Astronomy 11 — Fall 1998 (S.T. Myers)

Homework Set 7

Due Tuesday December 8, 1998 (5pm)

1. The flux from an astronomical source is usually measured in physical units of Watts per square meter (eg. the solar constant is 1370 W/m^2). However, the logarithmic scale of *magnitudes* is commonly used also. In fact, since our eyes are sensitive logaritmically to brightness, magnitudes were first introduced for stars by the Greek astronomer Hipparchus! We can define an apparent magnitude difference Δm for an apparent flux ratio

$$\Delta m = m_1 - m_2 = -2.5 \log(f_1/f_2)$$

between two stars of observed flux f_1 and f_2. This means that fainter stars have larger (more positive) magnitudes, and 5 magnitudes equals a factor of 100 in flux. The zero of the apparent magnitude scale (bolometric AB magnitudes) is the star Vega, which is set to $m = 0$. The faintest star you can see with your naked eye is around $m = 6$. **How many times fainter is the naked-eye limit than the brightest star in the sky ($m = -1.5$)?**

The zero point of the magnitude scale is given by the Sun: if the sun were at a distance of 10 parsecs, it would have an apparent magnitude of $m = +4.75$. **Since 1 parsec is 206265 AU, what is the apparent magnitude of the Sun from Earth at 1 AU distance?**

2. The apparent magnitude of a star if it were observed at the distance of 10 parsecs is known as the *absolute magnitude* M. From the above, we know that $M = +4.75$ for the Sun. Since the apparent flux goes as the inverse square of the distance, the apparent magnitude m of a star at distance d (parsecs) as compared to its magnitude M were it at 10 parsecs is given by the *distance modulus*:

$$m - M = 5 \ \log d - 5$$

Derive an expression for the distance modulus $m - M$ of a star which has an observed parallax p arcseconds. (Hint: if you have any trouble, read through section 11-2 of the textbook.)

Vega ($m = 0$) has a parallax of $p = 0\rlap{.}''1235$. **What is the distance to Vega in parsecs, and what is the absolute magnitude M of Vega?**

3. The final thing we need to do is relate magnitudes to luminosity. Since $M = +4.75$ for the Sun, and absolute magnitudes relate apparent magnitudes at the same distance the absolute magnitude gives the luminosity relative to the Sun

$$M = 4.75 - 2.5 \log(L/L_\odot).$$

What is the luminosity of Vega in $L_\odot$?

4. The parallax of EPS-451 was measured to be $p = 0\rlap{.}''05$ and the apparent magnitude as measured from the Daedalus observatory is $m = +7.25$. **What is the distance to EPS-451 in parsecs, and what is its absolute magnitude M?** (You should get the same distance as you did in the previous problem set and on the exam.)

From the previous homework set, you should have determined the orbital semimajor axes a for the planets of the EPS-451 system. Your photometric team tells you that they have measured the brightnesses of the planets relative to the star in both the optical and IR wavebands (see table below). Assume that these are bolometric luminosities, and that the visible light from the planets are due entirely to the reflected light from the planet, and the infrared (IR) is due to the thermal blackbody emission that is re-radiated by the planet at its effective temperature T_{eff}.

Assume that the planet is at a distance r from EPS-451. Then the flux incident at the planet is

$$F_{in} = \frac{L}{4\pi r^2}$$

where L is the luminosity of EPS-451. Since a planet of radius R both reflects and absorbs the incident radiation over its cross-sectional area πR^2, then we can calculate the reflected luminosity

$$L_{ref} = F_{in} \cdot A \cdot \pi R^2 = L \cdot A \cdot \frac{R^2}{4r^2}$$

and the absorbed luminosity (which equals the L_{em} re-radiated in the IR)

$$L_{abs} = F_{in} \cdot (1 - A) \cdot \pi R^2 = L \cdot (1 - A) \cdot \frac{R^2}{4r^2} = L_{em} = 4\pi R^2 \cdot \sigma T_{eff}^4$$

for an albedo A and effective blackbody temperature T_{eff}. Note T_{eff} will give the surface temperature if $G = 0$. **Solve the two equations for L_{abs} and L_{ref} to get expressions for the albedo A and the radius R of the planet.** (Hint: solve for A in terms of the ratio L_{ref}/L_{abs}, then use L_{ref} and A to solve for R. If you have difficulties, look at the discussion for determining sizes and albedos for asteroids in section 7-2 of the textbook.)

We can assume that the star and the planet are the same distance from the Earth (the Sun). **Use the visible and IR magnitude differences versus EPS-451 to solve for the albedo and radius of the planets, and also determine the effective temperature T_{eff} and again use these to estimate the type of planet.**

Orbit	Δm (vis)	Δm (IR)	R (km)	A	T_{eff} (K)	**probable type of planet**
1	23.911	21.016				
2	26.073	25.401				
3	26.051	25.656				
4	29.323	27.678				
5	23.295	23.338				
6	25.886	26.418				
7	28.634	29.074				
8	35.908	35.908				

5. To put these magnitudes in perspective, we note that the faintest objects that the Hubble Space Telescope can detect have an apparent magnitude of $m = 30$. This is also the limiting magnitude of the large ground-based telescopes (they are larger, but the atmospheric brightness limits them). **How many times brighter or fainter in flux are the brightest and the faintest of the above planets than the HST limit of $m = 30$?** (Use the apparent magnitude of EPS-451 given above, and the magnitude differences from the table.)

Remember that a telescope of diameter D will collect light with its geometric area $\pi D^2/4$, and thus the brightness of an image is proportional to D^2. **If the Hubble Space Telescope has $D = 2.4$ meters and the Daedalus telescope has a mirror diameter $D = 25$ meters, then what apparent magnitude at the Daedalus observatory will correspond to the limiting magnitude $m = 30$ of HST?**