain this rate of fusion? (Note: this is a balance between the resevoir of energy, the mass $0.8\,\mathrm{M}_\odot$ and the rate of energy expediture, the luminosity $0.4\,\mathrm{L}_\odot$.)

Compute the fusion rate in the Sun from its luminosity ($1\,\mathrm{L}_\odot$) and the lifetime from its mass ($1\,\mathrm{M}_\odot$), again assuming 10% of the mass is available for fusion. Do smaller stars live longer or shorter lives?

4. There are stars known as *white dwarfs* which have a mass of 1.4 $\mathrm{M}_\odot$ but a radius of only 2000 km, and are made almost entirely of carbon! This would be the remnant core from a more massive star that was able to fuse hydrogen to helium, followed by helium to carbon. The mass of a carbon-12 (^{12}C) nucleus is 12 amu exactly (this is what defines the atomic mass unit). Calculate the fraction of the original mass of twelve hydrogen nuclei that is turned into energy in the into a single carbon nucleus.

What is the total amount of energy generated by fusing the 1.4 $\mathrm{M}_\odot$ of hydrogen into carbon?

Estimate the total gravitational potential energy in the white dwarf. What is the ratio of the total amount of energy liberated in nuclear fusion to the potential energy liberated during the entire process of collapse of the core? (Hint: the total gravitational potential energy represents the energy generated by collapsing the mass to the current size from infinity.)

5. The process of turning four hydrogen nuclei (protons) into a single helium nucleus (two protons plus two neutrons) involves turning two of the protons into neutrons! This occurs via the *weak interaction*

$$\mathrm{p} + \mathrm{p} \rightarrow \mathrm{pn} + \nu + \mathrm{e}^+$$

where $\bar{\nu}$ is a neutrino, e^+ is a positron (postively charged anti-electron), and pn is a deuteron (heavy hydrogen nucleus). Neutrinos are massless or nearly massless particles that hardly interact with matter (this is what we mean by a "weak" interaction), and thus can escape the Sun and possibly be detected on Earth in huge neutrino detectors! If each fusion produces two neutrinos, how many neutrinos are generated by our Sun's fusion rate (calculated in problem 3 above) each second?

Compute the flux of neutrinos (neutrinos per second per square meter) at 1 AU from the Sun.

Note: This is the number of neutrinos that pass through the Earth, and our bodies, every second! However, they barely interact with matter so that over our entire lifetime, only on the order of one neutrino will have interacted with an atomic nucleus in our bodies! This is why neutrino detectors (like the Sudbury Neutrino Observatory that Penn is currently involved in building) are sooooooo huge.